Model-Free International Stochastic Discount Factors *

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Abstract

We characterize international stochastic discount factors (SDFs) in incomplete markets under various forms of market segmentation. Using 40 years of data on a cross-section of different countries, we estimate model-free SDFs and factorize them into permanent and transitory components. We find that large permanent SDF components help to reconcile the low exchange rate volatility, the exchange rate cyclicality, and the forward premium anomaly. However, under integrated markets, this entails highly volatile and very similar international SDFs. In contrast, segmented markets can generate less volatile and more dissimilar SDFs. Motivated by a simple model with constrained financiers who intermediate households' demand for international assets, we empirically document strong links between international model-free SDFs, Value-at-Risk constraints, and proxies of financial intermediaries' wealth.

Keywords: stochastic discount factor, exchange rates, market segmentation, market incompleteness, financial intermediaries.

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The simplest canonical models in international finance are at odds with many salient features of exchange rates and international asset prices. These can be summarized in three asset pricing puzzles: the low exchange rate volatility documented by Obstfeld and Rogoff (2001) and Brandt, Cochrane, and Santa-Clara (2006), the counter-cyclicality puzzle of Kollmann (1991) and Backus and Smith (1993), and the forward premium anomaly of Hansen and Hodrick (1980) and Fama (1984).

These puzzles spawned a large literature. One strand of the literature assumes integrated complete markets under various specifications of preferences and consumption dynamics. Another strand of the literature argues that some form of incompleteness may be useful to accommodate the puzzling features of exchange rates. Recently, however, Lustig and Verdelhan (2016) conclude that, even under incomplete spanning, the three puzzles cannot be jointly explained in an international consumption CAPM setting.[1]

In complete and integrated markets, the rate of appreciation of the real exchange rate (X) equals the ratio of the foreign (M_f) and domestic (M_d) stochastic discount factors (SDFs): $X = M_f/M_d$, i.e., the asset market view of exchange rates holds. Therefore, the common understanding in international financial economics is that under a departure from market completeness, the exchange rate is in general different from this ratio. Such a deviation from the asset market view can be captured by a multiplicative stochastic wedge as in Backus, Foresi, and Telmer (2001). In this paper, we show that deviations from the asset market view are either completely absent or very small in incomplete but integrated markets. Departures from the asset market view can be achieved via international market segmentation, which we document is key to quantitatively address salient features of international asset returns.

To this end, we propose a parsimonious model-free approach to characterize theoretically and to measure empirically the relations between international SDFs, exchange rate puzzles and the deviations from the asset market view of exchange rates. We develop our theory for

[1]The complete market assumption is used in the habit model of Stathopoulos (2017) to generate sizable currency risk premia. Similarly, Colacito and Croce (2013) employ recursive preferences with highly correlated international martingale components in a two-country complete market setting. Farhi and Gabaix (2016) rely on a complete market economy with time-additive preferences and a time-varying probability of rare consumption disasters. Gabaix and Maggiori (2015) provide a theory of exchange rate determination based on capital flows in financial markets with heterogenous trading technologies, while Chien, Lustig, and Naknoi (2015) propose a two-country stochastic growth model with segmented financial markets that generates smooth exchange rates and highly volatile stochastic discount factors.

economies where domestic and foreign investors have access to short- and long-term bonds, and aggregate equity. In order to characterize international SDFs without committing to a particular asset pricing model, we estimate different projections of international SDFs on the space of tradable returns for domestic and foreign investors.

Given the multitude of SDFs pricing returns in incomplete markets, we explore minimum dispersion SDFs, which minimize different notions of variability, e.g., the Hansen and Jagannathan (1991) SDF when we minimize the SDF variance or minimum entropy SDF projections. We prove that minimum entropy SDFs always imply the validity of the asset market view whenever foreign and domestic investors trade the same set of assets in integrated markets, i.e., when markets are symmetric. This finding is important, because it implies that a Backus, Foresi, and Telmer (2001)–type stochastic wedge only arises with respect to any other minimum dispersion SDF, including the tradable minimum variance SDFs. Using the latter SDFs, we show that exchange rate wedges are always interpretable as a measure of the amount of untradable exchange rate risk in international financial markets. Since under integrated markets untradable exchange rate risks and asset market view deviations are small, we break the strong link between international SDFs and exchange rates and allow for international market segmentation when studying the exchange rate puzzles.

More specifically, we examine the implications of different deviations from market symmetry, from highly segmented domestic and foreign markets to fully integrated markets, in which (risk-free) short- and long-term bonds and stocks are traded internationally. This allows us to quantify the trade-offs between a larger domestic and foreign SDF dispersion necessary to price a wider set of returns, the three exchange rate puzzles, and a deviation from the asset view. For our empirical analysis, we adopt the insights of Bansal and Lehmann (1997), Alvarez and Jermann (2005) and Hansen and Scheinkman (2009), in order to factorize international SDFs into a permanent and a transient component using long-term bonds.

We study eight benchmark currencies, namely the US dollar, the British pound, the Swiss franc, the Japanese yen, the euro (Deutsche mark before the introduction of the euro), the Australian dollar, the Canadian dollar, and the New Zealand dollar. The resulting seven exchange rates are expressed with respect to the US dollar as the domestic currency and the sample period spans January 1975 to December 2015. We summarize our empirical findings as follows.

We first document that our main empirical results are all consistent across the various choices of SDF projections used. Permanent (martingale) components of domestic and foreign SDFs across markets are highly volatile, irrespective of the degree of market segmentation, to the point that they actually dominate the overall SDF variability. The co-movement of permanent SDF components and long-term bond returns is positive, in order to match the typically negative local risk premia of long-term bonds. These features are consistent with previous evidence for the US market in, e.g., Alvarez and Jermann (2005).

In a setting where equity and long-term bond markets are segmented and investors are allowed to trade internationally only the riskless short-term bonds, we find the ensuing minimum dispersion SDFs to jointly address the three exchange rate puzzles. The low exchange rate volatility is explained by means of a volatile wedge between exchange rates and the ratio of foreign and domestic SDFs. The cyclicality puzzle is addressed because cross-country differences in transient SDF components are only weakly related to exchange rate returns. Carry trade premia are also in line with those observed in the data, because the international pricing constraints on risk-free bonds effectively force domestic and foreign SDFs to correctly reproduce the cross-section of currency risk premia. Moreover, the SDFs in this setting exhibit on average volatilities slightly higher than the local Sharpe ratios, whereas the correlation between domestic and foreign SDFs is far from perfect, ranging from 12% to 65%, giving rise to large deviations from the asset market view.

Under fully integrated international bonds and stock markets, we show that the three exchange rate puzzles are again all explained. However, the resulting SDF dispersions are significantly larger than under integrated short-term bond markets alone. At the same time, the correlations between international SDF projections are nearly perfect and the deviations from the asset market view uniformly small. These features are necessary in order to match the cross-sections of international equity premia and long-term bond risk premia together with the low exchange rate volatility. Compared to the integrated short-term bond markets case, the increase in minimum variance SDFs dispersion is on average 30%, where the highest variances are observed for the funding currencies. These large SDF dispersions suggest that it may be difficult to explain exchange rate puzzles in a structural model with fully integrated international bond and equity markets.

Our empirical analysis reveals that international market segmentation is the key economic driver of quantitatively relevant deviations from the asset market view, both under complete and incomplete markets. Therefore, market segmentation is also central for understanding different degrees of co-movement or similarity between international SDFs. To measure SDF similarity in our incomplete markets setting, we propose a novel SDF similarity index. This index complements existing measures of SDF co-dependence, such as Brandt, Cochrane, and Santa-Clara (2006) and Chabi-Yo and Colacito (2017). In the data, we find that SDF similarity is maximal for symmetrically integrated markets, while in presence of market asymmetries, SDF similarity is clearly reduced. To compare our results with other similarity measures, we calculate Chabi-Yo and Colacito (2017) SDF co-entropies using our model-free SDFs, without assuming the validity of the asset market view. Consistent with our similarity measures, we find that while co-movement of SDFs is only roughly 50% in asymmetric market settings, SDF co-movement is virtually perfect as one moves towards symmetric markets.

Finally, in quest of relating our model-free SDFs to economic fundamentals, we propose a stylized version of the Gabaix and Maggiori (2015) model which studies an economy with segmented short-term bond markets, in which households' asset demand is met by financial intermediaries. In such a setting, intermediaries' stochastic discount factor is shown to be linear in their wealth. Motivated by this framework, we relate our model-free SDFs to different proxies of financial constraints and financial intermediaries wealth, such as the VIX, a model-free implied volatility index extracted from S&P500 options, and the intermediaries capital ratio of He, Kelly, and Manela (2017). Depending on the market setting, we find that proxies of financial constraints and intermediaries' wealth alone can explain up to 50% of the variation of our model-free SDFs.

After a literature review, the rest of the paper is organized as follows. Section 1 provides the theoretical framework for our model-free selection of minimum dispersion SDFs in international financial markets. Section 2 describes our data and our main empirical findings, under various benchmark assumptions about the degree of international market segmentation. Motivated by a segmented market model where financiers intermediate households' demand for international assets, Section 3 studies the empirical relation between international model-free SDFs, Value-at-Risk constraints, and proxies of financial intermediaries' wealth. Section 4 is devoted to robustness checks regarding the SDF factorization into short- and long-term components. Section 5 concludes. An online appendix contains further results omitted in the paper.

Literature Review: Our paper contributes to the literature that studies the ability of market incompleteness to address various puzzles in international finance. Bakshi, Cerrato, and Crosby (2015) and Lustig and Verdelhan (2016) study preference-free SDFs in incomplete markets to address the weak link between exchange rates and macroeconomic fundamentals. The former impose "good deal" bounds on international SDFs to examine economies with a low amount of risk sharing and economically motivated pricing errors. Lustig and Verdelhan (2016) introduce a stochastic wedge between foreign and domestic SDFs and conclude that incomplete markets cannot jointly address the three exchange rate puzzles under a Consumption-CAPM framework. Adopting a minimum-dispersion SDF approach, we borrow from Almeida and Garcia (2012) and characterize the properties of international SDFs, without introducing any particular distributional assumption, while allowing them to be factorized into transient and permanent components. We demonstrate that long-run SDF components are key to reconcile stylized exchange rate facts in incomplete markets, especially the Backus-Smith puzzle. We further highlight theoretically the distinct economic roles of minimum entropy and minimum variance SDFs with respect to the asset market view. Similarly, we characterize empirically the trade-offs between SDF dispersion, SDF similarity, and market segmentation in international financial economies.

Another strand of literature studies structural models of exchange rate determination under different assumptions about market segmentation. Chien, Lustig, and Naknoi (2015) show that while limited stock market participation can reconcile highly correlated international SDFs with a low correlation in consumption growth, it is less successful in addressing the Backus and Smith puzzle. Alvarez, Atkeson, and Kehoe (2009) explain the Backus and Smith puzzle in a general equilibrium model with financial frictions and endogenous market participation. Gabaix and Maggiori (2015) study the disconnect puzzle and violations of UIP in a setting where specialized financiers intermediate households' asset demands in segmented markets. We contribute to this literature by jointly addressing the three exchange rate puzzles with a model-free approach and by quantifying empirically the large SDF dispersion and similarity trade-offs implied by settings with fully integrated versus segmented international financial markets. Theoretically, we prove that the asset market view always holds for symmetric markets with respect to minimum entropy SDFs. A key economic implication of this feature is that exchange rate wedges can always be factorized into tradable and untradable exchange rate risk components in incomplete markets. We document empirically that all minimum dispersion SDFs in symmetric markets are very similar and almost perfectly co-moving. At the same time, given a particular market segmentation structure, we are able to quantify the resulting degree of SDF similarity and co-dependence that may need to be generated by structural models assuming segmented markets.

Maurer and Tran (2016) construct minimum variance SDF projections on excess returns in incomplete continuous-time models, showing that the asset market view holds if and only if exchange rate risks can be disentangled in symmetric domestic and foreign asset markets, i.e. only in the absence of jump risk. Our approach is different and explicitly considers various model-free SDFs projections in discrete time. This allows us to show that the asset market view always holds for minimum entropy SDFs in symmetric economies, irrespective of the degree of market incompleteness and without making any additional assumptions on the distribution of asset returns. Similarly, considering different admissible SDF projections is key to decompose exchange rate risks into tradable and untradable risks from the perspective of domestic and foreign investors.

Empirically, the SDF factorization in permanent and transient components has been employed previously in various studies of international asset pricing, assuming the validity of the asset market view. Chabi-Yo and Colacito (2017) make use of co-entropies to characterize the horizon properties of SDFs co-movement. Lustig, Stathopoulos, and Verdelhan (2016) explore international long-term bond premia and conclude that the bond return parity condition holds when nominal exchange rates are stationary. The large co-movement of permanent components in these studies is a direct consequence of the assumed validity of the asset market view. Indeed, we demonstrate that regardless of the underlying degree of market incompleteness, an almost perfect co-movement of permanent SDF components emerges in all our symmetric international economies. While we document that the underlying SDF dispersion in symmetric settings may be difficult to explain using structural models with integrated (complete or incomplete) markets, we also quantify the SDF dispersion and similarity trade-offs in segmented market settings.

1 Preference-Free SDFs in International Markets

In this section, we introduce our model-free methodology for identifying international minimum dispersion SDFs in incomplete financial markets and for measuring their degree of similarity. One motivation for using minimum dispersion SDFs relies on the fact that they can be understood as optimal SDF projections generated by traded asset returns which also naturally bound the welfare attainable by marginal investors. In this sense, minimum dispersion SDFs constrain the best deals accessible to domestic and foreign investors. Moreover, minimum dispersion SDFs directly imply model-free constraints on the distribution of asset returns, such as asset pricing bounds on expected (log) returns and Sharpe ratios. These constraints need to be satisfied by any admissible international asset pricing model. We focus on three types of SDFs which are obtained by minimizing the SDF entropy, variance, and Hellinger divergence.

We denote by M_d and M_f generic domestic and foreign SDFs that price a vector of returns $\mathbf{R}_d$ and $\mathbf{R}_f$ from the perspective of domestic and foreign investors, i.e., in the domestic and foreign currency, respectively. Moreover, we denote by X the gross exchange rate return, where the exchange rate is defined as the domestic currency price of one unit of the foreign currency. In the following, we generally work with real SDFs, returns, and exchange rates. Whenever we use nominal exchange rates, we state this explicitly. The next definition introduces two central concepts for our work.

Definition 1. *(i) International financial markets are called* ***symmetric*** *whenever span*$(\mathbf{R}_d) =$ *span*$(\mathbf{R}_f X)$*, where span*$(\mathbf{R}_d)$ *(span*$(\mathbf{R}_f X)$*) is the linear span of portfolio returns generated by domestic returns (foreign returns converted in domestic currency). (ii) The* ***asset market view*** *of exchange rates is said to hold with respect to SDFs* M_d *and* M_f *if and only if*

$$X = M_f/M_d \ .$$

By definition, symmetric financial markets are integrated international financial markets, in which each foreign return is tradable by domestic investors through exchange rate markets, and vice versa. Consistent with this definition, we interpret a deviation from market symmetry as a particular form of market segmentation. One implication of market symmetry is that whenever domestic and foreign markets are complete, the asset market view of exchange rates holds. In this setting, there exists a unique SDF that can be expressed in different currency units by a simple change of numéraire via the exchange rate return. In other words, in complete and symmetric markets, domestic and foreign SDFs are numéraire invariant.

In incomplete or partially segmented markets, a deviation from the asset market view can arise, rendering the relation between domestic and foreign SDFs generally more complex. In principle, such a breakdown of the asset market view can be a pervasive feature that makes minimum dispersion SDFs not numéraire invariant in such settings. However, we show that minimum entropy SDFs always preserve their numéraire invariance, even under incomplete markets, implying that the asset market view always holds under market symmetry for these SDFs. This last property does not hold for any other minimum dispersion SDFs. Therefore, symmetric market settings are natural benchmarks to measure deviations from the asset market view in our context.

Intuitively, a deviation from the asset market view is intrinsically related to varying the co-movement and similarity properties of international SDFs consistently with the low exchange rate volatility. In order to quantify the trade-offs between a deviation from the asset market view, the amount of international SDF dispersion and the degree of SDF similarity in presence of a possible market segmentation, we specify a new family of model-free SDF similarity indices, which naturally complement existing indices of SDF co-movement in the literature.

1.1 Minimum-Dispersion SDFs

As is well known, stochastic discount factors can be thought of as investors' marginal utility of wealth. Using a preference-free approach, we study the properties of these SDFs in incomplete and possibly segmented markets by restricting the set of assets that investors can trade.

The vectors of domestic and foreign returns priced by M_d and M_f are $\mathbf{R}_d = (R_{d0}, \ldots, R_{dK_d})'$ and $\mathbf{R}_f = (R_{f0}, \ldots, R_{fK_f})'$, where R_{d0} and R_{f0} denote the risk-free returns in the domestic and foreign markets. In our empirical analysis, we take the United States (US) as the domestic market and the United Kingdom (UK), Switzerland (CH), Japan (JP), the European Union (EU), Australia (AU), Canada (CA) or New Zealand (NZ) as the foreign markets.[2] For each market $i = d, f$, we study the minimum-dispersion SDF that solves for parameter $\alpha_i \in \mathbb{R}\backslash\{1\}$

[2]Prior to the introduction of the Euro, we take Germany in its place.

the following optimization problem:

$$\begin{aligned} &\min_{M_i} \frac{1}{\alpha_i(\alpha_i - 1)} \log E[M_i^{\alpha_i}] \ , \\ &\text{s.t. } E[M_i \mathbf{R}_i] = \mathbf{1} \ ; \ M_i > 0 \ . \end{aligned} \tag{1}$$

The pricing restriction $E[M_i\mathbf{R}_i] = \mathbf{1}$ in equation (1), where $\mathbf{1}$ is a $(K_i + 1) \times 1$ vector of ones, ensures that the SDF satisfies the given pricing constraints, while the positivity constraint $M_i > 0$ ensures that it is indeed an admissible SDF.[3] The formulation in equation (1) depends on parameter α_i, which subsumes various SDF choices in incomplete markets economies. More specifically, different values of α_i assign different weights to the higher-order moments of the asset return distribution. For $\alpha_i = 2$, we obtain the minimum variance SDF underlying the well-known Hansen and Jagannathan (1991) bounds, while $\alpha_i = 0$ and $\alpha_i = 0.5$ yield the minimum entropy and minimum Hellinger divergence SDFs, respectively. These two SDFs also allow us to study the dependence of our findings on the higher-order moments of the asset return distribution.[4]

By construction, every minimum dispersion SDF corresponds to a different set of tight constraints on the moments of traded asset returns. Since asset returns are observable but SDFs are not, we can conveniently restate the minimum objective function in equation (1) into the maximum objective function in the following dual portfolio problem (see, e.g., Orlowski, Sali, and Trojani (2016), Proposition 4):

$$\begin{aligned} &\max_{\lambda_i} -\frac{1}{\alpha_i} \log E\left[R_{\lambda_i}^{\alpha_i/(\alpha_i - 1)}\right] \ , \\ &\text{s.t. } R_{\lambda_i} > 0 \ , \end{aligned} \tag{2}$$

where $R_{\lambda_i} = \sum_{k=1}^{K_i} \lambda_{ik} R_{ik} + (1 - \sum_{k=1}^{K_i} \lambda_{ik}) R_{i0}$ and λ_{ik} denotes the portfolio weight of asset k in market $i = d, f$. The first-order conditions (FOCs) associated with optimization problem (2)

[3] In case the risk-free return R_{i0} is traded, an equivalent formulation of problem (1) is:

$$\begin{aligned} &\min_{M_i} \frac{1}{\alpha_i(\alpha_i - 1)} \log E[(M_i/E[M_i])^{\alpha_i}] \ , \\ &\text{s.t.} \quad E[M_i \mathbf{R}_i] = \mathbf{1} \ ; \ M_i > 0 \ . \end{aligned}$$

[4] In line with Almeida and Garcia (2012), among others, the minimum-entropy SDF delivers a tight upper bound on the maximal expected log return with respect to the available traded assets, while the minimum-variance SDF delivers a tight upper bound on the maximal Sharpe ratio.

read:

$$E\left[R_{\lambda_i^*}^{-1/(1-\alpha_i)}(R_{ik}-R_{i0})\right]=0. \tag{3}$$

From optimal return $R_{\lambda_i^*}$ in equation (3), we obtain the minimum dispersion SDF, M_i^*, explicitly.

Proposition 1. *The minimum dispersion SDF in international financial markets is given by:*

$$M_i^* = R_{\lambda_i^*}^{-1/(1-\alpha_i)}/E[R_{\lambda_i^*}^{-\alpha_i/(1-\alpha_i)}] \ , \tag{4}$$

where $R_{\lambda_i^*}$ is the optimal portfolio return which solves the optimization problem (2). Moreover, due to the duality relation between problems (1) *and* (2)*, we have:*

$$\frac{1}{\alpha_i(\alpha_i-1)}\log E[M_i^{*\alpha_i}] = -\frac{1}{\alpha_i}\log E[R_{\lambda_i^*}^{\alpha_i/(\alpha_i-1)}]. \tag{5}$$

Proof: See Appendix.

Using different values of parameter α_i, we easily obtain different minimum dispersion SDFs via equation (5), denoted by $M_i^*(\alpha_i)$.

Example 1. The minimum entropy SDF is given by $M_i^*(0) = R_{\lambda_i^*}^{-1}$, with the corresponding optimal return from equation (3). The minimum Hellinger divergence SDF similarly follows as $M_i^*(1/2) = R_{\lambda_i^*}^{-2}/E(R_{\lambda_i^*}^{-1})$. Finally, the minimum variance SDF is given by $M_i^*(2) = R_{\lambda_i^*}/E(R_{\lambda_i^*}^2)$.

From Proposition 1 we obtain following bound for the distribution of any return R_{λ_i}:[5]

$$\frac{1}{\alpha_i(\alpha_i-1)}\log E[M_i^{\alpha_i}] \geq -\frac{1}{\alpha_i}\log E[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)}] \ . \tag{6}$$

While for $\alpha_i = 0$ and $\alpha_i = 2$ we obtain the well-known entropy and variance bounds, we also consider the case $\alpha_i = 1/2$, which yields the Hellinger bound:[6]

$$\log E\left[M_i^{1/2}\right] \leq \frac{\log E[R_{\lambda_i}^{-1}]}{2} \ . \tag{7}$$

[5]When there is no duality gap between the primal and dual solutions, i.e. for the optimal portfolio return $R_{\lambda_i^*}$, we retrieve equation (5).

[6]Kitamura, Otsu, and Evdokimov (2013) emphasize the optimal robustness features of Hellinger-type dispersion measures. We later show that Hellinger bounds naturally induce tight constraints on the first moment of transitory SDF components.

In the following, we explore the link between deviations from the asset market view and minimum dispersion SDFs in incomplete markets, under various forms of departure from market symmetry. We achieve this by adjusting the vector of traded returns $\mathbf{R}_d$ and $\mathbf{R}_f$ to reflect different degrees of international financial market segmentation, from settings of fully segmented markets to economies where domestic investors can freely trade foreign bonds and stocks by resorting to exchange rate markets.

1.2 SDF Components and Exchange Rate Wedges

Alvarez and Jermann (2005), Hansen and Scheinkman (2009), and Hansen (2012) show that SDF processes may be factorized into permanent and transitory components. The permanent component is a martingale which is used to characterize pricing over long investment horizons. The transitory component is related to the return on a discount bond of (asymptotically) long maturity. We factorize international SDFs into martingale (permanent) and transient components[7]

$$M_i = M_i^P M_i^T. \tag{8}$$

Theoretically, M_i^P is identifiable by the martingale normalization $E[M_i^P] = 1$, while the transient component can be written as the inverse of the return of the infinite maturity bond, i.e., $M_i^T := 1/R_{i\infty}$. In this setting, the normalization of the permanent component is ensured by requiring the return on the infinite maturity bond to be priced by SDF $M_i = M_i^P/R_{i\infty}$. Equivalently, $R_{i\infty}$ is defined as one of the components of return vector $\mathbf{R}_i$ in problem (1). Tradability of $R_{i\infty}$ obviously impacts the form of minimum dispersion SDFs and increases the SDF variability.

Since inequality (7) holds for any traded return, we obtain the following tight constraint on

[7]There are two important issues here. First, in general the decomposition in Alvarez and Jermann (2005) is not unique. However, the decomposition is unique under additional stochastic stability assumptions. The complete theoretical framework characterizing existence and uniqueness of the factorization is treated in Hansen and Scheinkman (2009) and Qin and Linetsky (2017), among others. Second, in the data we do not observe an infinite maturity bond return. While in the main text we approximate this return by the return of a long-term bond of finite maturity, in Section 4 we compute non-parametric estimates of permanent components using the sieve approach in Christensen (2017), which is based on the Hansen and Scheinkman (2009) eigenvalue decomposition. We find that both approaches lead to very similar results.

the expected transient SDF component:

$$\log E\left[(M_i^*(1/2))^{1/2}\right] \leq \frac{\log E[R_{i\infty}^{-1}]}{2} \ . \tag{9}$$

Therefore, the Hellinger minimum dispersion SDF directly reveals information about the expected size of transient SDF components, and vice-versa. In the following sections, we apply factorization (8) to quantify in a model-free way the relative importance of international transient and persistent SDF components for explaining salient features of exchange rates.

A useful property of minimum dispersion problem (1) is that it can flexibly incorporate simple forms of deviations from market symmetry, by adjusting the tradable returns $\mathbf{R}_d$ and $\mathbf{R}_f$ from the perspective of domestic and foreign investors. It is well-known that whenever international markets are complete, domestic and foreign SDFs are uniquely defined. As a consequence, all minimum dispersion SDFs are identical. Whenever international financial markets are also symmetric, the exchange rate return is uniquely given by the ratio of the foreign and domestic SDFs from the Euler equation pricing restrictions, i.e., the asset market view as stated in Definition 1 holds. In incomplete or segmented markets, a deviation from the asset market view can arise, wich can be parameterized by a wedge η between exchange rate returns and the ratio of foreign and domestic SDFs (see, e.g., Backus, Foresi, and Telmer (2001)):

$$\frac{M_f}{M_d}\exp(\eta) = X \ . \tag{10}$$

By construction, $\eta = 0$ in integrated and complete markets. Moreover, from expression (8) exchange rate wedges and persistent international SDF components are related via the following factorization:

$$\frac{M_f^P}{M_d^P}\frac{R_{d\infty}}{R_{f\infty}}\exp(\eta) = X. \tag{11}$$

Intuitively, when we extend the set of tradable assets from the perspective of domestic or foreign investors, the set of pricing constraints in problem (1) widens, the dispersion of optimal SDFs increases, and the size of the wedge usually shrinks, as markets tend to exhibit a lesser degree of segmentation. Crucially, we show in the next section that whenever domestic and foreign investors share the same set of assets in symmetric international markets, the wedge always vanishes with respect to the minimum entropy SDFs.

1.3 Minimum Dispersion SDFs, Changes of Numéraire and Asset Market View

Within our international markets environment, it is natural to expect that the properties of various minimum dispersion SDFs can be sensitive to a change of numéraire. Therefore, we now explore the effect of a change of numéraire on international minimum dispersion SDFs. We then link the properties of minimum dispersion SDFs under a change of numéraire to the asset market view of exchange rates.

Given a foreign SDF M_f for return vector $\mathbf{R}_f$, it is always the case that $M_d^e := M_f(1/X)$ is a SDF for the domestic currency-converted return vector $\mathbf{R}_d^e := \mathbf{R}_f X$. Symmetrically, $M_f^e := M_d X$ is a SDF for the foreign currency-converted return vector $\mathbf{R}_f^e := \mathbf{R}_d(1/X)$. Therefore, M_f (M_d) is a foreign (domestic) SDF for return vector $\mathbf{R}_f$ ($\mathbf{R}_d$) if and only if M_d^e (M_f^e) is a SDF for domestic- (foreign-) currency return vector $\mathbf{R}_d^e$ ($\mathbf{R}_f^e$). In other words, the numéraire transformation $\mathcal{N}_d^f : (M_d, \mathbf{R}_d) \longmapsto (M_f^e, \mathbf{R}_f^e)$ defines again a SDF when changing the numéraire from domestic to foreign-currency returns. Similarly, $\mathcal{N}_f^d : (M_f, \mathbf{R}_f) \longmapsto (M_d^e, \mathbf{R}_d^e)$ defines a new SDF under a change of numéraire from foreign to domestic-currency returns.

These numéraire transformations do not preserve in general the minimum dispersion property of a SDF, e.g., if M_d^* is a minimum dispersion SDF for return vector $\mathbf{R}_d$ then in general it does not follow that M_f^{*e} is a minimum dispersion SDF for $\mathbf{R}_f^e$. However, there exist market structures and dispersion measures for which the SDF minimum dispersion property is numéraire invariant. One obvious such situation emerges under complete markets. Indeed, in this case, domestic and foreign SDFs are uniquely defined and identical to the optimal SDFs under any dispersion criterion. Hence, it follows that M_d^{*e} (M_f^{*e}) is a uniquely defined SDF for return vector $\mathbf{R}_d^e$ ($\mathbf{R}_f^e$) and it is therefore also the minimum dispersion SDF.

As the complete market assumption is too restrictive for our analysis, we now address general properties of numéraire invariant minimum dispersion SDFs in incomplete markets.

1.3.1 Minimum Entropy SDFs and Numéraire Invariance

Without any additional assumption, there always exists a single numéraire invariant minimum dispersion SDF, the minimum entropy SDF ($\alpha_i = 0$). From equation (5), this SDF takes the form $M_i^*(0) = R_{\lambda_i^*}^{-1}$, with optimal portfolio weights λ_i^* that uniquely solves for $k = 1, \ldots, K_i$ the first-order conditions of optimization problem (2):

$$E[R_{\lambda_i^*}^{-1}(R_{ik} - R_{i0})] = 0 \; . \tag{12}$$

Therefore, $M_f^{*e}(0) := R_{\lambda_d^*}^{-1} X = (R_{\lambda_f^*}^e)^{-1}$, where $R_{\lambda_f^*}^e = R_{\lambda_d^*}(1/X)$ is a foreign return, solving for $k = 1, \ldots, K_d$ the first-order conditions

$$E[(R_{\lambda_f}^e)^{-1}(R_{fk}^e - R_{f0}^e)] = 0 \; . \tag{13}$$

Similarly, $M_d^{*e}(0) := R_{\lambda_f^*}^{-1}(1/X) = (R_{\lambda_d^*}^e)^{-1}$, where $R_{\lambda_d^*}^e := R_{\lambda_f^*} X$ is a domestic portfolio return, solves for $k = 1, \ldots, K_f$ the first order conditions:

$$E[(R_{\lambda_d}^e)^{-1}(R_{dk}^e - R_{d0}^e)] = 0 \; . \tag{14}$$

From these relations, we obtain the following numéraire invariance property.

Proposition 2. *[Numéraire Invariance of Minimum Entropy SDFs] Let either $i = d$, $j = f$ or $i = f$, $j = d$. $R_{\lambda_i^*}^{-1}$, with optimal weights λ_i^* uniquely solving the moment conditions (12), is a minimum entropy SDF for return vector $\mathbf{R}_i$ if and only if $(R_{\lambda_j^*}^e)^{-1}$, with optimal weights uniquely solving the moment conditions (13) or (14), is a minimum entropy SDF for the exchange rate-converted return vector $\mathbf{R}_j^e$.*

Proof: See Appendix.

Due to the functional form of minimum entropy SDFs, the optimal weights in portfolio returns $R_{\lambda_i^*}$ and $R_{\lambda_j^*}^e$ are identical, i.e., $\lambda_i^* = \lambda_j^*$. In this sense, minimum entropy SDFs M_i^* and M_j^{e*} are consistent with the idea of perfect sharing of the financial risks that are reflected in portfolios of returns $\mathbf{R}_i$ and $\mathbf{R}_j^e$ in domestic and foreign currencies. However, without additional assumptions, this risk sharing is not attainable by portfolios of traded returns, because minimum entropy SDFs are nonlinear transformations of asset returns. Moreover, note that while the numéraire invariance in Proposition 2 has been stated with respect to a change of numéraire associated with the exchange rate return X, it actually holds for any other well-defined change of numéraire, such as, a change of numéraire from real to nominal SDFs and returns.

As under the numéraire invariance in Proposition 2 we have in general $X = M_f^*(0)/M_d^{*e}(0)$, the asset market view of exchange rates always holds for minimum entropy SDFs associated with domestic and foreign return vectors $\mathbf{R}_d$ and $\mathbf{R}_f^e$. Therefore, given a set of domestic traded returns, we can always embed the asset market view in a corresponding international economy using minimum entropy SDFs. This is not true for other minimum dispersion SDFs, as their international counterparts will not entail minimum dispersion, since they are not numéraire invariant.

Corollary 1. *Let international financial markets be symmetric but incomplete and $\alpha_d = \alpha_f =: \alpha$. It then follows:*

(i) The asset market view of exchange rates holds with respect to minimum entropy SDFs ($\alpha = 0$): $X = M_f^*(0)/M_d^*(0)$.

(ii) The asset market view of exchange rates does not hold with respect to minimum dispersion SDFs different from minimum entropy SDFs: $X \neq M_f^*(\alpha)/M_d^*(\alpha)$ for $\alpha \neq 0$.

Minimum entropy SDFs are the only minimum dispersion SDFs consistent with the asset market view in symmetric markets under no additional assumption on the market structure. This implies that $M_i^*(0) = M_i^{*e}(0)$, for $i = d, f$. Moreover, as these SDFs are given by the same transformation of a common linear combination of returns in domestic and foreign currency, they are consistent with the idea of a perfect sharing of these financial risks under integrated international markets.

1.3.2 Minimum Variance SDFs and Numéraire Invariance

The only minimum dispersion SDF that is tradable with portfolios of asset returns is the minimum variance SDF ($\alpha_i = 2$). From equation (5), minimum variance SDFs take the form $M_i^* = R_{\lambda_i^*}/E[R^2_{\lambda_i^*}]$, with portfolio weights λ_i^* that uniquely solve for $k = 1, \ldots, K_i$ the first-order conditions in optimization problem (2):

$$E[R_{\lambda_i^*}(R_{ik} - R_{i0})] = 0 \ . \tag{15}$$

It is an immediate consequence of these first-order conditions that minimum variance SDFs are in general not numéraire invariant. Indeed, if $M_f^*(2) = R_{\lambda_f^*}/E[R^2_{\lambda_f^*}]$ is the foreign minimum variance SDF, then $M_d^{*e}(2) := M_f^*(2)(1/X)$ cannot be written as a linear combination of returns in vector $\mathbf{R}_d^e$. Therefore, it is not in general a minimum variance SDF for $\mathbf{R}_d^e$. Symmetric arguments show that $M_f^{*e}(2) := M_d^*(2)X$ is not in general the minimum variance SDF for $\mathbf{R}_f^e$.

One consequence of the previous findings is that minimum variance SDFs are not generally consistent with the asset market view of exchange rates. This implies that in general, $M_d^*(2) \neq M_d^{*e}(2)$ and $M_f^*(2) \neq M_f^{*e}(2)$. The origins of this violation are further understood by exploiting the numéraire invariance of minimum entropy SDFs in Proposition 2.

Corollary 2. *(i) The real exchange rate can always be decomposed as follows:*

$$X = \frac{M_f^*(2)}{M_d^{e*}(2)} \cdot \frac{1 + [M_f^*(0) - M_f^*(2)]/M_f^*(2)}{1 + [M_d^{*e}(0) - M_d^{*e}(2)]/M_d^{*e}(2)}\,, \tag{16}$$

$$= \frac{M_f^{*e}(2)}{M_d{}^*(2)} \cdot \frac{1 + [M_f^{*e}(0) - M_f^{*e}(2)]/M_f^{*e}(2)}{1 + [M_d^*(0) - M_d^*(2)]/M_d^*(2)}\,. \tag{17}$$

(ii) In symmetric markets, the following exchange rate decomposition holds:

$$X = \frac{M_f^*(2)}{M_d^*(2)} \cdot \frac{1 + [M_f^*(0) - M_f^*(2)]/M_f^*(2)}{1 + [M_d^*(0) - M_d^*(2)]/M_d^*(2)}\,, \tag{18}$$

$$= \frac{R_{\lambda_d^*}(2)}{R_{\lambda_f^*}(2)} \cdot \frac{1 + [R_{\lambda_d^*}(0) - R_{\lambda_d^*}(2)]/R_{\lambda_d^*}(2)}{1 + [R_{\lambda_f^*}(0) - R_{\lambda_f^*}(2)]/R_{\lambda_f^*}(2)}\,, \tag{19}$$

where $R_{\lambda_i^*}(\alpha_i)$ denotes the optimal portfolio return under dispersion parameter α_i in optimization problem (2).

Conveniently, we can compute the various decompositions in Corollary 2 from asset returns alone. Identity (18) clarifies that a deviation from the market view with respect to minimum variance SDFs is determined by the ratio of the relative projection errors of foreign and domestic minimum entropy SDFs on the space of foreign and domestic returns. Thus, a violation of the asset market view for minimum variance SDFs is the result of particular unspanned exchange rate risks, which are reproduced by the component of minimum entropy SDFs that cannot be replicated using basic asset returns. Similarly, the market view holds with respect to the minimum variance SDFs whenever minimum entropy SDFs are tradable in domestic and foreign markets, simply because in this case minimum variance and minimum entropy SDFs are identical.

Recalling that $R_{\lambda_i^*}(2)$ and $R_{\lambda_i^*}(0)$ are the returns of maximum Sharpe ratio and maximum growth portfolios in market $i = d, f$, Corollary 2 directly characterizes exchange rates also in terms of the tradable risk return trade-offs in international financial markets. The exchange rate return is larger when the domestic maximum Sharpe ratio return is higher than the foreign maximum Sharpe ratio return. This effect is produced by the first quotient on the RHS of equation (19) and can be interpreted as a tradable exchange rate effect due to the mean-variance trade-off between domestic and foreign markets. The exchange rate return is also higher when the excess return of the domestic maximal growth return relative to the maximum Sharpe ratio return is larger than the corresponding foreign excess return. This effect is summarized by the second quotient on the RHS of equation (19) and directly quantifies the risk-return trade-offs between domestic and foreign markets due to the higher moments of returns.

1.5 *SDFs Similarity and the Asset Market View*

An important literature in international finance attempts to quantify the amount of SDF co-dependence needed to explain the low volatility puzzle when the asset market view holds. For example, Brandt, Cochrane, and Santa-Clara (2006) introduce an SDF correlation index that is decreasing in the exchange rate variance.[8] In a related vein, Chabi-Yo and Colacito (2017) propose an index based on co-entropies, which incorporates higher-moment dependence and is decreasing in the exchange rate entropy. Here, we aim to quantify the degree of similarity between minimum dispersion SDFs, in presence of a possible deviation from the asset market view, in conceivably incomplete and segmented markets. Therefore, we propose a new class of SDF similarity indices.

1.5.1 *A Minimal SDF Similarity Index*

We introduce a novel SDF similarity index for incomplete markets, which is independent of the validity of the asset market view.

Definition 2. *Our index of SDF similarity is defined as follows:*

$$S(M_d, M_f) := \frac{E[\min(M_d, M_f)]}{\min(E[M_d], E[M_f])} \ . \tag{20}$$

By construction, $S(M_d, M_f) = S(M_f, M_d)$, i.e., (20) is a symmetric similarity index. Moreover, $0 \leq S(M_d, M_f) \leq 1$ and $S(M_d, M_f) = 1$ if and only if $M_d = M_f$ with probability one,

[8] Formally, this property holds for SDF correlations smaller than the minimum between the ratios of the two SDF volatilities.

i.e., international SDFs are identical.[9] In contrast, the two SDFs are more dissimilar when the expected minimum SDF is lower. This can be interpreted as follows. First, as each expected SDF is a bond price, in order to obtain a minimum SDF near to zero in some states, SDFs need to be sufficiently volatile. Second, in order to obtain a minimum SDF near to zero with a sufficiently large probability, SDFs need to be negatively associated. Therefore, settings of small SDF similarities tend to arise in economies with volatile and negatively associated SDFs. Obviously, this is in sharp contrast to the implications of settings of volatile SDFs that satisfy the asset market view, as in this case the only way to obtain a low exchange rate volatility is by means of (strongly) positively associated domestic and foreign SDFs.

While the definition of similarity index (20) is completely independent of the validity of the asset market view, a very useful property is that whenever the asset market view holds, this index is directly computable in a fully conditional way from at-the-money currency option prices alone.

Proposition 3. *If the asset market view of exchange rates holds, then*

$$\frac{E[M_d] - E[M_d \max(0, 1 - X)]}{\min(E[M_d], E[M_f])} = S(M_d, M_f) = \frac{E[M_f] - E[M_f \max(0, 1 - (1/X))]}{\min(E[M_d], E[M_f])} \ , \qquad (21)$$

where $\max(0, 1 - X)$ *(*$\max(0, 1 - (1/X))$*) is the payoff of an at-the-money put option on the spot exchange rate change* X *(*$1/X$*).*

Proof: See Appendix.

Under the conditions of Proposition 3, index (20) can be directly computed from the prices of at-the-money put options on spot exchange rates, without relying on time series information about the underlying set of traded asset returns. This feature offers a simple way of measuring empirically the *conditional* similarity dynamics and the cross-sectional similarity patterns, giving rise to sharp empirical predictions for the dynamic properties of model-based SDF specifications that assume the market view.[10] Whenever SDFs are specified to price real exchange rates and returns, a change of numéraire from real to nominal SDFs makes the model-implied similarity comparable to the similarity computed from nominal exchange rate option prices.[11]

[9] In the trivial case where both M_d and M_f are constants, the similarity index is equal to one.

[10] Appendix A discusses the differences between our SDF similarity index and other proposals in the literature.

[11] Recall from Proposition 3 that this change of numéraire does not alter the optimal return structure under-

1.5.2 Minimal Option-Implied SDF Similarity and Deviations from the Asset Market View

Whenever there is a deviation from the asset market view, the SDF numéraire invariance is not preserved and an asymmetry in the pricing properties of domestic and foreign exchange rate options emerges. In such settings, option prices reveal in general the conditional structure of a conceptually different similarity index, defined by:

$$\overline{S}(M_d, M_f) := \min(S(M_d, M_f^e), S(M_f, M_d^e)) \ , \tag{22}$$

where recall that $M_f^e := M_d X$ and $M_d^e := M_f(1/X)$. Thus, $\overline{S}(M_d, M_f)$ measures the minimal similarity between domestic and foreign SDFs with respect to their numéraire invariant foreign and domestic SDF counterparts, respectively.

By construction, $\overline{S}(M_d, M_f)$ is a symmetric similarity index such that $0 \leq \overline{S}(M_d, M_f) \leq 1$. Moreover, whenever the market view holds, $\overline{S}(M_d, M_f) = S(M_d, M_f)$, i.e., index (22) still measures the similarity of M_d and M_f. However, under a deviation from the asset market view, the index equals one if and only if $M_d = M_d X$ and $M_f = M_f(1/X)$ with probability one, i.e., the exchange rate return is constant and equal to one.

The explicit link between index (22) and the prices of a domestic and a foreign at-the-money put option on the spot exchange rate return is provided next.

Proposition 4. *Whenever the risk-free domestic and foreign returns $R_{f0}X$ and $R_{d0}(1/X)$ are traded, it follows:*

$$\overline{S}(M_d, M_f) = \frac{\min\left(E[M_d] - E[M_d \max(0, 1 - X)], E[M_f] - E[M_f \max(0, 1 - (1/X))]\right)}{\min(E[M_d], E[M_f])} \ , \tag{23}$$

where $\max(0, 1 - X)$ *(*$\max(0, 1 - (1/X))$*) is the payoff of an at-the-money put option on the spot exchange rate change X $(1/X)$.*

Proof: See Appendix.

Proposition 4 can be used empirically for various purposes. For instance, whenever the prices of domestic and foreign exchange rate at-the-money options are both observable, a comparison

lying, e.g., minimum entropy SDFs.

of indices (20) and (22) provides direct evidence on the existence of asset market view deviations, in terms of, e.g., their dynamic properties or their cross-sectional patterns across currencies. One can then compare these deviations to those implied by theoretical models that assume a violation of the asset market view.[12]

In our empirical analysis, we quantify the similarity $S(M_d, M_f)$ of minimum dispersion SDFs under various forms of financial market segmentation, which are consistent with both the three exchange rate puzzles and the observed option-implied similarity $\overline{S}(M_d, M_f)$.

2 Empirical Analysis

Using our model-free minimum-dispersion SDF approach, we can now characterize and quantify key properties of international SDFs under different assumptions about the degree of segmentation between domestic and foreign arbitrage-free financial markets. Full market integration arises for investors having access symmetrically to all assets, both in domestic and foreign markets: the risk-free return, the aggregate equity return, and the long-term bond return. We regard this setting as a natural benchmark, as equity, bond, and exchange rate risk premia are all matched by any minimum dispersion SDF under symmetric trading. In this setting, we are also able to naturally embed a situation where the asset market view exactly holds with respect to minimum entropy SDFs. Therefore, we can also quantify the deviations from the asset market view arising from the multiplicity of SDFs in incomplete markets.

We seek to answer a number of key questions. First, how much dispersion is necessary for SDFs to be able to match unconditional risk premia internationally? Second, in order to successfully address all three exchange rate puzzles, how are transient vs. permanent SDF components and exchange rate wedges connected? Third, how much similarity of minimum dispersion SDFs does a given market structure imply? Fourth, how is the degree of SDF similarity related to the option-implied similarity observed in the data? Studying these questions helps us to identify market structures that can be consistent not only with the well-known exchange rate puzzles, but also with a plausible degree of SDF dispersion and with the data-driven SDF similarity.

[12] In cases where option quotes are available only in the domestic or the foreign currency, Propositions 3 and 4 show that in general, i.e., in presence of deviations from the asset market view, the index computed from one of the equalities in (21) is always an upper bound for the similarity index $\overline{S}(M_d, M_f)$.

2.1 Data

We use monthly data between January 1975 and December 2015 from Datastream. We compute equity returns from the corresponding MSCI country indices' prices and risk-free rates from one-month LIBOR rates. In the main text, we follow Alvarez and Jermann (2005) and proxy transient SDF components by the inverse of the bond return with the longest maturity available, i.e., the ten-year (government) bonds in our case; see also Lustig, Stathopoulos, and Verdelhan (2016). We study in Section 4 a different non-parametric setup to estimate the permanent component of minimum dispersion SDFs, based on the approach in Christensen (2017). We find that the permanent component identified using finite maturity bond returns is almost perfectly correlated with the long-term component estimated non-parametrically.[13]

We study eight benchmark currencies: the US dollar (USD), the British pound (GBP), the Swiss franc (CHF), the Japanese yen (JPY), the euro (EUR) (Deutsche mark (DM) before the introduction of the euro), the Australian dollar (AUD), the Canadian dollar (CAD) and the New Zealand dollar (NZD).[14] The resulting seven exchange rates are expressed with respect to the USD as the domestic currency. Finally, over-the-counter currency options data is obtained from J. P. Morgan. Due to data limitations, our options sample starts in April 1993 and ends in April 2013. For every currency pair, we study one-month maturity at-the-money plain-vanilla European put options, quoted versus the US dollar.

[Insert Table 1 here]

We provide in Table 1 summary statistics for the different time-series. Panel A reports bond market summary statistics. We find that the CHF and the JPY feature low interest rates, in line with the intuition that they act as funding currencies in the carry trade, whereas the remaining ones can be regarded as investment currencies. Cross-sectional differences across countries arise with respect to unconditional long-term bond risk premia. To illustrate, (nominal) long-term risk premia in all countries are negative, but while in Japan and Switzerland they are -0.3% and

[13] In order to study whether the ten-year bond return is a valid proxy for the (unobservable) infinite maturity bond return, a more model-based approach may be also applied, e.g., based on a family of affine term structure models on countries' yields. Lustig, Stathopoulos, and Verdelhan (2016) do not obtain significant differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such a setting.

[14] Throughout the paper, the sample period ranges between January 1988 to December 2015 for New Zealand, due to data availability on the long-term bonds.

−1.02%, respectively, in the remaining countries they range between −2.07% (EU) and −6.06% (Australia). The fact that nominal returns on long-term bonds in local currencies are negative has been documented also in Lustig, Stathopoulos, and Verdelhan (2016). There are cross-sectional differences also with respect to unconditional equity premia, especially in the case of Japan relative to all other countries, which exhibits a substantially lower unconditional equity premium of 3.49% per year. New Zealand features the lowest cross-sectional average equity premia, but this is also a consequence of the restricted sample period. Switzerland displays the lowest market volatility with 15.42%, while the Euro-zone has the largest one (20.08%). These numbers imply a Sharpe ratio of 48% for Switzerland, which is close to the one in the US, and a much lower Sharpe ratio for Japan, 19%.[15] The unconditional average returns on exchange rates against the US dollar also display cross-sectional variation. The highest (positive) average return is obtained for the Swiss Franc (+2.96%), while the lowest (negative) average return follows for the Australian dollar (−0.86%). The cross-section of unconditional exchange rate volatilities does not exhibit significant variation, even though funding currencies, i.e., the Swiss franc and the Japanese yen, feature a higher volatility (12.12% and 11.32%, respectively), whereas the lowest is encountered for the Canadian dollar (6.78%). The last Panel reports inflation statistics for the countries in our sample. The highest average inflation rates are observed in New Zealand (5.57%) and in the UK (4.74%), while the lowest ones are those for Japan (1.57%) and Switzerland (1.76%).[16] In our empirical study, we deflate all domestic returns and exchange rates by the corresponding domestic Consumer Price Index, in order to obtain real returns and exchange rates.

The rich cross-sectional properties of international asset returns posit a challenge on domestic and foreign SDFs, which need to be consistent with observed exchange rate regularities. Using our model-free methodology, we quantify in the following sections the key trade-offs implied by different degrees of financial market segmentation in explaining these salient features.

[15] Using the whole sample period in the case of New Zealand would yield a Sharpe ratio close to the Japanese one.

[16] Note that the sample includes observations associated with The Great Inflation of the 1970s and early 1980s, also known as stagflation, when markets in general exhibited large inflation rates.

2.2 *Unrestricted International Trading*

We start our analysis with the least segmented market setting, in which investors can trade without restrictions domestic and foreign short- and long-term bonds, and stocks. The tradable vector of returns in market $i = d, f$ reads $\mathbf{R}_i = (R_{i0}, R_{i1}, R_{i\infty}, R^e_{i0}, R^e_{i\infty}, R^e_{i1})'$, where $R^e_{d0,t+1} := R_{f0,t+1}X_{t+1}$ ($R^e_{f0,t+1} := R_{d0,t+1}(1/X_{t+1})$) is the domestic (foreign) currency return of the foreign (domestic) risk free asset, $R^e_{d\infty,t+1} = R_{f\infty,t+1}X_{t+1}$ ($R^e_{f\infty,t+1} = R_{d\infty,t+1}(1/X_{t+1})$) is the domestic (foreign) currency return of the foreign (domestic) long-term bond, and $R^e_{d1,t+1} := R_{f1,t+1}X_{t+1}$ ($R^e_{f1,t+1} := R_{d1,t+1}(1/X_{t+1})$) is the domestic (foreign) currency return of the foreign (domestic) aggregate equity return. The estimated optimal portfolio return in market $i = d, f$ is given by:

$$\begin{aligned} R_{\hat{\lambda}^*_i,t+1} &= R_{i0,t+1} + \hat{\lambda}^*_{i1}(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i2}(R_{i\infty,t+1} - R_{i0,t+1}) \\ &\quad + \hat{\lambda}^*_{i3}(R^e_{i0,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i4}(R^e_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i5}(R^e_{i1,t+1} - R_{i0,t+1}) \ . \end{aligned} \tag{24}$$

Based on these returns, the time series of estimated minimum dispersion SDFs is obtained in closed-form from equation (5):

$$\hat{M}^*_{i,t+1} = \frac{R^{-1/(1-\alpha_i)}_{\hat{\lambda}^*_i,t+1}}{\hat{E}_i\left[R^{-\alpha_i/(1-\alpha_i)}_{\hat{\lambda}^*_i,t+1}\right]} \ , \tag{25}$$

where estimated portfolio weights in equation (24) are the unique solution of the exactly identified set of empirical moment conditions:

$$\hat{E}_i\left[R^{-1/(1-\alpha_i)}_{\hat{\lambda}^*_i}(R_{ik} - R_{i0})\right] = 0 \ , \tag{26}$$

with $k = 1, ..., K_i$ and $K_d = K_f = 5$ in this case. We estimate parameter vector $\hat{\lambda}^*_i$ in (26) using the exactly identified (generalized) method of moments. Note that as this setting implies symmetrically traded international returns, the market view of exchange rates holds by construction for the minimum entropy SDFs.[17]

[17] There are two additional market structures in which we can construct symmetrically traded international returns, namely when investors can trade the domestic and foreign risk-free bonds only, or when they can invest in both risk-free and long-term bonds domestically and abroad. As these frameworks are rather restrictive, the results are not reported here and are available upon request.

2.2.1 Minimum dispersion SDFs

Since the domestic and foreign risk-free rate, bond return, and equity return are all priced by domestic and foreign minimum dispersion SDFs, the risk premia of these returns are all matched by construction. In particular, the currency risk premia are also exactly matched and the forward premium anomaly is implicitly incorporated by the pricing properties of minimum dispersion SDFs. We report the summary statistics of minimum dispersion SDFs under unrestricted trading in Table 2 for each bilateral pair vis-à-vis the US.

As expected, since the risk free return R_{i0} is priced by the minimum dispersion SDF M_i^*, average minimum dispersion SDFs are virtually the same across different dispersion measures. The sample volatilities, on the other hand, display more dispersion across different values of α_i and are the lowest for $\alpha_i = 2$, by construction. The US NZ pair features the lowest volatilities (0.6 and 0.53, respectively), while the US CH pair has the highest volatility (0.87 and 0.83). One important first finding is that each of these volatilities exceeds by a large amount the equity Sharpe ratios in Table 1, which indicates a clearly tightened Hansen-Jagannathan bound under an unrestricted international trading.

[Insert Table 2 here]

To understand in more detail the properties driving the SDF dispersion, we decompose the SDFs into their transitory and permanent components. There are two interesting observations. First, the largest part of the SDF dispersion is generated by the permanent component, regardless of the country or dispersion measure considered. This is in line with the US evidence in Alvarez and Jermann (2005). Second, the correlation between all US and foreign SDFs are virtually perfect, which is a first broad indication of a very high SDF similarity under market symmetry. Indeed, while this high co-movement is fully expected for the mininum entropy SDFs, due to the low exchange rate volatility and their consistency with the asset market view in this setting, our findings show that it is a general feature of market symmetry, as it emerges also for all other minimum dispersion SDFs. Indeed, the lowest correlation between minimum dispersion SDFs, which is as large as 97.9%, is obtained for the US and NZ pair using the minimum variance SDFs.

2.2.2 *Exchange Rate Volatilities and Wedges*

The large SDF co-movement under fully integrated markets is related to the low exchange rate volatility puzzle in Brandt, Cochrane, and Santa-Clara (2006), who show that when the asset market view holds international SDFs need to be almost perfectly correlated to match the low exchange rate volatility. Our results show that such high correlations arise more broady in symmetric market settings also under a violation of the market view.

In our incomplete market setting, the large SDF correlation is related to the properties of the permanent SDF components. This seemingly perfect co-movement can be understood by rearranging terms in equation (11), to get the identity:

$$X_{t+1}\frac{R_{f\infty,t+1}}{R_{d\infty,t+1}} = \frac{M^P_{f,t+1}}{M^P_{d,t+1}}e^{\eta_{t+1}} \ . \tag{27}$$

As the variability of the LHS of equation (27) in the data is rather low, identity (27) can hold either under a low variability of both the ratio of permanent SDF components and the wedge, under a strong negative co-movement between the ratio of permanent SDF components and the wedge, or under a combination of these effects. A direct implication of these properties is that under the market view (i.e., $\eta_{t+1} = 0$), permanent components need to be almost perfectly positively related. In contrast, in a setting where the market view is not satisfied, a trade-off between the co-movement of the permanent SDF components and the long-run cyclicality of exchange rate wedges can emerge.

It follows that the high SDF dispersions in Table 2 can be empirically consistent with identity (27) only in presence of a sufficiently large wedge dispersion or when permanent SDF components are strongly positively correlated. To study this trade-off in detail, we compute wedge summary statistics from identity (27):

$$X_{t+1}\exp(-\eta_{t+1}) = \frac{R^{1/(1-\alpha_d)}_{\hat{\lambda}^*_d,t+1} R^{-1/(1-\alpha_f)}_{\hat{\lambda}^*_f,t+1}}{\hat{E}_d\left[R^{-\alpha_d/(1-\alpha_d)}_{\hat{\lambda}^*_d,t+1}\right]^{-1}\hat{E}_f\left[R^{-\alpha_f/(1-\alpha_f)}_{\hat{\lambda}^*_f,t+1}\right]} \ . \tag{28}$$

where the optimal returns are given in equation (24). Table 3 reports the wedge properties for minimum variance and minimum Hellinger SDFs alone, because the wedge resulting from minimum entropy SDFs vanishes by construction (see Corollary 1).

[Insert Table 3 here]

Consistent with the above intuition, we obtain a small wedge dispersion in all cases, which is an order of magnitude smaller than the domestic and foreign SDF dispersions. The wedge dispersions implied by minimum Hellinger divergence SDFs are also very small, which is a consequence of the fact that these SDFs are those most related to the minimum entropy SDFs. Finally, while the wedge dispersions implied by minimum variance SDFs are larger and in a few cases comparable to the volatility of exchange rates, they still are an order of magnitude smaller than the dispersions of miminum variance SDFs.[18]

Since the wedge dispersion contributes only mildly to the SDF dispersions, we focus next on the co-movement between martingale components. Table 4 reports the average correlations across different exchange rate pairs, showing that there is almost perfect co-movement across all currency pairs. Therefore, we conclude that in fully integrated markets, minimum dispersion SDFs are very similar and highly disperse, mainly due to their very similar and highly disperse permanent components.

[Insert Table 4 here]

2.2.3 Backus-Smith (1993) Puzzle

As the setting with fully integrated markets can incorporate both the forward premium anomaly and the low exchange rate volatility, we focus in this section on the exchange rate cyclicality by means of Backus and Smith (1993)-type regressions:

$$\begin{aligned} m_{f,t+1} - m_{d,t+1} &= \delta + \beta x_{t+1} + u_{t+1}, \\ m^U_{f,t+1} - m^U_{d,t+1} &= \delta^U + \beta^U x_{t+1} + u^U_{t+1}, \end{aligned}$$

for $U = T, P$, where we regress both the log difference between foreign and domestic SDFs and their transient and permanent components on the log exchange rate return x_{t+1}.

[18] Lustig and Verdelhan (2016) show that in order to match the low exchange rate volatility puzzle using equation (10), the wedge needs to covary positively (negatively) with the domestic (foreign) SDF, i.e., it needs to be pro-cyclical. Given the very low wedge dispersion, the cyclicality properties are not particularly insightful in the fully integrated market setting and we do not report them here to save space.

When the asset market view holds, the population point estimate from these regressions based on the overall SDFs is exactly one. This case emerges for the minimum entropy SDFs under fully integrated markets. More generally, when risk-free returns are traded internationally, Lustig and Verdelhan (2016) show that the same finding holds also under a deviation from the market view. Therefore, we expect similar results also for minimum variance and minimum Hellinger divergence SDFs. Finally, since the SDF variability in Table 2 is dominated by the permanent component, we anticipate analogous implications for regressions using the persistent SDF components.

[Insert Table 5 here]

Table 5 reports Backus-Smith (1993) estimated coefficients for the various country pairs. For all dispersion measures, the regressions with $m_{f,t+1} - m_{d,t+1}$ and $m^P_{f,t+1} - m^P_{d,t+1}$ produce estimated coefficients that are positive, highly significant, and close to one, with estimates based on the permanent component that are almost indistinguishable from the total SDF estimates. Turning to the regressions with transitory SDF components, we obtain estimated coefficients that are statistically not different from zero. Hence, we conclude that also the cyclicality puzzle can be explained in a setting of fully integrated markets, by a transitory component that is largely unrelated to exchange rate changes.

In summary, the fully integrated, but incomplete, market setting is consistent with the three exchange rate puzzles and with small or inexistent deviations from the asset market view. In this framework, martingale SDF components across countries are highly volatile and almost perfectly correlated, which is an indication of a large SDF similarity, while differences in transient SDF components are disconnected from exchange rate variations. The large SDF dispersion of fully integrated market settings is potentially a challenge for existing asset pricing models assuming the validity of the asset market view. We explore further the trade-offs between SDF dispersion, SDF similarity and deviations from the asset market view, by studying model-free minimum dispersion SDFs incorporating some form of market segmentation. A natural benchmark in this respect is a setting in which investors are allowed to trade internationally only short-term bonds.

2.3 Domestic Investors Trade Foreign Risk-Free Bonds

To lower the SDF dispersion, we now allow investors to trade internationally only the risk-free bonds. Hence, the vector of tradable real gross returns in the domestic (US) market reads $\mathbf{R}_d = (R_{d0}, R_{d1}, R_{d\infty}, R^e_{d0})'$, where $R^e_{d0,t+1} := R_{f0,t+1}X_{t+1}$ is the domestic currency return of the foreign risk free asset. Similarly, the vector of tradable real gross returns in the foreign market reads $\mathbf{R}_f = (R_{f0}, R_{f1}, R_{f\infty}, R^e_{f0})'$, where $R^e_{f0,t+1} := R_{d0,t+1}(1/X_{t+1})$ is the foreign currency return of the domestic risk free asset. Besides matching the risk premia on the domestic returns, minimum dispersion SDFs are still forced to match the risk premia on returns R^e_{d0} and R^e_{f0}. Therefore, they exactly match the exchange rate risk premium in the data, implicitly incorporating the forward premium anomaly.

Compared to the full integration case, the estimation of minimum dispersion SDFs in this economy is based on a reduced number of moment conditions, as the equity and long-term bonds are not traded internationally anymore, i.e., $K_d = K_f = 3$ in the set of moment conditions (26). Hence, the estimated optimal portfolio return in market $i = d, f$ reads:

$$R_{\hat{\lambda}^*_i,t+1} = R_{i0,t+1} + \hat{\lambda}^*_{i1}(R_{i1,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i2}(R_{i\infty,t+1} - R_{i0,t+1}) + \hat{\lambda}^*_{i3}(R^e_{i0,t+1} - R_{i0,t+1}) \ .$$

The closed-form expression for the estimated minimum dispersion SDF follows as in the full integration case by plugging this optimal return into equation (25).

2.3.1 Minimum Dispersion SDFs

Table 6 documents how the smaller set of tradable assets affects the properties of minimum dispersion SDFs. Due to the reduced set of pricing restriction on returns, the variability of minimum dispersion SDF decreases considerably, relative to the full integration case, across all currency pairs. For instance, the SDF variability in Switzerland drops by 40%, in Japan by 50%, and equivalently for New Zealand.

[Insert Table 6 here]

Similar to the full integration case, permanent SDF components are in all cases very volatile and positively correlated with the long-term bond return. Because of the lower dispersion of the permanent SDF components, these correlations are larger in absolute value, in order to match the negative long-term bond risk premia in local currencies.[19] More importantly, the correlation between US and foreign SDFs is on average much lower than in the fully integrated market setting and much more volatile across currency pairs: the largest correlation is only 65% for the USD/EUR pair ($\alpha_d = \alpha_f = 0$), while the smallest one is as low as 12% ($\alpha_d = \alpha_f = 0$) for the USD/GBP pair. This evidence reflects more dissimilar domestic and foreign SDFs linked to economically relevant asset market view deviations across currencies.

2.3.2 *Exchange Rate Volatility and Wedges*

Intuitively, the presence of more pronounced asset market view deviations in the setting with internationally traded risk-free bonds alone suggests a larger exchange rate wedge variability. Table 7 reports summary wedge statistics that support this intuition. Overall, we obtain wedges that are similarly volatile as minimum dispersion SDFs, with non-trivial higher moments that reflect the non-normality of exchange rate returns in a way that depends on the choice of minimum dispersion SDFs in incomplete markets.[20]

[Insert Tables 7 and 8 here]

Because of these deviations from the asset market view, the wedge now displays a non-trivial cyclicality with respect to the minimum dispersion SDFs. This is depicted in Table 8, where we observe a pro-cyclicality that is explained by a pronounced positive (negative) wedge correlation with the permanent components of domestic (foreign) SDFs. The co-movement with the transient component is instead typically weaker and of opposite sign.

[19] The correlation between the transient and permanent SDF components for an SDF M can be expressed as:

$$\mathrm{Corr}(M^T, M^P) = \frac{E[M] - E[M^T]}{\sqrt{\mathrm{Var}(M^T)}\sqrt{\mathrm{Var}(M^P)}} \ . \tag{29}$$

When the distribution of the transient SDF component is fixed by an observable proxy, the numerator is also fixed and the correlation increases in absolute value whenever the volatility of the permanent SDF component decreases.

[20] Especially for UK, CH and AU markets, we obtain wedges with fatter tails and opposite signs for skewness under parameter choices $\alpha_i = 2$ and $\alpha_i = 0, 0.5$, respectively. This last feature is a consequence of the fact that the minimum variance SDF does not capture the higher moment features linked to extreme exchange rate movements as strongly as the other two minimum dispersion SDFs.

In summary, the economy with internationally traded risk-free bonds alone can incorporate both the forward premium anomaly and the low exchange rate volatility, by means of a volatile exchange rate wedge and rather weakly correlated domestic and foreign SDFs.

2.3.3 Backus-Smith (1993) Puzzle

In line with the evidence in the last section, SDFs incorporating martingale components can support the low co-movement of cross-sectional differences in consumption growth and exchange rate returns, i.e., the exchange rate cyclicality properties. Table 9 quantifies these relations by reporting the point estimates of Backus-Smith (1993)-type regressions of log differences in minimum dispersion SDFs and martingale SDF components on real log exchange rate returns.[21]

Recalling the findings in Lustig and Verdelhan (2016), the population point estimate from these regressions for the overall SDFs is exactly one also under a deviation from the market view, whenever risk-free returns are traded internationally. Given the large fraction of permanent SDF variability in Table 6, similar implications have to hold also for regressions using the persistent SDF components in a setting with internationally traded risk-free bonds alone.

Table 9 shows that indeed all point estimates in the Backus-Smith (1993)-type regressions are significantly different from 0 and never significantly different from the target value of one.

[Insert Table 9 here]

In summary, the market setting with internationally traded risk-free bonds alone is consistent with the three exchange rate puzzles and with large deviations from the asset market view. In this setup, martingale SDF components across countries are volatile and only weakly correlated, while differences in transient SDF components are disconnected from exchange rate variations.

2.4 International Long-term Bond and Equity Risk Premia

The clearly distinct properties of minimum dispersion SDFs in Sections 2.2 and 2.3 can be understood economically in terms of the different sets of assets assumed to be traded interna-

[21] We do not report the regression of log differences in transitory SDF components since estimates remain the same, regardless of the degree of market segmentation and the dispersion measure used.

tionally under each setting. While both frameworks lead to potential "resolutions" of the three exchange rate puzzles, in a way that is consistent with a heterogenous market setting in which different investors may have a different access to international financial markets, their different degree of symmetry yields distinc interpretations for international asset prices.

Figure 1 reports the cross-sections of (real) international long-term bond and equity risk premia in the data, together with the risk premia implied by minimum entropy SDFs for the economy with internationally traded risk-free bonds alone. This economy is not constrained to price the cross-sections of international long term bond and equity returns.

[Insert Figure 1 and 2 here]

In our sample, international bond risk premia in USD are monotonically decreasing in the average interest rate differential, except for the New Zealand dollar. While the bond risk premia implied by USD minimum entropy SDFs are similarly monotonic, they also systematically overstate the actual risk premia in the data, especially for investment currencies. Analogously, the foreign currency minimum entropy risk premia of US bonds systematically overstate the risk premia in the data, especially for funding currencies. Interestingly, the minimum entropy bond risk premia in Figure 1 follow to a good extent the pattern of minimum entropy currency risk premia reported in Figure 2, which by construction exactly match the currency risk premia in the data. However, the actual long-term bond risk premia in Panel A of Figure 1 decrease faster as the interest rate differential increases. Finally, the economy with internationally traded risk-free bonds alone implies international average equity premia that are underestimated by minimum entropy risk premia, with large biases arising especially for CH, JP, UK and US international equity premia.

In summary, these differences in international asset risk premia under symmetric and partially segmented markets are related to the very different dispersion and co-movement properties of minimum dispersion SDFs in the presence of deviations from the asset market view.

2.5 SDF Similarity and Option-Implied Similarity

The different co-movement properties of international minimum dispersion SDFs under segmented and integrated markets point to different SDF similarity properties in these two settings. Therefore, quantifying the SDF similarity properties of integrated vs. segmented market settings can help to clarify empirically the hidden implications of structural models with different market segmentation structures. To this end, we first implement equation (23) using currency options.

2.5.1 Option-Implied Similarity

Figure 3 plots the time-series of option-implied SDF similarity indices $\overline{S}(M_d, M_f)$, using in all cases the USD as the domestic currency, while Table 10 reports similarity summary statistics.[22]

[Insert Figure 3 here]

We find that option-implied similarity indices across currencies feature a large degree of comovement and are on average near to their upper bound of one. Moreover, they exhibit interesting time-varying patterns with a tendency to simultaneously decrease in periods of global financial market turmoil. The decrease in similarity during periods of crisis is a natural consequence of the progressively more expensive at-the-money put exchange rate options; see again Proposition 3. These options offer protection against a depreciation of the foreign currency relative to the USD. Consistent with the evidence in Figure 3, the average option-implied SDF similarity across countries in Table 10 (Panel A) is about 0.988 and the cross-sectional standard deviation is very low.

2.5.2 Minimum Dispersion SDF Similarity

It is useful to compare the degree of option-implied SDF similarity to the similarity induced by model-free minimum entropy SDFs. To this end, recall that option-implied similarity index $\overline{S}(M_d, M_f)$ is comparable to SDF-based similarity index $S(M_d, M_f)$ only when the asset market view holds. In order to obtain SDF-based similarity indices corresponding to the nominal option-implied indices, we first convert real minimum dispersion SDFs to nominal SDFs,

[22] To take unconditional expectations as in equation (23), we can either average the values of $\overline{S}(M_d, M_f)$ directly or we can take averages for each component in the expression. Both yield very similar numbers.

by deflating real SDFs by the corresponding inflation rate.[23] We report the resulting SDF-based indices $\overline{S}(M_d, M_f)$ and $S(M_d, M_f)$ in Table 10 (Panel B) both for the integrated and the segmented market settings of the previous sections.

[Insert Table 10 here]

We find that across all currencies, SDF-based indices $\overline{S}(M_d, M_f)$ match very closely the corresponding option-implied similarities in the data, independent of the assumed market segmentation settings. This evidence suggests that all minimum dispersion SDFs fit unconditionally quite accurately the put option prices underlying the expression for index $\overline{S}(M_d, M_f)$ in Proposition 4.

In the market setting with unrestricted trading, the SDF similarity index $S(M_d, M_f)$ closely follows index $\overline{S}(M_d, M_f)$, because of the documented small deviations from the asset market view under symmetric markets. In contrast, the SDF similarity index $S(M_d, M_f)$ under internationally traded risk-free bonds alone is clearly lower, of about 0.941 on average across countries and dispersion measures. In summary, the unconditional option-implied similarity is reproduced quite well both by the integrated and the segmented market setting, which however imply very different SDF-similarities.

Our minimum dispersion SDFs also allow us to compute indices of international SDF co-dependence, without imposing the validity of the asset market view. We focus for brevity on the co-entropy-based index of Chabi-Yo and Colacito (2017):

$$\rho_{M_f,M_d} = 1 - \frac{L[M_f/M_d]}{L[M_f] + L[M_d]}, \tag{30}$$

where $L[x] \equiv \log(E[x]) - E[\log(x)]$ defines the entropy of a positive random variable x. Summary statistics for various minimum dispersion SDFs are reported in Table 11.

Consistent with the previous results, SDF co-entropies are close to being perfect whenever international financial markets are symmetric, i.e., when the asset market view holds with respect to minimum entropy SDFs and when deviations from it are small under the remaining minimum dispersion SDFs. In these settings, co-entropies across countries are never less than

[23]In the data, we find that SDF similarities computed from real and nominal SDFs are virtually identical.

95%. A different picture emerges in the segmented markets case. Co-entropies are particularly low for the funding currencies, with values below 33% and 20% for Switzerland and Japan, respectively.

[Insert Table 11 here]

In summary, various settings of integrated and segmented markets give rise to minimum dispersion SDFs compatible with the three exchange rate puzzles. However, these SDFs have very different implications for the amount of deviation from the asset market view and for the maximal Sharpe ratios attainable by domestic and foreign investors when optimally trading the minimum variance SDFs. In partially segmented markets, the large Sharpe ratios obtained by trading minimum variance SDFs that price international assets symmetrically may be interpreted as the optimal risk return trade-offs attainable only by a subset of particularly sophisticated financial intermediaries, who have privileged symmetric access to international financial markets. We discuss such an interpretation in more detail in the next section.

3 Financial Intermediaries' SDFs

With our model-free SDFs at hand, one natural question is: Whose SDF are we measuring? As a first illustration, we plot in Figure 4 scatter plots between our minimum entropy SDFs (in logs) and log consumption growth for each of the eight countries. Not very surprisingly, we find the correlation between the two series to be close to zero or mildly negative.

A growing empirical research documents the importance of financial intermediaries for asset prices (see, e.g., Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017), among others).[24] The role of these intermediaries seems particularly important in markets that feature complex financial assets, such as credit default swaps, sovereign bonds, and FX (see, e.g., Haddad and Muir (2017)). In the following, we link our model-free SDFs to various measures of financial intermediaries' wealth and constraints. To motivate this analysis, we first plot, in

[24]These papers study the intermediary Euler equation by means of empirical proxies of intermediaries' marginal utility and find that these proxies are able to explain the cross-section of different expected returns.

the upper panel of Figure 5, the twelve-month moving averages of our USD minimum-entropy SDFs across the various exchange rate parities in our sample.[25]

All USD SDFs across exchange rate parities are positively correlated and tend to spike together during crisis periods corresponding to major stock market events, such as the Black Monday in October 1987 or the Lehman Brother default in August 2008 (top panel). In the bottom panel, we plot the average US SDF together with the intermediary capital ratio from He, Kelly, and Manela (2017). This ratio is defined as the aggregate value of market equity divided by the sum of the aggregate market equity and the aggregate book debt of primary dealers who serve as counterparties of the Federal Reserve Bank of New York. The two series in the figure highlight a negative comovement between the levels of the capital ratio and the USD SDFs. We analyze the relationship between minimum dispersion SDFs and intermediaries capital ratios in more detail below.

[Insert Figures 4 and 5 here.]

In the following, we study a simple model of financial intermediation in segmented markets. We borrow from Gabaix and Maggiori (2015), where domestic and foreign households can only trade their own local short-term bond and do not have direct access to sovereign bonds. The role of financiers is to intermediate the market for sovereign bonds and thereby provide access to the FX carry trade.[26]

The economy consists of two countries, each populated by a unit mass of households and having its own currency. Households in each country can trade a one-period risk-free bond that is denominated in their respective local currency.[27] Denote by R_{dt} and R_{ft} the risk-free interest rates in the United States and the foreign country at time t. Without loss of generality, assume that R_{dt} is smaller than R_{ft}. The exchange rate S_t is defined as the quantity of dollars that

[25]For brevity, we only plot USD SDFs for the symmetric market setting. The corresponding foreign SDFs look very similar, as documented above.

[26]In line with our empirical analysis, one could also assume that financial intermediaries trade not just short- but also long-term bonds as well as equities. None of our results would change if we increase the set of tradable assets. To keep the notation simple, we follow Gabaix and Maggiori (2015) and allow for trade in short-term bonds only.

[27]Segmented money and bond markets are also the subject in Alvarez, Atkeson, and Kehoe (2002), where segmentation arises endogenously due to some fixed cost of trading. The assumption that some markets are segmented and can only be accessed by households via an intermediary is akin to He and Krishnamurthy (2013).

can be bought at time t by one unit of the foreign currency. Therefore, the gross exchange rate return in USD between period t and $t+1$ is $X_{t+1} = S_{t+1}/S_t$.

Households in each country have a downward-sloping demand for assets denominated in the other country's currency. Such demand may arise due to various reasons, such as trade or portfolio flows. In addition to households, the economy is populated by a unit mass of identical risk-neutral financiers, who can trade in the domestic bonds of *both* countries. Hence, while markets are segmented for households, they are symmetric for the financier. As such, the financiers act as intermediaries between investors in the two countries by taking the other side of their currency demands, at a profit. The representative financier enters the market with no capital of her own and takes at time t a long position of $-Q_t/S_t$ units of the foreign currency, funded by a short position of Q_t units of dollars. She unwinds this position at the end of period $t+1$. Consequently, her USD profit at time $t+1$ is

$$V_{t+1} = (R_{ft}X_{t+1} - R_{dt})\, Q_t. \tag{31}$$

A direct implication of equation (31) is that whenever $E_t[R_{ft}X_{t+1} - R_{dt}] > 0$, i.e., the Uncovered Interest rate Parity condition is violated, the representative financier wants to take infinitely large positions Q_t, unless some friction limits her ability to do so.

Intermediation frictions are modelled by assuming that the representative financier is subject to a Value-at-Risk constraint, in which the likelihood of making a negative profit cannot exceed some small threshold $0 < c_t << 1$.[28] The representative financier thus faces following optimization problem at time t:

$$\begin{aligned} \max_{Q_t} \quad & E_t[V_{t+1}] \\ \text{s.t.} \quad & \mathbb{P}_t(V_{t+1} \leq -\epsilon_t) \leq c_t, \end{aligned} \tag{32}$$

where ϵ_t is the Value-at-Risk of next period financier's wealth for confidence level c_t.

The Value-at-Risk constraint effectively limits the "risk-bearing capacity" of the financiers. When c_t is close to 1, the representative financier is essentially unconstrained and can take arbitrarily large currency positions. When c_t is small, the Value-at-Risk constraint binds and

[28] Adrian and Shin (2014) show that Value-at-Risk constraints similar to the one in this model can emerge as a result of a standard contracting framework with risk-shifting moral hazard.

the financier is restricted in her risk bearing capacity. Such a constraint is tighter in states of higher conditional risk, e.g., due to an increase in the anticipated volatility of exchange rate returns. In this sense, the Value-at-Risk constraint induces a downward-sloping demand curve for risk-taking by the financiers.[29]

In this setting, the financial intermediary SDF is a linear function of the intermediary's wealth.[30] This relationship motivates us to explain our model-free SDFs using proxies for the tightness of intermediaries' Value-at-Risk constraints and intermediaries' capital. For the former we use as proxy the VIX, a model-free implied volatility index of S&P500 options, whereas as a proxy of intermediaries' capital, we use the intermediary capital ratio of He, Kelly, and Manela (2017).[31]

Table 12 reports point estimates and $t-$statistics of linear regressions of our model-free SDFs under symmetric markets on changes in intermediaries' Value-at-Risk constraint and capital proxies. The upper panel collects the results for the US SDFs, while the lower panel reports results for the foreign SDFs. Consistent with the theoretical predictions, we find that all point estimates for the Value-at-Risk constraint proxy are positive and that those for the capital ratio are negative. The Value-at-Risk constraint is significant for all regressions except for the foreign New Zealand SDF. The capital ratio proxy is also significant in six SDF settings. With the exception of the USD-NZD currency pair, the explanatory power of these regressions is also quite large, with regression R^2s ranging from about 10% for USDJPY to about 45% for USDGBP.

[Insert Table 12 here]

Overall, this evidence is consistent with the intuition that the large dispersions of some

[29] Gabaix and Maggiori (2015) consider an alternative specification with a different constraint, in which the financiers are subject to a limited commitment friction that intensifies with the complexity of their balance sheets. Since both the Value-at-Risk constraint and the limited commitment constraint of Gabaix and Maggiori induce a downward-sloping demand for risk-taking by the financiers, they have similar implications for exchange rates and currency excess returns.

[30] Gabaix and Maggiori (2015) show that the financial intermediary SDF takes the form:

$$M_{t+1} = 1 - \Phi(V_{t+1}), \tag{33}$$

where Φ is a linear operator.

[31] The VIX has been used as a proxy of global intermediaries' leverage constraints in the work of Miranda-Agrippino and Rey (2015) or Bruno and Shin (2015), among many others.

model-free SDFs in symmetrically integrated bond and stock markets may be interpreted as a higher risk compensation available only to a subset of specialized intermediaries, which are subject to various forms of limits-to-arbitrage, captured by proxies of Value-at-Risk constraints and financial intermediaries' wealth.

4 Robustness of Long-Run SDF Factorizations

In our empirical analysis, we factorize minimum dispersion SDFs into transitory and permanent components, using as a proxy for the unobservable return of infinite maturity bonds the long, but finite-horizon bond return. While this is of course an approximation, note that this SDF decomposition can impact our main analysis only in terms of the exchange rate cyclicality patterns and the corresponding Backus and Smith (1993) regression results. Given the dominating role of martingale components in minimum dispersion SDFs, it is unlikely that this approximation error can substantially change our findings. Indeed, model-based evidence from estimated affine term structure models tends to support this intuition.[32] To address these issues, we follow Christensen (2017) and estimate non-parametrically the solution of the Perron-Frobenius eigenfunction problem that uniquely characterizes the long-run SDF factorization in Hansen and Scheinkman (2009). We defer exhaustive estimation details to the Online Appendix.

We report in Figure 6 the time-series of the estimated permanent SDF components identified with the approach in Section 2.2, together with those estimated with the non-parametric methodology.[33]

[Insert Figure 6 here]

The two series exhibit similar time series properties, with large common spikes during crises and recession periods, and are almost perfectly correlated.[34] While the unconditional dispersion of the two permanent components is similar and generates in both cases the largest fraction of the

[32]See again Lustig, Stathopoulos, and Verdelhan (2016), who do not obtain large differences between the yields of a hypothetical infinite maturity bond and a ten-year bond in such settings.

[33]For the sake of brevity, we only report results for two different currency pairs. The results look virtually the same for all other currency pairs.

[34]Correlations between permanent components under the two approaches are 0.96, 0.98, 0.98, 0.98, 0.98, 0.95 and 0.95 for the USDGBP, USDCHF, USDJPY, USDEUR, USDAUD, USDCAD and USDNZD pairs, respectively.

overall SDF volatility, the non-parametric component has a consistently slightly lower volatility than the overall SDF volatility. This implies a positive risk premium for the infinite maturity bond return resulting from the non-parametric SDF factorization.[35] Given the dominating role of permanent SDF components under both approaches, these findings imply overall unchanged exchange rate cyclicality patterns and Backus and Smith (1993) regression results, using either of the two proxies for the permanent SDF components.[36]

5 Conclusion

In this paper, we estimate model-free minimum dispersion SDFs to understand the asset pricing implications of different degrees of market segmentation in international financial markets. Since markets are incomplete and there exists many different SDFs, we explore various SDFs that minimize different measures of SDF dispersion: variance, entropy and Hellinger divergence. At the same time, we allow for a factorization of international SDFs into permanent (martingale) and transient components and for the potential presence of a stochastic wedge between exchange rates and the ratio of foreign and domestic SDFs.

If markets are complete and integrated, the asset market view of exchange rate holds and the change in real exchange rates is equal to the ratio of foreign and domestic SDFs. In incomplete or segmented markets, there can be deviations from the asset market view which are captured by a wedge. Theoretically, we show that minimum entropy SDFs always imply the validity of the market view of exchange rates in symmetric international markets, irrespective of the degree of market incompleteness. Using characterization, we then show that stochastic exchange rate wedges capture the amount of untraded risk in these economies. Finally, we propose a novel approach to measure the similarity of international SDFs, which extends known similarity indices to incomplete and possibly segmented markets.

Using a cross-section of developed countries, we document the following novel empirical findings. In order to jointly explain the exchange rate puzzles, the international SDFs have to exhibit large permanent components: While we find that permanent SDF components induce

[35] This finding for the sign of the risk premium of infinite maturity bond returns is consistent with the empirical evidence in Bakshi and Chabi-Yo (2012). The volatilities of the non-parametric permanent SDF components are reported in Table OA-1 of the Online Appendix.

[36] Detailed results are available from the authors upon request.

in all cases the largest fraction of SDF dispersion, we document that they are also necessary to imply an exchange rate cyclicality that is compatible with the Backus and Smith (1993) puzzle, i.e., the small or negative correlation between international consumption growth differentials and exchange rate changes. We then show that international minimum dispersion SDFs jointly explain the three exchange rate puzzles whenever domestic and foreign risk-free bonds can be traded internationally.

To quantify the implications of different degrees of market segmentation, we explore two benchmark cases. First, we consider the SDFs of sophisticated financial agents that can access all assets in the economy: short- and long-term bonds, as well as equity indices. In the second case, we study the SDFs of less sophisticated investors, who can trade internationally only the short-term bond. We document that while both these incomplete market settings are able to address the exchange rate puzzles, in the former the SDF dispersions and similarities are substantially larger. In contrast, a reasonable degree of market segmentation produces significantly lower SDFs dispersions and similarities.

We finally motivate the emergence of SDFs with structurally different properties under segmented markets, using a simple model with heterogenous trading technologies, in which some specialized financiers intermediate households' demand for international assets. In the model, financial intermediaries have symmetric access to international financial markets and they are subject to limits to arbitrage generated by Value-at-Risk constraints. Therefore, their SDFs directly depend on the financial intermediaries wealth. Empirically, we find strong links between model-free international SDFs and proxies of Value-at-Risk constraints or financial intermediaries capital, which can explain up to 50% of the SDF time-series variation. This evidence is compatible with the intuition that the large dispersions of some SDFs in symmetrically integrated markets may reflect a higher risk compensation available only to a subset of sophisticated financial intermediaries, who face different limits-to-arbitrage constraints.

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Appendix A Relation to other SDF Similarity Indices

The normalizing constant $\min(E[M_d], E[M_f])$ in the denominator of index (20) ensures both an upper bound of one and the model-free index representation in Proposition 3, which is based on the price of an at-the-money put on the *spot* exchange rate. Another natural approach can directly normalize the SDFs in definition (20), which gives the similarity index:

$$S(M_d/E[M_d], M_f/E[M_f]) := E[\min(M_d/E[M_d], M_f/E[M_f])] \ . \tag{A-1}$$

This index preserves all key properties of index (20), but it is not computable from the prices of options on spot exchange rates. It can be computed from the price of a single at-the-money put on the forward exchange rate, whenever option quotes on forward exchange rates are available.[37]

A relevant computation issue arises for the power similarity indices introduced in Orlowski, Sali, and Trojani (2016):

$$S_\alpha(M_d, M_f) := \frac{E[M_d^\alpha M_f^{1-\alpha}]}{E[M_d]^\alpha E[M_f]^{1-\alpha}} \ , \tag{A-2}$$

where $\alpha \in (0,1)$ parameterizes the index family. This family includes for $\alpha = 1/2$ the Hellinger similarity index proposed in Bakshi, Gao, and Panayotov (2017).[38] A key property of these indices is that under the asset market view they can be in principle computed from option information alone, whenever a continuum of out-of-the money exchange rate options with arbitrary strike price $K > 0$ is traded, i.e., under a complete exchange rate option market.[39] Unfortunately, the intrinsic incompleteness of exchange rate option markets makes such a computation of similarity index (A-2) very challenging. However, due to the following inequality:

$$S(M_d, M_f)\frac{\min(E[M_d], E[M_f])}{E[M_d]^\alpha E[M_f]^{1-\alpha}} \leq S_\alpha(M_d, M_f) \ , \tag{A-3}$$

we can always compute with Proposition 3 a lower bound for similarity index (A-2), using the price of a single at-the-money put on the spot exchange rate.[40]

[37] The following relations between the two indices hold: $0 \leq S(M_d/E[M_d], M_f/E[M_d]) \leq S(M_d, M_f) \leq 1$. Moreover, under validity of the asset market view, the same arguments as in the proof of Proposition 3 yield:

$$E[\min(M_d/E[M_d], M_f/E[M_f])] = 1 - \frac{1}{E[M_d]} E[M_d \max(0, 1 - XE[M_d]/E[M_f])] \ .$$

[38] Note that:

$$S_{1/2}(M_d, M_f) = E[(M_d/E[M_d])^{1/2}(M_f/E[M_f])^{1/2}] = 1 - \frac{E\left[((M_d/E[M_d])^{1/2} - (M_f/E[M_f])^{1/2})^2\right]}{2} \ .$$

The expectation on the RHS corresponds to Bakshi, Gao, and Panayotov (2017)'s Hellinger distance.

[39] Using standard replication formulas for nonlinear payoffs, this result follows from the identity:

$$S_\alpha(M_d, M_f) = \frac{1}{E[M_d]^\alpha E[M_f]^{1-\alpha}} E[M_d X^{1-\alpha}] \ .$$

[40] The bound follows from the inequality $x^\alpha y^{1-\alpha} \geq \min(x, y)$ for $\alpha \in (0, 1)$.

Appendix A.1 Similarity in Benchmark Models

The index $\overline{S}(M_d, M_f)$ measures similarity, rather than SDF co-movement or risk sharing. In the following, we illustrate this using some well-known benchmark models.

Lognormal SDFs and Exchange Rates. Whenever (M_d, X) and $(M_f, 1/X)$ are jointly lognormal, similarity can be directly computed from the Black and Scholes (1973)–Garman and Kohlhagen (1983) model-implied prices of the corresponding at-the-money put options:

$$E[M_d \max(0, 1-X)] = E[M_d]\mathcal{N}(-d_2) - E[M_f]\mathcal{N}(-d_1) =: BS_d(\sigma_x) \;, \tag{A-4}$$

$$E[M_f \max(0, 1-(1/X))] = E[M_f]\mathcal{N}(d_1) - E[M_d]\mathcal{N}(d_2) =: BS_f(\sigma_x) \;, \tag{A-5}$$

where

$$d_1 := \frac{-\log(E[M_d]/E[M_f]) + \frac{\sigma_x^2}{2}}{\sigma_x} \;;\; d_2 := \frac{-\log(E[M_d]/E[M_f]) - \frac{\sigma_x^2}{2}}{\sigma_x} \;, \tag{A-6}$$

with σ_x the volatility of log exchange rate returns. Hence, $\overline{S}(M_d, M_f)$ is maximal (minimal) for $\sigma_x \downarrow 0$ ($\sigma_x \uparrow \infty$). Given the identity of physical and implied volatilities under joint log-normality, these arbitrage-free settings are unlikely to generate simultaneously a low exchange rate volatility and a low option-implied SDF similarity. For example, such settings include jointly lognormal specifications of domestic and foreign SDFs under the asset market view.

Rare Disasters. One way to break the link between the realized and implied volatilities is to introduce time-varying rare disasters. We borrow from Farhi and Gabaix (2016) who model an economy with traded and nontraded goods in complete international financial markets. In this model, a disaster may happen in the world consumption of the tradeable good with probability p_t. The exogenous SDF in units of the world numéraire, is given as follows: if no disaster happens in $t+1$, $M^*_{t+1} = \exp(-R)$, for a parameter $R > 0$ which is related to the subjective discount rate and the expected growth rate of the tradable good. If a disaster happens, then $M^*_{t+1} = \exp(-R)B^{-\gamma}_{t+1}$, where $B_{t+1} > 0$ models the size of world disasters and γ is the coefficient of relative risk aversion. A key parameter in Farhi and Gabaix (2016) is a country's resilience to disasters, defined by:

$$H_{it} := H_{i*} + \hat{H}_{it} := p_t E^D_t[B^{-\gamma}_{t+1} F_{it+1} - 1], \tag{A-7}$$

where F_{it+1} is the future country's recovery rate and $E^D_t[\cdot]$ denotes the expectation conditional on a disaster state.[41] Thus, a relatively safe country has a high resilience H_{it}, while a relatively risky country has a low resilience. The stochastic discount factor for the nontraded good in country i depends on the time-varying resilience component $\hat{H}_{it}$. It is given by:

$$M_{it+1} = M^*_{t+1} \cdot \frac{\omega_{it+1}}{\omega_{it}} \cdot \frac{r_{ei} + \phi_{H_i} + \hat{H}_{it+1}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} \;, \tag{A-8}$$

where r_{ei} is the sum of the country's "steady state" interest rate for $\hat{H}_{it} = 0$ and a constant investment depreciation rate λ, ϕ_{H_i} is the resilience speed of mean reversion and ω_{it} the country's i export productivity. The price of a put option on the exchange rate follows from the asset market view and the joint lognormality of $r_{ed} + \phi_{H_d} + \hat{H}_{dt+1}$, $r_{ef} + \phi_{H_f} + \hat{H}_{ft+1}$ conditional on a no disaster state.[42] Therefore, the similarity index (22) is available in closed-form.

[41] By definition, H_{i*} and $\hat{H}_{it}$ are the constant and the time-varying components of a country's i resilience.

[42] See also Proposition 6 in Farhi and Gabaix (2016).

Proposition 5. *In the Farhi and Gabaix (2016) model, the similarity index (22) is given by:*

$$\overline{S}_t(M_d, M_f) = S(M_d, M_f) = \frac{E_t[M_{d,t+1}] - E_t[M_{d,t+1}\max(0, 1 - X_{t+1})]}{\min(E_t[M_{d,t+1}], E_t[M_{f,t+1}])} \ , \tag{A-9}$$

where the price of the domestic at-the-money put on the exchange rate is given by:

$$\begin{aligned} E_t[M_{d,t+1}\max(0, 1 - X_{t+1})] &= (1 - p_t) BS_d(\sigma_x) \\ &\quad + p_t E_t^D\left[G_{d,t}F_{d,t+1}\max\left(0, 1 - \frac{F_{f,t+1}G_{f,t}}{F_{d,t+1}G_{d,t}}\right)\right], \end{aligned} \tag{A-10}$$

with

$$G_{i,t} = \exp(g_{\omega_i})\frac{r_{ei} + \phi_{H_i} + \frac{1+H_{i*}}{1+H_{it}}\exp(-\phi_{H_i})\hat{H}_{it}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} \ , \tag{A-11}$$

and g_{ω_i} the constant growth rate of country i's productivity in no-disaster states.

Proof: See Appendix.

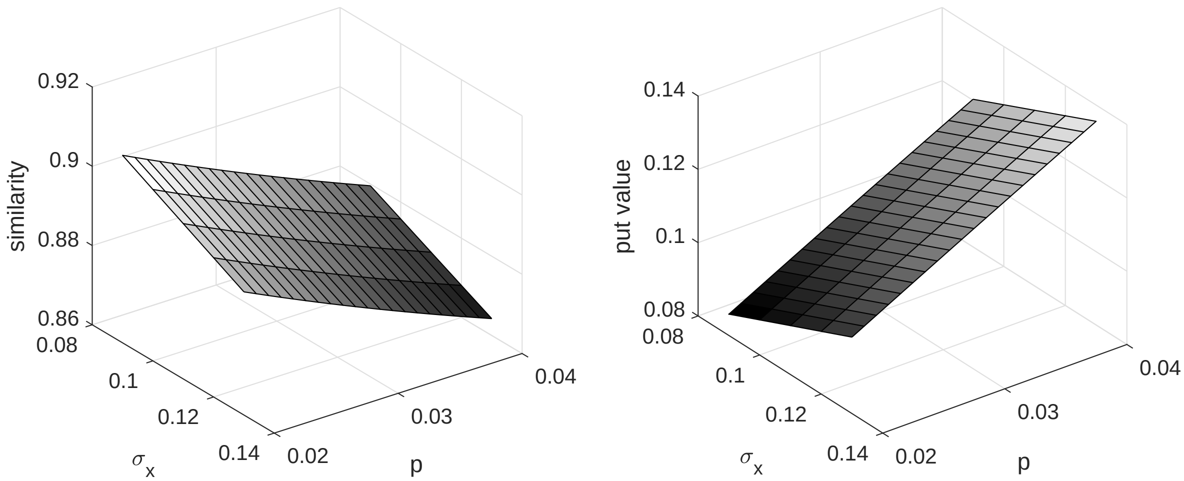
The first term on the RHS of equation (A-10) is the Black and Scholes (1973)–Garman and Kohlhagen (1983) price of a put option, weighted by the probability of no disaster. The second term is the put price conditional on a common disaster in domestic and foreign productivity, weighted by the probability of a world disaster. This term reflects the price for ensuring a large depreciation of foreign versus domestic non-traded goods, due to a larger decrease in foreign productivity when disasters occur. This component can create a disconnect between the minimal similarity index in Proposition 4 and the exchange rate volatility σ_x, which may be useful to induce low option-implied SDF similarities together with low exchange rate volatilities encountered in the data.

We can use the calibration parameters in Farhi and Gabaix (2016) to illustrate the range of similarity indices attainable in this model. We set $g_{\omega_d} = g_{\omega_f} = 0$, $\sigma_x \in [0.09, 0.13]$, $p_t \in [0.02, 0.04]$ and consider for simplicity the case where resilience in both countries is at its time-invariant level, i.e., $H_{it} = H_{i*}$ $(i = d, f)$, so that $G_{d,t} = G_{f,t} = 1$. Conditional on a disaster event, log recovery rates across countries are IID normally distributed, $\ln F_{it+1} \sim II\mathcal{N}(-\frac{\sigma_{F_i}^2}{2}, \sigma_{F_i}^2)$, with calibrated standard deviation $\sigma_{F_i} = 0.16$. Therefore,

$$E_t[F_{d,t+1}\max(0, 1 - F_{f,t+1}/F_{d,t+1})] = \mathcal{N}(\sigma_{F_i}/\sqrt{2}) - \mathcal{N}(-\sigma_{F_i}/\sqrt{2}) \ , \tag{A-12}$$

and the similarity index in Proposition 5 follows in closed-form. To gauge the effect of world disasters on the similarity index, we plot in Figure A-1 (left panel) the model-implied similarity index in Proposition 5 as a function of the disaster probability (p) and the volatility of exchange rates (σ_x). We notice that, on average, the similarity is around 0.88. As expected, when we increase the volatility of exchange rates (σ_x) or increase the probability of rare disasters (p), the similarity falls. This is explained by the fact that the price of the put option in the right panel of Figure A-1 concomitantly increases when the probability of world disasters (the volatility of exchange rates) increases.

Figure A-1. SDF Similarity in the Farhi and Gabaix (2016) model



The left panel plots the similarity index given in Proposition 5 as a function of exchange rate volatility (σ_x) and the disaster probability (p). The right panel plots the price of the at-the-money put option also as a function of the exchange rate volatility and the disaster probability. Calibrated values are as in Farhi and Gabaix (2016).

Appendix B Proofs and Derivations

Proof of Proposition 1. We study for any $\alpha_i \in \mathbb{R}\backslash\{1\}$ the following minimum divergence problem:[43]

$$\min_{M_i} \frac{1}{\alpha_i\,(\alpha_i-1)} \log E[M_i^{\alpha_i}],$$

subject to $E(M_i\mathbf{R}_i) = \mathbf{1}$ and $M_i > 0$. The Lagrange function is given by

$$\mathfrak{L}(M_i, \lambda_i, \nu_i) = \frac{\log E[M_i^{\alpha_i}]}{\alpha_i\,(\alpha_i-1)} + \mu_{i0}E[M_iR_{i0}-1] + \sum_{k=1}^{K_i} \mu_{ik}E[M_i\,(R_{ik}-R_{i0})] + \nu_i M_i,$$

with $\mu_i \in \mathbb{R}^{K_i+1}$ the vector of multipliers for the pricing constraints and ν_i the multiplier for the positivity constraint. As the optimal SDF needs to be strictly positive, ν_i vanishes almost surely and the first order conditions read:

$$\frac{M_i^{\alpha_i}}{\alpha_i-1} = M_i\left(\mu_{i0}R_{i0} + \sum_{k=1}^{K_i} \mu_{ik}\,(R_{ik}-R_{i0})\right) =: M_i\mu_{i0}R_{\lambda_i},$$

where we applied the renormalization $\lambda_i := \mu_{ik}/\mu_{i0}$, $k = 0, \ldots, K_i$. Taking expectations yields

$$E[M_i^{\alpha_i}] = (\alpha_i - 1)\,\mu_{i0}. \tag{A-13}$$

[43]The case $\alpha_i = 1$ can be treated separately in a similar way, but we do not consider it here as it is not necessary for the dispersion measures used in the main text.

Using (A-13) and plugging into the first order conditions, it follows

$$M_i = ((\alpha_i - 1)\,\mu_{i0} R_{\lambda_i})^{1/(\alpha_i - 1)}\,.$$

Therefore, the optimal SDF M_i^* and optimal return $R_{\lambda_i^*}$ are such that

$$\log E[(M_i^*)^{\alpha_i}] = \log E[R_{\lambda_i^*}^{\alpha_i/(\alpha_i-1)}]^{1-\alpha_i}.$$

In summary, we get the following inequality:

$$\frac{\log E[M_i^{\alpha_i}]}{\alpha_i\,(\alpha_i - 1)} \geq -\frac{\log E[M_i^{*\alpha_i}]}{\alpha_i\,(\alpha_i - 1)} = -\frac{1}{\alpha_i} \log E[R_{\lambda_i^*}^{\alpha_i/(\alpha_i-1)}]\,, \tag{A-14}$$

which shows that the convexity bound implied by the optimal portfolio return $R_{\lambda_i^*}$ is tight. The optimal SDF is given by

$$M_i^* = R_{\lambda_i^*}^{1/(\alpha_i-1)} / E[R_{\lambda_i^*}^{\alpha_i/(\alpha_i-1)}],$$

with

$$R_{\lambda_i^*} = R_{i0} + \sum_{k=1}^{K_i} \lambda_{ik}^* \,(R_{ik} - R_{i0}) > 0,$$

such that for any $k = 1, \ldots, K_i$

$$E[R_{\lambda_i^*}^{1/(\alpha_i-1)}\,(R_{ik} - R_{i0})] = 0.$$

Equivalently, the bound of the RHS of inequality (A-14) is obtained by solving the following maximization problem:

$$\max_{\lambda_i} -\frac{1}{\alpha_i} \log E[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)}]$$

such that

$$R_{\lambda_i} = \sum_{k=1}^{K_i} \lambda_{ik}\,(R_{ik} - R_{i0}) + R_{i0} > 0.$$

Indeed, this optimization problem is globally strictly concave and the first order conditions for $k = 1, \ldots, K_i$ are:

$$0 = \frac{\partial \log E[R_{\lambda_i}^{\alpha_i/(\alpha_i-1)}]}{\partial \lambda_i}\,|_{\lambda_i = \lambda_i^*} = E[R_{\lambda_i^*}^{1/(\alpha_i-1)}\,(R_{ik} - R_{i0})]. \tag{A-15}$$

Since (A-15) is identical to the pricing constraints for λ_i in the solution of the primal SDF optimization problem, the bounds are tight. This concludes the proof. □

Proof of Proposition 2. As $(R^e_{\lambda_f^*})^{-1}$ solves the moment conditions (13), it is the minimum entropy SDF for return vector $\mathbf{R}_f^e$. Hence, if $(R_{\lambda_d^*})^{-1}$ is the minimimum entropy SDF for return vector $\mathbf{R}_d$, i.e., it solves the moment condition (12), then $(R^e_{\lambda_f^*})^{-1}$ is the minimum entropy SDF for return vector $\mathbf{R}_f^e$. Symmetric arguments show that $M_d^{*e} = (R^e_{\lambda_d^*})^{-1}$ is the minimum entropy SDF for return vector $\mathbf{R}_d^e$ whenever M_f^* is the minimum entropy SDF for return vector $\mathbf{R}_f$. This concludes the proof. □

Proof of Proposition 3. Let the asset market view hold, i.e., $X = M_f / M_d$. It then follows:

$$E[\min(M_d, M_f)] = E[M_d \min(1, X)] = E[M_d] - E[M_d(1 - \min(1, X))]\,.$$

Moreover,

$$E[M_d(1-\min(1,X))] = E[M_d(1+\max(-1,-X))] = E[M_d \max(0,1-X)] \ .$$

The second equality in the statement of the proposition follows by symmetry. This concludes the proof. □

Proof of Proposition 4. By definition, we have:

$$S(M_d, M_d X) = \frac{E[M_d \min(1,X)]}{\min(E[M_d], E[M_d X])} \ . \tag{A-16}$$

Whenever the domestic return $R^e_{d0} := R_{f0}X$ is priced, it follows:

$$1 = E_t[M_{d,t+1}R^e_{d0,t+1}] \iff \frac{1}{R_{f0,t+1}} = E_t[M_{d,t+1}X_{t+1}] \ , \tag{A-17}$$

as $R_{f0,t+1}$ is the foreign risk-free rate. Taking unconditional expectations, we obtain $E[M_d X] = E[1/R_{f0}] = E[M_f]$. Therefore,

$$S(M_d, M_d X) = \frac{E[M_d \min(1,X)]}{\min(E[M_d], E[M_d X])} = \frac{E[M_d] - E[M_d \min(0,1-X)]}{\min(E[M_d], E[M_f])} \ ,$$

with the same arguments as in the proof of Proposition 3. By applying an identical approach with the foreign return $R^e_{0f} := R_{0d}(1/X)$, we obtain:

$$S(M_f, M_f(1/X)) = \frac{E[M_f] - E[M_f \min(0,1-(1/X))]}{\min(E[M_d], E[M_f])} \ .$$

This concludes the proof. □

Proof of Proposition 5. Given the underlying market completeness, the asset market view holds and the exchange rate return is given by:

$$X_{t+1} = \frac{M_{f,t+1}}{M_{d,t+1}} = \frac{\omega_{ft+1}/\omega_{ft}}{\omega_{dt+1}/\omega_{dt}} \cdot \frac{\frac{r_{ef}+\phi_{H_f}+\hat{H}_{ft+1}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\hat{H}_{dt+1}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} \ . \tag{A-18}$$

If follows that conditional on a disaster the exchange rate is:

$$X_{t+1} = \frac{M_{f,t+1}}{M_{d,t+1}} = \exp(g_{\omega_f} - g_{\omega_d})\frac{F_{ft+1}}{F_{dt+1}} \cdot \frac{\frac{r_{ef}+\phi_{H_f}+\frac{1+H_{f*}}{1+H_{ft}}\exp(-\phi_{H_f})\hat{H}_{ft}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\frac{1+H_{d*}}{1+H_{dt}}\exp(-\phi_{H_d})\hat{H}_{dt}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} \ , \tag{A-19}$$

where g_{ω_i} is a constant productivity growth of country i in no-disaster times. Similarly, conditional on no disasters, the exchange rate is:

$$X_{t+1} = \exp(g_{\omega_f} - g_{\omega_d}) \frac{\frac{r_{ef}+\phi_{H_f}+\hat{H}_{ft+1}}{r_{ef}+\phi_{H_f}+\hat{H}_{ft}}}{\frac{r_{ed}+\phi_{H_d}+\hat{H}_{dt+1}}{r_{ed}+\phi_{H_d}+\hat{H}_{dt}}} \ , \tag{A-20}$$

which is lognormally distributed, as it is the ratio of two jointly lognormal variables. If follows that conditional on a no disaster event M_d and X are jointly lognormal and we can apply the Black and Scholes (1973)–Garman and Kohlhagen (1983) formula. In summary, we obtain

$$E[M_{d,t+1} \max(0, 1 - X_{t+1})] = (1 - p_t) BS_d(\sigma_x) + p_t E_t^D [M_{d,t+1} \max(0, 1 - X_{t+1})] \ ,$$

where $E_t^D[\cdot]$ denotes expectations conditional on a disaster event. The explicit computation of the expectation on the RHS yields

$$E_t^D [M_{d,t+1} \max(0, 1 - X_{t+1})] = E_t \left[G_{d,t} F_{d,t+1} \max \left(0, 1 - \frac{G_{f,d} F_{f,t+1}}{G_{d,t} F_{d,t+1}} \right) \right] \ ,$$

where

$$G_{i,t} = \exp(g_{\omega_i}) \cdot \frac{r_{ei} + \phi_{H_i} + \frac{1+H_{i*}}{1+H_{it}} \exp(-\phi_{H_i}) \hat{H}_{it}}{r_{ei} + \phi_{H_i} + \hat{H}_{it}} \ . \tag{A-21}$$

This concludes the proof. $\square$

Appendix C Tables

Table 1
Data Summary Statistics

The table provides descriptive statistics for nominal domestic returns, exchange rates and CPI inflation for Switzerland, the Euro-zone (Germany before the introduction of the euro), the United Kingdom, Japan, the US, Australia, Canada and New Zealand. The sample period spans January 1975 to December 2015 (January 1988 to December 2015 for New Zealand) and the sampling frequency is monthly. Returns and inflation rates are annualized and displayed in percentages. In Panel A we report the annualized average returns for one-month risk-free bonds and ten-year government bonds. Panel B reports mean excess returns on equity, their volatility and the corresponding Sharpe ratios, computed as the ratio between the excess return and the return standard deviation. Panel C reports the annualized mean and standard deviation of exchange rates returns with respect to the US dollar. Panel D reports the average CPI inflation and its standard deviation.

	CHF	**EUR**	**GBP**	**JPY**	**USD**	**AUD**	**CAD**	**NZD**
				Panel A: Bonds				
1M	2.81	4.33	7.39	2.61	5.36	8.25	6.31	6.68
10Y	1.79	2.26	3.23	2.31	1.91	2.19	2.04	3.94
				Panel B: Excess stock returns				
Mean	7.39	6.89	6.23	3.49	7.08	5.71	5.15	0.84
Std	15.42	20.08	16.99	18.31	15.71	17.76	16.77	18.23
SR	48	34	37	19	45	32	31	5
				Panel C: Exchange rates				
Mean	2.96	0.03	-0.65	2.85		-0.86	-0.48	0.76
Std	12.12	10.56	10.20	11.32		10.93	6.78	11.92
				Panel D: Inflation				
Mean	1.76	2.22	4.74	1.57	3.69	4.83	3.71	5.57
Std	1.24	1.60	2.12	1.78	1.28	1.22	1.45	1.71

Table 2
Properties of SDFs (Unrestricted Trading)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha_i = 0$), Panel B for Hellinger SDFs ($\alpha_i = 0.5$) and Panel C for minimum variance SDFs ($\alpha_i = 2$), $i = d, f$, $j = d, f$, $i \neq j$. The SDFs are derived when international trading is unrestricted, i.e. the financial markets are fully integrated. There is an US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
	Panel A: $\alpha = 0$ (minimum entropy)													
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.917	0.948	1.048	0.951	0.814	0.707	0.774	0.725	1.029	1.065	0.823	0.827	0.681	0.625
$\sqrt{\text{Entropy}}(M_i)$	0.684	0.703	0.795	0.753	0.687	0.636	0.604	0.585	0.732	0.702	0.618	0.616	0.581	0.519
corr(M_i^T,M_i^P)	-0.454	-0.498	-0.407	-0.233	-0.519	-0.155	-0.549	-0.502	-0.411	-0.636	-0.506	-0.607	-0.317	-0.634
corr(M_i,M_j)		0.992		0.989		0.989		0.985		0.992		0.994		0.981
	Panel B: $\alpha = 0.5$ (Hellinger)													
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.784	0.805	0.925	0.880	0.720	0.677	0.661	0.647	0.823	0.843	0.688	0.682	0.620	0.547
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.856	0.882	0.991	0.903	0.791	0.688	0.741	0.687	0.908	0.943	0.775	0.779	0.657	0.612
$\sqrt{\text{Hellinger}}(M_i)$	0.706	0.725	0.823	0.780	0.695	0.646	0.599	0.599	0.752	0.768	0.634	0.631	0.590	0.526
corr(M_i^T,M_i^P)	-0.483	-0.534	-0.428	-0.243	-0.531	-0.152	-0.570	-0.524	-0.461	-0.716	-0.533	-0.641	-0.324	-0.646
corr(M_i,M_j)		0.990		0.989		0.989		0.984		0.990		0.994		0.980
	Panel C: $\alpha = 2$ (minimum variance)													
E[M_i]	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.739	0.754	0.873	0.834	0.699	0.658	0.639	0.622	0.776	0.791	0.659	0.655	0.600	0.535
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.803	0.824	0.930	0.853	0.763	0.670	0.711	0.659	0.839	0.874	0.733	0.735	0.632	0.595
corr(M_i^T,M_i^P)	-0.517	-0.587	-0.455	-0.268	-0.552	-0.169	-0.597	-0.564	-0.500	-0.775	-0.566	-0.683	-0.340	-0.665
corr(M_i,M_j)		0.989		0.988		0.989		0.984		0.988		0.993		0.979

Table 3

Wedge Summary Statistics (Unrestricted Trading)

This table reports the annualized mean, standard deviation, skewness and kurtosis of the wedge η, for $\alpha \in \{0.5, 2\}$. The domestic currency is the US dollar. The wedge is $\eta_{t+1} = \log\left(\frac{X_{t+1}M_{d,t+1}}{M_{f,t+1}}\right)$. The minimum dispersion SDFs account for the fact that domestic investors can trade any foreign asset.

	$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.000	0.022	-0.395	6.336	-0.007	0.059	-0.259	11.62
CH	-0.001	0.026	-1.277	8.426	-0.019	0.120	-6.146	68.40
JP	0.000	0.023	-1.286	7.608	-0.009	0.083	-4.483	36.18
EU	0.000	0.021	-0.065	5.814	0.000	0.064	-1.130	11.47
AU	0.000	0.019	0.424	8.003	0.005	0.075	2.110	24.82
CA	0.000	0.013	-0.488	6.080	-0.001	0.034	-0.538	5.772
NZ	-0.001	0.023	-2.695	17.23	-0.031	0.216	-15.61	268.3

Table 4

Correlation of Permanent SDF Components Across Exchange Rate Parities

This table reports the correlation between permanent components of domestic and foreign SDFs. The domestic SDF is the US one, whereas the foreign SDFs are those for the UK, CH, JP, EU, AU, CA and NZ. Standard errors are computed using a circular block bootstrap of size 10 with 10,000 simulations and reported in square brackets. Label *** denotes significance at the 1% level.

		UK	CH	JP	EU	AU	CA	NZ
	$\alpha = 0$	0.972***	0.984***	0.981***	0.978***	0.987***	0.976***	0.968***
		[0.019]	[0.007]	[0.004]	[0.007]	[0.006]	[0.005]	[0.006]
corr(M_d^P,M_f^P)	$\alpha = 0.5$	0.969***	0.984***	0.980***	0.976***	0.984***	0.975***	0.968***
		[0.013]	[0.004]	[0.004]	[0.006]	[0.005]	[0.005]	[0.005]
	$\alpha = 2$	0.968***	0.982***	0.977***	0.974***	0.980***	0.973***	0.965***
		[0.009]	[0.003]	[0.004]	[0.006]	[0.005]	[0.004]	[0.005]

Table 5
Backus-Smith (1993)-Type Regressions (Unrestricted Trading)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta x_{t+1} + u_{t+1}$, where small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of each component of the SDF on the log change in the real exchange rate: $m^U_{f,t+1} - m^U_{d,t+1} = \delta^U + \beta^U x_{t+1} + u^U_{t+1}$, where $U = P, T$ for permanent and transitory components, respectively. Standard errors are reported in square brackets. Label *** highlights significance at the 1% level.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.000***	1.060***	1.022***	1.000***	1.079***	1.233***
	[0.000]	[0.009]	[0.0261]	[0.000]	[0.009]	[0.043]
$\boldsymbol{\beta^P}$	1.085***	1.145***	1.065***	0.951***	1.030***	1.183***
	[0.068]	[0.067]	[0.0742]	[0.044]	[0.045]	[0.064]
$\boldsymbol{\beta^T}$	-0.084	-0.084	-0.084	0.049	0.049	0.049
	[0.068]	[0.068]	[0.068]	[0.044]	[0.044]	[0.044]

	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.000***	1.056***	1.107***	1.000***	1.050***	1.055***
	[0.000]	[0.009]	[0.033]	[0.000]	[0.008]	[0.027]
$\boldsymbol{\beta^P}$	1.083***	1.139***	1.189***	0.956***	1.006***	1.011***
	[0.053]	[0.053]	[0.065]	[0.046]	[0.046]	[0.056]
$\boldsymbol{\beta^T}$	-0.083	-0.083	-0.083	0.044	0.044	0.044
	[0.053]	[0.053]	[0.053]	[0.046]	[0.046]	[0.046]

	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.000***	1.042***	1.117***	1.000***	1.044***	1.031***
	[0.000]	[0.007]	[0.029]	[0.000]	[0.008]	[0.022]
$\boldsymbol{\beta^P}$	1.005***	1.047***	1.122***	1.027***	1.072***	1.059***
	[0.049]	[0.049]	[0.059]	[0.089]	[0.090]	[0.093]
$\boldsymbol{\beta^T}$	-0.005	-0.005	-0.005	-0.028	-0.028	-0.028
	[0.049]	[0.049]	[0.049]	[0.089]	[0.089]	[0.089]

	Panel G: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	1.000***	1.055***	1.409***
	[0.000]	[0.009]	[0.095]
$\boldsymbol{\beta^P}$	1.006***	1.061***	1.415***
	[0.038]	[0.038]	[0.098]
$\boldsymbol{\beta^T}$	-0.006	-0.006	-0.006
	[0.037]	[0.037]	[0.037]

Table 6

Properties of SDFs (Trading in Foreign Short-Term Bonds)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha_i = 0$), Panel B for Hellinger SDFs ($\alpha_i = 0.5$) and Panel C for minimum variance SDFs ($\alpha_i = 2$), $i = d, f$, $j = d, f$, $i \neq j$. There is an US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	**UK**	**US**	**CH**	**US**	**JP**	**US**	**EU**	**US**	**AU**	**US**	**CA**	**US**	**NZ**
	Panel A: $\alpha = 0$ (minimum entropy)													
$\mathrm{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.979	0.982	0.966	0.983	0.973	0.983	0.956
$\mathrm{Std}(M_i)$	0.611	0.723	0.769	0.674	0.722	0.364	0.645	0.487	0.603	0.821	0.634	0.514	0.539	0.380
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.707	0.824	0.843	0.698	0.800	0.379	0.732	0.535	0.702	0.931	0.729	0.641	0.585	0.466
$\sqrt{\mathrm{Entropy}}(M_i)$	0.520	0.550	0.659	0.598	0.666	0.362	0.575	0.461	0.515	0.702	0.533	0.457	0.486	0.359
$\mathrm{corr}(M_i^T, M_i^P)$	-0.586	-0.567	-0.503	-0.310	-0.527	-0.280	-0.578	-0.680	-0.593	-0.726	-0.566	-0.781	-0.372	-0.846
$\mathrm{corr}(M_i, M_j)$		0.122		0.374		0.460		0.646		0.585		0.396		0.536
	Panel B: $\alpha = 0.5$ (Hellinger)													
$\mathrm{E}[M_i]$	0.983	0.973	0.983	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$\mathrm{Std}(M_i)$	0.577	0.631	0.730	0.650	0.702	0.363	0.625	0.479	0.572	0.762	0.592	0.494	0.525	0.372
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.668	0.728	0.803	0.673	0.776	0.377	0.709	0.527	0.664	0.864	0.682	0.612	0.567	0.457
$\sqrt{\mathrm{Hellinger}}(M_i)$	0.533	0.568	0.676	0.610	0.674	0.363	0.587	0.466	0.528	0.717	0.547	0.466	0.496	0.363
$\mathrm{corr}(M_i^T, M_i^P)$	-0.617	-0.640	-0.526	-0.318	-0.540	-0.272	-0.593	-0.683	-0.623	-0.780	-0.602	-0.814	-0.377	-0.862
$\mathrm{corr}(M_i, M_j)$		0.154		0.351		0.469		0.627		0.538		0.430		0.523
	Panel C: $\alpha = 2$ (minimum variance)													
$\mathrm{E}[M_i]$	0.983	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
$\mathrm{Std}(M_i)$	0.555	0.581	0.699	0.630	0.681	0.359	0.608	0.471	0.551	0.728	0.568	0.474	0.512	0.366
$\mathrm{Std}(M_i^T)$	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
$\mathrm{Std}(M_i^P)$	0.637	0.666	0.766	0.651	0.747	0.376	0.684	0.518	0.634	0.816	0.648	0.579	0.549	0.446
$\mathrm{corr}(M_i^T, M_i^P)$	-0.652	-0.719	-0.554	-0.340	-0.564	-0.301	-0.617	-0.716	-0.656	-0.829	-0.638	-0.866	-0.393	-0.886
$\mathrm{corr}(M_i, M_j)$		0.166		0.276		0.492		0.586		0.480		0.451		0.503

Table 7

Wedge Summary Statistics (Trading in Foreign Short-Term Bonds)

The table reports sample mean, standard deviation, skewness and kurtosis of the wedge in Equation (10) ($\eta_{t+1} = \log((M_{d,t+1}X_{t+1})/M_{f,t+1})$), for dispersion measures $\alpha = 0, 0.5, 2$. The optimal derived SDFs account for the fact that domestic investors can trade the short-term foreign risk-free bond.

	$\alpha = 0$				$\alpha = 0.5$				$\alpha = 2$			
	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)	E[η]	Std(η)	Sk(η)	K(η)
UK	0.003	0.636	-0.646	13.55	0.005	0.665	-0.207	6.847	0.042	0.814	1.074	9.239
CH	-0.006	0.682	-0.367	6.270	-0.007	0.713	-0.174	4.647	-0.021	0.826	-0.019	3.724
JP	-0.123	0.545	1.446	8.938	-0.124	0.554	1.053	6.713	-0.149	0.612	-0.259	5.417
EU	-0.048	0.439	0.265	4.026	-0.048	0.455	0.058	3.629	-0.059	0.517	-0.554	5.011
AU	0.104	0.581	-0.181	5.573	0.106	0.614	0.050	4.898	0.129	0.716	1.051	6.714
CA	-0.036	0.490	0.148	9.963	-0.036	0.507	0.093	5.655	-0.040	0.561	0.305	5.082
NZ	-0.020	0.413	0.362	4.556	-0.021	0.419	0.265	4.045	-0.029	0.442	0.178	3.834

Table 8

Correlation Between Wedge and SDFs (Trading in Foreign Short-Term Bonds)

This table reports the correlation between the wedge η, the (log) domestic and foreign minimum entropy SDFs ($\alpha = 0$), as well as the log permanent and transient components of minimum entropy SDFs. Log SDFs are denoted by $m_i := \log M_i$ and log SDF components by $m_i^U := \log M_i^U$ ($i = d, f$ and $U = T, P$). Standard errors (SE) are computed using a circular block bootstrap of size 10 with 10000 simulations and reported in square brackets. *** denotes significance at the 1% level.

	corr(η, m_i)	SE	corr(η, m_i^P)	SE	corr(η, m_i^T)	SE
US	0.658***	[0.039]	0.651***	[0.039]	-0.356***	[0.049]
UK	-0.617***	[0.052]	-0.602***	[0.057]	0.338***	[0.053]
US	0.541***	[0.026]	0.569***	[0.029]	-0.431***	[0.038]
CH	-0.594***	[0.051]	-0.585***	[0.053]	0.077	[0.054]
US	0.728***	[0.039]	0.759***	[0.042]	-0.532***	[0.058]
JP	-0.201***	[0.054]	-0.200***	[0.056]	-0.029	[0.061]
US	0.552***	[0.047]	0.546***	[0.050]	-0.273***	[0.058]
EU	-0.296***	[0.084]	-0.324***	[0.909]	0.402***	[0.052]
US	0.257***	[0.062]	0.204***	[0.065]	0.087	[0.057]
AU	-0.685***	[0.049]	-0.714***	[0.044]	0.756***	[0.030]
US	0.606***	[0.077]	0.605***	[0.076]	-0.342***	[0.055]
CA	-0.426***	[0.120]	-0.470***	[0.111]	0.565***	[0.069]
US	0.523***	[0.031]	0.441***	[0.040]	0.278***	[0.039]
NZ	-0.465***	[0.075]	-0.508***	[0.071]	0.606***	[0.057]

Table 9

Backus-Smith (1993)-Type Regressions

(Trading in Foreign Short-Term Bonds)

This table reports the point estimates of a linear regression of the log difference between foreign and domestic SDFs on the log real exchange rate return: $m_{f,t+1} - m_{d,t+1} = \delta + \beta\, x_{t+1} + u_{t+1}$, where small-cap letters denote quantities in logs. We additionally report point estimates of a linear regression of the log difference of the permanent component of the SDF on the log real exchange rate return: $m^P_{f,t+1} - m^P_{d,t+1} = \delta^P + \beta^P x_{t+1} + u^P_{t+1}$. Standard errors are reported in square brackets. Labels ** and *** highlight significance at the 5% and 1% level, respectively.

	Panel A: US/UK			Panel B: US/CH		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.801***	0.918***	1.0683***	0.709***	0.849***	1.059***
	[0.274]	[0.287]	[0.352]	[0.253]	[0.265]	[0.307]
$\boldsymbol{\beta^P}$	0.886***	1.003***	1.153***	0.660**	0.799***	1.009***
	[0.316]	[0.329]	[0.391]	[0.276]	[0.288]	[0.329]
	Panel C: US/JP			Panel D: US/EU		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.937***	1.003***	1.114***	0.935***	0.975***	0.994***
	[0.229]	[0.222]	[0.245]	[0.184]	[0.191]	[0.217]
$\boldsymbol{\beta^P}$	1.019***	1.086***	1.197***	0.892***	0.931***	0.949***
	[0.249]	[0.253]	[0.276]	[0.213]	[0.219]	[0.245]
	Panel E: US/AU			Panel F: US/CA		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.981***	1.031***	1.087***	0.974***	1.039***	1.062***
	[0.231]	[0.244]	[0.284]	[0.312]	[0.323]	[0.357]
$\boldsymbol{\beta^P}$	0.986***	1.037***	1.093***	1.002***	1.067***	1.089**
	[0.262]	[0.275]	[0.315]	[0.383]	[0.395]	[0.429]

	Panel G: US/NZ		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 2$
$\boldsymbol{\beta}$	0.875***	0.933***	1.045***
	[0.186]	[0.189]	[0.199]
$\boldsymbol{\beta^P}$	0.881***	0.939***	1.051***
	[0.202]	[0.205]	[0.215]

Table 10

SDF Similarity Summary Statistics

Panel A provides descriptive statistics for SDF similarity measures computed from equation (20). Data starts in April 1993 and ends in April 2013. Panel B reports the average for the SDF similarity as implied by unrestricted international trading (I) and when international trading is restricted to short-term bonds (II). Minimum entropy, Hellinger and variance are obtained for $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 2$, respectively. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country.

		UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
	`Panel A : Option-Implied SDF Similarity Index`							
	Mean	0.990	0.987	0.987	0.989	0.987	0.991	0.987
	Std	0.003	0.003	0.004	0.004	0.005	0.004	0.005
	Skewness	-2.624	-1.412	-1.753	-1.501	-2.810	-2.198	-1.388
	Kurtosis	13.85	7.449	8.982	8.143	17.39	10.21	7.321
	`Panel B : Nominal Minimum Dispersion SDF Similarity Index`							
	`I : Unrestricted International Trading`							
$\alpha = 0$	$S(M_d, M_f)$	0.991	0.987	0.989	0.988	0.987	0.991	0.987
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.989	0.988	0.987	0.991	0.987
$\alpha = 0.5$	$S(M_d, M_f)$	0.990	0.987	0.988	0.988	0.986	0.985	0.987
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.989	0.988	0.986	0.992	0.987
$\alpha = 2$	$S(M_d, M_f)$	0.990	0.986	0.989	0.988	0.986	0.990	0.987
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.987	0.986	0.991	0.986
	`II : International Trading in Short-Term Bonds Only`							
$\alpha = 0$	$S(M_d, M_f)$	0.952	0.928	0.945	0.971	0.961	0.973	0.949
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.991	0.987
$\alpha = 0.5$	$S(M_d, M_f)$	0.951	0.926	0.945	0.970	0.960	0.972	0.947
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.993	0.985
$\alpha = 2$	$S(M_d, M_f)$	0.950	0.921	0.944	0.970	0.959	0.971	0.947
	$\bar{S}(M_d, M_f)$	0.991	0.987	0.988	0.988	0.986	0.991	0.986

Table 11
International Co-Entropies

The table reports the values of co-entropy as in Chabi-Yo and Colacito (2017), defined as $\rho_{M_f,M_d} = 1 - \frac{L[M_f/M_d]}{L[M_f]+L[M_d]}$, with $L[x] \equiv \log(E[x]) - E[\log(x)]$ being the entropy of the positive random variable x. We provide estimates using the minimum dispersion SDFs derived when international trading is restricted to short-term bonds (asymmetry) and when international trading is unrestricted (symmetry). Data starts in April 1993 and ends in April 2013. Minimum entropy, Hellinger and variance are obtained for $\alpha = 0$, $\alpha = 0.5$ and $\alpha = 2$, respectively. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country.

		UK	**CH**	**JP**	**EU**	**AU**	**CA**	**NZ**
$\alpha = 0$	Asymmetry	0.351	0.415	0.239	0.740	0.748	0.816	0.493
	Symmetry	0.983	0.988	0.980	0.962	0.979	0.982	0.982
$\alpha = 0.5$	Asymmetry	0.328	0.390	0.228	0.727	0.735	0.806	0.479
	Symmetry	0.980	0.986	0.978	0.958	0.976	0.977	0.979
$\alpha = 2$	Asymmetry	0.295	0.327	0.197	0.710	0.717	0.794	0.471
	Symmetry	0.977	0.982	0.952	0.955	0.964	0.975	0.817

Table 12
SDFs, Value-at-Risk Constraints and Financial Intermediary Wealth under Integrated Markets

The table reports estimated coefficients from regressing minimum entropy SDFs derived in Section 2.2 on changes in financial intermediary wealth by He, Kelly, and Manela (2017) and changes in VIX.

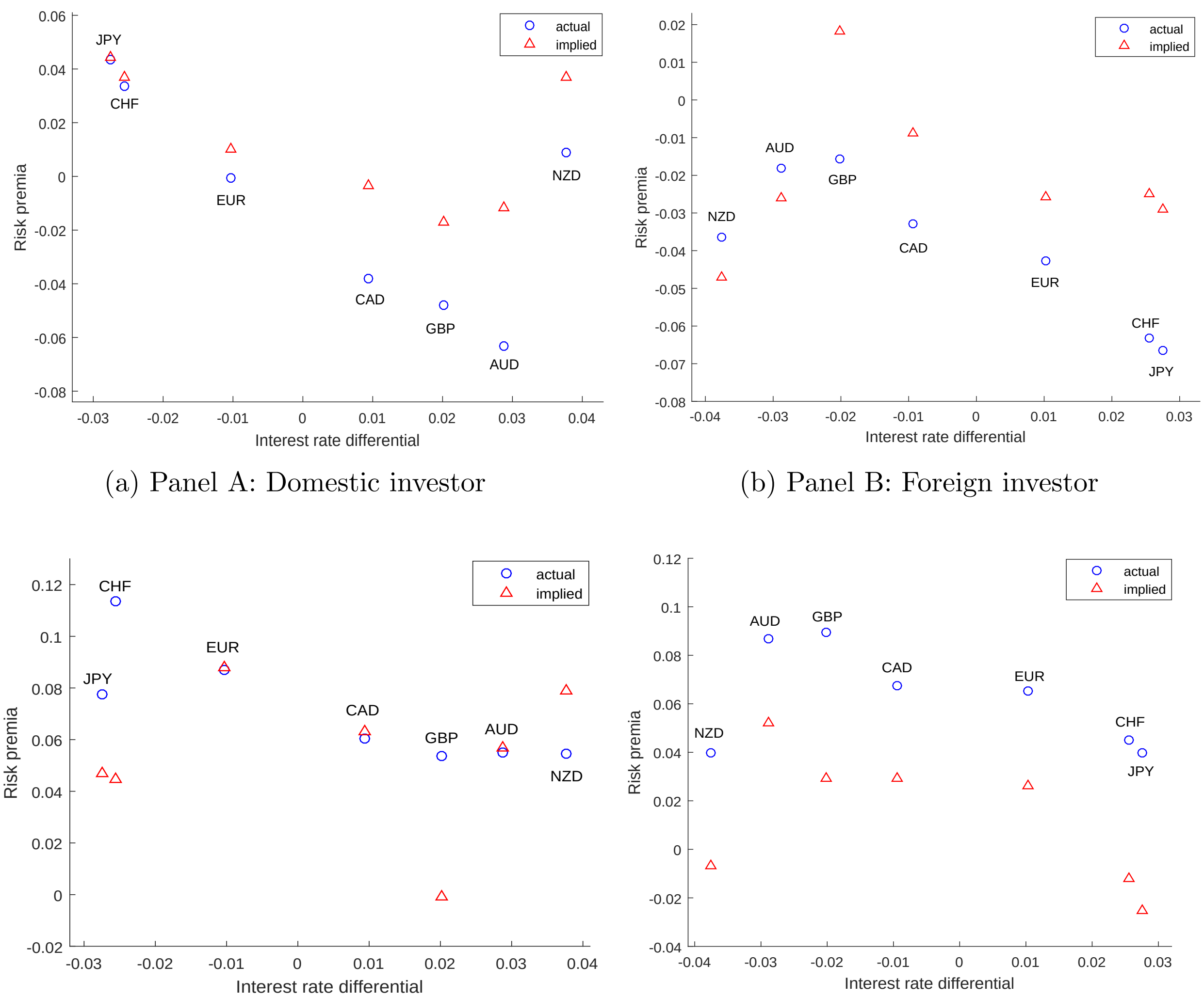
$$M_{t+1}^{i} = \alpha^{i} + \beta_{k}^{i}\ \Delta\text{intermediary wealth}_{t+1} + \beta_{v}^{i}\ \Delta\text{VIX}_{t+1} + \epsilon_{t+1}^{i},$$

where $i = d, f$ denotes domestic and foreign quantities. T-stats are computed using a circular block bootstrap of size 10 with $10,000$ simulations and reported in parenthesis.

	USDGBP	**USDCHF**	**USDJPY**	**USDEUR**	**USDAUD**	**USDCAD**	**USDNZD**
α	1.550	2.086	0.744	1.169	1.116	0.697	0.799
	(11.08)	(8.57)	(3.27)	(7.59)	(4.15)	(5.48)	(1.44)
β_k	-0.949	-1.587	-0.054	-0.437	-0.559	-0.153	-0.101
	(-6.31)	(-5.63)	(-0.12)	(-2.43)	(-1.60)	(-0.72)	(-0.12)
β_v	0.395	0.497	0.306	0.264	0.438	0.450	0.298
	(5.43)	(3.03)	(5.17)	(2.36)	(2.96)	(5.25)	(3.56)
R-Squared	0.46	0.35	0.09	0.24	0.20	0.31	0.04
	GBPUSD	**CHFUSD**	**JPYUSD**	**EURUSD**	**AUDUSD**	**CADUSD**	**NZDUSD**
α	1.552	2.125	0.776	1.104	0.916	0.751	0.824
	(12.05)	(9.41)	(4.05)	(10.19)	(3.79)	(6.89)	(1.71)
β_k	-0.903	-1.578	-0.087	-0.354	-0.335	-0.140	-0.044
	(-7.09)	(-6.12)	(-0.39)	(-2.42)	(-1.04)	(-0.86)	(-0.02)
β_v	0.347	0.450	0.308	0.247	0.412	0.383	0.215
	(5.67)	(3.30)	(5.11)	(2.87)	(3.22)	(5.29)	(0.22)
R-Squared	0.45	0.37	0.11	0.27	0.20	0.29	0.03

Appendix D Figures

Figure 1. International long-term bond and equity risk premia ($\alpha_i = 0$)



(a) Panel A: Domestic investor

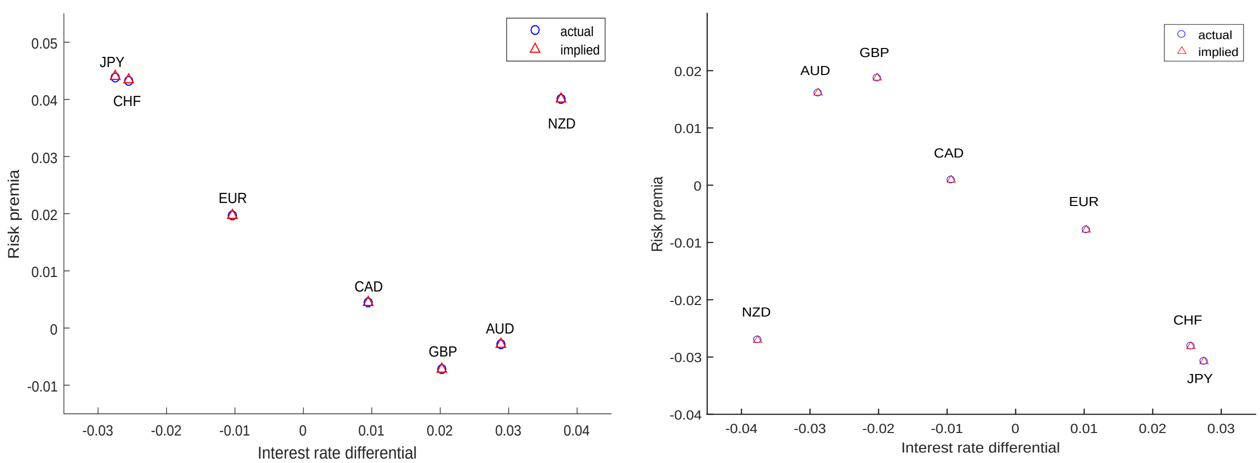
(b) Panel B: Foreign investor

(c) Panel C: Domestic investor

(d) Panel D: Foreign investor

The figure plots for $i = d, f$ various observed international risk premia against the average interest rate differential, computed as the difference between foreign and domestic nominal one-month LIBOR rates. The top panels plot the long-term bond risk premia $E[R^e_{i\infty,t+1}] - E[R_{i0,t+1}]$ and the risk premia $-\text{cov}(M_{i,t+1}/E[M_{i,t+1}], R^e_{i\infty,t+1} - R_{i0,t+1})$ under the minimum entropy SDF. The bottom panels plot the observed international equity premium $E[R^e_{i1,t+1}] - E[R_{i0,t+1}]$ and the equity risk premia $-\text{cov}(M_{i,t+1}/E[M_{i,t+1}], (R^e_{i1,t+1} - R_{i0,t+1}))$ under the minimum entropy SDF $M_{i,t+1}$. Panels A and C report risk premia for domestic investors ($i = d$); Panel B and D for foreign investors ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD. Data is monthly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

Figure 2. Currency risk premia ($\alpha_i = 0$)

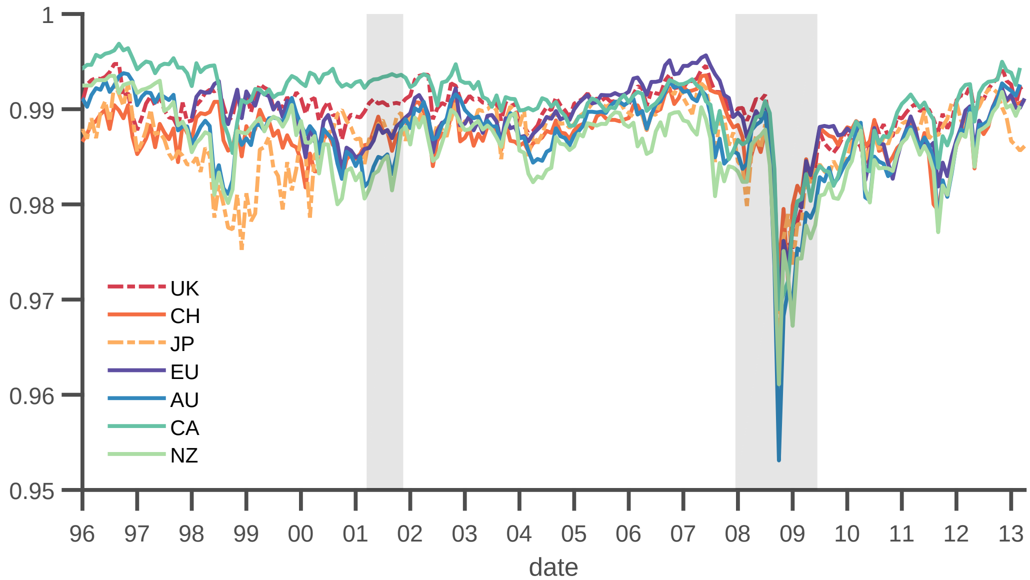


(a) Panel A: Domestic investor

(b) Panel B: Foreign investor

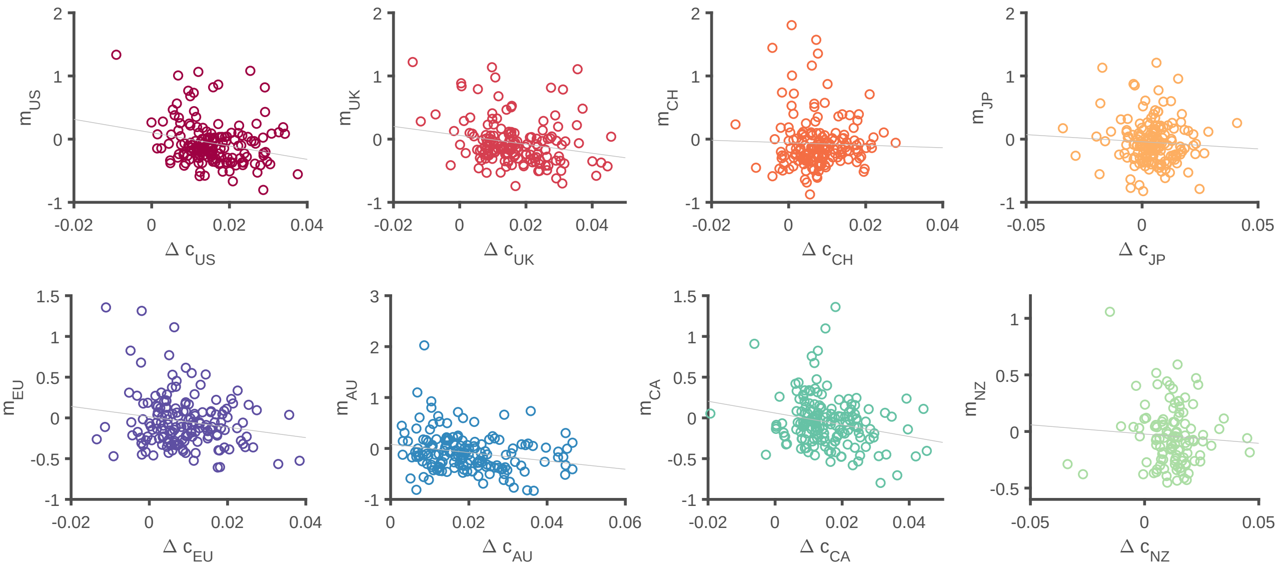
The figure plots for $i = d, f$ the observed exchange rate risk premium $E[R^e_{i0,t+1}] - E[R_{i0,t+1}]$ and the risk premium $-\text{cov}(M_{i,t+1}/E[M_{i,t+1}], R^e_{i0,t+1} - R_{i0,t+1})$ under the minimum entropy SDF $M_{i,t+1}$ against the average interest rate differential. Panel A reports the currency risk premium for the domestic investor ($i = d$), whereas Panel B for the foreign one ($i = f$). The domestic currency is the USD, while the foreign currencies are the GBP, the CHF, the JPY, the EUR, the AUD, the CAD and the NZD. Data is monthly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

Figure 3. Option-Implied SDF Similarity



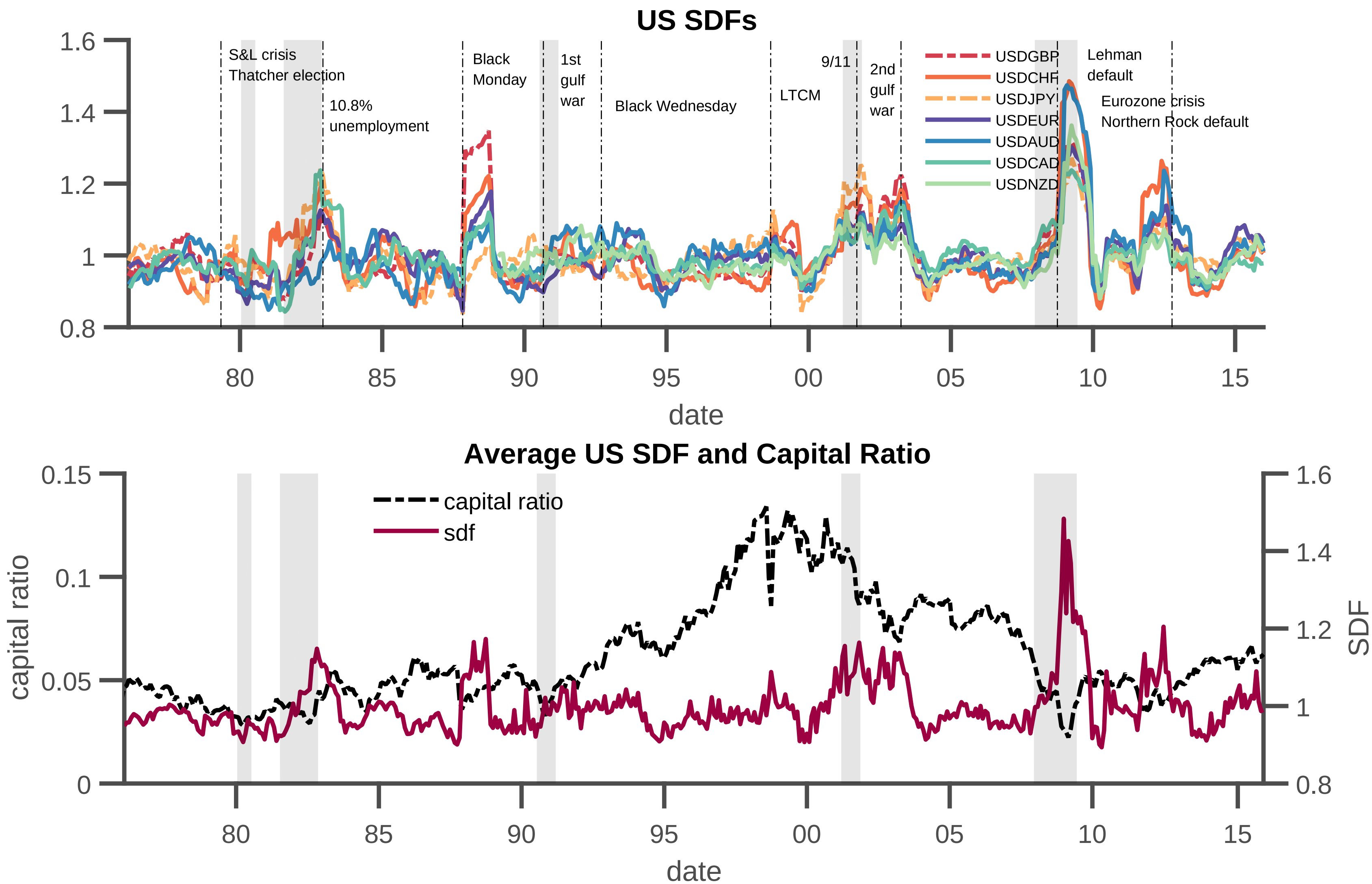
The figure plots option-implied SDF similarity measures from equation (21). Data is monthly and starts in January 1996 and ends in December 2013. AU = Australia, NZ = New Zealand, JP = Japan, CH = Switzerland, UK = United Kingdom, CA = Canada, EU = Eurozone. The US is always the domestic country. Gray shaded areas highlight recessions as defined by the NBER.

Figure 4. Log SDFs and log consumption growth



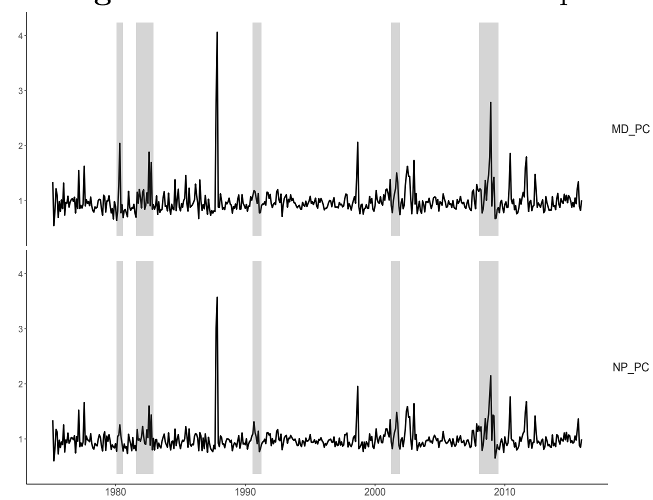
This figure plots (log) minimum entropy SDFs for the eight countries (y-axis) together with (log) consumption growth (x-axis) and the fitted least-square line. Data is quarterly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988.

Figure 5: US Stochastic Discount Factors and Intermediaries' Capital Ratio

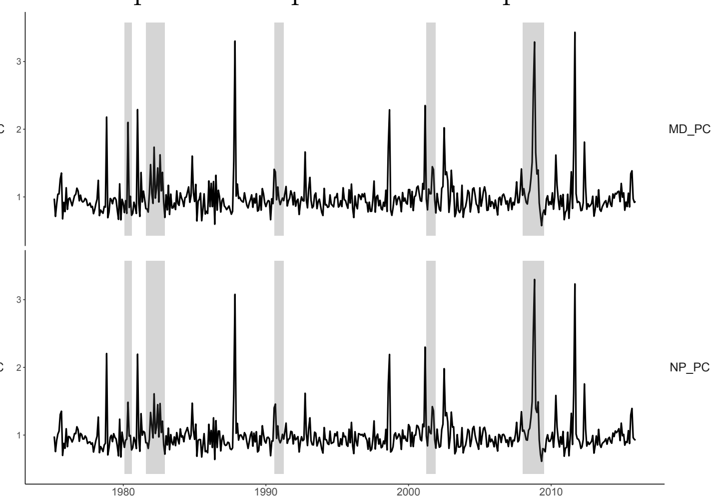


The figure plots US minimum entropy SDFs (upper panel) and the average SDF together with the capital ratio of He, Kelly, and Manela (2017) (lower panel). Data is monthly and runs from January 1975 to December 2015, except for New Zealand, for which the sample starts in January 1988. Gray shaded areas highlight recessions as defined by the NBER.

Figure 6. Time-series minimum dispersion and nonparametric permanent components



(a) USDGBP



(b) USDCHF

This figure plots the time-series of the minimum entropy permanent component (upper panel) and the nonparametric estimates of the permanent component (lower panel). The left panel depicts the domestic permanent components for the USDGBP currency pair, whereas the right panel illustrates the domestic permanent components of the USDCHF pair. Data is monthly and runs from January 1975 to December 2015. Gray shaded areas highlight recessions as defined by the NBER.

Online Appendix to "Model-Free International SDFs"
Not for Publication

Appendix OA-1 Non-parametric Estimates of SDF Components

In this section, we discuss the SDF factorization into transitory and permanent components. The seminal work of Hansen and Scheinkman (2009) characterizes SDFs decompositions in Markovian environments with a Perron-Frobenius eigenfunction problem. The permanent and transitory components are constructed from a given SDF process, the Perron-Frobenius eigenfunction and its eigenvalue. The eigenvalue determines the average yield on long-horizon payoffs and the eigenfunction characterizes the dependence of the prices on the Markov state. Sufficient conditions for existence of such an eigenfunction are provided in Hansen and Scheinkman (2009). Moreover, under an additional ergodicity assumption, the SDF factorization is unique and boils down to the factorization in Alvarez and Jermann (2005), which decomposes the SDF into discounting at the rate of return on the zero-coupon bond of asymptotically long maturity (the long bond) and a further risk adjustment via the martingale component.

We follow Christensen (2017) and estimate the solution to the Perron-Frobenius eigenfunction problem in Hansen and Scheinkman (2009) from time-series data on state variables and a given SDF process. For simplicity, we consider as a state variable the return on the long maturity bond and as a benchmark SDF the minimum entropy SDF estimated in Section 2.2. By deriving the eigenvalue and eigenfunction, we can reconstruct the time-series of the estimated non-parametric permanent and transitory components and compare them to our estimates based on the return of long, but finite horizon bond returns.

The estimation procedure builds upon a sieve approach, in which the infinite-dimensional eigenfunction problem is approximated by a low-dimensional matrix eigenvector problem. We use Hermite polynomials as basis functions and choose the smoothing parameter, i.e., the degree of the polynomial, k, equal to five. Under some regularity conditions, the solution to the eigenfunction and eigenvalue problem is unique and pinned down by the basis functions. We refer the interested reader to Christensen (2017) for the complete set of derivations and proofs. Here, $b^k \in L^2$ are the basis functions for the state variable X, $m(\cdot)$ denotes the SDF, ρ is the largest real eigenvalue and c_k is a vector in $\mathbb{R}^k$ such that the eigenfunction ϕ satisfies $\phi_k(x) = b^k(x)'c_k$. The sample counterparts of the matrices $\mathbf{G}_k$ and $\mathbf{M}_k$ are obtained by substituting the expected values with the sample averages, for a given time-series of the state variable $\{X_0, X_1, \dots, X_n\}$ and a given SDF process. In our case, the state variable is the return on the ten-year maturity bond and the SDF is given by the minimum entropy projection derived in Section 2.2.

Finally, given the eigenvector ρ and eigenfunction ϕ that solve the Perron-Frobenius problem, the permanent and transient components from time t to $t+\tau$ are given by:

$$\frac{M^P_{t+\tau}}{M^P_t} = \rho^{-\tau} \frac{M_{t+\tau}}{M_t} \frac{\phi(X_{t+\tau})}{\phi(X_t)}, \qquad \frac{M^T_{t+\tau}}{M^T_t} = \rho^{\tau} \frac{\phi(X_t)}{\phi(X_{t+\tau})}. \tag{A-1}$$

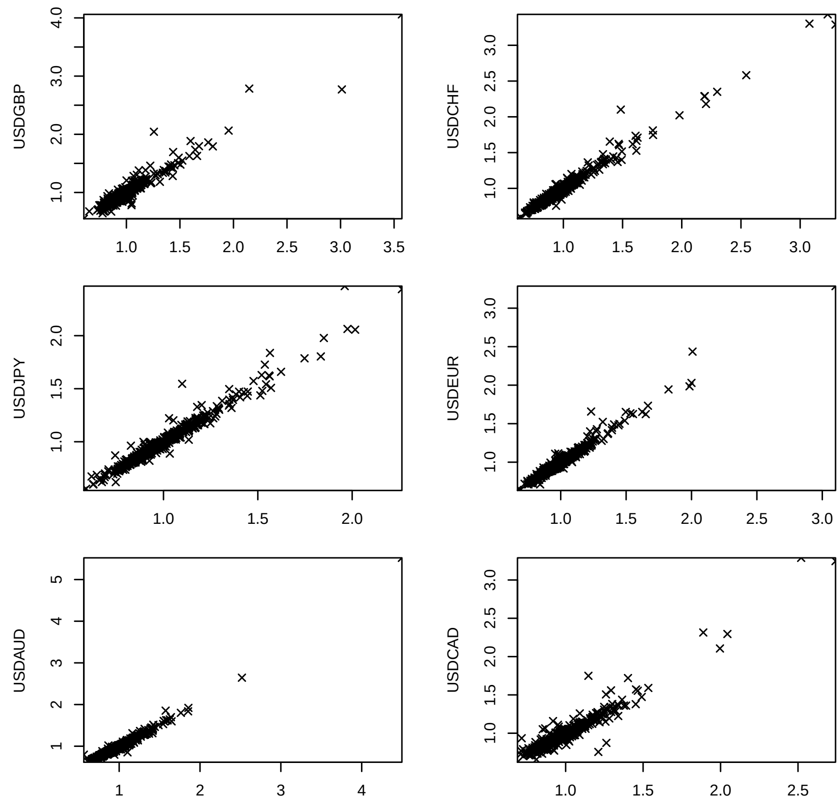
We summarize in Table OA-1 the properties of the non-parametric estimates of the permanent and transient components for the domestic minimum entropy SDFs. Consistent with our previous evidence, we find that the permanent component accounts for most of the variability of the overall SDF, whereas the transient component accounts for a much smaller fraction, which on average is close to the annualized volatility of our proxy for the long-term bond. These findings are supplemented by the almost perfect correlation between our permanent components and the corresponding non-parametric estimates. Such high correlations are illustrated by the scatter plots in Figure OA-1.

Table OA-1
Properties of SDFs components (Nonparametric estimates)

The table reports the annualized volatility of the nonparametric estimates of the domestic minimum-entropy SDF components ($\alpha_d = 0$). We use monthly data from January 1975 to December 2015.

	USDGBP	**USDCHF**	**USDJPY**	**USDEUR**	**USDAUD**	**USDCAD**	**USDNZD**
Std(M_d^P)	0.786	0.970	0.725	0.683	0.907	0.660	0.601
Std(M_d^T)	0.126	0.065	0.057	0.039	0.143	0.140	0.112

Figure OA-1. Minimum dispersion vs. nonparametric permanent SDF components



This figure plots the correlation between the nonparametric estimate of the permanent component (x-axis) and the minimum entropy permanent component (y-axis). Data is monthly and runs from January 1975 to December 2015.

Appendix OA-2 Financial Intermediary Wealth in Segmented Markets

The main text presents results where we regress model-free SDFs on proxies of intermediary wealth where the SDFs are estimated from symmetric markets. In this section, we present regression results when SDFs are estimated in segmented markets (see Section 2.3 in the main paper) in Table OA-2. Estimated coefficients are in line with those presented in the main paper (see Table 12). In particular, we find that higher changes in VIX load significantly and positively on SDFs while intermediary wealth loads negatively. We notice, however, slightly smaller R^2 compared to the symmetric trading case.

Table OA-2
SDFs, Value-at-Risk Constraints and Financial Intermediary Wealth under Segmented Markets

The table reports estimated coefficients from regressing minimum entropy SDFs derived in Section 2.3 on changes in financial intermediary wealth by He, Kelly, and Manela (2017) and changes in VIX:

$$M^i_{t+1} = \alpha^i + \beta^i_k \, \Delta\text{intermediary wealth}_{t+1} + \beta^i_v \, \Delta\text{VIX}_{t+1} + \epsilon^i_{t+1},$$

where $i = d, f$ denotes domestic and foreign quantities. T-statistics are computed using a circular block bootstrap of size 10 with $10,000$ simulations and reported in parenthesis.

	USDGBP	**USDCHF**	**USDJPY**	**USDEUR**	**USDAUD**	**USDCAD**	**USDNZD**
α	0.810	0.642	0.658	1.131	1.270	0.746	0.768
	(6.32)	(3.49)	(3.08)	(8.98)	(6.76)	(5.87)	(4.40)
β_k	-0.239	-0.280	-0.038	-0.404	-0.695	-0.150	-0.199
	(-1.35)	(-1.05)	(-0.02)	(-2.48)	(-2.91)	(-0.89)	(-0.80)
β_v	0.423	0.630	0.374	0.269	0.420	0.399	0.425
	(5.95)	(3.65)	(9.75)	(3.71)	(4.00)	(5.68)	(5.16)
R-Squared	0.36	0.37	0.20	0.27	0.32	0.33	0.29
	GBPUSD	**CHFUSD**	**JPYUSD**	**EURUSD**	**AUDUSD**	**CADUSD**	**NZDUSD**
α	1.842	2.539	1.194	0.969	0.955	0.756	0.689
	(15.06)	(13.18)	(10.70)	(11.02)	(5.01)	(6.87)	(2.39)
β_k	-0.896	-1.589	-0.125	-0.171	-0.294	-0.147	0.099
	(-8.06)	(-9.13)	(-1.48)	(-1.97)	(-1.48)	(-0.90)	(0.43)
β_v	0.052	0.051	-0.068	0.198	0.332	0.386	0.206
	(1.79)	(0.58)	(-1.58)	(4.07)	(5.55)	(5.23)	(3.64)
R-Squared	0.42	0.32	0.02	0.23	0.19	0.29	0.05