The volatility surface for real option analysis: The case of renewable and traditional energy projects

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Abstract

The aim of this paper is to provide a suitable method to estimate the volatility parameter for the purpose of real option project valuation, especially that of renewable and traditional energy projects. The method is based on the concept of implied volatility of financial options. Then, we obtain implied volatilities for renewable and traditional energy firms by using their debt-to-equity ratios instead of "moneyness" or the strike price used in the case of financial options. For a given debt-to-equity relation of the project, the implied volatility is obtained by employing the stochastic alpha-beta-rho (SABR) model. Our methodology may be extended to find the volatility of any real option project, subject to the availability of market data. Our empirical results show that the annual volatility for renewable energy projects ranged between 16.44% and 38.15% in the period from April 2014 to June 2016.

Keywords: Renewable energy investment, Real Options, Volatility Surface, SABR Model

***EFM* classification code:** 430 – Real Options

1. Introduction

Renewable sources of energy (e.g., biomass, hydropower, geothermal, wind power, solar power, etc.) are increasing their market share at the expense of other sources, but more progress needs to be made before renewables play an important role in slowing the effects of climate change. Nevertheless, of late, the valuation of renewable energy projects by employing real options is gaining in importance (Davis et al., 2003; Kumbaroğlu et al., 2008; Fernandes et al., 2011; Martínez-Ceseña and Mutale, 2011; Boomsma et al., 2012; Min et al., 2012; Detert and Kotani, 2013; Monjas-Barroso and Balibrea-Iniesta, 2013). The advantage of real options is the possibility of incorporating managerial flexibility in the valuation of projects with uncertainty, especially that of traditional and renewable energy projects.

However, one shortcoming that arises during the use of real options is the estimation of the volatility parameter. For new and renewable energy investments, the absence of historical and market data makes the volatility estimation challenging. Moreover, in real options literature, there is no consensus on the best method to employ for the calculation of this parameter. It is worth mentioning that in the reviewed literature, no proposal was found that aimed at providing traditional and renewable energy firms with a tool that would allow them to adequately compute the volatility. Several studies of renewable energy projects employ the volatility of the WTI price, electricity price, or some other commodity price, either directly or as an input in Monte Carlo simulations (Santos et al., 2014; Zhang et al., 2014; Ritzenhofen and Spinler, 2016; Zhang et al. 2017). However, the volatility of some projects is higher than that of commodity prices (Costa Lima and Suslick, 2006a).

The aim of this paper is to provide a suitable method to estimate the volatility parameter for firms in the traditional and renewable energy sectors and use real options to evaluate their projects. Depending on the debt-to-equity level of the project, the firm may use an implied volatility estimated from the market data of peers. Thus, the estimation procedure of implied volatility for real options resembles the methodology employed to calculate the implied volatility in financial options. In this paper, we employ different levels of debt-to-equity, rather than different values of the "moneyness," to derive the volatility surface under the real options framework. To the best of our knowledge, this is the first attempt to ascertain the project volatility depending on the capital structure of traditional and renewable energy projects, and we hope that this method will become the standard for the industry. In derivatives, one of the reasons for the volatility of the underlying price is the leverage relation; this is because the enterprise value is the sum of the stock and debt values. Thus, our proposal is a natural and straightforward approach to estimate the volatility for real options. The estimation and analysis of volatility is not only important for an accurate assessment of the project value, but also crucial for strategic decisions. This is because higher volatility may delay the investment decision (Dixit and Pindick, 1994) and increase the owner's value, but decrease the manager's value (Cui and Shibata, 2017).

The remainder of this paper is organized as follows. Section 2 presents a brief description of the framework of real options and volatility estimation. Section 3 introduces a description of the methodology employed to estimate the project volatility. Section 4 presents an analysis of the results of the research conducted. Section 5 presents the main conclusions of the paper.

2. Basics of real options and volatility estimation

2.1 Real options theory

Traditional valuation methods, such as the discounted cash flows (DCF), have limited applicability to new and renewable energy projects. For example, the DCF method does not take into account flexibility of investment decisions and is not suitable for energy projects owing to its high volatility. Thus, real options seem to be a more suitable method for the valuation of these types of projects (See Jang et al., 2013, and the references therein). According to Kogut and Kulatilaka (2001), a real option refers to “an investment decision that is characterized by uncertainty, the provision of future managerial discretion to be exercised at the appropriate time, and irreversibility.” Hence, real options provide the manager the possibility of taking actions of a strategic nature in the future, incorporating the concept of flexibility that is absent in traditional project valuation approaches. The tool for flexibility is based on the financial option theory. In financial options, the value of a call or put option, generally, depends on five variables: stock price, strike price, time to maturity, risk-free interest rate, and volatility. For plain vanilla options, the four first variables are known, right from the moment of signing the option contract. However, the most difficult variable to estimate is the volatility, since the dynamics of the value of the underlying asset will depend on this variable during the contract’s lifetime. There is a close relation between real options and financial options, and Table 1 presents the analogy between the main variables of a project and a call option, which is commonly observed in the literature on real options.

Table 1. Analogy between the variables of real option projects and financial call options

Project (Real option)	Variable	Call option (Financial option)
Present value of expected cash flows	S	Stock price
Present value of investment outlays	I	Strike price
Length of deferral time	T	Time to maturity
Time value of money (discount rate)	r	Risk-free rate
Volatility of project's return	σ	Volatility of stock returns

This analogy allows the implementation of an approach similar to the one used for implied volatility in the financial option industry for real option analysis (ROA). Although there are different types of real options, Trigeorgis (2000) lists the defer, alter operating scale, time-to-build, abandon, switch, and growth options as the most common ones. For a recent review of the literature on real option valuation in the renewable energy sector, please refer to Kozlova (2017).

Although ROA has been favorably received by the academic and practitioner communities, one of the main debates on its use centers around the estimation of the volatility to be incorporated in the valuation of the investment projects; this is because, in most cases, volatility is not an observable variable (Davis, 1998) and plays a determining role in the outcome of the investment evaluation process (Dotsis et al., 2012). Furthermore, there is no theoretical justification method for calculating the volatility parameter for real options (Lewis et al., 2008). The next subsection presents several methods to estimate this parameter for ROA.

2.2 Volatility estimation methods

Different methodologies have been proposed to estimate the volatility. The main methods used to estimate this parameter are market asset disclaimer (MAD) method, market proxy approach (MPA), and implied volatility. In the MAD method, it is assumed that the underlying asset is the project value without options and follows a geometric Brownian motion (GBM). Then, a Monte Carlo simulation is employed to estimate the volatility of the GBM. The main assumption is that present value of the project without options is the best unbiased estimator of the project's market value (Copeland and Antikarov, 2001). However, one of the disadvantages of this method is that the Monte Carlo simulation tends to overestimate the volatility of the cash flows (Godinho, 2006; Brandao et al., 2012).

On the other hand, MPA takes the volatility of stock returns of similar companies listed on the stock exchange and adjusts it by using the financial leverage ratio. If it is assumed that the enterprise value of the firm (V) is given by the sum of a firm's stock (E) and debt (D), then

$$V = E + D. \tag{1}$$

Moreover, the dynamics of the value of the firm is captured by

$$dV = \sigma_V V dZ, \tag{2}$$

where dZ is a Brownian motion. From equation (1), $E = V - D$, and by assuming that D is not stochastic, we get

$$\frac{dE}{E} = \frac{dV}{E}, \tag{3}$$

and replacing dV, the following equation is obtained

$$\frac{dE}{E} = \frac{\sigma_V V dZ}{E} = \frac{\sigma_V (E+D) dZ}{E}. \tag{4}$$

Using expression (2) for dV, we can assume that dynamics of stock value is given by

$$dE = \sigma_E E dZ. \tag{5}$$

By equating equation (4) and (5), we obtain

$$\frac{\sigma_V (E+D) dZ}{E} = \sigma_E dZ, \tag{6}$$

and by finding σ_V, we can obtain the volatility of the firm by MPA

$$\sigma_{MPA} = \frac{\sigma_E}{1+D/E}, \tag{7}$$

where σ_E is the equity volatility obtained from the market, and D/E is the debt-to-equity ratio. The expression in equation (7) will be employed as an initial value for the volatility estimation in our paper. The main disadvantage of this method is that volatility estimation could be distorted by different factors including financial bubbles and investors' overreaction.

A third method for the volatility estimation is the implied volatility approach. To estimate this parameter for financial options, the volatility that satisfies the BS option pricing formula is found to get the market price of the European option. In our case, the implied volatility for real options is the volatility that makes the value of a company equal to its market value. A similar approach was proposed by Brach and Paxson (2001), but the authors suggest that a stock with volatility similar to the analyzed project should be found. The practical disadvantage is that is difficult to find such a "twin" stock.

In financial options, it is common to obtain a graph of the implied volatility against the strike price or moneyness. For real options, we will use the debt-to-equity relation instead of the moneyness or strike price. To this end, our methodology is based on Merton's model (1974), in which the value of equity can be seen as a call option on the firm's assets. Under the assumption that the firm value follows a GBM, the debt and equity values satisfy the BS partial differential equation. The main assumption of our methodology is that the project volatility is the same as that of the firm.

As previously mentioned, one drawback of ROA is the assumption of constant volatility during the project's lifetime. Several studies propose a method to allow volatility to vary along the life of a real option (Cassimon et al., 2011; Ting et al., 2013; Čulík, 2016). A review of some of the abovementioned methods and a comparison of different approaches to estimate volatility for a case study can be found in Nicholls et al. (2014).

2.3 Implied volatility

The main assumption of the BS model is that volatility is constant. Thus, a plot of volatility against both strike price and time to maturity should be a flat surface under the BS assumptions. In fact, during the previous stock market crash of October 1987, the volatility surface for index options was "relatively" flat. However, after the crash, the volatility varies depending on the strike price level and changes over time. Thus, the volatility surface exhibits a "smile" or "smirk" shape. This raises concern in the financial industry and academics about valuing and hedging the financial options. One of the idea proposed to value similar options was to extract the volatility from the BS formula using the market price of the options instead of the theoretical BS price, and plugging this into the formula to value a similar option. For certain observed option prices in the market ($f_{i,mkt}$), the volatility is

$$\sigma_i = f_{i,BS}^{-1}\left(S_t, r, K_i, T_i, f_{i,mkt}\right). \tag{8}$$

This "traded" volatility is the implied volatility which is commonly plotted against the strike price and time to maturity, which results in the volatility smile.

Several approaches have been developed to model the volatility smile. The first approach models the stochastic dynamics of the stock price through a process that is more general than the GBM. The second approach models the implied volatility directly. Finally, a third approach is model-free and flexible. An example of the approach without a theoretical foundation is the vanna-volga method. There are several models based on the first approach. The first one is the local volatility model, where the realized volatility is allowed to vary deterministically according to time and the future stock price. In local volatility models, the stock price is the only stochastic factor. One of the most important local volatility models is the constant elasticity of variance (CEV) model introduced by Cox and Ross (1976). A second one is the stochastic volatility model, and the Heston (1993) model is an example. In contrast to local volatility models, stochastic volatility models account for two stochastic processes—one relates to stock price dynamics and the other to the volatility evolution. Moreover, the two processes may be correlated. The third type is referred to as the jump-diffusion model. This type was introduced by Merton (1976), and initially used to model a finite number of jumps. However, it is also possible to model an infinite number of jumps (Derman and Miller, 2016). The model employed in our paper to calibrate the implied volatility is the stochastic alpha-beta-rho (SABR) model, which can be classified as an extended local volatility model.

3. Methodology

The volatility estimation is performed by calculating the implied volatility of Merton (1976) model. The advantages of the model are its ability to provide accurate results and widespread acceptance among academics and practitioners. In this regard, Charitou et al.

(2013) point out that the model has been widely used to predict default probabilities and several structural models by employing iterative methods to estimate the value of the firm and its volatility. Our methodology is an extension of the one proposed in González-Echeverri et al. (2015), since our proposal results in the volatility surface, that is, different implied volatilities on many occasions over a period of time. The authors apply the implied volatility methodology to valuate a real option in the healthcare industry.

The approach is based on the Merton (1974) model, which establishes that the equity value corresponds to a call option on the assets of the firm. The firm value (V) is given by the following expression (as in equation [1])

$$V = E + D, \tag{9}$$

where E is the equity market value and D corresponds to the debt's face value. D and E satisfy the BS partial differential equation under the assumption that the firm value follows a GBM. Therefore, the equity value is given by the BS formula

$$E_t = V_t N(d_1) - De^{-r(T-t)} N(d_2), \tag{10}$$

where

$$d_1 = \frac{\ln(V_t/D) + \left(r + \sigma_V^2/2\right)\sqrt{T-t}}{\sigma_V\sqrt{T-t}}; \tag{11}$$

$$d_2 = d_1 - \sigma_V\sqrt{T-t}; \tag{12}$$

$N(d_1)$ and $N(d_2)$ represent the standard normal cumulative distribution functions; $T-t$ is the time to maturity; and σ_V corresponds to the firm volatility. The information required by this model is available from Bloomberg, except for σ_V and D. Hence, the following equation—obtained through Ito's lemma—is required.

$$\sigma_E E_t = N(d_1)\sigma_V V_t, \tag{13}$$

where σ_E corresponds to the equity volatility obtained from the market. Then, it is possible to solve two equations, that is, equations (10) and (13), and two unknowns σ_V and D. Our research interest is the firm volatility σ_V. The following steps describe the procedure for implementing our methodology.

STEP 1. Collection and debugging data

For renewable energy projects, information about stocks from the S&P/TSX Renewable Energy and Clean Technology Index[1] is used to estimate the implied volatility as of July 30, 2016. From the 19 stocks listed in the index, 15 stocks were considered due to information availability. For traditional energy projects, data on stocks from the S&P Global Oil Index[2] is considered for the period from January 1, 2006 to June 30, 2016. Out of the 120 stocks in the index, 57 are listed on the New York Stock Exchange, and for the purpose of our study, 22 stocks are considered due to data availability. For information about renewable energy stocks and oil stocks, please refer to Appendix A and Appendix B, respectively. For each stock, the data corresponding to variables shown in Table 2 are downloaded from Bloomberg

Table 2. Input Variables to Estimate the Implied Volatility

Variable	Symbol	Ticker/Function
Total debt to common equity	D/E	TOT_DEBT_TO_COM_EQY
Current market capitalization	E_t	CUR_MKT_CAP
Current enterprise value	V_t	CRNCY_ADJ_CURR_EV

[1] "The S&P/TSX Renewable Energy and Clean Technology Index measures performance of companies listed on the TSX whose core business is the development of green technologies and sustainable infrastructure solutions. Constituents are screened by Sustainalytics, one of the world's leading providers of environmental, social, and governance research and analysis." Source: us.spindices.com.

[2] "This index measures the performance of 120 of the largest, publicly-traded companies engaged in oil & gas exploration and extraction & production from around the world. It provides global institutional investors exposure to stocks drawn from constituents of the S&P Global BMI." Source: us.spindices.com.

Equity's Volatility	σ_E	HVT (Close-to-Close method)

Source: Bloomberg LP.

The data is imported into Matlab and the following variable is computed

$$\hat{\sigma}_0 = \frac{\sigma_E}{1+D/E}, \tag{14}$$

where $\hat{\sigma}_0$ is the initial seed employed to find the implied volatility; it is the same as the MPA volatility. For the equity volatility, we obtain the following close-to-close method for Equity variance (σ_E^2) from Bloomberg:

$$\sigma_E^2 = \frac{1}{(N-1)\Delta t}\sum_{n=1}^{N}\left[\ln\left(\frac{S_n}{S_{n-1}}\right) - \left(\frac{1}{N}\sum_{n=1}^{N}\ln\left(\frac{S_n}{S_{n-1}}\right)\right)\right]^2, \tag{15}$$

where N is the number of stock observations and S_n denotes the stock price. The observations are made after each interval of length (Δt), that is,

$$S_n = S(t_0 + n\Delta t). \tag{16}$$

This is an unbiased estimator for the variance parameter in a GBM with constant volatility and drift. Other methods to calculate historical equity volatility are available from Bloomberg.

STEP 2. Implied volatility estimation

By employing non-linear squares optimization tool from Matlab for each stock, the two unknowns—firm implied volatility (σ_V), and firm's debt (D)—from equations (10) and (13) are solved. The 10-year treasury bill rate, which is equal to 1.471%, is taken as the risk-free rate, and the time to maturity is 10 years.

STEP 3. Calibrating the implied volatility

The SABR model (Hagan et al., 2002) is the most common methodology employed in the financial industry to calibrate the implied volatility for derivatives, and it will be used in our paper for the firm's implied volatility. The advantage of the SABR model is that in it the volatility evolves over time, which is more realistic than assuming a constant volatility. As mentioned before, SABR model can be seen as an extended local volatility model. Following the Kienitz and Wetterau (2013) notation, the SABR model can be expressed through the following differential stochastic equations.

$$dS(t) = \sigma(t)S(t)^{\beta}dW(t) \qquad S(0) = S_0, \tag{17}$$

$$d\sigma(t) = \nu\sigma(t)dZ(t) \qquad \sigma(0) = \sigma_0, \tag{18}$$

$$dW(t)dZ(t) = \rho dt. \tag{19}$$

Hence, SABR is a CEV model that include stochastic volatility $\sigma(t)$, where $S(0)$ is the spot price of the underlying asset, and $\sigma(0)$ is the volatility of the spot value. The parameters ν, β, and ρ represent the volatility of the volatility process $\sigma(t)$, the asymmetry, and the correlation between the Brownian motions $dW(t)$ and $dZ(t)$, respectively. These parameters (ν, β, and ρ) are constant and must satisfy $\nu \geq 0$; $0 \leq \beta \leq 1$; and $-1 \leq \rho \leq 1$. By setting $\beta = 1$ and $\nu = 0$, we recover the traditional BS model.

Through the application of perturbation techniques, Skinner (2011) obtains an expression for the calculation of the implied volatility as follows

$$\sigma_{\text{SABR}}(K,T) \approx A\left(\frac{z}{x(z)}\right)B, \tag{20}$$

$$A = \frac{\sigma_0}{(SK)^{\frac{1-\beta}{2}}\left[1+\frac{(1-\beta)^2}{24}\log^2(S/K)+\frac{(1-\beta)^4}{1920}\log^4(S/K)+\cdots\right]}, \tag{21}$$

$$B = \left[1+\left(\frac{(1-\beta)^2\sigma_0^2}{24(S/K)^{1-\beta}}+\frac{\rho\beta\sigma_0}{4(SK)^{\frac{1-\beta}{2}}}\nu+\frac{2-3\rho^2}{24}\nu^2\right)T+\cdots\right], \tag{22}$$

$$z = \frac{\nu}{\sigma_0}(SK)^{\frac{1-\beta}{2}}\log(S/K), \tag{23}$$

$$x(z) = \log\left(\frac{\sqrt{1-2z\rho+z^2}+z-\rho}{1-\rho}\right). \tag{24}$$

The implementation is performed in Matlab by employing the function blackvolbysabr, wherein the variable inputs are the implied volatilities estimated in step 2, and their respective debt-to-equity relations, instead of the moneyness (or strike price) as in the case of financial options. For more details about SABR model, please see Hagan et al. (2002), Rebonato et al. (2009), and Kienitz and Wetterau (2013).

STEP 4. Volatility surface construction

For each date, implied volatilities are estimated by following steps 1 and 2, and these volatilities are calibrated by employing the SABR model according to step 3. For a given date, the graph of implied volatility against the debt-to-equity ratio can be obtained. By combining graphs for different dates, the volatility surface is obtained. Then, for a given date and leverage ratio of a project, the (implied) volatility can be obtained to estimate the value of the project. It is worth mentioning that we employ the date of implied volatility estimation rather than the time to maturity used in the case of financial options.

4. Empirical results

This section presents the results of applying the four-step procedure described in the methodology section. For renewable energy data, the volatility surface is obtained in Figure 1.

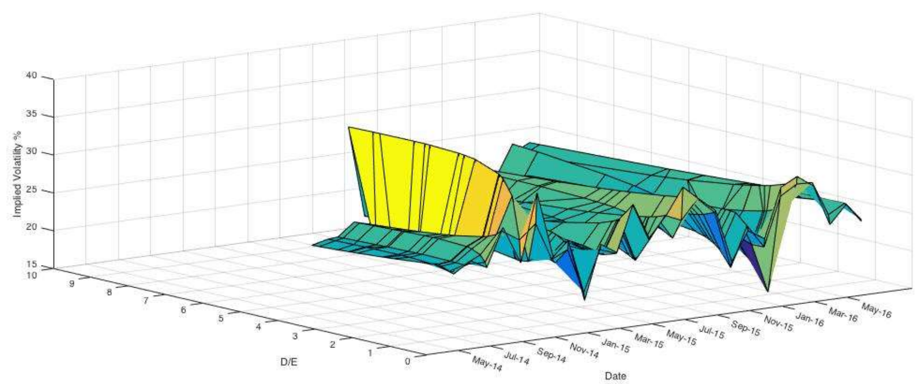

Figure 1. Volatility surface for renewable energy data (monthly basis)

Different values of implied volatility are obtained based on the analyzed period and the leverage ratio of the analyzed sample. The minimum implied volatility is 16.44% ($D/E = 0.45$), whereas the maximum implied volatility is 38.15% ($D/E = 4.60$). Monte Carlo simulations performed according to the procedure suggested by Martín-Barrera et al. (2016) determine a volatility of 30% for a renewable energy project. Kim et al. (2017) estimate a yearly volatility of 14.32% by using MAD approach for renewable energy projects in developing countries.

From Figure 1, it is seen that the implied volatility is higher when the ratio debt-to-equity (D/E) ratio is high, which is consistent with financial theory. For a given date, a volatility "smirk" pattern is also observed and this is similar to the shape formed when analyzing stock derivatives because the main inputs in both cases are the equity prices and their volatilities. On the contrary, a volatility "smile" is observed in foreign exchange derivatives, which is not the case here. The estimated parameters of the SABR model are presented in Figure 2.

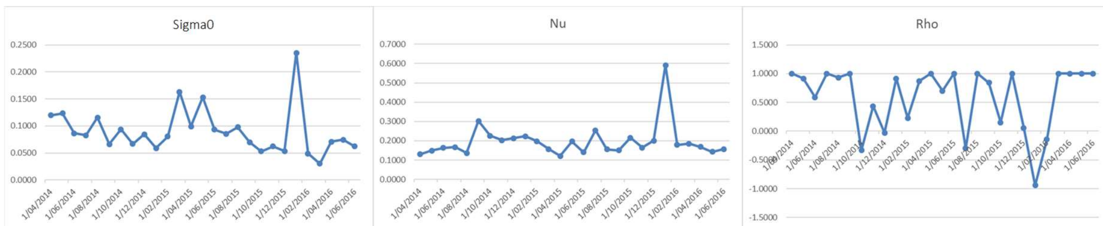

Figure 2. SABR estimated parameters for renewable energy data (monthly basis)

The ν parameter (referred to as the "volvol") is relatively stable during the sample period, but the volvol has its peak (around 0.6) in January 2016. In financial options, when the volvol parameter increases, the implied volatility increases for options deep in-the-money and deep out-of-the-money. This can be seen in Figure 3, where the estimated implied volatility increases more for the extreme debt-to-equity ratios from December 2015 to January 2016.

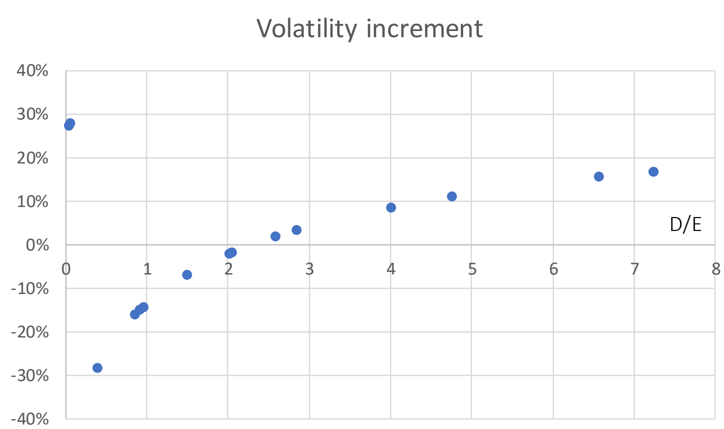

Figure 3. Variation of volatility from December 2015 to January 2016

The σ_0 parameter is interpreted as the initial volatility, when the volatility process starts. The maximum value is in January 2016 and is equal to 23.54%. The ρ parameter is the correlation between the two Brownian motions. The steeper the curve, the more negative is the ρ parameter, and this is noted in January 2016, when the correlation is equal to -0.94. The β parameter (called the backbone of implied volatility) is set as 0.5. The volatility surface obtained for the second set of data (oil data) is presented in Figure 4.

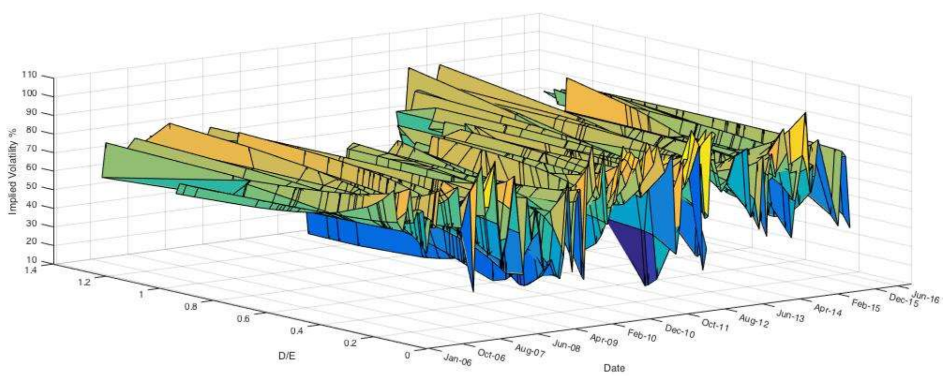

Figure 4. Volatility surface for traditional energy data (monthly basis)

From Figure 4, it can be seen that the minimum (maximum) implied volatility is 19.8% (107.1%) in October 2011 (January 2012). In a recent study on volatility of oil price, Abadie and Chamorro (2017) employ a long-term equilibrium volatility of 35.29% (February 2016) for valuation of an option to cover delay in crude oil production. Our results show a minimum (maximum) implied volatility of 59% (81.9%) in February 2016, a higher volatility compared to the study by Abadie and Chamorro (2017), and this difference can be noted in Figure 5. The latter presents a comparison of the minimum estimated implied volatility according to our proposed methodology and the historical 360-day volatility of WTI prices.

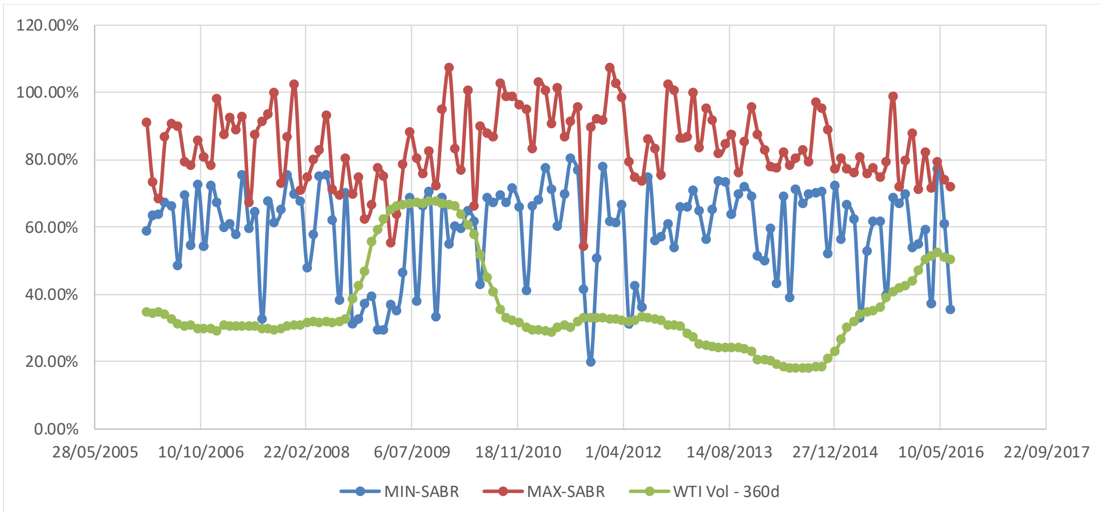

Figure 5. Comparison of minimum and maximum implied volatilities and historical oil volatility (monthly basis)

Figure 5 depicts the minimum and maximum implied volatilities obtained by the procedure described in the methodology section and the 360-day volatility of WTI obtained from Bloomberg on a monthly basis from January 2006 to June 2016. In general, the implied volatility for real option projects is higher than oil volatility, and this result is consistent with Costa Lima and Suslick (2006b). Finally, Figure 6 presents the parameter estimates of the SABR model for the oil case, and the interpretation is similar to that in the previous case.

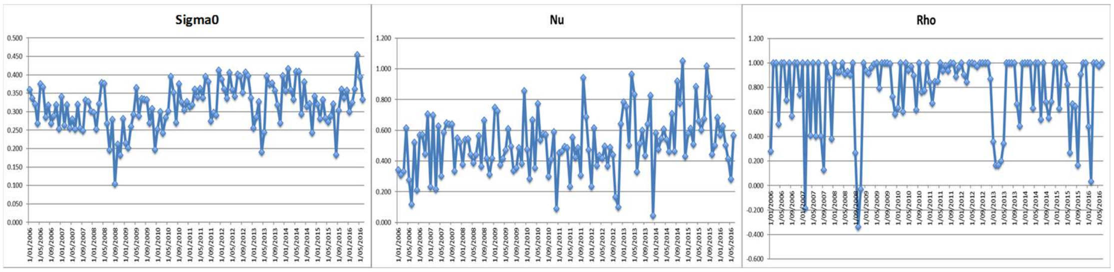

Figure 6. SABR estimated parameters for traditional energy data (monthly basis)

5. Conclusions

This paper proposed a novel method to estimate volatility for new and renewable energy projects by following the real options analysis. This method is also applicable to traditional energy projects, such as the oil-based projects that are extensively studied in the literature and other types of projects for which market data is available.

To the best of our knowledge, this is the first study to implement the volatility surface for renewable and traditional energy projects. The framework is based on the concept of implied volatility for financial options. We employed the debt-to-equity ratio for real options instead of the moneyness or strike price used in the case of financial options. To this end, we employed the SABR model to calibrate the implied volatilities. We described our proposal in a step-by-step procedure to be implemented to value renewable energy projects which involve flexibility in managerial decisions.

In our empirical results, we found an implied volatility range from 16.44% to 38.15% in the period from April 2014 to June 2016 for renewable energy projects. For oil energy projects, the implied volatility varies between 19.8% and 107.1% during the January 2006 to June 2016 period according to the leverage ratio used.

Future research can focus on the valuation of a real option as a case study involving renewable energy projects that employ the volatility estimation methodology proposed in our paper. Future research can also forecast the implied volatility proposed in our paper, since there is empirical evidence that extreme volatility in oil prices results in a decline in manufacturing activity (Elder and Serletis, 2011). Thus, the forecast of the implied volatility may help the manager take a decision on delaying or abandoning a project.

References

Abadie, L.M., & Chamorro, J.M. (2017). Valuation of real options in crude oil production. Energies 10 (8), 1-21.

Boomsma, T. K., Meade, N., & Fleten, S. E. (2012). Renewable energy investments under different support schemes: A real options approach. European Journal of Operational Research, 220(1), 225-237.

Brandao, L. E., Dyer, J. S., & Hahn, W. J. (2012). Volatility estimation for stochastic project value models. European Journal of Operational Research, 220(3), 642–648.

Brach, M.A. & Paxson, D.A. (2001) A gene to drug venture: Poisson options analysis. R&D Management, 31, 203–214.

Cassimon, D., Engelen, P. J., & Yordanov, V. (2011). Compound real option valuation with phase-specific volatility: A multi-phase mobile payments case study. Technovation, 31(5), 240-255.

Charitou, A., Dionysiou, D., Lambertides, N. y Trigeorgis, L. (2013). Alternative bankruptcy prediction models using option-pricing theory. Journal of Banking and Finance, 37(7), 2329–2341.

Copeland, T., & Antikarov, V. (2003). Real Options: A Practitioners Guide, Cengage Learning. New York.

Costa Lima, G.A. and Suslick, S.B. (2006a). Estimating the volatility of mining projects considering price and operating cost uncertainties. Resources Policy 31, 86–94.

Costa Lima, G.A. and Suslick, S.B. (2006b). Estimation of volatility of selected oil production projects. Journal of Petroleum Science and Engineering 54, 129–139.

Cox, J., Ross, S. (1976). The Valuation of Options for Alternative Stochastic Processes. Journal of Financial Economics 3 (1–2), 145–166.

Cui, X., Shibata, T. Investment strategies, reversibility, and asymmetric information. European Journal of Operational Research 263 (3), 1109-1122.

Čulík, M. (2016). Real options valuation with changing volatility. Perspectives in Science, 7, 10-18.

Davis, G. A. (1998). Estimating volatility and dividend yield when valuing real options to invest or abandon. The Quarterly Review of Economics and Finance, 38(3), 725-754.

Davis, G. A., & Owens, B. (2003). Optimizing the level of renewable electric R&D expenditures using real options analysis. Energy Policy, 31(15), 1589-1608.

Derman, E., Miller, M.B., (2016). The Volatility Smile. John Wiley and Sons. New Jersey: Hoboken.

Detert, N., & Kotani, K. (2013). Real options approach to renewable energy investments in Mongolia. Energy Policy, 56, 136-150.

Dotsis, G., Makropoulou, V., Markellos, R.N. (2012). Investment under uncertainty and volatility estimation risk. Applied Economics Letters 19, 133–137.

Elder, J., Serletis, A. (2011). Volatility in Oil Prices and Manufacturing Activity: An Investigation Of Real Options. Macroeconomic Dynamics 15 (Supplement 3), 379–395.

Fernandes, B., Cunha, J., & Ferreira, P. (2011). The use of real options approach in energy sector investments. Renewable and Sustainable Energy Reviews, 15(9), 4491-4497.

Godinho, P. (2006). Monte Carlo estimation of project volatility for real option analysis. Journal of Applied Finance, 16(1), 15–30.

González-Echeverri, G., Mora-Valencia, A., & Solano, J. G. (2015). Opciones reales aplicadas en redes integradas de servicios de salud empleando diferentes métodos de estimación de la volatilidad. Estudios Gerenciales, 31(136), 287-298.

Hagan, P. Kumar, D., Lesniewski, A. y Woodward, D. (2002). Managing smile risk. Wilmott Magazine, 84-108.

Heston, S. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. Review of Financial Studies 6 (2), 327–343.

Jang, Y-S, Lee, D-J, Oh, H-S. (2013). Evaluation of new and renewable energy technologies in Korea using real options. Int. J. Energy Res. 37, 1645–1656.

Kienitz, J. y Wetterau, D. (2013). Financial modeling: Theory, implementation and practice with Matlab source. Chichester: John Wiley and Sons.

Kim, K., Park, H., Kim, H. (2017). Real options analysis for renewable energy investment decisions in developing countries. Renewable and Sustainable Energy Reviews 75, 918-926.

Kogut, B., & Kulatilaka, N. (2001). Capabilities as real options. Organization Science, 12(6), 744-758.

Kozlova, M. (2017). Real option valuation in renewable energy literature: Research focus, trends and design. Renewable and Sustainable Energy Reviews 80, 180-196.

Kumbaroglu, G., Madlener, R., & Demirel, M. (2008). A real options evaluation model for the diffusion prospects of new renewable power generation technologies. Energy Economics, 30(4), 1882-1908.

Lewis, N. A., Eschenbach, T. G., & Hartman, J. C. (2008). Can we capture the value of option volatility? The Engineering Economist, 53(3), 230-258.

Martín-Barrera, G., Zamora-Ramírez, C., González-González, J.M. (2016). Application of real options valuation for analysing the impact of public R&D financing on renewable energy projects: A company's perspective. Renewable and Sustainable Energy Reviews 63, 292–301.

Martínez-Ceseña, E. A., & Mutale, J. (2011). Application of an advanced real options approach for renewable energy generation projects planning. Renewable and Sustainable Energy Reviews, 15(4), 2087-2094.

Merton, R. C. (1974). On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance, 29(2), 449–470.

Merton, R. C. 1976. Options Pricing When Underlying Stock Returns Are Discontinuous. Journal of Financial Economics 3, 125–144.

Min, K. J., Lou, C., & Wang, C. H. (2012). An exit and entry study of renewable power producers: A real options approach. The Engineering Economist, 57(1), 55-75.

Monjas-Barroso, M., & Balibrea-Iniesta, J. (2013). Valuation of projects for power generation with renewable energy: A comparative study based on real regulatory options. Energy Policy, 55, 335-352

Nicholls, Gillian M., Lewis, Neal A., Zhang, L., & Jiang, Z. (2014). Breakeven Volatility for Real Option Valuation. Engineering Management Journal 26 (2), 49-61

Rebonato, R., McKay, K. y White, R. (2009). The SABR/LIBOR market model: Pricing, calibration, and hedging for complex interest-rate derivatives. Chichester: John Wiley and Sons.

Ritzenhofen, I., & Spinler, S. (2016). Optimal design of feed-in-tariffs to stimulate renewable energy investments under regulatory uncertainty—A real options analysis. Energy Economics, 53, 76-89.

Santos, L., Soares, I., Mendes, C., & Ferreira, P. (2014). Real options versus traditional methods to assess renewable energy projects. Renewable Energy 68, 588-594.

Ting SHM, Ewald C-O, Wang W-K. (2013). On the investment-uncertainty relationship in a real option model with stochastic volatility. Math Social Sci 66, 22–32.

Trigeorgis, L. (1996). Real options: managerial flexibility and strategy in resource allocation. Cambridge: MIT Press.

Zhang, H., Li, L., Zhou, P., Hou, J., & Qiu, Y. (2014). Subsidy modes, waste cooking oil and biofuel: Policy effectiveness and sustainable supply chains in China. Energy Policy, 65, 270-274.

Zhang, H., Zhou, D.Q., Zhou, P., Chen, H.T. (2017). Optimal design of subsidy to stimulate renewable energy investments: The case of China. Renewable and Sustainable Energy Reviews 71, 873-883.

Appendix A. Stocks employed for renewable energy

Ticker	Company name
ATP CT Equity	Atlantic Power Corp
DRT CT Equity	DIRTT Environmental Solutions
INE CT Equity	Innergex Renewable Energy Inc
WPT CT Equity	Westport Fuel Systems Inc
BEP-U CT Equity	Brookfield Renewable Partners LP
CAS CT Equity	Cascades Inc
NFI CT Equity	New Flyer Industries Inc
CLR CT Equity	Clearwater Seafoods Inc
RNW CT Equity	TransAlta Renewables Inc
BLD CT Equity	Ballard Power Systems Inc
AXY CT Equity	Alterra Power Corp
NPI CT Equity	Northland Power Inc
BLX CT Equity	Boralex Inc
AQN CT Equity	Algonquin Power & Utilities Corp
NAL CT Equity	Newalta Corp

Source: Bloomberg LP.

Appendix B. Stocks employed for traditional energy

Ticker	Company name
OXY UN Equity	Occidental Petroleum Corp
NOV UN Equity	National Oilwell Varco Inc
PXD UN Equity	Pioneer Natural Resources Co
VLO UN Equity	Valero Energy Corp
SLB UN Equity	Schlumberger Ltd
TSO UN Equity	Tesoro Corp
HES UN Equity	Hess Corp
MRO UN Equity	Marathon Oil Corp
XEC UN Equity	Cimarex Energy Co
COG UN Equity	Cabot Oil & Gas Corp
FTI UN Equity	FMC Technologies Inc
EGN UN Equity	Energen Corp
ESV UN Equity	Ensco PLC
EOG UN Equity	EOG Resources Inc
RRC UN Equity	Range Resources Corp
NBR UN Equity	Nabors Industries Ltd
NE UN Equity	Noble Corp PLC
HAL UN Equity	Halliburton Co
RIG UN Equity	Transocean Ltd
MUR UN Equity	Murphy Oil Corp
DO UN Equity	Diamond Offshore Drilling Inc
NBL UN Equity	Noble Energy Inc

Source: Bloomberg LP.