

# Financial frictions and the role of endogenous human capital accumulation

## Preliminary and incomplete

Stylianos Asimakopoulos  
University of Stirling  
University of Bath

Aliya Kenjegalieva  
University of Stirling

### Abstract

This paper analyses how endogenous human capital accumulation affects the transmission mechanism of various financial shocks to the economy. A real business cycle model incorporating heterogeneous agents under endogenous human capital accumulation with banks and firms, is initially calibrated to match the key stylised facts and properties of the US economy. Examining the business cycle effects of various financial shocks we find interesting asymmetric effects to private consumption, income redistribution and skill premium (preliminary results).

Keywords: heterogeneous agents, financial frictions, endogenous human capital, skill premium

JEL Classification: E24, E32, E44, J24

Corresponding author: [stylianos.asimakopoulos@stir.ac.uk](mailto:stylianos.asimakopoulos@stir.ac.uk)

# 1 Introduction

Income redistribution and inequality has been examined extensively over the past years (see i.e. Goldin and Katz, 2008 and Hornstein et al., 2005 for reviews). In particular, there has been an upward trend on wage inequality since 1980 leading to its highest level since 1910 (see e.g. Acemoglu and Autor (2011) ).

The previous studies that use dynamic stochastic general equilibrium models have also shown the importance of including a banking sector (Goodhart et al (2006), Curdia and Woodford (2009), Dib (2010) and Iacoviello (2015)). However, here we extend that setup with the inclusion of endogenous human capital to assess its role to the economy over the business cycle.

Therefore, the aim of this study is to contribute to both of these research streams by providing a unified framework with banks, financial frictions and endogenous human capital. These two applications have not been jointly studied in the literature to date. In addition, as Matsuyama (2006) argues, less research has been done using "endogenous formation of class structure" assumptions. In our work we would like to fill this gap and allow for households to endogenously accumulate their human capital by obtaining college degrees or on-the-job training. Therefore we generate a single model taking into account both banking and human capital accumulation channel. Thus, we develop a modified DSGE model with banking following the Iacoviello (2015) paper and we introduce the human capital accumulation channel in a similar way as in Jones et al (1993), He and Liu (2008) and Peng (2012) papers.

When households are allowed to affect their human capital by educating/training themselves, it is implicitly assumed that they can change their productivity. Therefore, households can invest time and income to become more skilled and more productive in their jobs<sup>1</sup>. Productive households produce more goods in the short period affecting firms' production process. Higher production means higher profits for firms and higher wages for house-

---

<sup>1</sup>Cairo and Cajner (2014) claims that households with higher education levels and more training are more likely to find jobs in shorter periods and they are less likely to get fired.

holds. With higher wages households can invest more in their education and thus their human capital will rise further. This will help to shrink the wage inequality gap. Moreover they can consume and save more which will increase deposit accounts in banks and housing demand. Finally, higher saving rates mean that banks can invest more back to the economy.

With fast development and growth of industries there is a tendency for population to relocate in urban areas seeking for a better lifestyle and higher income. Historical examples can be seen from the labour movements in the US and UK. However movements to cities are challenging and require more time and investment especially in education. This can be explained by the fact that workers in bigger cities are more qualified/skilled where the workers from rural areas have to catch up by getting more knowledge and skills to compete in the labour market. By accumulating new skills and knowledge, attending additional trainings and pursue university degrees workers invest in their human capital.

As Galor and Moav (2003) state, the main driver for economic growth is no longer the physical capital but human capital. In order to compete in labour force, workers from rural areas have to invest in their human capital. Moreover, labour improvements in terms of productivity and skills has positive spill-over effects to the labour market. This can explain the relationship between economic development and wage distribution.

Some agents are more inclined to save when taking into account the level of income they earn. When there are credit constraints, inequality negatively effects the investment in human capital, however, stimulates aggregate savings which boosts economic growth. On the other hand, when wages rise and credit constraints are insufficiently binding, people are less interested in saving. Therefore the income inequality does not have a strong effect on growth.

The process of human capital accumulation confirms the increasing returns of investment in education. This motivates the importance of studies of this channel. Public policies, for example, aim to enhance the accumulation of human capital by subsidizing schools and colleges but also job training schemes. The aim of this study is assess the effects of human capital ac-

cumulation on the variations in wages in the labour market which shows variations in growth rates.

Human capital, as an aggregate wealth, essentially comes from humans themselves and its accumulation depends on the abilities and talents which affects the return rate. Individuals in the economy are responsible for each own human capital accumulation by dividing time, attending schools and on-the-job-trainings. Therefore higher human capital depends on its coverage around agents in an economy. We analyse human capital accumulation depending on how much time and income agents are willing to devote to education.

Similar to previous studies we present the model with households of two different preferences. Firstly, since households have different time preferences there are savers and borrowers. There are entrepreneurs who are good producers. It is essential to mention that unlike earlier literature capital stock occur in this model as housing estate. Mainly we assume that instead households buy and own houses while entrepreneurs purchase real estate. Therefore the logic behind the two is the same. However the main difference is that capital stock depreciates while housing in the current model is not. We also consider banks as a crucial player in an economy who provide loans and interest payments on households' deposit accounts. From previous works it is well known that banks can create, propagate and amplify financial shocks.

## **2 Related literature**

### **2.1 The importance of Banks**

Looking at the recent global financial crisis enriched models are needed to forecast and predict future crises. For decades econometricians around the world develop models which can fit data and produce realistic results. Dynamic general stochastic equilibrium models were developed for many decades once presented by Kydland and Prescott (1982) as a theory of real business cycles and by Rotemberg and Woodford (1997) as New-Keynesian

models. The difference between the two streams is that in New-Keynesian models it is assumed that monopolistically competitive firms set prices, which is costly. Classic dynamic stochastic general equilibrium (DSGE) models consist of two agents, i.e. households and firms. However, the importance of a banking sector has led to their inclusion in DSGE models as intermediaries between households and firms.

Banks can amplify and propagate shocks to real economy (i.e. Bernanke et al., 1999). The borrowers' decisions not to repay back loans they borrowed from commercial banks can be followed from different reasons like unable or unwilling to return loans anymore. This causes destructions in banks' activities. Banks cannot repay deposits to savers and also receive the part of their own capital they used for loans. This leads to the shocks creation and their distribution to the whole economy. When there are such surprises in an economy entrepreneurs are facing borrowing constraints. They don't produce enough and thus production decreases. When production declines firms need less workers to work for which increases unemployment rates.

Therefore credit markets are the vital source of propagation and amplification of financial shocks. Bankruptcies rise, declines of asset prices and banks fiasco actively play role in economy's depression. They add these credit-market imperfections into the model to analyse effects on an economy. They show how small shocks can significantly influence economy assuming that there are credit frictions. However, Cooley et al (2004) using models focusing only on the demand side of the credit market, they show the limited role of banks as the intermediaries between borrowers and lenders and that bank's capital does not have any influence on bank's ability to lend. In other words, lenders are suppliers of their savings, who lend to borrowers without interaction of intermediaries, such as, for example, banks.

Contrary to Cooley et al. (2004) paper, Meh and Moran (2004) show that banks have to rely on their capital when it comes to risky loans. They construct the model with banks and argue that banks can also face credit constraints. They also demonstrate the importance of firms' balance sheet. In their model they have two moral hazard problems. Firstly, as standard they assume that entrepreneurs produce goods. Trying to get more benefits,

they can make risky investments. Because these projects are risky, banks do not want to play any role, which makes firms invest their own money. Secondly, in some cases banks can still issue loans, however to save money, banks do not monitor their credits to entrepreneurs. Savers who have deposits in banks do not want to take that risk. As a result, savers require banks to invest their own capital in these risky projects and banks face financial constraints.

By introducing financial intermediaries in DSGE models researchers seek to forecast and prevent economic crunches rather than trying to manage them after they have already appeared. Looking at monetary policy's role and how banks are controlled can give clear understanding of shocks transmission. In their empirical work Angeloni and Faia (2009) confirm the banks' vibrant role in shocks transmission. They also prove the importance of the banks' capital and leverage ratios. Banks can redeploy assets in order to liquidate defaults. When banks issue loans to firm they rely that the firm's cash flow will be stable during the loan period. However firm cash flow is uncertain which creates uncertainty in bank's balance sheet. This is because bank's loan to deposit ratios raise can lead to runs of deposits while savers will be unsure about the liquidity of the bank. Therefore banks should mostly rely on their capital. Depending on their capital banks can issue more or less loans and set interest rates for their deposit accounts and loans.

Runs are dangerous for banks because it can weak their liquidity and stability in the credit market. If for some external reason depositors assume that a bank will might collapse or bankrupt they will take money from their accounts. This turns into huge amount of funds need to be given back to clients. The known fact that banks lend those deposit funds so they are unable to provide it back in full and at the time of immediate request. This creates further discredit and mistrust in banks making things in credit markets worse. This will also make banks be unable to issue new loans. When firms face borrowing constraints in obtaining new credits their production will immediately decline. Following decrease in production the growth of economy will fall too. This chain shows the importance of banking sector to rely in most on their capital rather than on savings accounts.

However there are papers that give opposite evidences of the role of a bank capital. They agree that bank capital has a sizeable impact on investment but argue that it is crucial in amplification of financial shocks. Geralli et al (2009), for example, show that a bank capital doesn't have much impact following monetary policy shocks. However if we turn our attention to the ratio of capital to assets then a bank capital decreases the impact of technology shocks to investment. But like other previous works this paper also shows banks' role in output decline during crises.

The paper by Christiano et al. (2008) also gives controversial results about banks role prior crisis. They argue that banks intermediation actually plays small role in creation of shocks and their further amplification to an economy. However monetary shocks created by banking sector have more persistent effects and that banks actually propagate these effects to rest of an economy. Banks invest in or issue loans to entrepreneurs, but the return of these loans are subject to risks or idiosyncratic shocks. These shocks can lead to negative impacts on entrepreneur's activity and his ability to repay a loan. This have a negative aftermath to banks which will not be able to return savers' deposits and any interests on it.

On the other hand there are empirical papers that show how bank capital can be the main reason of creating and amplifying shocks into a real economy. For example, Meh and Moran (2010) empirically demonstrate how financial and technological shocks are related to bank capital. They introduce new agents into the model – investors, who are depositors. They show that with less capital banks have to decrease their lending to firms which causes a decline in investment and output levels. Therefore banking sector not only propagates and amplifies shock but also it is an independent source of negative shocks.

Similar to Meh and Moran (2010) paper, Dib (2010) shows how financial shocks come from banking sector. The paper demonstrates how banks can further propagate impacts of shocks to an economy. He explains that there should be capital requirements for a bank capital so for banks be able to securely issue loans. He also illustrates that during booms banks issue more loans while during recessions they decrease credit supply. With his model he

proves that during expansionary monetary shock credit demand rises, since the loan cost is lower. This increases investment. Banks start raising their capital to satisfy this demand for loans. However the raise in bank capital is costly and thus issuing loans to entrepreneurs becomes more costly. Credit demand raises but less than previous and thus investment increases slowly too. Therefore the capital requirements help to dampen existed shocks.

Following the previous studies and particularly the paper by Goodhart et al (2006), deWalque et al (2010) paper develops the relationship between banks and entrepreneurs, using the lending injections into the credit market. However these injections can lead to the raise of inflation rate when is not financed from taxes. As in Dib (2010) they illustrate that capital requirements can extenuate financial instability of an economy, but it can lead to a negative impact on fluctuations in output in the long run. They also consider that the different maturity for deposits and loans can also play important role in creating shocks in banking sector.

As it was discussed in previous studies of banking sector in DSGE models, a banking capital is important for banks' stability and liquidity. Iacoviello (2015) introduces a model which can reasonably fit the US data. He produces a model with banks as intermediaries between households and entrepreneurs. However he introduces two types of households: impatient and patient. Patient households are savers and banks use these resources to lend to impatient households and entrepreneurs. Borrowers pledge their houses as collateral for loans and are issued loans by banks. When they default on their loans, banks can only receive the value of collateral. As the recent crisis shows banks have to sell houses for the lower prices which don't cover loan price and expenses. Thus borrowers pay back less than they agreed by contracts. The main idea here is to show how banking sector transfers and spills-over financial shocks which have persistent and large effects on real economy.

## **2.2 Human capital and wage inequality**

As we can see from previous studies of DSGE models, banking sector is an important component of these types of models. It plays a crucial role as a

shock propagator to economy. It is proven that banking sector not only can amplify and propagate financial shocks but also generate these shocks within the sector and affecting whole economy. What we would like to see in these models is how different households change the model's results if households come from different income groups. Therefore in this paper we want to investigate how households-borrowers and households-savers are affected by financial shocks when there is a banking sector in the model and endogenous human capital accumulation.

We assume that households in the economy are different: they have different preferences of their current and future consumption and savings. Most importantly households differ in the level of income, they can potentially earn and borrow to enlarge their consumption. We know from the early studies of the DSGE models that savers and borrowers are affected by different shocks including financial. But these studies do not assume the banking sector in an economy while also looking at the human capital accumulation. As we mentioned earlier, banking sector is the main resource and amplifier of financial shocks. As previous studies we want to see how financial shocks are created and propagated to other sectors. Moreover we would like to look at how the results of the model will change while we assume the banking sector and households accumulating news skills.

We assume that households can invest their money and time into accumulation of skills - their human capital. Savers will invest more money and time into their education, thus the return on their capital will be higher while borrowers will be getting less return. Later on the paper by Ben-Porath (1967) states that to accumulate human capital households have to improve their skills all the time. Other factors such as, for example, opportunity of gaining education or abilities, are all technology in the production function. Therefore as in our work he proves that the higher the human capital the bigger will be workers' earnings. Thus human capital can be assumed as a human wealth. Individuals in the model have some initial endowment. With time pass this endowment increases as they make savings and investments. So that their output is sum of their earnings and accumulated (produced) human capital. Human capital depreciates over time and this depreciation

is negative rather than positive.

In our model savers are subject to depreciation of their skills over time. Due to this reason they have to prove their skills and accumulate higher human capital so to compete with households-borrowers. Both these households' types are subject to the same depreciation rate. However the savers have higher wage rates meaning they can invest more into their human capital accumulation, while other households for these purposes have to borrow from banks. Therefore here we also look at the wage inequality.

From the previous studies it can be seen that the question of the income inequality was always a hot topic between researchers. For example, Paul Douglas (1926, 1930) was the first economist who observed the wage distribution. As he states in his works, clerical workers were substituted by new equipment which led to the decrease of their wages. The wage decrease was also caused by the increasing number of white-collar workers. Moreover the wages of uneducated and low-skilled workers were higher, which he assumed was caused by the decrease in numbers of immigrants.

Acemoglu (1998) also shows that there is a direct connection between workers' level of incomes and human capital levels they have. For example, technological change increases the number of skilled workers. There are couple of reasons for this. First of all, when the demand for skill workers higher firms will get higher profits. Secondly workers are willing to undertake college or university degree to improve their skills. However the last one requires considerable time to adjust. It is fair to say though that technological equipment is more complementary to those workers who are more skilled while low-skilled lose and earn less. Therefore this situation leads to rise in wage inequality.

Depending on different wage levels individuals are able to consume less or more. Mankiw (2000) suggests that households have different behaviour on smoothing their consumption. Households with low wealth are more likely to face binding borrowing constraints. They consume their total earnings left after income tax while making small or no savings. High wealth households smooth their consumption thinking about future generation and bequest. Also, these households are able to be issued loans and mortgages. If there is

interest rates rise borrowing becomes more expensive which means spenders' debt rises. However savers or higher wealth households are better off since the interest rate for savings rises too. This leads to bigger consumption gap between savers and borrowers. This can be another way of income inequality raise.

Lemieux (2006a) empirically tests how post-secondary education can increase one's wage and total wage distribution. The demand for highly educated workers is always higher. This makes their wages be higher comparing to those who, for example, have only primary or/and secondary education. This also shows where the growth in wage inequality and its distribution come out and the post-secondary education's main role in that. It is interesting that with the higher the education level the higher is wage. However, if comparing the wage gap of postgraduates and college graduates and college graduates and secondary school graduates the gap is higher for postgraduates. Therefore, there is return to education but the return is different for different levels of education.

Following the papers above Goldin and Katz (2008) also investigate wage inequality. They also find that the financial returns to education depend on the various factors, such as age, gender and race. However the level of education is most important one among that factors. Giving an example of US during 1920s it had more accessible education and thus higher wages across other countries. However given the different shocks in economy the technology change has two effects as it was previously said can only have negative effect. When the number of skilled workers rises and if there is a technological change then the wage of skilled workers can decrease. But if a technological change slows down in later decades the wages of skilled workers will start rising too.

### **3 Our model**

In this research we aim to contribute to both of these research streams by providing a unified framework with banks, financial frictions and endogenous human capital. These two applications have not been jointly studied in the

literature to date. In addition, as Matsuyama (2006) argues, less research has been done using "endogenous formation of class structure" assumptions. In the current work we are aiming to fill this gap and show that households endogenously accumulate their human capital by obtaining college degrees or on-the-job training. Therefore we generate a single model taking into account both banking and human capital accumulation channel. Thus, we develop a modified DSGE model with banking following the Iacoviello (2015) paper. We present the human capital accumulation channel in a similar way as in Jones et al (1993), He and Liu (2008) and Peng (2012) papers.

When households are allowed to affect their human capital by educating themselves then we can also assume that households can change their productivity. If households can invest in their education then they are more skilled and more productive in their jobs. Therefore, they improve their activities in the labour market. Cairo and Cajner (2014) claims that households with higher education levels and more training are more likely to find jobs in shorter periods and they are less likely to get fired. Productive households produce more goods in the short period. Thus, this affects firms' production process since households are suppliers of labour force. Higher production means higher profits for firms and higher wages for households. With higher wages households can invest more in their education and thus their human capital will rise further. This will help to shrink the wage inequality gap. Moreover they can consume and save more which will increase the number and amount of deposit accounts in banks. Higher saving rates mean that banks can invest more in an economy.

As it is mentioned above we present the model similar to the one in Iacoviello (2015) paper with two households types, mainly savers and borrowers. Both households provide labour to entrepreneurs and they both invest part of their income in purchasing houses. It is widely known that households differ by the level of their education. This directly effects the level of income they can potentially earn. We think of education as an activity of attending college or on-the job training which results in accumulating new skills, experience and additional knowledge. Therefore, we allow these two groups of households to accumulate human capital by investing in their education via

devoting their wages and time.

Piketty (2014) shows that income inequality consists of two parts: wage inequality and capital inequality. Households in our model differ in both wages, capital and the return on it. Therefore they face both wage inequality and capital inequality. In the present model we assume housing as a capital households can purchase depending on their income.

Households-borrowers invest less in their education due to smaller wages they earn comparing to households-savers. Thus their return from the human capital will be less than those savers get. Since savers are more educated and have higher wages, they are able to invest more although devoting less time in education. This also explains the fact that savers get higher returns on their education. Therefore in order to improve the level of their human capital borrowers have to take out loans from banks. Banks manage savers deposit accounts and issue loans to households-borrowers and entrepreneurs who produce the final goods and maximise their profit. Banks play essential role of intermediaries between savers and borrowers. In other words banks transfer financial resources between agents and over time.

The importance of this study is to see how households' productivity will change assuming there is a shock to their human capital accumulation channel. Moreover, we would like to assess the effects of this shock to the ability of households-borrowers to accumulate more skills and be more competitive with their savers counterparts in the labour market. This will have spill-over effects to wage and wealth inequality.

Moreover together with the redistribution shocks in the agents' budget constraints the shock to human capital accumulation channel can influence the stability of a banking system creating financial destructions in the economy. We use similar redistribution shocks as they are given in Iacoviello (2015). These redistribution shocks also act as borrowers' repayment shocks in the banks' budget constraint. We also include the borrowing capacity shocks in borrowing constraints of households-borrowers and entrepreneurs. It is shown that when these shocks appear borrowers are able to borrow more than the value of their collateral.

## 4 Model outline

The proposed model is a closed economy with four agents: savers, borrowers, entrepreneurs and banks. Households consume final goods and purchase houses and they are divided to savers and borrowers similar to patient and impatient households, as in Iacoviello (2015). Both types of households own houses. In addition, both type of households accumulate their human capital by investing their money and time in education, similar to Jones et al (1993), He and Liu (2008) and Peng (2012) papers. Banks and entrepreneurs in the model are presented as in Iacoviello (2015). The entrepreneurs produce the final good and maximise their profit. Banks in this model are given as in previous studies and are intermediaries between savers and borrowers. They accumulate households savings in deposit accounts and with adding their own capital issue loans to entrepreneurs and households-borrowers.

### 4.1 Households

Households in the model are represented by a continuum of infinitely living households of a unit mass. There are two types of households in our economy. First group represents savers, who have access to asset markets and own houses. Another group consists of households who are borrowers. They also own houses, however their consumption is smaller than that of savers due to the assumed difference in the time discount factor. The lifetime utility function of representative household is given by  $U = \sum_{t=0}^{\infty} \beta_j^t u(C_t, H_t, N_{H,t}, N_{H,t}^{ED})$  where  $\beta_j^t$  is the discount factor for a  $j$  agent at period  $t$  and  $0 < \beta_j^t < 1$ ,  $C_t$  is households consumption at period  $t$ ;  $H_t$  is housing; and  $N_{H,t}^{ED}$  and  $N_{H,t}$  is time spent in education and work respectively.  $u(\cdot)$  is strictly increasing, strictly concave and twice continuously differentiable.

#### 4.1.1 Households-Savers

Each period households-savers choose consumption  $C_{H,t}$ , housing  $H_{H,t}$ . and the time they spend in education  $N_{H,t}^{ED}$  and at work  $N_{H,t}$ . They maximize the following utility

$$\max E_0 \sum_{t=0}^{\infty} \beta_H^t [\log C_{H,t} + j \log H_{H,t} + \tau \log (1 - N_{H,t} - N_{H,t}^{ED})] \quad (1)$$

where  $\beta_H^t$  is discount factor for savers ( $0 < \beta_H^t < 1$ ).  $j$  is the parameter that shows the share in housing preference.  $\tau$  is the parameter which shows the supply of labour by savers.

Savers are subject to the following budget constraint:

$$\begin{aligned} C_{H,t} + I_{H,t}^{ED} + D_t + q_t (H_{H,t} - H_{H,t-1}) \\ = R_{H,t-1} D_{t-1} - W_{H,t} H C_{H,t-1} N_{H,t} \end{aligned} \quad (2)$$

At period  $t$  savers make spending for consumption, housing  $H_{H,t}$  and investments in education  $I_{H,t}^{ED}$ . They also have deposit accounts  $D_{S,t}$  in banks and they purchase houses at price  $q_t$ . The house prices are the same for all agents in the model. For their savings households receive  $R_{H,t}$  interest payments. They earn  $W_{H,t}$  wage for  $N_{H,t}$  worked hours.  $HC_{H,t-1}$  is the human capital accumulation.

**Human capital accumulation channel** We assume that households are able to accumulate new skills by attending trainings and obtaining further education. This allows them to receive higher returns from labour as they earn higher wages. The human capital accumulation channel is set up as in Jones et al (1993), He and Liu (2008) and Peng (2012):

$$HC_{H,t} = (1 - \delta_{SK}) HC_{H,t-1} + B \left[ (I_{H,t}^{ED})^\theta (HC_{H,t-1} N_{H,t}^{ED})^{(1-\theta)} \right]^\chi \quad (3)$$

Households invest  $I_{H,t}^{ED}$  amount for their education.  $B$  is the parameter of the production of new skills,  $\theta$  is the parameter which shows the importance of goods input in the transformation of skills and  $\chi$  is the parameter that shows the returns to scale. Human capital is also subject to depreciation over time, which is given by  $\delta_{SK}$ .

### 4.1.2 Households-Borrowers

Borrowers own houses and borrow, so that they invest enough to gain new skills or obtain education and thus increase their human capital level. At period  $t$  households-borrowers maximise their utility by choosing  $C_{S,t}$  consumption,  $H_{S,t}$  housing, time at work  $N_{S,t}$  and time in education/training  $N_{S,t}^{ED}$ :

$$\max E_0 \sum_{t=0}^{\infty} \beta_S^t \left[ \log C_{S,t} + j \log H_{S,t} + \tau \log (1 - N_{S,t} - N_{S,t}^{ED}) \right] \quad (4)$$

where  $\beta_S^t$  is households-borrowers discount factor and  $\beta_H^t > \beta_S^t$ .

They are subject to the budget constraint:

$$\begin{aligned} C_{S,t} + I_{S,t}^{ED} + q_t (H_{S,t} - H_{S,t-1}) + R_{S,t-1} L_{S,t-1} \\ = L_{S,t} + W_{S,t} H C_{S,t-1} N_{S,t} \end{aligned} \quad (5)$$

Borrowers invest  $I_{S,t}^{ED}$  into their education. They borrow  $L_{S,t}$  loans from banks for  $R_{S,t}$  interest rate.  $W_{S,t}$  is their wages rate and  $H C_{S,t-1}$  is their human capital accumulation in terms of productivity.

Households-borrowers are also subject to the borrowing constraint:

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1 - \rho_S) m_S A_{MS,t} \left( \frac{q_{t+1}}{R_{S,t}} H_{S,t} \right) \quad (6)$$

where  $\rho_S$  measures the slow adjustment of the borrowing constraint over time.  $m_S$  indicates that households are constraint by the amount of houses real prices.  $A_{MS,t}$  is the exogenous shock which affects the households borrowing ability.

**Human capital accumulation channel** As in case of households-savers, the households-borrowers accumulate human capital which improves their wage rates and helps them to compete in the labour market:

$$H C_{S,t} = (1 - \delta_{SK}) H C_{S,t-1} + B \left[ (I_{S,t}^{ED})^\theta (H C_{S,t-1} N_{S,t}^{ED})^{(1-\theta)} \right]^\chi \quad (7)$$

## 4.2 Banks

Banks in this model are intermediaries between savers and borrowers. They play a crucial role since the banking sector can create shocks and then propagate them to other sectors. Banks have the following utility function:

$$\max E_0 \sum_{t=0}^{\infty} \beta_B^t [\log C_{B,t}] \quad (8)$$

where  $\beta_B^t$  is the banks' discount factor and  $C_{B,t}$  is banks' consumption at period  $t$ .

Banks are subject to the budget constraint:

$$\begin{aligned} C_{B,t} + R_{H,t-1}D_{t-1} + L_t = \\ D_t + R_{S,t}L_{S,t-1} + R_{E,t}L_{E,t-1} \end{aligned} \quad (9)$$

where  $L_t = L_{S,t} + L_{E,t}$  measures total loans issued by banks. Banks cannot issue loans more than the capital they have for liquidity and stability reasons. Deposits of households are also included into the bank capital. Therefore banks are also subject to the following capital adequacy constraint:

$$\begin{aligned} L_t - D_t - E_t \varepsilon_{t+1} \geq \\ \rho_D (L_{t-1} - D_{t-1} - E_{t-1} \varepsilon_t) + (1 - \gamma)(1 - \rho_D) (L_t - E_t \varepsilon_{t+1}) \end{aligned} \quad (10)$$

where  $\rho_D$  is the parameter which shows the partial adjustment in bank capital and  $\gamma$  shows the long term target of capital-asset ratio.

## 4.3 Entrepreneurs

Entrepreneurs produce a final good by using housing and labour provided by households. They maximise the following welfare function:

$$\max E_0 \sum_{t=0}^{\infty} \beta_E^t [\log C_{E,t}] \quad (11)$$

where  $\beta_E^t$  is the goods producers' discount factor and  $C_{E,t}$  is entrepreneurs' consumption at period  $t$ .

They maximise their profit:

$$\Pi = Y_t - W_{H,t}HC_{H,t-1}N_{H,t} - W_{S,t}HC_{S,t-1}N_{S,t} - R_{V,t}q_tH_{E,t-1} - R_{E,t}L_{E,t-1} \quad (12)$$

where  $Y_t$  is the Cobb-Douglas production function and is given by the following form:

$$Y_t = Z_t H_{E,t-1}^v [HC_{H,t-1}N_{H,t}]^{(1-v)(1-\sigma)} [HC_{S,t-1}N_{S,t}]^{(1-v)\sigma} \quad (13)$$

where  $v$  is the share of housing of an entrepreneur in the production function and  $\sigma$  is the share of wage of households-borrowers.  $Z_t$  is total factor productivity.

Entrepreneurs are subject to the budget constraint:

$$\begin{aligned} C_{E,t} + q_t(H_{E,t} - H_{E,t-1}) + R_{E,t}L_{E,t-1} \\ + W_{H,t}HC_{H,t-1}N_{H,t} + W_{S,t}HC_{S,t-1}N_{S,t} = Y_t + L_{E,t} \end{aligned} \quad (14)$$

where  $C_{E,t}$  is entrepreneurs' consumption. They borrow loans  $L_{E,t}$  from banks and pay  $R_E$  interest payments. Entrepreneurs own real estate of  $H_{E,t}$ . When entrepreneurs borrow their total income after all payments should be less than the loan amount. Therefore they are restricted by their profit:

$$L_{E,t} \leq \rho_E L_{E,t-1} + (1 - \rho_E) A_{ME,t} \left[ \begin{array}{c} m_H E_t \left( \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} \right) - \\ m_N (W_{H,t}HC_{H,t-1}N_{H,t} + W_{S,t}HC_{S,t-1}N_{S,t}) \end{array} \right] \quad (15)$$

where  $\rho_E$  allows for slow adjustment over time,  $m_H$  is the real estate loan-to-value ratio,  $A_{ME,t}$  is an exogenous shock to entrepreneurs borrowing ability and the term  $m_N$  shows the wage bill paid in advance.

#### 4.4 Aggregate resource constraint

As it is assumed all goods produced in the economy should be consumed thus consumption and investment cannot exceed current production. This is shown in the standard aggregate resource constraint:

$$Y_t = C_t + I_t \quad (16)$$

where total consumption and total investment are respectively:

$$C_t = C_{H,t} + C_{S,t} + C_{E,t} + C_{B,t} \quad (17)$$

$$I_t = I_{H,t}^{ED} + I_{S,t}^{ED} \quad (18)$$

In addition, the time endowment that is available for all households is normalised to unity  $N_{H,t} + N_{S,t} = 1$ .

#### 4.5 Market Clearing Condition

The market clearing condition for housing is given by the following equation:

$$H_t = H_{S,t} + H_{U,t} + H_{E,t} = 1 \quad (19)$$

#### 4.6 Shocks

The exogenous shocks in the model follow AR(I) process

$$Z_t = \rho_Z Z_{t-1} + u_Z \quad (20)$$

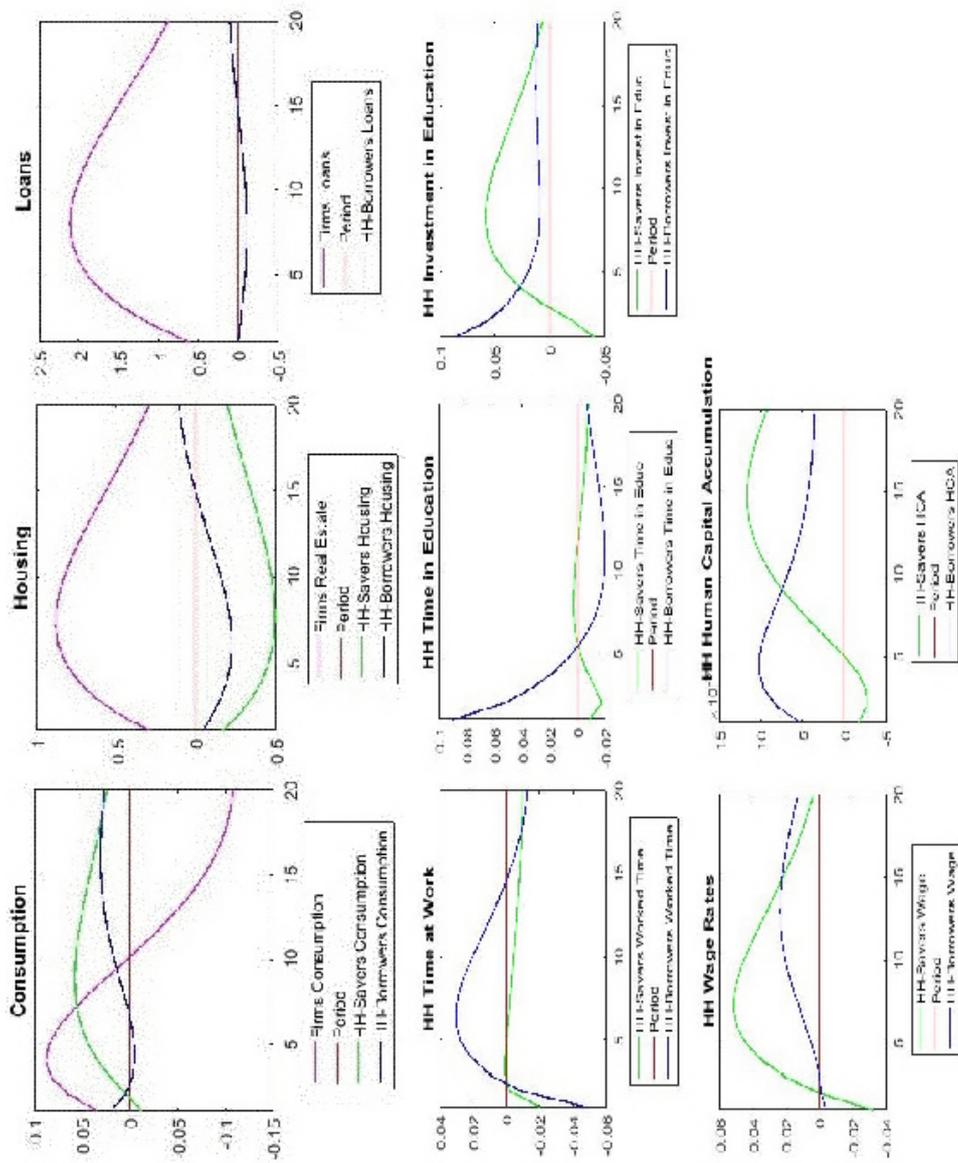
$$A_{ME,t} = \rho_{AME} A_{ME,t-1} + u_{ME} \quad (21)$$

$$A_{MS,t} = \rho_{AMS} A_{MS,t-1} + u_{MS} \quad (22)$$

## 5 Preliminary Results (early draft and not fully analysed)

### 5.1 Shock to firms' borrowing constraint ( $A_{ME,t}$ ):

- Firms
  - Firms can borrow more for the same value of their collateral
  - $\implies$  Purchasing of real estates, consumption and investment  $\uparrow \implies$  Production  $\uparrow$
  - Since Production  $\uparrow \implies$  Demand for labour increases and wages  $\uparrow$
- Households-Borrowers borrow less
  - Households-Borrowers get less wages  $\implies$  they have to work more hours  $\implies$  wages will slightly  $\uparrow$
  - $\implies$  spend less time in education  $\implies$  investment in education  $\downarrow$
  - However their human capital  $\uparrow$  because of raised working hours  $\implies$  more experience and better productivity.
- Households-Savers
  - Households-Savers worked hours  $\uparrow$  but slightly.
  - Households-Savers wages  $\uparrow \implies$  they consume and invest more in education so higher productivity  $\implies$  Human capital will steadily rise.
  - However Households will buy less houses

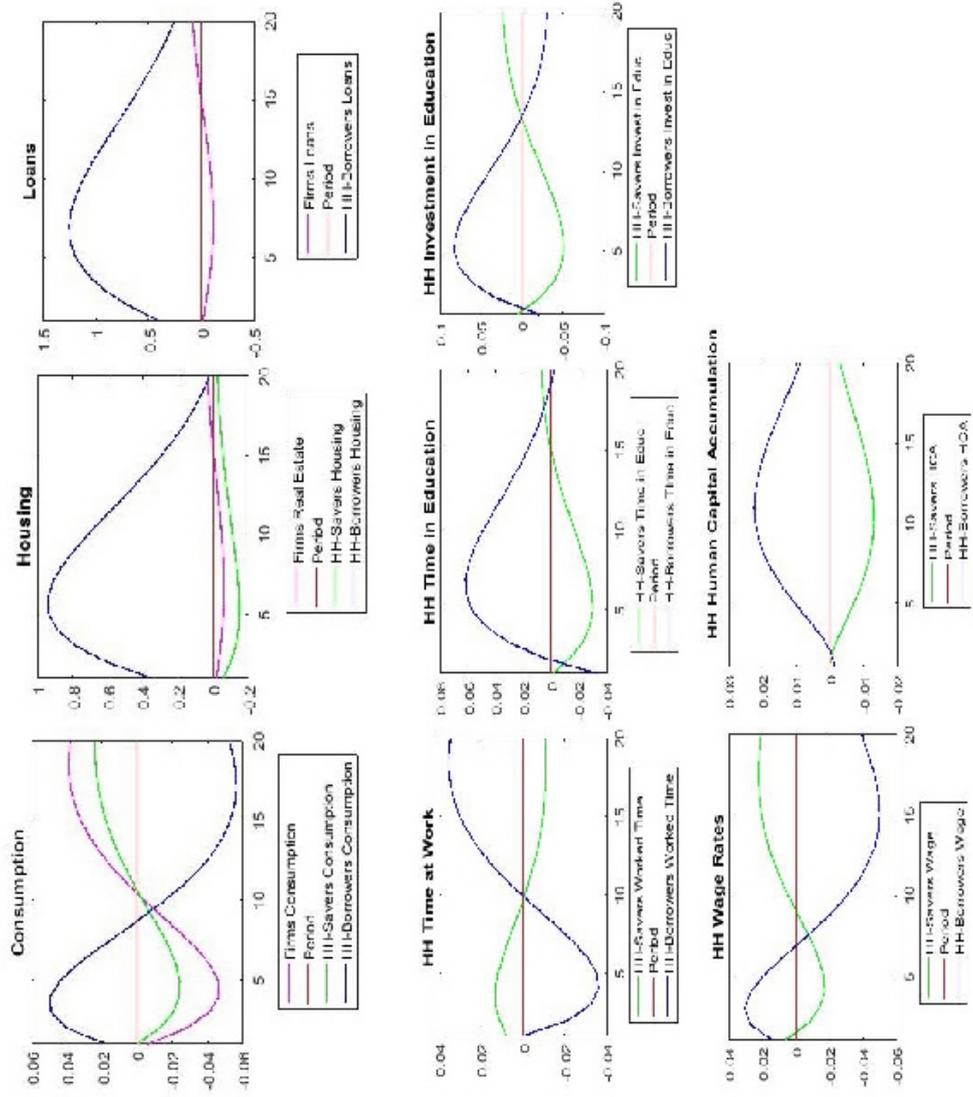


## 5.2 Shock to households-borrowers' borrowing constraint

$(A_{MS,t})$ :

- Firms
  - Firms borrowing  $\downarrow \implies$  Purchasing of real estates, consumption and investment  $\downarrow \implies$  Production  $\downarrow$
  - $\implies$  Households-Savers cannot afford higher consumption and housing
  
- Households-Borrowers are able to borrow more
  - $\implies$  spending and investment in education  $\uparrow \implies$  they can work less
  - $\implies$  spend more time in education  $\implies$  higher productivity  $\implies$  HC  $\uparrow \implies$  Households wages  $\uparrow$
  
- Households-Savers have to work more and since production  $\downarrow \implies$  wages  $\downarrow \implies$  spending will  $\downarrow$ 
  - $\implies$  spend less time in education  $\implies$  investment in education  $\downarrow \implies$  HC  $\downarrow$

As this shock fades away all the variables will return steadily to their initial levels.



## 6 Preliminary Conclusions (early draft and does not properly reflect the conclusions of the paper)

The single model with human capital accumulation gives the following results:

- when firms get more loans  $\implies$  production increases and it positively affects the whole economy. Even though the effect on investment in education is negative for Households-Borrowers, generally both Households productivity raises  $\implies$  Human Capital Accumulation  $\uparrow$  and both Households wages rise.
- when Households-Borrowers can borrow more  $\implies$  work less but invest more time and money in education  $\implies$  more competitive in the labour market and thus  $\uparrow$  Human Capital and get higher wages. However the economy is negatively affected, since firms borrowing  $\downarrow$  and their production and spending  $\downarrow$ . This will in turn negatively affect Households-Savers who have to work harder but still get smaller wages. Thus their investment and education time will  $\downarrow$  as well as their Human Capital.

## 7 References

To be completed