Credit risk characteristics of U.S. small business portfolios

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Abstract

We explore an inconsistency in the Basel Committee's Internal Ratings Based (IRB) rules: the IRB rules on corporate loans were calibrated to loan-level data, while the IRB rules on small business loans were calibrated with little, if any, information on small business loans.

We argue that the resulting IRB rules do not put small business and corporate loans on a level playing field. In order to treat both asset classes – small business and corporate loans – proportionately to their correlated credit risk, the IRB rules should require 45% lower capital requirements for small business loans than what the Basel Committee currently prescribes. This lowering of capital requirements on small business loans goes in the same direction as the SME Support Factor introduced by the European Commission in the current installment of the Basel regulation.

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1. INTRODUCTION

In an effort to safeguard the stability of financial institutions, capital requirements are widely used to prepare banks for correlated credit risk of borrowers. One type of capital requirements is governed by the Basel Committee IRB rules. This paper explores an inconsistency in the design of the Basel Committee IRB rules. The inconsistency comes from a shortage of actual historical data on small business loans in designing the IRB rules by the Basel Committee (Basel Committee on Banking Supervision (2005)). In particular, for corporate loans the Basel Committee calibrated the IRB rules to the available historical data. But faced with little to none historical information on small business loans, the Basel Committee made an educated guess. It calibrated the IRB rules such that the resulting capital requirements correspond to the actual capital levels and loss data a selected number of banks held prior to this regulation (Basel Committee on Banking Supervision (2005)). Consequently, the Basel Committee decided on a discount on capital requirements on small business loans. We explore the above inconsistency and the resulting discount to study if IRB rules accurately capture the difference in the correlated credit risk between small businesses loans and corporate loans.

Our contribution to the existing literature is threefold. First, we critically evaluate the inconsistency in IRB rules. The analysis adopts the Basel Committee's view on credit risk, which means it is confined to a world of the Basel Committee's asymptotic single factor model that underpins the IRB rules. The Basel Committee's asymptotic single factor model maps a firm's probability of default onto its IRB asset correlations and ultimately onto the IRB capital requirements. The purpose of the latter is to buffer the "unexpected losses". The unexpected losses together with the expected losses are the two types of losses identified by Basel. Unexpected losses come from correlated credit risk of borrowers and are covered by capital requirements, while the expected losses come from borrowers' stand-alone probabilities of default and are covered by pricing and provisioning. This analysis is simplistic in the sense that other potential drivers of unexpected losses, i.e. loss given default (LGD), or exposure at default (EAD), are treated as given and non-stochastic. In this context, we contribute by stipulating a fair and equal regulatory treatment of the correlated credit risk in small businesses loans and corporate loans.

To evaluate the IRB rules, we begin with the Basel Committee's asymptotic single factor model (Basel Committee on Banking Supervision (2005)) to derive our own asset correlation estimates. The Basel asymptotic single factor model underpins the IRB rules and gives our estimates a common denominator with the Basel Committee estimates. Equipped with default information on small business loans and corporate loans we estimate the model-implied asset correlations. The access to default information on small business loans is an important advantage over the Basel Committee on Banking Supervision (2005). We compare (1) the capital requirements computed with the Basel-prescribed IRB asset correlation to (2) the capital requirements computed with our own model-implied estimates of asset correlation. This exercise is repeated for two asset classes - small business and corporate loans. Corporate loans serve here as a reference point relative to which we compare the small business' ratio of the IRB to the model-implied capital requirements. Next, we study how large should be the additional discount on capital requirements on small business loans, such that on average the above ratio is equal for both asset classes.

In general, we find that the discount on capital requirements on small business loans is inadequate relative to corporate loans. We reach this conclusion based on the combined evidence that: (1) the Basel Committee formula significantly overstates the capital requirements on small businesses, and (2) no significant difference is found between our model-implied and Basel IRB capital requirements on corporate loans. Therefore, to put small businesses and their larger corporate counterparts on a level playing field, our data suggests a further 45% discount on capital requirements on small business loans. If applied, this discount would set the ratio of the IRB to the model-implied capital requirements equal for the small business and the corporate loans. In other words, such discount ensures that the Basel capital requirement is proportionate to the model-implied riskiness for both asset classes.

Secondly, we contribute to the current policy debate on the IRB capital requirements levels and special discounts on small business lending following the Capital Requirements Directive IV (European Parliament and the Council (2013)). Our results suggesting a discount on capital requirements on small business loans come hardly as a surprise to the small business lending industry, which views IRB capital requirements on small businesses as excessive (European Commission (2015)). This view is also reinforced by the actions of the European Commission, which in 2013 introduced a small and medium-sized enterprises (SME) Support Factor (European Parliament and the Council (2013)). The SME Support Factor discounts the capital requirements on small business loans by applying a 0.7619 factor to the capital requirements on small business loans, with the aim of stimulating the lending to small businesses. Moreover, studies like Dietsch et al. (2016) , or Dullmann and Koziol (2013) find evidence in support of a discount, or the SME Support Factor, in the context of European small business lending.

Lastly, we focus our attention on small businesses. In the U.S. small businesses employ 55% of the workforce and have provided two-thirds of the total net new jobs since 1970 (Small Business Administration, 2016). While some efforts were undertaken to shed light on credit risk of small businesses, these efforts are limited to aggregate measures of small business credit risk (Lee et al., 2009) or to loans originated under the U.S. Small Business Administration guarantee program (Glennon and Nigro, 2005). Unlike these earlier studies, our study performs an empirical analysis on a comprehensive dataset on small business defaults, covering quarterly observations from 2005 to 2010.

In principle, the discount on capital requirements on loans to small business should be followed by better lending conditions. For example, Behn et al. (2016) document that a 0.5 percentage point increase in capital requirements reduces a bank lending by 2.10 to 3.90 percentage point. Also, Brun et al. (2013) report a similar negative impact of capital requirements on bank lending. Next, Baker and Wurgler (2015) show that higher capital requirements have an unwelcomed effect and increase bank's cost of capital. An advantage of small business lending is shown in Figure 1, which illustrates that banks with a large share of small business loans enjoy lower capital requirements. In 2007, the top 25% of small business lenders enjoyed a 0.8 percentage point lower Tier 1 leverage capital ratio than the bottom 25%. This discount on capital requirements does not mean that banks are more prone to failure. During the recent crisis only 27 out of the top 25% of banks with mostly small business loans failed, comparing to 222 failed banks in the bottom 25%. But despite all the benefits banks enjoy from lending to small businesses, i.e. (1) lower capital requirements, (2) lower cost of capital (Baker and Wurgler, 2015), (3) lower failure rate of banks in the recent crisis, and (4) greater lending potential (Behn et al., 2016), bank lending to small businesses is more expensive than lending to large, corporate firms (Doove et al., 2015).

Importantly, it is not our objective to present the full and comprehensive treatment of small business exposures under IRB rules. Instead, we aim to compare two asset classes – small business loans and corporate loans – in relative terms. We try to pick a setting, which provides similar treatment of the two asset classes such that they differ only with respect to the retail or corporate category. It means we assume that both asset classes fall under the IRB rules, have their asset correlation coefficient computed according to the IRB rules, have their probabilities of default estimated from the data, have the same LGD parameters (which if we take ratios cancel out), are not secured by residential estate, and operate in the same period of 2005-2010, etc. It is true, that for small business loans, in absolute terms we find significant discrepancies between the IRB capital requirement and our estimates based on historical data. As we show later, we do not find such discrepancies for corporate loans. However, due to various shortcomings of the data or our method we are reserved to talk about the absolute levels of capital requirements. Instead, we focus on the relative differences in capital requirements between small business and corporate loans. Thus, our data suggests that small business loans demand about 45% lower capital requirements than what the Basel Committee currently prescribes. Such discount should ensure that the Basel required capital is proportionate to the model-implied riskiness for both asset classes.

Here, we would like to advocate a renewed calibration of capital requirements on small business loans. We are advocates of fair capital requirements rather than of lower capital requirements in general. In other words, we postulate relative changes in capital requirements rather than lowering the absolute level of capital requirements. In particular, we urge to calibrate the capital requirements on small business loans in accordance to the same rules as those on corporate loans, which is not the case at the moment. Given the now improved access to information on small business lending, the Basel Committee could improve on treatment of small business loans by recalibrating their small business capital requirements formula.

2. METHODOLOGY

In this section we recall the credit risk model used by the Basel Committee on Banking Supervision (2005) and devise a method to estimate it. The Basel Committee uses an asymptotic single factor credit risk model. Firms in this model are exposed to a single risk factor as well as to a firm-specific risk factor. The firm-specific risk factor is said to be uncorrelated between firms, leaving the single risk factor as the only source of correlation in the credit risk of borrowers. The single factor credit risk model offers an important advantage: the capital requirements are portfolio-invariant (Gordy, 2003). The portfolio-invariance means that a capital charge on a particular loan depends solely on the features of the particular borrower and not on a bank's portfolio composition. Although crude, the single factor credit risk model greatly simplifies the Basel capital regulation.[1]

To make our results comparable with Basel capital requirements on small business loans, we use the exact same version of the single factor model that is behind the Basel capital regulation. Thus, consider a portfolio of N small firms which are ordered into homogeneous borrower classes $k \in \{1,...,K\}$. This set of homogeneous borrower classes is categorized with respect to firm's characteristics, i.e. credit rating, industry, etc. Let latent variable $A_{i,t}$ denote the asset value of borrower i in borrower class k at time t which without loss of generality is standardized with mean zero. The asset value is driven by two independent components: a single risk factor x_t, common to all firms in the economy, and a firm-specific risk factor $\varepsilon_{i,t}$ per borrower i:

$$A_{i,t} = w_k x_t + \sqrt{1 - w_k^2}\,\varepsilon_{i,t} \qquad i \in k \qquad t = 1,...,T \tag{1}$$

[1] The measurement of credit risk in small businesses is notoriously difficult (Cole et al., 2004). And although more sophisticated empirical models of credit risk exist, including Cossin and Schellhorn (2007), Duffie et al. (2007), McNeil and Wendin (2007), Duffie et al. (2009), Berndt et al. (2010), limited data availability often precludes their use for informationally opaque small businesses loans.

where E[$x_t \varepsilon_{i,t}$]=0. The single risk factor x_t represents changes in the economic conditions common to all borrowers, while the firm-specific risk factor $\varepsilon_{i,t}$ stands for firm-specific risk. The weight w_k of the single risk factor measures the sensitivity of borrower i to the economic conditions. Given that any two firms classified into the same borrower class are sufficiently homogeneous, it is customary to assume that the common factor has an identical effect on these firms' asset values (McNeil and Wendin, 2007 and Gordy, 2000). It follows that the weight w_k is the same for borrowers in the same borrower class. Credit portfolio concentration risk depends heavily upon the magnitude with which borrowers respond to the single risk factor. The higher the firm's sensitivity to the single risk factor, the more responsive the asset value to unanticipated changes in the economic environment. In fact, the default dependency in a loan portfolio arises from co-movements in asset values that respond to the single risk factor.

In each period t, there are $N_{k,t}$ borrowers in class k which at time t are not in default on their payments. Because we are interested ultimately in the joint default occurrences, i.e. borrowers that simultaneously go into default, at time t we exclude borrowers that are already in default. Those $N_{k,t}$ borrowers can default on their payments between time t and t+1. In this model a borrower defaults if the asset value in (1) falls below a default threshold. This definition of default is also used in the structural models that date back to the work of Merton (1974) and Black and Cox (1976). Following the Basel Committee on Banking Supervision, we assume that common and firm-specific risk factors are both standard normally distributed. This implies that the asset value in (1) is standard normally distributed as well. Then, the default threshold is equal to $\Phi^{-1}(\bar{p}_k)$ where $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal cumulative distribution function and $\bar{p}_k$

is the unconditional probability of default in borrower class k. In mathematical terms, borrower $i \in k$ defaults if:

$$w_k x_{t+1} + \sqrt{1-w_k^2}\,\varepsilon_{i,t+1} < \Phi^{-1}\left(\bar{p}_k\right) \qquad \Leftrightarrow \qquad D_{i,t+1} = 1 \tag{3}$$

where $D_{i,t+1}$ denotes the default indicator of firm i. By definition $D_{i,t+1}$ takes value 1 if firm i defaults at time $t+1$ and 0 otherwise. From (3) it follows that if the economic conditions x_t are good, a firm defaults only if its firm-specific risk factor $\varepsilon_{i,t}$ is bad. Also, the asset correlation ρ_{ij} between two borrowers i and j comes from their dependence on the single risk factor:

$$\rho_{\mathrm{ij}} = \mathrm{Corr}\left[A_{i,t}, A_{j,t}\right] = w_k^2 \qquad i, j \in k \tag{4}$$

From the above relationship one can see that with an increase in w the borrowers become more correlated and show a greater correlated credit risk. A decrease in w suggests that the firm-specific risk is more dominant and the correlated credit risk of a loan portfolio decreases.

As we are interested in correlated credit risk, we study the probability of two borrowers simultaneously falling behind on their payments which is called the *joint probability of default*. We apply a method of moments estimator to estimate the sensitivities w, asset correlations ρ_{ij}, and ultimately of the capital requirements on small business loans. In the Appendix we derive the joint probability of default to be the probability of two borrowers simultaneously falling below the default threshold. Hence, the joint probability of default of borrowers i and j is:

$$p_{kk} \equiv P\left[D_{i,t+1} = 1, D_{j,t+1} = 1\right] = \int_{-\infty}^{\Phi^{-1}(\bar{p}_k)} \Phi\left(\frac{\Phi^{-1}\left(\bar{p}_k\right) - w_k^2 y}{\sqrt{1-w_k^4}}\right) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2} y^2\right) dy \qquad i, j \in k \tag{5}$$

where both borrowers i and j belong to borrower class k. The empirical analysis focuses on estimation of the sensitivities to the single risk factor (w_1,...,w_K). Recall that borrowers are assumed to be homogenous within a particular borrower class, implying that borrowers in the same class have homogenous (identical) sensitivity w_k to the single risk factor. The probability p_{kk} in Equation (5) measures the joint probability of default for two particular borrowers i and j. The probability p_{kk} can also be interpreted as a probability that a group of firms in class k fails to make a payment at the same time.

Equation (5) is at the center of the estimation procedure. The left hand side of Equation (5) gives the theoretical moment for the joint probability of default of borrowers in borrower class k. To estimate the sensitivities to the single risk factor (w_1,...,w_K), we minimize the distance between this theoretical moment for the joint probability of default and a sample moment.

The sample moment is obtained in the following way. Denote the observed default frequency in borrower class k at time t by $ODF_{k,t}$. The observed default frequency is equal to the rate at which borrowers in class k go into default: $ODF_{k,t} = \sum_{i \in k} D_{i,t+1} \Big/ N_{k,t}$. In the Appendix we show that for class k, for sufficiently large classes the sample moment for the joint probability of default corresponds to the historical average of the squared observed default frequencies in class k:

$$\hat{p}_{kk} = \quad = \frac{1}{T} \sum_{t=1}^{T} \left(ODF_{k,t} \right)^2 \qquad 6)$$

We refer to the expression in (6) as the *within borrower class* sample moment since it depicts the joint probability of default for borrowers in the same borrower classes.

The sensitivities (w_1,...,w_K) are estimated by application of method of moments applied to equation (5) using the sample moment in (6). It is a numerical optimization which minimizes the sum of squared errors between the population and sample moments over a domain of w.

To use this method of moments, banks need to monitor borrowers only at the class level and not at the firm level. The information they need to record are the observed default frequencies. Observed default frequencies are standard information recorded at any loan-granting financial institution, which facilitates easy application by small business finance providers of the approach proposed. In practice, the single-factor model is estimated solely on sub-portfolios composed of homogeneous borrowers (see Gordy, 2000 and Dietsch and Petey, 2002 and 2004). It is equivalent to the estimation of the *within borrower class* moments. Then, the estimate of the asset correlation ρ_{ij} is given by the square of the estimate of sensitivity w. This asset correlation ρ_{ij} is a crucial input into the computation of unexpected losses and corresponding capital requirements.

Gordy and Heitfield (2002) provide discussion on possible downward bias of the moment-based estimators in short time series. Our data with information between 2005 to 2010, might be considered as such short time series. However, since we avoid making statements in absolute terms, but rather we compare the estimates between two asset classes – small business and corporate loans – the downward bias goes for both asset classes in the same direction. Thus, for comparison of estimates for small business loans relative to estimates for corporate loans such bias loses importance.

3. DATA

The unique sample of small businesses is provided by Dun & Bradstreet and covers quarterly observations from 2005 to 2011. As explained below, we use this information to compute the default rates from 2005 to 2010. The six years of quarterly observations fulfills the Basel Committee on Banking Supervision (2006) condition under the paragraph 466, which sets the minimum length of the observation period used to five years. In total, the Dun & Bradstreet sample contains 500,000 U.S. small firms that were active at some point in time between 2005 and 2010. The panel is unbalanced and at each point in time it contains nearly 240,000 firms across all the credit ratings, industries and firm sizes in the U.S., which represent a cross section of the U.S. economy. A review of the businesses in Table 1 reveals that firms represent all the major U.S. industries with a high concentration in services (40.78%), retail trade (14.82%) and construction (13.61%). In this sample 56.59% of firms have fewer than 5 employees and 98.29% have fewer than 100 employees. The vast majority of records contains information on privately held firms (99.97%). The firms analyzed are located in all major U.S. regions with a higher concentration in California in the West, Texas in the Southwest and New York in the Northeast, representing 12.09%, 6.74% and 6.56% of the population, respectively.

[INSERT Table 1 HERE]

The panel contains also rich information on firms' actual borrowing and payment behavior, public detrimental information such as county court judgments, legal pre-failure events (receivership, bankruptcy, etc.), credit ratings and legal form, age, industry and firm's location. The sample covers about $19 billion of small business financial activity annually, providing a representative outlook on the U.S. economy. The average credit outstanding per firm is about $32 thousand with 24.19% of the exposures below $1 thousand and 99.75% below $1 million. Informational coverage of the U.S. economy is substantial with about 6,000 major firms (both financial and non-financial) reporting to Dun & Bradstreet. It includes loan and trade records stored by financial institutions and vendors.

In computing default rates, we adopt the Basel Committee view. In particular, the Basel Committee defines that a default takes place if the borrower is 90 days late or the payment of the exposure can be considered unlikely to be made. Dun & Bradstreet sample allows us to compute the defaults at the borrower-level as opposed to bank-exposure-level. This means once the borrower defaults on one if its loans in the Dun & Bradstreet sample, it is flagged as "in default" even though some of its multiple loans might be paid on time. Although, it is a simplification of the current practices, the paragraph 464 of the Basel Committee on Banking Supervision (2006) rules allows banks under certain circumstances to use external data in the computation of their default and loss parameters.

The default rates are computed on borrower class level and represent a share of financially sound firms that go into default at any point in time within 1 year. In particular, at time *t* we identify a group of firms in non-defaulted state. We track them over the next four quarters to see if their credit conditions deteriorate at any point in time; for example if their payments are 90 days overdue or written off, if any of the firms goes bankrupt or is downgraded to credit score 0 (default). If at any point in time during the next four quarters, the borrower is flagged as "in default", it counts as a default observation regardless if it recovers shortly after or it shows in the sample for shorter time than the four quarters. Then the default rate is the sum of those defaults over the initial weighted number of firms, where the weights are given by the ratio of number of quarters with valid observations over four. We repeat this procedure for each quarter. The most recent default rates we can compute from the Dun & Bradstreet sample is the default rate for December 2010 (we observe 2011 payment behavior and use it to compute the December 2010 default rate).

Figure 2 illustrates the resulting default rates for small businesses operating in one of the SIC divisions. Aside from the non-classified firms, manufacturing experiences the highest default rate of 17.48%. In the context of recession this high default rate is explained by the fact that consumers tend to abstain from new purchases and to repair the equipment they already own (consistent with lower default rate in services).

[INSERT Figure 2 HERE]

The homogeneous borrower classes are categorized with respect to two criteria: credit rating and industry. For the purpose of our study we adopt the D&B credit evaluation points (CPOINTS) as an indicator of a firm's credit rating. On this basis, in each quarter we construct the credit ratings as percentiles of the whole distribution such that the credit rating "1" contains the 10% most credit worthy borrowers and credit rating "10" the 10% least credit worthy borrowers. The industry is represented by ten major SIC industry divisions. With respect to industry, we group the firms into ten major SIC divisions.

4. RESULTS

In this section we examine the relative differences in regulatory treatment of the correlated credit risk in small businesses loans and corporate loans. Ultimately, we want to compare the capital requirements computed with the IRB asset correlation to the capital requirements computed with our model-implied estimates of asset correlation, and we want to compare it for two asset classes - small business and corporate loans. To this end, we begin by estimating the model-implied sensitivities to single risk factor w_k, and the model-implied asset correlations ρ_{ij}. Then, the model-implied asset correlations serve as input to the IRB regulatory formula that defines the capital requirements. Next, we study if small business loans need any further discount on capital requirements, which would set their Basel capital requirement and the model-implied riskiness in the same proportions as in the case of corporate loans. Also, to test the reliability of our results, we redefine borrower classes along other dimensions to see if the estimates of asset correlation can be driven by borrower classification.

4.1. RATIO OF IRB CAPITAL REQUIREMENTS TO MODEL-IMPLIED RISKINESS FOR SMALL BUSINESS LOANS

The empirical analysis begins by estimating the sensitivities to the single risk factor w_k. The sensitivity to the single risk factor w_k depicts how vulnerable a firm is to the economic conditions. The larger the estimate of w_k, the larger is the dependence on the surrounding environment and the greater its contribution to correlated credit risk. Low w_k, however, reveals that in case of small businesses the firm-specific characteristics have greater importance in shaping the credit risk. Given that small businesses correspond to a significant part of the U.S. economy, one might expect that their aggregate behavior follows economy swings. On the other hand, each small business has individual qualities and attributes, such as manager, location, business network, and faithful clients etc. that are relatively stable over the business cycle and often decisive. A bakery at the corner or a dentist downtown can do fine even during recession.

To that end, we classify borrowers with respect to conventional factors such as credit rating (used by the Basel Committee) and some less conventional as industry. The Basel Committee advises to classify borrowers at least according to their credit rating. A credit rating is expected to provide a sufficient degree of homogeneity among borrowers. We believe that adding one more dimension to classify the borrowers can only improve the homogeneity of borrowers within a borrower class and provides a sturdy analysis.

Table 2 reports the resulting estimates of sensitivities to the single risk factor w_k together with probabilities of default per credit rating and industry. The low estimates of sensitivities w_k suggest that small businesses credit risk is dominated by firm-specific risk. The single risk factor influences credit risk of small businesses only in a minimal way. We observe that the estimated sensitivities vary in the range 0.00-18.41% explaining only 0.00-3.39% of the asset variability. The remaining 96.61-100.00% of small business risk is due to changes in the firm-specific characteristics.[2] These results are striking, especially in the light of the crisis, which affected the whole economy with very few exceptions.

[INSERT Table 2 HERE]

[2] The low values of sensitivity parameters w_k remain robust to changes in the default definition to a less conservative one which considers only events of losses acquired by a debt holder.

Estimates of the asset correlation follow directly from the estimates of sensitivities to the single risk factor w_k. They are equal to the sensitivity w_k squared. In Table 3 we report the asset correlation estimates. Based on the Dun & Bradstreet sample we find evidence that the model-implied asset correlations are low and average around 0.41%. Mining and agriculture show the lowest asset correlation of 0.00% and retail trade businesses exhibit the highest statistically different from zero asset correlation of 0.78%. Most importantly, regardless of the small business' credit rating and industry, our estimates are significantly lower than any available estimates for corporate firms. For example, McNeil and Wendin (2007) report asset correlations between corporate firms ranging from 6.30%-10.90%. That is eight to fourteen times larger than our estimates for small businesses. These considerably lower asset correlations for small businesses have important regulatory consequences, which are discussed further in this section.

[INSERT Table 3 HERE]

To confront our results based on the Dun & Bradstreet sample with the Basel Committee asset correlation of small businesses, we refer back to the IRB capital requirements on small business loans. Under the Basel Committee's rules (Basel Committee on Banking Supervision, 2006), small business exposures are firms with an annual turnover below EUR 50 million. At the discretion of the national supervising authority, this criterion can be replaced with a criterion on total assets. Dependent on the aggregate exposure of a bank to the individual firm (or a group of connected firms) such small business exposures fall under corporate or retail exposures. If the aggregate exposure of a bank exceeds EUR 1 million, they fall under corporate exposures. In the corporate category, further discounts may apply, for example up to 20% based on the firm-size adjustment. Although, in general, capital requirements on corporate loan portfolio are higher than on small business loan portfolio. This brings us to the retail category. If the aggregate exposure is below EUR 1 million, the small business exposures fall under the retail category.[3] Within the retail category, unless secured by residential mortgages, or treated as qualifying revolving retail exposures (credit card product), it is reasonable to assume that much of the small business lending in Europe and the U.S. is treated as *other retail exposures*. The other retail exposure is the type of exposure we focus on and refer to as *small business loans*. The Basel Committee on Banking Supervision (2005) computes the asset correlation on other retail exposures following the formula:

$$\rho_{kk} = 0.03\frac{\left(1-e^{-35\cdot\bar{p}_k}\right)}{1-e^{-35}} + 0.16\left[1-\frac{\left(1-e^{-35\cdot\bar{p}_k}\right)}{1-e^{-35}}\right] \quad 7)$$

Table 2 reports the probabilities of default $\bar{p}_k$ necessary to compute the asset correlation in (7). In Table 3 we report the above IRB regulatory asset correlation next to our model-implied estimates of asset correlation based on the Dun & Bradstreet sample. By means of a *t*-test we find that, with the exception of mining and public administration, our model-implied asset correlation estimates are significantly below the the Basel Committee's small business asset correlation. This finding is confirmed in Panel B with a paired difference test and a Wilcoxon signed-rank test. Importantly, the information on loss given defaults (*LGD*s), and exposure at default (*EAD*) is irrelevant for the

[3] Other capital add-ons or cushions may apply which are beyond the scope of this paper. From this perspective, our paper offers a simplistic view of the capital requirement on small business loans. But as mentioned earlier we try to pick a setting, which provides similar treatment of the two asset classes such that they only differ with respect to the retail or corporate category. In such a setting, we assume neither of the category receives such add-ons or cushions.

tests in Table 3, as the ratios of asset correlation estimates $\rho_{ii,r}/\rho_{ii,m}$ are independent from *LGD*, and *EAD* parameters.

Once the asset correlation in (7) is computed, the capital requirement and an estimate of unexpected losses for small business exposures (under the other retail exposures) follow from the formula:

$$K = LGD \cdot \left\{ N\left[\left(\frac{1}{\sqrt{(1-\rho_{kk})}} \right) \cdot \Phi^{-1}(\bar{p}_k) + \left(\frac{\sqrt{\rho_{kk}}}{\sqrt{(1-\rho_{kk})}} \right) \cdot \Phi^{-1}(0.999) \right] - \bar{p}_k \right\} \qquad 8)$$

A bank that uses the Basel Committee IRB rules in its small business loan portfolio computes the asset correlation from (7) and then the capital requirement from (8). Next, we would like to confront the results based on historical data with the capital requirements required by the regulator. To this end, instead of using the asset correlation formula from (7) we insert our estimates of asset correlation from Table 3. We call this the model-implied required capital on small business loans. The main tests are shown in Table 4.

Results in Table 4 show that there are significant discrepancies in capital requirements implied by the Basel Committee and by the historical data. Importantly, as we show later, this is not the case for other asset classes i.e. for large corporate loans. At the bottom of the table, we report outcomes of the paired difference test and the Wilcoxon signed-rank test. These tests examine if the mean capital requirements required by the Basel Committee differ from the mean model-implied capital requirements. The tests suggest that the Basel Committee formula significantly overstates the capital requirements for all small business classes, compared to the model-implied capital requirements. Indeed, we observe that the capital requirements are on average 3.5 times higher than the data suggest. The more creditworthy borrowers are, the higher the capital requirements they suffer relative to their riskiness. For the most creditworthy borrowers the IRB regulatory formula overestimates the capital requirements even by a factor of eight, compared to the model-implied capital requirements. Panel C tests if the ratio of the two types of capital requirements is different from one. Importantly, this ratio of capital requirements K_r/K_m is independent from the LGD parameter used to compute the capital requirements, and confirms that the Basel Committee formula significantly overstates the capital requirements for small business loans, compared to the model-implied capital requirements.

[INSERT Table 4 HERE]

By requiring higher capital buffers than the small businesses correlated credit risk suggests, Basel may discourage lending to small businesses in favor of large corporate loans. An inadequate discount on capital requirements on loans to small business therefore may have dare consequences for small business access to finance. Higher capital requirements make banks safer and drive up their cost of capital. Baker and Wurgler (2015) provide evidence supporting that. But disproportionately higher capital requirements on loans to small businesses make banks disproportionally safer than, for example, capital requirements on large corporate loans. Hence, the Basel Committee may unintentionally encourage more financing of large corporate firms rather than of the small business economy. This is an undesirable outcome in the face of various stimulus programs and policies aiming to foster small business lending.

4.2. RATIO OF IRB CAPITAL REQUIREMENTS TO MODEL-IMPLIED RISKINESS FOR CORPORATE LOANS

Having confirmed that there are significant discrepancies between the IRB capital requirements and the model-implied capital requirements for small business loans we turn our attention to corporate loans. To this end, we employ the method from the methodology section to a portfolio of *corporate* debt. For corporate loans, we carefully examine if our method produces results that are in line with the Basel Committee regulatory framework. If for corporate loans we find the IRB capital requirements in line with the model-implied ones, it suggests that the Basel Committee formula significantly overstates the capital requirements for small business loans.

Thus, we use the public information on U.S. corporate default rates per credit rating provided by S&P. S&P reports the payment history of about 3,000 U.S. firms during a period of six years from 2005 through 2010 and covers a broad range of industries. Both S&P and our study weigh the default events by the number of borrowers rather than the nominal value of default. We exclude AAA and AA+ ratings from the analysis due to lack of defaults in those rating categories during the period analyzed. For consistency with the Basel methodology (Gordy, 2000 and 2003) the estimation of our model follows per sub-portfolio composed of borrowers from a homogenous borrower class.

Once we estimate the model-implied asset correlation and compute the IRB asset correlation according to paragraph 272 of the Basel Committee on Banking Supervision (2006) rules, the capital requirements on corporate loans follow from the formula:

$$K = LGD \cdot \left\{ N\left[\left(\frac{1}{\sqrt{(1-\rho_{kk})}} \right) \cdot \Phi^{-1}(\bar{p}_k) + \left(\frac{\sqrt{\rho_{kk}}}{\sqrt{(1-\rho_{kk})}} \right) \cdot \Phi^{-1}(0.999) \right] - \bar{p}_k \right\} \cdot \frac{1+(M-2.5)\cdot b}{1-1.5\cdot b} \quad 9)$$

where $b = [0.11852 - 0.05478 * \log(\bar{p}_k)]^2$ is the smoothed maturity adjustment and *M* is maturity. Panel A in Table 5 shows the resulting asset correlation estimates, which inform about the correlated credit risk. The table also shows the default rates, capital requirements on corporate exposures and the difference between our model and the Basel approach. In general, the results show that although corporate firms exhibit low probabilities of default relative to retail debt, they are heavily exposed to changes in economic conditions. The asset correlations vary between 3.83% and 22.18% and average at 15.01%, which confirms a substantial interdependence in corporate exposures. Most importantly, the capital requirements for corporate exposures implied by our estimates of asset correlations are in line with the IRB ones, which is shown both in panels A and B in Table 5. The paired difference test confirms that our model and the Basel Committee formula produce on average similar outcomes. We find no significant difference between the capital requirements computed according to the IRB formula and the ones computed using our estimates of asset correlation.

[INSERT Table 5 HERE]

4.3. RELATIVE ADJUSTMENT OF IRB CAPITAL REQUIREMENTS FOR SMALL BUSINESSES

Although, the above analysis reports the absolute values of asset correlation estimates, we would like to stress the relative differences between the small business and corporate asset classes. Having observed that the discrepancies between IRB capital requirements and our model-implied capital requirements is significant in case of small business but not in case of corporate loans, we ask how large should be the discount on capital requirement on small business loans to treat both asset classes proportionately to their correlated credit risk. In particular, the average ratio of capital requirements K_r/K_m on corporate loans in Table 5 is 1.76 while the average ratio of capital requirements K_r/K_m on small business loans in Table 4 is 3.53. To lower this ratio for small business loans to the corporate loans' level, the Basel Committee should require a further discount on small business loans of about 45%.

4.4.ROBUSTNESS: COMPOSITION OF BORROWER CLASSES

Lastly, we examine estimates of sensitivity w_k for borrowers classified according to alternative dimensions. Table 6 presents the results for borrowers classified with respect to ten credit ratings only, ten industry classes only, seven employee size classes only, and ten rating classes for a subsample of firms that operate in Massachusetts.

For example, in the last column, we single out one geographic region that is Massachusetts and we report estimates of sensitivities w_k for firms that operate in this state. Intuitively, it is expected that geographic proximity of small businesses would cause them to be more susceptive to the single risk factor. In other words, small businesses are expected to behave in a more correlated manner. Despite of the geographic proximity, our results show that the values of the sensitivity parameters w_k are low. This directly translates into low asset correlations which results in low correlated credit risk. The results show that even within one U.S. state the firm-specific risk in small business loans prevails.

[INSERT Table 6 HERE]

5. CONCLUDING REMARKS

This paper compares the asset correlations and IRB capital requirements to those implied by a single risk factor model for an extensive dataset of U.S. small businesses. We find, that the IRB formula demands higher capital requirements relative to riskiness of small businesses than for corporate loans. This can result in distorted lending and risk management practices by financial institutions, which hold small business loan portfolios. Thus, we advocate a renewed calibration of the IRB capital requirements on small business loans in order to guarantee a level playing field between small business and corporate loans. In our view, this regulatory shortcoming results from the overly-simplistic way in which the Basel Committee calibrated the asset correlations in portfolios of loans to small businesses.

Also, we provide an empirical analysis of a comprehensive panel of exposures to U.S. small businesses between 2005 and 2010. We find that in general small business risk is predominantly resulting from firm-specific risk, even after controlling for different definitions of the default event, geographical proximity, as well as industry and firm size heterogeneity. Our results show that small business risk is predominantly due to changes in the firm-specific characteristics.

Appendix

Given the vector of sensitivity parameters w, the distribution of a single default event in a borrower class k is given by:

$$p_i = P\left[D_{i,t+1} = 1\right] = P\left[A_{i,t+1} < \Phi^{-1}\left(\bar{p}_k\right)\right] = \int_{-\infty}^{\Phi^{-1}(\bar{p}_k)} f\left(A_{i,t+1}\right) \quad dA_{i,t+1} \qquad \text{A.1)}$$

where $f(\cdot)$ is a density function and in our application of the model takes the form of normal probability distribution function and $\Phi(\cdot)$ denotes the cumulative standard normal distribution function. By design, for any i and j where $i \neq j$, the probability distribution of a default event in which two borrowers fail to meet their payments is modeled as a bivariate normal distribution:

$$f_{ij}\left(A_{i,t}; A_{j,t}\right) = \frac{1}{2\pi \,|\Sigma|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2} A^T \Sigma^{-1} A\right\} \qquad \text{A.2)}$$

where $A = \begin{bmatrix} A_{i,t} \\ A_{j,t} \end{bmatrix}$

and $\Sigma = \begin{bmatrix} 1 & w_k^2 \\ w_k^2 & 1 \end{bmatrix}$

The above joint density of $A_{i,t}$ and $A_{j,t}$ can be transformed by standardizing the vector A and integrating out the effects of the risk factors. Consequently one will obtain the probability of an event in which both borrowers default at once:

$$p_{ij} \equiv P\left[D_{i,t+1} = 1, D_{j,t+1} = 1\right] = \int_{-\infty}^{\Phi^{-1}(\bar{p}_k)} \Phi\left(\frac{\Phi^{-1}\left(\bar{p}_k - w_k^2 y\right)}{\sqrt{1 - w_k^4}}\right) \cdot \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2} y^2\right) dy \qquad \text{A.3)}$$

The expression gives the population moment for joint probability of default. The sample moment is derived in the following way. We take the joint probability of default for two firms i and j from borrower classe k to be an average of all occasions in which both firms are simultaneously in default:

$$\hat{p}_{ij} = \frac{1}{T}\sum_{t=1}^{T}\left(D_{i,t+1} \cdot D_{j,t+1}\right) \tag{A.4}$$

Next, to arrive at sample moment of joint probability of default for the borrower class, we need to take an average over all possible pairs of firms in the borrower class:

$$\hat{p}_{kk} = \frac{1}{N_{k,t}\left(N_{k,t}-1\right)}\sum_{i\in k}\sum_{\substack{j\in k\\ j\neq i}}\frac{1}{T}\sum_{t=1}^{T}\left(D_{i,t+1} \cdot D_{j,t+1}\right) \tag{A.5}$$

where $N_{k,t}$ is the number of firms in borrower class k, which we assume to be sufficiently large. Now we change the order of summation, which gives us that the sample moment for joint probability of default is an average over time of the product of their observed default frequencies:

$$\hat{p}_{kk} = \frac{1}{T}\sum_{t=1}^{T}\frac{\sum_{i\in k} D_{i,t+1}}{N_{k,t}}\frac{\sum_{j\in k} D_{j,t+1}}{N_{k,t}}\frac{N_{k,t}}{N_{k,t}-1} \approx \frac{1}{T}\sum_{t=1}^{T}\frac{\sum_{i\in k} D_{i,t+1}}{N_{k,t}}\frac{\sum_{j\in k} D_{j,t+1}}{N_{k,t}} = \frac{1}{T}\sum_{t=1}^{T}\left(ODF_{kt}\right)^2 \tag{A.6}$$

The GMM estimator minimizes the distance between the population and sample moments with respect to the parameters *w*.

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Figure 1.

Categories of banks that lend to small businesses.

Banks are categorized into four equally populated groups with respect to their exposure to small business loans. Banks with higher exposure to small business lending enjoy lower capital requirements and even lower failure rates in the crisis. The number of failed banks decreases with the exposure to small business loans. Similarly, the Tier 1 leverage ratio decreases with the exposure to small business loans with the only exception in the fourth quantile. The exposure to small business loans is computed for year 2007 and is a share of C&I loans to small business with original amount of $100,000 or less. The share of C&I loans to small business is given from Call Reports as a ratio of item RCON5571 over RCON1766 for banks with item RCON6999 equal to *false*, or RCON1766 over RCON1766 otherwise. We match this information to the Failed Bank List provided by FDIC (September 2016). In the first quartile, 222 out of 1,795 banks failed since 2007. In the second quartile, 127 out of 1,883 banks failed. In the third quartile, 69 out of 1,890 banks failed. In the fourth quartile, only 27 out of 1,885 banks failed since 2007.

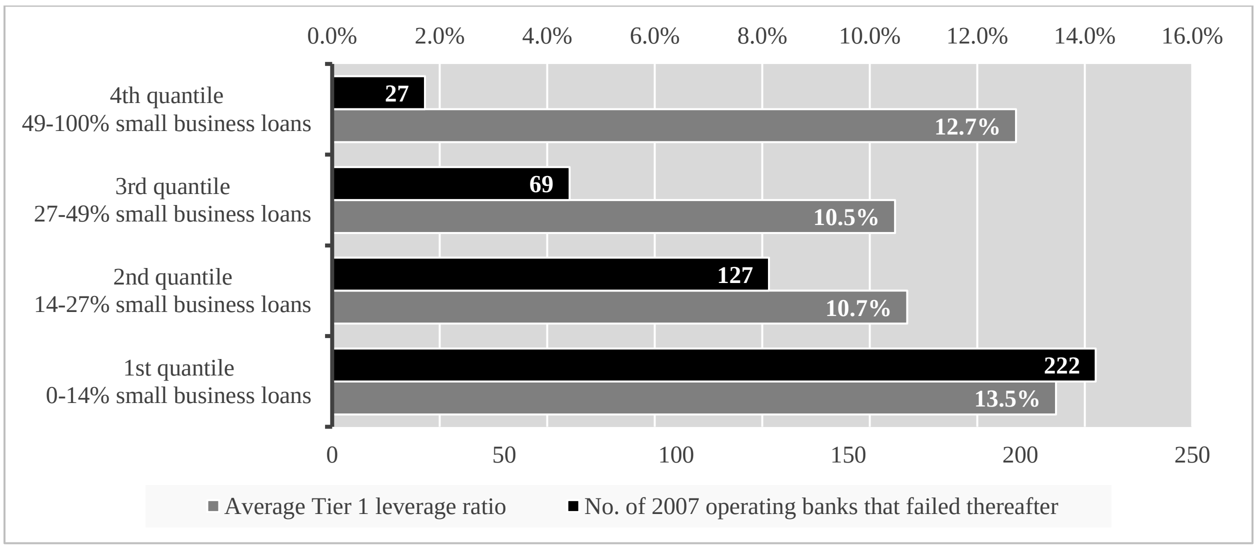

Figure 2.

Observed default frequencies by industry class.

The shaded area denotes the crisis phase as defined by NBER business cycle reference dates. The crisis is from December 2007 until June 2009.

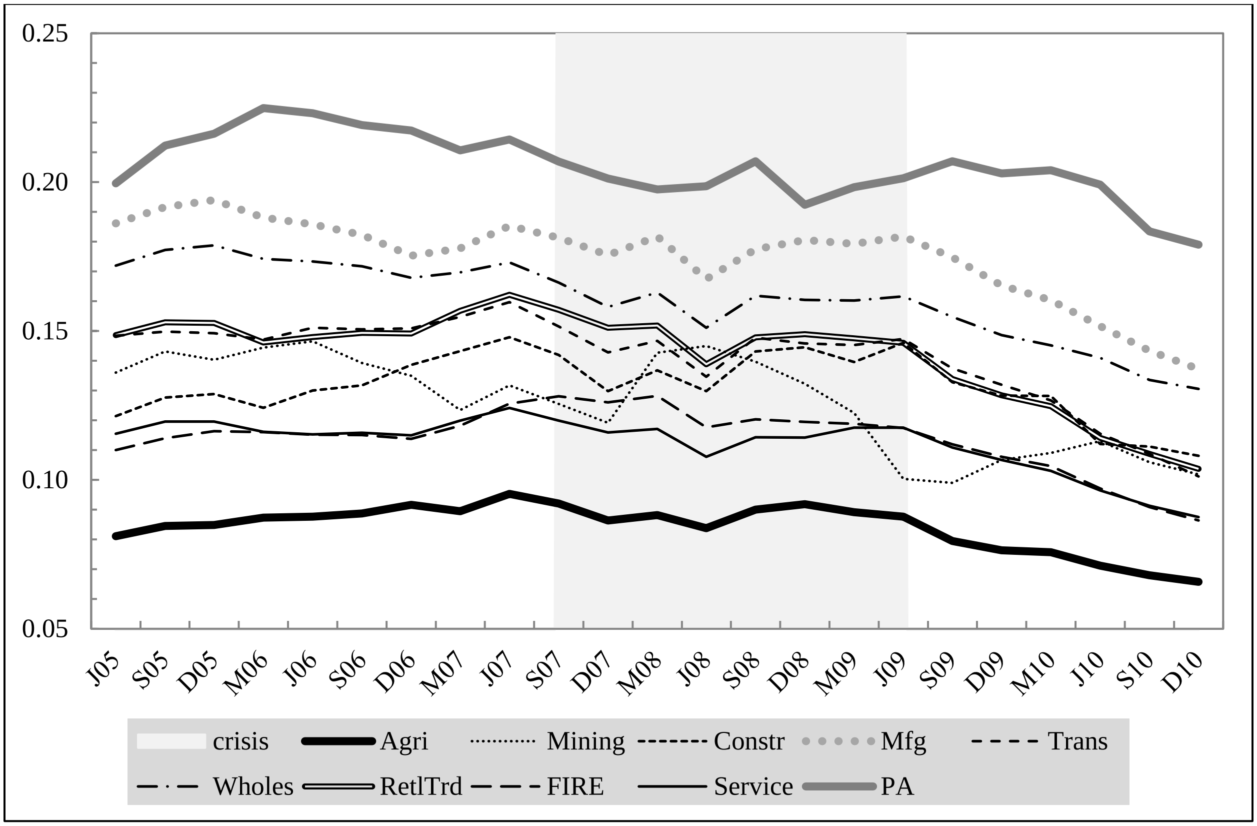

Table 1
Small businesses in the U.S.

The sample spans years 2005 to 2010. Each column reports an average over the sample period. The industries are abbreviated as: Agriculture, Forestry, Fishing (Agri), Mining (Mining), Construction (Constr), Manufacturing (Mfg), Transportation, Communications, Electric, Gas, and Sanitary Services (Trans), Wholesale Trade (Wholes), Retail Trade (RetlTrd), Finance, Insurance, and Real Estate (FIRE), Services (Service), Public Administration and non-classified (PA). Geographic regions are defined as: Central: IA, KS, MN, MO, NE, ND, SD; West: AK, AZ, CA, CO, HI, ID, MT, NV, NM, OR, UT, WA, WY; Northeast: CT, ME, MA, NH, NJ, NY, PA, RI, VT; Midwest: IL, IN, MI, OH, WI; Southeast: DE, DC, FL, GA, MD, VA, NC, SC, WV, AL, KY, MS, TN; Southwest: AR, LA, TX, OK.

	# firms	% total	min	max	default rate (%)
1. SIC division					
Agri	9,902	4.19	9,340	10,188	8.39
Mining	825	0.35	758	872	12.55
Constr	32,180	13.61	27,048	36,275	13.13
Mfg	16,382	6.93	14,155	18,278	17.48
Trans	8,123	3.44	6,963	9,046	14.12
Wholes	16,048	6.79	14,063	17,836	16.02
RetlTrd	35,032	14.82	29,552	39,993	14.19
FIRE	20,020	8.47	17,170	22,310	11.34
Service	96,379	40.78	85,672	104,065	11.19
PA	1,467	0.62	1,358	1,831	23.88
2. Firm size (employees)					
1-5	133,755	56.59	115,434	147,547	9.67
6-10	44,125	18.67	38,308	49,158	12.89
11-20	28,244	11.95	24,731	31,174	15.82
21-30	10,890	4.61	9,778	11,867	18.53
31-50	9,150	3.87	8,344	9,904	21.16
51-100	6,149	2.60	5,670	6,751	26.42
>100	4,043	1.71	3,700	4,446	35.98
3. Private	236,284	99.97	206,140	260,471	12.74
Public	74	0.03	50	117	42.58
4. Region					
Central	17,512	7.41	16,135	18,876	10.65
West	53,754	22.74	45,590	59,743	12.84
Northeast	49,437	20.92	43,212	54,240	12.37
Midwest	36,319	15.37	32,368	39,741	12.31
Southeast	55,219	23.36	47,174	61,552	14.00
Southwest	24,118	10.20	21,533	26,437	12.62
5. $ outstanding					
$0-500	38,530	16.30	29,436	48,676	5.78
$501-1,000	18,648	7.89	15,510	24,119	7.57
$1,001-2,000	22,880	9.68	19,990	27,531	9.15
$2,001-5,000	32,174	13.61	29,208	35,538	10.94
$5,000-15,000	48,536	20.54	42,366	52,458	12.67
$15,001-30,000	28,001	11.85	24,930	31,288	14.73
>$30,001	47,589	20.13	38,951	53,303	22.23
Total	236,358	100.00	206,196	260,590	12.74

Table 2
Sensitivities to the single risk factor and default rates per credit rating & industry borrower classes

Small businesses have low sensitivity to the single risk factor. The credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. Abbreviations for industries are explained in Table 1. Significance levels are denoted by * at the 90% level, ** at the 95% level and *** at the 99% level. Bootstrapped standard errors are in parentheses.

		Sensitivity w_k (%) and default rates $\bar{p}$ (%)									
		1	2	3	4	5	6	7	8	9	10
Agri	w_k	3.70	0.00	5.54	0.00	7.74*	8.39**	7.61*	6.86	6.34	6.14
		(4.05)	(3.73)	(3.96)	(3.90)	(4.27)	(4.04)	(4.53)	(4.52)	(4.54)	(4.80)
	$\bar{p}$	5.94	5.68	5.50	6.17	6.52	8.09	9.99	11.15	16.49	23.49
Mining	w_k	0.00	18.41	2.76	0.00	6.58	0.00	0.00	0.00	0.00	16.07
		(7.81)	(12.86)	(11.23)	(11.87)	(13.11)	(12.78)	(13.53)	(15.53)	(15.59)	(18.79)
	$\bar{p}$	9.16	9.05	8.71	11.96	13.34	15.03	18.33	17.21	23.32	31.57
Constr	w_k	5.49*	5.59**	4.84**	5.00**	6.44***	5.94***	6.14***	6.37***	7.82***	8.72***
		(2.92)	(2.23)	(2.16)	(2.20)	(1.79)	(1.83)	(1.71)	(1.61)	(1.47)	(1.53)
	$\bar{p}$	8.65	7.30	8.06	8.84	9.53	10.82	12.19	14.56	20.46	30.82
Mfg	w_k	5.85**	4.46	5.43**	6.15**	4.59	4.68*	6.69**	6.10**	7.06***	6.31**
		(2.76)	(3.07)	(2.76)	(2.60)	(2.84)	(2.70)	(2.70)	(2.37)	(2.25)	(2.64)
	$\bar{p}$	13.38	12.62	12.99	13.79	14.37	15.56	17.39	19.55	24.11	32.36
Trans	w_k	0.00	2.33	6.52	8.11*	0.00	5.61	5.15	7.41**	7.78**	8.97***
		(4.21)	(4.67)	(4.57)	(4.15)	(3.94)	(4.04)	(3.88)	(3.51)	(3.82)	(3.36)
	$\bar{p}$	10.32	10.65	10.71	10.81	11.06	11.67	13.34	15.01	18.06	26.01
Wholes	w_k	4.34	6.45**	2.19	5.74*	6.43**	2.57	6.49**	4.61	7.17***	6.29**
		(2.70)	(2.71)	(2.66)	(2.98)	(2.77)	(2.78)	(2.73)	(2.87)	(2.54)	(3.00)
	$\bar{p}$	13.11	11.89	12.77	13.86	14.10	14.76	16.49	18.49	22.90	30.26
RetlTrd	w_k	3.81	6.71***	6.30***	7.10***	7.70***	6.77***	7.14***	6.37***	6.59***	8.85***
		(2.62)	(2.33)	(1.95)	(1.69)	(1.67)	(1.67)	(1.57)	(1.41)	(1.33)	(1.45)
	$\bar{p}$	10.67	9.84	10.50	11.08	11.14	12.21	13.10	14.63	18.21	25.68
FIRE	w_k	3.77	4.75*	2.62	6.72***	4.63*	4.94*	8.68***	6.92***	7.63***	7.58**
		(2.67)	(2.73)	(2.47)	(2.22)	(2.73)	(2.73)	(2.43)	(2.52)	(2.89)	(3.01)
	$\bar{p}$	9.25	8.09	8.94	9.21	9.46	10.63	11.63	14.42	18.40	25.63
Service	w_k	3.68***	3.68***	4.19***	5.03***	4.99***	4.96***	5.40***	5.40***	5.68***	7.22***
		(1.23)	(1.17)	(1.03)	(1.01)	(0.99)	(0.94)	(0.91)	(0.93)	(0.94)	(1.00)
	$\bar{p}$	7.93	7.62	8.17	8.99	9.46	10.29	11.56	13.30	17.40	24.73
PA	w_k	7.36	0.00	0.00	5.77	5.56	5.93	11.83	12.00	6.18	9.69
		(4.77)	(5.65)	(6.59)	(7.77)	(8.59)	(8.58)	(8.39)	(8.68)	(8.45)	(7.46)
	$\bar{p}$	21.29	21.90	21.47	23.51	21.83	19.56	15.72	16.47	17.42	22.28

Table 3
Asset correlation per credit rating & industry

Small businesses are mainly subject to firm-level risk with low asset correlation. Panel A reports estimates of asset correlation implied by our data ($\rho_{ii,m}$) and implied by Basel Committee's IRB regulatory formula ($\rho_{ii,r}$). For each credit rating and industry, we find that the two asset values are significantly different from each other apart for some obligor classes in the mining industry and the public administration industry. Panel B tests if there is a significant difference in means of the estimates of asset correlation implied by our data ($\rho_{ii,m}$) and implied by Basel Committee's IRB regulatory formula ($\rho_{ii,r}$). Panel C tests if the ratio of the two types of estimates of asset correlation is different from one. The values reported cover the period from June 2005 to December 2010. The credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. Abbreviations for industries are explained in Table 1. Bootstrapped standard errors are in parentheses.

Panel A: Asset correlation ρ_{ii} within borrower class (%)

		1	2	3	4	5	6	7	8	9	10
Agri	$\rho_{ii,m}$	0.14	0.00	0.31	0.00	0.60	0.70	0.58	0.47	0.40	0.38
		(0.43)	(0.36)	(0.50)	(0.40)	(0.68)	(0.67)	(0.72)	(0.65)	(0.63)	(0.68)
	$\rho_{ii,r}$	4.62	4.78	4.90	4.50	4.33	3.77	3.39	3.26	3.04	3.00
	Diff.	-4.49	-4.78	-4.59	-4.50	-3.73	-3.06	-2.82	-2.79	-2.64	-2.63
	t-stat.	-10.39	-13.24	-9.27	-11.25	-5.47	-4.56	-3.92	-4.28	-4.17	-3.84
	$\rho_{ii,r}/\rho_{ii,m}$	33.83	NA	15.92	NA	7.23	5.35	5.87	6.93	7.56	7.98
Mining	$\rho_{ii,m}$	0.00	3.39	0.08	0.00	0.43	0.00	0.00	0.00	0.00	2.58
		(1.62)	(5.12)	(3.36)	(3.78)	(4.54)	(4.32)	(4.77)	(6.65)	(6.83)	(9.73)
	$\rho_{ii,r}$	3.53	3.55	3.62	3.20	3.12	3.07	3.02	3.03	3.00	3.00
	Diff.	-3.53	-0.16	-3.54	-3.20	-2.69	-3.07	-3.02	-3.03	-3.00	-0.42
	t-stat.	-2.18	-0.03	-1.05	-0.85	-0.59	-0.71	-0.63	-0.46	-0.44	-0.04
	$\rho_{ii,r}/\rho_{ii,m}$	NA	1.05	47.46	NA	7.21	NA	NA	NA	NA	1.16
Constr	$\rho_{ii,m}$	0.30	0.31	0.23	0.25	0.41	0.35	0.38	0.41	0.61	0.76
		(0.30)	(0.24)	(0.20)	(0.22)	(0.23)	(0.21)	(0.21)	(0.21)	(0.23)	(0.27)
	$\rho_{ii,r}$	3.63	4.01	3.77	3.59	3.46	3.29	3.18	3.08	3.01	3.00
	Diff.	-3.33	-3.70	-3.54	-3.34	-3.05	-2.94	-2.81	-2.67	-2.40	-2.24
	t-stat.	-10.93	-15.18	-17.76	-15.39	-13.05	-13.86	-13.48	-12.87	-10.41	-8.31
	$\rho_{ii,r}/\rho_{ii,m}$	12.03	12.84	16.09	14.35	8.36	9.35	8.44	7.58	4.92	3.94
Mfg	$\rho_{ii,m}$	0.34	0.20	0.29	0.38	0.21	0.22	0.45	0.37	0.50	0.40
		(0.30)	(0.30)	(0.30)	(0.32)	(0.27)	(0.25)	(0.35)	(0.28)	(0.31)	(0.31)
	$\rho_{ii,r}$	3.12	3.16	3.14	3.10	3.08	3.06	3.03	3.01	3.00	3.00
	Diff.	-2.78	-2.96	-2.84	-2.73	-2.87	-2.84	-2.58	-2.64	-2.50	-2.60
	t-stat.	-9.18	-9.83	-9.49	-8.50	-10.46	-11.17	-7.45	-9.61	-8.12	-8.28
	$\rho_{ii,r}/\rho_{ii,m}$	9.11	15.89	10.65	8.20	14.67	13.94	6.77	8.10	6.03	7.54
Trans	$\rho_{ii,m}$	0.00	0.05	0.43	0.66	0.00	0.32	0.27	0.55	0.60	0.80
		(0.48)	(0.59)	(0.65)	(0.67)	(0.42)	(0.51)	(0.47)	(0.50)	(0.57)	(0.57)
	$\rho_{ii,r}$	3.35	3.31	3.31	3.30	3.27	3.22	3.12	3.07	3.02	3.00
	Diff.	-3.35	-3.26	-2.88	-2.64	-3.27	-2.90	-2.86	-2.52	-2.42	-2.20
	t-stat.	-6.95	-5.57	-4.42	-3.92	-7.82	-5.68	-6.02	-4.99	-4.24	-3.89
	$\rho_{ii,r}/\rho_{ii,m}$	NA	61.19	7.77	5.01	NA	10.21	11.77	5.58	5.00	3.73

Wholes	$\rho_{ii,m}$	0.19	0.42	0.05	0.33	0.41	0.07	0.42	0.21	0.51	0.40
		(0.24)	(0.34)	(0.20)	(0.34)	(0.35)	(0.23)	(0.34)	(0.27)	(0.35)	(0.34)
	$\rho_{ii,r}$	3.13	3.20	3.15	3.10	3.09	3.07	3.04	3.02	3.00	3.00
	Diff.	-2.94	-2.79	-3.10	-2.77	-2.68	-3.01	-2.62	-2.81	-2.49	-2.61
	t-stat.	-12.51	-8.31	-15.48	-8.07	-7.72	-13.30	-7.64	-10.21	-7.05	-7.58
	$\rho_{ii,r}/\rho_{ii,m}$	16.63	7.70	65.79	9.42	7.48	46.65	7.21	14.23	5.85	7.59
RetlTrd	$\rho_{ii,m}$	0.14	0.45	0.40	0.50	0.59	0.46	0.51	0.41	0.43	0.78
		(0.21)	(0.30)	(0.24)	(0.24)	(0.26)	(0.22)	(0.22)	(0.18)	(0.18)	(0.26)
	$\rho_{ii,r}$	3.31	3.42	3.33	3.27	3.26	3.18	3.13	3.08	3.02	3.00
	Diff.	-3.17	-2.96	-2.93	-2.76	-2.67	-2.72	-2.62	-2.67	-2.59	-2.22
	t-stat.	-15.39	-9.75	-12.16	-11.51	-10.19	-12.20	-11.78	-14.89	-14.67	-8.63
	$\rho_{ii,r}/\rho_{ii,m}$	22.85	7.59	8.39	6.48	5.50	6.94	6.14	7.59	6.97	3.83
FIRE	$\rho_{ii,m}$	0.14	0.23	0.07	0.45	0.21	0.24	0.75	0.48	0.58	0.58
		(0.21)	(0.26)	(0.17)	(0.29)	(0.26)	(0.28)	(0.41)	(0.34)	(0.42)	(0.43)
	$\rho_{ii,r}$	3.51	3.76	3.57	3.52	3.47	3.32	3.22	3.08	3.02	3.00
	Diff.	-3.37	-3.54	-3.50	-3.07	-3.26	-3.07	-2.47	-2.61	-2.44	-2.43
	t-stat.	-16.39	-13.42	-20.21	-10.69	-12.49	-11.11	-6.00	-7.59	-5.76	-5.58
	$\rho_{ii,r}/\rho_{ii,m}$	24.69	16.70	51.86	7.78	16.22	13.59	4.28	6.45	5.18	5.22
Service	$\rho_{ii,m}$	0.14	0.14	0.18	0.25	0.25	0.25	0.29	0.29	0.32	0.52
		(0.09)	(0.09)	(0.09)	(0.10)	(0.10)	(0.09)	(0.10)	(0.10)	(0.11)	(0.15)
	$\rho_{ii,r}$	3.81	3.90	3.75	3.56	3.47	3.35	3.23	3.12	3.03	3.00
	Diff.	-3.67	-3.77	-3.57	-3.31	-3.23	-3.11	-2.93	-2.83	-2.71	-2.48
	t-stat.	-42.19	-43.15	-41.03	-32.69	-32.10	-33.03	-29.68	-27.67	-25.07	-17.06
	$\rho_{ii,r}/\rho_{ii,m}$	28.14	28.87	21.36	14.10	13.96	13.63	11.05	10.72	9.39	5.76
PA	$\rho_{ii,m}$	0.54	0.00	0.00	0.33	0.31	0.35	1.40	1.44	0.38	0.94
		(0.72)	(0.83)	(1.14)	(1.64)	(2.02)	(1.95)	(2.18)	(2.29)	(1.97)	(1.61)
	$\rho_{ii,r}$	3.01	3.01	3.01	3.00	3.01	3.01	3.05	3.04	3.03	3.01
	Diff.	-2.47	-3.01	-3.01	-2.67	-2.70	-2.66	-1.65	-1.60	-2.65	-2.07
	t-stat.	-3.44	-3.62	-2.64	-1.62	-1.33	-1.37	-0.76	-0.70	-1.34	-1.28
	$\rho_{ii,r}/\rho_{ii,m}$	5.55	NA	NA	9.01	9.73	8.57	2.18	2.11	7.93	3.20

Panel B: Test for difference in means between $\rho_{ii,m}$ and $\rho_{ii,r}$

	Test statistics	*p*-value
Paired *t*-test	44.79	0.00
Wilcoxon signed-rank test	8.68	0.00

Panel C: Test for difference of the ratio $\rho_{ii,r}/\rho_{ii,m}$ from one

t-test	8.75	0.00

Table 4
Capital requirements on small business loans

Panel A shows that the Basel Committee significantly overstates the small business asset correlation and hence also the accompanying capital requirements. Per each credit rating and industry, we find that the two capital requirements values are significantly different from each other apart for some obligor classes in mining industry and public administration industry. Panel B tests if there is a significant difference in means of the capital requirements implied by our data (K_m) and implied by the Basel Committee's IRB regulatory formula (K_r). Panel C tests if the ratio of the two types of capital requirements is different from one. Credit rating buckets are constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk level. The sample spans years 2005 to 2010. K_m (%) stands for the model-implied capital requirements computed with the IRB regulatory formula but with our estimates of asset correlation, K_r (%) stands for the IRB regulatory charges. We take the asset correlation as in Table 3 and assume LGD = 0.50. The LGD parameter does not affect the ratio K_r/K_m. Abbreviations for industries are explained in Table 1. Bootstrapped standard errors are in parentheses.

Panel A: The capital requirements on small businesses sub-portfolios

		Credit rating									
		1	2	3	4	5	6	7	8	9	10
Agri	K_m	0.73	0.00	1.07	0.00	1.78	2.27	2.35	2.25	2.64	3.09
		(0.91)	(0.80)	(0.88)	(0.90)	(1.13)	(1.25)	(1.57)	(1.65)	(2.05)	(2.56)
	K_r	6.01	5.98	5.96	6.04	6.09	6.33	6.71	6.97	8.21	9.48
	Diff.	-5.28	-5.98	-4.89	-6.04	-4.31	-4.06	-4.37	-4.72	-5.57	-6.40
	t-stat.	-5.78	-7.47	-5.57	-6.74	-3.81	-3.26	-2.78	-2.87	-2.72	-2.50
	K_r/K_m	8.23	NA	5.55	NA	3.43	2.79	2.86	3.10	3.11	3.07
Mining	K_m	0.00	6.33	0.71	0.00	2.41	0.00	0.00	0.00	0.00	9.56
		(2.76)	(5.77)	(4.39)	(5.50)	(6.51)	(6.57)	(7.48)	(8.68)	(9.10)	(10.80)
	K_r	6.54	6.52	6.45	7.16	7.49	7.88	8.59	8.36	9.46	10.34
	Diff.	-6.54	-0.19	-5.74	-7.16	-5.08	-7.88	-8.59	-8.36	-9.46	-0.78
	t-stat.	-2.37	-0.03	-1.31	-1.30	-0.78	-1.20	-1.15	-0.96	-1.04	-0.07
	K_r/K_m	NA	1.03	9.06	NA	3.10	NA	NA	NA	NA	1.08
Constr	K_m	1.48	1.34	1.23	1.36	1.89	1.88	2.11	2.46	3.73	5.00
		(0.85)	(0.58)	(0.59)	(0.65)	(0.58)	(0.63)	(0.64)	(0.67)	(0.75)	(0.92)
	K_r	6.44	6.20	6.33	6.48	6.61	6.90	7.22	7.78	9.00	10.28
	Diff.	-4.96	-4.87	-5.10	-5.12	-4.73	-5.02	-5.10	-5.31	-5.27	-5.29
	t-stat.	-5.83	-8.33	-8.68	-7.92	-8.13	-7.97	-7.99	-7.89	-6.98	-5.77
	K_r/K_m	4.35	4.64	5.16	4.77	3.51	3.67	3.42	3.16	2.41	2.06
Mfg	K_m	2.13	1.53	1.92	2.29	1.71	1.84	2.88	2.79	3.62	3.63
		(1.07)	(1.13)	(1.05)	(1.05)	(1.13)	(1.12)	(1.24)	(1.14)	(1.21)	(1.55)
	K_r	7.50	7.32	7.40	7.59	7.73	8.00	8.40	8.83	9.57	10.39
	Diff.	-5.37	-5.79	-5.48	-5.30	-6.02	-6.16	-5.52	-6.04	-5.95	-6.76
	t-stat.	-5.02	-5.13	-5.22	-5.07	-5.33	-5.49	-4.46	-5.30	-4.91	-4.35
	K_r/K_m	3.52	4.79	3.85	3.32	4.51	4.35	2.92	3.17	2.64	2.86

Trans	K_m	0.00	0.69	2.07	2.66	0.00	1.86	1.85	2.96	3.46	4.83
		(1.40)	(1.61)	(1.62)	(1.52)	(1.35)	(1.47)	(1.52)	(1.51)	(1.82)	(1.90)
	K_r	6.78	6.86	6.87	6.89	6.95	7.09	7.49	7.88	8.54	9.82
	Diff.	-6.78	-6.17	-4.80	-4.24	-6.95	-5.23	-5.64	-4.92	-5.07	-4.99
	t-stat.	-4.85	-3.84	-2.97	-2.79	-5.15	-3.55	-3.70	-3.26	-2.79	-2.63
	K_r/K_m	NA	9.98	3.32	2.60	NA	3.81	4.05	2.66	2.46	2.03
Wholes	K_m	1.52	2.19	0.73	2.13	2.44	0.95	2.71	2.01	3.59	3.53
		(1.00)	(1.00)	(0.96)	(1.19)	(1.13)	(1.10)	(1.22)	(1.31)	(1.34)	(1.73)
	K_r	7.43	7.15	7.35	7.61	7.67	7.82	8.21	8.63	9.40	10.24
	Diff.	-5.91	-4.95	-6.62	-5.48	-5.23	-6.87	-5.50	-6.62	-5.81	-6.71
	t-stat.	-5.89	-4.96	-6.92	-4.62	-4.63	-6.24	-4.53	-5.04	-4.33	-3.89
	K_r/K_m	4.88	3.26	10.06	3.57	3.15	8.24	3.03	4.30	2.62	2.90
RetlTrd	K_m	1.15	2.02	1.97	2.33	2.56	2.35	2.61	2.47	2.91	4.73
		(0.84)	(0.77)	(0.67)	(0.61)	(0.62)	(0.64)	(0.63)	(0.59)	(0.63)	(0.82)
	K_r	6.86	6.68	6.83	6.96	6.97	7.22	7.43	7.79	8.57	9.78
	Diff.	-5.71	-4.66	-4.86	-4.63	-4.41	-4.87	-4.82	-5.32	-5.66	-5.04
	t-stat.	-6.78	-6.04	-7.30	-7.56	-7.11	-7.65	-7.67	-9.00	-8.99	-6.13
	K_r/K_m	5.95	3.31	3.47	2.99	2.73	3.07	2.85	3.16	2.95	2.07
FIRE	K_m	1.03	1.20	0.69	1.93	1.31	1.52	3.01	2.67	3.43	4.02
		(0.78)	(0.75)	(0.70)	(0.70)	(0.83)	(0.90)	(0.94)	(1.06)	(1.39)	(1.67)
	K_r	6.56	6.33	6.49	6.55	6.60	6.85	7.08	7.74	8.61	9.77
	Diff.	-5.52	-5.13	-5.81	-4.61	-5.29	-5.33	-4.08	-5.07	-5.18	-5.76
	t-stat.	-7.10	-6.83	-8.34	-6.55	-6.37	-5.89	-4.33	-4.80	-3.73	-3.45
	K_r/K_m	6.35	5.26	9.45	3.39	5.04	4.51	2.36	2.90	2.51	2.43
Service	K_m	0.90	0.87	1.06	1.38	1.42	1.49	1.78	1.94	2.42	3.75
		(0.32)	(0.30)	(0.28)	(0.30)	(0.31)	(0.31)	(0.32)	(0.36)	(0.43)	(0.55)
	K_r	6.31	6.25	6.35	6.50	6.60	6.78	7.07	7.48	8.40	9.66
	Diff.	-5.41	-5.38	-5.29	-5.12	-5.18	-5.28	-5.29	-5.54	-5.98	-5.90
	t-stat.	-16.93	-18.13	-18.81	-17.07	-16.92	-17.25	-16.38	-15.27	-14.01	-10.70
	K_r/K_m	7.00	7.15	6.00	4.71	4.65	4.53	3.98	3.85	3.47	2.57
PA	K_m	3.56	0.00	0.00	2.89	2.68	2.71	5.11	5.33	2.65	4.90
		(2.45)	(2.93)	(3.46)	(4.32)	(4.72)	(4.51)	(4.15)	(4.39)	(4.25)	(4.08)
	K_r	9.14	9.24	9.17	9.49	9.23	8.83	8.04	8.20	8.41	9.30
	Diff.	-5.58	-9.24	-9.17	-6.59	-6.55	-6.13	-2.92	-2.88	-5.76	-4.40
	t-stat.	-2.27	-3.15	-2.65	-1.53	-1.39	-1.36	-0.70	-0.66	-1.35	-1.08
	K_r/K_m	2.56	NA	NA	3.28	3.44	3.26	1.57	1.54	3.17	1.90

Panel B: Test for difference of the ratio $\rho_{ii,r}/\rho_{ii,m}$ from one

	LGD	Test statistics	*p*-value
Paired *t*-test	0.50	41.65	0.00
Wilcoxon signed-rank test	0.50	-8.68	0.00

Panel C: Test for difference of the ratio K_r / K_m from one (not affected by LGD)

t-test	All possibilities	14.53	0.00

Table 5

Capital requirements on corporate loans in the U.S.

Our estimates of capital requirements on corporate loans are in line with the Basel Committee. The default rates $\overline{p}$ are an average over time of observed default frequencies. Estimation of asset correlation $\rho_{ii,m}$ within a borrower class is based on a sample of annual default rates provided by S&P. The sample spans years 2005 to 2010. Monte Carlo standard errors are in parentheses. K_m stands for the model-implied capital requirements computed with the IRB regulatory formula but with our estimates of asset correlation, K_r stands for the IRB regulatory charges. We assume LGD = 0.50 and effective maturity M = 3. The LGD and M parameters do not affect the ratio K_r/K_m. Panel B displays tests for a difference between our estimates and the IRB regulatory approach.

Panel A: The capital requirements on corporate sub-portfolios

	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
$\overline{p}$ (%)	0.19	0.17	0.13	0.16	0.16	0.16	0.24	0.30
$\rho_{ii,m}$ (%)	20.72	20.28	19.34	11.57	20.11	11.54	9.57	13.20
	(6.33)	(6.31)	(6.40)	(4.60)	(6.46)	(4.40)	(3.78)	(4.60)
$\rho_{ii,r}$ (%)	22.94	23.04	23.24	23.08	23.08	23.10	22.65	22.32
Diff.	-2.22	-2.76	-3.90	-11.51	-2.97	-11.56	-13.08	-9.12
t-stat.	0.35	0.44	0.61	2.50	0.46	2.63	3.46	1.98
$\rho_{ii,r}/\rho_{ii,m}$	1.11	1.14	1.20	1.99	1.15	2.00	2.37	1.69
K_m (%)	3.64	3.33	2.73	1.69	3.22	1.66	1.77	2.89
	(1.25)	(1.17)	(1.00)	(0.72)	(1.16)	(0.68)	(0.73)	(1.09)
K_r (%)	4.14	3.92	3.45	3.83	3.83	3.79	4.72	5.31
Diff.	0.50	0.58	0.72	2.15	0.61	2.13	2.95	2.42
t-stat.	0.40	0.50	0.71	2.99	0.53	3.13	4.06	2.23

	BB+	BB	BB-	B+	B	B-	CCC/C
$\overline{p}$ (%)	0.68	0.44	0.47	1.61	2.92	6.24	23.97
$\rho_{ii,m}$ (%)	13.80	11.31	3.83	15.02	16.81	22.18	15.92
	(4.50)	(4.05)	(1.86)	(4.52)	(5.14)	(5.45)	(6.00)
$\rho_{ii,r}$ (%)	20.56	21.65	21.47	17.36	14.79	12.53	12.00
Diff.	-6.76	-10.34	-17.64	-2.34	2.02	9.65	3.92
t-stat.	1.50	2.55	9.48	0.52	-0.39	-1.77	-0.65
$\rho_{ii,r}/\rho_{ii,m}$	1.49	1.91	5.61	1.16	0.88	0.56	0.75
K_m (%)	4.93	3.08	1.20	8.82	13.33	23.38	25.81
	(1.66)	(1.14)	(0.50)	(2.56)	(3.71)	(4.72)	(5.09)
K_r (%)	7.59	6.30	6.55	10.16	11.88	14.92	22.24
Diff.	2.66	3.23	5.35	1.34	-1.45	-8.47	-3.57
t-stat.	1.60	2.82	10.67	0.52	0.39	1.79	0.70

Panel B: Test for difference in means between K_r and K_m

	LGD	M	Test statistics	*p*-value
Paired *t*-test	0.50	3	0.88	0.39
Wilcoxon signed-rank test	0.50	3	-1.48	0.14

Table 6
Sensitivity to the single risk factor for broad borrower classes

The small business sensitivity to the single risk factor is low regardless of the borrower classification. In column (1), the credit rating is constructed to represent deciles of the firms' risk distribution where 1 represents the lowest and 10 the highest credit risk. Column (2) contains sensitivities with respect to industry classification. Column (3) contains sensitivities with respect to firm size classification. Column (4) contains estimates of sensitivities for borrowers, operating in Massachusetts and classified with respect to credit rating. Abbreviations for industries are explained in Table 1. Significance levels are denoted by * at the 90% level, ** at the 95% level and *** at the 99% level. Bootstrapped standard errors are in parentheses.

Sensitivities to single risk factor w_k (%)							
(1)		(2)		(3)		(4)	
credit rating	w_k	*industry*	w_k	*firm size*	w_k	*credit rating for MA*	w_k
1	4.31 ***	Agri	4.71 ***	1-5	6.10 ***	1	10.32
	(0.64)		(1.45)		(0.30)		(7.79)
2	4.73 ***	Mining	5.28	6-10	5.24 ***	2	5.65
	(0.66)		(4.81)		(0.50)		(6.52)
3	4.51 ***	Constr	5.02 ***	11-20	4.21 ***	3	6.29
	(0.60)		(0.58)		(0.60)		(6.37)
4	5.41 ***	Mfg	5.45 ***	21-30	5.57 ***	4	5.64
	(0.56)		(0.81)		(1.03)		(6.17)
5	5.20 ***	Trans	6.33 ***	31-50	5.69 ***	5	4.35
	(0.55)		(1.22)		(1.12)		(6.03)
6	5.02 ***	Wholes	5.32 ***	51-100	7.72 ***	6	1.46
	(0.55)		(0.83)		(1.30)		(5.59)
7	5.93 ***	RetlTrd	6.82 ***	>100	8.53 ***	7	9.17
	(0.53)		(0.59)		(1.53)		(6.31)
8	5.82 ***	FIRE	5.34 ***			8	9.34
	(0.53)		(0.81)				(6.54)
9	6.14 ***	Service	4.81 ***			9	4.15
	(0.55)		(0.34)				(5.46)
10	7.07 ***	PA	2.52			10	5.23
	(0.62)		(2.49)				(5.93)