Ignorance is Bliss?
Anonymous Lending with Roll Over Risk*

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Abstract

We provide a model of banks' short-term funding and study the conditions influencing roll over risk. Our model reproduces the major differences between U.S. and euro short-term funding markets. Not anonymous, short-term markets are prone to "early runs" on low quality borrowers, which impairs welfare. In anonymous, short-term markets "late runs" on low quality borrowers emerge, anonymity reduces banks' roll over risk, and welfare improves. An insurance mechanism, that transfers wealth from high to low quality borrowers, always delays runs, and it is thus welfare improving.

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1 Introduction

Banks heavily rely on short-term funding, which exposes them to runs, roll over risk, and wider financial contagion. The stability of funding markets crucially depends on the market structure (Martin et al., 2014a). In the United States, from 2008 to 2010, short-term funding in the form of tri-party and bilateral repos[1] declined by 40% (Copeland et al., 2014) and 34% (Krishnamurthy et al., 2014), respectively, exacerbating the 2007–2009 financial crisis. In Europe, short-term funding in the form of centrally cleared repos *increased* by 14%, and the safest segment of the Euro repo market even behaved as a shock absorber, in the sense that repo lending increased with risk, while spreads, maturities, and haircuts remain stable (Mancini et al., 2016).

The opposite responses of the U.S. and euro repo markets to crisis periods, and the social costs and benefits that they entail, raise at least two important questions. What are the market features and institutional settings that can reduce banks' roll over risk? Is there a market design that can ensure the stability of short-term funding markets? This paper seeks to address these questions by developing a model of short-term funding that captures the main differences between U.S. and euro repo markets.

There exist two major differences between repo markets in the United States and Europe, namely anonymity and insurance. In U.S. repo markets, lenders and borrowers typically know each others' identity; lending is not anonymous.[2] Furthermore, in tri-party and bilateral repos, lenders are directly exposed to counterparty risk, and there is typically no clear or prearranged procedure for the orderly distribution of losses in case of default of a borrower; no "insurance mechanism" is in place. In Europe, repo markets are set up essentially in the opposite way. The vast majority of interbank repos occurs via central counterparty (CCP)-based markets that are set up as anonymous electronic platforms, where lenders and borrowers do not observe each others' identity; lending is

[1] A repurchase agreement (repo) is essentially a collateralized loan based on a simultaneous sale and forward agreement to repurchase securities at the maturity date. In tri-party repos, a third party organizes the settlement and collateral management. A main tri-party agent active in the United States and Europe is JP Morgan (European Central Bank, 2012). Bilateral repos constitute the traditional over-the-counter market, in which the borrower and the lender trade directly with each other.

[2] Lenders and borrowers often establish long term relations in the form of tri-party and bilateral repos. Copeland et al. (2012) estimate the size of the various segments of the U.S. repo market as of May 2012 and find that the value of all repos was about $3 trillion, with approximately two-thirds from the tri-party market, both interdealer and investor-to-dealer, and one-third from the bilateral market. The estimated size of total repos as of July 2008 was $6.1 trillion.

anonymous.[3] In addition, the CCP eliminates direct counterparty risk as it has various lines of defense and clear rules to protect itself and clearing members in case of default of a counterparty; an insurance mechanism is in place.[4]

This paper develops a model of short-term funding and studies the conditions under which runs can occur in various short-term markets. In particular, we study whether anonymity reduces roll over risk. A priori it is unclear how anonymity would impact funding markets. On the one hand, anonymity can be detrimental to funding markets, as it prevents the assessment of counterparty risk. On the other hand, anonymity could allow low quality borrowers "to pull" with high quality borrowers, enhancing the stability of funding markets.

A key insight of our analysis is that non-anonymous short-term markets are prone to "early runs" on low quality borrowers, which impairs welfare. In anonymous short-term markets two equilibria outcomes emerge. Because "late runs" on low quality borrowers become possible, welfare improves and anonymity reduces banks' roll over risk. An insurance mechanism, that effectively transfers wealth from high quality borrowers to low quality borrowers, it always delays runs, and it is thus welfare improving. Or very simply, anonymity can delay runs, which is welfare improving in times of liquidity shortage, but it comes at a welfare cost in good times as resources are inefficiently allocated.

Our model features an illiquid long-term technology, that banks can finance through short-term loans, as well as learning of lenders and borrowers about the technology type. We analyze how the illiquidity of the technology and agents' learning impact banks' roll over risk and lead to various equilibrium outcomes.

Our paper contributes to various streams of the literature. Recent research highlights the importance of the market structure for the fragility of funding markets. Martin et al. (2014a)

[3]With an estimated volume of more than EUR 5.6 trillion (International Capital Market Association, 2012), the size of the European repo market is of similar magnitude to U.S. repo markets. The Euro interbank repo market can be divided into three parts: CCP-based, bilateral, and triparty. CCP-based repos constitute the majority in the Euro interbank repo market. From 2009 to 2013 the market share of CCP-based repos increased from 42% to 71%, whereas the share of bilateral repos declined from 50% to 19%. The share of tri-party repos remained relatively constant at around 10% (European Central Bank, 2013).

[4]For instance, at Eurex Repo, the market is structured in a way that a lender does not learn about the default of a borrower. The lender and any other market participant can only be affected by the default if the CCP has to draw on the clearing fund. This occurs after position closeout of the participant in default, liquidation of collateral of the participant in default, exhaustion of the clearing fund contribution of the participant in default, and after the CCP, Eurex Clearing, runs out of reserves; www.eurexclearing.com/clearing-en/risk-management/lines-of-defense/. LCH.Clearnet, another important CCP, has a similar water fall procedure in case of default of a clearing member.

study expectations-driven runs in triparty and bilateral repo markets, and find that lenders stop funding borrowers with expected risky collateral or high counterparty risk. Martin et al. (2014b) show that systemic runs can occur, for example, when the banking system as a whole is exposed to short-term funding risk. This stream of the literature focuses on bilateral versus triparty repos. We focus on anonymous lending via CCPs versus not anonymous bilateral repos, and analyze the impact of anonymity on short-term funding markets.

Another stream of the literature studies the link between information revelation and roll over risk. An important message from that literature is that disclosing bank-specific information during crisis stabilizes the financial system, while retaining information is optimal during normal times (Bouvard et al., 2015; Goldstein and Leitner, 2015). We contribute to that literature by studying the information revelation in funding markets, and uncovering the beneficial aspects of not disclosing borrowers' information.

A third stream of the literature studies the role of CCPs in derivatives markets. Biais et al. (2016) show that appropriately designed margin deposits and clearing arrangements can mitigate counterparty risk. Duffie and Zhu (2011) find that there may be destabilizing effects from introducing a CCP. When banks forgo bilateral netting opportunities across different contracts by trading through an anonymous CCP, banks may actually increase their overall counterparty risk, making the financial system more fragile. While this literature focuses on derivatives markets, we focus on the role of CCPs in short-term funding markets.

2 Model

Consider an economy with two types of risk neutral agents, three periods $t = 0, 1, 2$ and no discounting.

Borrowers There are two borrowers with access to a long term technology. Alternatively, there is a storage technology of which the net return is normalized to 0. Borrowers invest i_0 at $t = 0$ in the long term technology which yields a gross return of R^ω at $t = 2$, where $\omega \in \{L, H\}$. With probability β the long term technology yields a high payoff R^H and with probability $1 - \beta$ a low payoff R^L. In either state the long term technology yields a positive net return which is larger in the high state than in the low state, i.e. $R^H > R^L > 1$. If the long term technology is liquidated at $t = 1$, it delivers a gross return of λ. Early liquidation is costly in the sense that it yields a negative gross return $\lambda < 1$. At the time of initial investment, $t = 0$, borrowers do not know their project's quality. They just know that they have a positive net return project in expectation with $\beta R^H + (1 - \beta)R^L > 1$. Borrowers only learn about the exact quality of their project at $t = 1$. It is reasonable to assume that investors do not have full certainty about their investment's return at the time of investment but learn about its return over the course of the investment. Furthermore, we restrict attention to the case where there is a high type borrower and a low type borrower in the economy. This is the case which generates (i) the asymmetric information problem and (ii) makes insurance relevant. The idea of asymmetric information here is that if agents observe the borrowers' identity they can learn the borrowers' asset price

Lenders There are two generations of $2m$ lenders[5] with an initial endowment of 1 unit of cash who live for one period, i.e. they are born at t and exit the economy at $t + 1$. When lenders exit the economy at date t, they consume, c_t, both their initial endowment and their returns such that they obtain a utility $U_t = c_t$. We denote the initial cash endowment of each generation of lenders by $2m_t$. A priori, $m_0 = m_1 = m$ so that first generation lenders could be fully replaced by second generation lenders. For the remainder the subscript t will be omitted for m since it has the same size at both dates. It is not interesting to consider a case in which there is more cash available at $t = 1$ than at date $t = 0$ because it is impossible to increase the investment in the long term technology beyond the investment level of $t = 0$. This is due to the idea of a long term investment. The investment requires two periods to mature. Lenders can only invest their endowment into a loan with the borrower or into a storage technology which yields a net return which we normalize to 0. This implies that lenders themselves could not undertake the investment in the long term technology. As it is common in this literature, we assume that the long term technology is only available to a specialized group of investors, the agent, which in this case are the borrowers. A key element of this model is that second generation lenders receive an unexpected, observable liquidity shock f at $t = 1$. It is unexpected because none of the agents anticipates the shock at $t = 0$,

[5] An alternative way of interpreting the two generations of lenders is one set of lenders who live for two periods and roll over their loans.

when taking the investment decision[6]. In order to prevent any coordination issues, we assume that all second generation lenders are hit equally by the liquidity shock so that, at $t = 1$, the total endowment of cash available in the economy is $2m(1 - f)$. In the analysis we restrict attention to the case in which there is enough cash available to fully finance at least one of the borrowers, i.e. $2m(1 - f) > m$ and $f < \frac{1}{2}$. If there was less cash available, the high type would always get all the finance and the low type nothing. Moreover, there would be no reason for insurance between high and low type borrower since it would be socially optimal to invest everything into the high type borrower's project. The liquidity shock is observable at $t = 1$ by all the agents in the economy. The idea is that one morning Financial Market participants wake up and read on the news that the CDS market has suffered a severe loss, i.e. an aggregate shock on all Financial Market participants. Therefore, they are able to evaluate how much money is available for borrowers to roll over their loans.

Institutional Differences In this economy, there are two ways for borrowers and lenders to agree on a loan. Either bilaterally or through a central clearing platform. These two setups are inspired by the US versus the European system of interbank lending. In the US, two thirds of interbank lending is conducted via bilateral or tri-party[7] loans while in Europe 70 % of interbank lending is done over central clearing platforms. There are two key features which distinguish the US interbank system from the European one. First, in bilateral and tri-party lending lenders can observe their counterparty which is not possible on a central clearing platform, i.e. there is asymmetric information on a central clearing platform. In terms of the model, this means, in the bilateral setup, the lender observes the borrower project's quality while on the central clearing platform lenders do not. Second, central clearing platforms in Europe have an insurance mechanism. Meaning that when a borrower defaults on the loan, the other platform participants have to contribute to the bailout of the loan. In reality there exists a so called clearing fund to which all participants contribute up front. Over time, participants on the central clearing platform take on both roles, lender and borrower. The change of roles is something we are not concerned with in

[6]The liquidity shock is very close in spirit to the idea of depositors running on a bank as in Diamond and Dybvig (1983). It is also close to the idea of global games in which a liquidity parameter in a stationary equilibrium is perturbed like in Martin et al. (2014 a,b).

[7]In the following we refer to both of them as bilateral lending. For this paper anonymity is the important difference and both bilateral and tri-party lending is non-anonymous.

this article. When a participant defaults on the loan, first the defaulting participant's collateral and contribution to the clearing fund are ceased and then all the other participants have to contribute on a pro-rata basis. For modelling purposes we are just considering a classic insurance mechanism through which the borrower with the good project subsidizes the borrower with a bad project. In principle, the intermediary in tri-party lending could set up a similar insurance mechanism which did not happen in reality though. It is worth emphasizing that this article does not look at the selection of participants into one or the other institution but is a mere comparison of the two.

Contracts At $t = 0$, borrowers and and first generation lenders agree on a loan with a loan amount ℓ_0 and a gross loan rate of c_1. The gross loan rate is per lender and ℓ_0 is the total amount of loan each borrower receives at $t = 0$ from all lenders together. The gross loan rate c_1 is paid at $t = 1$ to each of the lenders and they consume it straight away. In other words, lenders consume their initial endowment plus the net return from the loan. The contract signed by lenders and borrowers consists of the two components (c_1, ℓ_0).

At $t = 1$, first generations lenders each get a gross return of c_1 from the loan and consume it. Furthermore, borrowers learn about their project's type which is R^ω with $\omega \in \{L, H\}$. At the same time, each second generation lender receives a liquidity shock $f \in [0, 1]$ to the initial endowment of 1 unit of cash so that the total cash endowment of the economy is $2m(1 - f)$. The loan contract between second generation lenders and borrowers is defined by the loan amount ℓ_1 and the gross loan rate c_2. Borrowers can repay first generation lenders through the new loan from second generation lenders or by liquidating a part $z \leq \ell_0$ of the long term technology which yields a gross return $\lambda < 1$.

In $t = 2$ the payoff from the long term technology realizes and borrowers repay the loans plus an interest which amounts to c_2. The sequence of events is summarized in figure 1.

3 Equilibrium description

The equilibrium characterized is a Perfect Bayesian Nash equilibrium. In case of non-anonymous lending, i.e. in the bilateral or tri-party setting, there is no information asymmetry and hence lenders are able to perfectly discriminate between borrower types. With anonymous lending there is asymmetric information about the borrower's type. This can lead to two types of equilibria. An

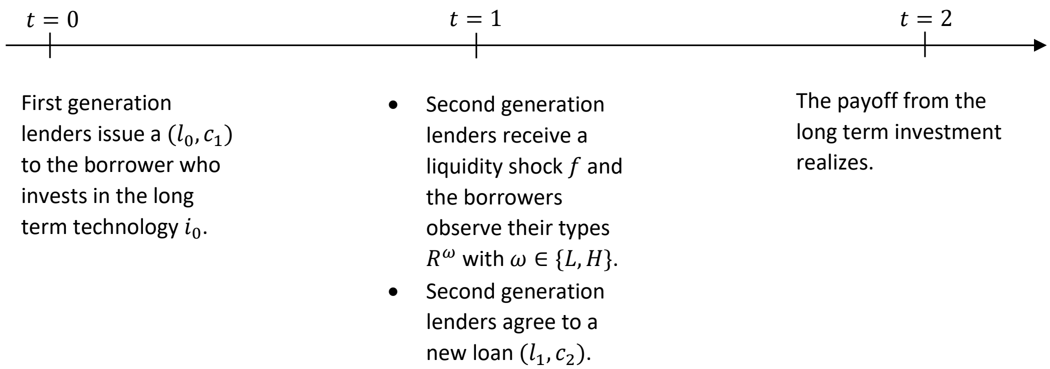

Figure 1: Timeline of the model

equilibrium in which both borrower types obtain the same loan, a pooling equilibrium. And an equilibrium in which borrowers use the loan contract to signal their type, a separating equilibrium. For the main part of the paper it is assumed that borrowers behave competitively at $t = 1$ as there is too little liquidity in the economy relative to what borrowers would need to fully roll over their long term project. The assumption seems reasonable as a liquidity shortage should induce competition on the borrower side but at the same time it is merely for analytical clarity and can easily be relaxed. For $t = 0$ it is necessary to require that first generation lenders do not have all the bargaining power. This is necessary since first generation lenders have a first mover advantage and thus could, by correctly anticipating borrowers and second generation lenders decisions at $t = 1$ extract all the profit. This would lead to no roll over at $t = 1$ as first generation lenders and borrowers at $t = 0$ do not anticipate the liquidity shock at $t = 1$ and hence first generation lenders would ask for rates which borrowers would only be able to repay by liquidating a too large chunk of the long term investment. It would be too large in the sense that second generation lenders would not provide borrowers with loans as they would anticipate not to be repaid since too much of the long term investment is liquidated. In economic terms, it is required that at $t = 0$, the economy is not at the verge of collapse and lenders are still eager to provide loans. This seems a reasonable description of reality as lenders in normal times are in need for investment and therefore compete to finance positive NPV projects. Notice that for most part of the following analysis it is not assumed that first generation lenders are in perfect competition. It is however needed that they do not hold all the market power.

4 Non-anonymous loans

In anonymous lending, to which for the remainder of the paper it will be referred to as bilateral lending for ease of exposition, lenders can perfectly discriminate between borrower types. More precisely, at $t = 1$, second generation lenders can observe the borrowers' project quality and hence negotiate the loan contract accordingly. Firstly, the equilibrium in bilateral lending is described and secondly, a welfare analysis is provided.

4.1 The loan contract in bilateral lending

The solution specifies gross repo rate and loan amount $(c_2^\omega, \ell_1^\omega)$ and is obtained from backward induction. Since at $t = 2$ agents do not take any decisions but only realize payoffs, the analysis starts from $t = 1$. Second generation lenders, after having been hit by a liquidity shock, negotiate a loan with borrowers who learned about their project's quality. In the non-anonymous setup, lenders can observe borrowers' identity and thus learn their projects' quality too. It is like once you know the identity of your counterparty you know their asset price.

In equilibrium, second generation lenders are willing to issue a loan if their net return is positive:

$$c_2^\omega \geq 1. \tag{1}$$

Lenders prefer to issue a loan to the high type, if $c_2^H \geq c_2^L$[8]. Observe, some lenders are better off than others because they lend to the high type borrower. Since lenders are identical there is no obvious way how to attribute different types of borrowers to lenders. For the purpose of this study the distribution of wealth among lenders is not relevant since only aggregate wealth of lenders is considered.

Lenders provide loans up to capacity to the high type, $\ell_1^H = i_0 = m$ and only the rest, i.e. $\ell_1^L = 2m(1-f) - m = (1-2f)m$ to the low type borrower. The capacity limit of the loan at $t = 1$ is given by the initial investment into the long term technology i_0 at $t = 0$. Since at $t = 0$, neither borrowers nor lenders know the quality of the projects, the total endowment of cash available at

[8]In the further analysis, it is shown that this is true in equilibrium.

$t = 0$ is split evenly between the two lenders so that each borrower invests $i_0 = m$ into the long term technology[9]. How much a borrower is able to roll over at $t = 1$ depends on the rate promised to first generation lenders c_1. Formally this is given by the following feasibility constraint:

$$\lambda z_1^\omega + \ell_1^\omega - mc_1 = 0 \tag{2}$$

$$z_1^\omega = \frac{mc_1 - \ell_1^\omega}{\lambda} \tag{3}$$

The feasibility constraint assures that at $t = 1$ a borrower has enough money to repay the first generation lenders. The feasibility constraint holds with equality because the borrower wants to liquidate as little as possible of the long term technology since it generates a negative net return with $\lambda < 1$. From equation 3, it is clear that the higher the rate promised to first generation lenders c_1 the more the borrower has to liquidate. In addition, the higher the liquidation cost of the long term technology, i.e. the smaller λ, the more of the long term technology has to be liquidated. This is truly the cost of early liquidation.

Borrowers trade off the gain from the return of the long term technology against the cost of the loan at $t = 2$. Since the storage technology's net return is normalized to 0, borrowers' outside option is 0. The trade-off of a borrower with a project of quality ω is summarized as follows:

$$R^\omega(i_0 - z_1^\omega) - c_2^\omega \ell_1^\omega \geq 0 \tag{4}$$

With $z_1^L = \frac{mc_1 - \ell_1^L}{\lambda}$, $\ell_1^H = m$ and $\ell_1^L = (1 - 2f)m$, it is clear that participation in inequality 4 is more binding for the L-type than for the H-type. Hence, in a competitive borrower market, both types of borrowers compete up to the point at which the L-type borrower's marginal profit equals the marginal cost:

$$c_2^L = R^L(1 - \frac{c_1 - (1 - 2f)}{\lambda})\frac{1}{1 - 2f} \tag{5}$$

This is the gross repo rate for which the L-type borrower makes zero net profit. c_2^L is also a measure for the split of the profit accruing from the L-type's project. In this case all the profit goes to the lenders of the L-type. It is intuitive that it decreases in the size of the liquidity shock to second

[9]This is more formally shown in the appendix.

generation lenders[10] as more of the long term project has to be liquidated early which is costly with $R^L > 1 > \lambda$. The net profit from the L-project decreases in the repo rate promised to first generation lenders. Since first generation lenders have a first mover advantage they can, if they have all the bargaining power at $t = 0$, extract all the rent and leave second generation lenders and borrowers with zero net profit. As argued above, this possibility is ruled out but it helps to illustrate a classic result from maturity transformation. Lenders with an one period horizon can benefit from a two period technology.

Clearly, the L-type borrower cannot compete in repo rates beyond c_2^L. So if the H-type borrower offers a rate $c_2^H = c_2^L + \epsilon$ where ϵ is arbitrarily small but positive, lenders compete for this rate. In other words, at this point the H-type borrower has all the bargaining power vis-à-vis the lenders. Which drives down the H-type borrower's repo rate to $c_2^H = c_2^L$. Notice this satisfies the initially postulated condition $c_2^H \geq c_2^L$ which is required for the H-type borrower receiving a full roll-over of the loan $\ell_1^H = m$ and the L-type lender receives what is left as cash endowment in the economy $\ell_1^L = (1 - 2f)m$.

There are different ways to show the existence of an equilibrium in the non-anonymous setting. The general idea however is the same for all. It answers to the question, "How big can a liquidity shock to second generation lenders be before the L-types project delivers a too low return to secure agents' participation?". The route taken here to prove existence at the same time illustrates the limit to first generation lenders to extract rent from borrowers and second generation lenders.

Lenders are only willing to provide the low type borrower with a loan if they can make a positive profit. They provide a one unit loan in return for a gross return of c_2^L:

$$c_2^L \geq 1. \tag{6}$$

The largest possible c_1 attainable for first generation lenders is when $c_2^L = 1$ and the L-type makes zero profit:

$$R^L(i_0 - z_1^L) - c_2^L \ell_1^L = 0 \tag{7}$$

$$c_1 = 1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}). \tag{8}$$

[10]For $c_1 \geq 1$ which has to be satisfied for first generation lenders to issue loans.

At $t = 0$, first generation lenders do not anticipate the liquidity shock f to second generation lenders. Therefore the last term in the expression 8 equals zero. If first generation lenders would charge a repo rate of $c_1 = 1 + \lambda - \frac{\lambda}{R^L}$, the L-type borrower would always default at $t = 1$. This is why it is required that first generation lenders do not have full bargaining power at $t = 0$ and hence cannot extract all the profit. The amount of bargaining power they cannot have amounts at least to the last term in 8: $-2f(1 - \frac{\lambda}{R^L})$.

On the other hand, the lowest possible rate required by first generation lenders is $c_1 = 1$. Therefore there exists a range of equilibria with

$$1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}) \geq c_1 \geq 1. \tag{9}$$

This leads back to the question "How big can a liquidity shock to second generation lenders be before the L-types project delivers a too low return to secure agents' participation?". Notice that neither the L-type borrower nor second generation lenders make profits for the upper bound of expression 9. Therefore the return from the L-type project is completely exhausted if

$$1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}) = 1, \tag{10}$$

$$f^{Bil} = \frac{R^L - 1}{R^L - \lambda}\frac{\lambda}{2}. \tag{11}$$

Observe f^{Bil} is increasing in both the return to the long term technology when liquidated at maturity, R^L, and when liquidated early, λ. It is intuitive that the larger the return at maturity, R^L, the more the borrower can liquidate at $t = 1$ without affecting the debt repayment at $t = 2$ and thus the less loan is needed from second generation lenders. Similarly, the larger the return of early liquidation λ, the less of the long term technology the borrower has to liquidate in order to repay first generation lenders at $t = 1$ which in turn implies the less the repayment of second generation lenders is affected at $t = 2$.

The threshold f^{Bil} defines the existence of the equilibrium in non-anonymous lending in which both types of borrowers are able to roll over their loans. Recall the assumption about funds available in the economy at date $t = 1$, $2m > 2m(1 - f) > m$ which implies that $\frac{1}{2} > f > 0$. In addition, observe that with $\lambda < 1$, $\frac{R^L-1}{R^L-\lambda}\frac{\lambda}{2} < \frac{1}{2}$.

If the liquidity shock on second generation lenders is sufficiently large, i.e. $\frac{1}{2} > f > f^{Bil}$, second generation lenders provide loans only to the high type borrower, (c_2^H, ℓ_1^H)[11], since they anticipate that the low type borrower is not able to repay at $t = 2$. This is what the literature widely refers to as a *run* on borrowers. In this case the low type borrower has to liquidate all of the long term technology but since early liquidation is costly, the low type borrower is not able to fully repay the first generation lenders. Therefore part of the lenders' initial endowment is lost. In other words, the economy incurs an overall loss. The loss to the economy is quantified more precisely in the following subsection.

Before, the main finding of this subsection is summarized in the following proposition.

Proposition 4.1 (Non-anonymous lending). *If the unexpected liquidity shock on second generation lenders is small, $f^{Bil} \geq f$, both types of borrowers can roll over their loans. When the liquidity shock is large, $\frac{1}{2} \geq f > f^{Bil}$, there is a run on the low type borrower and only the high type borrower is able to roll over the loan.*

At the run threshold f^{Bil}, a marginal increase in the liquidity shock f leads to a credit crunch similar to credit rationing in Stiglitz and Weiss (1981) with the major difference that the credit rationing in this case is due to roll-over risk and not to adverse selection[12].

4.2 Welfare in non-anonymous lending

Since the objective of this article is to evaluate the resilience of different interbank lending institutions against an unexpected liquidity shock, the welfare analysis has necessarily to be conducted from an ex-post perspective. Welfare is simply defined as the sum of net returns of all agents in the economy, i.e. lenders and borrowers.

The following expression aggregates net returns for both lenders and borrowers in case there is no run, $f^{Bil} \geq f$:

[11] Since the H-type borrower has all the bargaining power in this case, the H-type loan contract is $c_2^H = 1$ and $\ell_1^H = m$. It is worrisome to see that in times of a run, interest rates can drop sharply which makes the market look very liquid when in fact it is only liquid for the H-type borrower but has collapsed for the L-type borrower.

[12] The authors are grateful to David R. Skeie for pointing this out.

$$
\begin{aligned}
W^{Bil}_{f<f^{Bil}} = & \underbrace{R^H(i_0 - z^H_1) - c^H_2 \ell^H_1}_{\text{Net return H-type borrower}} + \underbrace{R^L(i_0 - z^L_1) - c^L_2 \ell^L_1}_{\text{Net return L-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \\
& + \underbrace{\lambda z^H_1 + \ell^H_1 - c_1\ell_0}_{\text{cashflow at t=1 H-type borrower}} + \underbrace{\lambda z^L_1 + \ell^L_1 - c_1\ell_0}_{\text{cashflow at t=1 L-type borrower}} \\
& + \underbrace{2c_1\ell_0 - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{c^H_2 \ell^H_1 + c^L_2 \ell^L_1 - 2(1-f)m}_{\text{Net return 2nd generation lenders}} . \\
& = R^H(i_0 - z^H_1) + R^L(i_0 - z^L_1) + 2c_1\ell_0 - 2m - 2(1-f)m \qquad (12) \\
& = (R^H + R^L - 2)m - (\frac{R^H + R^L}{\lambda} - 2)(c_1 - 1)m - 2f(\frac{R^L}{\lambda} - 1)m \qquad (13)
\end{aligned}
$$

It makes sense that absent a liquidity shock, i.e. $f = 0$, and a perfectly competitive lender market at $t = 0$, implying that $c_1 = 1$, the economy generates a net return of $(R^H + R^L - 2)m$. It is just the gross return of the two projects available in this economy net of the initial investment.

In equation 12, it is interesting to see that welfare does not depend on transfers between borrowers and second generation lenders c^{ω}_2. It is therefore true what was previously postulated that the competitive situation at $t = 1$ and thus the distribution of profits is only of minor importance for this study. Furthermore, observe in expression 13 that welfare is decreasing in the repo rate paid to first generation lenders for every $c_1 > 1$ since $\frac{R^H+R^L}{\lambda} > 2$. The explanation is that, even without a liquidity shock, second generation lenders can at most replace the initial investment. If first generation lenders however demand more than the initial investment, i.e. $c_1 > 1$ per lender, some of the long term project has to be liquidated in order to repay first generation lenders. A first policy implication is therefore, it is important that the interbank market has sufficient liquidity so that lenders behave competitively which keeps rates low and in turn reduces the roll over risk.

It comes at no surprise that welfare in expression 13 decreases in the liquidity shock f. And even more so the larger the difference between the L-type's return R^L and the early liquidation return λ. The welfare loss stemming from the liquidity shock is driven by the L-type's project because it is the L-type borrower only who is affected by the liquidity shock. Recall, the H-type borrower receives full roll over.

When the liquidity shock is sufficiently large, $f > f^{Bil}$, there is a run on the low type borrower so that he is forced to liquidate the long term technology early. The high type borrower holds the long term technology until maturity and thus can reap the full profits of it. In the following the net returns of lenders and borrowers in case of a run are summarized:

$$
\begin{aligned}
W_{Bil,f>f^{Bil}} = & \underbrace{R^H i_0 - c_2^H \ell_1^H}_{\text{Net return H-type borrower}} + \underbrace{\lambda i_0 - \lambda m}_{\text{Net return L-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \\
& + \underbrace{\lambda z_1^H + \ell_1^H - c_1 \ell_0}_{\text{cashflow at t=1 H-type borrower}} + \underbrace{c_2^H \ell_1^H - m + (1-2f)m - (1-2f)m}_{\text{Net return 2nd generation lenders}} + \underbrace{c_1 \ell_0 - m + \lambda \ell_0 - m}_{\text{Net return 1st generation lenders}} \\
& = R^H (i_0 - z_1^H) - m + c_1 \ell_0 - m + \lambda \ell_0 - m \qquad (14) \\
& = (R^H + \lambda - 2)m - (\frac{R^H}{\lambda} - 1)(c_1 - 1)m \qquad (15)
\end{aligned}
$$

From expression 15 it is clear that the economy in case of the L-type borrower's bankruptcy generates the gross return from the H-type's project and the early liquidation value of the L-type's project net of the initial investment. In addition welfare decreases the more first generation lenders' repo rate exceeds the initial per lender loan. In case of the L-type's bankruptcy, only the H-type is affected by a large c_1 as the L-type's can at most repay the value of early liquidation λm. Welfare in case of a run on the L-type is independent of the size of the liquidity shock since, by assumption, there is always enough cash endowment in the economy to roll over the H-type borrower's loan.

Just like in the case of no run in expression 12, welfare in case of a run does not depend on transfers from borrowers to second generation lenders as becomes apparent from expression 14.

The results of the welfare analysis are summarized in the following proposition.

Proposition 4.2 (Welfare in non-anonymous lending)**.** *As long as the liquidity shock is sufficiently small, $f^{Bil} \geq f$, both types of borrowers manage to roll over their loans and thus total welfare is $W_{Bil,f\leq f^{Bil}}$. As soon as there is a run on the low type borrower, i.e. $\frac{1}{2} \geq f > f^{Bil}$, total welfare becomes $W_{Bil,f>f^{Bil}}$.*

It is straightforward to show that at $f = f^{Bil}$, the no run welfare $W_{Bil,f\leq f^{Bil}}$ yields larger welfare than the run welfare $W_{Bil,f>f^{Bil}}$ if $c_1 < \frac{R^L - \lambda^2}{R^L - \lambda}$. This is interesting as it implies if the promised return to first generation lenders is too large, it is welfare improving to let the L-type borrower default. For the remainder of this paper the focus is however on the case in which it is welfare improving to have both borrowers roll over their loans.

5 Anonymous central clearing

On central clearing platforms, such as the EUREX in Frankfurt, participants cannot observe their counterparty but only observe, usually on a PC screen, interest rates, loan amounts and the category of collateral. The purpose of this study is to focus on the first two aspects of the loan contract, the loan amount and the interest rate. One of the two institutional differences is asymmetric information on the central clearing platform. Lenders cannot observe their counterparty's type. In practice this means lenders do not have information such as stock price, earning's reports, ad-hoc announcements etc. since they do not know the identity of their counterparty[13].

This section develops Perfect Bayesian Equilibrium in which the borrowers' identities are not revealed to lenders. Both types of borrowers offer the same loan contract consisting of gross loan rate and loan amount. Therefore, lenders cannot infer the borrowers' types from the loan contract. This characterizes a pooling equilibrium which is fully described in the following subsection.

5.1 Anonymous loan contract

As it is a dynamic game, it is solved by backward induction. At $t = 2$ no decisions are taken and only payoffs realize. So that we can move backward to $t = 1$. The subgame at $t = 1$ is a game of incomplete information due to the asymmetric information about the borrowers' types. Second generation lenders, after having been hit by a liquidity shock, negotiate a loan with borrowers who learned about their project's quality. Borrowers trade off the gain from the return of the long term technology against the cost of the loan at $t = 2$. The borrower's gross return at $t = 2$ consists of the per unit return R^{ω} times the amount initially invested at $t = 0$, i_0, reduced by the amount needed for repayment of first generation lenders z_1^P where the superscript indicates the Pooling equilibrium. In other words, it must be the same for both types of borrowers. Otherwise lenders could infer the borrower type. The borrower's cost of the loan c_2^P is the same for both types of

[13] It is worth thinking about whether the lenders' types are relevant as well. In fact, lenders might become so liquidity constrained that they do not want to supply loans anymore. This study however is a first step towards understanding the effects of the institutional differences on interbank lending. And therefore it is only interesting to study a case where there is at least some supply of loans. Which for modeling purposes we assume to be non-stochastic. In a repeated game of interbank lending, more sophisticated shocks, both on the demand and supply side might be interesting to study but is outside the scope of this analysis.

borrowers by the same argument as before. Since the storage technology's net return is normalized to 0, borrowers' outside option is 0. The trade-off of a borrower with a project of quality ω is summarized as follows:

$$R^{\omega}(i_0 - z_1^P) - c_2^P \ell_1^P \geq 0 \tag{16}$$

At $t = 1$, borrowers have to be able to repay first generation lenders. They have two sources from which they can repay them, new loans from second generation lenders ℓ_1^P and liquidation of the long term technology z_1^L with a return λ. Borrower's repayment of first generation lenders is summarized in the following feasibility constraint:

$$\lambda z_1^P + \ell_1^P - c_1 \ell_0 = 0 \tag{17}$$

The feasibility constraint holds with equality since it is costly to liquidate the long term technology and hence borrowers only do so up to the point at which they can repay first generation lenders. From the feasibility constraint in equation 17, it follow that the amount liquidated from the long term technology is

$$z_1^P = \frac{\ell_0 c_1 - \ell_1^P}{\lambda} \tag{18}$$

In a pooling equilibrium both types of borrower offer to pay the same loan rate to second generation lenders. And lenders are willing to provide loans to borrowers if their net return is positive

$$c_2^P \geq 1. \tag{19}$$

If lenders' participation constraint in inequality 19 is satisfied, then the total loan amount available to both borrowers amounts to $2m(1 - f)$. Since agents are risk neutral, projects have positive NPV and the outside option is normalized to zero, it is rational to fully invest the cash endowment in the long term projects. In addition, lenders cannot distinguish between borrower types and therefore both borrowers receive identical loan amounts $\ell_1^P = (1-f)m$. From equation 18, it directly follows that borrowers have to liquidate $z_1^P = \frac{c_1-(1-f)}{\lambda}m$ from the long term technology.

A priori it is not clear that borrowers prefer $\ell_1^P = (1-f)m$ over other possible loan amounts ℓ_1'. The high type in particular might prefer to get the full roll over and pay a slightly higher rate in return. The largest loan the high type would be interested in is full roll over $\ell_1' = \ell_1^H = i_0 = m$[14]. In order to make sure the high type borrower does not want to deviate from the equilibrium quantity ℓ_1^P, the equilibrium payoff is compared to the payoff from deviation

$$R^H(i_0 - z_1^P) - c_2^P \ell_1^P \geq R^H(i_0 - z_1') - c_2' \ell_1' \quad \forall \quad \ell_1' \in [0, i_0] \tag{20}$$

It is clear that the H-type lender would prefer a larger loan amount than ℓ_1^P. And hence the question is for which rate c_2^P, the H-type would still prefer the pooling contract (c_2^P, ℓ_1^P). By the same reasoning, lenders' beliefs in the appendix are specified. I.e. if they observe a different loan amount than the equilibrium one ℓ_1^P, they believe to be facing a H-type borrower and thus request the H-type loan rate c_2^H obtained in the separating equilibrium in expression 39.

The individual rationality constraint 20 therefore can be rewritten as:

$$R^H(i_0 - z_1^P) - c_2^P \ell_1^P \geq R^H(i_0 - z_1^H) - c_2^H \ell_1^H \text{ with } c_2^H = \frac{R^L}{\lambda}(1 + \lambda - c_1) \tag{21}$$

$$\frac{R^L}{\lambda(1-f)}(1 + \lambda - c_1) - f\frac{R^H}{\lambda(1-f)} \geq c_2^P \tag{22}$$

Inequality 22 states that the H-type lender prefers the contract (c_2^P, ℓ_1^P) over the contract (c_2^H, ℓ_1^H) if the loan rate paid in equilibrium c_2^P is sufficiently small relative to the loan rate paid otherwise c_2^H. In addition, condition 22 implies that also the L-type borrower does not want to increase the loan size. Further below it is shown why the L-type also does not want to deviate to a lower loan amount ℓ_1^L. Notice the implicit assumption that both in separating equilibrium and pooling equilibrium the same repo rate c_1 is paid to first generation lenders.

It turns out that condition 22 is more binding than the condition obtained from the L-type's participation constraint, the alternative rate on which borrowers could pool[15]. In other words, the rate which would leave the L-type borrower with zero profit is not small enough to make the

[14] In the appendix it is shown that it is indeed optimal at $t = 0$ for lenders to invest all their endowment in the two borrowers and for the two borrowers to invest everything in their positive NPV projects. Intuitively it is clear that lenders would like to do so because their outside option yields a lower return. Since there is no Moral Hazard on the borrowers' side, they are risk neutral and have positive NPV projects no matter which type they turn out to be, they prefer to invest the entire loan into the long term technology.

[15] They would never pool on the H-type's rate as the L-type would earn negative profit at that rate.

H-type borrower not deviate. In fact, from the L-type borrower's participation constraint it can be obtained:

$$R^L(i_0 - z_1^P) - c_2^P \ell_1^P \geq 0 \tag{23}$$

$$\frac{R^L}{\lambda(1-f)}(1 + \lambda - c_1 - f) \geq c_2^P \tag{24}$$

Condition 22 is more binding than condition 24. In economic terms this means that the repo rate in the pooling equilibrium has to be so low for both types participate that even the L-type borrower makes positive profit differently from the separating equilibrium. This is also the reason why the L-type borrower does not only not want to deviate to a higher loan amount ℓ_1^H but also not to a lower loan amount ℓ_1^L.

Participation of the H-type borrower is implied by the individual rationality constraint in 22 and the participation constraint of the L-type borrower in 24.

Consequentially, the highest rate second generation lenders can charge from borrowers is $c_2^P = \frac{R^L}{\lambda(1-f)}(1 + \lambda - c_1) - f\frac{R^H}{\lambda(1-f)}$. Since c_2^P is decreasing in c_1, the largest rate first generation lenders can charge is when second generation lenders are indifferent between lending or not, i.e. $c_2^P = 1$. The lowest rate first generation lenders are willing to accept is $c_1 = 1$. Notice, first generation lenders have the first mover advantage but do not observe the liquidity shock f.

There are different ways to show the existence of an equilibrium in the non-anonymous setting. The general idea however is the same for all. It answers to the question, "How big can a liquidity shock to second generation lenders be before the L-types project delivers a too low return to secure agents' participation?". The route taken here to prove existence at the same time illustrates the limit to first generation lenders to extract rent from borrowers and second generation lenders.

There exists a pooling equilibrium for the range $1 + \lambda - \frac{\lambda}{R^L} - f\frac{R^H - \lambda}{R^L} \geq c_1 \geq 1$ for $c_2^P = \frac{R^L}{\lambda(1-f)}(1 + \lambda - c_1) - f\frac{R^H}{\lambda(1-f)}$ and $\ell_1^P = (1-f)m$. Observe that for value of c_1 in the above given range, the equilibrium exists if $f^{Pool} = \frac{R^L - 1}{R^H - \lambda}\lambda \geq f$. There is only bankruptcy in case of a pooling equilibrium if $f^{Pool} < \frac{1}{2}$ which leads to the following condition on the early liquidation return $\lambda < \frac{R^H}{2R^L - 1}$. It is interesting to note that there exist parameters in which there is no bankruptcy in the pooling equilibrium, i.e. when $\frac{R^H}{2R^L - 1} > 1 \quad \Rightarrow \quad 2R^L - R^H > 1$.

Existence of the pooling equilibrium is different from the existence of the separating equilibrium/non-anonymous lending which is defined by $f^{Sep} = f^{Bil}$. In fact, the pooling equilibrium can withstand larger liquidity shocks if early liquidation is sufficiently costly, $2R^L - R^H > \lambda$. Notice that from the two conditions on the early liquidation return λ, the following two cases follow:

$$\lambda < \begin{cases} \frac{R^H}{2R^L - 1} & \text{if } 2R^L - R^H > 1 \\ 2R^L - R^H & \text{if } 2R^L - R^H < 1. \end{cases} \tag{25}$$

In fact this study is only interesting in the case early liquidation of the long term technology is costly, otherwise runs would never be an issue. Also for the welfare comparison later in this study, it turns out that the interesting case is when liquidation cost are sufficiently large, i.e. λ sufficiently small.

If the liquidity shock on second generation lenders is sufficiently large, i.e. $\frac{1}{2} > f > f^{Pool}$, second generation lenders provide loans only to the H-type borrower, (c_2^H, ℓ_1^H)[16], since they anticipate that the low type borrower is not able to repay at $t = 2$. This is what the literature widely refers to as a *run* on borrowers. In this case the low type borrower has to liquidate all of the long term technology but since early liquidation is costly, the low type borrower is not able to fully repay the first generation lenders. Therefore part of the lenders' initial endowment is lost.

The following proposition summarizes the main findings in case of anonymous lending.

Proposition 5.1 (Anonymous loan contract)**.** *If the unexpected liquidity shock on second generation lenders is small, $f^{Pool} \geq f$, both types of borrowers can roll over their loans with a loan contract (c_2^P, ℓ_1^P). When the liquidity shock is large, $\frac{1}{2} \geq f > f^{Pool}$, there is a run on the low type borrower and only the high type borrower is able to roll over the loan at a loan contract (c_2^H, ℓ_1^H).*

5.2 Welfare in anonymous lending

The objective is to evaluate the resilience of different interbank lending institutions against an unexpected liquidity shock. Therefore the welfare analysis is conducted from an ex-post perspective. Welfare is the sum of net returns of all agents in the economy, i.e. lenders and borrowers.

The results of the welfare analysis are summarized in the following proposition[17].

[16] Since the H-type borrower has all the bargaining power in this case, the H-type loan contract is $c_2^H = 1$ and $\ell_1^H = m$.
[17] The formal derivation is referred to the appendix

Proposition 5.2 (Welfare in anonymous lending)**.** *For a sufficiently small liquidity shock, $f^{Pool} \geq f$, both types of borrowers roll over their loans and total welfare is*

$$W_{Pool,f \leq f^{Pool}} = ((R^H + R^L - 2) - \frac{f}{\lambda}(R^H + R^L - 2\lambda))m. \tag{26}$$

If the low type borrower faces a run, $\frac{1}{2} \geq f > f^{Pool}$, total welfare becomes

$$W_{Pool,f > f^{Pool}} = (R^H + \lambda - 2)m. \tag{27}$$

In case of no run, welfare in equality 26 is decreasing linearly in the size of the liquidity shock f. If there is no liquidity shock $f = 0$, total welfare amounts to $(R^H + R^L - 2)m$ which is the maximum net return this economy can generate. Recall, positive net returns solely accrue from long term investment which generates $R^H + R^L$ per unit of cash invested. And the initial endowment of the economy is $2m$. When the liquidity shock f increases up to f^{Pool}, the cost of early liquidation of the long term technology, due to too little lending from second generation lenders, has eaten up the entire gain from the L-type borrower's long term investment plus some of the return from the H-type borrower's return. In the comparison of equilibria, it will become clear that this is due to the fact that the run is delayed. As soon as f grows beyond f^{Pool}, it can be seen from equation 27, that the low type borrower's is generating a negative net return since all of the long term investment is liquidated prior to maturity at a gross return of $\lambda < 1$. It is noteworthy, that at the threshold at which lenders run on the low type borrower, f^{Pool}, welfare without a run is only larger than welfare with a run, i.e. $W_{Pool,f \leq f^{Pool}} > W_{Pool,f > f^{Pool}}$, if the return from early liquidation of the long run technology is not too large $2 - R^L > \lambda$.

6 Anonymous lending with signaling

In the second possible equilibrium on the central clearing platform, borrowers can use the loan contract to credibly signal their types. The idea is that the high type borrower is willing to pay a higher loan rate in exchange for a larger loan. This implies there are two different, incentive compatible loan contracts for each borrower type: $(c_2^\omega, \ell_1^\omega)$ with $\omega \in \{L, H\}$. Where ℓ_1^ω is the loan amount a borrower ω receives from all the lenders at $t = 1$ and c_2^ω is the per lender gross loan rate from borrower ω. The gross loan rate contains the loan repayment of 1 unit of cash plus the interest to each lender. Quite naturally, the high type borrower receives loans up to the amount which allows to leave all the money invested in the long term technology at date $t = 1$, i.e. $\ell_1^H = i_0$. It is not possible for borrowers to top up the investment in the long term technology at date $t = 1$. Therefore, at $t = 1$, borrowers at most ask for a loan amounting to the initial investment. In the appendix, lenders' beliefs are specified such that whenever lenders observe a different loan amount ℓ_1' but the equilibrium one ℓ_1^ω, they believe it is the high type who wants to deviate. Therefore it follows from the individual rationality constraint of the H-type borrower:

$$R^H(i_0 - z_1^H) - c_2^H \ell_1^H \geq R^H(i_0 - z_1') - c_2' \ell_1' \tag{28}$$

$$\ell_1^H \geq \ell_1' \quad \forall \quad \ell_1' \in [0, i_0] \tag{29}$$

$$\text{with } z_1^H = \frac{\ell_0 c_1 - \ell_1^H}{\lambda}, z_1' = \frac{\ell_0 c_1 - \ell_1'}{\lambda}, c_2' = c_2^H \tag{30}$$

The only ℓ_1^H satisfying the above condition is $\ell_1^H = i_0$. At $t = 0$, when borrowers take the decision on the long term investment and engage in a loan contract with first generation lenders, the total amount of money available in the economy is invested in the long term projects because no matter the project type, the net return is positive $R^H > R^L > 1$. Ex-ante borrowers' types are unobservable for any agent in the economy and thus the money is split equally among the two borrowers, $\ell_0 = m$. Borrowers fully invest the loan into the long term technology $i_0 = \ell_0 = m$ since it maximizes their return[18].

With $\ell_1^H = i_0 = m$, the low type borrower receives the money left in the economy, $\ell_1^L = 2m(1 - f) - \ell_1^H$. This requires that $c_2^H \geq c_2^L$. In terms of total welfare this is the best scenario since it leaves as much money as possible in the long term technology.

With lending resources being scarce at $t = 1$ due to the liquidity shock, borrowers compete for funding. The H-type borrower competes with the L-type borrower for funds.

The L-type borrower's profit is smaller than the H-type borrower's profit with $\ell_1^H > \ell_1^L$ and

[18] The optimal choice at $t = 0$ are more formally derived in the appendix E.

$R^H > R^L$. Therefore borrowers compete up to the point at which the L-type borrower breaks even:

$$R^L(i_0 - z_1^L) - c_2^L \ell_1^L = 0 \tag{31}$$

$$c_2^L = R^L(1 + \frac{1}{\lambda} - \frac{c_1}{\lambda} - 2\frac{f}{\lambda})\frac{1}{1-2f} \tag{32}$$

The net profit from the L-project to second generation lenders decreases in both the liquidity shock to second generation lenders and the repo rate promised to first generation lenders c_1[19]. The reason is the same for both liquidity shock f and repayment c_1. The larger they are the more of the long term project has to be liquidated at the intermediate stage which is costly with $R^L > 1 > \lambda$. If however there is no liquidity shock, $f = 0$, and the L-type borrower only has to repay initial investment, $c_1 = 1$, then second generation lenders can reap the full return from the L-type project, $c_2^L = R^L$. For the remainder of this section it is assumed that there is always a non-zero liquidity shock $f > 0$. This assumption will become clearer as the H-type borrower's rate is discussed.

First generation lenders have a first mover advantage. They can, if they have all the bargaining power at $t = 0$, extract all the rent and leave second generation lenders and borrowers with zero net profit. The largest possible c_1 attainable for first generation lenders is when $c_2^L = 1$ and the L-type borrower makes zero profit:

$$R^L(i_0 - z_1^L) - c_2^L \ell_1^L = 0 \tag{33}$$

$$c_1 = 1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}). \tag{34}$$

At $t = 0$, first generation lenders do not anticipate the liquidity shock f to second generation lenders. Therefore the last term in the expression 34 equals zero. If first generation lenders would charge a repo rate of $c_1 = 1 + \lambda - \frac{\lambda}{R^L}$, the L-type borrower would always default at $t = 1$. This is why it is required that first generation lenders do not have full bargaining power at $t = 0$ and hence cannot extract all the profit. The amount of bargaining power they cannot have amounts at least to the last term in 34: $-2f(1 - \frac{\lambda}{R^L})$. In principle, the separating equilibrium can exist for

[19]For $c_1 \geq 1$ which has to be satisfied for first generation lenders to issue loans.

different values of repo rates

$$1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}) \geq c_1 \geq 1. \tag{35}$$

A general conclusion can be drawn about the state of the economy at date $t = 0$. For borrowers to be able to roll over profitable projects at the interim stage, lenders' market power at the initial investment stage has to be sufficiently low. In other words, it is good to have a liquid interbank lending market so that lenders face sufficient competition in order to keep interest rates low and facilitate roll over at the interim stage. Below it is shown that this result is true from a total welfare perspective as well.

The H-type borrower pays the rate which the L-type borrower is just not willing to pay. This is the rate obtained from the L-type borrower's incentive compatibility constraint[20], differently from the non-anonymous case[21], which ensures that the L-type borrower does not want to choose the H-type contract:

$$R^L(i_0 - z_1^L) - c_2^L \ell_1^L \geq R^L(i_0 - z_1^H) - c_2^H \ell_1^H \tag{36}$$

$$c_2^H \geq R^L(1 + \frac{1}{\lambda} - \frac{c_1}{\lambda}) \tag{37}$$

Lenders compete for the H-type loan if

$$R^L(1 + \frac{1}{\lambda} - \frac{c_1}{\lambda}) > c_2^L. \tag{38}$$

The latter condition is satisfied if $0 < f < \frac{1}{2}$. This is true by the assumptions above. Therefore the H-type borrower pays exactly the rate

$$c_2^H = R^L(1 + \frac{1}{\lambda} - \frac{c_1}{\lambda}). \tag{39}$$

It is straightforward to show that the H-type loan contract (c_2^H, ℓ_1^H) satisfies both the H-type

[20]For the given set of lender beliefs, the L-type borrower's incentive compatibility constraint is identical to the individual rationality constraint which ensures that the L-type borrower does not want to choose a different quantity ℓ_1'.

[21]There it is enough that the H-type borrower is willing to pay an ϵ more than the L-type borrower and in equilibrium even pays the same rate as the L-type borrower due to lender competition for the H-type loan.

borrower's participation constraint and incentive compatibility constraint.

To conclude the equilibrium analysis, existence of the equilibrium is shown. The general idea is to ask the following question, "How big can a liquidity shock to second generation lenders be before the L-types project delivers a too low return to secure agents' participation?". Due to its lower return, it is the L-type project which delivers the binding condition. For existence it is irrelevant whether lenders or the borrower receive the return. Therefore the analysis is simplified by giving zero net profit to first generation lenders, $c_1 = 1$ and the L-type borrower. Then it remains to check at which size of liquidity shock second generation lenders earn zero net profit:

$$\begin{aligned} c_2^L =& R^L(1 - \frac{c_1 - (1-2f)}{\lambda})\frac{1}{1-2f} \geq 1 \qquad (40) \\ & \frac{R^L - 1}{R^L - \lambda}\frac{\lambda}{2} \geq f. \qquad (41) \end{aligned}$$

Call $f^{Sep} = \frac{R^L-1}{R^L-\lambda}\frac{\lambda}{2}$ the threshold for which the separating equilibrium on the CCP exists. Recall the assumption $\frac{1}{2} \geq f > 0$. With $\lambda < 1$, it is straightforward that the left hand side of expression 41 is always smaller than $\frac{1}{2}$.

The findings are summarized in the following proposition.

Proposition 6.1 (Anonymous lending with signaling)**.** *If the unexpected liquidity shock on second generation lenders is small, $f^{Sep} \geq f > 0$, both types of borrowers can roll over their loans with loan contracts (c_2^H, ℓ_1^H) and (c_2^L, ℓ_1^L). When the liquidity shock is large, $\frac{1}{2} \geq f > f^{Sep}$, there is a run on the low type borrower and only the high type borrower is able to roll over the loan at a loan contract*[22] $(c_2^H = 1, \ell_1^H = m)$.

6.1 Welfare in anonymous lending with signaling

The welfare analysis of the separating equilibrium is identical to the one in the non-anonymous case since loan quantities are identical and loan rates only represent transfers which leaves welfare unaffected.

Therefore we directly state the following proposition[23].

Proposition 6.2 (Welfare in lending with signaling)**.** *As long as the liquidity shock is sufficiently*

[22] Which is different from the separating equilibrium loan contract due to the bargaining power of the H-type.
[23] Refer for the derivations to the appendix.

small, $f^{Sep} \geq f \geq \frac{R^L-1}{R^H-\lambda}\frac{\lambda}{2}$*, and both types of borrowers manage to roll over their loans, total welfare is*

$$W_{Sep,f\leq f^{Sep}} = (R^H + R^L - 2)m - (\frac{R^H + R^L}{\lambda} - 2)(c_1 - 1)m - 2f(\frac{R^L}{\lambda} - 1)m. \tag{42}$$

As soon as there is a run on the low type borrower, i.e. $\frac{1}{2} \geq f > f^{Sep}$*, total welfare becomes*

$$W_{Sep,f>f^{Sep}} = (R^H + \lambda - 2)m - (\frac{R^H}{\lambda} - 1)(c_1 - 1)m. \tag{43}$$

Welfare in a separating equilibrium without a run, $W_{Sep,f\leq f^{Sep}}$, is identical to welfare in non-anonymous lending without a run because the amounts rolled over for H-type borrower and L-type borrower respectively are identical at the intermediate stage. Notice, welfare is not affected by the different interest rates in the separating equilibrium and the non-anonymous equilibrium.

In case of a run, welfare is identical among all three different equilibria as only the H-type borrower is able to roll over the loan and the L-type borrower has to liquidate all of the long term project which is costly since $R^L > 1 > \lambda$.

7 Resilience against liquidity shocks

One of the two key findings is that on an anonymous interbank lending platform larger liquidity shocks can be absorbed[24]. The reason is, when lenders cannot observe the borrowers' types, they provide the same loan to either type when borrowers have to roll over. If instead lenders learn borrowers' types, they discriminate between them by giving a larger loan to the better type. It is the L-type project which determines the existence of the equilibria for two reasons. One, the L-type project generates a lower return and two, in equilibrium the L-type project receives at most as much funding as the H-type project. In other words, in comparison between the anonymous setup and the non-anonymous setup, the L-type borrower receives a larger loan in the anonymous setup and thus can promise repayment at the final date for a larger liquidity shock than in the non-anonymous setup.

[24]To simplify the model, it is assumed that the liquidity shock hits second generation lenders. It is conjectured that the same results hold true when borrowers have an endowment out of which they finance the long term project at the initial stage and then at the interim stage are hit by a liquidity shock which forces them to seek outside finance.

8 Anonymity and welfare

The comparison of bilateral lending and lending via a central clearing platform is on the basis of welfare. The difference between the two platforms is anonymity, i.e. whether lenders can observe the borrower's type. In the previous sections it is shown if lenders *cannot* observe the borrower's type they run later. Recall from the characterization of equilibria in the previous sections that in case of central clearing there are two types of equilibria. An equilibrium in which borrowers signal their types to lenders and an equilibrium in which both borrower types make the same loan and hence lenders cannot observe the borrowers' types. In non-anonymous lending, lenders can always observe borrowers' identities which for the purpose of this study is identical to borrowers' types.

When borrowers have to roll over their debt at $t = 1$, there is not enough money available in the economy for both borrowers to fully roll over their debt. In case lenders can observe borrowers' types, they prefer to lend to high type borrowers first since they earn a larger return. After the high type borrower has fully rolled over the loan, the remaining money in the economy is lent to the low type borrower which implies that the low type borrower has to liquidate some of the long term technology early which comes at a cost. At $t = 1$, if the liquidity shock to second generation lenders is too large, meaning that the low type borrower has to liquidate too much of the long term technology to repay first generation lenders, second generation lenders do not provide a new loan to the low type borrower. Because lenders correctly anticipate that the low type borrower will not be able honor their loans at the final date $t = 2$.

In case of anonymity, when lenders cannot differentiate between the high type and low type borrower, they provide equal size loans to either of them. This way the low type lender gets a larger loan in comparison to when there is no anonymity and thus can repay first generation lenders by liquidating less of the long term technology. Therefore, lenders are more likely to provide loans to the low type borrower since there is enough money left in the long term technology to repay their loans at $t = 2$. In other words, the run on the low type borrower occurs later with anonymity.

The following proposition summarizes this result.

Proposition 8.1 (Runs with anonymity)**.** *Lenders run later in anonymous lending than when they can observe the borrowers' types,* $f^{Sep} = f^{Bil} < f^{Pool}$ *if* $2R^L - R^H > \lambda$.

Turning to welfare, it can generally be said that since lenders run later in anonymous lending, costly liquidation of the long term technology occurs later and hence affects welfare less. The welfare functions obtained in the previous sections are plotted against the size of the liquidity shock in figure 2.

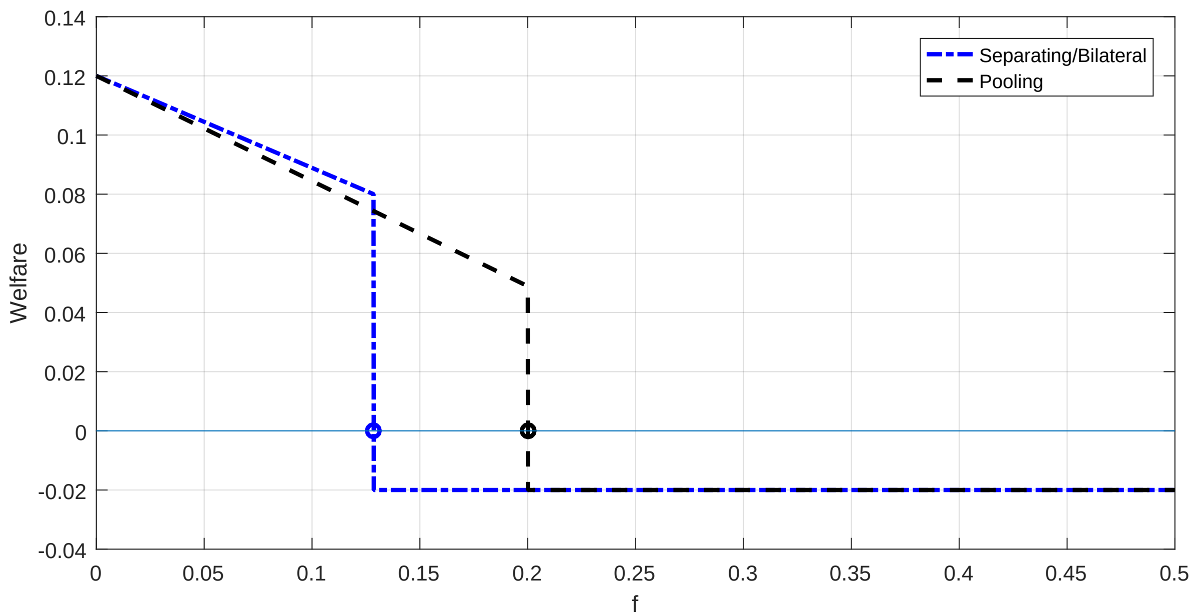

Figure 2: Welfare and anonymity. Parameters: $R^H = 1.08$, $R^L = 1.04$, $c_1 = 1$, $\lambda = 0.9$, $m = 1$

As seen in the propositions defining welfare in the non-anonymous setup and the anonymous setup with and without signaling, welfare is decreasing in the size of the liquidity shock. The intuition is the larger the liquidity shock at the time of roll over, the less money remains invested in the long term technology which only yields positive NPV at maturity. In addition, if early liquidation of the long term technology is sufficiently costly, $2R^L - R^H > \lambda$, the run on the low type borrower occurs for a larger liquidity shock f in anonymous lending than in non-anonymous and anonymous lending with signaling.

The following proposition summarizes the main result of the paper.

Proposition 8.2 (Welfare Comparison). *Welfare in anonymous lending with signaling and in non-anonymous lending without a run are always larger than welfare in purely anonymous lending. Anonymous lending however is more resilient against liquidity shocks.*

The policy advice from this result is to adopt an anonymous central clearing platform in order to create a more shock-resilient interbank lending market. This however might come at a welfare cost as better projects receive less financing[25]. Clearly, the policy maker cannot steer the economy to a separating or a pooling equilibrium respectively in case the anonymous central clearing platform is adopted. In comparison to non-anonymous lending, the central clearing platform performs at least as good. In fact, if the pooling equilibrium occurs, the interbank market becomes more shock resilient at a potential welfare cost and if the separating equilibrium occurs, the central clearing platform performs equally well as the non-anonymous market. This is to say there is no cost at all for the policy maker to adopt the central clearing platform.

Generally it can be stated that if early liquidation is very costly, i.e. λ is small, the pooling equilibrium is always welfare improving as it can absorb a larger liquidity shock which prevents the economy from incurring large liquidation cost. Large liquidation cost can be associated with liquidity dry-ups or fire-sales in the market for the long term technology. It can also mean that the long term technology's return requires specialist knowledge and hence can only be reaped by few specialist investors.

9 Interest rates and loan amounts

This paper is primarily concerned with runs in interbank lending and the welfare conclusions thereof. Interest rates are only of secondary interest as they do not impact neither welfare nor the point at which runs occur. There are however some interesting insights this paper can deliver from the interest rates obtained.

Recall the assumption that there is competition among borrowers for scarce resources of funding at the intermediate stage. In addition, for the above equilibria to exist, it is required that borrowers are not perfectly competitive at the initial investment stage because otherwise they would promise a too high rate to first generation lenders. It would be too high in the sense that they would not be able to roll over their loans at the interim stage and instead default on the first loans. It makes sense that there is some competition among first generation lenders since their outside option is worse, by assumption, than investment in the borrowers' projects. And hence they cannot fully

[25]It is difficult to find a general welfare metric to compare anonymous to non-anonymous lending. One way is to take the integral of the welfare functions over f. The difficulty stems from the unanticipated liquidity shock. It is not clear how f is distributed and thus how weights of losses and gains should be attributed. Assuming each f occurs with equal probability, simulations show that the larger the early liquidation cost, the more beneficial the shock resilience of the anonymous central clearing platform.

reap their first mover advantage.

Consider first the equilibria in which no run occurs. In case of non-anonymous lending the loan contracts are as follows:

$$\text{Gross loan rates: } c_2^H = c_2^L = R^L(1 - \frac{c_1 - (1-2f)}{\lambda})\frac{1}{1-2f} \tag{44}$$

$$\text{Loan amounts: } \ell_1^H = m, \ell_1^L = (1-2f)m \tag{45}$$

$$\text{for } 1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}) \geq c_1 \geq 1 \tag{46}$$

In case of anonymous lending with signaling the loan contracts are as follows:

$$\text{Gross loan rates: } c_2^H = R^L(1 - \frac{c_1 - 1}{\lambda}), c_2^L = R^L(1 - \frac{c_1 - (1-2f)}{\lambda})\frac{1}{1-2f} \tag{47}$$

$$\text{Loan amounts: } \ell_1^H = m, \ell_1^L = (1-2f)m \tag{48}$$

$$\text{for } 1 + \lambda - \frac{\lambda}{R^L} - 2f(1 - \frac{\lambda}{R^L}) \geq c_1 \geq 1 \tag{49}$$

And in case of anonymous lending the loan contracts are as follows:

$$\text{Gross loan rates: } c_2^P = R^L(1 - \frac{c_1 - 1}{\lambda})\frac{1}{1-f} - f\frac{R^H}{\lambda(1-f)} \tag{50}$$

$$\text{Loan amounts: } \ell_1^P = (1-f)m \tag{51}$$

$$\text{for } 1 + \lambda - \frac{\lambda}{R^L} - f(\frac{R^H - \lambda}{R^L}) \geq c_1 \geq 1 \tag{52}$$

In case of a run, there is only the H-type borrower received funding:

$$\text{Gross loan rate: } c_2^H = 1 \tag{53}$$

$$\text{Loan amount: } \ell_1^H = m \tag{54}$$

For ease of exposition, it is assumed that lenders are perfectly competitive at $t = 0$ and hence charge $c_1 = 1$.

In a non-anonymous market interbank lending is performed through bilateral or tri-party contracts as it used to be the case for most interbank loans in the US. There exists one loan rate for both borrower types.

When there is separation on the anonymous central clearing platform, there is a liquidity premium paid by the H-type borrower for receiving a larger loan.

In case of pooling, the rate paid by borrowers is smaller than the rate by the H-type borrower in a separating equilibrium but it is, for a sufficiently large liquidity shock, larger than the one paid by the L-type borrower. This again resembles the notion of a liquidity premium.

From the liquidity supply side, the fact that the H-type borrower can fully roll over the loan, resembles a flight to safety phenomenon.

It might seem odd that in case of a run, the interest rate drops dramatically. The sharp drop however occurs only for the H-type borrower. The L-type borrower instead faces a rate which essentially goes up to infinity as lenders no longer provide loans. The side of the H-type borrower with very low interest rates and excess liquidity[26] resembles very much the post-Lehman era in which investors are holding large amounts of liquidity and are searching for profitable investment paired with flight to safety. The model predicts that this implies that other, though less, profitable projects are foregone. Empirically this is less obvious to prove.

10 The Social Planner's solution

There are two frictions in this model. First, second generation lenders are hit by a liquidity shock at the interim stage of the borrowers' long term project. The liquidity shock to second generation lenders is a threat to the financing of the borrowers' long term project and early liquidation thereof is very costly for the economy. The second friction is the asymmetric information between lenders and borrowers about the borrowers' type. But if lenders can observe borrowers' identity they costlessly learn the borrowers' type. This represents a situation in which borrowers learn about their stock price over time. And in case lenders know the identity of the borrower they learn the borrowers' stock price too.

The social planner observes the borrowers' types at $t = 1$ and can allocate funds from second generation lenders optimally to the borrowers. Clearly, it is relatively more costly to liquidate the project of the high type borrower as the foregone profit is larger. Therefore the social planner fully rolls over the H-type borrower's loan and gives only what is left to the L-type borrower. This maximizes the return of the economy. Bankruptcy of the L-type borrower is very costly for the

[26]Since the H-type borrower needs at most $i_0 = m$ and the remaining $(1 - 2f)m$ are not invested.

overall economy and would occur for a relatively low level of liquidity shock to the second generation lenders. The social planner can commit to the promise to second generation lenders at $t = 1$ to take away some of the H-type borrower's profit and give it to lenders of the L-type borrower. This way the L-type borrower becomes resilient against larger liquidity shocks to second generation lenders.

The social planner wants to maximize total welfare which is the sum of lenders' and borrowers' utilities:

$$\begin{aligned} W^{SP} = & \underbrace{R^H(i_0 - z_1^H) - c_2^H \ell_1^H}_{\text{Net return H-type borrower}} + \underbrace{R^L(i_0 - z_1^L) - c_2^L \ell_1^L}_{\text{Net return L-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \\ & + \underbrace{\lambda z_1^L + \ell_1^L - c_1 m}_{\text{cashflow at t=1 L-type borrower}} + \underbrace{\lambda z_1^H + \ell_1^H - c_1 m}_{\text{cashflow at t=1 H-type borrower}} \\ & + \underbrace{2c_1 m - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{c_2^H \ell_1^H + c_2^L \ell_1^L - 2(1-f)m}_{\text{Net return 2nd generation lenders}} . \end{aligned} \tag{55}$$

It is interesting to consider welfare from an ex-post perspective because the liquidity shock to second generation lenders which causes the shortage of funding for the borrowers' long term projects only becomes known at $t = 1$.

The social planner wants to liquidate as little as possible of the long term technology. Therefore the borrowers' repayment to first generation lenders is done with the new loan from second generation lenders and only if that is not sufficient, borrowers liquidate a part of the long term project. Obviously as little as possible since early liquidation is very costly:

$$\lambda z_1^\omega + \ell_1^\omega - c_1 \ell_0 \stackrel{!}{=} 0 \tag{56}$$

$$z_1^\omega = \frac{c_1 \ell_0 - \ell_1^\omega}{\lambda} \tag{57}$$

The larger the cost from early liquidation, i.e. the smaller λ, or the larger the return promised to first generation borrowers, the more has to be liquidated from the long term technology.

The total amount of liquidity available at $t = 1$ to roll over the borrowers' debt is.

$$2(1-f)m \stackrel{!}{=} \ell_1^H + \ell_1^L \tag{58}$$

For the social planner, the question is how to distribute the funds such that it maximizes total welfare in case of no run and at the same time allows the largest liquidity shock possible. It will be shown that these two objectives are aligned.

In order to study how funds are optimally split between the H-type and the L-type borrower, the following additional notation is adopted:

$$\ell_1^{\omega} = (1 - \alpha^{\omega} f)m. \tag{59}$$

The parameter α^{ω} will be helpful to decide how to split funding between the two borrowers. With the relationship from expression 58, should have the following properties:

$$\alpha^H + \alpha^L = 2 \text{ and } \alpha^{\omega} > 0. \tag{60}$$

For the remainder of this analysis, suppose that the initial investment i_0 equals the total loan obtained at $t = 0$ ℓ_0. Meaning that borrowers invest all the loan in the long term project. Since there is no problem of moral hazard, there is no reason why borrowers should not do so given they have a positive NPV project for sure. Moreover, suppose that first generation lenders provide their entire endowment as loan to borrowers, $\ell_0 = m$. The implicit assumption here is that their outside option yields at most the same return. To simplify exposition, the net return of the outside option is set to 0. This is without loss of generality.

Then total welfare simplifies to:

$$W^{SP} = R^H(1 + \frac{1 - \alpha^H f}{\lambda})m + R^L(1 + \frac{1 - \alpha^L f}{\lambda})m - 2(1 - f)m - 2m - (\frac{R^H + R^L}{\lambda} - 2)mc_1 \tag{61}$$

With $(\frac{R^H+R^L}{\lambda} - 2) > 0$, it is optimal for the social planner to set $c_1 = 1$, the minimum value. The lower the amount the borrower has to repay at the intermediate stage, the less refinancing needs the borrower has and consequently the less of the long term technology has to be liquidated.

Then, with $\alpha^L = 2 - \alpha^H$, the welfare further simplifies to

$$W^{SP} = R^H(1 - \frac{\alpha^H f}{\lambda})m + R^L(1 - \frac{(2-\alpha^H)f}{\lambda})m - 2(1-f)m \tag{62}$$
$$= (R^H + R^L - 2)m - (\frac{(R^H - R^L)\alpha^H + 2R^L}{\lambda} - 2)fm \tag{63}$$

From the latter expression it is obvious that welfare without a run is maximized when the loan of the H-type is fully rolled over, $\alpha^H = 0$, and the L-type receives the remaining liquidity $\ell_1^L = (1-2f)m$.

The maximal liquidity shock the economy can absorb is when total welfare becomes zero:

$$(R^H + R^L - 2)m - (\frac{(R^H - R^L)\alpha^H + 2R^L}{\lambda} - 2)fm = 0 \tag{64}$$
$$\frac{R^H + R^L - 2}{(R^H - R^L)\alpha^H + 2R^L - 2\lambda}\lambda = f \tag{65}$$

The smaller the share of liquidity given to the L-type borrower, α^H, the larger can be the liquidity shock to the second generation lender before there is a run on the L-type borrower. Meaning the split of liquidity, α^H, which maximizes welfare in case of no bankruptcy and the split of liquidity which makes the economy most resilient against a bankruptcy coincide at $\alpha^H = 0$.

For any f larger than this threshold, the L-type borrower goes bankrupt and only the H-type borrower rolls over the loan.

Recall the assumption $0 \leq f \leq \frac{1}{2}$. With the socially optimal $\alpha^H = 0$, there is only bankruptcy if $\frac{R^H+R^L-2}{R^L-\lambda}\frac{\lambda}{2} < \frac{1}{2}$, i.e. if and only if the cost of early liquidation is sufficiently small, $\frac{R^L}{R^H+R^L-1} > \lambda$.

Consequently the social planner adopts the following strategy:

- $0 \leq f \leq \frac{R^L-1}{R^L-\lambda}\frac{\lambda}{2}$, $\alpha^H = 0$,
- $\frac{R^L-1}{R^L-\lambda}\frac{\lambda}{2} < f \leq min\{\frac{1}{2}, \frac{R^H+R^L-2}{R^L-\lambda}\frac{\lambda}{2}\}$, $\alpha^H = 0$ and $\tau = 2m(\frac{R^L}{\lambda} - 1)f - (R^L - 1)m$

where τ is a transfer from H-type borrower to L-type borrower at $t = 2$ promised to second generation lenders at $t = 1$.

The idea is that, for $\frac{R^L-1}{R^L-\lambda}\frac{\lambda}{2} < f$, the L-type borrower needs a promised transfer at $t = 2$ from the H-type in order to stay in business:

$$R^L(i_0 - z_1^L) - c_2^L \ell_1^L + \tau \geq 0 \tag{66}$$

$$\tau \geq 2m(\frac{R^L}{\lambda} - 1)f - (R^L - 1)m \text{ with } c_2^L = c_1 = 1 \text{ and } \ell_1^L = (1-2f)m \tag{67}$$

At $t = 2$, the H-type borrower can at most transfer the following surplus:

$$R^H(i_0 - z_1^H) - c_2^H \ell_1^H - \tau \geq 0 \tag{68}$$

$$\tau \leq (R^H - 1)m \text{ with } c_2^H = c_1 = 1 \text{ and } \ell_1^H = m \tag{69}$$

Consequently, the transfer to the L-type is bound from above by the H-type borrower's capacity to contribute:

$$(R^H - 1)m \geq \tau \geq 2m(\frac{R^L}{\lambda} - 1)f - (R^L - 1)m \tag{70}$$

The H-type borrower can keep the L-type borrower up to a liquidity shock $f^{social} = \frac{R^H + R^L - 2}{R^L - \lambda}\frac{\lambda}{2}$. It comes as no surprise that this threshold coincides with the threshold obtained from setting the total welfare surplus equal to zero.

There is no problem of feasibility like in depositor insurance.

The social planners' welfare over the range of liquidity shocks is depicted in figure 3.

It shows that the right amount of liquidity provision at $t = 1$ combined with the appropriate transfers allows the social planner to absorb the largest liquidity shock possible.

11 Insurance

On the central clearing platform, there exists an insurance mechanism. In case of a borrower's default, it allows the central clearing platform to draw funds from the solvent borrower to subsidize the insolvent borrower.

11.1 Pooling equilibrium with insurance

In the following the result of insurance in the pooling equilibrium is derived.

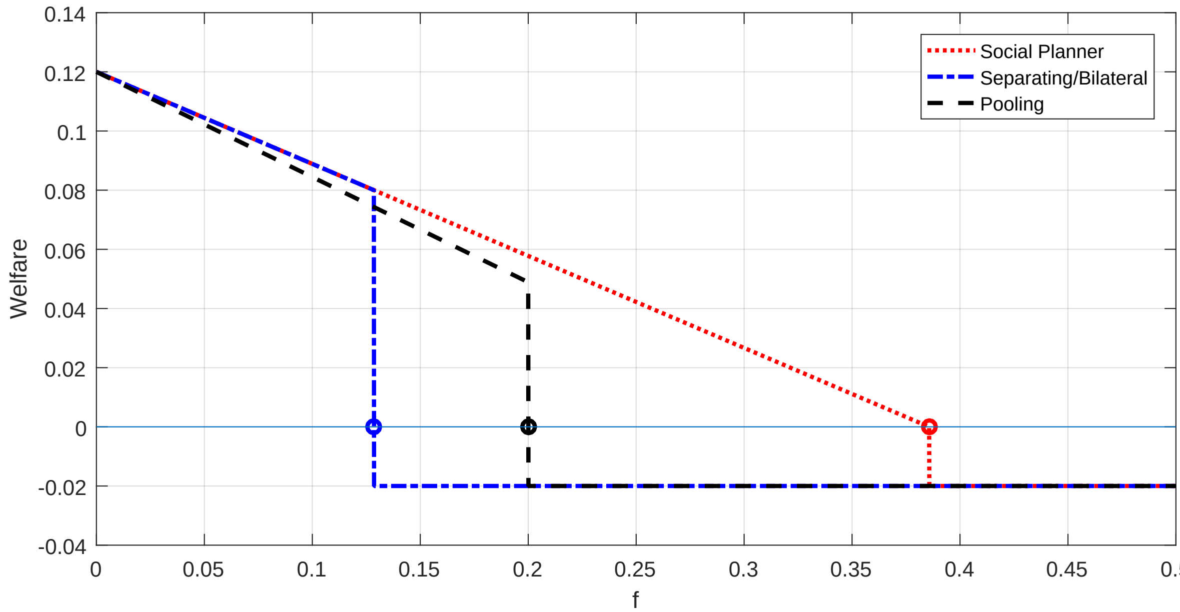

Figure 3: Welfare and anonymity. Parameters: $R^H = 1.08$, $R^L = 1.04$, $c_1 = 1$, $\lambda = 0.9$, $m = 1$

If lenders consider both borrower types as one entity which is jointly reliable to repay their loans and they cannot distinguish the types, borrowers can roll over loans at $t = 1$ for a liquidity shock determined by the following inequality:

$$(R^H + R^L)(i_0 - z_1^P) - 2c_2^P \ell_1^P \geq 0 \tag{71}$$

$$\frac{(R^H + R^L)(\lambda - f)}{2\lambda(1-f)} \geq c_2^P \tag{72}$$

When the latter condition holds with equality, borrowers make zero net profit.

Lenders require to be repaid at least their initial loan:

$$c_2^P \geq 1 \tag{73}$$

To satisfy the two conditions,

$$\frac{(R^H + R^L)(\lambda - f)}{2\lambda(1-f)} \geq c_2^P \geq 1 \tag{74}$$

$$\frac{R^H + R^L - 2}{R^H + R^L - 2\lambda}\lambda \geq f \tag{75}$$

At the point where the latter condition holds with equality, lenders make zero net profit since $c_2^P = 1$.

It is straightforward to check that $\frac{R^H+R^L-2}{R^H+R^L-2\lambda}\lambda > \frac{R^L-1}{R^H-\lambda}\lambda$. That is, insurance allows for an even larger liquidity shock in comparison to the pooling equilibrium without insurance. This is illustrated in figure 4.

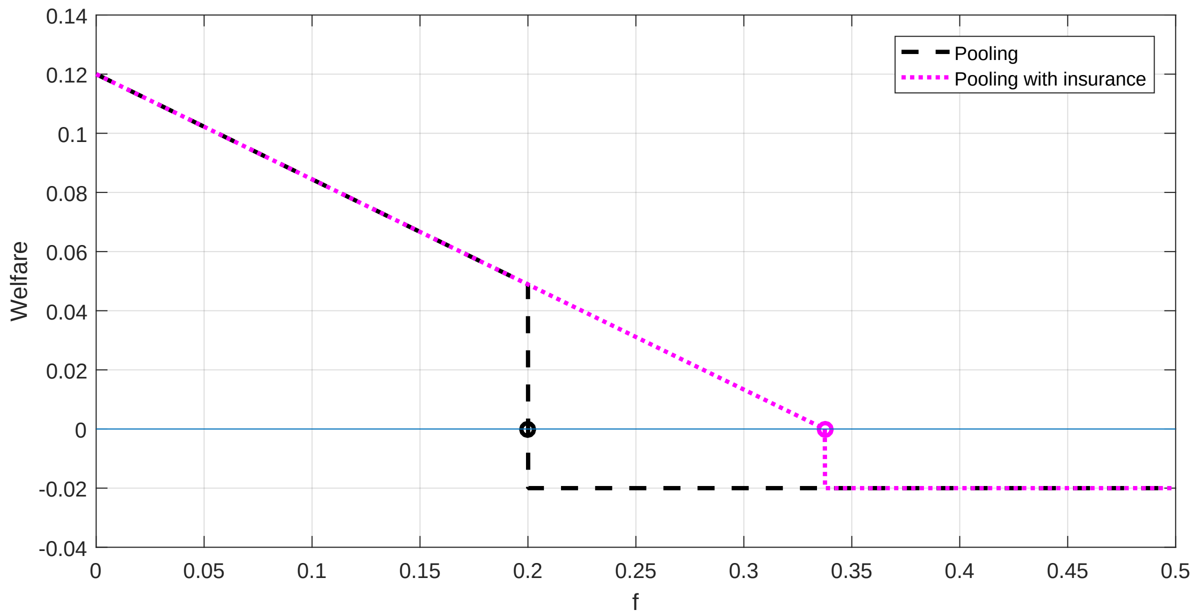

Figure 4: Pooling with insurance. Parameters: $R^H = 1.08$, $R^L = 1.04$, $c_1 = 1$, $\lambda = 0.9$, $m = 1$

Figure 4 illustrates with the insurance mechanism in place, the economy can resist to a shock which leaves the agents in the economy with zero net profit. This is however not individually rational. In fact, the CCP has to impose the following transfers. Therefore the CCP needs to know the borrowers' types which seems a realistic assumption. Then, with $c_2^P = \frac{(R^H+R^L)(\lambda-f)}{2\lambda(1-f)}$, the transfer from the H-type to the L-type borrower amounts to $\tau = \frac{R^H-R^L}{2}(1-\frac{f}{\lambda})$ since,

$$R^L(i_0 - z_1^P) - c_2^P \ell_1^P + \tau = 0 \tag{76}$$

and

$$R^H(i_0 - z_1^P) - c_2^P \ell_1^P - \tau = 0 \tag{77}$$

11.2 Separating with insurance

The insurance result in case of anonymous lending with signaling can at the extreme replicate the social planner's solution. It requires that the H-type borrower transfers all the profit to the L-type lenders and that lenders make zero net profit.

$$R^H(i_0 - z_1^H) - c_2^H \ell_1^H + R^L(i_0 - z_1^L) - c_2^L \ell_1^L \geq 0 \tag{78}$$

With $c_1 = 1$, $c_2^H = c_2^L = 1$, $\ell_1^H = m$ and $\ell_1^L = (1-2f)m$, expression 78 can be rewritten as:

$$\frac{R^H + R^L - 2}{(R^H - R^L)\alpha^H + 2R^L - 2\lambda}\lambda \geq f \tag{79}$$

The transfer from the H-type borrower to the L-type lenders and the giving up of profit of the lenders is not incentive compatible but would need to be enforced by the CCP. It is still interesting to consider this extension as CCPs around the world have clearing funds which do exactly that, a redistribution of losses in case of bankruptcy of one of the participants.

In figure 5, the line for Separating with insurance is identical to the welfare function in case of a social planner. In other words, insurance in a separating equilibrium can replicate the social planner solution.

11.3 Insurance comparison

Since in the separating equilibrium, financing resources are optimally allocated at the intermediate stage, there is more revenue available in an economy with a separating equilibrium. In other words, there is more profit to redistribute from the H-type borrower to the L-type borrower. The promise at $t = 1$ of the H-type borrower's return at $t = 2$ to L-type lenders is enough to allow the L-type borrower to continue borrowing. The L-type borrower actually receives a transfer from the H-type borrower at $t = 2$.

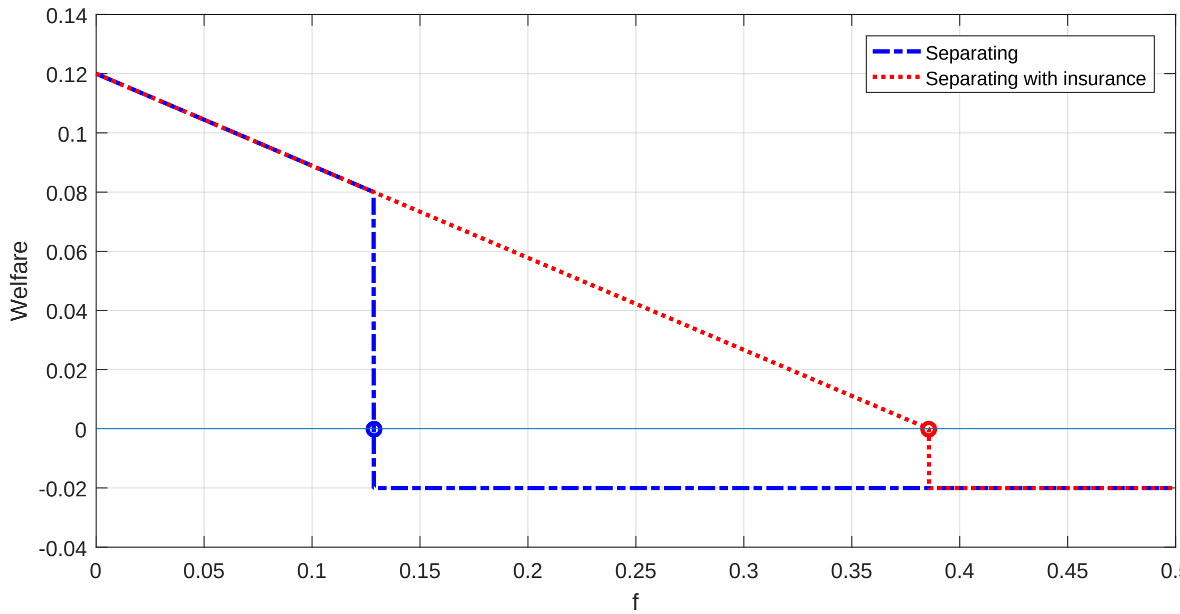

Figure 5: Separating with insurance. Parameters: $R^H = 1.08$, $R^L = 1.04$, $c_1 = 1$, $\lambda = 0.9$, $m = 1$

12 Discussion

12.1 Collateral

In this model, lenders are risk neutral and hence only care about the expected return but not the risk of it. Therefore, lenders are indifferent between securing the loan with collateral or not. Moreover, from the perspective of first generation lenders there is no risk since the liquidity shock of second generation lenders and thus borrowers' capacity to repay their loans is unanticipated.

Adding collateral to the economy, would delay the runs symmetrically across all equilibria. At $t = 1$, borrowers can substitute missing loans from second generation lenders with the return made from selling collateral. If borrowers' endowment with collateral is large enough, they can always stave off a run. This is the inherent idea of collateral: collateral provides additional safety to lenders. If the collateral value however is small relative to second generation lenders' liquidity shock, liquidating collateral still cannot absorb borrowers' missing funding and hence a run still occurs. Given borrowers are ex-ante equally endowed with collateral, the results obtained above qualitatively do not change. A relatively small collateral value can have several reasons. Loans might be undercollateralised from the start or there is a sudden drop in collateral value. If collateral value and the liquidity shock to lenders are positively correlated, the result is particularly dramatic as both sources of funding the long-term technology at the intermediate date are unavailable.

If instead collateral is correlated with borrowers' types, collateral can be used as a signaling device. The study of collateral as signaling device is beyond the scope of this paper and thus deferred to future research.

12.2 Two borrowers with opposite types

In order to model borrower heterogeneity, a realistic feature, in a simple two borrower setup, it makes sense to have one borrower adopt the opposite type of the other one. In the following, it is explained how this realistic feature generates interesting results.

It is true that separating and pooling equilibrium in case of anonymous lending would exist also with one borrower who can be of two types. Also the non-anonymous contract would exist with one borrower. Interest rates would, due to a different competitive setup be different. But the determination of interest rates is not the main objective of this paper and indeed, welfare and the point at which lenders run are unaffected by the size of the loan rates. In fact, with one borrower only, the same difference in points at which lenders run in the different equilibria would occur. This is to say, the main result holds with one borrower too.

So why have two borrowers? It allows to give a role to insurance. With two different borrowers, the H-type can subsidize the L-type borrower. This is a feature of Repo lending at the EUREX in Frankfurt.

12.3 Cost of separation

In signaling models, the separating equilibrium comes at a cost because in order to separate types the L-type has to get a smaller quantity for a smaller price. Also in this model the L-type gets a smaller loan for a smaller interest rate. It however does not impose a cost in comparison to non-anonymity or pooling equilibrium since the liquidity shock reduces funds for all equilibria. Then the separating equilibrium is constructed such that it uses all of the reduced funds just like the pooling equilibrium and the non-anonymous equilibrium. In other words, due to the liquidity shock there is not enough funding for both types anyway.

13 Conclusion

There is a lively discussion among politicians and academics alike whether central clearing platforms are beneficial. This paper is the first to compare a lending setup with and without a central clearing platform. It becomes clear that a central clearing platform can provide resilience against runs but this sometimes comes at a welfare cost.

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A Bilateral: Optimal loan in $t = 0$

We have to make sure that $\ell_0 = m$ satisfies the lenders' and borrowers' participation. For lenders, $c_1 = 1$ trivially satisfies first generation lenders' participation. Therefore all lenders want to participate in the loan, i.e. $\ell_0 = m$.

Borrowers, at $t = 0$ do not know their types and hence evaluate the expected payoff:

$$\begin{aligned} \beta(R^H i_0 - c_2^H \ell_1^H) + (1-\beta)(R^L i_0 - c_2^L \ell_1^L) + \ell_0 - i_0 + \ell_1 - c_1 m \geq 0 \\ \beta(R^H - c_2^H) + (1-\beta)(R^L - c_2^L) > 0 \end{aligned}$$

With $i_0 = \ell_0 = m$ and $\ell_1^H = \ell_1^L = m$, the latter condition is satisfied by construction.

B Pooling: Beliefs

The beliefs in a pooling equilibrium are specified as follows:

$$\mu = Pr(R^H | \ell_1) = \begin{cases} \beta & \text{if } \ell_1 = \ell_1^P \\ 1 & \text{else} \end{cases} \tag{80}$$

The intuition for the off-equilibrium belief is, a deviation from the equilibrium path by the borrower is punished with a higher loan rate by the lender.

C Pooling: Optimal loan contract at $t = 0$

We have to make sure that $i_0 = \ell_0 = m$ satisfies the lenders' and borrowers' participation. For lenders, $c_1 = 1$ trivially satisfies first generation lenders' participation.

Borrowers, at $t = 0$ do not know their types and hence evaluate the expected payoff:

$$\beta(R^H i_0 - c_2^P \ell_0) + (1-\beta)(R^L i_0 - c_2^P \ell_0) + \ell_0 - i_0 + \ell_1^P - c_1 \ell_0 \geq 0 \tag{81}$$

$$\beta R^H + (1-\beta) R^L - c_2^P > 0 \tag{82}$$

The latter condition is satisfied by construction.

D Pooling: Welfare

Ex-post welfare is given as the sum of utilities of all agents:

$$\begin{aligned}
W^{Pool} &= \underbrace{(R^H + R^L)(i_0 - z_1^P) - 2c_2^P \ell_1^P}_{\text{Net return both borrowers}} \qquad \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \\
&+ \underbrace{2(\lambda z_1^P + \ell_1^P - \ell_0 c_1)}_{\text{cashflow at t=1 both borrowers}} + \underbrace{2c_1\ell_0 - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{2c_2^P \ell_1^P - 2(1-f)m}_{\text{Net return 2nd generation lenders}} . \\
&= (R^H + R^L)(i_0 - z_1^P) - 2(1-f)m + 2c_1\ell_0 - 2m \qquad (83) \\
&= (R^H + R^L - 2)m - (\frac{R^H + R^L}{\lambda} - 2)(c_1 - 1 + f)m \qquad (84)
\end{aligned}$$

If there is a run instead ex-post welfare is:

$$W^{Pool}_{f>f^{Pool}} = \underbrace{R^H(i_0 - z_1^H) - c_2^H \ell_1^H}_{\text{Net return H-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \tag{85}$$

$$+ \underbrace{\lambda z_1^L - c_1^L \ell_0}_{\text{cashflow at t=1 L-type borrower}} + \underbrace{\lambda z_1^H + \ell_1^H - c_1^H \ell_0}_{\text{cashflow at t=1 H-type borrower}} \tag{86}$$

$$+ \underbrace{(c_1^L + c_1^H)\ell_0 - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{c_2^H \ell_1^H - m}_{\text{Net return 2nd generation lenders}} . \tag{87}$$

Since, the L-type borrower defaults, first generation lenders who face a L-type borrower can at most pay $c_1^L = \lambda < 1$ on the unit invested. That is they make a loss. Whereas first generation lenders facing a H-type borrower earn the agreed rate $c_1^H = c_1$. Second generation lenders who are not providing loans to the H-type borrower invest in the outside option of which the net return is normalized to 0.

Welfare in case of a run, which only the H-type borrower withstands, is identical in pooling equilibrium and separating equilibrium

$$\begin{aligned}
W^{Pool}_{f>f^{Pool}} &= R^H(i_0 - z_1^H) - m + c_1^H \ell_0 - m + \lambda\ell_0 - m \qquad (88) \\
&= (R^H + \lambda - 2)m - (\frac{R^H}{\lambda} - 1)(c_1 - 1)m. \qquad (89)
\end{aligned}$$

E Separating: Optimal loan contract at $t = 0$

We have to make sure that $i_0 = \ell_0 = m$ satisfies the lenders' and borrowers' participation. For lenders, $c_1 = 1$ trivially satisfies first generation lenders' participation.

Borrowers, at $t = 0$ do not know their types and hence evaluate the expected payoff:

$$\beta(R^H(i_0 - z_1^H) - c_2^H m) + (1-\beta)(R^L(i_0 - z_1^L) - c_2^L(1-2f)m) + \ell_0 - i_0 + \beta\ell_1^H + (1-\beta)\ell_1^L - c_1 m \geq 0 \tag{90}$$

$$\beta(R^H - c_2^H) + (1-\beta)(R^L - c_2^L) > 0 \tag{91}$$

The latter condition is satisfied by construction.

F Separating: Beliefs

Lenders' beliefs are specified:

$$\mu = Pr(R^H|\ell_1) = \begin{cases} 1 & \text{if } \ell_1 = \ell_1^H \\ 0 & \text{if } \ell_1 = \ell_1^L \\ 1 & \text{else} \end{cases} \tag{92}$$

The intuition for the off-equilibrium belief is, a deviation from the equilibrium path by the borrower is punished with a higher loan rate by the lender.

F.1 Separating: Welfare

Given there is no run, ex-post welfare is given as the sum of utilities of all agents:

$$
\begin{aligned}
W^{Sep}_{f<f^{Sep}} = & \underbrace{R^H(i_0 - z_1^H) - c_2^H \ell_1^H}_{\text{Net return H-type borrower}} + \underbrace{R^L(i_0 - z_1^L) - c_2^L \ell_1^L}_{\text{Net return L-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \\
& + \underbrace{\lambda z_1^L + \ell_1^L - c_1 \ell_0}_{\text{cashflow at t=1 L-type borrower}} + \underbrace{\lambda z_1^H + \ell_1^H - c_1 \ell_0}_{\text{cashflow at t=1 H-type borrower}} \\
& + \underbrace{2c_1\ell_0 - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{c_2^H \ell_1^H + c_2^L \ell_1^L - 2(1-f)m}_{\text{Net return 2nd generation lenders}} . \\
& = R^H(i_0 - z_1^H) + R^L(i_0 - z_1^L) + 2c_1\ell_0 - 2m - 2(1-f)m \quad (93) \\
& = (R^H + R^L - 2)m - (\frac{R^H + R^L}{\lambda} - 2)(c_1 - 1)m - 2f(\frac{R^L}{\lambda} - 1)m \quad (94)
\end{aligned}
$$

For the same level of bargaining power at $t = 1$, i.e. identical c_1, the separating equilibrium can produce the same level of welfare for any given level of f. Since however, the separating equilibrium exists on a smaller range of f, it generates always less welfare than the social optimum.

If there is a run instead ex-post welfare is:

$$
\begin{aligned}
W^{Sep}_{f>f^{Sep}} = & \underbrace{R^H(i_0 - z_1^H) - c_2^H \ell_1^H}_{\text{Net return H-type borrower}} + \underbrace{2\ell_0 - 2i_0}_{\text{cashflow at t=0 both borrowers}} \quad (95) \\
& + \underbrace{\lambda z_1^L - c_1^L \ell_0}_{\text{cashflow at t=1 L-type borrower}} + \underbrace{\lambda z_1^H + \ell_1^H - c_1^H \ell_0}_{\text{cashflow at t=1 H-type borrower}} \quad (96) \\
& + \underbrace{(c_1^L + c_1^H)\ell_0 - 2m}_{\text{Net return 1st generation lenders}} + \underbrace{c_2^H \ell_1^H - m}_{\text{Net return 2nd generation lenders}} . \quad (97)
\end{aligned}
$$

Since, the L-type borrower defaults, first generation lenders who face a L-type borrower can at most $c_1^L = \lambda < 1$ on the unit invested. That is they make a loss. Whereas first generation lenders facing a H-type borrower earn the agreed rate $c_1^H = c_1$. Second generation lenders who are not providing loans to the H-type borrower invest in the outside option of which the net return is normalized to 0.

$$W^{Sep}_{f>f^{Sep}} = R^H(i_0 - z_1^H) - m + c_1^H \ell_0 - m + \lambda \ell_0 - m \tag{98}$$

$$= (R^H + \lambda - 2)m - (\frac{R^H}{\lambda} - 1)(c_1 - 1)m \tag{99}$$

G Separating: Insurance

In case of the separating equilibrium, the joint profit of both borrowers can be used to stave off the run on the low type borrower:

$$R^H(i_0 - z_1^H) - c_2^H m + R^L(i_0 - z_1^L) - c_2^L m(1 - 2f) = 0 \tag{100}$$

For a fixed high type loan rate, c_2^H, the low type borrower is now able to repay up to:

$$c_2^L = \frac{(R^H + R^L - c_2^H)m - R^L \frac{2f}{\lambda} m}{(1 - 2f)m} \tag{101}$$

Therefore the low type borrower is able to repay lenders if:

$$\frac{(R^H + R^L - c_2^H)m - R^L \frac{2f}{\lambda} m}{(1 - 2f)m} \geq 1$$

$$f^{Sep,Ins} = \frac{R^H + R^L - c_2^H - 1}{R^L - \lambda} \frac{\lambda}{2} \geq f \tag{102}$$

For any $R^H > c_2^H$, $\frac{R^H + R^L - c_2^H - 1}{R^L - \lambda} \frac{\lambda}{2} > \frac{R^L - 1}{R^L - \lambda} \frac{\lambda}{2}$.