Structural Recovery of Face Value at Default

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Abstract

We carefully study the transmission mechanisms from default-free rates to corporate bond prices within structural models of endogenous default risk. The transmission critically depends on whether the model is value-based or EBIT-based, on the assumptions made for the drift of the state variable, and on the way the residual value at default is shared among bondholders. The recovery assumption is crucial: Recovery of Face Value, which entails receiving the same share of residual value at default regardless of the remaining maturity, greatly helps explaining the empirical evidence on bond-price sensitivities to interest rates.

JEL Classification: G12, G13, G33.

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1 Introduction

Three important stylized facts characterize the observed market valuation of corporate bonds that have either a speculative grade or a near-to-default outlook. (1) Same-seniority bonds with different remaining maturities have similar prices in the period leading to the default date (see for example the evidence in Figure 1 for Worldcom, Enron, and Lehman Brothers Holdings). (2) Junk bonds have low empirical durations (e.g. Cornell and Green (1991)). (3) The sensitivity of credit spreads to interest rates is negative and decreasing as credit quality declines (e.g. Duffee (1998), Collin-Dufresne et al. (2001); a similar evidence for CDS rates is reported by Ericsson et al. (2009), Bedendo et al. (2011) and Narayanan and Uzmanoglu (2018)). The significance of these facts for the risk management of corporate high-yield debt is conspicuous. However, to the best of our knowledge, their joint investigation has not been attempted yet.

[Figure 1 about here.]

We provide a unifying explanation of the three stylized facts by means of a structural model of endogenous corporate default that accounts for a heterogeneous finite-maturity debt composition and that employs either a value-based or a cashflow-based state variable. Chiefly, our joint explanation hinges on the five channels that render the corporate bond prices dependent on the default-free interest rate: i) discounting future payments; ii) the drift of the state variable that triggers default; iii) the endogenous default barrier; iv) the endogenous residual value at default; v) the recovery assumption that rules the share of residual value received at default by a maturity-specific bondholder.

We find that the channel v) is of great importance. We make the realistic assumption of Recovery of Face Value at default (RFV), which means that, at default, holders of bonds of the same issuer and seniority receive a share of the residual firm value equal to the ratio between the single-bond face value and the total notional amount of outstanding same-seniority debt, regardless of the remaining maturity. An alternative assumption is Recovery of Treasury (RT), which assumes that, at default, bondholders receive a share of the residual firm value equal to the ratio between the value of the default-free single-bond counterpart and the sum of the values of the default-free counterparts of the outstanding bonds. The RFV assumption is much more consistent with typical bond indenture language and with US bankruptcy law than the RT assumption.[1] Chiefly, we emphasize that RFV is distinctly relevant in explaining the three stylized facts. RFV implies by construction that same-seniority near-to-default bonds with different remaining maturities are very closely priced (fact (1)). As RFV renders the share of residual value recovered at default independent from the default-free rate, it makes junk bond prices less dependent on the default-free rate (fact (2)). Since the credit spread is given by the difference between the promised yield and the default free rate, the subdued interest-rate sensitivity of the junk bonds' promised yield implies that the sensitivity of the credit spread to the default-free rate becomes markedly negative (fact (3)). We highlight that the additional source of junk bond price stability, as the default-free rate increases, is the fall in the probability of default due to a possibly increasing drift of the state variable (channel ii)) and to a decreasing default barrier (channel iii)).

Our analysis brings a novel contribution to the existing literature on structural models of corporate default risk. Leland and Toft (1996) study endogenous corporate default risk in the presence of finite-maturity debt to shed light on the risk management of high-yield bonds. However, they restrict their analysis only to a specific state variable (the asset value) with a particular drift (default-free rate minus payout rate). Chiefly, they do not discuss the distinct transmission mechanisms that cause the bond price to depend on the default-free rate and their relative importance across the credit-quality spectrum. While the endogenous-default literature that considers

[1] RFV is a direct consequence of the debt acceleration clause in typical bond indentures. The claim acceleration clause implies that, at default, the principal amount and the accrued interest of the bonds outstanding is immediately payable (the accrued interest is also immediately payable but it is dwarfed by comparison with the principal amount). RFV is also in line with US bankruptcy law. The stage of the Chapter 11 process that most directly affects the relative recovery values of bonds within the same class is the classification of claims as proposed in the reorganization plan. Only substantially similar claims can be put into the same class, so that same-seniority claims are very likely to be grouped in the same class. Yet, within the same class, no specific provisions are made for dealing with the bond-specific contractual details, such as the maturity.

as the key state variable the EBIT level with an interest-rate independent drift has been growing (e.g. Goldstein et al. (2001), Hackbarth and Mauer (2011), Christensen et al. (2014) and Cai et al. (2017)),[2] our bond risk management results suggest caution in the use of a drift of the EBIT process that is totally unrelated to the default-free rate. We show that such a use brings about credit spreads at odds with the fact (3) as their interest-rate sensitivities are positive and decreasing in the credit quality. This is because, given a fixed initial EBIT level with an objective-probability drift that remains put, the default barrier as well as the objective probability of default increase in the default-free rate.

Another key contribution of our work is the comparison between different recovery assumptions in the context of a structural model of credit risk. Indeed, such a comparison has been made only within intensity-based models (e.g. Duffie and Singleton (1999), Delianedis and Lagnado (2002) and Bakshi et al. (2006)). Huang and Huang (2012) do not consider RFV in their study of credit spread underestimation across several structural credit risk models. While using RFV in their credit spread investigation, Feldhütter and Schaefer (2016) focus on exogenous structural default risk with a single bond outstanding. Several structural credit risk models (for example, in Longstaff and Schwartz (1995), Cathcart and El-Jahal (1996), Briys and De Varenne (1997) and Hsu et al. (2010)) rely on the RT recovery form. We show that, by rendering the share of residual value recovered at default interest-rate sensitive, RT constitutes an unpalatable source of bond price dependence on the default-free rate.

The remainder of the paper proceeds as follows. Section 2 describes the model, introduces the different specifications proposed and identifies the potential transmission mechanisms of the interest rate over bond prices. Section 3 presents a numerical analysis assessing the interest-rate dependence of bond prices, the bond duration and the credit spread sensitivity to interest rates. Section 4 concludes. Technical details are relegated to an Appendix.

[2] Flor (2008) develops a structural model where the firm value depends on both the processes for the asset value and the EBIT, which are allowed to have a non-unit correlation.

2 The model

In this section we describe the structural model of credit risk that allows to take into account different recovery assumptions. We describe the EBIT-based and the more traditional value-based specifications. In the first specification, the EBIT process determines the aggregate value of corporate claims, whereas, in the second one, the state variable is the value of firm's unlevered assets. We show how to introduce a heterogeneous debt structure, obtained with a continuous issuance of bonds with initial maturity T, and find the endogenous barrier maximizing the equity value at default. We then introduce the two different recovery hypotheses at default, RFV and RT. Finally, we discuss the transmission mechanisms of the interest rate over bond prices.

2.1 EBIT-based specification with finite-maturity debt

Under the risk-neutral probability measure, we assume that the EBIT process has the following dynamics:

$$\frac{\mathrm{d}\delta\left(t\right)}{\delta\left(t\right)} = \mu_{\delta}\mathrm{d}t + \sigma\mathrm{d}z_{t}, \tag{1}$$

where μ_δ is the constant drift of the EBIT growth process, for which we consider the two following specifications: i) $\mu_\delta = \mu - \sigma\lambda$, where μ is the drift under the objective probability measure and λ is the market price of risk; and ii) $\mu_\delta = r + \Lambda - \sigma\lambda$, where r is the constant interest rate, Λ is an offset of the objective-probability drift over the risk-free rate, and $\sigma\lambda$ is the risk premium. The first specification is rather standard in the literature of EBIT-based models, since the work by Goldstein et al. (2001), while the second is less common and accounts for the fact that the growth prospects of the earnings tend to follow the interest rate.[3] We develop the model by considering a generic drift μ_δ, introducing the two aforementioned specifications in the subsequent numerical analysis.

[3] Titman and Tsyplakov (2007) define a similar process for the price of the unique good produced by a firm. In particular, they model the market price of the production good as a geometric Brownian motion with risk-neutral drift $r - \alpha$, where r is the interest rate and α is a convenience yield.

The corporate tax rate is τ, while bankruptcy costs are a fraction α of the after-tax claim on EBIT at the moment of bankruptcy. The before-tax claim over the whole EBIT flow at time t can be obtained by integrating the risk-neutral expectation of the discounted EBIT flow:

$$\mathcal{V}(t) = \mathrm{E}_t \left[\int_t^{\infty} e^{-r(s-t)} \delta(s) \, \mathrm{d}s \right] = \frac{\delta(t)}{r - \mu_\delta}. \tag{2}$$

As a consequence, the claim on the EBIT flow has the same dynamics as $\delta(t)$:

$$\frac{\mathrm{d}\mathcal{V}(t)}{\mathcal{V}(t)} = \mu_\delta \mathrm{d}t + \sigma \mathrm{d}z_t, \tag{3}$$

and then, from (2), the instantaneous total return on the claim turns out to be:

$$\frac{\mathrm{d}\mathcal{V}(t) + \delta(t)\, \mathrm{d}t}{\mathcal{V}(t)} = r\mathrm{d}t + \sigma \mathrm{d}z_t. \tag{4}$$

In Appendix we show that the levered firm value can be written as:

$$v(\delta) = \frac{1-\tau}{r-\mu_\delta}\delta - \alpha \frac{1-\tau}{r-\mu_\delta}\delta_B \left(\frac{\delta}{\delta_B}\right)^{-x} + \tau \frac{C}{r}\left(1 - \left(\frac{\delta}{\delta_B}\right)^{-x}\right), \tag{5}$$

where $x = \frac{1}{\sigma^2}\left[\left(\mu_\delta - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu_\delta - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$.

The first term represents the after-tax value of the claim on firm's EBIT flow, the second term the expected present value of bankruptcy costs, while the third term represents the expected present value of the tax benefits over interest payments. The expression is formally equivalent to that found by Leland (1994), once considered that the quantity $\frac{1-\tau}{r-\mu_\delta}\delta$ represents the after-tax value of the claim on future EBIT, i.e. $(1-\tau)\mathcal{V}$, and that the exponent x in our model has a different expression, as it depends on the parameters of the EBIT process.

We draw from the work by Leland and Toft (1996), where the firm issues at each moment in time a density of principal of debt equal to $p = P/T$ with initial maturity T. The coupon is fixed and its density is $c = C/T$. Bonds are issued at par, so that the capital inflow associated to the issuance of debt nets out with the principal repayment of expired bonds. The density of debt value $d(\delta; \delta_B, t)$, for a bond with residual maturity equal to t, is equal to:

$$d(\delta; \delta_B, t) = \int_0^t e^{-rs} c(t) \left[1 - F(s; \delta, \delta_B)\right] \mathrm{d}s + e^{-rt} p(t) \left[1 - F(t; \delta, \delta_B)\right] + \int_0^t e^{-rs} \rho(t) (1-\tau) \mathcal{V}_B f(s; \delta, \delta_B) \mathrm{d}s,$$

where $\mathcal{V}_B = (1-\tau) \frac{\delta_B}{r - \mu_\delta}$ is the value of the claim on EBIT when δ equals the endogenously chosen default barrier δ_B, $f(t; \delta, \delta_B)$ is the probability density function of the time of first passage of δ through the barrier δ_B and $F(t; \delta, \delta_B)$ is the corresponding cumulative distribution function (cumulative failure probability). $\rho(t)$ is the density of residual value received in the event of bankruptcy by the holders of bonds with residual maturity t. Integrating:

$$d(\delta; \delta_B, t) = \frac{c(t)}{r} + e^{-rt} \left[p(t) - \frac{c(t)}{r}\right] \left[1 - F(t; \delta, \delta_B)\right] + \left[\rho(t)(1-\tau)\mathcal{V}_B - \frac{c(t)}{r}\right] G(t),$$

where:

$$F(t) = \mathcal{N}(h_1(t)) + {}^{-2ba}\mathcal{N}(h_2(t)), \tag{6}$$

$$G(t) = e^{-b(a-z)} \mathcal{N}(q_1(t)) + e^{-b(a+z)} \mathcal{N}(q_2(t)), \tag{7}$$

and:

$$h_1(t) = \frac{-b - a\sigma^2 t}{\sigma\sqrt{t}}, \quad h_2(t) = \frac{-b + a\sigma^2 t}{\sigma\sqrt{t}}, \quad q_1(t) = \frac{-b - z\sigma^2 t}{\sigma\sqrt{t}}, \quad q_2(t) = \frac{-b + z\sigma^2 t}{\sigma\sqrt{t}},$$

$$a = \frac{\mu_\delta - \frac{\sigma^2}{2}}{\sigma^2}, \qquad b = \log\left(\frac{\delta}{\delta_B}\right), \qquad z = \frac{\sqrt{(a\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

The solution is formally identical (up to the drift term in a and for the terms $1-\tau$ in (5)) to that in Leland and Toft (1996). The value of the total coupon C, and therefore of the density of coupon c, is obtained by considering an issuance at par of the debt with a maturity equal to T, that is by imposing $d(T) = p$. The endogenous default barrier, expressed in terms of EBIT, δ_B, maximizes the equity value at default and, again, can be derived in a similar fashion to Leland and Toft (1996):

$$\delta_B = \frac{r - \mu_\delta}{1 - \tau} \frac{\frac{C}{r}\left(\frac{A}{rT} - B\right) - A\frac{P}{rT} - \tau x \frac{C}{r}}{1 + \alpha x - (1 - \alpha) B}, \tag{8}$$

where $x = a + z$ and the coefficients A and B are equal to:

$$A = 2ae^{-rT}\mathcal{N}\left(a\sigma\sqrt{T}\right) - 2z\mathcal{N}\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}}n\left(z\sigma\sqrt{T}\right) + \frac{2e^{-rT}}{\sigma\sqrt{T}}n\left(a\sigma\sqrt{T}\right) + z - a, \tag{9}$$

and

$$B = -\left(2z + \frac{2}{z\sigma^2 T}\right)\mathcal{N}\left(z\sigma\sqrt{T}\right) - \frac{2}{\sigma\sqrt{T}}n\left(z\sigma\sqrt{T}\right) + z - a + \frac{1}{z\sigma^2 T}. \tag{10}$$

2.2 Value-based specification with finite-maturity debt

A value-based model considers the unlevered firm's assets, V, as state variable. The risk-neutral dynamics of V is:

$$\frac{\mathrm{d}V(t)}{V(t)} = \mu_V \mathrm{d}t + \sigma \mathrm{d}z_t, \tag{11}$$

where μ_V is a constant drift parameter for which we consider two different specifications: i) as in e.g. Leland (1994) $\mu_V = r - \delta$ (δ is a constant payout ratio that incorporates taxes and dividend payments), which is consistent with the assumption that the firm's unlevered assets are tradable; and ii) $\mu_V = \mu - \sigma\lambda$.

As derived in Leland and Toft (1996), the firm value is in this case given by:

$$v(V,V_B,t) = (1-\tau)V - \alpha(1-\tau)V_B\left(\frac{V}{V_B}\right)^{-x} + \tau\frac{C}{r}\left(1-\left(\frac{V}{V_B}\right)^{-x}\right), \tag{12}$$

where $x = \frac{1}{\sigma^2}\left[\left(\mu_V - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu_V - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right]$. The value of a bond with a density of principal $p(t)$, a density of coupon $c(t)$ and maturity t is:

$$d(V,V_B,t) = \frac{c(t)}{r} + e^{-rt}\left[p(t) - \frac{c(t)}{r}\right]\left[1 - F(t;V,V_B)\right] + \left[\rho(t)V_B - \frac{c(t)}{r}\right]G(t).$$

The expressions for $F(t)$ and $G(t)$ are respectively given in (6) and (7), where the only difference is that in this case a and b are related to the parameters of the process V rather than the EBIT process, and thus:

$$a = \frac{\mu_V - \frac{\sigma^2}{2}}{\sigma^2}, \qquad b = \log\left(\frac{V}{V_B}\right).$$

The optimal default barrier is given by:

$$V_B = \frac{\frac{C}{r}\left(\frac{A}{rT} - B\right) - A\frac{P}{rT} - \tau x\frac{C}{r}}{1 + \alpha x - (1-\alpha)B}, \tag{13}$$

where again $x = a + z$ and A and B are respectively given in (9) and (10).

2.3 Accounting for different recovery policies at default

In the work by Leland and Toft (1996) the recovery assumption is the Recovery of Face Value (RFV). This is implicit in the assumption of a constant density for the fraction of asset value received by a bondholder whose bond has a maturity equal to t, that is:[4]

$$\rho(t) = \frac{1-\alpha}{T}. \tag{14}$$

[4] See equations (2) and (3) in Leland and Toft (1996).

In order to make a comparison between different recovery assumptions, we want to consider also the case of Recovery of Treasury (RT), which corresponds to the case where the density for the fraction of asset value received in case of default is proportional to the treasury value of the promised payments. The density of value of a default-free coupon bond, with density of principal $p = P/T$, density of coupon $c = C/T$ and maturity t, is given by:

$$\frac{c}{r} + e^{-rt}\left(p - \frac{c}{r}\right).$$

Integrating from 0 to T, one obtains the total valuation of a default-free debt with the same payment schedule of the corporate defaultable debt:

$$\int_0^T \left(\frac{c}{r} + e^{-rt}\left(p - \frac{c}{r}\right)\right) \mathrm{d}t = T\left(\frac{c}{r} + \left(p - \frac{c}{r}\right)\frac{1 - e^{-rT}}{rT}\right).$$

The density of recovery $\rho(t)$ must be thus equal to the ratio of the density of present value corresponding to a bond of maturity t and the integral above, multiplied by the scalar that accounts for bankruptcy costs, $1 - \alpha$:

$$\rho(t) = \frac{1}{T}\frac{\frac{c}{r} + e^{-rt}\left(p - \frac{c}{r}\right)}{\frac{c}{r} + \left(p - \frac{c}{r}\right)\frac{1-e^{-rT}}{rT}}\left(1 - \alpha\right). \tag{15}$$

We want to compare the pricing properties of coupon bonds when two different redistribution policies, RFV and RT, are applied in case of default. Remember that the yield-to-maturity $y(t)$ of a continuous-coupon bond is implicitly defined through the following relation:

$$d = \int_0^t e^{-y(t)s} c \mathrm{d}s + e^{-y(t)t}p = \frac{1 - e^{-y(t)t}}{y(t)}c + e^{-y(t)t}p.$$

The recovery rate of a bond of maturity t is given by $\frac{\rho(t)V_B}{p}$ for the value-based case and by $\frac{\rho(t)\mathcal{V}_B}{p}$ for the EBIT-based case, being thus constant for RFV and maturity-dependent for RT.

2.4 Main drivers of interest rate sensitivity of bond prices

Before developing a numerical analysis, in the table below we identify, for each of the four model specifications considered, the potential transmission mechanisms of variations of the interest rate r over corporate bond prices. i) The interest rate has a discounting effect over any future payment, such that an increase of r determines a bond price drop. For a non-defaultable bond, which in our model is obtained for $\delta \to \infty$ or $V \to \infty$, this is the only effect at play. ii) For the models where the drift of the state variable depends on the interest rate, an increase of r tends to make default less likely, by increasing the drift away from any lower boundary (including the default barrier). iii) The default barrier, in terms either of asset value or EBIT, depends on r in every model because it is endogenously chosen by the shareholders. The exact direction of variation is not trivial to be identified ex-ante from (8) and (13) and will be studied numerically in the next section. iv) In EBIT-based models, the value of the claim on future EBIT is $\mathcal{V} = \frac{\delta}{r-\mu_\delta}$ and thus depends on the interest rate, unless μ_δ shifts one-to-one with r. This dependence does not show in value-based models, as the barrier is directly expressed in terms of asset value. And, finally, v) when the recovery assumption is RT, the residual value after default, which is $(1-\alpha)\,V_B$ for value-based models and $(1-\alpha)\,(1-\tau)\,\mathcal{V}_B$ for EBIT-based models, is redistributed to bondholders proportionally to the risk-free valuation of promised payments, which does depend on r. The following table-like synopsis summarizes the five transmission mechanisms across model types and recovery forms.

Propagation channel of interest rate r over bond prices				
Model family	Value-based (state variable V)		EBIT-based (state variable δ)	
Drift rate of the state variable	$\mu_V = \mu - \sigma\lambda$	$\mu_V = r - \delta$	$\mu_\delta = \mu - \sigma\lambda$	$\mu_\delta = r + \Lambda - \sigma\lambda$
Cashflow discounting	✓	✓	✓	✓
Drift of variable triggering default		✓		✓
Default barrier (V_B or δ_B)	✓	✓	✓	✓
Residual value given V_B or δ_B			✓	
Share of residual value	✓ (RT only)	✓ (RT only)	✓ (RT only)	✓ (RT only)

The combination of the aforementioned factors determines in a nontrivial fashion the sensitivity of corporate bond prices to interest rate, which we study for each of the proposed specifications of the model in the following section.

3 Numerical analysis

In this section, after discussing the base case parameter values, we perform a numerical analysis aimed at assessing the interest rate sensitivities of bond prices and credit spreads for the different specifications of the model proposed, identifying the transmission mechanisms and pointing out the relevant implications for bond portfolio management.

3.1 Parameter values

We take the base case parameter values shown in Table 1a. The constant interest rate r is equal to 0.03. For the specifications with a constant drift μ for the asset value or EBIT processes, we consider a value of 0.06. When the risk-neutral drift of the asset value process V is $r - \delta$, the constant payout ratio δ is chosen to be equal to 0.025. When the drift of the EBIT process shifts one-to-one with the interest rate r, we consider an offset $\Lambda = 0.03$, so that at the base case value for μ_δ is the same in both specifications. The initial asset value V_0 as well as the initial value of the claim on future after-tax EBIT $(1-\tau)\,\mathcal{V}_0$ are chosen to be equal to 1.[5] The market price of risk, λ, is equal to 0.25. The fractional bankruptcy costs are $\alpha = 0.35$, which is consistent with the observation by Andrade and Kaplan (1998) and Huang and Huang (2012) that the value of 0.5, often employed in the literature of structural models of credit risk, is too high. The bond maturity at issuance $T = 30$ and the corporate tax rate $\tau = 0.35$.

According to the credit rating of the bond considered, we rely on the values in Table 1b for the volatility σ of the processes for the unlevered firm's assets or for the EBIT, as well as for the leverage ratios (equal to the ratio $\frac{P}{V_0}$ for the value-based specifications and $\frac{P}{(1-\tau)\mathcal{V}_0}$ for the EBIT-based specifications), which we take from the median values estimated by Feldhütter and Schaefer (2016) over the period 1985–2015.

[Table 1 about here.]

3.2 Dependence of bond prices on the interest rate

Figure 2 shows the relation between the interest rate r and the bond price across the four model specifications considered. It is important to stress that we are considering bonds that have been issued at par for the interest rate at its base case value, which pins down the amount of the coupon c, and that only after issuance the interest rate r is modified. We show the numerical results for both the recovery assumptions RFV and RT, considering both an investment-grade firm (credit rating A) and a non-investment-grade firm (credit rating B).

[Figure 2 about here.]

The price of the A-rated bond mainly depends on the risk-neutral valuation of its promised payments, as the credit risk of such a bond is very low. This is confirmed by the very low default

[5] If the initial value of the claim on future after-tax EBIT is $(1-\tau)\,\mathcal{V}_0$, then the initial value for the EBIT process is $\delta_0 = \frac{r-\mu_\delta}{1-\tau}\mathcal{V}_0$.

probability shown in Figure 3. The consequence is that there is a marked negative price sensitivity to interest rate variations, showing also the typical (positive) convexity of default-free bond prices. This is verified for all four model specifications considered, as well as for both the recovery assumptions RFV and RT, as the recovery assumption is relevant only in case of default.

[Figure 3 about here.]

For B-rated firms, the behavior of the bond prices depends on the specification considered. For both value-based models and the EBIT-based model with an r-dependent drift, respectively in Panels (a), (b) and (d) of Figure 2, and especially for the RFV assumption, the slope of the bond price is much lower in absolute value than the slope for the A-rated bond, and the convexity almost disappears. This phenomenon is partly documented by Leland and Toft (1996), as they focus only on the specific asset-based case with $\mu_V = r - \delta$ (see our Panel (b)) and shun from an exact investigation of the different r-dependence channels that contribute to the phenomenon itself. The channels we have highlighted in Section 2.4 are five: a cashflow-discounting effect, a drift effect on the probability of default, a separate effect on the default probability due to the endogeneity of the default barrier, an effect of the endogenous barrier on the residual value at default and, finally, a recovery-form effect (RFV vs RT).

The subdued dependence of the bond price on r that is evident in Panels (a), (b) and (d) of Figure 2 stems from the dominance of the endogenous default barrier effect on the default probability. As r increases, the default barriers (see the corresponding panels in Figure 4) decrease, thus depressing the probability of default (see the corresponding panels in Figure 3). This key effect is compounded by an increasing drift μ_V and μ_δ away from the default barrier in Panels (b) and (d). Notice that, as far as bond pricing is concerned, the endogenous default barrier effect on the default probability overwhelms the slightly negative dependence on r of the residual value at default (see Panels (a), (b) and (f)). The conspicuous recovery-form effect (see Figures 2 and 3) is discussed later in the section.

[Figure 4 about here.]

Importantly, the intriguing results in the Panels (c) of Figures 2, 3 and 4 highlight the relevance of a careful discussion of the different r-dependence channels. The EBIT-based Leland and Toft (1996) model with drift $\mu_\delta = \mu - \sigma\lambda$ in the Panels (c) is consistent with a markedly negative slope and with a significant convexity of junk bond prices: the behavior of the B-rated bond price is almost indistinguishable from that of the A-rated bond. Our multi-channel analysis provides a ready explanation of this peculiar phenomenon. Differing from the other panels of Figure 3, Panel (c) shows that the probability of default is an increasing function of r. This stems from the fact that the optimal default barrier δ_B is an increasing function of r, as can be noticed in Figure 4, which is due to the presence of the multiplicative term $r - \mu_\delta$ in (8). The term $r - \mu_\delta$ is independent of r for the specification where $\mu_\delta = r + \Lambda - \sigma\lambda$ and is not present for the value-based barrier V_B in (13). It is also noteworthy that, similarly to what happens for the asset-based default barrier V_B, the value of the claim on the after-tax EBIT at default, $(1 - \tau)\,\mathcal{V}_B$, is a decreasing function of r for all the specifications considered. For an increasing r, the combination of a higher default probability (see Panel (c) of Figure 3) and a lower residual value at default (see Panel (e) of Figure 4) makes the EBIT-based specification with $\mu_\delta = \mu - \sigma\lambda$ very sensitive to interest rate variations even for junk bonds, thus rendering their price dependence on r more similar to that obtained for investment-grade bonds, with a strong negative slope and a positive convexity. This has strong implications when this kind of model, often invoked by the recent literature of structural models of credit risk, is employed in bond portfolio management.

Finally, we look at the impact of the recovery assumption. As can be noticed in Figure 2, there is a tangible difference in terms of pricing between the two recovery forms for non-investment-grade bonds. For the B-rated bonds, the RFV assumption always implies a lower price sensitivity to interest rate variations. This effect is explained by looking at Figures 5 and 6, which show the fraction $\rho\,(t)$ of the before-default value received by holders of bonds with maturity t, respectively for $t = 30$ years (the bond with the longest maturity) and for $t = 5$ years. The RFV assumption makes long-maturity bonds, whose default-free counterparts have values that are very negatively affected by a rise in interest rates, to receive the same r-independent fraction of the residual value at default of short-maturity bonds. This conspicuously reduces the net sensitivity to variations in r of long-maturity bonds. In contrast, under the RT assumption, the long-maturity (short-maturity) bondholders receive a lower (higher) fraction of the residual value at default when r is high. This makes, for both value-based and EBIT-based models, long-maturity bonds priced with the RFV assumption definitely less sensitive to interest rate variations than bonds priced employing the RT assumption.

[Figure 5 about here.]

[Figure 6 about here.]

3.3 Modified duration vs. classical modified duration

In this section we study the implications of the previous findings in the context of bond portfolio management, where the interest rate sensitivities of bond prices, measured in terms of duration, are crucial. Portfolio managers with investment-grade benchmarks are often allowed out-of-benchmark allocations to high-yield debt. Thus, understanding the effect such allocations have on the overall portfolio duration is important. It is widely acknowledged that the default-free rate sensitivity of high-yield securities is not necessarily what their promised cash flows imply. Indeed, it is well known empirically that the duration of a bond tends to decline as default becomes imminent. For example, Cornell and Green (1991) show that the empirical duration (i.e., the return realized per unit of default-free yield change) of low-grade bonds is lower than that for high-grade bonds and suggest that this may be partly due to the fact that coupons are higher for low-grade bonds.[6] We

[6] Dynkin et al. (2004) confirm this finding with daily data over the period August 1998 - September 2004. They regress daily price returns of whole-letter-grade components of the Lehman Investment-Grade and High-Yield Credit indices against daily changes in the 10-year U.S. Treasury yield.

show that the specification of the structural model and the recovery form assumed deeply affects the model sensitivities to default-free rates, in particular for low-grade bonds. We consider the negative semi-elasticity of the model bond price d with respect to the default-free interest rate r, known as the modified duration (MD):

$$\mathrm{MD} = -\frac{1}{d}\frac{\partial d}{\partial r}.$$

The classical modified duration (CMD), instead, is the negative semi-elasticity of the model bond price with respect to its promised yield. Given the bond price d and its promised yield y, the credit spread s is:

$$s = y - r.$$

The modified duration and classical modified duration in this case are related by this important equation:

$$-\frac{1}{d}\frac{\partial d}{\partial r} = -\frac{1}{d}\frac{\partial d}{\partial y}\left(1 + \frac{\partial s}{\partial r}\right),$$

that is:

$$\mathrm{MD} = \mathrm{CMD} \cdot \left(1 + \frac{\partial s}{\partial r}\right). \tag{16}$$

[Figure 7 about here.]

[Figure 8 about here.]

[Figure 9 about here.]

[Figure 10 about here.]

For investment-grade credit quality, we show in Figures 7 (value-based specifications) and 8 (EBIT-based specifications) the bond price, the modified and classical modified durations, and the credit spread sensitivity to the interest rate. The modified durations MD are similar for the value-based (Figure 7) and the EBIT-based specification with $\mu_\delta = r + \Lambda - \sigma\lambda$ (Figure 8b), while MD is higher for the EBIT-based specification with $\mu_\delta = \mu - \sigma\lambda$ (Figure 8a), albeit the CMD being very similar in all four cases. For what concerns the B-rated bond, which durations are shown respectively in Figures 9 (value-based) and 10 (EBIT-based), the MD for the same recovery form are again roughly aligned for the value-based models and the EBIT-based specification with $\mu_\delta = r + \Lambda - \sigma\lambda$, while the MD are substantially higher (about twice as much for a 30-year bond) for the EBIT-based specification with $\mu_\delta = \mu - \sigma\lambda$. Note that, again, the CMD are very similar in all four cases. The higher MD obtained for the EBIT-based specification with $\mu_\delta = \mu - \sigma\lambda$ is a direct consequence of the higher r-dependence of the bond price, which we documented in Figure 2c.

It is then interesting to consider the differences between the two different recovery forms, RT and RFV. For the A-rated bonds, given the specification for the drift, the modified durations are essentially the same for the two recovery forms, due to default being an unlikely event. However, as the credit quality decreases, the modified durations implied by the two recovery forms diverge. When default is close, bondholders expect to receive imminently the recovery payment specified by the recovery form. Bondholders receive a fixed share of the residual value at default under RFV (see $\rho(t)$ in (14)), whereas they receive a default-free-rate-dependent share under RT (see $\rho(t)$ in (15)). In our example for the non-investment-grade bond and as long as the maturity considered is sufficiently long (at least 5 years), the modified bond duration decreases substantially more under RFV than under RT. Hence, by specifying a default-free-rate-insensitive share of the residual value at default, structural RFV greatly helps matching the stylized fact that empirical durations for low-grade bonds are low.

As shown in (16), there is an important quantitative link between duration and the sensitivities of credit spreads to the default-free rate. An empirical measure of such sensitivities is provided by the slope coefficients in the regressions of changes in corporate yield spreads on changes in

Treasury bond rates performed by Duffee (1998) and by Collin-Dufresne et al. (2001). Duffee (1998) considers monthly data (January 1985 through March 1995) for noncallable bonds issued by US industrial, utility, and financial firms, and performs regressions by rating group in which the effect of the default-free term structure slope is controlled for. He finds that corporate bond yield spreads move inversely with short default-free rates (the 3-month Treasury yield) and that the inverse relation is stronger for lower-quality bonds. Collin-Dufresne et al. (2001) consider monthly data (July 1988 through December 1997) for noncallable bonds issued by US industrial firms, and perform regressions by leverage, as well as by rating group, in which they control for the effect of several structural model determinants of credit spread changes (including the default-free term structure slope). Consistent with the empirical findings of Duffee (1998), they find that an increase in the default-free rate (the 10-year Benchmark Treasury yield) lowers the credit spread for all bonds. Furthermore, the sensitivity to default-free rates increases monotonically across both leverage and rating groups.[7] In summary, there is robust empirical evidence that the regression coefficient of the credit spread changes on the default-free rate level changes becomes more negative as credit quality decreases.

In the following, we assess how the model specification and the choice of the recovery form affect the ability of the structural model to match this empirical evidence. We start by rearranging (16) to note that the spread-rate slope is related to MD and CMD (see also Acharya and Carpenter (2002), Equation 19, p. 1370):

$$\frac{\partial s}{\partial r} = \frac{\mathrm{MD}}{\mathrm{CMD}} - 1. \tag{17}$$

If the modified duration is low, then the credit spread sensitivity $\frac{\partial s}{\partial r}$ must be quite negative. Different model specifications as well as different recovery assumptions have significant implications for the model-implied modified duration.

[7] Dynkin et al. (2004) focus on more recent daily data and find similar results in performing regressions by rating group on 10-year Treasury yield changes.

[Figure 11 about here.]

We show in Figure 11 the model-implied credit spread sensitivities to default-free rates for a 30-year bond versus S&P rating groups (the range is B through AAA). These are represented for the four specifications and the two recovery forms considered, and are shown alongside the empirical sensitivities estimated by Collin-Dufresne et al. (2001).[8] The value-based and EBIT-based specifications that let the drift depend on the interest rate r seem to be those that better match the negative empirical sensitivities (Figure 11b and 11d), while the value-based specification with drift $\mu_V = \mu - \sigma\lambda$ (Figure 11a) provides negative sensitivities that are lower in absolute value than the empirical ones measured for B-rated bonds. By contrast, the EBIT-based model with drift $\mu_\delta = \mu - \sigma\lambda$ is completely at odds with the empirical evidence, as it provides positive credit spread sensitivities to the interest rate across all credit ratings considered. This is documented also, for all residual maturities from 0 to 30 years, in the upper-right graphs of Figures 8a and 10a, and stems from the values of the modified durations (bottom-left graphs), which are higher than the respective classical modified durations (bottom-right graphs). This fact entails via (17) a positive credit spread sensitivity to r.

For the three specifications providing a spread sensitivity to r with a negative sign, the RFV assumption significantly helps reproducing the empirically-observed sharp increase in the slope magnitude for non-investment-grade bonds by squeezing the magnitude of MD relative to CMD. Hence, the credit spread sensitivity evidence adds to the unpalatability of the RT assumption.

4 Conclusion

The observed market prices of non-investment-grade corporate bonds lead to three robust stylized facts that are of significant interest for bond risk managers. Firstly, market valuations of near-

[8] In Figure 11, the empirical credit spread sensitivities are the slope coefficients in the regressions of changes in long-dated corporate yield spreads on changes in the 10-year Treasury yields performed by Collin-Dufresne et al. (2001) across the credit ratings (see the fifth row in their Table III, Panel C, p. 2189).

to-default bonds of the same firm and seniority are substantially equal, irrespectively of their residual maturities. Secondly, junk bonds have a significantly lower duration than their default-free counterparts. And, thirdly, junk bonds are characterized by a negative credit spread sensitivity to interest rates, which tends to decrease with the lowering of the credit rating. How interconnected are these seemingly unrelated stylized facts?

We aim at providing a joint explanation of the three facts by using a structural endogenous-default model of corporate credit risk to examine a firm that issues a debt structure which, at a given point in time, is heterogeneous in terms of residual bond maturity. We take the state variable to be either the unlevered asset value or the EBIT. We identify five different channels of propagation of the interest rate onto bond prices: i) a cashflow-discounting effect, which is the unique mechanism at play in default-free bonds and is dominant for investment-grade bonds; ii) an effect on the drift of the default-triggering state variable, which makes the default less likely when the drift rate increases corresponding to an increase of the default-free rate; iii) an effect on the endogenous default barrier, which tends to decrease in level and makes the default less likely for all specifications considered, except when, for an EBIT-based specification, the drift is assumed to be independent of the default-free rate; iv) an effect on the residual value of the firm at default, which tends to decrease for an increasing default-free rate; and v) an effect due to the recovery form, that is the assumption on how the residual value at default is shared among the bondholders.

We find that the channel v) is of great relevance. We implement two different recovery forms, Recovery of Face Value (RFV) and Recovery of Treasury (RT). While both forms have been widely used in structural credit risk models, their bond risk management implications have not been studied yet. RFV is realistic as it is grounded in typical bond indenture language and in US bankruptcy law. Importantly, RFV not only provides a natural explanation for the observed similar valuation of near-to-default bonds (namely the first stylized fact) but it is also crucial in explaining the empirical evidence related to hedging high-yield bonds against default-free rate changes (low duration and large negative credit spread sensitivity to interest rates, namely the other two stylized facts). By contrast, RT is much less palatable, as it warps bond prices and their sensitivities to the interest rate by assuming a share of the residual value at default that depends on the single-bond residual maturity as well as on the default-free rate.

We show that the effects on the drift of the state variable (channel ii)) and on the level of the endogenous default barrier (channel iii)) are also tangible. In particular, in EBIT-based models with a constant drift of the state variable, we identify an undesirable interest-rate dependence of the default barrier and of the objective probability of default, which are both increasing functions of the default-free rate. Distortions can follow (e.g. a positive credit spread sensitivity to interest rates, which tends to even increase with the lowering of the credit rating).

Our findings have direct relevance for the risk management of low-credit quality corporate debt and of related derivatives. Chiefly, our analysis carefully quantifies how much the price sensitivity to the default-free rate for non-investment-grade corporate debt is affected by the boundary conditions engendered by the recovery assumption. Our results in the presence of a constant default-free rate provide useful reference points, around which future research that keeps full account of interest rate risk can be articulated.

Appendix

Firm value for EBIT-based specification

Due to the time-independent nature of the problem that we consider, the fair value X of any claim written on the EBIT process must satisfy the following ODE:

$$rX(t) = \Pi(t) + X_\delta(t)\,\mu_\delta \delta(t) + \frac{1}{2} X_{\delta\delta}(t)\,\sigma^2 \delta^2(t)\,, \qquad (18)$$

where $\Pi(t)$ is the payout rate of the claim. The solution depends on the particular value of $\Pi(t)$ and on the boundary conditions for the specific claim to be priced. The general solution to the homogeneous problem

$$rX = X_\delta \mu_\delta \delta + \frac{1}{2} X_{\delta\delta} \sigma^2 \delta^2,$$

is:

$$X_{GS} = A_1 \delta^{-x} + A_2 \delta^{-y},$$

where:

$$x = \frac{1}{\sigma^2}\left[\left(\mu_\delta - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu_\delta - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right],$$

and

$$y = \frac{1}{\sigma^2}\left[\left(\mu_\delta - \frac{\sigma^2}{2}\right) - \sqrt{\left(\mu_\delta - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\right].$$

Consider the value of the levered firm, v, in the case where the total coupon paid to bondholders is constant and equal to C. We assume that the value of the firm at the default barrier δ_B is equal to the after-tax value of the claim over the EBIT flow minus the bankruptcy costs:

$$v(\delta_B) = (1-\alpha)(1-\tau)\frac{\delta_B}{r-\mu_\delta}.$$

The value of the firm, for a very large δ, is equal to the after-tax value of the claim over the EBIT flow plus the tax savings for interest (coupon) payments:

$$v(\delta \to \infty) = (1-\tau)\frac{\delta}{r-\mu_\delta} + \tau\frac{C}{r}.$$

This quantity can be taken as a particular solution of the ODE (18), corresponding to the particular case where the payout is $\Pi(t) = \tau C + (1-\tau)\delta(t)$. This gives the solution provided in (5).

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Table 1: Model parameters.

(a) Base case model parameters

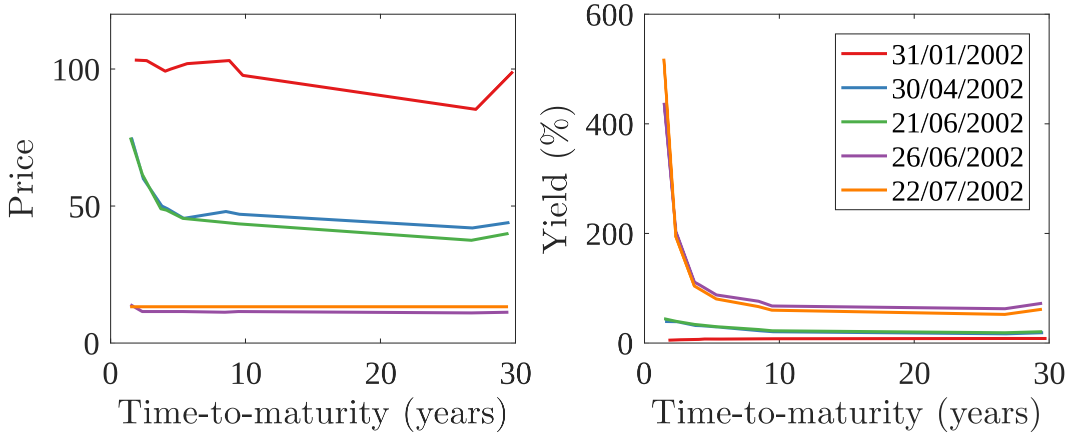
Model family	Value-based (state variable V)		EBIT-based (state variable δ)	
Drift rate of the state variable	$\mu_V = \mu - \sigma\lambda$	$\mu_V = r - \delta$	$\mu_\delta = \mu - \sigma\lambda$	$\mu_\delta = r + \Lambda - \sigma\lambda$
Nominal interest rate: r	0.03	0.03	0.03	0.03
Growth prospects: μ	0.06		0.06	
Offset of growth prospects: Λ				0.03
Payout ratio: δ		0.025		
Market price of risk: λ	0.25	0.25	0.25	0.25
Initial asset value: V_0	1	1		
Initial all-equity value: $(1-\tau)\,\mathcal{V}_0$			1	1
Tax rate: τ	0.35	0.35	0.35	0.35
Bankruptcy costs: α	0.35	0.35	0.35	0.35
Maturity of newly issued debt: T	30	30	30	30

(b) Volatility of EBIT / unlevered firm's assets and leverage ratios for different credit ratings

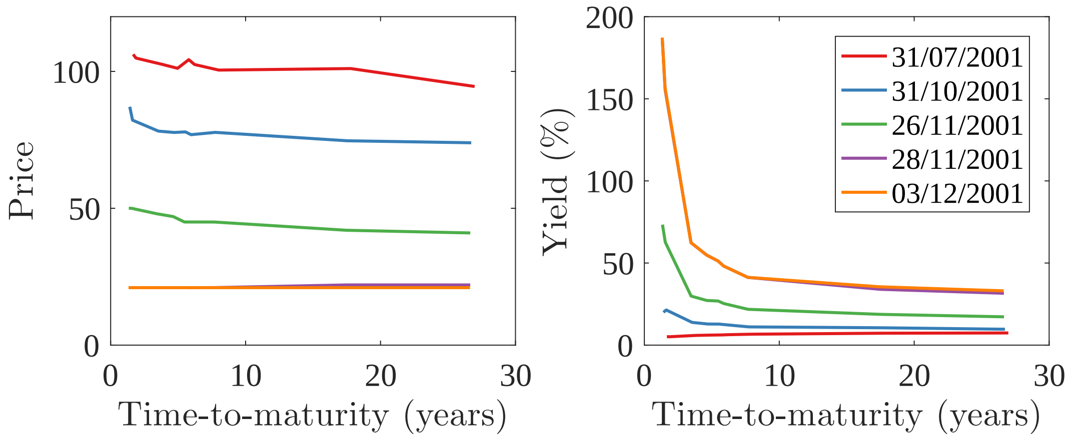
S&P credit rating	EBIT/asset volatility (σ)	Leverage ratio ($\frac{P}{V_0}$)
AAA	0.23	0.07
AA	0.24	0.11
A	0.24	0.17
BBB	0.27	0.25
BB	0.30	0.37
B	0.32	0.53

Figure 1: Bond prices and yields before Worldcom's, Enron's and Lehman's defaults for several observation dates.

(a) Worldcom Inc.



(b) Enron Corp.



(c) Lehman Brothers Holdings Inc.

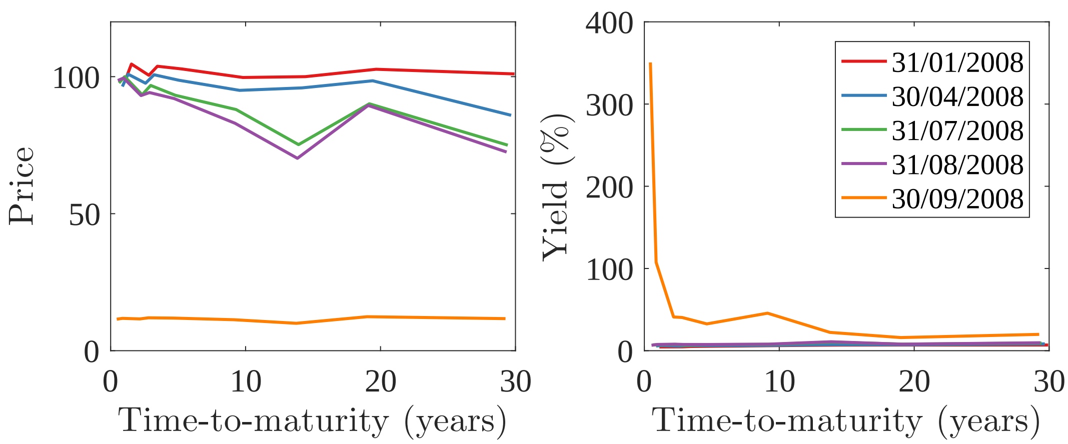


Figure 2: 30-year bond price as a function of the interest rate.

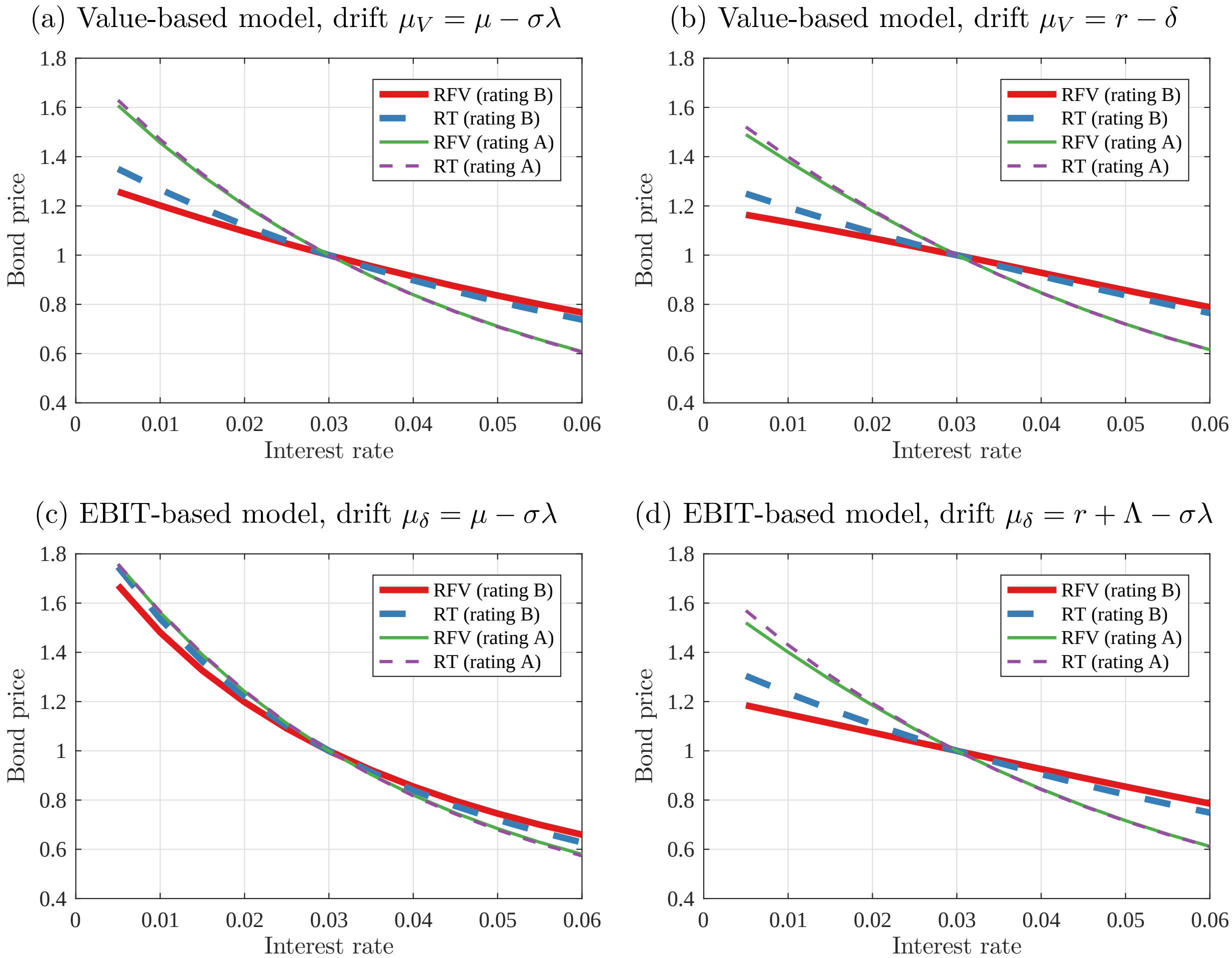
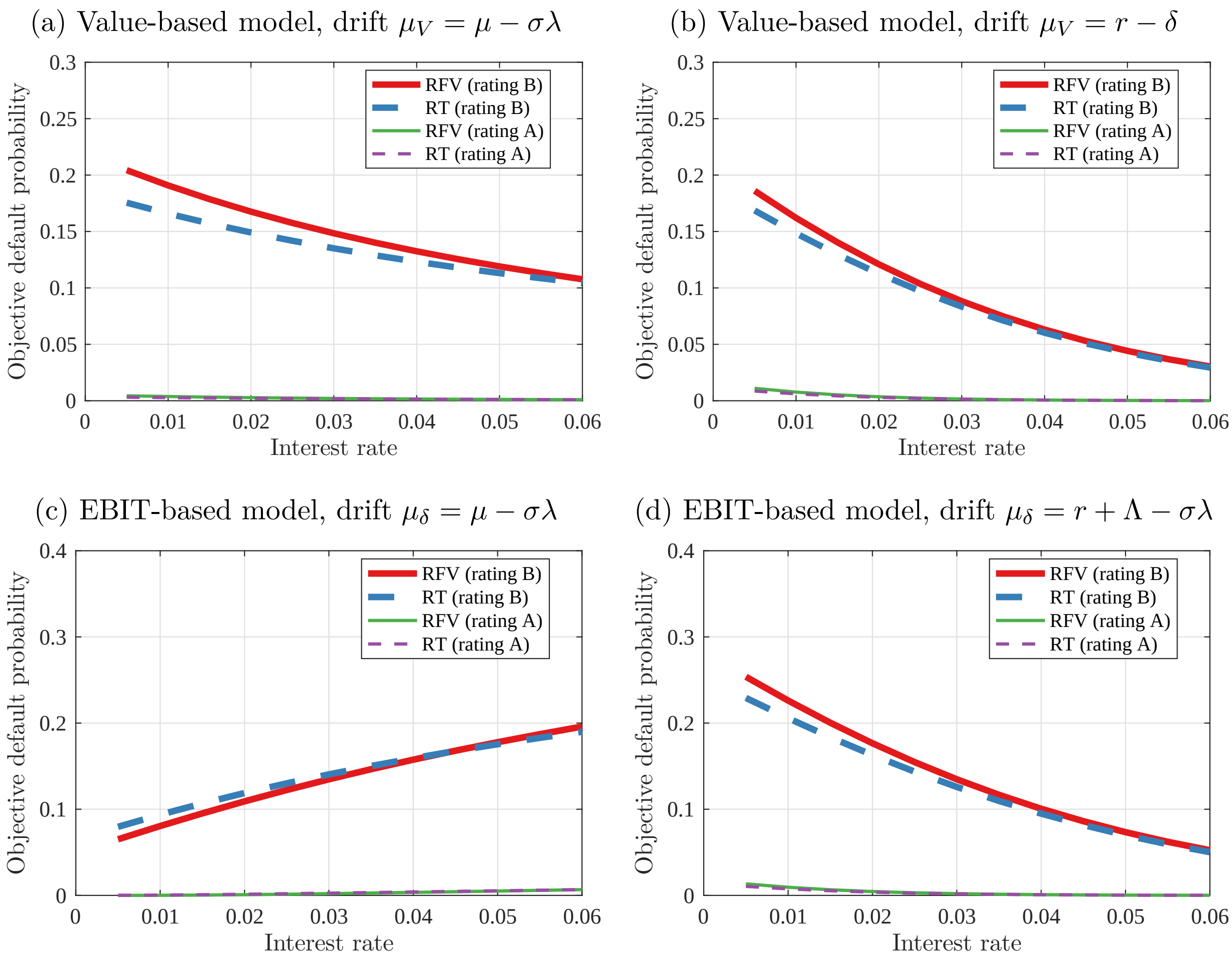


Figure 3: 30-year bond default probability as a function of the interest rate.



(a) Value-based model, drift $\mu_V = \mu - \sigma\lambda$

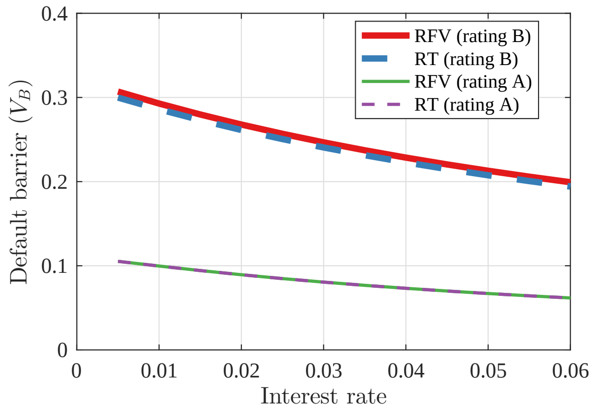
(b) Value-based model, drift $\mu_V = r - \delta$

(c) EBIT-based model, drift $\mu_\delta = \mu - \sigma\lambda$

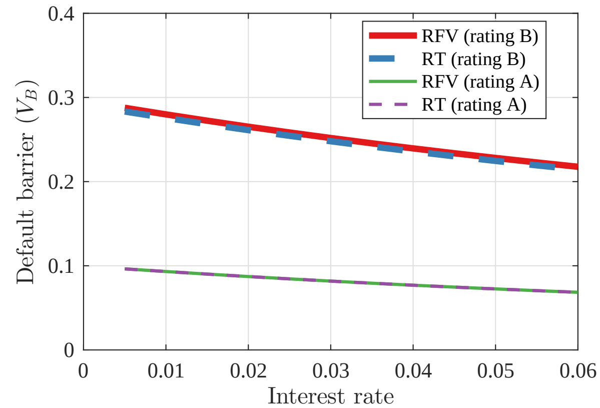
(d) EBIT-based model, drift $\mu_\delta = r + \Lambda - \sigma\lambda$

Figure 4: Default barrier and pre-default value as a function of the interest rate.

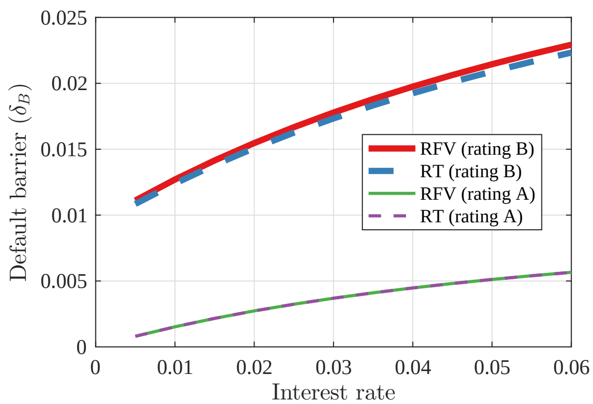
(a) Default unlevered assets value (V_B) for value-based model, drift $\mu_V = \mu - \sigma\lambda$



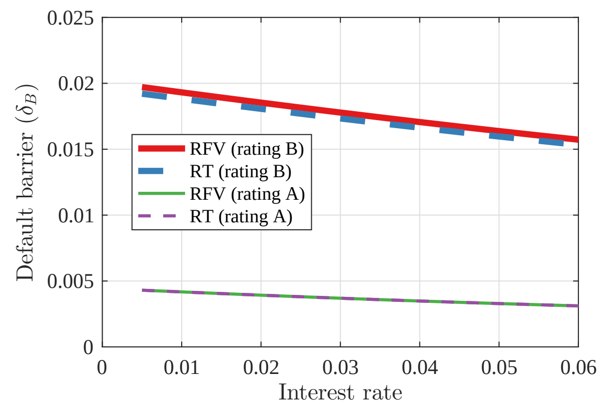
(b) Default unlevered assets value (V_B) for value-based model, drift $\mu_V = r - \delta$



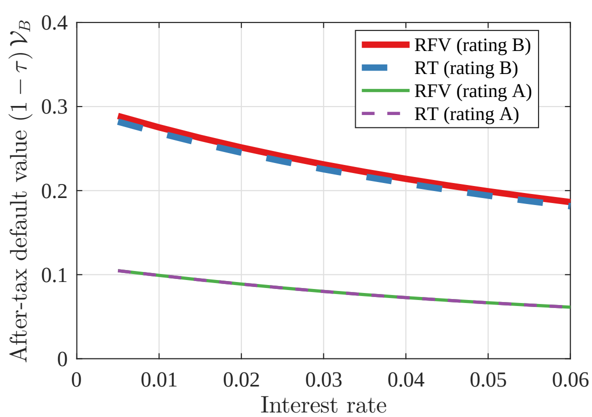
(c) Default EBIT (δ_B) for EBIT-based model, drift $\mu_\delta = \mu - \sigma\lambda$



(d) Default EBIT (δ_B) for EBIT-based model, drift $\mu_\delta = r + \Lambda - \sigma\lambda$



(e) After-tax default value $(1-\tau)\,\mathcal{V}_B$ for EBIT-based model, drift $\mu_\delta = \mu - \sigma\lambda$



(f) After-tax default value $(1-\tau)\,\mathcal{V}_B$ for EBIT-based model, drift $\mu_\delta = r + \Lambda - \sigma\lambda$

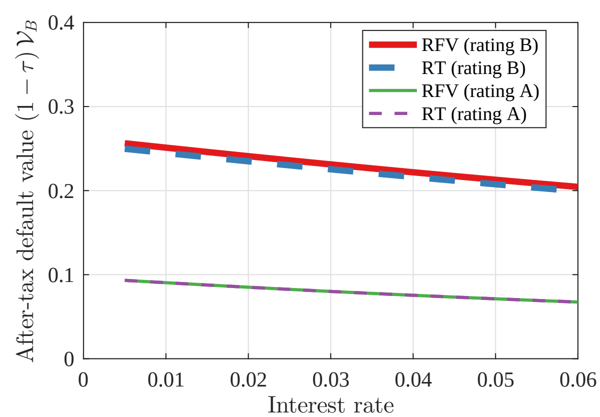
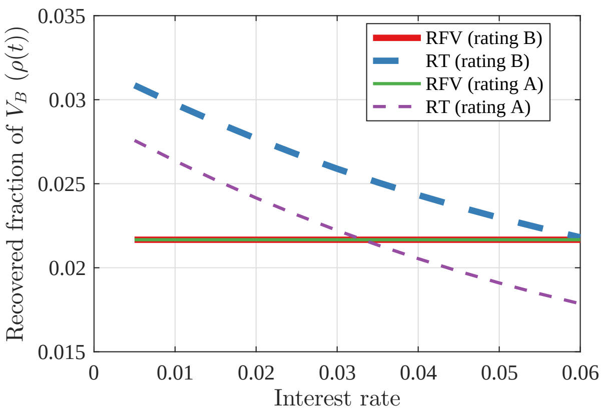


Figure 5: Share of residual value as a function of the interest rate for a 30-year bond ($t = 30$ years).

(a) Value-based models



(b) EBIT-based models

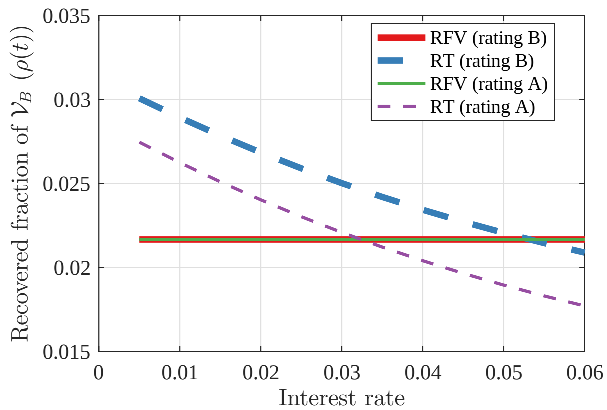
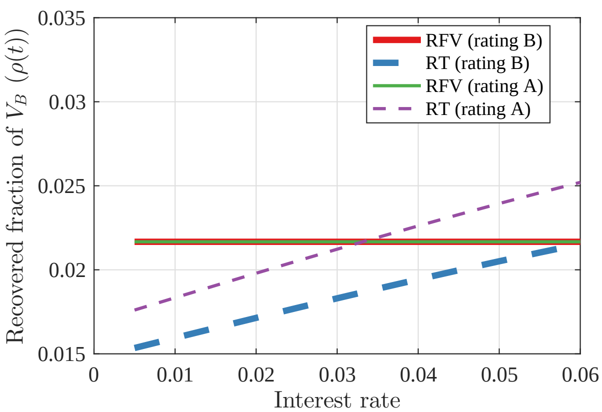


Figure 6: Share of residual value as a function of the interest rate for a 5-year bond ($t = 5$ years).

(a) Value-based models



(b) EBIT-based models

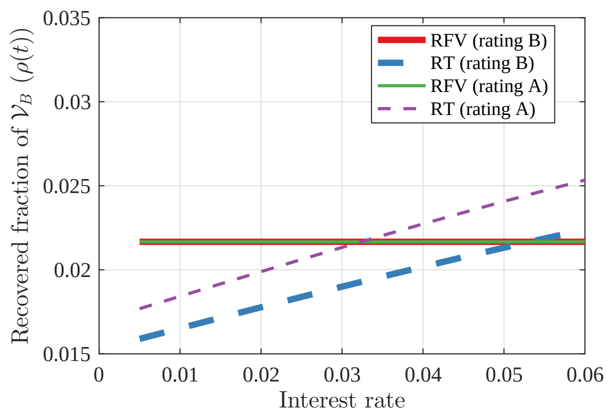
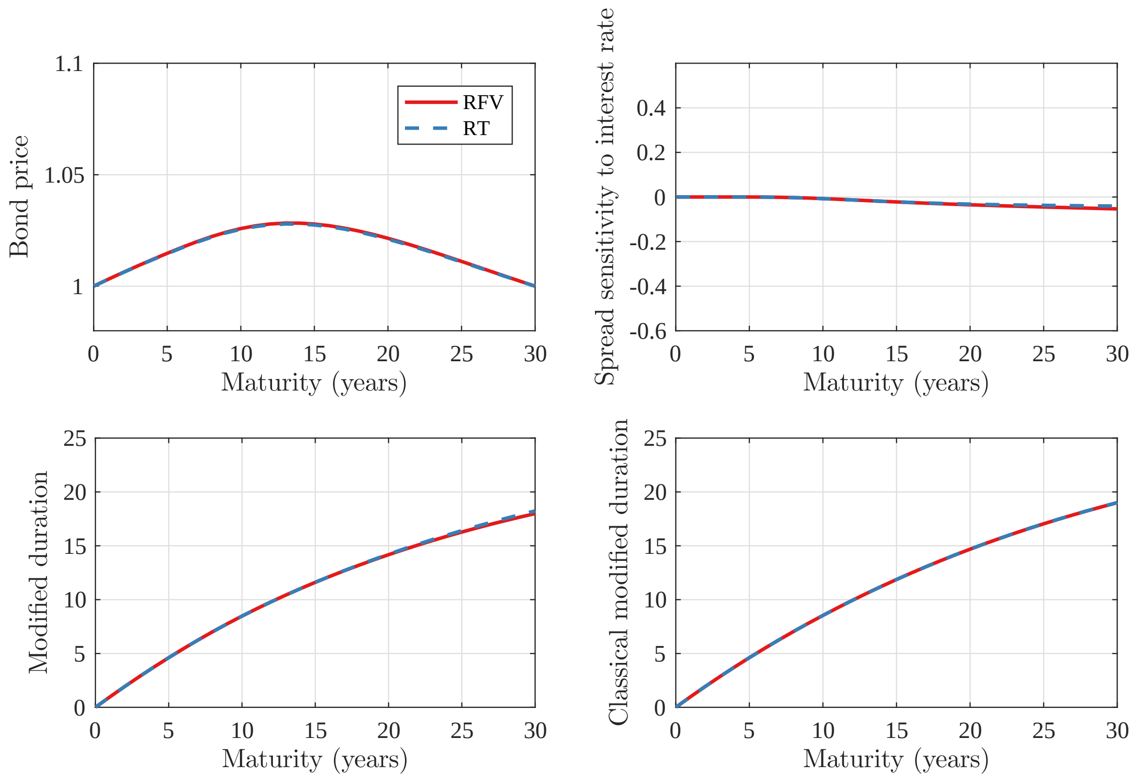


Figure 7: A-rated issue, value-based model.

(a) Drift $\mu_V = \mu - \sigma\lambda$



(b) Drift $\mu_V = r - \delta$

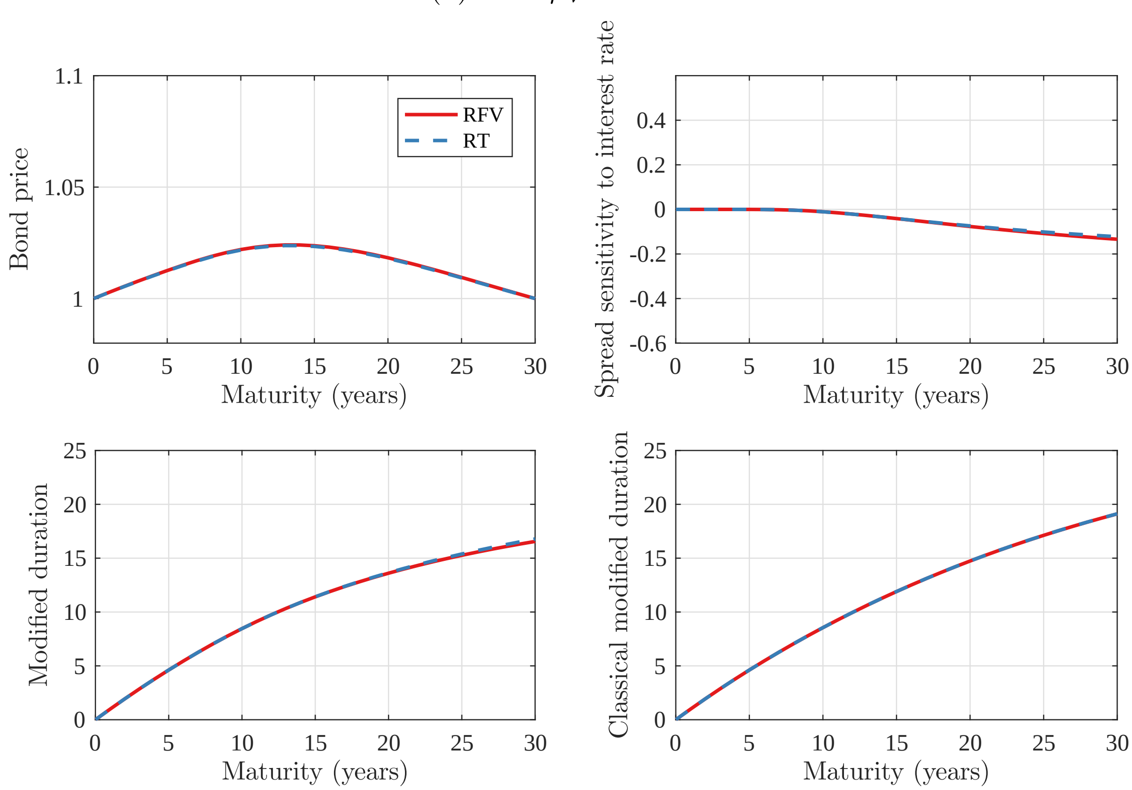
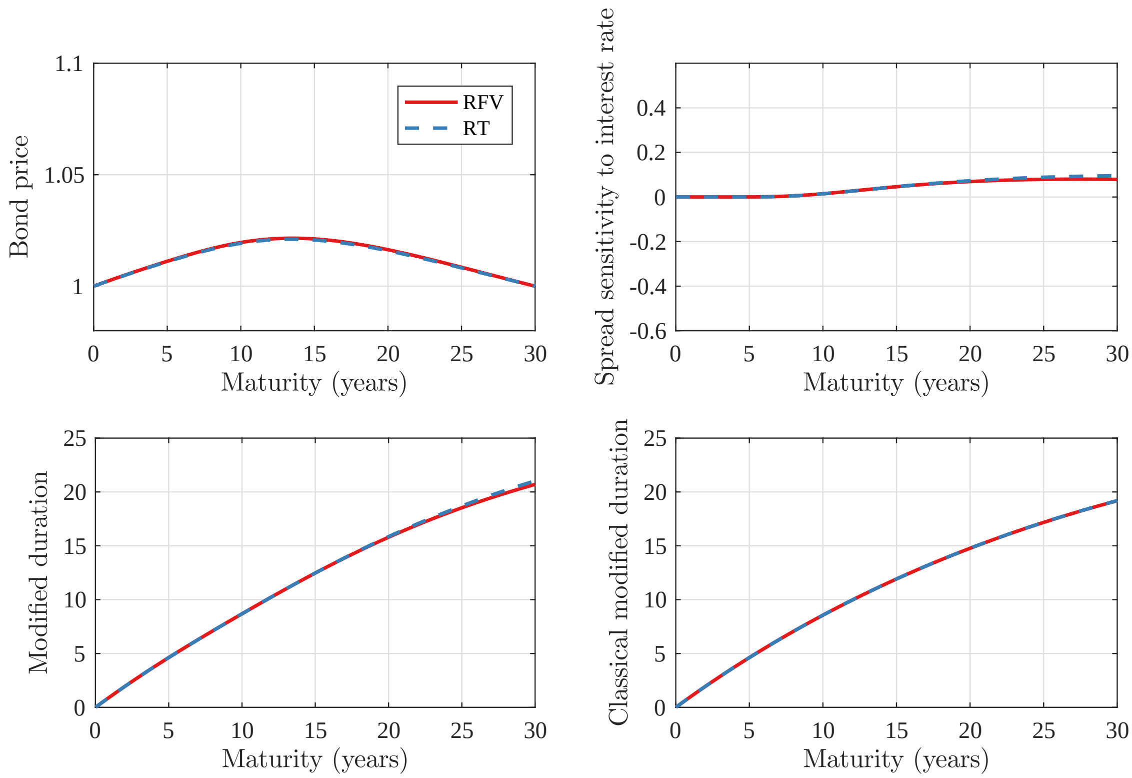


Figure 8: A-rated issue, EBIT-based model.

(a) Drift $\mu_\delta = \mu - \sigma\lambda$



(b) Drift $\mu_\delta = r + \Lambda - \sigma\lambda$

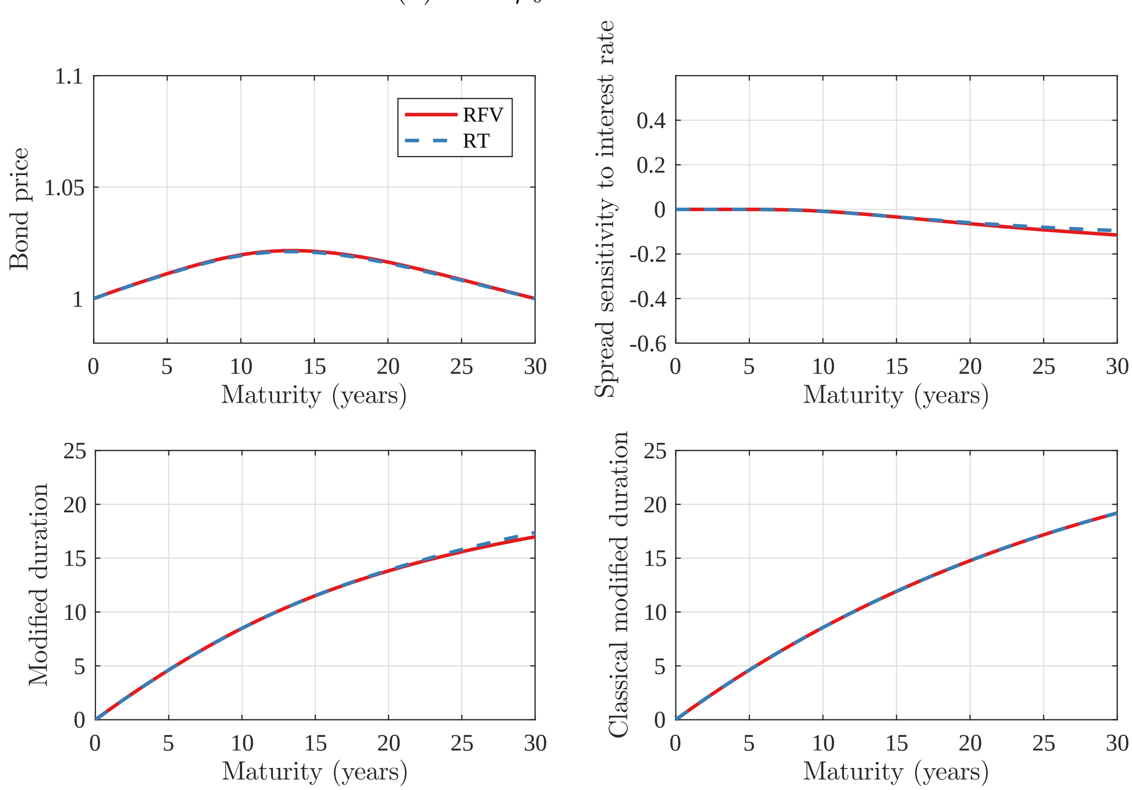
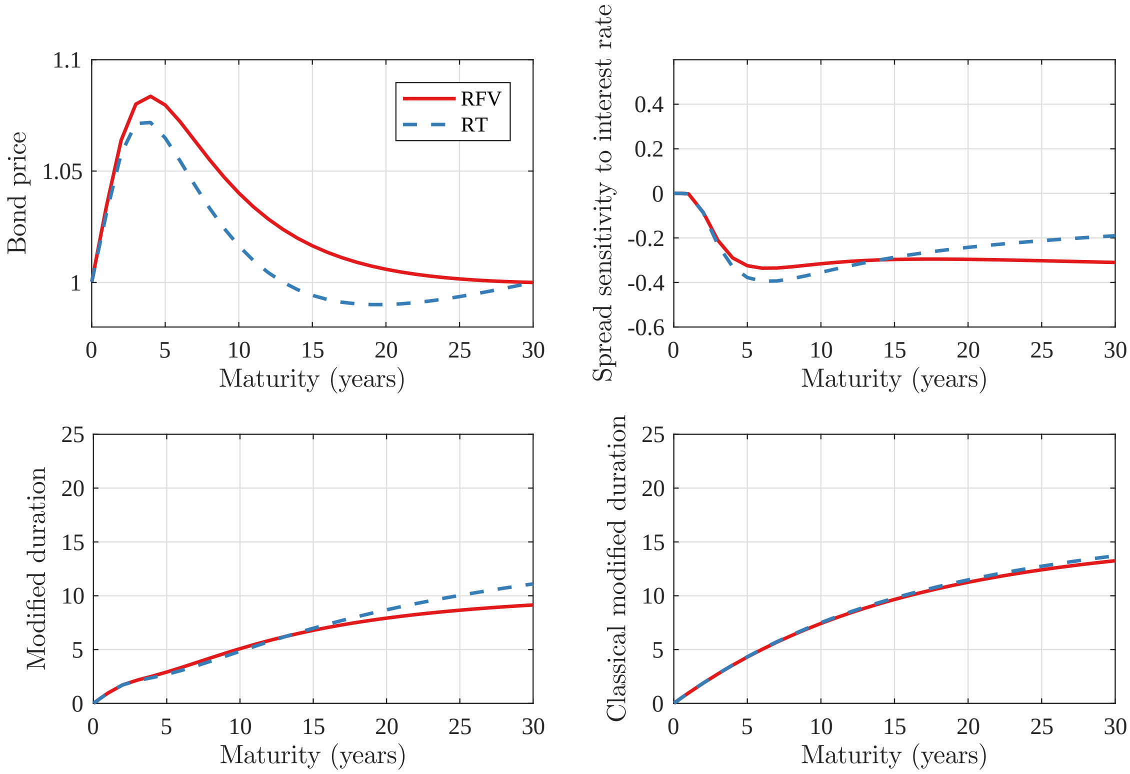


Figure 9: B-rated issue, value-based model.

key bond risk management

(a) Drift $\mu_V = \mu - \sigma\lambda$



(b) Drift $\mu_V = r - \delta$

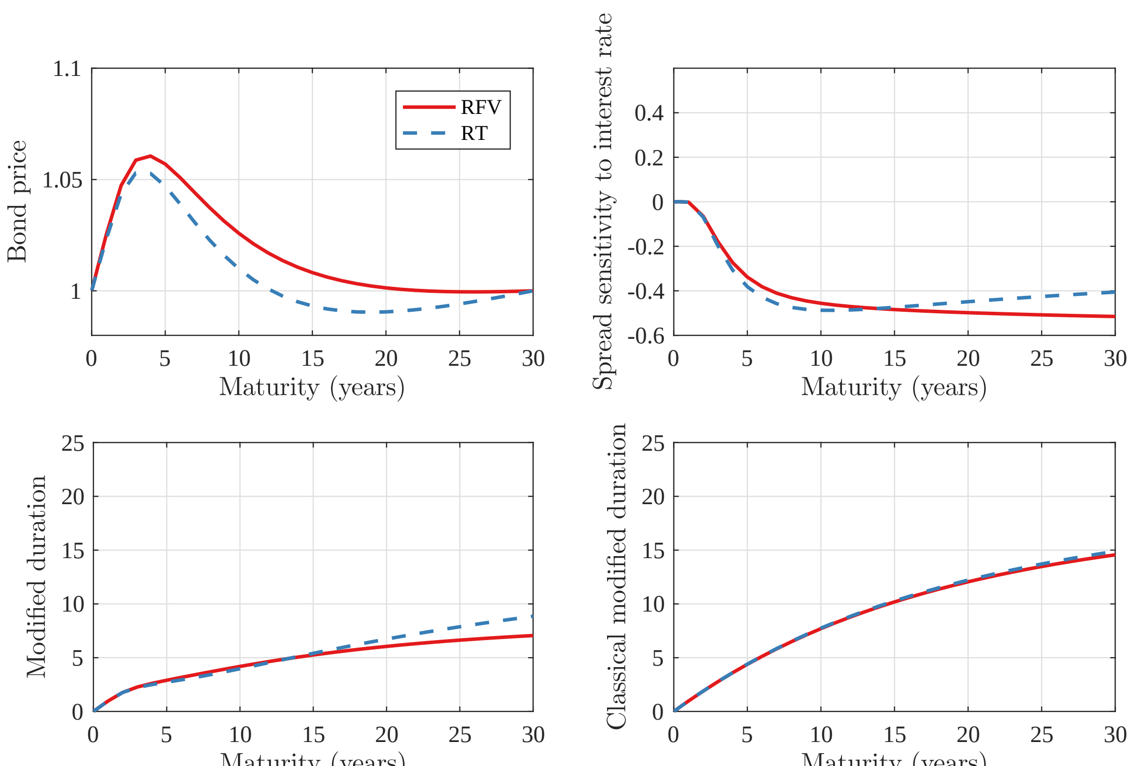
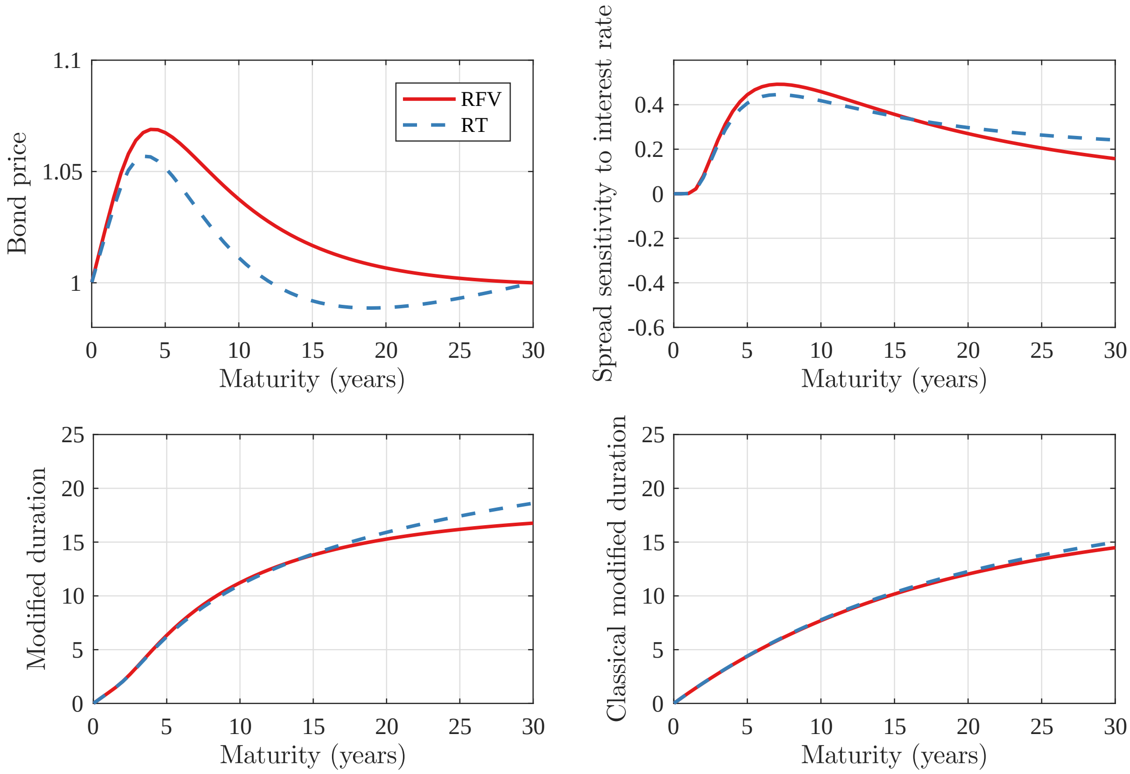


Figure 10: B-rated issue, EBIT-based model.

(a) Drift $\mu_\delta = \mu - \sigma\lambda$



(b) Drift $\mu_\delta = r + \Lambda - \sigma\lambda$

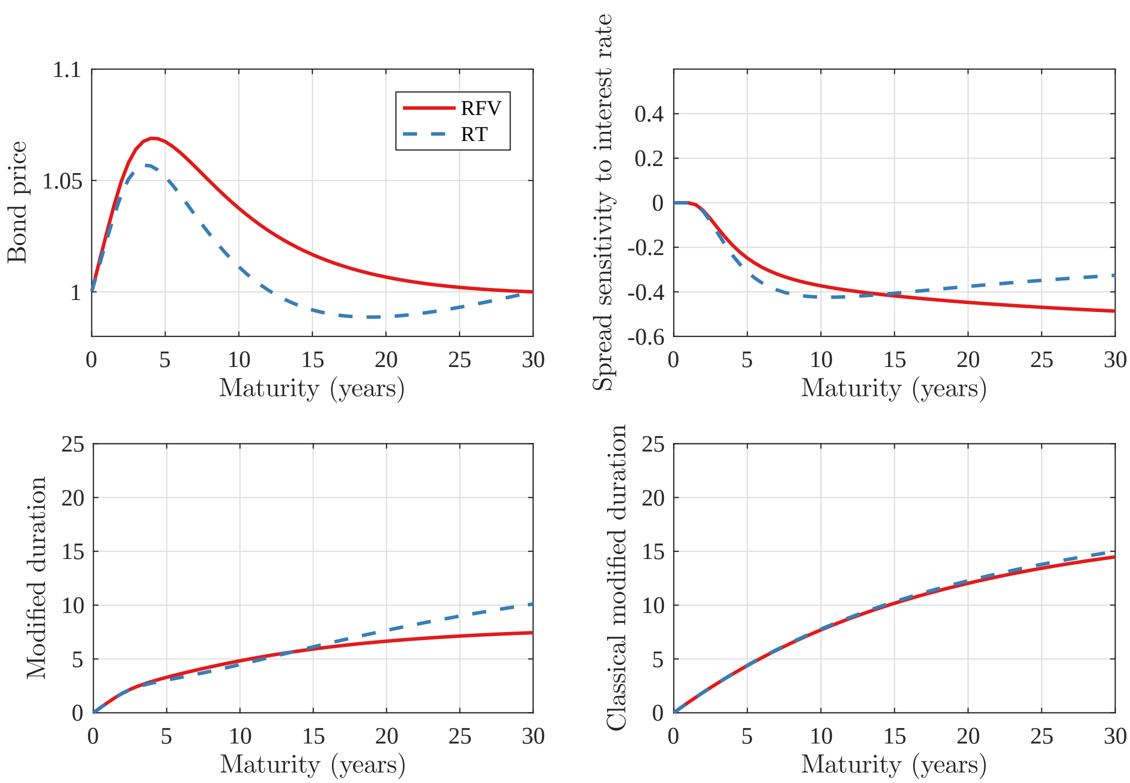
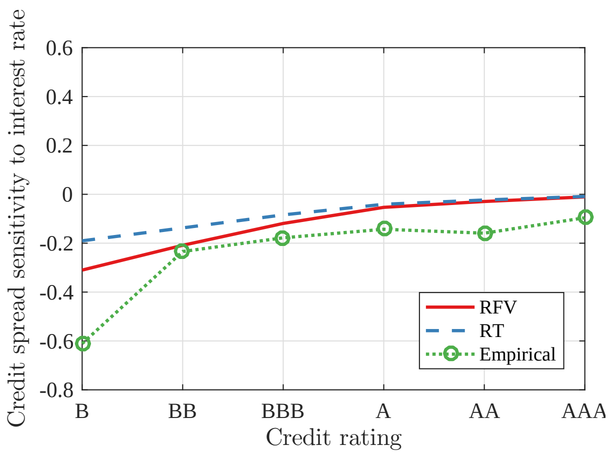
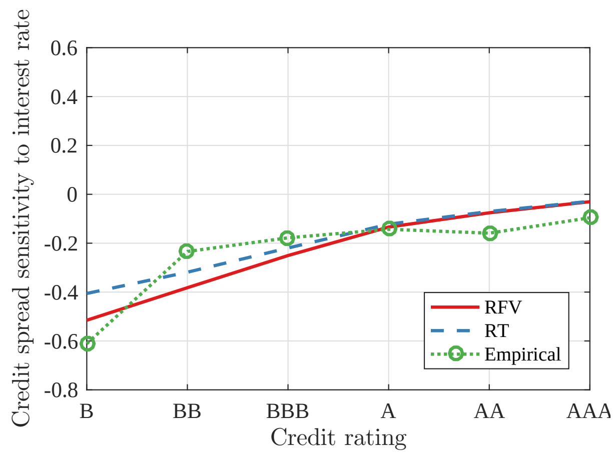


Figure 11: Credit spread sensitivity to interest rate.

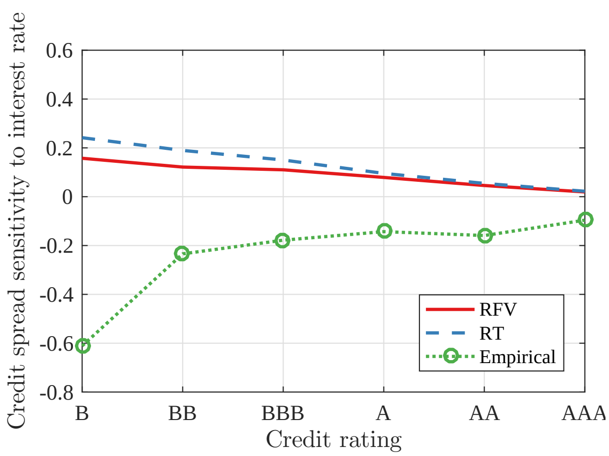
(a) Value-based model, drift $\mu_V = \mu - \sigma\lambda$



(b) Value-based model, drift $\mu_V = r - \delta$



(c) EBIT-based model, drift $\mu_\delta = \mu - \sigma\lambda$



(d) EBIT-based model, drift $\mu_\delta = r + \Lambda - \sigma\lambda$