The Dynamic Allocation of Funds in Diverse Financial Markets Using a State-dependent Strategy: Application to Developed and Emerging Equity Markets

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Abstract

This study implements a state-dependent strategy including both developed and emerging markets. We seek to highlight the diversification benefits that these markets are able to offer. First, we test the level of integration of emerging markets with the world markets. Second, informed by the integration analysis, we implement a dynamic asset allocation strategy; the efficiency of this strategy is validated by conducting an out-of-sample performance. We find that a number of emerging markets expose time-varying integration relative to the world markets and that market-timing potentially adds value to portfolio performance and provides diversification benefits. Hence, investors can optimize the return on their investment by diversifying their portfolio towards emerging markets. The empirical outcomes of this study have practical implication for risk assessment of portfolios and asset allocation decision across emerging markets.

Keywords: Asset allocation, state-dependent strategy, emerging markets

1. Introduction

Investors are aware that markets may undergo abrupt shifts. This may be related to the business cycle, as the economy swings between stable conditions to uncertain conditions. Evidence of such behaviour has been reported for stock returns (Hamilton & Susmel, 1994), interest rates (Gray, 1996), inflation (Kumar & Okimoto, 2007) and commodity prices (Heaney, 2006).

Change in financial markets behaviour present significant challenges for both risk management and portfolio selection. If international financial markets are more correlated with each other in bad times than in normal times (Longin & Solnik, 2001), this will lead to poor forecasts for the portfolio performance when markets decline.[1] Previous studies have shown that asymmetric correlations caused by extremely large shocks are statistically significant and that state-dependent models have the potential to capture the degree of asymmetric correlation observed in historical data (Ang & Bekaert, 2002). Given such evidence, it may be preferable to manage risk and select portfolio on the basis of state-dependent strategy.

State-dependent model (SDM) is introduced by Goldfeld and Quandt (1973), later popularized by Hamilton (1989). SDM allows the data to be drawn from different distributions (states) where the process is modelled by probabilities of switching between different states. Based on volatility of returns, a degree of probability is assigned that the process will either remain in the same state or will transition to another state in the next period.

SDMs have stimulated interest in international asset allocation decision and portfolio selection. Ang and Bekaert (2004) document how the presence of state-dependency in normal and bear markets can be utilized in global asset allocation setting, using six international equity indices. They find that a state-dependent strategy is superior to standard static mean-variance strategy as the model potentially captures different distribution of asset returns based on the business cycle. Guidolin and Timmermann (2007) find evidence of four separate states, characterized as crash, slow growth, bull and recovery states using U.S. stocks, bonds and T-bills and confirm the economic importance of accounting for the presence of state dependency in asset allocation decision. More recently, Dou, Gallagher, Schneider, and

[1] As stated by Hamilton & Susmel (1994) *''extremely large shocks arise from quite different causes and have different consequences for subsequent volatility than do small shocks''*. This causes high correlation and volatility in equity markets during market turbulence.

Walter (2014) accommodates SDMs across different regions and sectors. They find that state-dependant asset allocations outperform the traditional static asset allocation while optimal allocation across sectors provides greater benefit compared to international diversification.

The international diversification of an investment portfolio presents both challenge and opportunity for equity portfolio management. While currently we appear to be witnessing a movement towards greater integration of international markets, sufficient segmentation exists across emerging markets as to provide diversification benefits. This potential can be expected to increase as markets become more accessible and their underlying economies develop. Previous research in relation to the merits of a state-dependent strategy in emerging markets is scarce. Given the significant economic growth and increasing access to these markets for foreign investment, allocation of funds across these markets is a fundamental component in portfolio performance. This study contributes to the empirical literature in this field by demonstrating how the concept of a state-dependent correlation with mainstream markets can be made exploitable in an asset allocation programme. Thus, we demonstrate how the level of integration of emerging markets with the world capital markets can potentially add value to portfolio performance and provide diversification benefits for international investors. Our intention is to investigate the switching behaviour of financial markets using a SDM that allows the asset allocation decision to be dependent on an identified state. More precisely, we address the following question: does a state-dependent strategy offer portfolio diversification benefit and improve asset allocation decision when we include emerging markets into portfolio? We follow Ang and Bekaert's (2004) approach that analyses the asset allocation strategy for developed equity markets. However, we extend their approach to the universe of developed and emerging equity markets. Ang and Bekaert (2004) find that state-dependent strategy can potentially outperform the static mean-variance strategy because they capture different distribution of portfolio returns during different time. But they point out that the outperformance of state-dependent strategies may be related to a historical period (in their case 1975-2000). Moreover, they indicate that the state-dependent portfolio need not be home biased and in any practical implication of a SDM, the optimal portfolio are likely to be more internationally diversified.

In this study, we apply SDM on six international equity indices including both developed and emerging markets informed by MSCI country dataset. In this way, we seek to provide a comparable performance level between emerging and developed equity markets. We find that a number of emerging markets expose time-varying integration relative to the world capital markets and that market-timing can potentially outperform portfolio performance and provide diversification benefits. Our empirical results suggest that the presence of two states and two tangency portfolios is superior to a single unconditional tangency portfolio. More precisely, the Sharpe ratio increased from 0.51 to 0.70 by holding the optimal tangency portfolio with state-dependent strategy in out-of-sample portfolio. In other words, investors can optimize the returns on their investment by diversifying their portfolio towards emerging markets (i.e. emerging Asia and emerging Europe).

The remaining of this study is as follows. Section 2 reviews the previous literature. Section 3 describes data and Section 4 gives an overview on the SDM and the model estimation. Section 5 presents the empirical findings and compares the performance of state-dependent portfolios with the static mean-variance optimal portfolios. In section 6, we carry out a practical implementation to check if the results are robust in out-of-sample performance. Section 6 is concluding and remarks.

2. Literature on dynamic allocation of fund under state-dependant approach

International diversification of investment portfolios and the allocation of funds across regions are crucial for investors. The benefit of portfolio diversification initiated by Grubel (1968) who finds that an internationally diversified portfolio brings higher return and lower risk in comparison to a purely domestic selection of portfolio. Portfolio optimization is the most developed and practiced approach to assess the optimal decision in allocation of funds.

The mean-variance approach developed by Markowitz (1952) is the foundation for portfolio selection. The approach selects the optimal portfolio by calculating the risk-return trade-off utilizing the estimated mean vector and covariance matrix of portfolio returns. One of the benefits of the Markowitz approach is that there are no limitations on the asset classes that can be incorporated. For instance, equities and fixed-incomes can be added to the model to achieve the optimal portfolio. However, the Markowitz approach is a one-period approach without stochastic specifications. The model also assumes that asset returns are formed on a stationary process with mean and covariance matrix of returns being constant over a specific period.

A wide range of research argues that asset returns follow a more complex process with multiple-periods with changes in market conditions relating to the business cycle, and each associated with a different distribution of returns. This empirical characteristic of asset returns highlights the necessity of a dynamic model in the allocation of funds decision so as to account for different distributions of returns across different time horizons.

The SDM generates a practical measure of addressing the shifting correlation conditions. In this study, we follow the standard Markowitz portfolio approach to allow the shifting nature of covariance matrix under separate market situations. Indeed, one of the serious concerns emerged during the 2008 global financial crisis (GFC), and more broadly in any market turbulence, was the sudden increase in correlations that arise leading to an ensuing lack of diversification in investment portfolios.

There have been a number of studies detecting the presence of multiple states in financial markets, more precisely in stock markets. A notable example is Hamilton and Susmel (1994) who apply Markov-switching model by analysing US weekly stock returns to describe volatility in stock returns whereby high volatility state to some extent is related to economic conditions. Their analysis supports previous findings that negative shocks lead to higher volatility than would positive shocks of the same magnitude, known as volatility clustering. These studies generally specify that the high (low) return of the stock market subject to low (high) volatility.

The increase in correlation between international financial markets, which has arisen during bad times, raises questions about the advantage of international diversification in optimal portfolios. Ang and Bekaert (2002) were among the first to address this issue by developing a dynamic portfolio selection for US investors utilizing a Markov-switching approach that could account for high correlation and volatilities during market turbulence. They show that international diversification can still benefit investors while allowing for state-dependency in international financial markets.

Graflund and Nilsson (2003) address the questions of how investors' perception of the state of the economy affects the dynamic portfolio decision in four major markets, US, UK, Germany and Japan with a Markov-switching approach. Their findings pinpoint the economic influence of accounting for the presence of state dependency, as taking a specific state into account affects the portfolio decision. Their findings are robust across the four influential economies of investigation.

Honda (2003) investigates the dynamic portfolio choice in which the mean returns of a risky asset depend on an unobservable state variable of the economy. The investor evaluates the prevailing state by observing past and current stock prices. He finds that the optimal portfolio of a long-term investment horizon can be essentially different from optimal portfolio of a short-term investment horizon. He also finds that the level of investor's risk aversion, the estimation of asset returns, and the prevailing state are key factors in investor's optimal portfolio decision.

Ang and Bekaert (2004) extend a Markov-switching approach for a broader international asset allocation strategies relying on changes in the systematic risk for six major equity returns. They use a two-state Markov-switching model to estimate returns and covariance matrix providing superior risk-adjusted returns in the content of an optimal international equity portfolio. Further, they argue that substantial wealth was achieved when investors switched between diverse assets such as equities, bonds and cash. In particular, market-timing profits are possible because high volatility states are contemporaneous with periods of high interest rate. [2]

Guidolin and Timmermann (2007) identify four states, characterized as crash, slow growth, bull and recovery states, in the joint returns series of stock and bond markets using Markov-switching model. They show that optimal allocations of fund vary significantly across different states and change over time as investors reassess their estimates of the state probabilities and each state has an intuitive interpretation. The out-of-sample forecasting method conducted in their study supports the economic justification for the presence of state-dependent in the allocation of funds.

Tu (2010) suggests a Bayesian framework for constructing portfolio that takes into account state-dependent model together with asset pricing model uncertainty and parameters uncertainty. The sample set consists of investable assets including the risk-free asset, the value-weighted Centre for Research in Security Prices market index portfolio, the size factor portfolio, the value factor portfolio and the Fama and French 25 portfolio sorted on size and book-to-market. His findings reveal that the economic value of accounting for state-dependent model is substantially different from the commonly used single-state models and it should be considered instead in portfolio selection regardless of any concerns about model or parameters estimates.

Ang and Timmermann (2011) survey state-dependent models with application in finance literature to model interest rate, equity returns, exchange rates and asset allocation. They conclude that the switching behavior in financial markets lead to potentially large consequences for investors' optimal portfolio selection.

Kritzman, Page, and Turkington (2012) consider Markov-switching models to forecast the asset returns in market turbulence, inflation and economic growth. They find that state-

[2] They stated that ''in a persistent high-volatility market, the model told the investor to switch primarily to cash. Large market-timing benefits are possible because high-volatility states tend to coincide with periods of relatively high interest rate''.

dependent asset allocation substantially improve the portfolio performance in comparison to the static asset allocation.

Bae, Kim, and Mulvey (2014) investigate the presence of state dependency in various asset classes, namely the S&P500 index, US government bonds and the Goldman-Sachs commodity index, as an additional asset class into portfolio. They develop a stochastic program to optimize portfolio selection employing the Markov-switching approach. Their analysis confirms the findings of earlier researchers that accounting for state-dependency information help portfolios avoid risk during left-tail events.

More recently Dou et al. (2014) extend Ang and Bekaert's (2004) approach to a diverse range of regions and sectors. They find that state-dependant allocation of fund adds value to the standard optimal portfolio, supporting the prior findings by other researcher. Additionally, optimal allocation across sectors provides an alternative to international diversification of fund allocation.

Jiang, Liu, and Tse (2015) test a dynamic investment strategy applying a Markov-switching approach using the international iShares exchange-traded funds. They find that a dynamic investment strategy outperforms the standard mean-variance strategy and this can be more practical and even incorporate into highly frequent trading process such as exchange-traded funds.

Pereiro and González-Rozada (2015) use a state-dependent model known as self-exciting threshold autoregressive model, to identify the price changes in a large number of emerging and developed markets. They show that such a model has the potential to improve the accuracy of long-term financial forecast. However, they did not check whether taking state-dependent into account could properly optimize asset allocation programme.

Nystrup, Hansen, Madsen, and Lindström (2015) examine whether a state-dependent investment strategy can effectively respond to changes in financial markets, in an effort to benefit over the long-term horizon investment in comparison to standard approaches. They confirm the validity of their investment strategy of switching between stocks and bonds.

One conclusion from these findings is that the potential benefit of state-dependent based asset allocation is achievable, provided sufficient information about the prevailing state and future changes. However, investors should consider this benefit with caution. First, because not all of these findings consider transaction costs involved in switching between different assets. This is essential as frequent rebalancing can possibly outweigh the benefit of dynamic investment strategy. Nevertheless Nystrup et al. (2015) state that even with the inclusion of some level of transaction cost, the dynamic investment strategy can be profitable.

Second, the practical tests on a dynamic investment strategy have been limited to relatively developed financial markets by focusing on different asset classes such cash, bond and equity index. As these markets are relatively integrated, the dynamic asset allocation that switches between diversified assets in these markets may not purely reflect the success that can be possibly achieved by investors especially during bad times. In this study, we argue that investors can switch their funds to emerging markets as an alternative to invest in a safe asset or a bond during bad times, which recommended by many researchers.

3. Data

In this study we aim to investigate the performance of emerging market indices and whether time-varying systematic risks can improve the estimated expected returns hence portfolio performance. For this purpose, we used the Morgan Stanley Capital International (MSCI) emerging markets classification for three regions (Asia, Europe and Latin America). We justify our selection of MSCI indexes by the fact that they constitute a reliable benchmark measure of market performance. In addition, they have also been widely applied in several previous studies. As a benchmark measure, we also analyse the MSCI indexes for developed markets (Europe, North America and Pacific) enabling us to have comparable investigation on equity indices (Table 1).

[Insert Table 1 here]

We obtain data from the Thomson Reuters Financial DataStream. The indices cover weekly returns in the period ranging from 3 January 2001 till 30 December 2015 which gives us an evaluation of equity indices before and after the subprime crisis, providing a reliable measure for the performance of the models when we look at emerging markets. We use weekly data to avoid the problem of non-synchronous trading and short-term correlations due to noise with higher frequencies such as daily data.[3] Using low-frequency data also allows us to isolate cyclical variations and to better analyse state dependency across time. Hamilton and Susmel (1994) suggest that state-dependent heterscedasticity is more appropriate for low-frequency data such as weekly and monthly data. Moreover, according to Aloui and Jammazi (2009) state dependency can be detected more clearly across time using low frequency. This is further proved by Walid, Chaker, Masood, and Fry (2011) who employ SDM to investigate the dynamic linkage between stock price volatility and exchange rate changes on emerging

[3] Nonsynchronous trading can cause correlations between two independent assets, when there is not any. This in turns affects portfolios and risk management.

markets. On the other hand, most of the previous studies on state-dependent asset allocation strategy use monthly return data; as recommended by Dou et al. (2014), one extension would be to investigate the switching behaviour of asset returns on a weekly basis, which might convey more timely information, especially during the beginning of the high volatility state when the necessity of diversification is more highlighted.

We calculated returns as the logarithmic of total returns indexes. Our proxy for the risk-free rate is the weekly 3-months US LIBOR rate. For the world financial markets index, we use the MSCI world total return index. These rates are used to evaluate equity indexes performances using international capital asset pricing model (ICAPM).

4. Description of the Model

The parameters estimation of SDM consists of two steps. First, we estimate the state-dependent expected returns and standard deviation of the world market returns. From that we distinguish between high volatility and low volatility of the world markets based on the realization of the state probabilities including both ex-ante and ex-post probabilities. Second, the expected excess returns for each region are estimated based on identified state of the world market returns but separate from the estimation of the world return parameters. Hence, the information in individual regions does not influence the world return generating process.

4.1 State-dependent Model – World market Returns

The intrinsic idea underlying SDM is that there exist two states of economy, when the first one corresponds to normal period (high return – low volatility) and second one is related to time of uncertainty (low return – high volatility).[4] These two states may offer different investment opportunities and hence different asset allocations over time as the investors' perceptions change depending on the underlying state probabilities. We investigate whether state-dependent mean-variance efficient (MVE) portfolio across different states can potentially outperform the mean-variance optimal portfolio. To demonstrate this hypothesis empirically, we set the excess return series in a state-dependent framework. To maintain the parsimony of the model we follow Ang and Bekaert (2004) approach who assume that the expected excess returns in each region is linear to its beta with respect to world market returns. In another word, we assume that the expected excess return for each region drives

[4] Several studies extended the model to more number of states, see for example Guidolin and Timmermann (2007).

from the world expected excess return based on market volatility. The equation for the world equity market in excess of risk free rate is then defined as:

$$r_t^w = \mu_{s_t}^w + \sigma_{s_t}^w \varepsilon_t^w \qquad (1)$$

where $\mu_{s_t}^w$ is the world conditional expected return and $\sigma_{s_t}^w$ is the world conditional variance (volatility). The assumption is that the world expected returns and volatility can take two different values depending on the realization of the two unobserved state variables, s_t, which indicates the world market condition. Then we assume that the excess returns series have two unobserved states, state1 and 2.[5] Subsequently, $\mu_{s_t}^w$ and $\sigma_{s_t}^w$ can take different values according to realization of the state variable s_t.

The economic explanation underlying this assumption is the phase of the world economic cycle. As a result, the equity markets can be defined by high uncertainty and less returns (bear market) and low uncertainty and higher returns (bull market).

In order to complete this process, the likelihood function should be characterized such as to maximize the parameters of this function. In conducting SD-ICAPM in this study, the parameters estimation is carried out by adopting expectation maximisation (EM) algorithm of Hamilton (1990) (see Appendix B for further explanation on Expectation Maximisation Algorithm).

The state variable s_t follows a two-state Markov chain process with constant transition probabilities:

$$\begin{pmatrix} P_{11} & 1 - P_{11} \\ 1 - P_{22} & P_{22} \end{pmatrix}$$

where, the probability of remaining in the same state next time depends only on the current state. If the current state is State 1, P_{11} denotes for the probability of staying in the first state (i.e. $1 - P_{11}$ denotes for the probability of transitioning to another state). Likewise, if the current state is State 2, P_{22} denotes the probability of remaining in State 2 and $1 - P_{22}$ denotes for the probability of transitioning to another state (see Appendix A for further explanation on Markov chain process).

With this alteration in the model, the world expected returns and volatility can be varying through time.

Assuming that an investor knows the current state, the conditional expected returns and conditional volatility for the world market returns in the next period would be:

[5] The number of states is restrictive since including more state may result in extreme computational issues.

$$e_1^w = P_{11}\mu_{s_t=1}^w + (1 - P_{11}) \tag{2}$$

$$\Sigma_1^w = P_{11}(\sigma_{s_t=1}^w)^2 + (1 - P_{11})(\sigma_{s_t=2}^w)^2 + P_{11}(1 - P_{11})\left[\mu_{s_t=2}^w - \mu_{s_t=1}^w\right]^2 \tag{3}$$

If the current state is State 1, $s_t = 1$ and would be:

$$e_2^w = (1 - P_{22})\mu_{s_t=1}^w + P_{22}\mu_{s_t=2}^w \tag{4}$$

$$\Sigma_2^w = (1 - P_{22})(\sigma_{s_t=1}^w)^2 + P_{22}(\sigma_{s_t=2}^w)^2 + P_{22}(1 - P_{22})\left[\mu_{s_t=2}^w - \mu_{s_t=1}^w\right]^2 \tag{5}$$

If the current state is State 2, $s_t = 2$.

e_1^wdenotes for the world conditional expected returns in State 1. If State 1 realizes, the investor assigns the expected returns to be e_1^w. Likewise, if the investor realizes that the world market is in State 2, he considers e_2^w as the expected returns. To estimate this expected returns, the investor applies $(1 - P_{22})$ and P_{22} to weight the expected returns.

For instance, when the investor knows that the world market is in State 1 today, the expected return for the next period depends on investor's expectation for the state realization at time $t + 1$. Therefore, the investor weights the possible realization of expected returns, $\mu_{s_t}^w$, based on related probabilities.

Similar to the conditional mean, the conditional variance changes across states. When investor realizes that the world market is in State 1 at time t, he expects that the first state will carry on with probability P_{11} and assigns a probability of $(1 - P_{11})$ for transitioning to another state (i.e. State 2). In fact, the first element in equation (3) and (5) is a weighted average of conditional variance across the two states. The second element is an additional jump, which arises because the conditional mean is different across the two states.

In case that $P_{11} = 1 - P_{22}$, the assumption of state structure is not fitted to the expected returns since they are identical through different states. However, an empirical estimation of state persistent has been documented in previous studies (Ang & Bekaert, 2002, 2004; Guidolin & Timmermann, 2008).

4.2 State-dependent Model for International CAPM

State-dependent ICAPM (SD-ICAPM) is considered the return generating process to be affected by state variable $\mathrm{s_t}$ which identifies the process at each point in time based on the realization of the state probability.[6] The underlying assumption is as follows: if $\mathrm{s_t} = 1$ means

[6] Following Ang and Bekaert (2004) the SD-ICAPM is conditional on smoothed probabilities of the high (low) market volatility state being lower (higher) than 50 per cent. More precisely, the expected return for each region is calculated as a product of estimated betas and world market expected returns with smoothed probabilities of high (low) market volatility being lower (higher) than 50 per cent.

that the process is in state 1 and we assume that $s_t = 2$ when the process is in state 2. In the other words, the return generating process can be sufficiently modelled by SD-ICAPM in which one state is subjected to normal volatility ($s_t = 1$) and the other is high volatility ($s_t = 2$).

To characterize this model to generate the expected returns, the modified linear ICAPM specification with state dependency is applied by allowing the parameters to be time varying.

$$r_{i,t} = \alpha_{i,s_t} + \beta_{i,s_t}(r_{m,t}) + \varepsilon_{i,s_t} \qquad \varepsilon_{i,s_t} \sim N(0, \sigma^2_{i,s_t}) \tag{6}$$

where α_{i,s_t} denotes state-dependent alphas, β_{i,s_t} denotes betas and ε_{i,s_t} is idiosyncratic volatilities for markets excess returns based on the realization of the state probability (see Appendix C for further explanation on filtered and smoothed probabilities).

When the state probability is realized, equation (6) can be defined as:

$$r_{i,t} = \alpha_{i,1} + \beta_{i,1}(r_{m,t}) + \varepsilon_{i,1} \tag{7}$$

when state probability $p_t > 0.5$, or

$$r_{i,t} = \alpha_{i,2} + \beta_{i,2}(r_{m,t}) + \varepsilon_{i,2} \tag{8}$$

when state probability $p_t < 0.5$.

More precisely, we assume that s_t= 1 denotes a low variance state and s_t= 2 denotes a high variance state. Then σ^2_{i,s_t} is defined as the conditional variance [7] of residuals where: $\sigma^2_{i,2} > \sigma^2_{i,1}$.

4.3 Asset Allocation Strategy

This section explains the asset allocation by implementing the SDM for developed and emerging equity markets. In order to carry out the asset allocation strategy, we apply the mean-variance optimization following Ang and Bekaert (2004).

To estimate the expected returns and variance-covariance matrices associated with each state, we define the vector of conditional expected returns for each region to depend on state i, $e_{s_{t=i}}$, where i implies current state according to smoothed probabilities. We allow the variance-covariance associated with each state to be Σ_i.

Because the world expected returns switches between two states, the expected returns for each region, given by $\alpha_i + \beta_i e_i^w$ vary across states. We have e_i^w defined in equation (2) and (4) and with $\alpha_i$and β_i as vectors defined in equation (7) and (8) as the parameters of SD-ICAPM for the six regions. Therefore, the expected returns for each region are specified as:

$$e_{s_{t=i}} = \alpha_{s_{t=i}} + \beta_{s_{t=i}} e_i^w \tag{9}$$

[7] We could either have conditional mean or conditional variance model.

where the expected returns for each region vary depending on their different alphas and betas with respect to the realization of state probabilities for world market returns, smoothed probabilities.

The variance-covariance matrix has three elements. First, there is an idiosyncratic volatility, σ_i, for each region that we obtain by matrix v_i for the state i:

$$v_i = \begin{bmatrix} (\bar{\sigma}_i)^2 & 0 \\ 0 & (\bar{\sigma}_i)^2 \end{bmatrix} \quad (10)$$

where v_i is a matrix of zeros with $(\bar{\sigma}_i)^2$ along the diagonal. Second, the difference in systematic risk, β_i, through different regions and their correlations are given by the world market variance and the betas similar to a normal model:

$$\Omega_i = (\beta_i \beta_i')(\sigma_i^w)^2 + v_i \quad (11)$$

Since the variance of the world market and betas for the next period, time $t + 1$, relies on the realization of the current state, time t, we obtain two possible variance matrices for the expected returns next period.

Third, because the covariance matrix accounts for state structure, it is associated with the realization of the current state. As a result, the covariance matrix has additional jump component to the conditional variance matrix, which again arises because the conditional means are different across two states. Therefore, the conditional covariance matrix associated with each state is defined as:

$$\Sigma_1 = P_{11}\Omega_1 + (1 - P_{11})\Omega_2 + P_{11}(1 - P_{11})(e_1 - e_2)(e_1 - e_2)' \quad (12)$$

$$\Sigma_2 = (1 - P_{22})\Omega_1 + P_{22}\Omega_2 + P_{22}(1 - P_{22})(e_1 - e_2)(e_1 - e_2)' \quad (13)$$

where Σ_1 is the conditional covariance matrix if the current state is State 1 and Σ_2 is the conditional covariance matrix if the current state is State 2.

To perform mean-variance optimization, we need to specify the risk-free rate. In this regards, for each period, we assume the risk-free rate to be weekly 3-months US LIBOR rate, hence, the risk-free rate will be vary over time.

The SDM provides two optimal tangency portfolios (for all the equity regions) that investors can select, depending on the state realization. One obvious issue as indicated in the literature is that (i) the mean-variance portfolios based on historical data may be quite unbalanced, and (ii) the rational investors do not apply straight forward portfolio weights (Black & Litterman, 1992; Green & Hollifield, 1992). One practical solution, therefore, is to impose constraint on asset allocation program as recommended by Ang and Bekaert (2004) for future studoies. For instance, Dou et al. (2014) perform two alternative constraints on SDM. The short-sale constraint requires the optimal portfolio weights to be positive; while the benchmark constraint keeps the asset allocations close to their average market capitalization (e.g. not more than 10 per cent deviate from market capitalization).

4.4 Performance Measurement

A numbers of measurements have been introduced to assess equity return performance, often depending on the type of risk measure under consideration. The main performance measure is the Sharpe Ratio (SR) (Sharpe, 1966).

The interaction between average asset returns and its standard deviation from the mean forms a criterion for rational investors to develop optimal strategies and make marginal profit between different possible investments. This statistics is adopted to assess the asset performance. The Sharpe's (1966) ratio, refers to this measurement:

$$SR = \frac{(r_{i,t} - r_{f,t})}{\sigma(r_{i,t})} \tag{14}$$

where $r_{i,t}$ refers to the returns on portfolio i and $r_{f,t}$ denotes for the risk-free rate. This ratio compares the arbitrage profit by risk associated to the investment on a portfolio. In practice, the higher the ratio implies better performance of the portfolio. In a case that the SR is positive (negative), the asset i outperforms (underperforms) given the risk-free rate. Indeed, excess return is related with higher risk if the ratio ranged from 0 to 1, whereas it related to lower risk if the ratio was higher than one.

5. Empirical Results

In order to achieve reasonable outcomes, the development process should take into consideration the distributional characteristics of financial data. Table 2, Panel A, presents the summary statistics of a sample set. The first part summarizes characteristics of the excess return series for each of the equity regions. The following properties of data are worth considering. First, we note that markets with marginal excess returns do not necessarily present higher volatility suggesting that the risk-return trade-off may not be present. Second, negative skew implies that the return distribution is skewed to the left and has long left tail. This indicates that large negative returns are most likely to happen. However, the excess return distributions are not heavy unconditional skewed, except for Latin America and emerging Europe. Third, as a common factor of financial time series, these markets exhibit high level of kurtosis than normal value of 3. Accordingly, the distributions of excess return series are leptokurtic and hence are non-Gaussian. Furthermore, Jarque-Bra test statistics, advocated that excess return series are not well estimated by normal distribution. We perform unit root test of Dickey and Fuller (1981) on logarithmic of excess returns. The associated test statistics is also presented in Table 2, Panel B. The result show that all the excess return series are integrated to the order of 1, since the result of ADF test statistics are less than critical value. Estimates from the correlation matrix of excess return series, Panel C, for the world markets and the six equity regions suggest that excess returns series have very different degree of correlation with correlation coefficient ranging from 0.75 to 0.94, for emerging Asia and Europe respectively.

[Insert Table 2 here]

Figure 1, shows the volatility clustering of logarithmic excess returns. Some of these equity markets experience spikes of volatility at similar time during world events such as the 2001 September 11 terrorist attack, the 2003 Internet bubble, GFC during 2008-2009 and more recent market fall due to European sovereign debt crisis in 2011.

[Insert Figure 1 here]

Table 2, Panel D summarizes the results of unconditional ICAPM, without switching condition, estimated by OLS and Newey-West HAC standard errors were computed (Newey & West, 1987). The necessary condition for the model is: intercept term (α) must be zero. Then we assume that the market is integrated with the world financial system if β=1 and is segmented if β=0. First, the preliminary results show, for the unconditional ICAPM, the $\hat{\alpha}$'s are not significantly different from zero at conventional significance level. Second, $\hat{\beta}$'s in all of the markets are estimated at high level of significance and their magnitudes are appeared economically reasonable. However, the value of $\hat{\beta}$'s for Emerging Europe (1.40), Europe (1.15) and Latin America (1.28) imply volatility at par with world market and suggesting strong level of integration with world markets.

5.1 Model Estimation and Results

Table 3, Panel A, includes the estimation results for the mean-variance model for the world equity markets, equation (1). We consider the first state as a normal period, where the world equity markets have yield 0.35 per cent (18.20% per year) with 1.37 per cent (9.87% per year) volatility. On the other hand, when the world markets are in state 2, high volatile state, it is expected to yield a negative return of -0.50 per cent (-26% per year) and higher volatility of 3.63 per cent (26.17% per year).

[Insert Table 3 here]

Since the historical data are adopted to characterize the excess returns, it is expected that the model would generate reasonably poor forecast about expected returns. This is reasonable as changing in financial leverage, which cause by negative shocks, stimulates the volatility and hence expected returns Black (1976) and Christie (1982). More specifically, high (low) returns and low (high) volatility states are associated with the existence of bull and bear markets (Ang & Bekaert, 2004; Dou et al., 2014; Liu, Margaritis, & Wang, 2012). The results of this study are consistent with the above findings that obtained negative correlations between state-dependant volatility and returns.

The estimated transitional probabilities are $P_{11} = 0.96$ and $P_{22} = 0.93$ which implies that once the market is in state 1 today, it will remain in the same state the next period with an average 96% of the time. Accordingly, there is only 4% likelihood that the market switches into a high volatile state (state 2). Similarly, there is only 7% likelihood that it will switch out of the high volatile state, meaning that either of these states are persistent. As Hamilton (1990) noted, we can use these transition probabilities to measure the approximate time duration in which the world market system stays in a given state by calculating the maximum number of corresponding periods, define as, $P(S_{t+n} = i, S_{t+n-1} = i, \ldots, S_{t+1} = i | S_t = i) > 0.5$.

Then the expected duration of remaining in each state can be estimated as $\frac{1}{1-P_{11}}$, where P_{11} is estimated transitional probability. The expected duration of being in each of these states are approximately 27 and 14 weeks respectively (Table 3, Panel A).

Table 3, Panel B, contains the estimation results for the SD-ICAPM from equation (7) and (8). First, the $\hat{\alpha}$'s are not significant at conventional level of significance. Second, $\hat{\beta}$'s are estimated with high level of significance in both states and economically reasonable. $\hat{\beta}$'s for emerging Europe, Europe and North American regions increase significantly in state 2 supporting the hypothesis that the equity markets are more integrated with each other during bear market. This is in line with the results achieved by Ang and Bekaert (2002) and Longin and Solnik (2001) who indicate that international equity markets are more correlated with each other in bear markets than in bull markets. However, this is not the case for all the equity regions. For example, the Pacific region has a beta of 0.96 during the normal period but much lower systematic risk (0.76) in the bear market. Besides, it seems that the low beta for the Pacific region is being offset by a large negative alpha in State 2 indicating that the asset in this region may be priced locally. This indicates that the underperformance of the Pacific region during a bear market is much more related to idiosyncratic events since the Pacific region has the lowest average returns in the data (Dou et al., 2014).

Overall, we find strong evidence for state-dependent beta coefficients. This implies that the estimated beta from the unconditional ICAPM underestimates the risk premium under high volatile states while overestimating the risk premium under low volatile states. The SD-ICAPM allows the market risk premium to be drawn from two distinct states in order to characterize the instability of beta, consistent with previous studies. This in fact enables us to achieve more precise expected returns during different time periods that will give a reliable forecast about the portfolio performance.

Panel A of Table 4 shows the estimated expected returns computed using equation (9) with data from January 2001 till December 2015 for six equity regions. The expected excess returns may seem high during normal time while negative during world market turbulence. However, they are conditional on the realization of a bull and bear markets. And because betas are greater than 1 for emerging Asia, emerging Europe and Latin America, the expected excess returns are quite high in these regions. In the bear market, State 2, expected excess returns are significantly lower and negative, with Pacific, emerging Europe having the lowest expected excess returns. Since the historical data are used, it is expected that high beta regions to have lower expected returns from SD-ICAPM. In fact, the expected returns for emerging Europe estimated by the model are the highest of all the regions in the normal state but by far lowest in the bear market.

[Insert Table 4 here]

Panel B of Table 4 reports the covariance and correlation matrix for each state obtained from equation (10) – (13). State 2 is high volatility state, which yield higher correlation on average. The average correlation has increased from 0.55 to 0.74. This is in line with the results achieved by Ang and Bekaert (2002) and Longin and Solnik (2001) who indicate that international equity markets are more correlated with each other in bear markets than in bull markets.

In addition, the estimation procedure generates classification about prevailing state in each period. Panel A of Figure 2 illustrates the total returns of $1 invested in the six regions during the sample period. Panel B of Figure 2 is the ex-ante (filtered) and ex-post (smoothed) state probabilities. The ex-ante probability is the probability that the state next week will be the low-volatility world market state, given past and current information up to time t; the ex-post probability is the probability that the state next time will be the low-volatility world market state, given all the information available in the sample period. The high-volatility markets are considerable during 2001 till early 2003 and also 2008 till 2009. The dotted lines show the two economic recessions, the 2001 Dot-Com Bubble and the GFC, during the sample period reported by the National Bureau of Economic Research (NBER). Overall, the unconditional probability of the normal state, bull market, is 66 per cent (see Appendix A, equation (6.a)).

[Insert Figure 2 here]

The results in Table 3 and 4 along with the plots in Figure 2 give complementary description of the existence of two states for the world market, highlighting the fact that we need to account for the existence of at least two states when we look at the portfolio performance and asset allocation strategy in financial markets.

5.2 State-dependent Asset Allocation Performance

Figure 3 illustrates the implementation of SDM for asset allocation practice. The solid line shows the mean-standard deviation frontier when unconditional ICAPM is used to estimate the expected returns. The other two frontiers are obtained from SDM in the two states. The one on top of Figure 3 is for normal state, State 1. It is obvious that the risk-return relationship is better in State 1 than the unconditional frontier. This is because the investor is now accounting less likelihood to the bear market, high-volatility, for the next period.

In practice, the presence of two states and two tangency portfolios can provide State-dependent investment opportunity, which is prevailing to a single unconditional tangency portfolio. More precisely, as indicated in Figure 3, the Sharpe ratio relatively improved from 0.1591, by holding average market caps, to 0.1718, by holding the optimal tangency portfolio. However, the optimal tangency portfolio remains almost at the same level of average market caps when we use unconditional MVE. In State 2, the unconditional portfolio generates a Sharpe ratio of $|0.4693|$, which could marginally improve when holding the optimal tangency portfolio for the high-volatility state. In other words, investors can minimize the lost on an investment if they diversify their portfolio towards less volatile markets when the world market is in high-variance state.

[Insert Figure 3 here]

6. Practical Implementation

To demonstrate whether the state-dependent asset allocation adds value to standard mean-variance optimization, we estimate the returns of these two strategies both in-sample and out-of-sample performance. The state-dependent model estimated up to time t, and the state-dependent weights were calculated from information available up to time t, the estimation date. The test started with $1 investment in January 2001 till December 2008 for in-sample and January 2009 till December 2015 for out-of-sample period. Portfolio weight re-estimated every week which is consistent with data frequency. The performance criterion is the *ex post* Sharpe ratio realized by various strategy.

The state-dependent strategy required the risk-free rate and the realization of the state. For the risk-free rate, we used weekly US LIBOR rate. To derive the state, we assumed that the investor compute the state probability from current information, which is a by-product of estimating the SDM. If the state probability was larger than 50 per cent for State 1, the investor classified the state as 1, otherwise it was classified as 2. This calculation did not require any further data input as explained by Ang and Bekaert (2004).

Panel A Table 5 reports the in-sample average returns, standard deviations and Sharpe ratio estimated by the static mean-variance, state-dependent strategies and the MSCI world index for all equity asset allocation with no constraints and short-sale constraint. Over the in-sample period, the state-dependent strategy yields higher average returns and less volatility in comparison to the world market returns and the static strategy with no constraint and short-sale constraint scenarios. Although the Sharpe ratio has negative sign for the world returns, static strategy and state-dependent strategy but it indicates a better portfolio performance when state-dependent strategy is used. But the Sharpe ratio for state-dependent strategy outperforms when the short-sale constraint is imposed, possibly because short-sale constraint restrict weights to be positive.

Over the out-of-sample period and with no constraint, the state-dependent strategy's average returns is 15.70 per cent, which is higher than the average returns of the static portfolio (14.10 per cent) and the world market returns (10.78 per cent), Panel B of Table 5. The state-dependent portfolio's Sharpe ratio increased compared to the world market portfolio and static strategy. The state-dependent strategy did well because during this sample period, all equity markets recorded better returns as the world markets passed the financial crisis. In fact, under both no constraint and short-sale constrain, the state-dependent strategy delivers higher Sharpe ratios in the out-of-sample period, compared to the world market returns and the static strategy.

[Insert Table 5 here]

Figure 4 shows how wealth accumulated over time with different strategies before and after GFC. Panel A shows that the state-dependent strategy performed relatively well but not very differently during GFC. However, it is over the last five years that the state-dependent strategy particularly outperformed the static strategy. Given that the results in this example may be closely related to historical period, the success of the state-dependent strategy presented here is not necessarily a proof of future success. For instance, not all investors would rather choose a relatively large short positions imposed by the model.

[Insert Figure 4 here]

7. Concluding and Remarks

This study adds further investigation to the literature in asset allocation decision and portfolio selection process by providing a comparable analysis on the behaviour and the performance of asset returns in both developed and emerging equity markets. We also contribute to the asset allocation literature by extending state-dependent asset allocation strategy to emerging equity markets.

Using the MSCI country dataset for both developed and emerging equity regions, we find that a number of emerging market regions expose time-varying integration relative to the world capital markets and that market-timing can potentially outperform portfolio performance and provide diversification benefits. Overall, we find strong evidence for state-dependent beta coefficients. This implies that the estimated beta from the unconditional ICAPM underestimates the risk premium under high volatile states while overestimating the risk premium under low volatile states. The SD-ICAPM allows the market risk premium to be drawn from two distinct states in order to characterize the instability of beta. This in fact enables us to achieve more precise expected returns during different time periods that will give a more reliable measure about the portfolio performance.

In addition, our empirical results suggest that the presence of two states and two tangency portfolios that account for different distribution of asset returns is superior to a single unconditional tangency portfolio. More precisely, the Sharpe ratio improved from 0.51 to 0.70 by holding the optimal tangency portfolio with state-dependent strategy in out-of-sample portfolio. In other words, investors can optimize the returns on their investment by diversifying their portfolio towards emerging markets (i.e. emerging Asia and emerging Europe).

One important conclusion is that state-dependent strategies have the potential to outperform because they set up a selective portfolio in a bear market that hedge against high correlations and low returns. This conclusion remains reliable in the presence of short-sale constraint because this portfolio essentially tilts the allocations toward the lowest-volatility assets. In addition, the state-dependent strategy need not be home biased; in this practical example, we involved more internationally diversified portfolios by including emerging markets into portfolio returns. We show that diversification across emerging markets gives higher benefit to international investors.

The implementation of the state-dependent strategy can further be improved by incorporating the following extensions. First, we only consider the equity markets in this study primarily because we wanted to have a comparable performance between equity markets in developed and emerging markets. For example, Guidolin and Timmermann (2007) use U.S. stocks, bonds and T-bills for the presence of state dependency in asset allocation decision. Further research can look at implication for the performance of bonds in emerging and developed markets.

Second, in this study we use the world market returns (an endogenous variable) to derive the volatility in equity returns which is known as constant probabilities. One extension is to allow for time-varying transition probabilities. In this case, the transition probabilities can be drawn as a function of predetermine variable (an exogenous variable) such as economic variables. For example, Ang and Bekaert (2004) allow the interest rate to influence the transition probabilities. A time-varying state-dependent model is more flexible and produces more parameters. Hence, a state-dependent asset allocation that trade based on an economic variable may offer additional values.

Finally, this study applies SD-ICAPM model where beta is the only factor that characterized the expected returns. Another possible extension is to formulate expected returns from factor models such as Fama and French three factor model or incorporate macroeconomic variables such as inflation that can influence equity returns.

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Table 1. Composition of International Equity Markets

The developed and emerging equity markets in each region based on MSCI equity market classification.

Americas		Europe		Asia-Pacific	
Emerging	Developed	Emerging	Developed	Emerging	Developed
Brazil	US	Czech Republic	Austria	China	Australia
Chile	Canada	Greece	Belgium	India	Hong Kong
Colombia		Hungary	Denmark	Indonesia	Japan
Mexico		Poland	Finland	Korea	New Zealand
Peru		Russia	France	Malaysia	Singapore
		Turkey	Germany	Philippines	
			Ireland	Taiwan	
			Italy	Thailand	
			the Netherlands		
			Norway		
			Portugal		
			Spain		
			Sweden		
			Switzerland		
			the UK		

Table 2. Descriptive statistics on weekly excess returns

Panel A, reports summary statistics of weekly excess returns and are dominated in US dollars. The returns are in excess of, for each region, 3-month secondary market US LIBOR rates from logarithmic of total return as weekly frequency. The sample period for the regional returns is from January 3st 2001 till December 25st 2015. ADF in Panel B stands for augmented Dickey-Fuller unit root test statistics. The correlation matrix of excess return series for the world markets and the regional markets reports in Panel C. Panel D is the result from unconditional ICAPM. Standard errors are in parenthesis.

A. Sample moments	World	Emerging Asia	Pacific	Emerging Europe	Europe	Latin America	North America
Mean	0.0005	0.0014	0.0003	0.0009	0.0005	0.0012	0.0006
Maximum	0.0902	0.2030	0.1400	0.2066	0.1051	0.1077	0.1027
Minimum	-0.1751	-0.1874	-0.1725	-0.3762	-0.1558	-0.4041	-0.1698
Std. Dev.	0.0245	0.0326	0.0262	0.0455	0.0301	0.0391	0.0243
Skewness	-0.8941	-0.4061	-0.5332	-1.3471	-0.6803	-1.7959	-0.7193
Kurtosis	8.0501	7.3696	6.5301	12.3317	6.0909	17.7782	8.2908
Jarque-Bera	0.0005	0.0014	0.0003	0.0009	0.0005	0.0012	0.0006
B. Unit root test							
ADF (Log returns)	-43.23*	-45.01*	-61.09*	-53.42*	-45.84*	-43.77*	-56.41*
C. Correlation matrix							
Emerging Asia	0.7430						
Pacific	0.7593	0.7533					
Emerging Europe	0.7523	0.6876	0.6376				
Europe	0.9372	0.6825	0.6929	0.7343			
Latin America	0.7996	0.6849	0.6240	0.7816	0.7447		
North America	0.9457	0.5926	0.5918	0.6318	0.8121	0.7166	
D. ICAPM - OLS estimation							
Alpha		0.0008	-0.0001	0.0001	-0.0001	0.0005	0.0001
		(0.0008)	(0.0006)	(0.0011)	(0.0004)	(0.0008)	(0.0003)
Beta		0.9885	0.8137	1.3999	1.1544	1.2789	0.9389
		(0.0319)	(0.0250)	(0.0439)	(0.0154)	(0.0344)	(0.0115)
AIC		-4.8105	-5.2992	-4.1711	-6.2676	-4.6588	-6.8412

Table 3. State-dependent equity model parameters estimations

Panel A, reports the results for equation (1) where μ_1 and μ_2 are the conditional mean (expected returns) and σ_1 and σ_2 are the conditional variances (volatility) for the world equity returns in State 1 and 2 respectively. P_{11} and P_{22} are transitional probabilities to stay in the same state and the expected duration for the world market returns. Panel B reports the parameters estimation for regional returns from equation (7) and (8). All the parameters are presented on weekly basis. Standard errors are in parenthesis.

A.	Estimates	μ_1	μ_2	σ_1	σ_2	P_{11}	P_{22}
		0.0035	-0.0050	0.0137	0.0363	0.9624	0.9306
		(0.0006)	(0.0022)				
	Expected duration					27	14

B.		Emerging Asia	Pacific	Emerging Europe	Europe	Latin America	North America
	State 1						
	Alpha	0.0013	0.0003	0.0007	-0.0001	-0.0001	0.0000
		(0.0008)	(0.0006)	(0.0011)	(0.0004)	(0.0008)	(0.0003)
	Beta	1.0986	0.9674	1.3347	1.1479	1.3834	0.8963
		(0.0566)	(0.0450)	(0.0777)	(0.0275)	(0.0581)	(0.0193)
	Idiosyncratic volatility	0.0166	0.0175	0.0140	0.0241	0.0085	0.0180
		0.0013	0.0003	0.0007	-0.0001	-0.0001	0.0000
	AIC	-5.2446	-5.7020	-4.6089	-6.6856	-5.1895	-7.3906
	State 2						
	Alpha	-0.0009	-0.0021	-0.0005	-0.0001	0.0009	0.0006
		(0.0017)	(0.0013)	(0.0024)	(0.0008)	(0.0019)	(0.0006)
	Beta	0.9490	0.7611	1.4128	1.1562	1.2541	0.9533
		(0.0467)	(0.0357)	(0.0648)	(0.0226)	(0.0523)	(0.0176)
	Idiosyncratic volatility	0.0277	0.0280	0.0214	0.0388	0.0135	0.0313
	AIC	-4.3062	-4.8415	-3.6529	-5.7625	-4.0822	-6.2590

Table 4. State-dependent equity model estimation results

The state-dependant excess returns, Panel A, are from equation (9) and covariance of excess returns Panel B, are driven from estimates of equation (10) - (13). The correlations in Panel B are shaded. In Panel C, we computed the mean-variance efficient tangency portfolio weights by using an interest rate of 1.87 per cent, which is the average 3-months T-bill rate over the sample period. The MSCI average shows the average MSCI world index weight for each region across sample. All the numbers are annualized.

		Emerging Asia	Pacific	Emerging Europe	Europe	Latin America	North America
A.	State-dependant excess returns						
	State 1	0.2502	0.1767	0.2599	0.1857	0.2243	0.1491
	State 2	-0.2649	-0.2851	-0.3511	-0.2710	-0.2428	-0.1874
B.	State-dependent covariance and correlations						
	State 1						
	Emerging Asia	0.0306	0.6081	0.5556	0.5648	0.5720	0.4422
	Pacific	0.0160	0.0209	0.4511	0.5710	0.4551	0.4515
	Emerging Europe	0.0235	0.0163	0.0539	0.6069	0.6572	0.4153
	Europe	0.0145	0.0121	0.0208	0.0199	0.6556	0.6488
	Emerging Latin America	0.0208	0.0141	0.0318	0.0190	0.0402	0.5958
	North America	0.0090	0.0075	0.0118	0.0106	0.0136	0.0119
	State 2						
	Emerging Asia	0.0982	0.8209	0.7394	0.7238	0.7294	0.6432
	Pacific	0.0635	0.0610	0.7182	0.7413	0.6976	0.6438
	Emerging Europe	0.1052	0.0805	0.2062	0.7742	0.8283	0.6978
	Europe	0.0707	0.0570	0.1095	0.0971	0.7728	0.8541
	Emerging Latin America	0.0892	0.0672	0.1468	0.0939	0.1523	0.7550
	North America	0.0515	0.0406	0.0810	0.0680	0.0753	0.0654
C.	Tangency portfolio weight						
	MSCI average market cap	0.0732	0.1151	0.0073	0.2076	0.0140	0.5828
C1.	No constraints						
	State 1	0.3285	0.0995	0.0656	0.1053	-0.0613	0.4624
	State 2	-0.0807	-0.7620	0.0061	0.8773	0.0192	0.9400
	Unconditional	0.0400	-0.3058	0.1654	0.7310	0.1844	0.1849
C2.	Short-sale constraint						
	State 1	0.3202	0.1035	0.0471	0.0907	0.0000	0.4384
	State 2	0.0000	0.0000	0.0000	0.0000	0.0600	0.9400
	Unconditional	0.0000	0.0000	0.1220	0.5675	0.1507	0.1599

Table 5. In-sample and out-of-sample performance of all equity portfolios

Mean, standard deviation and Sharpe ratio, equation (14), of both in-sample and out-of-sample returns based on state-dependent model and static strategy. All the returns and standard deviations are annualized and reported in percentages.

		No constraints			Short constraint		
		World	Static	State-dependent	World	Static	State-dependent
A.	In-sample performance 2001-2008						
	Mean returns (%)	-0.60	-1.18	0.52	-0.60	-1.12	2.67
	Standard deviation (%)	18.11	24.08	20.81	18.11	21.41	16.48
	Sharpe ratio	-0.14	-0.13	-0.06	-0.14	-0.14	0.05
B.	Out-of-sample performance 2009-2015						
	Mean returns (%)	10.78	14.10	15.70	10.78	12.64	13.59
	Standard deviation (%)	17.01	24.08	19.64	17.01	21.41	15.82
	Sharpe ratio	0.52	0.51	0.70	0.52	0.50	0.74

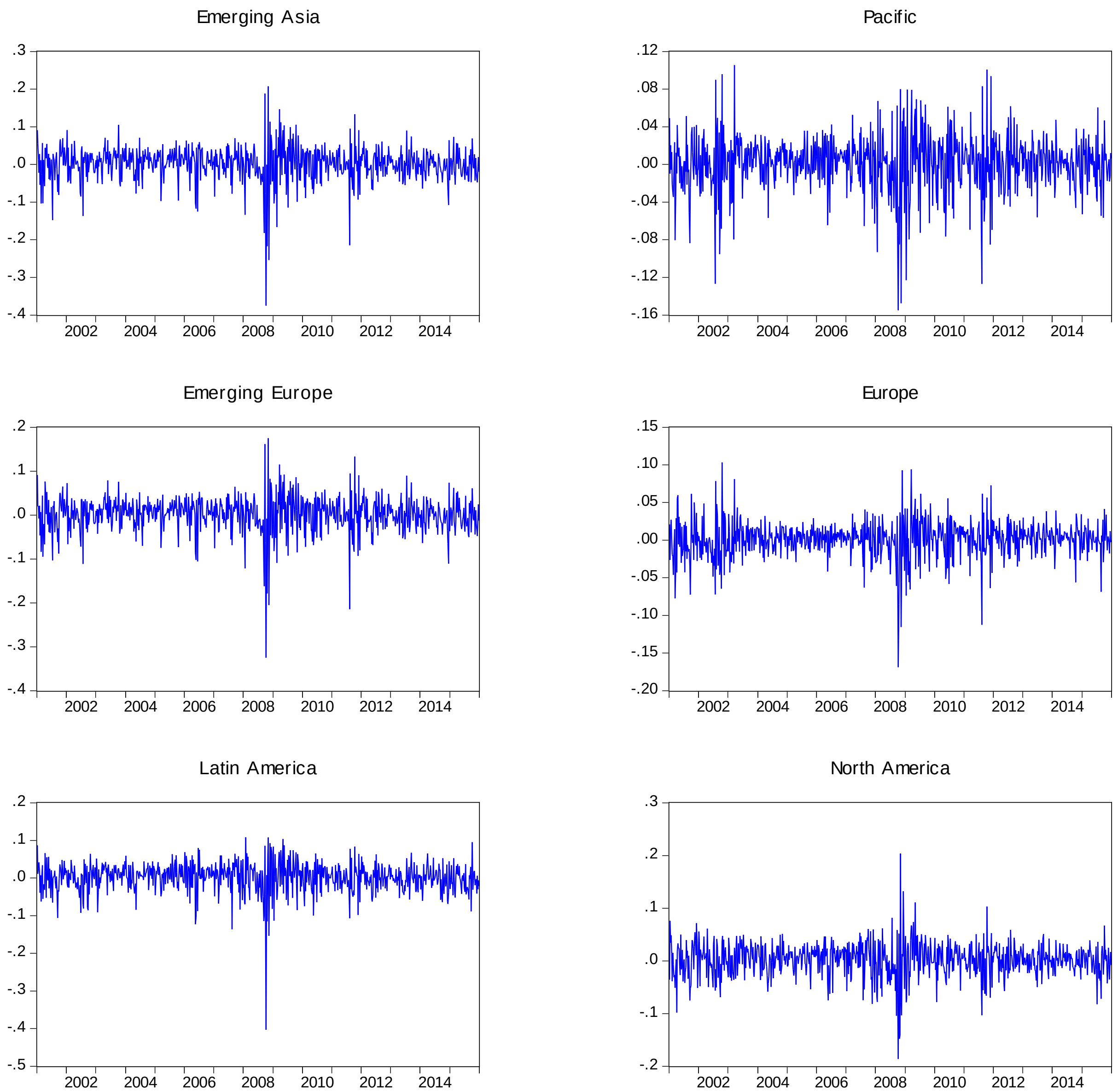
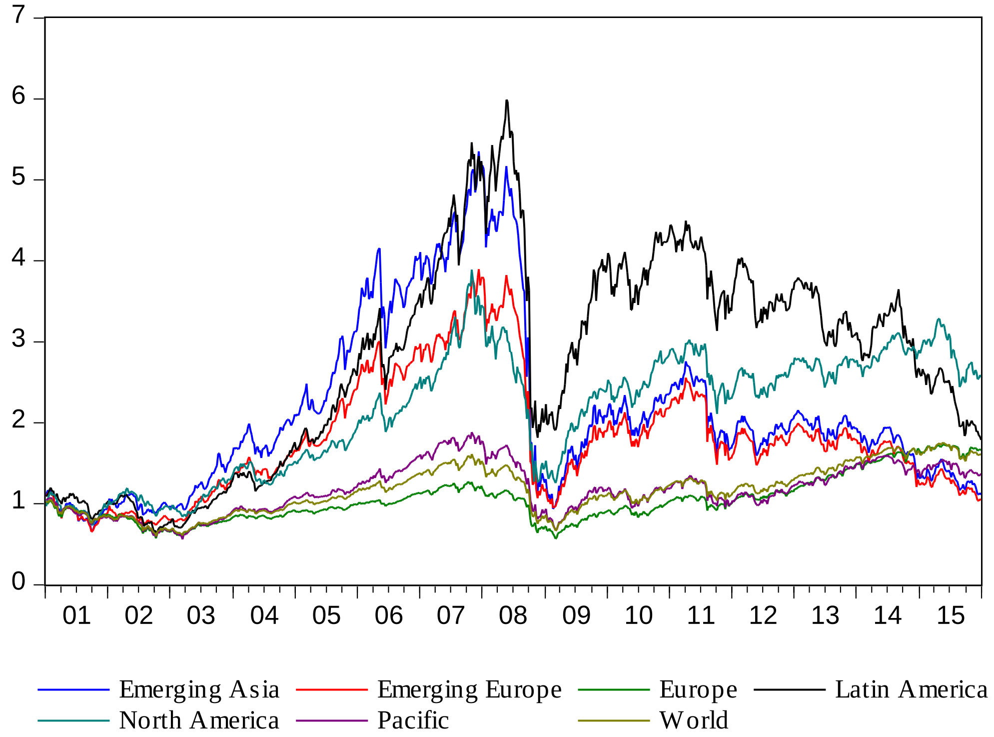


Figure 1: Plot of logarithmic excess returns, showing volatility clustering for developed and emerging equity regions.

A. Cumulated returns of $1 invested in the six regions January 2001- December 2015



B. Ex-ante and ex-post state probabilities of being in normal state (State 1)

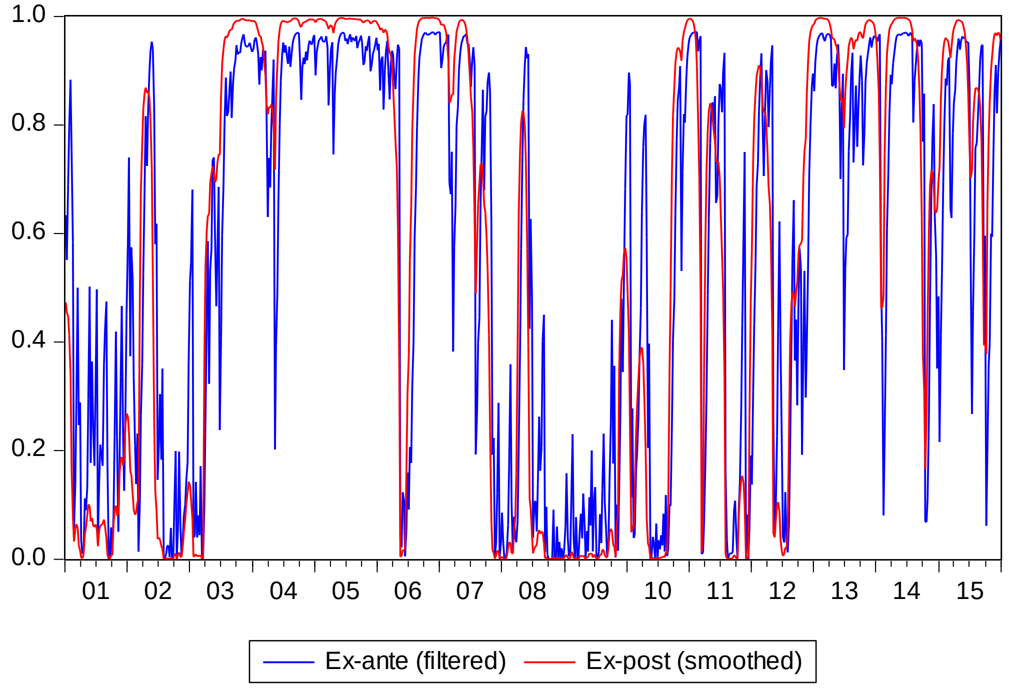


Figure 2: Cumulated historical returns and ex-ante and ex-post probabilities

Panel A shows the total returns of $1 invested in the six regions over the sample period. Panel B shows the ex-ante (filtered) and ex-post state probabilities. The ex-ante probability is the probability, given current information, and the ex-post probability is the probability, given all of the information present in data sample, that the state next week will be the world low-variance, the normal state.

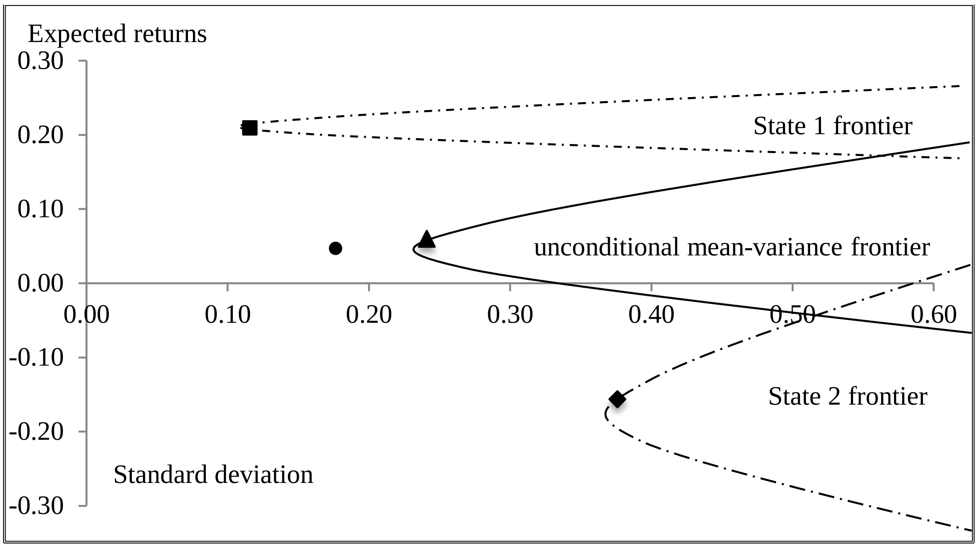
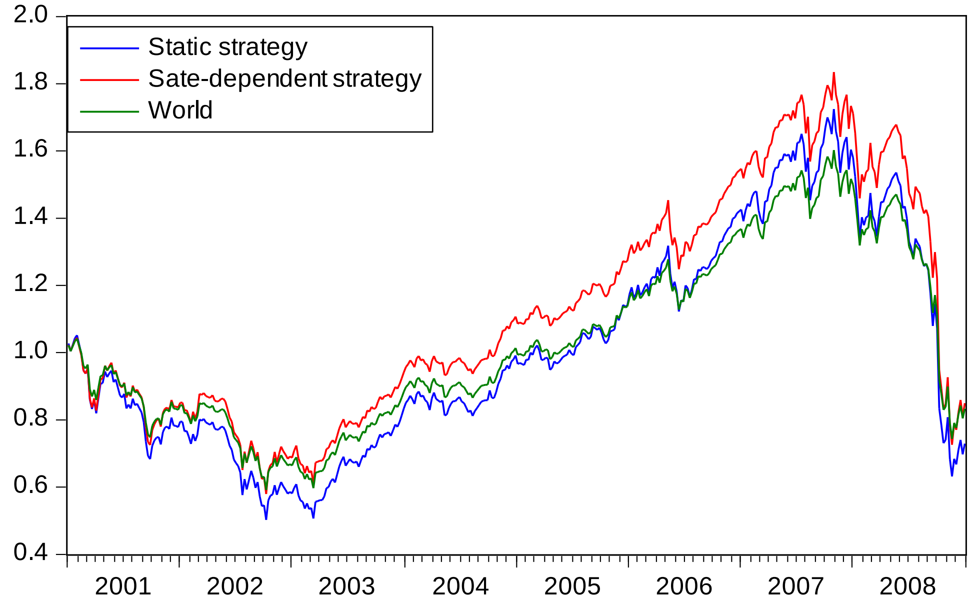


Figure 3: Mean-standard deviation frontier, 2001-2015

● World market portfolio (Sharpe ratio = 0.1611), ■ MVE State 1 (Sharpe ratio = 1.6550), ♦ MVE State 2 (Sharpe ratio = -0.4693), ▲ MVE Unconditional (Sharpe ratio = 0.1718)

The expected returns are estimated from ICAPM in equation (3) and SD-ICAPM in equation (13) by using an average interest rate of 1.87 per cent. All the mean and standard deviation are annualized.

A. In-sample wealth for various strategies, January 2001- December 2008



B. Out-of-sample wealth for various strategies, January 2009- December 2015

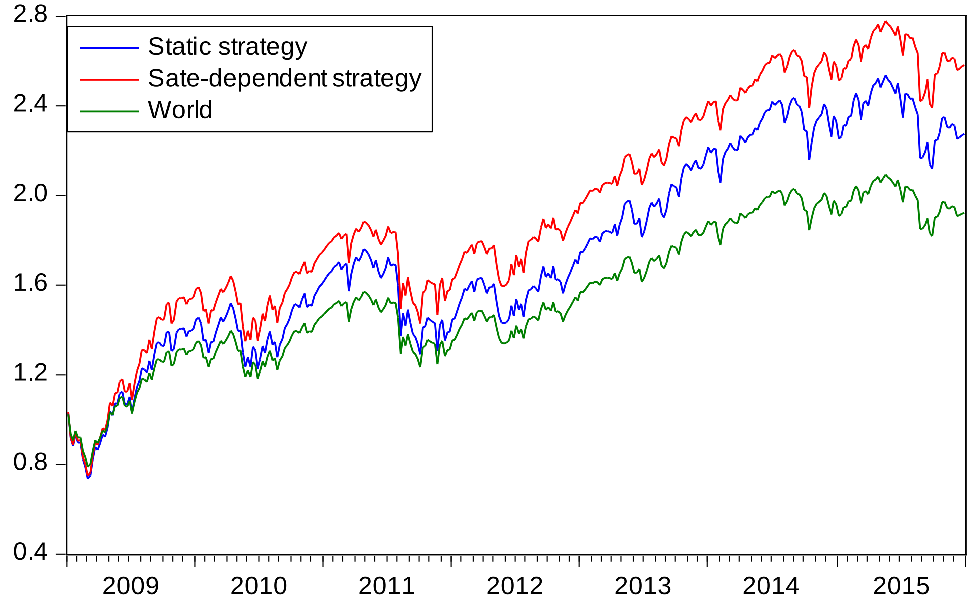


Figure 4: In-sample (Panel A) and out-of-sample (Panel B) wealth for all equity model

Panel A shows the in-sample wealth for the value of $1 invested at January 2001 till December 2008 for the state-dependent strategy asset allocation model for the six regions with no constraint, compared with a static mean-variance strategy and the returns for the world markets. Panel B shows the out-of--sample wealth for the value of $1 invested at January 2009 till December 2015 for the state-dependent strategy asset allocation model for the six regions with no constraint, compared with a static mean-variance strategy and the returns for the world markets.

Appendix A. Markov Chains

Consider s_t as a random variable that could get the value of 1, 2… N. The assumption is that the probability that s_t takes a particular value of j only depends on the previous value s_{t-1} so that:

$$P\{s_t = j|s_{t-1} = i, s_{t-2} = 2, \ldots\} = P\{s_t = j|s_{t-1} = i\} = p_{ij} \quad (1.a)$$

This process is so-called an N-state Markov chain with transition probability defines as p_{ij}. Where the transition probability (p_{ij}) takes the probability that being in state j depends on state i (where $p_{i1} + p_{i2} + \cdots + p_{iN} = 1$) (Hamilton 1994, p.678).

Now consider the case of two states of Markov chain in level where $s_t = 1$ and $s_t = 2$ defines as the unobserved states with low variance and high variance respectively where the transition probability between the states is followed by Markov chain of order one:

$$\begin{cases} P\{s_t = 1|s_{t-1} = 1\} = p_{11} \\ P\{s_t = 1|s_{t-1} = 2\} = 1 - p_{11} \\ P\{s_t = 2|s_{t-1} = 2\} = p_{22} \\ P\{s_t = 2|s_{t-1} = 1\} = 1 - p_{22} \end{cases} \quad (2.a)$$

It is sometime more suitable to write the transition probability in the form of matrix. Where in the case of two states, the transition probability takes the following form:

$$P\{s_t = j|s_{t-1} = i\} = \begin{bmatrix} p_{i1} \\ p_{i2} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} \quad (3.a)$$

Where : p_{ij}= p(S_t=j | S_{t-1}=i)

The solution to find the unconditional probability of each state is to $|P - \lambda I_N| = 0$ (Where I_N is 2×2 identity matrix in the case of two states). Following the process given by (Hamilton 1994, p. 683), the unconditional probability that the process is in state 1 at any given time is:

$$P\{s_t = 1\} = \frac{1-p_{22}}{2-p_{11}-p_{22}} \quad (4.a)$$

Similarly we could obtain the same value for state 2:

$$P\{s_t = 2\} = \frac{1-p_{11}}{2-p_{11}-p_{22}} \quad (5.a)$$

To estimate the expected duration of being in each state, the occupation time is calculated as follows:

$$\sum_{k=1}^{T} k\, {p_{11}}^{k-1}(1 - p_{11}) = (1 - p_{11})^{-1} \quad (6.a)$$

$$\sum_{k=1}^{T} k\, {p_{22}}^{k-1}(1 - p_{22}) = (1 - p_{22})^{-1} \quad (7.a)$$

The significance of this application is that the occupation time of a typical event can be calculated from the estimation of maximum likelihood parameters and then compare this with historical average duration of the event (Hamilton, 1989).

Appendix B. Expectation Maximisation Algorithm

In conducting MS-ICAPM in this study, the parameters estimation is carried out by adopting expectation maximisation (EM) algorithm of Hamilton (1990). The estimation procedure is outlined below:

The purpose is to perform a model with two states as the outcome of an unobserved two-state Markov chain where s_t is independent from ε_t (residuals) in both subsample. Now consider $r_{i,t}$ as observed variable.

If the process follows by state $s_t = j$ at time t then the conditional density of r_{it} will take the form of:

$$f\,(r_{it} \mid s_t = j, r_{mt}\,;\theta) \qquad (1.b)$$

where θ is defined as a set of parameters $(\theta \equiv \alpha_1\,, \alpha_2\,, \beta_1, \beta_2, \sigma_1^2\,, \sigma_2^2)'$ determining the conditional density. If the process is in state 1, the observed variable r_{it} is drawn from a $N(\mu_1, \delta_1^2)$ distribution. Alternatively, the process is in state 2 then r_{it} has been drawn from a $N(\mu_2, \delta_2^2)$ distribution. Therefore, the density of r_{it} conditional on the random variable s_t=j is equation (1.b).

In this case θ consists of α_1 , α_2 , $\beta_1, \beta_2, \sigma_1^2$ and σ_2^2 and the two densities function considering N=2 are:

$$\eta_t = \begin{bmatrix} f\,(r_{it} \mid s_t = 1,\ r_{mt}\ ;\theta) \\ f\,(r_{it} \mid s_t = 2,\ r_{mt}\ ;\theta) \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\{\frac{-(r_{it} - \alpha_1 - \beta_1\, r_{mt})^2}{2\sigma_1^2}\} \\ \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\{\frac{-(r_{it} - \alpha_2 - \beta_1\, r_{mt})^2}{2\sigma_2^2}\} \end{bmatrix} \qquad (2.b)$$

We assume that the conditional density, function (2.b), relies only on the previous state (smoothed probability).

Then the log likelihood function can be defined by getting log of equation (1.b):

$$\log\{f(r_{it} \mid s_t{=}j, r_{mt}\,;\theta)\} = \log f\,(r_{i1};\theta) + \sum_{t=2}^{T} \log f\,(r_{it} \mid r_{mt}\,;\ \theta) \qquad (3.b)$$

That is given the numerical[8] ability to the equation (1.b) to estimate the log likelihood function regarding to the unknown parameters $(\alpha_1\,,\alpha_2\,, \beta_1, \beta_2\,, \sigma_1^2\,, \sigma_2^2)$ (Hamilton 1994, p. 133)

[8] In addition to unknown parameters this model involves unobserved latent variable (Markov model). Therefore, expected-maximization algorithm will be performed.

Appendix C. Filtered and Smoothed Probabilities

Following Hamilton (1994) we can derive the unconditional probability the process will be in state 1 at any given time is:

$$p(s_t = 1) = \frac{1-q}{(1-p)+(1-q)} \tag{1.c}$$

Obviously, the unconditional probability that the process will be in state 2 would be 1 minus p.

Now the joint distribution of the two probabilities is:

$$\begin{aligned} p(s_t, s_{t-1}|Y_{t-1}; X_t) &= p(s_t|s_{t-1}, Y_{t-1}; X_{t-1}) \times p(s_{t-1}|Y_{t-1}; X_{t-1}) \\ &= p(s_t|s_{t-1}) \times p(s_{t-1}|Y_{t-1}; X_{t-1}) \end{aligned} \tag{2.c}$$

The first line in equation (2.c) is given by Bayes Theorem and the second is given by independent principle of Markov chain. The transition probability $p(s_t|s_{t-1})$ and the filter probability $p(s_{t-1}|Y_{t-1}; X_{t-1})$, are known at time t, we can compute $p(s_t, s_{t-1}|Y_{t-1}; X_t)$.

Summarizing s_{t-1} from equation (2.c), we get the conditional probability of s_t.

$$p(s_t|Y_{t-1}; X_t) = \sum_{s_{t-1}}^{2} p(s_t, s_{t-1}|Y_{t-1}; X_t) \tag{3.c}$$

The joint distribution of y_t and s_t at time t can be computed:

$$p(y_t,\ s_t|Y_{t-1}; X_t) = f\ (y_t|\ s_t, Y_{t-1}; X_t)p(s_t|Y_{t-1}; X_t) \tag{4.c}$$

The first part on the right hand side of equation (4.c) is the likelihood function and the second part is from equation (3.c) so that equation (4.c) can also computed. As a result the filter probability, the prevailing state at each point in time is given by:

$$p(s_t|Y_t; X_t) = \frac{p(y_t, s_t|Y_{t-1};X_t)}{p(y_t|Y_{t-1};X_t)} = \frac{f(y_t|\ s_t,Y_{t-1};X_t)p(s_t|Y_{t-1};X_t)}{\sum_{s_{t-1}}^{2} f(y_t|\ s_t,Y_{t-1};X_t)p(s_t|Y_{t-1};X_t)} \tag{5.c}$$

The filtering probability, ex-ante, is the probability given past and current information up to time t. Alternatively, we can use all the information available in the sample period, ex-post, to derive about historical state that the process was in at time t. It is therefore more intuitive to employ all the information available up to time T rather than t.

Similarly, the smoothed probability, given all the information available up to time T is as followed:

$$p(s_t|Y_T; X_T) = \sum_{s_{T-1}}^{2} p(s_T, s_t|Y_T; X_T) \qquad t = 1,2, \dots, T \tag{6.c}$$