Left-Tail Momentum: Limited Attention of Individual Investors and Expected Equity Returns*

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ABSTRACT

This paper documents a significantly negative cross-sectional relation between left-tail risk and future returns on individual stocks trading in the U.S. and international countries. We find that the left-tail risk anomaly is stronger for stocks that are more likely to be held by retail investors and that receive less investor attention, underscoring the importance of investor clientele and inattention mechanisms. We also provide an alternative explanation showing that individual investors underestimate the persistence in left-tail risk and overprice stocks with large recent losses. Thus, low returns in the left-tail of the distribution persist into the future causing left-tail return momentum.

Keywords: left-tail risk, momentum, equity returns, retail investors, investor inattention

JEL Codes: G10, G11, G12.

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1. INTRODUCTION

Although the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) has been the dominant paradigm in the asset pricing literature, the question of whether left-tail risk plays a special role in determining the cross-section of expected returns has also received attention of financial economists since decades. The concept of safety-first investors introduced by Roy (1952), the emphasis made by Markowitz (1959) on semi-variance as a risk metric and the efforts of authors such as Arzac and Bawa (1977) and Bawa and Lindenberg (1977) to incorporate lower partial moments of empirical return distributions in asset pricing models are milestones in the advancement of this line of research. The prospect theory of Kahneman and Tversky (1979) also contributes to this literature with its central concept of loss aversion hinged on the idea that investors make decisions based on the losses and gains on their portfolios rather than the expected outcomes and they have asymmetric value functions with different slopes and curvatures for losses and gains.

Ang, Chen, and Xing (2006), Kelly and Jiang (2014), Van Oordt and Zhou (2016), Chabi-Yo, Ruenzi, and Weigert (2017), and Lee and Yang (2017) are some studies in the literature that exclusively focus on the concept of systematic tail risk (or left-tail beta). These studies focus on individual stock exposure to extreme market downturns and test whether left-tail beta predicts cross-sectional variation in future stock returns. They do not examine the magnitude or probability of large negative losses realized on the left-tail of the return distribution, proxied by value-at-risk (VaR) and expected shortfall (ES). We aim to fill this gap by providing a comprehensive investigation of the relation between the left-tail risk proxies (VaR, ES) and the cross-section of equity returns.

The positive trade-off between risk and expected return is one of the most fundamental concepts in financial economics. Risk-averse investors demand higher compensation in the form of higher expected return to hold financial securities with higher risk and uncertainty. Translated to the left-tail risk framework, investors would be expected to pay lower prices for stocks with higher left-tail risk for accepting a higher probability and magnitude of large losses and, consequently, they would expect to earn higher returns from stocks with higher left-tail risk. We test this conjecture and reach a conflicting conclusion. We estimate left-tail risk using two standard metrics; value-at-risk and expected shortfall which measure, respectively, a decrease in an asset's value at a certain probability and the average magnitude of the losses conditional that the loss is lower than a certain threshold. Univariate portfolio analyses show that stocks with high (low) left-tail risk have low (high) future returns. This finding contradicts with the well-celebrated positive risk-return trade-off. The left-tail risk anomaly continues to persist in bivariate portfolio-level analyses and multivariate cross-sectional regressions after controlling for idiosyncratic volatility and various other firm characteristics and risk factors that are known to predict the cross-section of equity returns. Moreover, the negative relation between left-tail risk and expected returns is robust to alternative measures of left-tail risk widely used in the risk management literature. We also provide evidence outside of the U.S. equity market and test whether the anomaly is observed in an international setting. We again find that stocks with higher left-tail risk earn significantly lower expected returns in various country groupings.

We explain the anomalous negative relation between left-tail risk and expected returns by focusing on the cross-sectional persistence of left-tail risk. We first establish that left-tail risk is a highly persistent equity characteristic. If this persistence is underestimated by investors, they are likely to overprice securities that experience large losses recently and they get negatively surprised when these large losses drift into the future. In other words, investors anticipate short-term mean-reversion in left-tail risk and extrapolate past left-tail risk too soon into the future or not at all such that they expect stocks with high past left-tail risk to have a lower future left-tail risk and vice versa. Our empirical results are consistent with this explanation and suggest that the left-tail risk anomaly is stronger for those equities that have experienced large daily losses recently. Moreover, the anomaly is strongest for those stocks with large daily losses both during the portfolio formation month and the preceding month, indicating that investors are overconfident in their consideration of the mean-reversion in left-tail risk. Next, motivated by the idea that retail investors would be more likely to underestimate the persistence in left-tail risk, we test and find that individual (institutional) investors are more (less) active in high left-tail risk stocks and the magnitude of the left-tail risk anomaly is stronger for those equities with lower institutional ownership. Finally, we test a complementary hypothesis that the limited attention of retail investors can provide a channel through which stock prices underreact to the information embedded in negative price shocks for stocks with high left-tail risk. Specifically, we show that the left-tail risk anomaly is more pronounced for stocks that receive less investor attention and that are more likely to be held by retail investors, indicating the importance of the investor inattention mechanism and investor clientele effect. These findings provide a behavioral explanation for the anomaly, which we term as the left-tail return momentum, the phenomenon of large losses to persist into the future. We also show that the left-tail return momentum cannot be explained by long-established low-risk anomalies (i.e., the idiosyncratic volatility puzzle, betting-against-beta) or demand for lottery-like stocks. Finally, we present evidence that the negative relation between left-tail risk and expected equity returns is not driven by earnings announcement returns.

The remainder of the paper is organized as follows. Section 2 describes the data. Section

3 discusses the empirical methodology. Section 4 presents the empirical results. Section 5 provides behavioral explanations for the core findings. Section 6 tests alternative explanations of left-tail momentum. Section 7 presents a battery of robustness tests. Section 8 provides international evidence for left-tail momentum. Section 9 concludes.

2. DATA

Daily and monthly equity data for returns, shares outstanding and volume of shares are obtained from the Center for Research in Security Prices (CRSP). Balance sheet data come from Compustat. The risk-free rate used to calculate excess returns is the interest rate on one-month U.S. T-bills and is available at the Federal Reserve database. Monthly excess returns on the market (MKT), size (SMB), value (HML), and momentum (MOM) factors of Fama and French (1993) and Carhart (1997) are obtained from Kenneth French's online data library. Monthly excess returns on the profitability (RMW) and investment (CMA) factors of Fama and French (2015) are also obtained from Kenneth French's data library. Monthly returns on the liquidity risk factor (LIQ) of Pastor and Stambaugh (2003) are from Lubos Pastor's website. Institutional holdings data come from Thompson-Reuters' Institutional Holding (13F) database. Analyst coverage data or the number of analysts following each stock is obtained from IBES.

The sample used throughout this study covers the period from 1962 to 2014. Each month, we include all U.S.-based common stocks trading on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and NASDAQ with an end-of-month stock price of $5 or more in our sample to make sure that the results are not driven by small and illiquid stocks.[1] The final sample contains 3,038 equity observations per month. Our univariate tests which examine the relation between left-tail risk and expected equity returns utilize a total of about 1.9 million firm-month observations.

3. EMPIRICAL METHODOLOGY

Left-tail risk is the key variable of interest in our analyses. The first left-tail risk metric is value-at-risk (VaR) that measures how much the value of an investment can decline over a given time period with a given probability. For example, if the given time period is one month and the given probability is 1%, then the VaR measure would be an estimate of a decrease in the investment's value that could occur with a 1% probability over the next month. To put it differently, losses greater than the VaR measure should occur less than 1% of the time during the next month. In our empirical analyses, we use the lower tail of the actual empirical distribution to

[1] As will be discussed in Section 7.4, our results are similar when use all stocks trading at NYSE, AMEX, and NASDAQ (i.e., full CRSP universe).

calculate a non-parametric measure of value-at-risk following Bali, Demirtas and Levy (2009). Specifically, VaR is calculated as the 1st (VaR1) or 5th (VaR5) percentile of the daily returns over the past one year (250 trading days) as of the end of month *t* with the restriction that at least 200 non-missing return observations should exist in the past year. Since the maximum likely loss values obtained using this method are negative, we multiply the 1st and 5th percentile values by -1 so that higher values of VaR correspond to higher levels of left-tail risk.

Alternative measures of left-tail risk can be obtained from the left tail of the empirical distribution of equity returns. Expected shortfall (ES), originally proposed by Artzner, Delbean, Eber and Heath (1999), is one of the most popular measures of left-tail risk among financial institutions and regulators. ES is defined as the conditional expectation of a loss given that the loss is beyond the VaR threshold. For example, if the loss probability level for the VaR measure is 1%, ES can be interpreted as the average loss in the worst 1% of cases. We consider the 1% and 5% expected shortfall as alternative proxies for left-tail risk and define ES as the average of the observations that are less than or equal to the 1st (ES1) or 5th (ES5) percentile of the daily returns for each stock during the past year (250 trading days) as of the end of month *t* with the restriction that at least 200 non-missing return observations should exist in the past year. In a similar fashion to VaR, we multiply these average large losses by -1 so that higher values of ES correspond to higher levels of left-tail risk. We use these risk metrics calculated at the end of month *t* to explain the cross-section of stock returns observed during month *t*+1 (and longer horizons) so that there is no look-ahead bias in our empirical analyses.

To test the relation between left-tail risk and expected stock returns, we first conduct a univariate portfolio analysis by sorting individual stocks based on various left-tail risk metrics and compare the relative performances of the highest and the lowest left-tail risk portfolios. Specifically, decile portfolios are formed every month between July 1962 to December 2014 by sorting equities based on their left-tail risk measures, where decile 1 contains stocks with the lowest left-tail risk and decile 10 contains stocks with the highest left-tail risk. We also check whether the excess return differences between the extreme left-tail risk deciles can be explained by standard asset pricing models. In our analysis, we use two alternative five-factor models to calculate abnormal returns (or alphas). The first model, abbreviated as the FFCPS model, incorporates the standard market, size, value and momentum factors of Fama-French (1993) and Carhart (1997) and augments these factors by the liquidity risk factor of Pastor and Stambaugh (2003). The second factor model, abbreviated as the FF5 model, has been recently proposed by Fama and French (2015) and adds the profitability and investment factors to the market, size and value factors of Fama and French (1993).[2] To determine whether the excess return differences between extreme left-tail risk deciles survive after accounting for these factor models, we calculate the monthly returns to the zero-cost portfolio that buys the equities with the highest left-tail risk and sells the equities with the lowest left-tail risk, regress this time-series of returns on the five factors incorporated into both factor models, and test whether the intercept terms obtained from these regressions are significant.

A significant relation between left-tail risk and expected stock returns can be explained by the correlation between left-tail risk and another firm-specific attribute which is known to explain the cross-section of equity returns. Alternatively, the lack of a relation between left-tail risk and expected stock returns can be attributed to the possibility that left-tail risk and another firm-specific attribute which is correlated with left-tail risk both impact expected returns but the effects are in the opposite direction and subsume each other. Thus, we also use dependent and independent double sorts on various firm-specific attributes and left-tail risk to have a deeper understanding of the trade-off between left-tail risk and expected returns. For dependent (conditional) bivariate sorts, each month, we sort stocks into decile portfolios based on various firm-specific characteristics. Next, we sort stocks into additional deciles based on various left-tail risk metrics in each firm-specific characteristic decile. For each first-stage sorting variable, this bivariate analysis provides 100 conditionally double-sorted portfolios. Portfolio 1 is the combined portfolio of stocks with the lowest left-tail risk in each firm-specific characteristic decile, whereas portfolio 10 is the combined portfolio of stocks with the highest left-tail risk in each firm-specific characteristic decile. For bivariate independent sorts, each month, all stocks are grouped into decile portfolios based on independent ascending sorts of both a firm characteristic and a left-tail risk measure. The intersections of each of the decile groups are used to form 100 portfolios. We then investigate whether the abnormal return difference between the extreme left-tail risk deciles is significant after controlling for firm characteristics and risk factors in bivariate portfolios.

We also run firm-level multivariate cross-sectional regressions to test the robustness of our findings from univariate and bivariate portfolio-level analyses. The regression procedure follows Fama and MacBeth (1973) in which one-month-ahead excess equity returns are regressed on left-tail risk measures and various control variables in each month of the sample period. Then, the time-series of slope coefficients for the independent variables are averaged and tests of statistical significance are performed using the Newey-West (1987) standard errors with optimal number of lags to take autocorrelation and heteroscedasticity into account. Asparouhova, Bessembinder and

[2] As will be discussed in Section 6.1, we run a battery of robustness checks using alternative factor models, such as the Q-factor model of Hou, Xue, and Zhang (2015) as well as the extensions of FFCPS, FF5, and Q factor models with the betting-against-beta, idiosyncratic volatility, and lottery demand factors.

Kalcheva (2013) point out that microstructure noise in security prices biases the results of empirical asset pricing specifications. Thus, they suggest performing each monthly cross-sectional regression using a weighted-least squares (WLS) specification where each return is weighted by the observed gross return on the same security in the prior period. In the results section, we present findings for both OLS and WLS specifications.

Our sorting variables in the bivariate portfolio-level analysis and the control variables in the multivariate regression analysis fall into two main categories. First, we use several firm-specific characteristics that have been shown to affect equity returns by the prior literature. Fama and French (1992) propose that the size and the book-to-market equity ratio of a firm have a significant relation with expected returns. Therefore, we calculate the natural logarithm of each stock's market capitalization and its book-to-market equity (BM) ratio at the end of each month and use them to predict one-month-ahead excess returns. Next, to control for the medium-term momentum effect of Jegadeesh and Titman (1993), we measure the momentum return (MOM) of each stock as its cumulative return during the past 11 months after skipping one month. We also control for the short-term reversal (SR) effect of Jegadeesh (1990) by controlling for the one-month lagged stock return. Amihud (2002) shows that there exists an expected return premium to stocks that are more illiquid, thus, we calculate Amihud illiquidity measure, defined as the absolute daily return divided by the daily dollar trading volume averaged over all trading days in each month for each stock. Next, following Ang, Hodrick, Xing and Zhang (2006) which uncover a negative relation between idiosyncratic volatility (IVOL) and expected equity returns, we calculate idiosyncratic volatility as the standard deviation of the residuals from a regression of excess stock returns on the excess market return in each month. Finally, motivated by Bali, Cakici, and Whitelaw (2011) who identify a role for lottery demand in asset pricing, we calculate MAX as the average of the five highest daily returns of each stock in each month.[3,4] We require that at least 15 non-missing return observations should exist in a month when we calculate IVOL and MAX.

We also control for several different measures of risk. Each of these measures is calculated at the end of month t using daily return data from the one-year period covering months t-11 through t, inclusive. We also require a minimum of 200 valid daily equity return observations for all risk measures. First, we calculate the standard market beta as the ratio of the covariance

[3] There is theoretical and empirical evidence that investors have a preference for lottery-like assets, i.e., assets with a relatively small probability of a large payoff (e.g., Barberis and Huang (2008), Kumar (2009), Bali et al. (2011, 2017), Hwang and Green (2012), Han and Kumar (2013), Barberis, Mukherjee, and Wang (2016), and Kumar, Page and Spalt (2016)).

[4] We also measure lottery demand as the maximum daily return of each stock in each month and find that the results from this alternative proxy for lottery demand are very similar to those reported in our tables.

between daily excess returns of a stock and daily excess market returns to the variance of daily excess market returns during the past year. Second, we calculate the downside beta of each stock as the ratio of the covariance between daily excess returns of a stock and daily excess market returns to the variance of daily excess market returns on the days that the market's excess return is less than the average market excess return during the past year, following Bawa and Lindenberg (1977) and Ang, Chen, and Xing (2006). Third, co-skewness, shown by Harvey and Siddique (2000) to be negatively related to expected equity returns, is calculated as the slope coefficient of the squared excess market return term from a regression of the daily excess returns of a stock on the daily excess market returns and the squared daily excess market returns in the past year.

4. EMPIRICAL RESULTS

4.1 Descriptive Statistics

Table 1 presents descriptive statistics along with correlation measures for the variables used in this study. Statistics in Panel A of Table 1 are computed as the time-series averages of the cross-sectional values. We present the mean, standard deviation, 25^{th} percentile, median, 75^{th} percentile, minimum, maximum, skewness and kurtosis statistics for left-tail risk metrics and other firm-specific attributes. VaR1 has a mean and median equal to 6%, implying that there is only 1% probability that the average daily loss that a typical firm experiences in the prior year exceeds 6%. The minimum value of VaR1 is 1% and the maximum value is 26%, indicating that there has been a sample firm for which the 1^{st} percentile of daily returns during the past year corresponds to -26%. VaR5 has a monthly mean and median equal to 4% and 3%, respectively, which are mechanically less than those for VaR1 since the latter metric extracts information from further on the left tail of the empirical return distribution. VaR1 has a mildly positively skewed and leptokurtic distribution with a skewness statistic of 1.31 and a kurtosis statistic of 8.82. The empirical distribution of VaR5 is more well-behaved in terms of being closer to normality with respect to that of VaR1. Turning our focus to the expected shortfall metrics, ES1 has a mean and median value of 8% and 7%, respectively. The mean and median values for ES5 are equal to 5% and again less than those of ES1 in a mechanical fashion. The central tendency statistics for expected shortfall metrics are naturally higher than those for the corresponding value-at-risk metrics because returns used to calculate expected shortfall measures have upper bounds that are determined by the value-at-risk measures. Similar to the value-at-risk measures, ES1 and ES5 have mildly positively skewed and leptokurtic distributions with the latter variable being closer to normality.[5]

[5] The interested reader may wish to consult Table 1 for the descriptive statistics of the control variables.

Panel B of Table 1 includes the time-series averages of the cross-sectional correlations for all variables. First, we find that there is a strong positive correlation between the left-tail risk metrics with the correlation coefficients varying between 0.73 and 0.96. Indeed, motivated by this observation, we first present results for our tests using VaR1 and defer the results of the analyses using the other three left-tail risk metrics to the robustness section. Second, some firm-specific characteristics exhibit a mild correlation with the left-tail risk metrics. Specifically, smaller firms, stocks with higher market betas, higher downside betas, higher idiosyncratic volatilities, and stronger lottery-like features also have higher left-tail risk. Third, the correlation matrix indicates that larger firms have, on average, higher market betas, lower book-to-market ratios and lower idiosyncratic volatilities. Finally, there is a highly significant, positive correlation between idiosyncratic volatility and lottery demand.

4.2 Univariate Portfolio Analysis

In this section, we perform univariate portfolio-level analysis, where deciles are formed every month by sorting stocks based on their value-at-risk metrics at the 1% level and one-month-ahead returns are calculated for each decile to test whether the zero-cost portfolio that takes a long position in stocks with the highest valıue-at-risk and a short position in stocks with the lowest value-at-risk generates a significant return.

Table 2 presents the time-series averages of one-month-ahead excess returns for each of the VaR1-sorted deciles. In Panels A and B, we present results for value-weighted and equal-weighted portfolio returns, respectively. Panel A shows that stocks in the lowest value-at-risk decile (Portfolio 1) have a monthly value-weighted average excess return of 47 basis points. The excess returns decrease starting with portfolio 8, where portfolios 8 and 9 have an average excess return of 51 and 31 basis points, respectively. The sharpest decline in excess returns occurs in portfolio 10 which contains the stocks with the highest value-at-risk. For this decile, the average excess return equals -31 basis points. The average return difference between the extreme value-at-risk deciles is -0.78% with a significantly negative *t*-statistic of -2.34, indicating that equities with higher left-tail risk have significantly lower expected excess returns.

Next, we examine whether the significantly negative return difference between the highest and lowest value-at-risk deciles is robust after we control for the pricing factors of FFCPS and FF5 models are accounted for. The abnormal returns (alphas) obtained from the FFCPS model exhibit a decreasing pattern moving from equities with the lowest value-at-risk to those with the highest value-at-risk. Portfolio 1 has a FFCPS alpha of 7 basis points, whereas portfolio 10 has a FFCPS alpha of -87 basis points per month. The FFCPS alpha to the zero-cost portfolio is equal to -0.94% per month with a t-statistic of -4.42 which is both economically and statistically significant. The abnormal returns from the FF5 model paint a similar picture. Portfolio 1 has an FF5 alpha of -3 basis points and portfolio 10 has an FF5 alpha of -65 basis points per month. The FF5 alpha difference between the highest and lowest value-at-risk deciles is -0.63% per month and again both economically and statistically significant with a t-statistic of -3.75. The factor model analysis reveals two main conclusions. First, the finding that equities with higher value-at-risk earn lower one-month-ahead returns cannot be explained by commonly used factors. Second, this finding is driven by the underperformance of stocks with high left-tail risk, implying that investors overprice securities with higher left-tail risk and, consequently, experience significantly negative abnormal returns in the future. The tendency of large losses to persist into the future suggest the existence of a left-tail return momentum.

Panel B of Table 2 presents results for the equal-weighted portfolios and the findings are similar to those of Panel A. The excess return difference between the extreme value-at-risk deciles is equal to -0.66% per month and significantly negative. The corresponding FFCPS and FF5 alpha spreads between deciles 1 and 10 are -0.80% per month (t-stat. = -5.20) and -0.62% per month (t-stat. = -4.56), respectively. Thus, the underperformance of the stocks with the highest value-at-risk is visible in equal-weighted portfolios as well.[6]

We also investigate the long-term predictive power of left-tail risk by calculating the monthly returns and alphas of the value-at-risk deciles from two to twelve months after portfolio formation. The results are presented in Table II of the online appendix. During the second month after portfolio formation, the decile that contains the stocks with the highest (lowest) value-at-risk has a value-weighted return of -18 (47) basis points. The difference is equal to -65 basis points and significant with a t-statistic of -2.12. Similarly, the zero-cost strategy has a return of -59 basis points with a t-statistic of -2.04 during the third month after portfolio formation. The predictive power of left-tail risk on future returns diminishes as one moves further away from the portfolio formation month and becomes insignificant after the sixth month. These results show that the negative cross-sectional relation between left-tail risk metrics and future returns is not just a one-month affair.

4.3 Average Portfolio Characteristics

We now investigate which firm-specific attributes can potentially explain the anomalous significantly negative relation between value-at-risk and expected equity returns uncovered in the previous section. To do so, we again sort stocks based on their VaR1 metrics into deciles each

[6] Going forward, we present the main findings from the value-weighted portfolios to emphasize that the results are not driven by small stocks. All our results are robust to using equal-weighted portfolios.

month and report the time-series averages of the cross-sectional means for various firm-specific characteristics for each decile. The results are presented in Table 3.

First, by construction the value-at-risk measures increase mechanically moving from portfolio 1 to portfolio 10. The mean VaR1 for portfolio 1 is 0.0268, meaning that the 1st percentile of daily returns during the past year is equal to -2.68% for the representative firm in the decile which contains the stocks with the lowest left-tail risk. Similarly, for the average firm in the decile which contains the stocks with the highest left-tail risk, the 1st percentile of daily returns in the prior year corresponds to -11.68%. The average market beta for portfolio 1 (portfolio 10) is 0.40 (1.26), indicating that equities with higher value-at-risk are more sensitive to market movements. Companies with higher value-at-risk tend to be significantly smaller and have lower book-to-market equity ratios. The average momentum return for the lowest (highest) value-at-risk decile is equal to 16% (8%), whereas the one-month lagged return for the lowest (highest) value-at-risk decile is equal to 1% (-1%). For both return measures, the difference between the extreme value-at-risk portfolios is statistically significant. Equities with higher value-at-risk tend to be less liquid, have significantly higher idiosyncratic volatilities and exhibit stronger lottery-like characteristics. The average co-skewness measure for equities in portfolio 10 is significantly less negative (or large in absolute magnitude) than that of equities in portfolio 1. Finally, stocks with higher left-tail risk are also more sensitive towards downward movements in the value of the market portfolio.

Prior literature suggests that the firm-specific attributes considered in Table 3 are instrumental in determining the cross-section of expected equity returns. Specifically, equities with higher market betas and downside betas, lower market capitalizations, higher book-to-market equity ratios, higher momentum returns, lower one-month lagged returns, lower liquidity, lower co-skewness, lower idiosyncratic volatility and lower lottery demand tend to have higher expected returns. Considering these prior findings in the literature and the patterns that the firm-specific attributes exhibit across the value-at-risk deciles, one can see that some of these attributes may drive the significantly negative relation between left-tail risk and expected returns. For example, equities with higher left-tail risk have lower book-to-market ratios and momentum returns and the positive relation between these two firm characteristics and expected returns may drive the negative relation between left-tail risk and expected returns (see, e.g., Fama and French (1992, 1993) and Jegadeesh and Titman (1993)). Furthermore, market beta, idiosyncratic volatility and lottery demand are positively related to left-tail risk and negatively related to expected returns which may be the cause of the left-tail risk anomaly (see, e.g., Ang, Hodrick, Xing, and Zhang (2006), Bali, Cakici, and Whitelaw (2011), and Frazzini and Pedersen (2014)). We further analyze these possibilities in the bivariate portfolios and multivariate Fama-MacBeth regressions presented in the next two subsections.

4.4 Bivariate Portfolio Analysis

The negative relation between left-tail risk and equity returns in the univariate portfolios presented in Table 2 may be observed because a firm-specific characteristic that is correlated with value-at-risk has a significant impact on expected stock returns. To test whether this is the case, we use two-stage dependent sorts based on various firm-specific attributes and value-at-risk. Table 4 presents the results from the bivariate portfolio analysis for value-weighted decile returns. Panel A and B present results for FFCPS and FF5 alphas, respectively.

The findings in Panel A suggest that, for dependent sorts on all first-stage sorting variables, the FFCPS alphas exhibit a declining pattern across deciles. For example, the first row shows that, when market beta is used as the first-stage sorting variable, portfolio 1 has a FFCPS alpha of 4 basis points, whereas portfolio 10 has a FFCPS alpha of -66 basis points. The alpha difference between the highest and lowest value-at-risk deciles is -0.69% with a t-statistic of -6.43. Similar results are observed for the other first-stage sorting variables. The FFCPS alpha differences between the extreme value-at-risk deciles vary between -41 basis points with a t-statistic of -3.09 (for lottery demand) and -85 basis points with a t-statistic of -5.01 (for short-term reversal).

When one focuses on the results for FF5 alphas, Panel B shows that the declining pattern in FFCPS alphas across the value-at-risk deciles continues to be observed. The alpha differences between the extreme value-at-risk deciles vary between -37 basis points with a t-statistic of -3.05 (for lottery demand) and -79 basis points with a t-statistic of -8.67 (for downside beta). These results indicate that even after controlling for various firm characteristics and risk factors in bivariate portfolios, there is a strong negative relation between VaR1 and future returns. In other words, left-tail return momentum cannot be explained by other cross-sectional return predictors. Moreover, this relation is driven by the underperformance of stocks with high value-at-risk because the alphas for portfolio 10 are negative and highly significant without exception, whereas the corresponding alphas for portfolio 1 are mostly insignificant.[7]

4.5 Firm-Level Cross-Sectional Regression Analysis

We run firm-level cross-sectional regressions for each month, where the dependent variable is the one-month-ahead returns on each stock and the independent variables are lagged value-at-risk and various firm-specific control variables. Each monthly regression is estimated

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[7] Results for independent sorts are presented in Table I of the online appendix and are similar to those for dependent sorts.

using either the ordinary least squares (OLS) method or a weighted least squares (WLS) methodology following Asparouhova, Bessembinder and Kalcheva (2013) where each observed return is weighted by one plus the observed prior return on the stock. Panels A and B of Table 5 present the results from the OLS and WLS estimations, respectively.

In the first column of Panel A, VaR1 has a significantly negative coefficient of -0.0782 with a t-statistic of -2.49 in a univariate regression specification. The economic magnitude of the associated effect is similar to that documented in Table 2 for the univariate decile portfolios based on VaR1. As reported in Table 3, the spread in average VaR1 between portfolios 10 and 1 is 0.09 = (0.1168 – 0.0268), and multiplying this spread by the average slope of -0.0782 yields an estimated monthly premium of 70 basis points.

Columns 2 to 11 augment the univariate regression by adding an extra firm-specific attribute among the independent variables one at a time. The coefficients of value-at-risk are estimated in the range of -0.0455 and -0.1244 in these specifications and they are all significantly negative with t-statistics between -2.32 and -7.25. Regression (11) which controls for all firm characteristics and risk attributes, shows that the slope coefficient of value-at-risk is negative and highly significant with a value of -0.0458 and t-statistic of -2.32. These results show that left-tail risk has distinct, significant information orthogonal to market beta, downside beta, idiosyncratic volatility, lottery demand, co-skewness, illiquidity, and past return characteristics and it is a strong and robust predictor of future equity returns.

Similar results are observed in Panel B for the WLS regressions. In the univariate specification of the first column, VaR1 has a significantly negative coefficient of -0.0987 with a t-statistic of -3.10. Incorporating additional control variables to the specification does not subsume the negative relation between left-tail risk and one-month-ahead equity returns. In columns 2 to 11, the coefficient of VaR1 varies between -0.0656 and -0.1468 with t-statistics ranging from -3.27 to -8.24. In other words, the anomalous negative relation between left-tail risk and expected returns continues to hold after other determinants of cross-sectional equity returns are controlled for in a more comprehensive way.

Several observations are worth mentioning regarding the control variables. As seen from the OLS regressions, the negative relation between firm size and equity returns and the positive relation between book-to-market equity ratio and expected returns is clearly observable. In column 11, firm size has a coefficient of -0.0010 with a t-statistic of -3.53 and book-to-market equity ratio has a coefficient of 0.0014 with a t-statistic of 2.17. Second, the short-term reversal effect is strongly visible in the estimation results with coefficients between -0.0656 and -0.0728 and t-statistics between -6.69 and -7.23. Third, there is a strong negative relation between idiosyncratic volatility and one-month-ahead equity returns with a t-statistic of -5.14 for the IVOL coefficient in the regression (10). Fourth, in tabulated results, we observe that there is also a strong negative relation between lottery demand and expected returns when MAX is included in the specification rather than IVOL. The coefficient of MAX is -0.1156 with a t-statistic of -5.32. Both IVOL and MAX become insignificant when they are included simultaneously in the regression due to the high level of multicollinearity between them. Finally, the other firm-specific characteristics, namely the market beta, illiquidity, co-skewness, and downside beta do not display a significant relation with expected stock returns. These results also apply to the WLS estimates.

4.6 Transition Matrix

In this section, we present results regarding the cross-sectional persistence of left-tail risk. In Table 6, we investigate this issue by examining the average 12-month-ahead portfolio transition matrix for our sample firms.[8] Specifically, we present the average probability that a stock in decile *i* (defined by the rows) in one month will be in decile *j* (defined by the columns) in the subsequent 12 months. All the probabilities in the matrix should be approximately 10% if the evolution for value-at-risk for each stock is random and the relative magnitude of left-tail risk in one period has no implication about the relative left-tail risk values in the subsequent period. However, Table 6 shows that 52% of stocks in the lowest value-at-risk decile in a certain month continue to be in the same decile 12 months later. Similarly, 33% of the stocks in the highest value-at-risk decile in a certain month continue to be in the same decile 12 months later. Moreover, the stocks have a 54% probability of being in deciles 9 and 10, which exhibit higher left-tail risk in the portfolio formation month and lower returns in the subsequent month. These results overall suggest that left-tail risk is a highly persistent equity characteristic.

Theory suggests that investors would pay higher (lower) prices for stocks that have exhibited lower (higher) left-tail risk in the past with the expectation that this behavior will persist in the future. However, the analyses of the previous sections show the opposite to be true and that investors overprice securities with the highest value-at-risk. If the expectation of value-at-risk was a characteristic that evolved randomly through time, we would expect no relation between left-tail risk and future stock returns. The fact that left-tail risk is persistent and it has an anomalous relation with the cross-section of expected returns suggests the possibility that investors underestimate the magnitude of the cross-sectional persistence uncovered in this section. We delve further into this possibility in the next section.

[8] Since VaR1 is estimated using daily returns over the past 12 months, we investigate the 12-month-ahead cross-sectional persistence of left-tail risk to avoid the issue of monthly overlapping observations that would induce artificial persistence.

5. BEHAVIORAL EXPLANATIONS OF THE CORE FINDINGS

5.1 Delta VaR Analysis

In this section, we propose a behavioral explanation for the finding that equities with higher left-tail risk have lower expected returns. The explanation is based on the conjecture that stocks with higher left-tail risk have experienced large losses during the recent period and investors underestimate the probability of these losses to persist. In other words, they overestimate the mean-reversion in left-tail risk. As a result, they end up paying high prices for such stocks and experience lower returns when the losses continue into the future. To test this idea, we calculate the change in value-at-risk for each stock between months *t* and *t*-1 and use these changes in value-at-risk measures in bivariate portfolio analyses to see whether they have any implications on month *t*+1 returns.

We define DeltaVaR as VaR1 at the end of month *t* minus VaR1 at the end of month *t*-1. DeltaVaR can be either negative, zero or positive at a certain month for each stock. A negative DeltaVaR indicates that value-at-risk at the end of portfolio formation month *t* is less than the value-at-risk at the end of month *t*-1. We calculate value-at-risk from the daily returns observed during the prior year. Thus, a negative DeltaVaR means that the return observation that corresponds to the 1st percentile of daily returns in the year preceding the end of month *t* is less than the 1st percentile of daily returns in the year preceding the end of month *t*-1. In other words, the stock must have experienced a large non-recent crash during month *t*-12. Conversely, a positive DeltaVaR means that value-at-risk at the end of month *t* is greater than the value-at-risk at the end of month *t*-1. The stock should have experienced a large daily loss recently, namely during month *t*. If DeltaVaR is zero, the return observation that corresponds to the 1st percentile of daily returns in the prior year observed at the end of month *t* should have been observed any time between months *t*-11 and *t*-1, inclusive. We have already demonstrated that left-tail risk is a persistent equity characteristic. Thus, we expect equities that have experienced a large daily loss in the portfolio formation month to continue to experience such large losses in the future. Among the stocks with high value-at-risk at the end of month *t*, those that have experienced a crash more recently have a higher probability of experiencing a similar crash in the next month. Therefore, we expect the anomalous negative relation between value-at-risk and one-month-ahead returns to be more pronounced for stocks that have experienced a large daily loss in the portfolio formation month, i.e, stocks with a positive DeltaVaR.

To test our conjecture, we first sort stocks into five VaR1 quintiles at the end of month *t*. Next, within each value-at-risk quintile, we separate the stocks into three groups based on whether their DeltaVaR values are negative, zero or positive. Then, we look at the excess and abnormal return differences between the stocks in the highest and lowest VaR quintiles for each DeltaVaR group.

The results are presented in Panel A of Table 7. For those stocks with negative DeltaVaR values or stocks that have experienced their large losses in the more distant past, the excess return difference between the extreme VaR1 quintiles is -55 basis points with an insignificant t-statistic of -1.61. Similarly, for those stocks with zero DeltaVaR, the excess return difference between VaR1 quintiles 5 and 1 is equal to -34 basis points and insignificant with a t-statistic of -1.22. However, when DeltaVaR is positive, the excess return to the zero-cost portfolio is equal to -90 basis points per month with a t-statistic of -2.72. A similar pattern is also observed for the alpha spreads. For example, when DeltaVaR is negative, the FF5 alpha for the zero-cost portfolio is equal to -45 basis points per month with a t-statistic of -2.18. However, the absolute magnitude of the alpha is much higher when DeltaVaR is positive with a value of -79 basis points per month and a t-statistic of -3.50. These results can be interpreted in the following way. For the stocks in the highest VaR quintile, stocks that are most susceptible to experience a large loss in the subsequent month are those that have experienced a large recent loss at month *t* due to the high level of persistence in left-tail risk. Investors underestimate this persistence or overestimate the level of mean-reversion and, thus, overprice those securities with high left-tail risk and recent capital losses. When this persistence materializes and stocks that have crashed in month *t* continue to crash in month *t*+1, the negative relation between left-tail risk measured in month *t* and one-month-ahead equity returns becomes visible and the left-tail return momentum phenomenon emerges.

We push this analysis one step further and investigate the returns to equity groupings based on lagged DeltaVaR in addition to DeltaVaR. *Lagged* DeltaVaR is defined as VaR1 at the end of month *t*-1 minus VaR1 at the end of month *t*-2. A negative lagged DeltaVaR means that the return observation that corresponds to the 1st percentile of daily returns in the year preceding the end of month *t*-1 is less than the 1st percentile of daily returns in the year preceding the end of month *t*-2. In other words, the stock must have experienced a large non-recent crash during month *t*-13. Conversely, a positive lagged DeltaVaR means that value-at-risk at the end of month *t*-1 is greater than the value-at-risk at the end of month *t*-2. The stock should have experienced a large daily loss during month *t*-1. In the analysis conducted in Panel B of Table 7, we first sorts stocks into quintiles based on their VaR1 at the end of month *t*. Then, we group the stocks in each quintile into nine groups based on whether their DeltaVaR and lagged DeltaVaR is negative, zero or positive. If both DeltaVaR and lagged DeltaVaR for a stock are negative, this would imply that the stock did not experience a large enough daily loss during the past two months compared to its daily losses from month *t*-13 to *t*-2. We anticipate the negative relation between value-at-risk and one-month-ahead returns to be the weakest for this group of stocks. If both DeltaVaR and lagged DeltaVaR for a stock are positive, this would imply that the stock experienced a large daily crash in both of the last two months compared to its daily losses from month *t*-13 to *t*-2. The negative relation between value-at-risk and expected returns should be most pronounced for this group of stocks.

Panel B of Table 7 shows that when both DeltaVaR and lagged DeltaVaR are negative, the average return difference between VaR1 quintiles 1 and 5 is only 6 basis points with a t-statistic of 0.14. The corresponding FFCPS and FF5 alphas are -0.42% and 0.23% per month, both of which are statistically insignificant in line with our expectations. Furthermore, the return spreads between the extreme VaR quintiles increase in absolute value when DeltaVaR is negative but lagged DeltaVaR is positive. These are the stocks that experienced a relatively large daily loss in month *t*-1 but not in month *t*. For this group, although not significant at conventional levels, the excess returns to the zero-cost portfolio is -83 basis points per month, much larger in absolute value compared to the group for which lagged DeltaVaR is negative.

When we focus on stocks with a positive DeltaVaR, interesting patterns emerge. First, we look at the stocks for which lagged DeltaVaR is negative. These are the stocks that have experienced a relatively large daily loss in month *t* but not in month *t*-1. We again observe no significant excess or abnormal return differences between the highest and lowest VaR quintiles for this group of equities. However, the return spreads increase in absolute value uniformly as lagged DeltaVaR first becomes zero and then positive. When DeltaVaR and lagged DeltaVaR are both positive, the excess return to the zero-cost portfolio is -1.32% with a t-statistic of -3.33. The corresponding alphas are -1.43% and -1.11% with t-statistics of -3.92 and -3.34 for the FFCPS and FF5 models, respectively. These are the largest alpha values observed in the table for any group of stocks. Stocks with a positive DeltaVaR and a positive lagged DeltaVaR are those with large daily losses both in month *t* and month *t*-1. Due to the persistence of left-tail risk, they are also the stocks that are most likely to experience a large daily loss in the subsequent month. Investors underestimate this likelihood and they are negatively surprised when the large losses occur. As stocks with recent large losses continue to experience further losses, returns in the left-tail of the empirical distribution exhibit momentum.

These results are also reminiscent of the disposition effect suggested by Shefrin and Statman (1985). The disposition effect refers to the greater propensity of investors to sell stocks that have risen in value rather than fallen in value since purchase. Grinblatt and Han (2005) suggest that the disposition effect causes price underreaction to information. Frazzini (2006) finds a stronger a disposition effect in the trading of individual investors, compared to institutional investors. Barberis and Xiong (2009, 2012) draw on the prospect theory of Kahneman and Tversky (1979) to model the conditions under which the disposition effect could be observed. In our context, stocks with high value-at-risk are more likely to have fallen in value since purchase and investors who refrain from realizing their losses could prefer to hold on to these stocks. Thus, the effect of recent crashes will not be fully reflected in equity prices resulting in left-tail momentum. Such behavioral biases are more likely to be observed for retail investors and subsequent sections investigate this point.

5.2 Institutional Ownership

The previous section conjectures that the negative relation between left-tail risk and expected returns is driven by the persistence in left-tail risk and investors' underestimating the likelihood of recent large losses to continue in the future. In this section, we investigate the differing levels of institutional ownership among stocks with high and low value-at-risk. We test three hypotheses. First, we investigate whether the level of institutional ownership is lower for those stocks whose left-tail risk is high and are more likely to earn negative returns in the next month. This would be true if institutional investors are better able to capture the persistence in left-tail risk and shy away from those stocks which have experienced recent large losses. Second, we explore whether institutional investors adjust their holdings of equities by taking large left-tail events into account. Third, we test whether the magnitude of the negative relation between value-at-risk and expected returns is larger for those stocks in which retail investors are more active compared to those stocks in which institutional investors are more active.

In Panel A of Table 8, we present the time-series averages of cross-sectional means for percentage institutional ownership (INST) for equity deciles formed via a univariate sort based on VaR1. The results show that equities with higher value-at-risk are more likely to be held by individual investors. The percentage institutional ownership is equal to 42% for portfolio 1. In contrast, for portfolio 10 which includes the equities with the highest value-at-risk, the percentage institutional ownership drops to 36%. The difference in institutional holdings between the extreme value-at-risk quintiles is highly significant with a t-statistic of 3.90. In Panel B of Table 8, we present the time-series averages of three- to twelve-month changes in INST for the value-at-risk deciles. We find that the percentage institutional ownership increases by 29% for stocks with the lowest value-at-risk and decreases by 4% for stocks with the highest value-at-risk three months after portfolio formation. The difference between the extreme deciles is significant with a t-statistic of 3.12 and this predictability persists up to nine months.

Next, we analyze the strength of the left-tail risk anomaly across institutional ownership portfolios using a bivariate sort analysis. At the end of each month *t*, all stocks in the sample are independently grouped into quintiles based on an ascending sort of the level of institutional ownership and deciles based on an ascending sort of VaR1. In Panel C of Table 8, we present the time-series averages of the one-month-ahead excess returns for each of the 50 resulting intersection portfolios, as well as the excess return difference between the extreme value-at-risk deciles, five-factor alphas and associated t-statistics.

For the quintile for which the level of institutional ownership is the highest (INST5), we do observe a decreasing pattern of returns as one moves from the lowest VaR1 quintile (70 basis points) to the highest VaR1 quintile (0 basis points). The excess return, FFCPS alpha and FF5 alpha to the zero-cost portfolio are -0.69%, -0.77% and -0.44% per month, respectively. Among these, only the FFCPS alpha is significant at the 5% level. The magnitude of the excess and abnormal return to the zero-cost portfolio that buys stocks with the highest value-at-risk and sells stocks with the lowest value-at-risk increases uniformly in absolute value as one moves towards the stocks for which the level of institutional holdings is lowest (INST1). For those stocks in which retails investors are most active, stocks in the lowest VaR decile earn a one-month ahead excess return of 61 basis points, whereas stocks in the highest VaR decile earn a one-month ahead excess return of -79 basis points per month. The average return difference is economically and statistically significant; -1.40% per month with a t-statistic of -3.13. Similarly, the corresponding alpha spreads are -1.51% and -0.93% per month with t-statistics of -3.76 and -2.45 for the FFCPS and FF5 models, respectively. These alphas are about twice as high in absolute magnitude with respect to those observed in the highest institutional ownership quintile. Collectively, these results suggest that the left-tail risk anomaly is much stronger for equities that are more likely to be held by retail investors.

5.3 Limited Attention of Individual Investors

Market reactions to extremely large plunges in individual stock prices can generate important insights on how the market processes information about left-tail price shocks on the information efficiency of the equity market. There are two potential market frictions that prevent public information from being incorporated into security prices: limited investor attention and illiquidity. There has been an increasing body of empirical evidence suggesting that investor inattention can lead to underreaction to information. These studies show that, due to limited investor attention, stock prices underreact to public information about stock fundamentals and characteristics.[9]

We focus on large negative price shocks that generate left-tail risk, which are harder to interpret by average investors compared to the direct and well-defined information events studied in the previous literature (e.g., new products, earnings news, demographic information, or information about related stocks). Consistent with Hirshleifer, Hsu, and Li (2013) who emphasize that investors would have more difficulty in processing information that is less tangible, we think that the elusive nature of left-tail risk thus makes the investors' attention constraints more likely to be binding. These constraints would be even more binding for retail investors who are more active in high left-tail risk equities compared to institutional investors. As a result, the stock market can underreact to persistence in left-tail risk. Moreover, as indicated in the model of Peng and Xiong (2006), an investor who optimizes the amount of attention allocation would allocate more attention to systematic shocks and less to stock-specific shocks (in some cases even completely ignoring them). Thus, a strong case can be made for underreaction to stock-level left-tail price shocks based on theories of investor attention.

The investor attention theory predicts that the degree of underreaction to left-tail risk, as measured by its return predictability, should be more pronounced for stocks that receive less investor attention. This can be tested by dividing our sample into subgroups based on investor inattention proxies such as analyst coverage, firm size and illiquidity. We also test for the existence of investor clientele effects by exploring whether left-tail return momentum is more pronounced for stocks commonly held by retail investors. In particular, retail investors are more likely to hold positions in stocks with higher idiosyncratic volatility and higher lottery demand (e.g., Kumar (2009), Han and Kumar (2013), and Bali et al. (2017)). Thus, the negative relation between left-tail risk and future returns should be observed more visibly in these stocks. To investigate these conjectures, first, we sort stocks into terciles based on the aforementioned firm characteristics. Next, we divide each firm characteristic tercile into quintiles based on VaR1. We test whether the strength of the left-tail return momentum exhibits a pattern across the firm characteristic terciles. Table 9 reports FFCPS alphas for each portfolio and the alpha difference between the extreme left-tail risk quintiles in each tercile.[10]

[9] See, e.g., Huberman and Regev (2001), Hirshleifer and Teoh (2003), Hirshleifer, Hou, Teoh, and Zhang (2004), Hou and Moskowitz (2005), Peng (2005), Barber and Odean (2008), Cohen and Frazzini (2008), Hirshleifer, Lim, and Teoh (2009), Da, Engelberg, and Gao (2011), Hirshleifer, Hsu, and Li (2013), Bali, Peng, Shen, and Tang (2014), Da, Gurun and Warachka (2014), and Han, Hirshleifer, and Walden (2017).

[10] Results for FF5 alphas are presented in Table III of the online appendix and are similar to those for FFCPS alphas.

Panel A of Table 9 shows that, for the tercile which includes stocks with the lowest analyst coverage, the alpha spread between the extreme value-at-risk quintiles is -97 basis points with a t-statistic of -4.62. However, the analogous alpha spread for the tercile which includes stocks with the highest analyst coverage is only -22 basis points with an insignificant t-statistic of -0.74. In Panel B, the alpha spread between the extreme value-at-risk quintiles is significantly negative with a value of -93 basis points and a t-statistic of -6.55 for the smallest size tercile, whereas the same spread is insignificant for the biggest size tercile. Panel C shows that although the left-tail risk anomaly is visible in each illiquidity tercile, it is strongest for the stocks with the lowest liquidity with an alpha difference of -83 basis points and a t-statistic of -6.57. Finally, Panels D and E reveal that idiosyncratic volatility and lottery demand also affect the magnitude of the left-tail risk anomaly and stocks with the highest idiosyncratic volatility (lottery demand) exhibit the largest alpha spread in absolute magnitude with a value of -1.17% (-1.23%) and a t-statistic of -5.02 (-5.21) between the extreme VaR quintiles. These results collectively suggest that the negative relation between value-at-risk and expected returns is stronger for stocks that are more likely to be exposed to limited investor attention and held by retail investors.

In Table 10, we present firm-level cross-sectional regressions that provide evidence supporting the investor inattention and clientele effects. Specifically, we re-estimate the monthly Fama-MacBeth (1973) regressions of Table 5 by augmenting the specifications by the fraction of shares held by institutional investors for each stock (INST) and the interaction of this variable with value-at-risk. Our goal is two-fold. First, we test the bivariate sort results in Panel B of Table 8 showing that the left-tail return momentum is stronger for stocks that are more likely to be held by retail investors in a multivariate setting. Second, we treat INST as a proxy for investor attention since stocks held by institutional investors attract more attention from the investment community. Table 9 indicates that the left-tail return momentum is more pronounced for stocks that receive less investor attention. Hence, we expect the coefficient on the interaction term in Table 10 to be significantly positive since a higher value of INST indicates a lower concentration of retail investors and a higher level of investor attention. Since INST is highly correlated with firm size which is already included in the regressions, we orthogonalize this variable with the logarithm of market value of equity for each firm via monthly cross-sectional regressions and use these orthogonalized values (OINST) in the specifications in Table 10. Panel A shows that, in the specification which only includes VaR1, OINST and their interaction term, value-at-risk has a negative coefficient of -0.1061 with a significant t-statistic of -3.30. The higher absolute value of this coefficient with respect to the coefficient of VaR1 in the univariate specification in Panel A of Table 5 indicates that the left-tail return momentum manifests itself more strongly for stocks that are subject to the limited attention of retail investors. More importantly, the coefficient of the interaction term between value-at-risk and institutional ownership is significantly positive with a coefficient of 0.1063 and a t-statistic of 1.98. In other words, the left-tail return momentum becomes stronger for stocks that are more likely to be held by individual investors. These patterns persist when additional control variables are included in the specifications. In column 11 of Table 10, the coefficient of VaR1 is significantly negative with a value of -0.0562 (t-statistic = -3.26) and the coefficient of the interaction term is significantly positive with a value of 0.1254 (t-statistic = 2.44). Overall, these results indicate that even after controlling for a large number of stock return predictors, the left-tail momentum remains stronger for stocks largely held by individual investors and for stocks that receive less investor attention.

6. TESTING ALTERNATIVE EXPLANATIONS OF LEFT-TAIL MOMENTUM

6.1 Alternative Factor Models

In this section, we test whether the significantly negative returns of the zero-cost portfolio which is long in equities with high left-tail risk and short in equities with low left-tail risk can be explained by alternative factor models or long-established persistent anomalies. One criticism that could be brought up against our results is that the left-tail return momentum is another manifestation of low-risk anomalies in the literature. Ang, Hodrick, Xing and Zhang (2006) find a negative relation between idiosyncratic risk and expected stock returns. Similarly, Frazzini and Pedersen (2014) document that high-beta assets require lower risk-adjusted returns than low-beta assets. To investigate whether the left-tail return momentum can be explained by these effects, we extend the characteristics-based bivariate and multivariate tests of the previous sections and investigate whether the returns to the zero-cost portfolio constructed based on left-tail risk survive in asset pricing models that incorporate factors that reflect idiosyncratic volatility and betting against beta. We also extend this analysis to account for the lottery demand effect in Bali, Cakici and Whitelaw (2011) and incorporate a lottery-demand based factor following Bali, Brown, Murray and Tang (2017).

We use three baseline asset pricing models in our tests. Two of these models are already described in Section 3; the FFCPS model with the market, size, value, momentum and liquidity factors of Fama and French (1993), Carhart (1997), and Pastor and Stambaugh (2003), and the FF5 model with the market, size, value, profitability and investment factors of Fama and French (2015). The third model, Q factor model, is borrowed from Hou, Xue and Zhang (2015) which adds size, investment, and profitability factors to the market factor. Hou et al. (2015) measure investment as the annual percentage change in total assets and profitability as the ratio of income before extraordinary items to one-quarter-lagged book value of equity. After performing a triple

2x3x3 sort on size, investment and profitability, the investment factor (IA) is calculated as the monthly difference between the simple average of the returns of the six lowest and six highest investment portfolios. Similarly, the profitability factor (ROE) is calculated as the monthly difference between the simple average of the returns of the six highest and six lowest profitability portfolios. Next, we augment each of these three baseline asset pricing models with a betting against beta factor (BAB), lottery-demand factor (FMAX) and idiosyncratic volatility factor (FIVOL). BAB is borrowed from Frazzini and Pedersen (2014). This factor is constructed by forming two portfolios, one holding stocks with market betas that are below the median beta and the other holding stocks with above-median betas. BAB factor return is equal to the excess return of the low-beta portfolio minus the excess return of the high beta portfolio. For the lottery-demand factor, we sort all stocks into two groups at the end of each month based on their market capitalization with the breakpoint determined by the median market capitalization of stocks traded on the NYSE. We also independently sort all stocks in our sample into three groups using MAX based on the NYSE breakpoints. The intersection of the two size and three MAX groups constitute six portfolios. FMAX factor is the difference in the average return of the two value-weighted high lottery demand portfolios and the average return of the two value-weighted low lottery demand portfolios. FIVOL factor is calculated in an analogous manner by repeating the identical methodology using idiosyncratic volatility (IVOL) rather than MAX.

Table 11 presents the results of time-series factor regressions where the monthly value-weighted excess return differences between the highest value-at-risk decile and the lowest value-at-risk decile are regressed on the contemporaneous values of the asset pricing factors. Panel A reports results for the FFCPS model augmented sequentially by the BAB, FMAX and FIVOL factors. Panels B and C replace the baseline model with the FF5 model and Q-model, respectively. Our main interest is the regression intercepts (alpha) that should equal zero if the factor models adequately describe the left-tail return momentum phenomenon.

We first focus on the first columns of each panel which present results for the baseline models. The FFCPS and FF5 results in the first two panels are repetitions of the results in the last column of Panel A of Table 2 and indicate that the regression alphas are significantly negative. Panel C of Table 11 also shows that the Q-model of Hou, Xue and Zhang (2015) does not fully account for the left-tail return momentum and leaves a significant portion of the zero-cost portfolio return unexplained. The intercept term from this regression is equal to -0.57% with a significant t-statistic of -2.57. Next, we look at the second, third and fourth columns of each panel which add the betting against beta, lottery demand and idiosyncratic volatility factors to the baseline regressions. We observe that the return on the zero-cost portfolio loads negatively on BAB and positively on both FMAX and FIVOL, which is consistent with the correlation statistics in Panel B of Table 1 that stocks with higher left-tail risk have higher market beta, higher idiosyncratic volatility, and stronger lottery-like features. More importantly, all the regression intercepts continue to be significantly negative regardless of what baseline model and additional pricing factor is used. The only marginally statistically significant intercept belongs to the Q-model augmented by BAB where the intercept is still economically substantial with a monthly alpha of 39 basis points. These results collectively suggest that a variety of asset pricing models which are designed to mimic the common variation in returns related to a multitude of firm characteristics do not account for the left-tail return momentum.

6.2 Earnings Announcements

Daily equity returns are sensitive to information-related events such as earnings announcements. A highly unfavorable earnings event can induce a highly negative return which would shift the daily return distribution of a stock towards the left and impact the left-tail risk metrics. Ball and Brown (1968) find that cumulative abnormal returns tend to drift up for good news firms and down for bad news firms after earnings are announced. Thus, bad earnings news will increase left-tail risk for a stock and also be followed by negative returns which may drive the negative relation between left-tail risk and future returns that we uncover in this study. To make sure that left-tail return momentum does not simply capture the lower leg of the post-earnings announcement drift, we put special emphasis into earnings announcement returns in this section.

Our strategy to distinguish between left-tail momentum and post-earnings announcement drift is omitting equity returns during a three-day window around earnings announcements for each stock throughout the sample period when we calculate value-at-risk metrics. By omitting the earnings announcement returns, we ensure that the left-tail risk proxies for our sample stocks are not affected by negative earnings news. In Table 12, we repeat our univariate portfolio analysis in Table 2 using these recalculated value-at-risk measures. The results show that stocks with the highest value-at-risk have a 54 basis points lower return compared to stocks with the highest value-at-risk and this difference is statistically significant with a t-statistic of -1.99. The associated FFCPS and FF5 alphas are -83 and -45 basis points with t-statistics of -4.07 and -2.77, respectively. These results collectively suggest that left-tail momentum cannot be explained by post-earnings announcement drift.

7. ROBUSTNESS TESTS

7.1 Orthogonalization with respect to Idiosyncratic Volatility and other Firm Characteristics

One asset pricing relation that is related to our results is the negative relation between idiosyncratic volatility and expected stock returns proposed by Ang et al. (2006). Table 2 shows that the average correlation between IVOL and the left-tail risk metrics vary between 0.58 and 0.67. To make sure that the left-tail momentum phenomenon is not driven by the idiosyncratic volatility puzzle, we use bivariate sorts that keep idiosyncratic volatility flat across value-at-risk deciles in Table 4 and we also control for idiosyncratic volatility in the cross-sectional regressions of Table 5. In this section, we conduct a robustness check by orthogonalizing value-at-risk via monthly cross-sectional regressions of VaR1 on IVOL. The residual terms from these regressions are denoted as VaR1_orth. We also go one step further and utilize an alternative orthogonalization by running monthly cross-sectional regressions of VaR1 on all firm-specific attributes used in this study. Next, we regress one-month-ahead returns on orthogonalized value-at-risk and all control variables and observe whether VaR1_orth still has a significantly negative coefficient. The results are presented in Table 13.

The first three regressions in Table 13 present the results from OLS estimations where VaR1 is orthogonalized with respect to only IVOL. These specifications differ among themselves based on whether they include IVOL and MAX separately or simultaneously in the set of independent variables. We find that the slope coefficients of orthogonalized value-at-risk are between -0.0405 and -0.0458 with t-statistics between -2.15 and -2.32. Similar results are observed for the WLS estimations in specifications (4) to (6). In these regressions, VaR1_orth has slope coefficients between -0.0638 and -0.0672 with t-statistics between -3.26 and -3.35. These findings indicate that the left-tail momentum is a distinct phenomenon than the idiosyncratic volatility puzzle. The last six regressions in Table 13 orthogonalize value-at-risk with respect to IVOL and all other firm characteristics. Both the OLS and WLS results uncover significantly negative coefficients on residual value-at-risk without exception. We conclude that the negative relation between left-tail risk and expected equity returns cannot be explained by any of the equity characteristics employed in our study.

7.2 Skipping a Month between Portfolio Formation Month and Holding Period

In this subsection, we investigate whether microstructure issues affect our main finding regarding the existence of left-tail momentum. Although we control for a wide variety of variables in our bivariate and multivariate tests, many of these variables use information from the portfolio formation month and this can drive the negative relation between left-tail metrics and one-month-ahead equity returns. Given the persistence in left-tail risk uncovered earlier, our main findings should not be affected by skipping a month between portfolio formation month and holding period. Thus, in Table IV of the online appendix, we examine the relation between value-at-risk and future equity returns after skipping one month and focusing on two-month-ahead portfolio returns. For value-weighted decile returns presented in Panel A, we observe that both excess returns and five-factor alphas decrease as one moves from portfolio 1 to portfolio 10. The FFCPS (FF5) alpha difference between the extreme value-at-risk deciles equals -83 (-51) basis points with a t-statistic of -4.50 (-3.48). For the equal-weighted decile returns presented in Panel B, similar patterns emerge. The FFCPS (FF5) alpha difference between the extreme value-at-risk deciles equals -59 (-64) basis points with a t-statistic of -2.17 (-2.21).

7.3 Effect of Outliers

To examine the effect of outliers on the negative relation between left-tail risk and future returns, we truncate our sample at the 1% level by removing the stocks with the highest and lowest VaR1 each month and repeat our univariate analysis in Table 2. The results are presented in Table V of the online appendix. In Panel A which presents value-weighted portfolio returns, the excess return difference between the extreme value-at-risk deciles is equal to -74 basis points with a t-statistic of -2.27. The five-factor alphas associated with this return difference are highly significantly negative. Similarly, for equal-weighted portfolio returns presented in Panel B, the return to the zero-cost portfolio is -60 basis points with a t-statistic of -2.19 and the five-factor alphas again suggest a strong left-tail momentum effect. Finally, rather than calculating weighted-average portfolio returns, we focus on the time-series of the *median* stock returns in each portfolio-month in Panel C to minimize the effect of outliers on average return spreads. The findings reveal that our results are robust after changing the central tendency statistic from weighted-average to median, as the return and alphas of the zero-cost portfolio continue to be significantly negative.

7.4 Full CRSP Universe and NYSE Breakpoints

In the univariate portfolio return analysis of Section 4.2 presented in Table 2, we exclude the common stocks with an end-of-month stock price less than $5 from our sample to ensure that small and illiquid stocks do not drive the results. We also calculate the decile cut-off points using the VaR1 measures of all the stocks in this sample such that there is an even number of stocks in each portfolio in a particular month. In this section, we carry out additional analysis to show that our results are robust to these choices. Table VI of the online appendix presents five-factor alphas for the value-weighted returns of the equity deciles and the zero-cost portfolio calculated using alternative empirical approaches. In the first two columns of this table, we present results for the original sample which excludes low-priced stocks, however, the VaR1 cut-off points are obtained from only NYSE-traded stocks. In the third and fourth columns, no price screens are utilized and the decile cut-off points are calculated using all the stocks in this full CRSP universe. In the last two columns, we again use the full CRSP universe as our sample, however, the decile breakpoints are obtained from only NYSE-traded stocks. The results indicate that the left-tail momentum effect remains significant regardless of the methodological choice. Both FFCPS and FF5 alphas tend to decline as one moves to portfolios with higher value-at-risk and the abnormal returns to the zero-cost portfolio vary between -0.80% and -1.07% per month with highly significant *t*-statistics.

7.5 Subsample Analysis

In this section, we investigate whether the strength of the left-tail momentum phenomenon varies with the state of the economy. To answer this question, we divide the full sample period into two based on several macroeconomic indicators that signal whether the economy is experiencing unfavorable conditions or high uncertainty. First, we split our sample based on the level of the Chicago National Activity Index (CFNAI). This monthly index is a weighted average of numerous indicators that gauge overall economic activity and is constructed to have an average value of zero and a standard deviation of one. Since a negative index value corresponds to growth below trend, we interpret those months during which CFNAI is less than zero to be bad states of the economy. Second, we use the macroeconomic uncertainty index developed by Jurado, Ludvigson and Ng (2015, JLN) and consider those months during which the JLN index level is greater than its sample median to be periods of high economic uncertainty. Third, we utilize another measure of economic distress, the default spread (DEF) defined as the yield difference between BAA-rated and AAA-rated corporate bonds. After splitting our sample into two using these indicator variables one at a time, we calculate the monthly value-weighted excess return and five-factor alphas to the zero-investment portfolio that is long (short) in equities with high (low) value-at-risk. The results are presented in Table VII of the online appendix.

The findings indicate that the negative relation between left-tail risk and equity returns is stronger during bad economic conditions, proxied by low economic activity (CFNAI < 0), high economic uncertainty (JLN > Median), and high default risk (DEF > Median). First, when the CFNAI index is negative, the FFCPS alpha is equal to -1.35% with a t-statistic of -3.60. On the other hand, when the CFNAI index is non-negative, the FFCPS alpha is reduced by half to -0.66% with a t-statistic of -2.97. Second, during months of high economic uncertainty as indicated by the JLN index, the FFCPS alpha is equal to -1.03% with a t-statistic of -3.44, whereas the magnitude of the alpha reduces to -0.76% with a t-statistic of -3.20 during months of low economic uncertainty. Finally, the results for default spread reveal that the FFCPS alpha to the zero-cost portfolio is -1.92% with a t-statistic of -5.15 when the economy is under relative distress, whereas the alpha is only -0.46% with a t-statistic of -2.19 during months of relative tranquility. Similar patterns apply to the raw return and the FF5 alpha of the zero-cost portfolio.

7.6 Alternative Left-Tail Risk Metrics

The correlation matrix in Table 1 highlights the high degree of correlation between alternative left-tail risk metrics considered in the study. Thus, our empirical treatment up to this point only utilized the value-at-risk metric at the 1% level. In this section, we repeat the univariate portfolio analysis of Table 2 for value-at-risk at the 5% level and expected shortfall at the 1% and 5% levels to ensure that our findings are robust to the choice of the left-tail risk metric. The results are presented in Table 14.[11]

Panel A of Table 14 displays the mean value-at-risk, value-weighted excess return and five-factor alphas for deciles formed based on a univariate sort using VaR5. The average VaR5 values for the deciles are mechanically lower than the average VaR1 values presented in Table 3. There again exists a decreasing pattern in excess returns as one moves towards the higher value-at-risk portfolios with the sharpest drop occurring for portfolio 10. Portfolio 1 has a one-month-ahead excess return of 51 basis points, whereas portfolio 10 has a one-month-ahead excess return of -23 basis points with the difference being -0.74% per month and significantly negative with a t-statistic of -2.01. The abnormal return to the zero-cost portfolio is -0.93% per month with a t-statistic of -3.94 for the FFCPS model and -0.54% per month with a t-statistic of -2.89 for the FF5 model. Once again, the alphas indicate that the negative relation between left-tail risk and expected equity returns (or left-tail return momentum) is driven by the stocks in the highest left-tail risk decile.

The results for expected shortfall at the 1% and 5% levels are presented in Panels B and C of Table 14 and they closely follow those for value-at-risk. The expected shortfall measures incorporate return observations from further to the left tail of the empirical return distributions, thus, their average values for each decile are higher than those for the corresponding value-at-risk metrics. The decreasing pattern of excess and abnormal returns as one moves from low to high left-tail risk stocks is also evident in these two panels. For ES1, the 10-1 alpha spreads are -0.51% and -0.39% per month with t-statistics of -3.08 and -2.68 when the FFCPS and FF5 models are used, respectively. For ES5, the analogous alpha values are -0.95% and -0.71% per month with t-statistics of -4.48 and -4.05, respectively. Again, the negative relation between left-tail risk and

[11] Although we only present results for the univariate portfolio analysis with value-weighted returns for the alternative left-tail risk metrics in the main body of the paper, we run a battery of additional tests and present the results in the online appendix. Table VIII presents results for the univariate portfolio analysis with equal-weighted returns.

one-month-ahead equity returns is mainly driven by the underperformance of stocks in the highest expected shortfall deciles.

8. INTERNATIONAL EVIDENCE

In addition to the analyses presented for the U.S. stocks, this section investigates whether the negative relation between left-tail risk and expected returns is significant in the cross-section of stocks trading in international equity markets. For the international analysis, we collect data from various markets around the world excluding the U.S. and we group them in alternative ways to see whether our main result holds globally.[12] The equity return data are obtained from Datastream. Daily returns for each stock are calculated using a daily total return index which is adjusted for stock splits and dividend payments. We utilize returns denominated in US dollars to make the returns comparable across countries, eliminate the effect of exchange rate risk on returns and reflect the effect of different inflation rates across countries through purchasing power parity following Lee (2011). We follow other international equity market studies such as Bekaert, Harvey and Lundblad (2007), Hou, Karolyi and Kho (2011) and Karolyi, Lee and van Dijk (2012) to screen the data and omit some of the data errors in Datastream that have been reported in the prior literature. Specifically, we select stocks only from major exchanges defined as those in which the majority of equities in a given country are traded and exclude stocks with special features such as depository receipts, real estate investment trusts, and preferred stocks. We retain all data for defunct stocks in the sample to avoid survivorship bias. We set the highest and lowest 1% of daily returns in each country-month to be missing and winsorize market value of equity at the 1% level in each country-month to deal with extreme observations. We also drop any day from the sample as a non-trading day if more than 90% of stocks in a given exchange have zero returns (based on return indices denominated in local currencies) on that day. The international dataset covers the period from January 1988 to December 2014. This sample includes 3.3 million firm-month observations for which left-tail risk proxies can be calculated.

Table 15 presents value-weighted excess return and abnormal return comparisons between equity deciles formed based on VaR1 for various country groupings. In this analysis, we combine all equities in each subsample, thus stocks in each decile may come from a multitude of countries. Panel A presents results for the full international sample. The average magnitudes of VaR1 for the value-at-risk deciles is comparable to those observed for the U.S. The mean excess return difference between the extreme value-at-risk deciles is -1.44% with a t-statistic of -2.67. The

[12] Our international stock sample covers Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland and the UK.

magnitude of this difference is almost twice as high as that observed in Panel A of Table 2 for the U.S. stocks. To check whether this significant return difference can be explained by various international asset pricing factors, we employ two models. First, we adjust for the global market, size, value and momentum factors detailed in Asness and Frazzini (2013) and these factors are obtained from Andrea Frazzini's data library. Second, we adjust for the global market, size, value, profitability and investment factors of Fama and French (2017) obtained from Kenneth French's data library. The alphas from the regressions of monthly excess return differences between the extreme value-at-risk deciles on the two sets of pricing factors suggest that the negative relation between left-tail risk and future stock returns for developed markets is not explained by the factor models. The monthly alpha spreads from the factor models of Asness-Frazzini (2013) and Fama-French (2017) turn out to be very similar both in economic and statistical significance; -1.42% (t-stat. = -3.96) and -1.42% (t-stat. = -3.63), respectively. These results are again driven by the underperformance of high value-at-risk stocks as in the U.S. with portfolio 10 having abnormal returns of -1.39% and -1.53% per month with t-statistics of -4.03 and -4.16 for the Asness-Frazzini (AF) and Fama-French (FF) models, respectively.

Next, we focus on two subsamples among developed markets, namely G10 and G7 countries. Leaving the U.S. aside, countries included in G10 are Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland and the United Kingdom, whereas Belgium, Netherlands, Sweden and Switzerland are not included in G7. The results for the univariate analysis for these two subsamples are reported in Panels B and C of Table 15. For G10 countries, the zero-cost portfolio has AF and FF alphas of -0.91% and -0.93% per month with t-statistics of -3.10 and -2.69, respectively. For G7 countries, the zero-cost portfolio has AF and FF alphas of -0.77% and -0.82% per month with t-statistics of -2.60 and -2.37, respectively. Finally, in Panel D, we focus on the members of the European Union. In this set of markets, the average VaR1 values are lower than those for the U.S. indicating a lower level of left-tail risk. However, the momentum in left-tail returns manifested in the significantly negative relation between left-tail risk and expected returns is intact. The abnormal returns to the zero-cost portfolio are -0.98% and -0.83% per month with t-statistics of -2.78 and -2.25 for the AF and FF models, respectively.

9. CONCLUSION

This study investigates the relation between left-tail risk and the cross-section of expected stock returns. According to the rational asset pricing theory, investors would pay lower prices for equities with higher left-tail risk and, thus, such stocks are expected to earn higher future returns. We test this conjecture by constructing various left-tail risk measures for all U.S. stocks between

1962 and 2014 and investigating whether these left-tail risk measures predict one-month-ahead equity returns. Our univariate portfolio-level analysis reveals a significant relation between left-tail risk and expected stock returns; however, to the opposite direction than predicted by the theory. We find that stocks in the highest left-tail risk quintile have statistically and economically lower future returns than stocks in the lowest left-tail risk quintile. The bivariate portfolio-level analyses and multivariate cross-sectional regressions suggest that this anomalous result is not explained by idiosyncratic volatility and other firm-specific characteristics that are correlated with left-tail risk. The negative relation between left-tail risk and expected returns is robust to using alternative measures of left-tail risk and remains highly significant in international equity markets and for various country groupings.

After establishing the cross-sectional persistence of left-tail risk, we provide a behavioral explanation for this anomaly. If investors underestimate the persistence in left-tail risk, it is possible that they overprice securities with high left-tail risk or securities that have experienced recent large daily losses. When these recent large daily losses repeat themselves in the future, equities with higher current left-tail risk earn lower returns in the subsequent month. As a result, returns in the left-tail of the empirical return distribution will persist into the future causing left-tail return momentum. We test and find strong empirical support for this explanation by grouping stocks based on whether the change in their left-tail risk measures in the last two months are negative, zero or positive. Two consecutive monthly positive changes in left-tail risk implies that the stock has experienced large daily losses in each of the past two months. The left-tail risk anomaly is strongest for this group of stocks supporting the conjecture that recent losses are underestimated by investors. We also find that institutional (individual) investors are less (more) likely to be active in equities with high left-tail risk. Thus, our results indicate that retail investors with limited attention are more likely to underestimate the persistence in left-tail risk and the anomaly is more pronounced for equities with high ownership of retail investors. We also find that left-tail return momentum is stronger for the type of stocks that are more likely to be held by retail investors and that receive less investor attention, underscoring the importance of investor inattention and clientele mechanisms. Moreover, we show that the left-tail return momentum cannot be explained by highly persistent low-risk anomalies. Finally, omitting earnings announcement days from the sample does not impact the negative relation between left-tail risk and expected returns.

REFERENCES

Amihud, Yakov, 2002. Illiquidity and stock returns: cross-section and time series effects. *Journal of Financial Markets* 5, 31–56.

Ang, Andrew, Joseph Chen, and Yuhang Xing, 2006. Downside risk. *Review of Financial Studies* 19, 1191–1239.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006. The cross-section of volatility and expected returns. *Journal of Finance* 61, 259–299.

Artzner, Philippe, Freddy Delbaen, Jean-Marc Eber, and David Heath, 1999. Coherent measures of risk. *Mathematical Finance* 9, 203–228.

Arzac, Enrique R., and Vijay S. Bawa, 1977. Portfolio choice and equilibrium in capital markets with safety-first investors. *Journal of Financial Economics* 4, 277–288.

Asness, Cliff, and Andrea Frazzini, 2013. The devil in HML's details. *Journal of Portfolio Management* 39, 49–68.

Asparouhova, Elena, Hendrik Bessembinder, and Ivalina Kalcheva, 2013. Noisy prices and inference regarding returns. *Journal of Finance* 68, 665–714.

Bali, Turan G., Stephen J. Brown, Scott Murray, and Yi Tang, 2017. A lottery demand-based explanation of the beta anomaly. *Journal of Financial and Quantitative Analysis* 52, 2369–2397.

Bali, Turan G., Nusret Cakici, and Robert F. Whitelaw, 2011. Maxing out: stocks as lotteries and the cross-section of expected returns. *Journal of Financial Economics* 99, 427–446.

Bali, Turan G., K. Ozgur Demirtas, and Haim Levy, 2009. Is there an intertemporal relation between downside risk and expected returns? *Journal of Financial and Quantitative Analysis* 44, 883-909.

Bali, Turan G., Lin Peng, Yannan Shen, and Yi Tang, 2014. Liquidity shocks and stock market reactions. *Review of Financial Studies* 27, 1434–1485.

Ball, Ray, and Philip Brown, 1968. An empirical evaluation of accounting income numbers. *Journal of Accounting Research* 6, 159-178.

Barber, Brad M., and Terrance Odean, 2008. All that glitters: The effect of attention on the buying behavior of individual and institutional investors. *Review of Financial Studies* 21, 785–818.

Barberis, Nicholas, and Ming Huang, 2008. Stocks as lotteries: the implications of probability weighting for security prices. *American Economic Review* 98, 2066–2100.

Barberis, Nicholas, Abhiroop Mukherjee, and Baolian Wang, 2016. Prospect theory and stock returns: an empirical test. *Review of Financial Studies* 29, 3068–3107.

Barberis, Nicholas, and Wei Xiong, 2009. What drives the disposition effect? An analysis of a long-standing preference-based explanation. *Journal of Finance* 64, 751–784.

Barberis, Nicholas, and Wei Xiong, 2012. Realization utility. *Journal of Financial Economics* 104, 251–271.

Bawa, Vijay S., and Eric B. Lindenberg, 1977. Capital market equilibrium in a mean-lower partial moment framework. *Journal of Financial Economics* 5, 189–200.

Bekaert, Geert, Campbell R. Harvey, and Christian Lundblad, 2007. Liquidity and expected returns: lessons from emerging markets. *Review of Financial Studies* 20, 1783–1831.

Carhart, Mark M., 1997. On persistence in mutual fund performance. *Journal of Finance* 52, 57–82.

Chabi-Yo, Fousseni, Stefan Ruenzi, and Florian Weigert, 2017. Crash sensitivity and the cross-section of expected returns. *Journal of Financial and Quantitative Analysis*, forthcoming.

Cohen, Lauren, and Andrea Frazzini, 2008. Economic links and predictable returns. *Journal of Finance* 63, 1977–2011.

Da, Zhi, Joey Engelberg, and Pengjie Gao, 2011. In search of attention. *Journal of Finance* 66, 1461–1499.

Da, Zhi, Umit Gurun, and Mitch Warachka, 2014. Frog in the pan: continuous information and momentum. *Review of Financial Studies* 27, 2171–2218.

Fama, Eugene F., and Kenneth R. French, 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and Kenneth R. French, 2015. A five-factor asset pricing model. *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and Kenneth R. French, 2017. International test of a five-factor asset pricing model. *Journal of Financial Economics* 123, 441–463.

Fama, Eugene F., and James D. MacBeth, 1973. Risk, return and equilibrium: empirical tests. *Journal of Political Economy* 81, 607–636.

Frazzini, Andrea, 2006. The disposition effect and underreaction to news. *Journal of Finance* 61, 2017–2046.

Frazzini, Andrea, and Lasse H. Pedersen, 2014. Betting against beta. *Journal of Financial Economics* 111, 1–25.

Grinblatt, Mark, and Bing Han, 2005. Prospect theory, mental accounting and momentum. *Journal of Financial Economics* 78, 311–339.

Han, Bing, David Hirshleifer, and Johan Walden, 2017. Social transmission bias and investor behavior. Working Paper, SSRN eLibrary.

Han, Bing, and Alok Kumar, 2013. Speculative retail trading and asset prices. *Journal of Financial and Quantitative Analysis* 48, 377–404.

Harvey, Campbell R., and Akhtar Siddique, 2000. Conditional skewness in asset pricing tests. *Journal of Finance* 55, 1263–1295.

Hirshleifer, David, Kewei Hou, Siew Hong Teoh, and Yinglei Zhang, 2004. Do investors overvalue firms with bloated balance sheets? *Journal of Accounting and Economics* 38, 297–331.

Hirshleifer, David, Po-Hsuan Hsu, and Dongmei Li, 2013. Innovative efficiency and stock returns. *Journal of Financial Economics* 107, 632–654.

Hirshleifer, David, Sonya Seongyeon Lim, and Siew Hong Teoh, 2009. Driven to distraction: extraneous events and underreaction to earnings news. *Journal of Finance* 64, 2289–2325.

Hirshleifer, David, and Siew Hong Teoh, 2003. Limited attention, information disclosure, and financial reporting. *Journal of Accounting and Economics* 36, 337–386.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015. Digesting anomalies: an investment approach. *Review of Financial Studies* 28, 650-705.

Hou, Kewei, G. Andrew Karolyi, and Bong-Chan Kho, 2011. What factors drive global stock returns? *Review of Financial Studies* 24, 2527–2574.

Hou, Kewei, and Tobias Moskowitz, 2005. Market frictions, price delay, and the cross-section of expected returns. *Review of Financial Studies* 18, 981–1020.

Huberman, Gur, and Tomer Regev, 2001. Contagious speculation and a cure for cancer: a non-event that made stock prices soar. *Journal of Finance* 56, 387–396.

Hwang, Byoung-Hyoun, and T. Clifton Green, 2012. IPOs as lotteries: skewness preference and first-day returns. *Management Science* 58, 432–444.

Jegadeesh, Narasimhan, 1990. Evidence of predictable behavior of security returns. *Journal of Finance* 45, 881–898.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993. Returns to buying winners and selling losers: implications for stock market efficiency. *Journal of Finance* 48, 65–91.

Jurado, Kyle, Sydney C. Ludvigson, and Serane Ng, 2015. Measuring uncertainty. *American Economic Review* 105, 1177–1216.

Kahneman, Daniel, and Amos Tversky, 1979. Prospect theory: an analysis of decision under risk. *Econometrica* 47, 263–291.

Karolyi, G. Andrew, Kuan-Hui Lee, and Mathijs A. van Dijk, 2012. Understanding commonality in liquidity around the world. *Journal of Financial Economics* 105, 82–112.

Kelly, Bryan, and Hao Jiang, 2014. Tail risk and asset prices. *Review of Financial Studies* 27, 2841–2871.

Kumar, Alok, 2009. Who gambles in the stock market? *Journal of Finance* 64, 1889–1933.

Kumar, Alok, Jeremy K. Page and Oliver G. Spalt, 2016. Gambling and comovement. *Journal of Financial and Quantitative Analysis* 51, 85–111.

Lee, Kuan-Hui, 2011. The world price of liquidity risk. *Journal of Financial Economics* 99, 136–161.

Lee, Kuan-Hui, and Cheol-Won Yang, 2017. The world price of tail risk. Working Paper, Seoul National University Business School.

Lintner, John, 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics* 47, 13–37.

Markowitz, Harry, 1959. Portfolio selection. New Haven: Yale University Press.

Mossin, Jan, 1966. Equilibrium in a capital asset market. *Econometrica* 34, 768–783.

Newey, Whitney K., and Kenneth D. West, 1987. A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica* 55, 703–708.

Pastor, Lubos, and Robert F. Stambaugh, 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111, 642–685.

Peng, Lin, 2005. Learning with information capacity constraints. *Journal of Financial Quantitative Analysis* 40, 307–329.

Peng, Lin, and Wei Xiong, 2006. Investor attention, overconfidence and category learning. *Journal of Financial Economics* 80, 563–602.

Roy, Arthur D., 1952. Safety first and the holding of assets. *Econometrica* 20, 431–449.

Sharpe, William F., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance* 19, 425–442.

Shefrin, Hersh, and Meir Statman, 1985. The disposition to sell winners too early and rise losers too long. *Journal of Finance* 40, 777–790.

van Oordt, Marten R.C., and Chen Zhou, 2016. Systematic tail risk. *Journal of Financial and Quantitative Analysis* 51, 685–705.

Table 1. Descriptive Statistics and Correlation Matrix

This table presents the descriptive statistics and correlation matrix for various left-tail risk metrics and firm-specific variables. *VaR1* is the 1% value-at-risk that corresponds to -1 times the 1st percentile of daily returns in the past year. *VaR5* is the 5% value-at-risk that corresponds to -1 times the 5th percentile of daily returns in the past year. *ES1* is the 1% expected shortfall and calculated as -1 times the average of the returns below the 1st percentile of daily returns in the past year. *ES5* is the 5% expected shortfall and calculated as -1 times the average of the returns below the 5th percentile of daily returns in the past year. *Beta* is the market beta of each stock with respect to the value-weighted market index calculated from daily returns during the past year. *Size* is the logarithm of market value of equity. *BM* is the ratio of the book value of equity to the market value of equity. *Momentum* is the cumulative return of a stock during the past 11 months after skipping one month. *STR* is the return of a stock in the previous month. *Illiq* is Amihud's illiquidity ratio calculated as the absolute daily return of a stock divided by its daily dollar trading volume averaged over the month. *Coskew* is the co-skewness calculated as the coefficient of the squared excess market return term from a regression of the daily excess returns of a stock on the daily excess market returns and the squared daily excess market returns in the past year. *Betadown* is the downside beta calculated as the sensitivity of each stock towards the value-weighted market index during the days that the excess market return has been below its mean during the past year. *IVOL* is the standard deviation of error terms calculated from a market model which uses daily returns in each month. *MAX* measures lottery demand as the average of the five highest daily returns of each stock in each month. Panel A presents the mean, standard deviation, 25th percentile, median, 75th percentile, minimum, maximum, skewness and kurtosis statistics for each variable. Statistics are computed as the time-series averages of the monthly cross-sectional means. Panel B presents the time-series averages of the monthly cross-sectional correlations between the variables.

Panel A. Descriptive Statistics

	Mean	St Dev	25th Per	Median	75th Per	Min	Max	Skew	Kurt
VaR1	0.06	0.03	0.04	0.06	0.08	0.01	0.26	1.31	8.82
VaR5	0.04	0.01	0.03	0.03	0.05	0.00	0.12	0.74	4.16
ES1	0.08	0.04	0.05	0.07	0.10	0.01	0.42	1.84	10.09
ES5	0.05	0.02	0.04	0.05	0.07	0.01	0.18	0.98	5.02
Beta	0.85	0.58	0.44	0.78	1.19	-0.92	3.31	0.61	3.67
Size	5.21	1.70	3.96	5.05	6.31	0.87	11.55	0.43	2.94
BM	0.77	0.50	0.41	0.68	1.01	0.08	3.25	1.44	6.45
MOM	0.17	0.42	-0.08	0.10	0.32	-0.74	5.44	2.65	27.29
STR	0.01	0.11	-0.05	0.01	0.06	-0.54	0.98	1.01	14.54
Illiq	0.87	2.00	0.04	0.17	0.72	0.00	18.04	4.50	31.35
Coskew	-4.28	18.50	-14.09	-3.66	5.95	-115.36	114.73	-0.10	8.24
Betadown	0.95	0.71	0.46	0.86	1.35	-1.87	4.46	0.56	4.37
IVOL	0.02	0.01	0.01	0.02	0.03	0.00	0.17	2.79	35.39
MAX	0.03	0.02	0.02	0.03	0.04	0.00	0.21	2.18	24.32

Table 1 (continued)

Panel B. Correlation Matrix

	VaR1	VaR5	ES1	ES5	Beta	Size	BM	MOM	STR	Illiq	Coskew	Betadown	IVOL	MAX
VaR1	1.00													
VaR5	0.88	1.00												
ES1	0.85	0.73	1.00											
ES5	0.96	0.94	0.90	1.00										
Beta	0.31	0.35	0.28	0.33	1.00									
Size	-0.22	-0.23	-0.17	-0.23	0.29	1.00								
BM	-0.08	-0.09	-0.09	-0.09	-0.13	-0.20	1.00							
MOM	-0.11	-0.09	-0.15	-0.12	0.07	0.09	0.02	1.00						
STR	-0.08	-0.06	-0.10	-0.08	-0.01	0.06	0.02	0.01	1.00					
Illiq	0.06	0.07	0.05	0.07	-0.08	-0.13	0.08	-0.03	-0.01	1.00				
Coskew	-0.07	-0.06	-0.06	-0.07	0.06	0.11	0.00	-0.07	0.00	-0.01	1.00			
Betadown	0.31	0.33	0.28	0.33	0.79	0.15	-0.10	0.10	-0.01	-0.06	-0.38	1.00		
IVOL	0.63	0.66	0.58	0.67	0.15	-0.26	-0.06	-0.03	0.01	0.09	-0.07	0.18	1.00	
MAX	0.59	0.64	0.51	0.62	0.23	-0.15	-0.06	-0.02	0.34	0.07	-0.05	0.23	0.87	0.88

Table 2. Univariate Portfolio Analysis

This table presents return comparisons between equity deciles formed monthly based on VaR1 for the stocks in our sample. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports the one-month-ahead excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panel A presents results for value-weighted portfolio returns. Panel B presents results for equal-weighted portfolio returns. VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Value-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.47	0.61	0.58	0.56	0.57	0.57	0.62	0.51	0.31	-0.31	-0.78
	(3.28)	(3.59)	(3.14)	(2.72)	(2.49)	(2.20)	(2.25)	(1.70)	(0.90)	(-0.77)	(-2.34)
FFCPS Alpha	0.07	0.09	0.02	-0.07	-0.05	-0.08	-0.01	-0.16	-0.39	-0.87	-0.94
	(0.92)	(1.62)	(0.35)	(-0.99)	(-0.56)	(-0.88)	(-0.12)	(-1.34)	(-3.16)	(-5.02)	(-4.42)
FF5 Alpha	-0.03	0.02	-0.08	-0.09	-0.06	0.00	0.10	-0.03	-0.15	-0.65	-0.63
	(-0.43)	(0.32)	(-1.66)	(-1.31)	(-0.81)	(-0.05)	(1.07)	(-0.31)	(-1.35)	(-4.52)	(-3.75)

Panel B. Equal-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.71	0.81	0.85	0.88	0.92	0.89	0.88	0.68	0.52	0.05	-0.66
	(4.73)	(4.65)	(4.42)	(4.20)	(4.01)	(3.59)	(3.37)	(2.36)	(1.64)	(0.15)	(-2.34)
FFCPS Alpha	0.25	0.25	0.21	0.19	0.19	0.13	0.11	-0.09	-0.22	-0.56	-0.80
	(2.99)	(3.63)	(3.19)	(3.11)	(3.07)	(1.99)	(1.93)	(-1.36)	(-2.85)	(-5.50)	(-5.20)
FF5 Alpha	0.13	0.10	0.06	0.05	0.05	0.01	0.03	-0.09	-0.18	-0.49	-0.62
	(1.84)	(1.75)	(1.04)	(0.90)	(0.78)	(0.25)	(0.56)	(-1.44)	(-2.17)	(-4.35)	(-4.56)

Table 3. Portfolio Characteristics

This table presents portfolio characteristics for equity deciles formed monthly based on VaR1 for the stocks in our sample. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports the time-series averages of the monthly averages for VaR1 and various firm-specific attributes for each decile. The last column shows the differences for the firm-specific attributes between deciles 10 and 1 and the associated t-statistics. VaR1 and the firm-specific attributes are defined in Table 1.

	Port1	**Port2**	**Port3**	**Port4**	**Port5**	**Port6**	**Port7**	**Port8**	**Port9**	**Port10**	**High-Low**
VaR1	0.0268	0.0366	0.0428	0.0485	0.0543	0.0605	0.0677	0.0764	0.0881	0.1168	0.0900
											(99.43)
Beta	0.40	0.58	0.67	0.74	0.81	0.88	0.97	1.06	1.16	1.26	0.86
											(55.79)
Size	6.05	6.05	5.82	5.54	5.30	5.08	4.86	4.68	4.47	4.19	-1.86
											(-48.11)
BM	0.92	0.85	0.82	0.82	0.84	0.84	0.84	0.82	0.78	0.75	-0.22
											(-14.41)
MOM	0.16	0.17	0.17	0.18	0.18	0.18	0.19	0.18	0.16	0.08	-0.08
											(-6.30)
STR	0.01	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	-0.01	-0.02
											(-8.54)
Illiq	0.45	0.51	0.64	0.78	0.95	1.10	1.28	1.57	1.90	3.35	2.90
											(22.15)
Coskew	-0.83	-1.73	-2.30	-2.94	-3.63	-4.31	-5.06	-5.83	-7.14	-9.08	-8.25
											(-23.21)
Betadown	0.42	0.61	0.72	0.80	0.88	0.98	1.08	1.20	1.33	1.48	1.05
											(57.72)
IVOL	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.04	0.03
											(81.18)
MAX	0.01	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.04	0.05	0.04
											(70.75)

Table 4. Bivariate Portfolio Analysis

This table presents results from the value-weighted bivariate portfolios based on dependent double sorts of various firm-specific attributes and VaR1. For the dependent sorts, first, decile portfolios are formed every month based on a firm-specific attribute for the stocks in our sample. Next, additional decile portfolios are formed based on VaR1 within each firm-specific attribute decile. Portfolio 1 is the combined portfolio of stocks with the lowest value-at-risk in each firm-specific attribute decile. Portfolio 10 is the combined portfolio of stocks with the highest value-at-risk in each firm-specific attribute decile. Panels A and B report one-month-ahead five-factor FFCPS and FF5 alphas for each decile, respectively. The last columns in each panel show the differences of monthly alphas between VaR1 deciles 10 and 1 for each firm-specific attribute. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 and firm-specific attributes are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. FFCPS Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Beta	0.04	0.04	0.02	0.02	0.04	-0.11	-0.17	-0.25	-0.28	-0.66	-0.69
	(0.64)	(0.63)	(0.45)	(0.33)	(0.55)	(-1.48)	(-2.04)	(-3.31)	(-3.61)	(-7.31)	(-6.43)
Size	0.26	0.28	0.24	0.23	0.18	0.11	0.03	-0.05	-0.24	-0.58	-0.84
	(3.15)	(3.58)	(3.47)	(3.35)	(2.82)	(1.80)	(0.45)	(-0.79)	(-3.19)	(-6.00)	(-5.42)
BM	0.11	0.08	0.10	0.07	0.03	0.07	0.05	0.02	-0.37	-0.60	-0.71
	(1.37)	(1.40)	(1.59)	(0.92)	(0.38)	(0.85)	(0.45)	(0.13)	(-2.92)	(-3.56)	(-3.43)
MOM	0.15	0.07	0.07	0.03	-0.01	-0.22	-0.11	-0.17	-0.40	-0.67	-0.83
	(2.17)	(1.17)	(1.13)	(0.40)	(-0.15)	(-2.87)	(-1.40)	(-2.01)	(-3.63)	(-5.33)	(-4.83)
STR	0.10	0.08	0.02	0.01	-0.03	0.02	-0.05	-0.19	-0.29	-0.75	-0.85
	(1.56)	(1.41)	(0.35)	(0.16)	(-0.51)	(0.20)	(-0.56)	(-2.15)	(-2.58)	(-5.78)	(-5.01)
Illiq	0.13	0.11	0.12	0.09	0.06	0.00	-0.07	-0.22	-0.35	-0.71	-0.84
	(1.64)	(1.53)	(1.77)	(1.41)	(0.94)	(-0.04)	(-1.37)	(-3.61)	(-5.07)	(-8.08)	(-5.93)
Coskew	0.07	0.03	0.03	-0.03	-0.10	-0.05	-0.13	-0.19	-0.42	-0.67	-0.73
	(0.94)	(0.72)	(0.66)	(-0.50)	(-1.48)	(-0.75)	(-1.43)	(-1.90)	(-3.73)	(-5.38)	(-4.47)
Betadown	0.08	0.03	0.02	0.05	-0.05	-0.08	-0.05	-0.27	-0.34	-0.72	-0.81
	(1.60)	(0.50)	(0.36)	(0.70)	(-0.75)	(-1.27)	(-0.68)	(-3.48)	(-4.53)	(-7.59)	(-7.31)
IVOL	-0.03	-0.08	-0.16	-0.11	-0.24	-0.17	-0.22	-0.34	-0.26	-0.49	-0.46
	(-0.34)	(-1.26)	(-2.76)	(-1.86)	(-3.57)	(-2.22)	(-2.79)	(-3.95)	(-2.81)	(-5.02)	(-3.46)
MAX	0.03	-0.05	-0.09	-0.14	-0.13	-0.12	-0.15	-0.23	-0.31	-0.38	-0.41
	(0.53)	(-1.03)	(-1.53)	(-1.95)	(-1.89)	(-1.63)	(-1.77)	(-2.63)	(-3.25)	(-3.84)	(-3.09)

Table 4 (continued)

Panel B. FF5 Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Beta	-0.02	-0.03	-0.06	-0.06	-0.06	-0.18	-0.22	-0.28	-0.31	-0.68	-0.67
	(-0.31)	(-0.50)	(-1.17)	(-0.99)	(-0.91)	(-2.78)	(-3.13)	(-4.23)	(-4.43)	(-8.03)	(-7.11)
Size	0.15	0.13	0.11	0.07	0.03	-0.02	-0.06	-0.05	-0.21	-0.51	-0.65
	(2.04)	(1.90)	(1.80)	(1.16)	(0.42)	(-0.30)	(-1.13)	(-0.93)	(-2.82)	(-5.01)	(-4.95)
BM	-0.06	-0.10	-0.02	-0.05	-0.04	-0.06	-0.06	-0.12	-0.37	-0.59	-0.55
	(-0.86)	(-1.62)	(-0.31)	(-0.65)	(-0.47)	(-0.62)	(-0.57)	(-0.87)	(-2.91)	(-3.44)	(-2.91)
MOM	-0.02	-0.10	-0.09	-0.11	-0.15	-0.26	-0.12	-0.17	-0.39	-0.50	-0.48
	(-0.34)	(-1.72)	(-1.60)	(-1.62)	(-2.16)	(-3.62)	(-1.42)	(-2.10)	(-4.00)	(-4.54)	(-3.52)
STR	0.01	0.03	0.00	-0.02	-0.05	0.01	0.04	-0.08	-0.13	-0.64	-0.65
	(0.23)	(0.61)	(0.06)	(-0.45)	(-0.86)	(0.20)	(0.48)	(-1.01)	(-1.35)	(-5.50)	(-4.63)
Illiq	0.01	-0.03	-0.07	-0.08	-0.10	-0.12	-0.14	-0.21	-0.31	-0.66	-0.67
	(0.10)	(-0.53)	(-1.23)	(-1.25)	(-1.71)	(-2.26)	(-2.52)	(-3.47)	(-4.04)	(-7.09)	(-5.56)
Coskew	0.01	-0.04	-0.01	-0.08	-0.16	0.00	-0.08	-0.07	-0.27	-0.54	-0.55
	(0.20)	(-0.88)	(-0.12)	(-1.40)	(-2.59)	(0.02)	(-0.93)	(-0.75)	(-2.86)	(-4.79)	(-3.92)
Betadown	0.05	0.02	0.00	-0.05	-0.08	-0.18	-0.14	-0.32	-0.42	-0.74	-0.79
	(1.26)	(0.54)	(-0.05)	(-0.90)	(-1.41)	(-2.90)	(-2.12)	(-4.32)	(-5.41)	(-8.25)	(-8.67)
IVOL	-0.08	-0.11	-0.16	-0.05	-0.16	-0.10	-0.12	-0.25	-0.16	-0.45	-0.37
	(-1.41)	(-1.83)	(-2.95)	(-0.98)	(-2.41)	(-1.55)	(-1.53)	(-2.96)	(-1.89)	(-5.23)	(-3.31)
MAX	-0.01	-0.01	-0.09	-0.08	-0.06	-0.06	-0.11	-0.16	-0.28	-0.38	-0.37
	(-0.12)	(-0.26)	(-1.87)	(-1.28)	(-0.89)	(-0.84)	(-1.43)	(-1.91)	(-3.12)	(-3.91)	(-3.05)

Table 5. Firm-Level Cross-Sectional Regression Analysis

This table presents results from the cross-sectional regressions of future equity returns on VaR1 and various control variables. Panel A presents results estimated for one-month-ahead returns using the ordinary least squares (OLS) methodology. Panel B presents results estimated for one-month-ahead returns using a weighted least squares (WLS) methodology following Asparouhova, Bessembinder and Kalcheva (2013) where each observed return is weighted by one plus the observed prior return on the stock. Reported coefficients are time-series averages from monthly Fama-MacBeth (1973) regressions and the associated t-statistics are reported using the Newey-West (1987) procedure. Average R-squared statistics for each regression are presented in the last row. VaR1 and firm-specific characteristics are defined in Table 1.

Panel A. OLS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
VaR1	-0.0782	-0.0634	-0.1244	-0.0907	-0.0683	-0.0826	-0.0881	-0.0868	-0.0827	-0.0455	-0.0458
	(-2.49)	(-2.52)	(-7.25)	(-4.47)	(-3.47)	(-4.04)	(-4.14)	(-4.11)	(-3.89)	(-2.32)	(-2.32)
Beta		-0.0005	0.0019	0.0013	0.0000	0.0002	0.0000	0.0001	0.0014	0.0015	0.0016
		(-0.38)	(1.36)	(0.99)	(0.02)	(0.14)	(0.01)	(0.07)	(0.81)	(0.85)	(0.99)
Size			-0.0015	-0.0010	-0.0009	-0.0008	-0.0009	-0.0009	-0.0009	-0.0010	-0.0010
			(-4.78)	(-3.54)	(-3.46)	(-2.79)	(-3.05)	(-3.02)	(-2.91)	(-3.41)	(-3.53)
BM				0.0008	0.0009	0.0014	0.0014	0.0015	0.0015	0.0015	0.0014
				(1.60)	(1.87)	(2.20)	(2.10)	(2.16)	(2.19)	(2.26)	(2.17)
MOM					0.0050	0.0016	0.0013	0.0011	0.0011	0.0012	0.0012
					(2.85)	(0.65)	(0.55)	(0.46)	(0.45)	(0.48)	(0.49)
STR						-0.0718	-0.0728	-0.0727	-0.0725	-0.0704	-0.0656
						(-7.19)	(-7.23)	(-7.26)	(-7.28)	(-7.00)	(-6.69)
Illiq							-0.0001	-0.0001	-0.0001	-0.0001	-0.0001
							(-0.40)	(-0.44)	(-0.51)	(-0.47)	(-0.37)
Coskew								0.0000	-0.0001	-0.0001	0.0000
								(-0.36)	(-1.27)	(-1.19)	(-0.98)
Betadown									-0.0015	-0.0014	-0.0011
									(-1.54)	(-1.47)	(-1.21)
IVOL										-0.1438	-0.0517
										(-5.14)	(-0.65)
MAX											-0.0826
											(-1.41)
Intercept	0.0119	0.0114	0.0196	0.0149	0.0130	0.0128	0.0139	0.0137	0.0135	0.0149	0.0151
	(7.09)	(6.87)	(7.82)	(5.87)	(5.38)	(4.97)	(5.19)	(5.10)	(5.00)	(5.50)	(5.62)
Avg. R^2	0.0251	0.0429	0.0489	0.0541	0.0646	0.0810	0.0866	0.0891	0.0915	0.0946	0.0971

Table 5 (continued)

Panel B. WLS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
VaR1	-0.0987	-0.0898	-0.1468	-0.1159	-0.0917	-0.0888	-0.0931	-0.0910	-0.0870	-0.0672	-0.0656
	(-3.10)	(-3.55)	(-8.24)	(-5.73)	(-4.64)	(-4.35)	(-4.37)	(-4.32)	(-4.10)	(-3.35)	(-3.27)
Beta		-0.0001	0.0021	0.0015	0.0002	0.0004	0.0002	0.0002	0.0014	0.0014	0.0018
		(-0.06)	(1.55)	(1.17)	(0.17)	(0.29)	(0.13)	(0.16)	(0.84)	(0.81)	(1.07)
Size			-0.0014	-0.0009	-0.0009	-0.0008	-0.0009	-0.0009	-0.0009	-0.0010	-0.0010
			(-4.51)	(-3.30)	(-3.24)	(-2.78)	(-3.09)	(-3.07)	(-2.99)	(-3.30)	(-3.40)
BM				0.0009	0.0010	0.0013	0.0014	0.0015	0.0015	0.0016	0.0015
				(1.77)	(2.06)	(2.11)	(2.09)	(2.17)	(2.19)	(2.30)	(2.21)
MOM					0.0057	0.0024	0.0021	0.0019	0.0019	0.0019	0.0020
					(3.21)	(1.00)	(0.88)	(0.78)	(0.78)	(0.79)	(0.81)
STR						-0.0670	-0.0683	-0.0681	-0.0679	-0.0658	-0.0582
						(-6.69)	(-6.77)	(-6.79)	(-6.82)	(-6.50)	(-5.91)
Illiq							-0.0001	-0.0002	-0.0002	-0.0002	-0.0001
							(-0.82)	(-0.88)	(-0.97)	(-1.02)	(-0.91)
Coskew								0.0000	0.0000	0.0000	0.0000
								(-0.06)	(-1.10)	(-1.01)	(-0.95)
Betadown									-0.0014	-0.0013	-0.0011
									(-1.44)	(-1.35)	(-1.23)
IVOL										-0.0790	0.0700
										(-3.00)	(0.94)
MAX											-0.1334
											(-2.36)
Intercept	0.0124	0.0119	0.0197	0.0150	0.0131	0.0131	0.0142	0.0140	0.0138	0.0146	0.0148
	(7.56)	(7.28)	(7.63)	(5.85)	(5.36)	(4.99)	(5.26)	(5.19)	(5.11)	(5.43)	(5.54)
Avg. R^2	0.0246	0.0425	0.0484	0.0541	0.0649	0.0821	0.0877	0.0903	0.0927	0.0958	0.0983

Table 6. Transition Matrix

This table presents transition probabilities for VaR1 for the stocks in our sample at a lag of 12 months. At each month *t*, all stocks are sorted into deciles based on an ascending ordering of VaR1. The procedure is repeated in month *t*+12. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. For each VaR1 decile in month *t*, the percentage of stocks that fall into each of the month *t*+12 VaR1 decile is calculated. The panels present the time-series averages of these transition probabilities. Each row corresponds to a different month *t* VaR1 portfolio and each column corresponds to a different month *t*+12 VaR1 portfolio. VaR1 is defined in Table 1.

	Port1	**Port2**	**Port3**	**Port4**	**Port5**	**Port6**	**Port7**	**Port8**	**Port9**	**Port10**
Port1	52%	22%	11%	6%	4%	2%	1%	1%	1%	1%
Port2	23%	26%	19%	12%	8%	5%	3%	2%	1%	1%
Port3	12%	21%	20%	16%	12%	8%	5%	3%	2%	1%
Port4	6%	14%	18%	17%	14%	11%	8%	5%	4%	2%
Port5	4%	9%	14%	16%	16%	14%	11%	8%	6%	3%
Port6	2%	6%	9%	13%	16%	15%	14%	11%	8%	6%
Port7	1%	3%	6%	10%	13%	15%	16%	15%	12%	9%
Port8	1%	2%	4%	6%	10%	14%	17%	18%	16%	13%
Port9	0%	1%	2%	4%	7%	11%	15%	19%	21%	20%
Port10	0%	1%	1%	2%	4%	7%	11%	17%	24%	33%

Table 7. Delta VaR Analysis

This table presents value-weighted return comparisons between equity quintiles formed based on the monthly change in VaR1 (Delta VaR) for the stocks in our sample. Delta VaR is defined as the difference between VaR1 in month *t* and month *t*-1. Lagged Delta VaR is defined as the difference between VaR1 in month *t*-1 and month *t*-2. In Panel A, value-at-risk quintiles are formed each month and stocks are classified based on whether their Delta VaR is negative, zero or positive into three groups in each VaR1 quintile. In Panel B, the stocks are classified based on whether their Delta VaR and Lagged Delta VaR are negative, zero or positive into nine groups in each VaR1 quintile. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 5 is the portfolio of stocks with the highest value-at-risk. The table reports the one-month-ahead value-weighted excess returns for each quintile. The last three columns show the differences of monthly excess returns and two distinct alpha metrics. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Sorts based on Delta VaR

	Port1	Port2	Port3	Port4	Port5	High-Low	FFCPS Alpha	FF5 Alpha
DeltaVaR<0	0.62	0.57	0.83	0.78	0.07	-0.55	-0.92	-0.45
						(-1.61)	(-3.74)	(-2.18)
DeltaVaR=0	0.49	0.56	0.61	0.66	0.15	-0.34	-0.50	-0.25
						(-1.22)	(-3.16)	(-1.90)
DeltaVaR>0	0.53	0.61	0.54	0.15	-0.36	-0.90	-1.04	-0.79
						(-2.72)	(-3.92)	(-3.50)

Panel B. Sorts based on Delta VaR and Lagged Delta VaR

	Lagged Delta VaR	Port1	Port2	Port3	Port4	Port5	High-Low	FFCPS Alpha	FF5 Alpha
DeltaVaR<0	Negative	0.58	0.86	0.82	0.90	0.60	0.06	-0.42	0.23
							(0.14)	(-0.96)	(0.56)
	Zero	0.49	0.61	0.77	0.77	-0.06	-0.56	-0.77	-0.41
							(-1.49)	(-2.45)	(-1.74)
	Positive	0.59	0.51	0.57	0.04	-0.25	-0.83	-1.06	-0.60
							(-1.59)	(-1.86)	(-1.16)
DeltaVaR=0	Negative	0.57	0.58	0.73	0.74	0.37	-0.20	-0.51	-0.25
							(-0.56)	(-1.51)	(-0.71)
	Zero	0.45	0.53	0.65	0.67	0.20	-0.26	-0.39	-0.16
							(-0.88)	(-2.18)	(-1.01)
	Positive	0.48	0.67	0.60	0.44	-0.30	-0.78	-0.86	-0.74
							(-0.88)	(-3.31)	(-3.30)
DeltaVaR>0	Negative	0.64	0.65	0.62	0.53	0.34	-0.30	-0.52	-0.10
							(-0.67)	(-1.26)	(-0.26)
	Zero	0.53	0.64	0.56	0.16	-0.44	-0.95	-1.05	-0.88
							(-2.94)	(-3.96)	(-3.51)
	Positive	0.73	0.99	0.31	0.23	-0.54	-1.32	-1.43	-1.11
							(-3.33)	(-3.92)	(-3.34)

Table 8. Institutional Ownership

This table presents the institutional ownership level for equity deciles formed monthly based on VaR1 and the results of bivariate portfolio analyses of the relation between one-month-ahead equity returns and value-at-risk after controlling for the level of institutional ownership. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. Panel A presents the average institutional holdings for each decile where *INST* denotes the fraction of total shares outstanding that are owned by institutional investors as of the end of the last fiscal quarter end or prior to month *t*. Panel B presents the average change in institutional holdings for each decile for time horizons between 3 months and 12 months. Panel C presents the one-month-ahead value-weighted excess returns to portfolios that are independently sorted with respect to VaR1 deciles and institutional holdings quintiles. The last six rows of Panel C show the differences of monthly excess returns and two distinct alphas between VaR deciles 10 and 1 within each institutional holdings quintile. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Level of Institutional Holding

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High - Low	t-stat
INST	0.42	0.48	0.48	0.48	0.47	0.46	0.45	0.44	0.41	0.36	-0.05	(-3.90)

Panel B. Change in Institutional Holding

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low	t-stat
△ INST (3 months)	0.29	0.36	0.47	0.54	0.55	0.66	0.69	0.62	0.53	-0.04	-0.34	(-3.12)
△ INST (6 months)	0.53	0.69	0.87	1.02	0.98	1.13	1.99	0.36	0.94	0.02	-0.51	(-2.93)
△ INST (9 months)	0.79	1.02	1.27	1.40	1.47	1.61	1.65	1.58	1.34	0.27	-0.52	(-2.34)
△ INST (12 months)	1.04	1.35	1.65	1.78	1.89	2.04	1.21	2.80	1.72	0.58	-0.46	(-1.79)

Panel C. Returns to Double-Sorted Portfolios

	INST1	INST2	INST3	INST4	INST5
Port1	0.61	1.03	0.72	0.77	0.70
Port2	0.38	0.86	0.99	0.78	0.85
Port3	0.80	0.59	1.17	0.80	0.74
Port4	0.42	0.96	0.91	0.92	0.79
Port5	0.52	0.71	0.90	0.90	0.56
Port6	0.58	0.50	0.55	0.82	0.89
Port7	0.73	0.58	0.91	0.74	0.85
Port8	0.56	0.60	0.90	0.72	0.96
Port9	0.38	0.48	0.40	0.39	0.59
Port10	-0.79	-0.19	-0.17	0.00	0.00
High-Low	-1.40	-1.22	-0.89	-0.78	-0.69
	(-3.13)	(-2.40)	(-1.83)	(-1.91)	(-1.85)
FFCPS Alpha	-1.51	-1.55	-0.90	-0.71	-0.77
	(-3.76)	(-3.47)	(-2.60)	(-2.64)	(-2.66)
FF5 Alpha	-0.93	-0.88	-0.54	-0.34	-0.44
	(-2.45)	(-1.96)	(-1.61)	(-1.25)	(-1.56)

Table 9. Limited Attention of Individual Investors

This table presents results from the value-weighted portfolios based on bivariate sorts of various firm-specific attributes and VaR1. First, tercile portfolios are formed every month based on a firm-specific attribute for the stocks in our sample. Next, additional quintile portfolios are formed based on VaR1 within each firm-specific attribute tercile. The table reports one-month-ahead five-factor FFCPS alphas for each quintile. The last rows in each panel show the differences of monthly alphas between VaR1 quintiles 5 and 1 for each firm-specific attribute tercile. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). Panels A, B, C, D and E present results for analyst coverage, firm size, illiquidity, idiosyncratic volatility and lottery demand, respectively. VaR1 and firm-specific attributes are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Analyst Coverage

	CVRG1	**CVRG2**	**CVRG3**
Port1	0.16	0.16	0.13
	(1.37)	(1.62)	(1.33)
Port2	-0.13	0.00	-0.01
	(-1.05)	(0.00)	(-0.08)
Port3	-0.15	-0.01	0.12
	(-1.13)	(-0.08)	(1.09)
Port4	-0.43	-0.11	0.00
	(-2.83)	(-0.71)	(0.01)
Port5	-0.81	-0.72	-0.15
	(-5.38)	(-2.72)	(-0.64)
High-Low	-0.97	-0.88	-0.22
	(-4.62)	(-2.77)	(-0.74)

Panel B. Firm Size

	SIZE1	**SIZE2**	**SIZE3**
Port1	0.47	0.23	0.03
	(4.22)	(3.04)	(0.39)
Port2	0.38	0.17	0.08
	(3.62)	(2.31)	(1.74)
Port3	0.19	0.12	-0.03
	(1.67)	(1.71)	(-0.50)
Port4	-0.07	-0.07	0.00
	(-0.69)	(-1.04)	(-0.01)
Port5	-0.46	-0.58	-0.28
	(-4.36)	(-5.57)	(-2.11)
High-Low	-0.93	-0.81	-0.31
	(-6.55)	(-5.23)	(-1.61)

Table 9 (continued)

Panel C. Illiquidity

	ILLIQ1	ILLIQ2	ILLIQ3
Port1	0.04	0.10	0.16
	(0.59)	(1.38)	(1.73)
Port2	0.04	0.09	0.14
	(0.90)	(1.23)	(1.34)
Port3	-0.01	0.05	0.01
	(-0.25)	(0.68)	(0.11)
Port4	-0.04	-0.12	-0.21
	(-0.41)	(-1.71)	(-1.88)
Port5	-0.36	-0.59	-0.66
	(-2.51)	(-5.52)	(-5.95)
High-Low	-0.40	-0.69	-0.83
	(-2.03)	(-4.61)	(-6.57)

Panel D. Idiosyncratic Volatility

	IVOL1	IVOL2	IVOL3
Port1	0.11	0.05	-0.13
	(1.23)	(0.61)	(-0.94)
Port2	0.01	-0.07	-0.28
	(0.19)	(-1.05)	(-1.95)
Port3	0.08	-0.07	-0.54
	(1.17)	(-0.76)	(-3.10)
Port4	-0.07	0.10	-0.83
	(-0.92)	(0.91)	(-4.28)
Port5	0.02	-0.13	-1.30
	(0.25)	(-1.01)	(-6.95)
High-Low	-0.08	-0.18	-1.17
	(-0.57)	(-1.05)	(-5.02)

Panel E. Lottery Demand

	MAX1	MAX2	MAX3
Port1	0.09	0.07	-0.15
	(1.09)	(1.06)	(-1.22)
Port2	0.02	-0.15	-0.17
	(0.20)	(-2.34)	(-1.24)
Port3	0.15	-0.06	-0.41
	(2.16)	(-0.63)	(-2.48)
Port4	0.17	0.00	-0.71
	(2.02)	(-0.01)	(-4.21)
Port5	0.15	-0.17	-1.38
	(1.30)	(-1.17)	(-6.65)
High-Low	0.06	-0.24	-1.23
	(0.41)	(-1.37)	(-5.21)

Table 10. Firm-Level Cross-Sectional Regressions with Institutional Ownership Interaction

This table presents results from the cross-sectional regressions of future equity returns on VaR1 and various control variables. Regressions are estimated for one-month-ahead returns using the ordinary least squares (OLS) methodology. Reported coefficients are time-series averages from monthly Fama-MacBeth (1973) regressions and the associated t-statistics are reported using the Newey-West (1987) procedure. Average R-squared statistics for each regression are presented in the last row. VaR1 and firm-specific characteristics are defined in Table 1. OINST denotes the fraction of total shares outstanding that are owned by institutional investors as of the end of the last fiscal quarter end or prior to month *t* orthogonalized by the logarithm of market value of equity (SIZE). VaR1×OINST denotes the interaction term between this variable and value-at-risk.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
VaR1	-0.1061	-0.0936	-0.1222	-0.1164	-0.0940	-0.1056	-0.1052	-0.1049	-0.1039	-0.0577	-0.0562
	(-3.30)	(-3.86)	(-5.57)	(-5.95)	(-5.07)	(-5.50)	(-5.28)	(-5.45)	(-5.35)	(-3.30)	(-3.26)
OINST	-0.0038	-0.0034	-0.0043	-0.0050	-0.0066	-0.0057	-0.0055	-0.0054	-0.0055	-0.0055	-0.0055
	(-1.20)	(-1.19)	(-1.47)	(-1.67)	(-2.20)	(-1.84)	(-1.75)	(-1.72)	(-1.75)	(-1.74)	(-1.75)
VaR1×OINST	0.1063	0.0962	0.1030	0.1062	0.1545	0.1330	0.1336	0.1292	0.1293	0.1271	0.1254
	(1.98)	(1.91)	(2.02)	(2.02)	(3.02)	(2.56)	(2.55)	(2.48)	(2.49)	(2.44)	(2.44)
Beta		-0.0002	0.0015	0.0017	0.0000	0.0004	0.0002	0.0003	0.0024	0.0025	0.0028
		(-0.10)	(0.84)	(0.91)	(0.01)	(0.20)	(0.10)	(0.14)	(1.05)	(1.10)	(1.30)
Size			-0.0009	-0.0009	-0.0009	-0.0008	-0.0009	-0.0009	-0.0009	-0.0011	-0.0011
			(-2.92)	(-2.86)	(-2.83)	(-2.53)	(-2.79)	(-2.85)	(-2.72)	(-3.31)	(-3.47)
BM				0.0004	0.0005	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005
				(1.25)	(1.63)	(1.70)	(1.92)	(1.85)	(1.87)	(1.68)	(1.66)
MOM					0.0061	0.0059	0.0058	0.0058	0.0058	0.0059	0.0060
					(3.39)	(3.01)	(2.94)	(2.93)	(2.98)	(3.01)	(3.09)
STR						-0.0282	-0.0282	-0.0283	-0.0284	-0.0264	-0.0248
						(-6.43)	(-6.41)	(-6.50)	(-6.52)	(-6.08)	(-4.46)
Illiq							-0.0002	-0.0003	-0.0003	-0.0002	-0.0002
							(-2.01)	(-2.07)	(-2.10)	(-1.47)	(-1.36)
Coskew								0.0000	-0.0001	-0.0001	0.0000
								(0.06)	(-1.02)	(-0.95)	(-0.69)
Betadown									0.0023	0.0022	-0.0017
									(-1.74)	(-1.68)	(-1.36)
IVOL										-0.1761	-0.1742
										(-6.39)	(-2.94)
MAX											-0.0207
											(-4.04)
Intercept	0.0139	0.0132	0.0186	0.0180	0.0161	0.0164	0.0174	0.0174	0.0170	0.0189	0.0193
	(6.42)	(6.08)	(6.12)	(6.24)	(5.88)	(5.78)	(5.95)	(5.93)	(5.81)	(6.35)	(6.53)
Avg. R^2	0.0284	0.0454	0.0497	0.0488	0.0570	0.0625	0.0643	0.0659	0.0674	0.0691	0.0707

Table 11. Testing Alternative Explanations of Left-Tail Momentum

This table presents results from the time-series regressions of the monthly value-weighted excess return differences between extreme VaR1 deciles on various asset pricing factors. VaR1 is defined in Table 1. Portfolio 1 (Portfolio 10) is the portfolio of stocks with the lowest (highest) value-at-risk. The dependent variable in each regression is the excess return of Portfolio 1 minus the excess return of Portfolio 10 calculated monthly. In Panel A, the baseline FFCPS model includes the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). In Panel B, the baseline FF5 model includes the market, size, value, profitability and investment factors of Fama and French (2015). In Panel C, the baseline Q-model includes the market, size, investment and profitability factors of Hou, Xue and Zhang (2015). The results for these baseline models are presented in the first column of each panel. The second column of each panel augments the baseline models by the betting against beta (BAB) factor of Frazzini and Pedersen (2014). The third column of each panel augments the baseline models by a factor that captures lottery demand (FMAX). The fourth column of each panel augments the baseline models by a factor that captures idiosyncratic volatility (FIVOL). FMAX (FIVOL) factor is calculated based on 2x3 independent sorts on firm size and MAX (IVOL) using NYSE breakpoints, respectively. The table presents the intercepts, coefficient estimates and the associated Newey-West (1982) adjusted t-statistics. R-squared statistics for each regression are presented in the last row.

Panel A. Extending the FFCPS Model with BAB, MAX, and IVOL Factors

	(1)	(2)	(3)	(4)
Constant	-0.0094	-0.0065	-0.0041	-0.0042
	(-4.42)	(-2.76)	(-2.78)	(-2.93)
MKT	0.6208	0.6579	0.1246	0.0744
	(7.71)	(10.67)	(2.60)	(1.48)
SMB	1.2077	1.2163	0.6109	0.4116
	(12.12)	(14.27)	(8.48)	(5.07)
HML	-0.7539	-0.4508	-0.1997	-0.1358
	(-5.87)	(-4.60)	(-3.26)	(-2.30)
WML	-0.3088	-0.1939	-0.2019	-0.1708
	(-3.60)	(-3.08)	(-3.91)	(-3.42)
PS	-0.1057	-0.0771	-0.0487	-0.1042
	(-1.55)	(-1.33)	(-1.09)	(-2.04)
BAB		-0.5707		
		(-4.82)		
FMAX			1.3102	
			(16.10)	
FIVOL				1.3976
				(17.87)
R^2	0.7124	0.7573	0.8506	0.8557

Table 11 (continued)

Panel B. Extending the FF5 Model with BAB, MAX, and IVOL Factors

	(1)	(2)	(3)	(4)
Constant	-0.0063	-0.0042	-0.0038	-0.0044
	(-3.75)	(-2.35)	(-2.74)	(-3.19)
MKT	0.5356	0.5769	0.1401	0.0943
	(7.79)	(11.44)	(2.80)	(1.80)
SMB	0.9821	1.0342	0.5751	0.3867
	(10.09)	(10.94)	(7.08)	(4.45)
HML	-0.4006	-0.2640	-0.0861	-0.0731
	(-3.12)	(-2.44)	(-1.11)	(-0.81)
RMW	-1.0575	-0.8313	-0.3718	-0.2875
	(-7.47)	(-6.28)	(-3.11)	(-2.70)
CMA	-0.8722	-0.6783	-0.3670	-0.2164
	(-4.69)	(-4.75)	(-3.00)	(-1.73)
BAB		-0.4977		
		(-4.95)		
FMAX			1.2207	
			(15.63)	
FIVOL				1.3333
				(15.70)
R^2	0.7494	0.7820	0.8430	0.8483

Panel C. Extending the Q Factor Model with BAB, MAX, and IVOL Factors

	(1)	(2)	(3)	(4)
Constant	-0.0057	-0.0039	-0.0038	-0.0049
	(-2.57)	(-1.82)	(-2.41)	(-3.07)
MKT	0.6565	0.6811	0.1386	0.0950
	(8.52)	(12.01)	(2.63)	(1.69)
ME	0.8753	0.9597	0.4685	0.2874
	(6.81)	(9.17)	(5.81)	(3.20)
IA	-1.3338	-0.9462	-0.4695	-0.2440
	(-7.66)	(-6.52)	(-4.49)	(-2.67)
ROE	-0.7249	-0.5002	-0.2729	-0.2066
	(-4.38)	(-3.83)	(-3.03)	(-2.12)
BAB		-0.5875		
		(-4.74)		
FMAX			1.3289	
			(14.25)	
FIVOL				1.4427
				(15.97)
R^2	0.6894	0.7401	0.8433	0.8478

Table 12. Omitting Earnings Announcement Returns

This table presents return comparisons between equity deciles formed monthly based on VaR1 for the stocks in our sample. VaR1 measures are calculated after omitting the earnings announcement returns from the sample. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports the one-month-ahead excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015. VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.46	0.59	0.59	0.54	0.53	0.66	0.54	0.56	0.30	-0.19	-0.65
	(3.13)	(3.49)	(3.26)	(2.62)	(2.30)	(2.50)	(2.02)	(1.86)	(0.84)	(-0.49)	(-1.99)
FFCPS Alpha	0.06	0.07	0.04	-0.07	-0.10	0.01	-0.08	-0.11	-0.44	-0.77	-0.83
	(0.79)	(1.28)	(0.87)	(-1.01)	(-1.43)	(0.06)	(-0.82)	(-0.87)	(-3.01)	(-4.72)	(-4.07)
FF5 Alpha	-0.04	0.00	-0.05	-0.09	-0.12	0.11	0.06	0.03	-0.19	-0.49	-0.45
	(-0.65)	(-0.05)	(-1.02)	(-1.42)	(-1.80)	(1.15)	(0.68)	(0.29)	(-1.65)	(-3.59)	(-2.77)

Table 13. Orthogonalization with respect to Idiosyncratic Volatility and other Firm Characteristics

This table presents results from the cross-sectional regressions of future equity returns on orthogonalized VaR1 and various control variables. In regressions (1)-(6), orthogonalization is done by running a contemporaneous regression of VaR1 on only IVOL. In regressions (7)-(12), orthogonalization is done by running a contemporaneous regression of VaR1 on all control variables. VaR1_orth is the residual term from these regressions. Regressions (1)-(3) and (7)-(9) present results estimated using the ordinary least squares (OLS) methodology. Regressions (4)-(6) and (10)-(12) present results estimated for one-month-ahead returns using a weighted least squares (WLS) methodology following Asparouhova, Bessembinder and Kalcheva (2013) where each observed return is weighted by one plus the observed prior return on the stock. Reported coefficients are time-series averages from monthly Fama-MacBeth (1973) regressions and the associated t-statistics are reported using the Newey-West (1987) procedure. Average R-squared statistics for each regression are presented in the last row. VaR1 and firm-specific characteristics are defined in Table 1.

		OLS IVOL			**WLS IVOL**			**OLS All Variables**			**WLS All Variables**	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
VaR1_orth	-0.0455	-0.0405	-0.0458	-0.0672	-0.0638	-0.0656	-0.0459	-0.0458	-0.0458	-0.0664	-0.0657	-0.0656
	(-2.32)	(-2.15)	(-2.32)	(-3.35)	(-3.30)	(-3.27)	(-2.33)	(-2.32)	(-2.32)	(-3.31)	(-3.27)	(-3.27)
Beta	0.0015	0.0020	0.0016	0.0014	0.0018	0.0018	0.0006	0.0012	0.0010	0.0002	0.0007	0.0008
	(0.85)	(1.20)	(0.99)	(0.81)	(1.11)	(1.07)	(0.36)	(0.71)	(0.55)	(0.10)	(0.38)	(0.45)
Size	-0.0010	-0.0009	-0.0010	-0.0010	-0.0009	-0.0010	-0.0008	-0.0007	-0.0008	-0.0006	-0.0006	-0.0007
	(-3.41)	(-3.18)	(-3.53)	(-3.30)	(-3.21)	(-3.40)	(-2.50)	(-2.42)	(-2.64)	(-2.05)	(-2.08)	(-2.21)
BM	0.0015	0.0014	0.0014	0.0016	0.0015	0.0015	0.0016	0.0015	0.0015	0.0016	0.0016	0.0016
	(2.26)	(2.13)	(2.17)	(2.30)	(2.17)	(2.21)	(2.19)	(2.12)	(2.13)	(2.28)	(2.20)	(2.21)
MOM	0.0012	0.0012	0.0012	0.0019	0.0020	0.0020	0.0011	0.0011	0.0011	0.0020	0.0020	0.0020
	(0.48)	(0.50)	(0.49)	(0.79)	(0.80)	(0.81)	(0.43)	(0.44)	(0.43)	(0.78)	(0.79)	(0.78)
STR	-0.0704	-0.0608	-0.0656	-0.0658	-0.0580	-0.0582	-0.0695	-0.0609	-0.0636	-0.0644	-0.0578	-0.0556
	(-7.00)	(-5.70)	(-6.69)	(-6.50)	(-5.45)	(-5.91)	(-6.81)	(-5.70)	(-6.48)	(-6.26)	(-5.41)	(-5.62)
Illiq	-0.0001	0.0000	-0.0001	-0.0002	-0.0001	-0.0001	-0.0001	-0.0001	-0.0001	-0.0002	-0.0002	-0.0002
	(-0.47)	(-0.29)	(-0.37)	(-1.02)	(-0.84)	(-0.91)	(-0.44)	(-0.31)	(-0.33)	(-1.04)	(-0.93)	(-0.93)
Coskew	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(-1.19)	(-1.04)	(-0.98)	(-1.01)	(-0.73)	(-0.95)	(-0.95)	(-0.84)	(-0.76)	(-0.81)	(-0.59)	(-0.77)
Betadown	-0.0014	-0.0014	-0.0011	-0.0013	-0.0012	-0.0011	-0.0014	-0.0013	-0.0011	-0.0013	-0.0012	-0.0012
	(-1.47)	(-1.49)	(-1.21)	(-1.35)	(-1.26)	(-1.23)	(-1.39)	(-1.35)	(-1.13)	(-1.38)	(-1.27)	(-1.27)
IVOL	-0.1971		-0.1058	-0.1605		-0.0100	-0.1761		-0.0623	-0.1278		0.0479
	(-4.98)		(-1.27)	(-4.20)		(-0.13)	(-5.51)		(-0.77)	(-4.20)		(0.64)
MAX		-0.1606	-0.0826		-0.1397	-0.1334		-0.1474	-0.1021		-0.1175	-0.1566
		(-5.43)	(-1.41)		(-4.80)	(-2.36)		(-5.90)	(-1.74)		(-4.85)	(-2.77)
Intercept	0.0149	0.0143	0.0151	0.0146	0.0144	0.0148	0.0149	0.0146	0.0151	0.0145	0.0146	0.0148
	(5.50)	(5.42)	(5.62)	(5.43)	(5.47)	(5.54)	(5.45)	(5.51)	(5.62)	(5.35)	(5.51)	(5.54)
Avg. R^2	0.0946	0.0946	0.0971	0.0958	0.0960	0.0983	0.0945	0.0948	0.0971	0.0958	0.0961	0.0983

Table 14. Alternative Left-Tail Risk Metrics

This table presents return comparisons between equity deciles formed monthly based on VaR5, ES1 and ES5 for the stocks in our sample. Portfolio 1 (Portfolio 10) is the portfolio of stocks with the lowest (highest) value-at-risk. The table reports the average left-tail risk metrics, one-month-ahead value-weighted excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panel A presents results for value-at-risk at the 5% level. Panel B presents results for expected shortfall at the 1% level. Panel C presents results for expected shortfall at the 5% level. The left-tail risk metrics are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. VaR5

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR5	0.0150	0.0218	0.0256	0.0290	0.0325	0.0362	0.0405	0.0456	0.0522	0.0661	
Excess Return	0.51	0.53	0.61	0.54	0.57	0.59	0.54	0.42	0.24	-0.23	-0.74
	(3.47)	(3.35)	(3.32)	(2.66)	(2.56)	(2.31)	(1.86)	(1.29)	(0.67)	(-0.55)	(-2.01)
FFCPS Alpha	0.08	0.03	0.05	-0.02	-0.11	-0.08	-0.12	-0.22	-0.47	-0.85	-0.93
	(1.02)	(0.49)	(0.91)	(-0.32)	(-1.42)	(-0.81)	(-1.06)	(-1.88)	(-2.91)	(-4.35)	(-3.94)
FF5 Alpha	0.02	-0.07	-0.03	-0.12	-0.12	-0.05	-0.01	-0.07	-0.26	-0.52	-0.54
	(0.32)	(-1.22)	(-0.48)	(-1.84)	(-1.71)	(-0.53)	(-0.10)	(-0.71)	(-1.93)	(-3.22)	(-2.89)

Panel B. ES1

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
ES1	0.0342	0.0465	0.0547	0.0625	0.0705	0.0794	0.0895	0.1024	0.1214	0.1763	
Excess Return	0.47	0.61	0.55	0.56	0.61	0.62	0.47	0.48	0.28	0.01	-0.46
	(3.20)	(3.64)	(2.96)	(2.69)	(2.70)	(2.49)	(1.71)	(1.69)	(0.88)	(0.02)	(-1.80)
FFCPS Alpha	0.05	0.08	-0.02	-0.08	-0.02	0.04	-0.15	-0.21	-0.33	-0.47	-0.51
	(0.64)	(1.44)	(-0.36)	(-1.17)	(-0.27)	(0.49)	(-1.30)	(-2.10)	(-2.84)	(-3.71)	(-3.08)
FF5 Alpha	-0.05	0.01	-0.07	-0.10	0.04	0.07	-0.05	-0.09	-0.19	-0.44	-0.39
	(-0.83)	(0.23)	(-1.26)	(-1.79)	(0.54)	(0.97)	(-0.45)	(-0.89)	(-2.00)	(-3.82)	(-2.68)

Table 14 (continued)

Panel C. ES5

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
ES5	0.0238	0.0319	0.0372	0.0421	0.0470	0.0523	0.0584	0.0657	0.0754	0.0971	
Excess Return	0.49	0.59	0.57	0.54	0.59	0.60	0.55	0.53	0.19	-0.36	-0.85
	(3.43)	(3.50)	(3.15)	(2.58)	(2.49)	(2.28)	(1.93)	(1.74)	(0.54)	(-0.90)	(-2.50)
FFCPS Alpha	0.07	0.07	0.04	-0.13	-0.06	-0.04	-0.10	-0.11	-0.55	-0.88	-0.95
	(0.91)	(1.32)	(0.64)	(-1.81)	(-0.71)	(-0.43)	(-0.97)	(-0.98)	(-3.73)	(-5.28)	(-4.48)
FF5 Alpha	0.00	-0.04	-0.06	-0.15	-0.07	0.01	0.04	0.03	-0.28	-0.71	-0.71
	(-0.02)	(-0.78)	(-1.04)	(-2.26)	(-0.93)	(0.07)	(0.36)	(0.30)	(-2.32)	(-4.87)	(-4.05)

Table 15. Evidence from International Equity Markets

This table presents return comparisons between equity deciles formed monthly based on VaR1 for various country groupings. Portfolio 1 (Portfolio 10) is the portfolio of stocks with the lowest (highest) value-at-risk. The table reports the average value-at-risk metrics, one-month-ahead value-weighted excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. Global AF alphas are calculated after adjusting for the market, size, value and momentum factors of Asness and Frazzini (2013). Global FF alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2017). Panels A-D present results for the full international sample (Panel A), including 23 developed countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland and the UK); G10 countries excluding US (Panel B); G7 countries excluding US (Panel C); and European Union countries (Panel D). VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Full International Sample

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR1	0.0214	0.0341	0.0402	0.0452	0.0502	0.0557	0.0623	0.0705	0.0831	0.1178	
Excess Return	0.47	0.47	0.38	0.26	0.05	0.02	-0.11	-0.17	-0.21	-0.97	-1.44
	(2.72)	(1.89)	(1.28)	(0.80)	(0.16)	(0.06)	(-0.27)	(-0.34)	(-0.38)	(-1.59)	(-2.67)
AF Alpha	0.04	-0.08	-0.22	-0.36	-0.59	-0.57	-0.58	-0.59	-0.57	-1.39	-1.42
	(0.35)	(-0.84)	(-2.03)	(-3.46)	(-5.57)	(-4.30)	(-3.42)	(-2.63)	(-2.06)	(-4.03)	(-3.96)
FF Alpha	-0.11	-0.22	-0.33	-0.43	-0.65	-0.61	-0.70	-0.77	-0.80	-1.53	-1.42
	(-0.98)	(-2.08)	(-2.95)	(-3.83)	(-5.79)	(-4.48)	(-3.93)	(-3.00)	(-2.67)	(-4.16)	(-3.63)

Panel B. G10 excluding US

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR1	0.0214	0.0335	0.0392	0.0437	0.0482	0.0529	0.0582	0.0647	0.0743	0.1096	
Excess Return	0.49	0.42	0.35	0.18	0.07	0.01	-0.21	-0.19	-0.16	-0.58	-1.06
	(3.07)	(1.73)	(1.25)	(0.57)	(0.21)	(0.02)	(-0.51)	(-0.42)	(-0.32)	(-1.07)	(-2.23)
AF Alpha	0.09	-0.13	-0.25	-0.43	-0.54	-0.59	-0.68	-0.59	-0.48	-0.82	-0.91
	(0.83)	(-1.18)	(-2.38)	(-3.74)	(-4.46)	(-4.76)	(-4.34)	(-2.97)	(-2.10)	(-3.11)	(-3.10)
FF Alpha	-0.06	-0.28	-0.35	-0.50	-0.61	-0.60	-0.75	-0.79	-0.65	-0.99	-0.93
	(-0.51)	(-2.58)	(-3.02)	(-4.00)	(-4.77)	(-4.70)	(-4.65)	(-3.57)	(-2.56)	(-3.11)	(-2.69)

Table 15 (continued)

Panel C. G7 excluding US

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR1	0.0215	0.0339	0.0398	0.0444	0.0491	0.0538	0.0591	0.0654	0.0753	0.1123	
Excess Return	0.39	0.36	0.31	0.11	-0.02	-0.06	-0.22	-0.20	-0.14	-0.57	-0.96
	(2.41)	(1.53)	(1.10)	(0.35)	(-0.05)	(-0.16)	(-0.56)	(-0.46)	(-0.28)	(-1.08)	(-2.03)
AF Alpha	-0.03	-0.18	-0.28	-0.50	-0.65	-0.63	-0.71	-0.62	-0.49	-0.80	-0.77
	(-0.23)	(-1.72)	(-2.64)	(-4.40)	(-5.38)	(-4.50)	(-4.23)	(-2.81)	(-2.12)	(-3.10)	(-2.60)
FF Alpha	-0.13	-0.35	-0.36	-0.56	-0.09	-0.67	-0.82	-0.82	-0.68	-0.94	-0.82
	(-1.07)	(-3.11)	(-2.93)	(-4.58)	(-5.16)	(-4.70)	(-4.65)	(-3.20)	(-2.56)	(-3.03)	(-2.37)

Panel D. European Union

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR1	0.0178	0.0295	0.0357	0.0400	0.0438	0.0479	0.0532	0.0604	0.0703	0.0931	
Excess Return	0.44	0.51	0.55	0.43	0.40	0.36	0.29	0.21	0.04	-0.53	-0.98
	(2.75)	(2.02)	(1.86)	(1.32)	(1.11)	(0.96)	(0.69)	(0.47)	(0.09)	(-0.88)	(-1.84)
AF Alpha	0.04	-0.05	-0.04	-0.22	-0.20	-0.30	-0.31	-0.30	-0.36	-0.93	-0.98
	(0.34)	(-0.33)	(-0.30)	(-1.41)	(-1.19)	(-1.77)	(-1.66)	(-1.39)	(-1.39)	(-2.64)	(-2.78)
FF Alpha	-0.11	-0.27	-0.32	-0.48	-0.43	-0.49	-0.44	-0.36	-0.30	-0.94	-0.83
	(-0.78)	(-1.85)	(-2.11)	(-3.01)	(-2.60)	(-2.92)	(-2.56)	(-1.70)	(-1.26)	(-2.55)	(-2.25)

Online Appendix for

"Left-Tail Momentum: Limited Attention of Individual Investors and Expected Equity Returns"

Yigit Atilgan, Turan G. Bali, K. Ozgur Demirtas, and A. Doruk Gunaydin

List of Tables:

Table I. Bivariate Portfolio Analysis with Independent Sorts

This table presents return comparisons between equity quintiles formed based on independent double sorts of various firm-specific attributes and VaR1. For the independent sorts, all stocks are grouped into decile portfolios based on independent ascending sorts of both a firm-specific attribute and VaR1 each month. The intersections of each of the decile groups are used to form the portfolios. Panels A and B report one-month-ahead five-factor FFCPS and FF5 alphas for each decile, respectively. The last columns in each panel show the differences of monthly alphas between VaR1 deciles 10 and 1 for each firm-specific attribute. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 and firm-specific attributes are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. FFCPS Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Beta	0.17	0.15	0.08	0.04	-0.06	-0.03	-0.05	-0.25	-0.25	-0.66	-0.96
	(1.00)	(1.79)	(1.04)	(0.53)	(-0.76)	(-0.37)	(-0.76)	(-3.00)	(-3.13)	(-5.41)	(-3.75)
Size	0.29	0.29	0.25	0.24	0.24	0.14	0.11	-0.10	-0.22	-0.65	-0.95
	(3.55)	(3.92)	(3.49)	(3.77)	(3.89)	(2.18)	(1.69)	(-1.38)	(-2.68)	(-5.72)	(-5.44)
BM	0.14	0.16	0.04	-0.02	0.03	-0.02	0.05	-0.15	-0.43	-0.64	-0.82
	(1.64)	(2.38)	(0.57)	(-0.23)	(0.45)	(-0.27)	(0.51)	(-1.53)	(-4.04)	(-4.74)	(-4.29)
MOM	0.27	0.16	-0.02	0.02	-0.08	-0.07	-0.05	-0.28	-0.35	-0.73	-0.95
	(2.33)	(2.60)	(-0.27)	(0.38)	(-1.05)	(-0.85)	(-0.50)	(-2.71)	(-2.97)	(-4.70)	(-3.82)
STR	0.08	0.13	0.05	0.03	0.04	-0.04	-0.04	-0.21	-0.34	-0.84	-0.87
	(0.82)	(1.96)	(0.82)	(0.51)	(0.54)	(-0.50)	(-0.42)	(-2.07)	(-3.01)	(-5.92)	(-3.58)
Illiq	0.14	0.15	0.11	0.11	0.06	0.01	-0.03	-0.23	-0.36	-0.81	-0.94
	(1.61)	(2.19)	(1.73)	(1.75)	(0.87)	(0.16)	(-0.49)	(-3.51)	(-4.59)	(-7.77)	(-5.49)
Coskew	0.08	0.06	-0.02	-0.03	-0.01	-0.05	-0.01	-0.17	-0.35	-0.71	-0.89
	(0.82)	(1.09)	(-0.27)	(-0.58)	(-0.11)	(-0.68)	(-0.13)	(-1.76)	(-2.83)	(-5.71)	(-4.15)
Betadown	0.31	-0.08	-0.02	-0.04	-0.07	-0.08	-0.06	-0.29	-0.38	-0.76	-1.03
	(0.99)	(-0.87)	(-0.28)	(-0.59)	(-0.90)	(-1.03)	(-0.78)	(-3.66)	(-4.36)	(-5.73)	(-2.64)
IVOL	0.21	0.18	-0.02	-0.07	-0.05	-0.14	-0.02	-0.16	-0.38	-0.53	-0.59
	(1.66)	(1.55)	(-0.19)	(-0.89)	(-0.59)	(-1.94)	(-0.24)	(-1.77)	(-4.02)	(-3.73)	(-2.13)
MAX	0.12	0.07	0.02	0.02	-0.02	-0.04	0.04	-0.18	-0.08	-0.34	-0.52
	(1.21)	(0.90)	(0.27)	(0.36)	(-0.25)	(-0.61)	(0.50)	(-1.88)	(-0.71)	(-2.40)	(-2.27)

Table I (continued)

Panel B. FF5 Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Beta	-0.04	0.07	-0.06	-0.09	-0.18	-0.14	-0.12	-0.25	-0.26	-0.65	-1.14
	(-0.31)	(0.95)	(-0.77)	(-1.56)	(-2.66)	(-1.84)	(-2.07)	(-3.90)	(-3.25)	(-6.61)	(-6.55)
Size	0.18	0.15	0.11	0.12	0.10	0.05	0.05	-0.08	-0.18	-0.58	-0.78
	(2.34)	(2.40)	(1.76)	(2.05)	(1.64)	(0.80)	(0.85)	(-1.37)	(-2.20)	(-4.53)	(-4.80)
BM	-0.07	-0.01	-0.09	-0.10	-0.06	-0.03	0.03	-0.09	-0.34	-0.48	-0.51
	(-1.04)	(-0.19)	(-1.42)	(-1.34)	(-0.79)	(-0.42)	(0.29)	(-0.87)	(-3.37)	(-3.22)	(-2.80)
MOM	0.05	-0.03	-0.18	-0.15	-0.17	-0.12	-0.11	-0.24	-0.26	-0.54	-0.66
	(0.53)	(-0.54)	(-2.72)	(-2.43)	(-2.58)	(-1.52)	(-1.24)	(-2.43)	(-2.43)	(-3.95)	(-3.30)
STR	-0.05	0.02	-0.05	-0.02	0.01	-0.01	0.02	-0.10	-0.19	-0.67	-0.64
	(-0.63)	(0.31)	(-0.88)	(-0.39)	(0.21)	(-0.11)	(0.25)	(-1.15)	(-2.03)	(-5.54)	(-3.27)
Illiq	0.00	0.00	-0.06	-0.02	-0.09	-0.10	-0.11	-0.26	-0.32	-0.75	-0.72
	(0.01)	(0.05)	(-1.09)	(-0.39)	(-1.47)	(-1.72)	(-1.96)	(-4.00)	(-3.87)	(-6.77)	(-4.78)
Coskew	-0.03	0.00	-0.08	-0.11	-0.05	0.00	0.06	-0.09	-0.20	-0.59	-0.62
	(-0.32)	(0.08)	(-1.30)	(-1.99)	(-0.84)	(0.05)	(0.71)	(-1.07)	(-1.88)	(-4.63)	(-3.62)
Betadown	0.18	-0.10	-0.11	-0.11	-0.16	-0.15	-0.12	-0.31	-0.39	-0.77	-1.32
	(0.88)	(-1.05)	(-1.64)	(-1.88)	(-2.23)	(-1.91)	(-1.71)	(-3.87)	(-4.79)	(-6.39)	(-4.16)
IVOL	0.12	0.12	-0.13	-0.12	-0.12	-0.12	0.00	-0.14	-0.34	-0.50	-0.53
	(0.93)	(1.07)	(-1.57)	(-1.66)	(-1.66)	(-1.60)	(-0.06)	(-1.65)	(-3.25)	(-3.68)	(-2.22)
MAX	0.02	-0.01	-0.10	-0.03	-0.09	-0.04	0.06	-0.12	-0.03	-0.31	-0.44
	(0.24)	(-0.12)	(-1.63)	(-0.57)	(-1.43)	(-0.49)	(0.74)	(-1.33)	(-0.34)	(-2.38)	(-2.23)

Table II. Longer-Term Portfolio Returns

This table presents longer-term return comparisons between equity deciles formed monthly based on VaR1 for the stocks in our sample. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports monthly excess returns and two distinct five-factor alphas for each decile from two to twelve months ahead after portfolio formation. Panels A, B and C present results for excess returns, FFCPS alphas and FF5 alphas, respectively. The last column in each panel shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Excess Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
t+2	0.47	0.60	0.59	0.57	0.58	0.56	0.59	0.51	0.30	-0.18	-0.65
	(3.21)	(3.56)	(3.17)	(2.84)	(2.53)	(2.19)	(2.20)	(1.72)	(0.89)	(-0.48)	(-2.12)
t+3	0.48	0.59	0.57	0.58	0.57	0.55	0.57	0.47	0.28	-0.12	-0.59
	(3.24)	(3.47)	(3.07)	(2.85)	(2.49)	(2.16)	(2.10)	(1.60)	(0.85)	(-0.33)	(-2.04)
t+4	0.49	0.59	0.58	0.58	0.57	0.55	0.56	0.47	0.29	-0.06	-0.55
	(3.30)	(3.43)	(3.08)	(2.82)	(2.49)	(2.22)	(2.09)	(1.59)	(0.89)	(-0.17)	(-1.97)
t+5	0.50	0.58	0.58	0.57	0.57	0.56	0.55	0.46	0.28	-0.01	-0.52
	(3.36)	(3.36)	(3.06)	(2.77)	(2.48)	(2.23)	(2.04)	(1.55)	(0.86)	(-0.03)	(-1.92)
t+6	0.51	0.58	0.58	0.57	0.57	0.55	0.54	0.44	0.29	0.05	-0.47
	(3.38)	(3.34)	(3.04)	(2.75)	(2.50)	(2.21)	(2.02)	(1.51)	(0.91)	(0.13)	(-1.81)
t+7	0.52	0.58	0.58	0.58	0.57	0.55	0.53	0.43	0.31	0.10	-0.42
	(3.36)	(3.29)	(2.97)	(2.77)	(2.48)	(2.19)	(1.98)	(1.48)	(0.98)	(0.29)	(-1.66)
t+8	0.52	0.57	0.58	0.58	0.57	0.54	0.53	0.44	0.32	0.15	-0.37
	(3.33)	(3.20)	(2.97)	(2.74)	(2.46)	(2.16)	(1.97)	(1.50)	(1.01)	(0.44)	(-1.53)
t+9	0.52	0.56	0.57	0.58	0.57	0.52	0.51	0.44	0.32	0.19	-0.33
	(3.28)	(3.15)	(2.91)	(2.71)	(2.45)	(2.09)	(1.92)	(1.53)	(1.03)	(0.57)	(-1.41)
t+10	0.52	0.55	0.57	0.56	0.56	0.52	0.50	0.44	0.34	0.24	-0.28
	(3.24)	(3.05)	(2.86)	(2.62)	(2.41)	(2.07)	(1.85)	(1.54)	(1.12)	(0.74)	(-1.25)
t+11	0.52	0.55	0.57	0.56	0.56	0.53	0.49	0.44	0.36	0.27	-0.25
	(3.17)	(3.04)	(2.85)	(2.61)	(2.40)	(2.09)	(1.81)	(1.53)	(1.19)	(0.85)	(-1.14)
t+12	0.51	0.55	0.56	0.55	0.55	0.51	0.48	0.43	0.37	0.29	-0.22
	(3.12)	(3.00)	(2.78)	(2.56)	(2.37)	(2.06)	(1.78)	(1.52)	(1.23)	(0.94)	(-1.03)

Table II (continued)

Panel B. FFCPS Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
t+2	0.04	0.08	0.02	-0.04	-0.05	-0.08	-0.03	-0.16	-0.37	-0.78	-0.83
	(0.66)	(1.71)	(0.39)	(-0.68)	(-0.78)	(-1.19)	(-0.34)	(-1.54)	(-3.46)	(-5.37)	(-4.50)
t+3	0.05	0.07	0.01	-0.02	-0.04	-0.08	-0.05	-0.18	-0.37	-0.73	-0.78
	(0.76)	(1.73)	(0.20)	(-0.41)	(-0.75)	(-1.20)	(-0.62)	(-1.88)	(-3.69)	(-5.56)	(-4.63)
t+4	0.05	0.07	0.01	-0.02	-0.05	-0.07	-0.06	-0.17	-0.36	-0.68	-0.74
	(0.92)	(1.68)	(0.24)	(-0.47)	(-0.85)	(-1.16)	(-0.80)	(-1.95)	(-3.71)	(-5.59)	(-4.69)
t+5	0.07	0.06	0.02	-0.02	-0.05	-0.06	-0.07	-0.18	-0.36	-0.64	-0.71
	(1.22)	(1.60)	(0.44)	(-0.51)	(-0.89)	(-0.99)	(-1.04)	(-2.16)	(-3.95)	(-5.56)	(-4.76)
t+6	0.07	0.06	0.01	-0.02	-0.04	-0.07	-0.08	-0.19	-0.36	-0.60	-0.67
	(1.33)	(1.67)	(0.41)	(-0.49)	(-0.82)	(-1.23)	(-1.22)	(-2.39)	(-3.95)	(-5.39)	(-4.67)
t+7	0.07	0.06	0.01	-0.01	-0.04	-0.07	-0.10	-0.21	-0.33	-0.55	-0.62
	(1.44)	(1.71)	(0.28)	(-0.25)	(-0.81)	(-1.31)	(-1.43)	(-2.62)	(-3.75)	(-5.14)	(-4.48)
t+8	0.07	0.05	0.02	-0.01	-0.04	-0.08	-0.10	-0.20	-0.32	-0.51	-0.58
	(1.45)	(1.56)	(0.45)	(-0.20)	(-0.86)	(-1.51)	(-1.55)	(-2.61)	(-3.72)	(-5.01)	(-4.35)
t+9	0.07	0.04	0.01	-0.01	-0.04	-0.09	-0.11	-0.19	-0.32	-0.47	-0.53
	(1.40)	(1.33)	(0.39)	(-0.18)	(-0.76)	(-1.65)	(-1.70)	(-2.50)	(-3.76)	(-4.82)	(-4.18)
t+10	0.06	0.03	0.01	-0.01	-0.04	-0.09	-0.13	-0.19	-0.30	-0.42	-0.48
	(1.40)	(0.95)	(0.37)	(-0.37)	(-0.87)	(-1.69)	(-2.06)	(-2.54)	(-3.61)	(-4.56)	(-3.97)
t+11	0.06	0.03	0.01	-0.01	-0.04	-0.09	-0.14	-0.20	-0.29	-0.40	-0.45
	(1.30)	(0.99)	(0.41)	(-0.32)	(-0.91)	(-1.70)	(-2.22)	(-2.80)	(-3.61)	(-4.44)	(-3.83)
t+12	0.05	0.03	0.01	-0.02	-0.05	-0.10	-0.14	-0.21	-0.27	-0.37	-0.43
	(1.18)	(1.05)	(0.17)	(-0.47)	(-1.01)	(-1.83)	(-2.32)	(-2.91)	(-3.52)	(-4.27)	(-3.66)

Panel C. FF5 Alphas

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
t+2	-0.04	0.01	-0.06	-0.07	-0.03	-0.02	0.07	-0.02	-0.15	-0.55	-0.51
	(-0.77)	(0.15)	(-1.43)	(-1.45)	(-0.57)	(-0.24)	(0.91)	(-0.21)	(-1.60)	(-4.55)	(-3.48)
t+3	-0.04	0.00	-0.06	-0.05	-0.03	-0.02	0.05	-0.03	-0.17	-0.49	-0.45
	(-0.74)	(-0.06)	(-1.65)	(-1.11)	(-0.53)	(-0.27)	(0.69)	(-0.39)	(-1.92)	(-4.53)	(-3.41)
t+4	-0.03	-0.01	-0.05	-0.05	-0.03	-0.01	0.04	-0.03	-0.15	-0.44	-0.41
	(-0.68)	(-0.26)	(-1.47)	(-1.10)	(-0.59)	(-0.22)	(0.65)	(-0.39)	(-1.85)	(-4.42)	(-3.34)
t+5	-0.02	-0.01	-0.04	-0.05	-0.03	0.00	0.03	-0.03	-0.16	-0.40	-0.37
	(-0.55)	(-0.26)	(-1.09)	(-1.14)	(-0.57)	(-0.05)	(0.45)	(-0.44)	(-2.01)	(-4.19)	(-3.20)
t+6	-0.02	-0.01	-0.04	-0.05	-0.02	-0.01	0.01	-0.04	-0.15	-0.35	-0.33
	(-0.44)	(-0.35)	(-1.08)	(-1.21)	(-0.52)	(-0.24)	(0.22)	(-0.51)	(-1.99)	(-3.92)	(-3.01)
t+7	-0.02	-0.01	-0.04	-0.03	-0.02	-0.01	0.01	-0.04	-0.13	-0.30	-0.28
	(-0.43)	(-0.38)	(-1.18)	(-0.86)	(-0.52)	(-0.24)	(0.13)	(-0.63)	(-1.74)	(-3.46)	(-2.62)
t+8	-0.02	-0.02	-0.03	-0.03	-0.02	-0.02	0.01	-0.04	-0.12	-0.27	-0.25
	(-0.47)	(-0.60)	(-0.97)	(-0.68)	(-0.53)	(-0.30)	(0.17)	(-0.58)	(-1.62)	(-3.25)	(-2.42)
t+9	-0.02	-0.02	-0.03	-0.02	-0.02	-0.02	-0.01	-0.03	-0.12	-0.23	-0.21
	(-0.44)	(-0.69)	(-0.90)	(-0.53)	(-0.44)	(-0.47)	(-0.09)	(-0.46)	(-1.65)	(-2.89)	(-2.15)
t+10	-0.02	-0.03	-0.03	-0.03	-0.02	-0.03	-0.02	-0.03	-0.10	-0.18	-0.16
	(-0.53)	(-1.08)	(-0.86)	(-0.86)	(-0.55)	(-0.50)	(-0.36)	(-0.42)	(-1.36)	(-2.37)	(-1.69)
t+11	-0.03	-0.03	-0.03	-0.02	-0.02	-0.02	-0.03	-0.04	-0.09	-0.16	-0.13
	(-0.74)	(-1.08)	(-0.79)	(-0.65)	(-0.48)	(-0.37)	(-0.45)	(-0.59)	(-1.26)	(-2.08)	(-1.40)
t+12	-0.03	-0.03	-0.03	-0.03	-0.02	-0.03	-0.03	-0.04	-0.08	-0.14	-0.11
	(-0.82)	(-1.00)	(-0.92)	(-0.72)	(-0.49)	(-0.53)	(-0.51)	(-0.67)	(-1.23)	(-1.87)	(-1.20)

Table III. Investor Clientele Effect with FF5 Alphas

This table presents return comparisons between equity portfolios formed based on bivariate sorts of various firm-specific attributes and VaR1. First, tercile portfolios are formed every month based on a firm-specific attribute for the stocks in our sample. Next, additional quintile portfolios are formed based on VaR1 within each firm-specific attribute tercile. The table reports one-month-ahead five-factor FF5 alphas for each quintile. The last rows in each panel show the differences of monthly alphas between VaR1 quintiles 5 and 1 for each firm-specific attribute tercile. FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panels A, B, C, D and E present results for analyst coverage, firm size, illiquidity, idiosyncratic volatility and lottery demand, respectively. VaR1 and firm-specific attributes are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Analyst Coverage

	CVRG1	CVRG2	CVRG3
Port1	-0.01	0.01	0.00
	(-0.05)	(0.16)	(0.02)
Port2	-0.25	-0.13	-0.06
	(-2.26)	(-1.19)	(-0.58)
Port3	-0.15	-0.02	0.12
	(-1.15)	(-0.22)	(1.15)
Port4	-0.22	0.02	0.22
	(-1.45)	(0.13)	(1.72)
Port5	-0.51	-0.40	0.10
	(-3.47)	(-1.78)	(0.52)
High-Low	-0.50	-0.42	0.14
	(-2.48)	(-1.59)	(0.62)

Panel B. Firm Size

	SIZE1	SIZE2	SIZE3
Port1	0.34	0.09	-0.08
	(3.09)	(1.42)	(-1.36)
Port2	0.25	0.00	-0.02
	(2.54)	(-0.04)	(-0.49)
Port3	-0.02	-0.02	-0.03
	(-0.23)	(-0.44)	(-0.48)
Port4	-0.22	-0.10	0.07
	(-2.13)	(-1.67)	(0.95)
Port5	-0.56	-0.49	0.00
	(-4.57)	(-4.76)	(0.00)
High-Low	-0.89	-0.58	0.08
	(-6.84)	(-4.45)	(0.54)

Table III (continued)

Panel C. Illiquidity

	ILLIQ1	ILLIQ2	ILLIQ3
Port1	-0.05	-0.07	0.04
	(-1.01)	(-1.09)	(0.43)
Port2	-0.02	-0.12	0.00
	(-0.47)	(-1.80)	(-0.05)
Port3	0.03	-0.11	-0.21
	(0.57)	(-1.55)	(-2.10)
Port4	0.14	-0.13	-0.39
	(1.60)	(-1.97)	(-3.63)
Port5	-0.05	-0.50	-0.83
	(-0.44)	(-4.55)	(-7.65)
High-Low	0.00	-0.43	-0.87
	(0.01)	(-3.44)	(-7.04)

Panel D. Idiosyncratic Volatility

	IVOL1	IVOL2	IVOL3
Port1	0.03	-0.08	-0.08
	(0.47)	(-1.06)	(-0.60)
Port2	-0.04	-0.08	-0.03
	(-0.68)	(-1.13)	(-0.24)
Port3	-0.02	0.00	-0.30
	(-0.30)	(-0.02)	(-2.06)
Port4	-0.11	0.21	-0.52
	(-1.69)	(2.15)	(-3.47)
Port5	-0.09	0.05	-1.12
	(-0.76)	(0.42)	(-6.36)
High-Low	-0.12	0.13	-1.04
	(-0.82)	(0.81)	(-4.88)

Panel E. Lottery Demand

	MAX1	MAX2	MAX3
Port1	0.01	-0.05	-0.01
	(0.19)	(-0.85)	(-0.11)
Port2	-0.04	-0.12	0.10
	(-0.60)	(-1.89)	(0.73)
Port3	0.06	-0.04	-0.13
	(0.90)	(-0.45)	(-0.91)
Port4	0.04	0.05	-0.39
	(0.42)	(0.05)	(-2.66)
Port5	-0.02	-0.06	-1.11
	(-0.19)	(-0.48)	(-6.03)
High-Low	-0.04	-0.01	-1.10
	(-0.25)	(-0.08)	(-5.13)

Table IV. Skipping a Month between Portfolio Formation Month and Holding Period

This table presents return comparisons between equity deciles formed monthly based on VaR1 for the stocks in our sample. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports the two-month-ahead excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panel A presents results for value-weighted portfolio returns. Panel B presents results for equal-weighted portfolio returns. VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Value-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.47	0.60	0.59	0.57	0.58	0.56	0.59	0.51	0.30	-0.18	-0.65
	(3.21)	(3.56)	(3.17)	(2.84)	(2.53)	(2.19)	(2.20)	(1.72)	(0.89)	(-0.48)	(-2.12)
FFCPS Alpha	0.04	0.08	0.02	-0.04	-0.05	-0.08	-0.03	-0.16	-0.37	-0.78	-0.83
	(0.66)	(1.71)	(0.39)	(-0.68)	(-0.78)	(-1.19)	(-0.34)	(-1.54)	(-3.46)	(-5.37)	(-4.50)
FF5 Alpha	-0.04	0.01	-0.06	-0.07	-0.03	-0.02	0.07	-0.02	-0.15	-0.55	-0.51
	(-0.77)	(0.15)	(-1.43)	(-1.45)	(-0.57)	(-0.24)	(0.91)	(-0.21)	(-1.60)	(-4.55)	(-3.48)

Panel B. Equal-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	1.13	1.23	1.25	1.31	1.32	1.26	1.28	1.08	0.98	0.54	-0.59
	(7.59)	(7.04)	(6.45)	(6.31)	(5.81)	(5.18)	(4.85)	(3.81)	(3.09)	(1.60)	(-2.17)
FFCPS Alpha	0.72	0.83	0.84	0.90	0.91	0.86	0.87	0.67	0.57	0.13	-0.59
	(4.82)	(4.69)	(4.32)	(4.30)	(3.99)	(3.48)	(3.28)	(2.34)	(1.79)	(0.38)	(-2.17)
FF5 Alpha	0.68	0.79	0.79	0.85	0.83	0.78	0.80	0.58	0.48	0.04	-0.64
	(4.77)	(4.65)	(4.21)	(4.36)	(3.93)	(3.44)	(3.22)	(2.15)	(1.53)	(0.12)	(-2.21)

Table V. Effect of Outliers

This table presents return comparisons between equity deciles formed monthly based on VaR1 after VaR1 is truncated at the 1% level each month. Portfolio 1 is the portfolio of stocks with the lowest value-at-risk and Portfolio 10 is the portfolio of stocks with the highest value-at-risk. The table reports the one-month-ahead excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panel A presents results for value-weighted portfolio returns. Panel B presents results for equal-weighted portfolio returns. Panel C presents results for median returns. VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. Value-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.48	0.61	0.57	0.55	0.58	0.57	0.61	0.52	0.37	-0.26	-0.74
	(3.25)	(3.62)	(3.11)	(2.64)	(2.54)	(2.19)	(2.27)	(1.68)	(1.08)	(-0.66)	(-2.27)
FFCPS Alpha	0.05	0.10	0.01	-0.09	-0.04	-0.08	-0.03	-0.13	-0.33	-0.83	-0.88
	(0.66)	(1.99)	(0.24)	(-1.24)	(-0.46)	(-0.90)	(-0.25)	(-1.06)	(-2.71)	(-4.67)	(-4.00)
FF5 Alpha	-0.03	0.01	-0.07	-0.10	-0.05	-0.01	0.07	0.01	-0.10	-0.61	-0.57
	(-0.54)	(0.24)	(-1.37)	(-1.46)	(-0.67)	(-0.09)	(0.73)	(0.07)	(-0.92)	(-4.14)	(-3.34)

Panel B. Equal-Weighted Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.71	0.81	0.85	0.89	0.92	0.89	0.88	0.70	0.54	0.12	-0.60
	(4.61)	(4.62)	(4.38)	(4.20)	(4.01)	(3.61)	(3.38)	(2.41)	(1.71)	(0.34)	(-2.19)
FFCPS Alpha	0.22	0.25	0.21	0.20	0.19	0.13	0.11	-0.08	-0.20	-0.51	-0.73
	(2.72)	(3.68)	(3.06)	(3.19)	(3.05)	(2.02)	(1.85)	(-1.26)	(-2.48)	(-5.29)	(-4.93)
FF5 Alpha	0.11	0.09	0.05	0.05	0.05	0.02	0.02	-0.09	-0.16	-0.44	-0.55
	(1.59)	(1.58)	(0.98)	(0.93)	(0.80)	(0.28)	(0.49)	(-1.41)	(-1.95)	(-4.08)	(-4.18)

Table V (continued)

Panel C. Median Returns

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
Excess Return	0.34	0.39	0.39	0.35	0.32	0.22	0.10	-0.16	-0.47	-1.13	-1.46
	(2.36)	(2.36)	(2.14)	(1.76)	(1.50)	(0.93)	(0.40)	(-0.59)	(-1.62)	(-3.62)	(-6.05)
FFCPS Alpha	-0.11	-0.14	-0.20	-0.30	-0.35	-0.49	-0.62	-0.91	-1.15	-1.72	-1.61
	(-1.36)	(-2.00)	(-2.87)	(-4.37)	(-5.04)	(-6.89)	(-9.01)	(-12.34)	(-12.75)	(-15.26)	(-11.28)
FF5 Alpha	-0.21	-0.30	-0.37	-0.46	-0.53	-0.65	-0.78	-0.96	-1.20	-1.70	-1.50
	(-2.70)	(-4.67)	(-5.89)	(-7.68)	(-7.67)	(-9.40)	(-11.92)	(-10.85)	(-10.68)	(-13.20)	(-11.36)

Table VI. Results from Full CRSP Universe and NYSE Breakpoints

This table presents return comparisons between equity deciles formed based on VaR1 for the stocks in our sample. The value-weighted decile portfolios are formed every month from 1962 to 2014. Portfolio 1 (Portfolio 10) is the portfolio of stocks with the lowest (highest) value-at-risk. The table reports two distinct five-factor alphas associated with the value-weighted returns for each decile. The last row shows the differences in alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). In the first and second columns, the original sample which excludes stocks with an end-of-month price of less than $5 is used and decile cut-off points are calculated using only NYSE-traded stocks. In the third and fourth columns, the full CRSP universe is used and decile cut-off points are calculated using all CRSP stocks. In the fifth and sixth columns, the full CRSP universe is used and decile cut-off points are calculated using only NYSE-traded stocks. VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

	Sample with Price Screen Using NYSE breakpoints		**Full CRSP Sample Using CRSP breakpoints**		**Full CRSP Sample Using NYSE breakpoints**	
	FFCPS Alpha	**FF5 Alpha**	**FFCPS Alpha**	**FF5 Alpha**	**FFCPS Alpha**	**FF5 Alpha**
Port 1	0.13	0.27	0.06	-0.03	0.10	0.23
	(2.07)	(4.18)	(0.93)	(-0.65)	(1.63)	(3.80)
Port 2	0.02	-0.08	0.09	-0.01	0.08	-0.04
	(0.33)	(-1.39)	(1.93)	(-0.24)	(1.30)	(-0.79)
Port 3	0.02	0.11	-0.03	-0.04	-0.01	-0.11
	(0.39)	(-2.04)	(-0.59)	(-0.75)	(-0.12)	(-2.04)
Port 4	-0.04	-0.11	0.10	0.06	-0.02	-0.13
	(-0.49)	(-1.71)	(1.38)	(0.85)	(-0.30)	(-1.87)
Port 5	0.06	-0.10	0.03	0.11	0.11	-0.08
	(0.73)	(-1.23)	(0.36)	(1.34)	(1.24)	(-1.06)
Port 6	-0.09	-0.23	-0.03	0.07	-0.09	-0.22
	(-1.14)	(-2.87)	(-0.27)	(0.75)	(-0.94)	(-2.56)
Port 7	-0.22	-0.24	-0.25	-0.06	-0.27	-0.24
	(-2.36)	(-2.67)	(-2.12)	(-0.63)	(-2.94)	(-2.76)
Port 8	-0.24	-0.23	-0.51	-0.27	-0.17	-0.17
	(-2.08)	(-2.21)	(-3.16)	(-2.15)	(-1.57)	(-1.60)
Port 9	-0.34	-0.29	-0.97	-0.71	-0.26	-0.30
	(-3.01)	(-2.58)	(-4.49)	(-4.12)	(-1.89)	(-2.15)
Port 10	0.67	-0.70	-1.01	-0.88	-0.77	-0.84
	(-4.21)	(-4.20)	(-4.51)	(-4.15)	(-4.23)	(-4.55)
High - Low	-0.80	-0.96	-1.07	-0.84	-0.87	-1.06
	(-4.16)	(-4.73)	(-4.24)	(-3.73)	(-4.18)	(-4.97)

Table VII. Subsample Analysis

This table presents value-weighted returns and five-factor alphas to the zero-cost portfolio that buys stocks in the highest VaR1 decile and sells stocks in the lowest VaR1 decile each month during various subsamples. CFNAI refers to the level of the Chicago National Activity Index. JLN refers to the macroeconomic uncertainty index created by Jurado, Ludvigson and Ng (2015, JLN). DEF refers to the default spread defined by the yield difference BAA-rated and AAA-rated corporate bonds. The full sample period is divided into two based on whether CFNAI is less than or greater than zero, JLN is greater than or less than its sample median or DEF is greater than or less than its sample median. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). VaR1 is defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

	Return	**FFCPS Alpha**	**FF5 Alpha**
CFNAI < 0	-1.30	-1.35	-0.89
	(-2.15)	(-3.60)	(-3.22)
CFNAI ≥ 0	-0.55	-0.66	-0.46
	(-1.54)	(-2.97)	(-2.15)
JLN > Median	-1.21	-1.03	-1.66
	(-2.23)	(-3.44)	(-2.48)
JLN ≤ Median	-0.30	-0.76	-0.56
	(-0.91)	(-3.20)	(-2.65)
DEF > Median	-1.27	-1.92	-1.23
	(-1.83)	(-5.15)	(-3.50)
DEF ≤ Median	-0.58	-0.46	-0.37
	(-1.52)	(-2.19)	(-2.08)

Table VIII. Equal-Weighted Univariate Portfolio Returns for Alternative Left-Tail Risk Metrics

This table presents return comparisons between equity deciles formed based on VaR5, ES1 and ES5 for the stocks in our sample. The decile portfolios are formed every month from 1962 to 2014. Portfolio 1 is the portfolio of stocks with the lowest left-tail risk and Portfolio 10 is the portfolio of stocks with the highest left-tail risk. The table reports the average left-tail risk metrics, one-month-ahead equal-weighted excess returns and two distinct five-factor alphas for each decile. The last column shows the differences of monthly excess returns and alphas between deciles 10 and 1. FFCPS alphas are calculated after adjusting for the market, size, value and momentum factors of Fama and French (1993) and Carhart (1997) and the liquidity factor of Pastor and Stambaugh (2003). FF5 alphas are calculated after adjusting for the market, size, value, profitability and investment factors of Fama and French (2015). Panel A presents results for value-at-risk at the 5% level. Panel B presents results for expected shortfall at the 1% level. Panel C presents results for expected shortfall at the 5% level. The left-tail risk metrics are defined in Table 1. Newey-West (1987) adjusted t-statistics are presented in parentheses.

Panel A. VaR5

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
VaR5	0.0150	0.0218	0.0256	0.0290	0.0325	0.0362	0.0405	0.0456	0.0522	0.0661	
Excess Return	0.74	0.78	0.85	0.86	0.85	0.92	0.84	0.77	0.53	0.05	-0.68
	(4.88)	(4.46)	(4.49)	(4.21)	(3.81)	(3.74)	(3.23)	(2.61)	(1.66)	(0.15)	(-2.27)
FFCPS Alpha	0.28	0.20	0.23	0.20	0.12	0.16	0.06	-0.02	-0.20	-0.58	-0.86
	(3.22)	(2.84)	(3.55)	(2.92)	(1.90)	(2.63)	(0.87)	(-0.26)	(-2.68)	(-4.77)	(-4.87)
FF5 Alpha	0.17	0.07	0.08	0.03	-0.03	0.02	-0.05	-0.02	-0.17	-0.43	-0.60
	(2.25)	(1.06)	(1.35)	(0.45)	(-0.58)	(0.41)	(-0.82)	(-0.26)	(-2.31)	(-3.55)	(-4.11)

Panel B. ES1

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
ES1	0.0342	0.0465	0.0547	0.0625	0.0705	0.0794	0.0895	0.1024	0.1214	0.1763	
Excess Return	0.71	0.85	0.87	0.92	0.91	0.93	0.80	0.63	0.45	0.15	-0.56
	(4.68)	(4.78)	(4.46)	(4.26)	(3.86)	(3.75)	(2.94)	(2.19)	(1.47)	(0.46)	(-2.23)
FFCPS Alpha	0.23	0.26	0.23	0.21	0.07	0.18	0.01	-0.13	-0.28	-0.43	-0.66
	(2.91)	(3.82)	(3.60)	(3.47)	(2.58)	(3.21)	(0.12)	(-1.87)	(-3.67)	(-4.94)	(-4.91)
FF5 Alpha	0.11	0.12	0.07	0.08	0.05	0.09	-0.04	-0.13	-0.23	-0.45	-0.57
	(1.63)	(2.09)	(1.20)	(1.46)	(0.91)	(1.83)	(-0.76)	(-2.04)	(-2.77)	(-4.32)	(-4.50)

Table VIII (continued)

Panel C. ES5

	Port1	Port2	Port3	Port4	Port5	Port6	Port7	Port8	Port9	Port10	High-Low
ES5	0.0238	0.0319	0.0372	0.0421	0.0470	0.0523	0.0584	0.0657	0.0754	0.0971	
Excess Return	0.71	0.82	0.86	0.87	0.91	0.92	0.85	0.76	0.51	0.00	-0.71
	(4.77)	(4.65)	(4.57)	(4.14)	(4.03)	(3.75)	(3.13)	(2.65)	(1.59)	(0.00)	(-2.44)
FFCPS Alpha	0.25	0.24	0.25	0.17	0.19	0.15	0.05	0.01	-0.26	-0.59	-0.83
	(2.89)	(3.35)	(3.93)	(2.54)	(2.93)	(2.41)	(0.77)	(0.20)	(-3.12)	(-5.55)	(-5.14)
FF5 Alpha	0.14	0.10	0.08	0.03	0.03	0.04	-0.02	-0.03	-0.17	-0.52	-0.66
	(1.86)	(1.70)	(1.44)	(0.50)	(0.57)	(0.60)	(-0.36)	(-0.57)	(-2.05)	(-4.44)	(-4.67)