Infrastructure investment: Application of real options analysis to the Ponta Delgada airport expansion

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Abstract: (146 words)

Ponta Delgada airport is the most important air transport infrastructure in the autonomous region of the Azores, both socially and economically. The increase in airport activity that resulted from the partial liberalization of the air transport sector that occurred in 2015 has increased the pressure on its capacity, both by increasing the mobility of the local population and by increasing the number of visitors. This study aims to evaluate the value of the Ponta Delgada airport expansion, taking into consideration the future growth opportunities that come from an increase in tourism activity. A model developed by Smit (2003) was applied, with demand following a binomial process. Using a real options model, it was possible to maximize the value of the option, since the decision to invest only occurs if the scenario is favourable. Additionally, a maximum value for investment was determined, holding all other parameters constant.

Keywords: expansion; option; real options; binomial process; value.

Introduction and literature review (1403 words)

The paradigm of air transport in the Azores has substantially changed in 2015 with the withdrawal of public service obligations related to regular air services, which allowed the entry and operation of low cost airlines in Ponta Delgada airport (Couto, Pimentel, and Ponte 2017).

According to data from the Azores Statistics Regional Service (SREA), there have been significant volume increases in passenger traffic at the airports of the Azores beginning in 2015, compared to the faint variations that were previously shown. The growth rates in the number of passengers embarking and disembarking are above 20% in 2015 and 2016. In 2016, Ponta Delgada airport was responsible for handling 54.8% of all passengers who embarked, disembarked, or were in transit via the Azores airports. It should be noted that only airports present on the islands of Santa Maria, São Miguel, Terceira, Faial, and Pico handle international flights. Therefore, the transport of passengers from the other Azorean islands on international routes presupposes a transfer of passengers to one of the referenced islands, by air or sea, reinforcing the importance of Ponta Delgada airport due to its size and quantity of routes.

Chambers (2007) mentions that the degree of success of a project in the transport sector is strongly associated with the demand for tourism and business, economic indicators such as GDP or *per capita* income, the price of gas and fuel, and general changes in consumer preferences. In other words, the trips are a means to an end, not a motivation in themselves.

Tourism is an important focus for many regional and local political actors, as it has a high potential for value creation, and a significant multiplier effect throughout the economy, both downstream and upstream (Couto, Pimentel, and Ponte 2017). Although there had been a trend of growth in the number of guests in the Azores prior to the liberalization of airspace in 2015, this event had a significant impact on tourist demand. Note that the growth rates of guests in tourist accommodation units in 2015 and 2016 were 27.7% and 23.7%, respectively.

Since Ponta Delgada airport is the main gateway for visitors to the Azores, it is expected that activity at the Ponta Delgada airport will increase in the coming years due to the expected increase in the number of tourists in the Azores, and São Miguel in particular. It is possible to envision a scenario where the maximum capacity of the airport is reached, resulting in an inefficient response to the increased demand. This expected increase in passenger traffic in Ponta Delgada can place new constraints on the airport infrastructure. In 2009, when ANA, Aeroportos de Portugal, S.A. was still a public company, it made an investment in this airport of approximately 30 million euros. At that time, the airport had 960,000 passengers and that investment was intended expand its capacity to 1.2 million passengers. Today, it is important to assess whether it is justifiable to implement an additional investment in the Ponta Delgada airport to avoid constraints and disruptions that could impair the quality of service when this volume is exceeded.

Traditional methods of investment valuation assume passivity on the part of management, since the projected scenario is the most likely given the strategy previously outlined for company operations. These methods may not take into account market dynamics marked by change, uncertainty, and competitive interactions, which may change the cash flows initially estimated (Trigeorgis 1995). According to Putten and MacMillan (2004), the real options method allows recapturing some of the value lost through these conservative methods for evaluation, since different scenarios are designed for the operation. According to Brach (2003), this happens because real options analysis assumes that managers can mitigate the risk over time, which means that volatility is seen as a positive factor that increases value.

Culik (2015) identified several methods in financial theory that are applicable to the quantification of the price of options, from analytical equations (Black-Scholes model), network models (binomial, trinomial, multinomial), or simulation (Monte Carlo) of partial differential equations (finite difference method). Generally, any of these approaches can be used to quantify real options but, given their specific features, the network models are the most commonly used; they are easy to calculate and interpret, easily adaptable to most real options problems, and are useful for evaluating different types of options (call, put, etc.). Moreover, strategic decisions are made in discrete time rather than continuous time, and they permit evaluation of real multinomial options (a wider range of decisions are available in a given period of time), real options with multiple sources of risk, and real options based on variable parameters (volatility, strike price, risk-free interest rate, etc.).

Pimentel, Azevedo-Pereira and Couto (2012) developed a model to determine the optimal time to invest in high-speed transport based on an analysis in continuous time with stochastic demand and stochastic investment spending. Kerr (2014) used the real options approach to assess forest investment by Brazilian cellulose industry companies and the optimal time to harvest, based on a method of fully implicit finite differences, considering that wood prices followed a stochastic process. Kostrova et al. (2016) developed a numerical method to evaluate American-type real options, which are the most difficult and complex to resolve, as they may be exercised at any time. The method was based on Monte Carlo simulation and network models, exemplifying with a practical case the adoption, collection, and conversion option evaluation of permanent energy crops on a farm.

For investment in transport infrastructure, particularly investment in airport infrastructure, Smit (2003) states that these investments can lead to economic growth in a region. In addition, they represent benefits to shareholders and opportunities for companies related to the infrastructure in question, which assigns added importance to the government, either because they establish the regulatory framework, or because they often have a key role in providing the infrastructure. Thus, the option value is not fully allocated to investors, but is shared by other players in the region where the investment is made.

There is a great degree of uncertainty associated with investments in transport infrastructure, particularly regarding the estimation of costs and traffic forecasts, which are essential for determining the viability and value of projects. Regarding the estimation of costs, Chambers (2007) cites a Flyvbjerg (2002) study, which concluded that nine out of ten transport infrastructure projects cost more than originally planned, and this reality is global and insensitive to time. Additionally, he adds that, when carried out by the public sector, the projects show a greater disparity between the predicted values and actual values, not only in terms of construction costs, but also in terms of time until completion. On the other hand, poor traffic forecasting can be even more damaging, since it can lead to the construction of a more expensive infrastructure with a low utilization rate.

Regarding specific to investments in air transport infrastructure, Chambers (2007) identifies several sources of uncertainty, such as changes in the economy, which can have both positive and negative effects. Other sources include the number of trips for business and leisure; regulatory and technological changes that impact, for example, airport design; competition with other forms of transport, as alternatives such as high-speed transport are developed; restructuring of airlines, with particular emphasis on the growth of low cost carriers; public support and drastic changes of opinion; and the growth of airport multi systems.

For the application of valuation models using real options, it is important to refer to Smit's (2003) work to assess the expansion of a European airport. The author explains that investments in infrastructure require more careful scrutiny and analysis than what is offered by discounted cash flow methods. Consequently, he developed an evaluation system that combines real options and game theory using a binomial model to capture not only a more effective assessment of an individual investment, but also the trade-off between the flexibility of management and competitive pressures to invest for different investment strategies. Pereira, Rodrigues, and Armada (2006) developed a model to assess the optimal time to invest in the construction of a new airport using two stochastic factors (number of passengers and net cash flow per passenger) and shocks (random events with positive or negative impacts on airport revenues). More recently, Xiao et al. (2016) modelled the choice of airport capacity *a priori* (i.e. to invest immediately) and the real option value for future expansion based on different objectives (maximization of profits or social welfare).

The airport expansion

Methodology (870 words)

This study employs the model developed by Smit (2003) to assess the expansion option value of Ponta Delgada airport. This model evaluates the value of the airport expansion as a set of options, using a two-step procedure, and combines real options analysis with game theory, since it assumes the existence of two airports in the same region to capture flights in a duopoly regime. Thus, Smit's (2003) model assigns the value of airport infrastructure based on the potential to improve the airport's strategic position relative to its main competition, since the infrastructure investment will increase the airport's capacity to take advantage of future growth opportunities.

However, since Ponta Delgada airport operates in a monopoly, there is no other airport in São Miguel that can compete to capture the market share of this airport. The expansion option value will be in the increased revenue that the airport will capture by increasing its capacity to meet the increased demand, not by increasing its strategic position relative to other local competition. Therefore, it is necessary to adapt Smit's (2003) model to the situation of the airport in question.

Smit (2003) begins by assessing the value of the assets in place. Thus, the analysis starts with the definition of the 'local market' for flights. Ponta Delgada airport is responsible for capturing the number of passengers intending to land or leave from São Miguel island. Since there are no other airports on São Miguel island, the market share of Ponta Delgada airport is 100%. The demand for the Ponta Delgada airport is uncertain and it is assumed that it follows a binomial process.

To determine the value of assets in place in Ponta Delgada airport, their value is considered to be the airport value on each node of the event tree with no opportunity for expansion, future government restrictions, or competitive interactions that may result from this expansion. The total operating cash flow at time t is given by:

$$TCF_t = Q_t \cdot CF_t \tag{1}$$

To use the binomial structure, it is necessary to calculate the neutral risk probability from total operating cash flow of the event tree with no options or competitive interaction, using the risk-free rate.

$$p \equiv \frac{(1+R_f)TCF_t - TCF^{-}_{t+1}}{TCF^{+}_{t+1} - TCF^{-}_{t+1}} \tag{2}$$

In the end (time T), the residual value of Ponta Delgada is obtained without any competitive interaction or restrictions, assuming a perpetuity of annual operating cash flow from assets in place (no expansion), growing at a constant rate, g, discounted at a constant risk-adjusted rate, k (the weighted average cost of capital).

However, airports could face constraints in the growth of the number of flights due to the capacity of assets in place. Therefore, the number of flights without restrictions, and the corresponding free cash flow, can be depleted when there are high levels of demand due to capacity or environmental constraints. The author states that there may be changes both in the growth rate and the cost of capital opportunities over the terminal nodes of the binomial tree if there are restrictions or interactions.

In the backward evaluation, the equivalent-certain values can be consistently updated to the risk-free rate, to determine the value of assets in place. When returning to the current state, the airport value is calculated using the risk neutral probability and the present expected value in the up and down states.

$$V = \frac{p \cdot V^{+} + (1-p) \cdot V^{-}}{(1+R_f)^{\Delta t}} \quad (3)$$

The value obtained is added to the present value of expected operating cash flows occurring in the period between the nodes.

For the Ponta Delgada airport expansion, management has, in each period, the option to invest in the next investment module if demand is high or defer the investment if demand is low, since demand is exogenous. It is assumed that, in each period, the investment creates additional capacity measured in number of flights per year. The cash flow value is the present value of the incremental cash flows from the additional capacity. Thus, according to the standard real options analysis, this expansion option reflects a call option on this additional cash flow value. The exercise price will be equal to the required amount of investment, I, to create the additional capacity. In the absence of competitive interactions, the result is a nonlinear payoff given by:

$$C_i = \text{Max} \, [\, \Delta V_t - I_t, 0 \,] \quad (4)$$

In this way, and using the backward valuation, the Ponta Delgada airport expansion value for taking advantage of future growth opportunities can be calculated. However, it should be noted that, as stated by the author, the incremental value of a large value investment may involve interactive competitions with competitors, which the author develops in his model, considering a value-capture game for flights. In this sense, these interactions introduce a discontinuity in the option payoff due to investments made in previous years, before the effective capture of flights. These competitive interactions make the airport's capacity not only depend on the evolution of the annual demand for flights, but also on competitor’s investments. So, it could happen that an investment to increase the airport's capacity temporarily results in low occupancy rates of the infrastructure until the demand has effectively grown to the expected levels. In the absence of direct competitive interactions in Ponta Delgada, it will be assumed that the investment to be made by the airport in each period corresponds to the evolution of demand in the following period to maximize airport efficiency.

Data (945 words)

The demand for Ponta Delgada airport has a strong correlation with tourism development in the Azores, not only on the island of São Miguel, but throughout the entire archipelago. A linear regression between the number of guests that are hosted in tourist accommodation units in the Azores and the passenger traffic at the airports of the Azores has been estimated, based on the behaviour of these variables over the past 10 years. Using simple linear regression, the regression coefficients were estimated by the ordinary least squares method (Seltman 2015). The obtained model is:

$$\hat{Y}_j = 35{,}05 + 5{,}17X_{1j} \tag{5}$$

The coefficient of determination was close to 1, which points to excellent model quality; in other words, the dependent variable (passenger traffic at the airports of the Azores) is explained almost entirely by the independent variable (number of guests hosted in tourist accommodation units in the Azores). Since the p-value is approximately 0 using Student's t-test, the model is statistically significant.

To project the number of guests staying in tourist accommodation units in the Azores, the growth rate estimated by Turismo de Portugal in the Strategy Tourism 2027 document was used as a reference; in an optimistic scenario, they estimate that the number of overnight stays in hotels will grow at an average rate of 6.1% annually. Adopting this rate as a reference for calculating the growth of overnight stays in the Azores, and assuming the average stay remains the same (about 3.03 nights), the growth rate in the number of guests will also be 6.1%.

With this data, and based on the estimated regression, it is possible to estimate the movement of passengers that will occur in the Azores airports and Ponta Delgada airport, and the number of flights to be realized at the Ponta Delgada airport in 2017 and following years until 2027. In 2016, the proportion of passengers at Ponta Delgada airport in relation to the total of the Azores was 56.2%; the assumption is that this relationship will continue in the future. This appears to be justified by the commitment of the various agents to promote the tourist destination 'Azores' as a whole, while seeking to expose the differentiating characteristics of each island. To estimate the number of flights, which is the critical variable of the model, it is estimated that the average occupancy of each plane at Ponta Delgada airport is about 95 passengers, maintaining, in a prudent analysis, the average occupation confirmed in 2016. By 2028, it is estimated that there will be a residual increase in the number of flights, assuming an annual growth rate of 1%.

[Insert Table 1 here]

In each period, the number of flights could increase to the projected values or remain unchanged, resulting in a parameter d (downward movement) with a value of 1. This assumption is also made by Smit (2003), and is justified by growth trend in the number of flights in the past. Indeed, in the case of Ponta Delgada airport, it is very unlikely that there will be a decrease in the number of flights in the future. The liberalization of air space in 2015, with a consequently lower cost of travel, and the development of the Azores as a more attractive tourist destination will promote airport activities.

Based on the 2016 Annual Report of ANA, an operating cash flow (total activity in Portugal) of 348,434 thousand euros was recorded. Since aviation activity of ANA corresponds to 70.8% of total turnover recorded this year, the operating cash flow was adjusted by this coefficient to better reflect the actual cash flow realized by aircraft movements. Given that the number of flights in Portugal in 2016 was 358,981, the estimated operating cash flow per flight is 687 euros.

The constant risk-adjusted rate is given by the weighted average cost of capital (WACC), where the cost of equity is estimated using the capital asset pricing model (CAPM) (Perold 2004). The beta parameter was estimated at 0.81 based on the regression between the daily returns of VINCI, Airports International, S.A. shares (the company that owns ANA) and the daily returns of the CAC 40 index (the French market where shares of VINCI Airports International, SA are listed) between 2000 and 2016. The risk-free rate of 2.903% was obtained using the average of the Portuguese government's 10-year bond yield over the last three years, after the reopening of the markets to Portuguese government debt issues. For the market risk premium, Fernandez and Linares (2014) estimated that the market risk premium used in Portugal was 8.5%. Thus, by applying the CAPM model, the expected return on equity is 9.8%. The cost of debt was calculated at 2.7% based on Ross, Westerfield and Jaffe's (2010) methodology. Considering the values of assets, equity, and debt of ANA in 2016, and a tax rate on profits of 18.30% (the official tax rate), the constant risk-adjusted rate is 3.9%.

However, the Ponta Delgada airport may be operating at the limit of its capacity, which means that it may not be capturing all the potential cash flows that were estimated. It is assumed that the limit of Ponta Delgada airport capacity is 16,000 annual flights, which means that more aircraft cannot be moved when this volume is reached.

It is estimated that capturing each incremental flight requires an investment of 9.526 euros, based on the average occupancy of planes in 2016. This value results from the last investment made by ANA in 2009 of 30 million euros for an increase in capacity of 300,000 passengers. Therefore, to capture additional flights, it would be necessary to invest 100 euros per passenger moved. It should be noted that the investment required to increase airport capacity will result in incremental cash flows in perpetuity.

Results (272 words)

The application of the model developed by Smit (2003) to assess the value of the assets in place at the airport resulted in an estimate of 393.6 million euros in 2016.

[Insert Figure 1 here]

To capture future growth opportunities, the expansion value is approximately 23.1 million euros.

[Insert Figure 2 here]

In fact, there is a limitation in capturing value from assets in place due to the maximum airport capacity, which will already be achieved in 2017 if demand increases as the projections suggest. On the other hand, the present value of growth opportunities exactly captures this value, since the annual investments will increase the capacity of the airport infrastructure. On each node of the event tree, it was assessed whether it would pay to carry out the investment based on the resulting incremental value of the additional cash flow obtained by eliminating the capacity restriction. This considers the value of management flexibility, which naturally increases the option value, in contrast to a traditional method of discounted cash flows.

[Insert Table 2 here]

To complement the results, the maximum investment amount to obtain a new flight that could be borne to obtain a null option was estimated. This analysis is introduced because the investment value estimated may not reflect the real value of the investment needed to upgrade the Ponta Delgada airport. Holding everything else constant, the maximum amount that can be spent to obtain a null option is 11,939 euros per flight. In this scenario, airport management is indifferent in 2016 to exercising the option from a purely financial perspective. Any investment value exceeding this limit will also result in a null option, but will result from a decision not to invest.

[Insert Figure 3 here]

Final considerations (93 words)

This study applied the model developed by Smit (2003) to the Ponta Delgada airport. The airport expansion option value was evaluated using the real options model to capture future growth opportunities that may arise from the development of tourism in the Azores. It was assumed that demand follows an exogenous binomial process. It was possible to maximize the incremental value, since it considered the value of management's flexibility to only make the decision to invest if the scenario was favourable. In addition, a maximum investment limit was estimated, holding all other parameters unchanged.

References

Brach, M. 2003. *Real Options in Practice.* John Wiley & Sons, Inc.

Chambers, R. 2007. *Tackling Uncertainty in Airport Design: A Real Options Approach*. (Master's Thesis, Massachusetts Institute of Technology, Engineering Systems Division, Technology and Policy Program).

Copeland, T., and P. Tufano. 2004. A Real-World Way to Manage Real Options. *Harvard Business Review* 82(3): 90-99.

Couto, G., P. Pimentel, and J. Ponte. 2017. Tourism Development Potential in an Insular Territory: The Case of Ribeira Grande in the Azores. *Journal of Tourism Research & Hospitality* 6(2).

Čulík, M. 2015. Real options valuation with changing volatility. *Perspectives in Science* 7, 10-18.

Fernandez, P., and P. Linares. 2014. "Market Risk Premium Used in 88 Countries in 2014: A Survey with 8.228 Answers." SSRN, IESE Business School, Madrid.

Kerr, R. 2014. Competitiveness of the Brazilian Pulp Industry: A Real Options Analysis of a Forestry Investment. *Práticas de Contabilidade e Gestão* 2(1), 8-30, São Paulo.

Kostrova, A., W. Britz, U. Djanibekov, and R. Finger. 2016. "Monte-Carlo Simulation and Stochastic Programming in Real Options Valuation: The Case of Perennial Energy Crop Cultivation." University of Bonn, Institute for Food and Resource Economics.

Mun, J. 2006. *Real Options Analysis: Tools and Techniques for Valuing Strategic Investments and Decisions.* John Wiley & Sons, Inc.

Pereira, P., A. Rodrigues, and M. Armada. 2007. The Optimal Timing for the Construction of an Internacional Airport. SSRN Eletronic Journal.

Perold, A. 2004. The Capital Asset Pricing Model. *Journal of Economic Perspectives* 18 (3): 3-24.

Pimentel, P., J. Azevedo-Pereira, and G. Couto. 2012. High-speed rail transport valuation. *The European Journal of Finance* 18(1-2): 167-183.

Putten, A., and I. MacMillan. 2004. Making Real Options Really Work. *Harvard Business Review* 82(12): 134-142.

Ross, S., R. Westerfield, and J. Jaffe. 2010. Corporate Finance 9^{th} Edition. *McGraw-Hill/Irwin.*

Seltman, H. (2015). "Experimental Design and Analysis." Carnegie Mellon University, Pittsburgh.

Smit, H. (2003). Infrastructure Investment as a Real Options Game: The Case of European Airport Expansion. *Financial Management* 5-35.

Trigeorgis, L. (1995). *Real Options in Capital Investment: Models, Strategies, and Applications.* Londres: Greenwood Publishing Group.

Xiao, Y., X. Fu, T. Oum, and J. Yan, J. 2016. "Modelling Airport Capacity Choice with Real Options." The University of Sydney, Institute of Transport and Logistics Studies.

Table 1 - Projection of the number of guests in the Azores, movement of passengers in the Azores and São Miguel, and number of flights to be realized in Ponta Delgada airport

Years	Guests in the Azores	Passengers in the Azores	Passengers in Ponta Delgada	Flights in Ponta Delgada airport
2016	509,051	2,646,975	1,486,851	15.609
2017	540,051	2,793,219	1,568,999	16,471
2018	572,938	2,963,315	1,664,544	17,474
2019	607,828	3,143,769	1,765,909	18,539
2020	644,843	3,335,212	1,873,446	19,667
2021	684,112	3,538,214	1,987,531	20,865
2022	725,772	3,753,783	2,108,564	22,136
2023	769,969	3,982,375	2,236,968	23,484
2024	816,858	4,224,886	2,373,191	24,914
2025	866,602	4,482,166	2,517,709	26,431
2026	919,375	4,755,113	2,671,028	28,041
2027	975,362	5,044,682	2,833,684	29,748

Table 2 - Model application results

Value of assets in place	Present value of growth opportunities (PVGO)	Total airport value
393,642,798.75 €	23,085,811.98 €	416,728,610.73 €

Figure 1 - Binomial tree with the value of assets in place

											395,785,385 €
										395,613,758 €	
									395,446,974 €		395,785,385 €
								395,284,895 €		395,613,758 €	
							395,127,389 €		395,446,974 €		395,785,385 €
						394,974,326 €		395,284,895 €		395,613,758 €	
					394,825,583 €		395,127,389 €		395,446,974 €		395,785,385 €
				394,681,036 €		394,974,326 €		395,284,895 €		395,613,758 €	
			394,540,567 €		394,825,583 €		395,127,389 €		395,446,974 €		395,785,385 €
		394,404,062 €		394,681,036 €		394,974,326 €		395,284,895 €		395,613,758 €	
	394,271,408 €		394,540,567 €		394,825,583 €		395,127,389 €		395,446,974 €		395,785,385 €
393,642,799 €		394,404,062 €		394,681,036 €		394,974,326 €		395,284,895 €		395,613,758 €	
	393,770,441 €		394,540,567 €		394,825,583 €		395,127,389 €		395,446,974 €		395,785,385 €
		393,900,342 €		394,681,036 €		394,974,326 €		395,284,895 €		395,613,758 €	
			394,030,878 €		394,825,583 €		395,127,389 €		395,446,974 €		395,785,385 €
				394,158,401 €		394,974,326 €		395,284,895 €		395,613,758 €	
					394,274,874 €		395,127,389 €		395,446,974 €		395,785,385 €
						394,362,733 €		395,284,895 €		395,613,758 €	
							394,383,759 €		395,446,974 €		395,785,385 €
								394,254,924 €		395,613,758 €	
									393,796,026 €		395,785,385 €
										392,616,124 €	
											389,867,249 €
2016	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**

Figure 2 - Present value of growth opportunities

2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
											347,234,994 €
										307,760,472 €	
									270,384,434 €		304,585,014 €
								234,996,181 €		267,386,237 €	
							201,490,801 €		232,165,211 €		264,383,194 €
						169,768,877 €		198,817,598 €		229,329,532 €	
					139,736,193 €		167,244,517 €		196,139,818 €		226,489,008 €
				111,303,473 €		137,352,263 €		164,715,709 €		193,457,329 €	
			84,386,117 €		109,052,037 €		134,964,010 €		162,182,326 €		190,769,994 €
		58,903,966 €		82,259,694 €		106,796,402 €		132,571,311 €		159,644,234 €	
	34,781,063 €		56,895,497 €		80,129,189 €		104,536,445 €		130,174,036 €		157,101,296 €
23,085,812 €		32,883,894 €		54,883,063 €		77,994,484 €		102,272,041 €		127,772,052 €	
	21,284,054 €		30,982,872 €		52,866,547 €		75,855,458 €		100,003,061 €		125,365,222 €
		19,477,717 €		29,077,884 €		50,845,832 €		73,711,985 €		97,729,372 €	
			17,665,715 €		27,168,815 €		48,820,796 €		71,563,936 €		95,450,837 €
				15,845,821 €		25,255,546 €		46,791,312 €		69,411,178 €	
					14,013,331 €		23,337,956 €		44,757,252 €		67,253,574 €
						12,158,179 €		21,415,918 €		42,718,483 €	
							10,258,662 €		19,489,305 €		40,674,869 €
								8,267,861 €		17,557,982 €	
									6,084,162 €		15,621,814 €
										3,487,349 €	
											- €

Figure 3 - Binomial tree with payoffs considering the possible maximum investment value to obtain a null option

											347,234,994 €
										303,639,938 €	
									262,616,413 €		304,585,014 €
								224,012,988 €		263,502,227 €	
							187,687,141 €		224,843,084 €		264,383,194 €
						153,504,733 €		188,464,854 €		225,668,469 €	
					121,339,516 €		154,233,203 €		189,237,989 €		226,489,008 €
				91,072,669 €		122,021,700 €		154,957,226 €		190,006,415 €	
			62,592,364 €		91,711,351 €		122,699,562 €		155,676,671 €		190,769,994 €
		35,793,350 €		63,190,162 €		92,345,832 €		123,372,976 €		156,391,407 €	
	10,576,570 €		36,352,731 €		63,783,878 €		92,975,990 €		124,041,813 €		157,101,296 €
0 €		11,099,855 €		36,908,145 €		64,373,394 €		93,601,701 €		124,705,941 €	
	491,465 €		11,619,284 €		37,459,477 €		64,958,588 €		94,222,835 €		125,365,222 €
		977,762 €		12,134,747 €		38,006,609 €		65,539,334 €		94,839,259 €	
			1,457,120 €		12,646,129 €		38,549,419 €		66,115,503 €		95,450,837 €
				1,925,824 €		13,153,309 €		39,087,781 €		66,686,962 €	
					2,375,942 €		13,656,168 €		39,621,565 €		67,253,574 €
						2,790,407 €		14,154,579 €		40,150,640 €	
							3,132,339 €		14,648,412 €		40,674,869 €
								3,321,897 €		15,137,536 €	
									3,186,077 €		15,621,814 €
										2,349,838 €	
											0 €
2016	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	**2024**	**2025**	**2026**	**2027**