Factor Based Commodity Investing

Athanasios Sakkas[1], Nikolaos Tessaromatis[2]

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Abstract

A multi-factor commodity portfolio combining the high momentum, low basis and high basis-momentum commodity factor portfolios outperforms significantly, economically and statistically, widely used commodity benchmarks. We find evidence that a variance timing strategy applied to commodity factor portfolios improves the return to risk trade-off of unmanaged commodity portfolios. In contrast, dynamic commodities strategies based on commodity return prediction models provide little value added once variance timing has been applied to commodity portfolios.

***JEL* Classification:** G10, G11, G12, G23

[1] Athanasios Sakkas is an Assistant Professor of Finance at the Southampton Business School, University of Southampton, Southampton, UK. *E-mail address*: a.sakkas@soton.ac.uk

[2] Nikolaos Tessaromatis is a Professor of Finance, EDHEC Business School and EDHEC Risk Institute, Nice, France. *E-mail address*: nikolaos.tessaromatis@edhec.edu

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1. Introduction

There is growing evidence that commodity prices can be explained by a small number of priced commodity factors. Commodity portfolios exposed to commodity factors earn significant risk premiums, in addition to the premium offered by a broadly diversified commodity index. We adopt a factor-based investment approach to create a diversified portfolio of commodity factors and examine the efficiency gains achieved compared to widely used commodity benchmarks. Assuming that commodity risk premiums are time varying, we also explore the possible benefits from dynamic strategies that rotate between commodity factors based on commodity volatility timing and commodity return forecasting models.

Research shows that commodity investment strategies based on exposures to commodity fundamental characteristics such as the basis, momentum, inflation, liquidity, skewness, open interest, value outperform commercially available commodity indices such as the S&P GSCI or a passive equally weighted index of all commodities.[3] Fuertes, Miffre and Fernandez-Perez (2015) study the benefits from strategy combination that explores the imperfect correlation between the returns of momentum, term structure and idiosyncratic volatility strategies while Fernandez-Perez, Miffre and Fuertes (2017) examine the performance of combining eleven long/short commodity strategies (styles) in a commodity portfolio using a portfolio construction methodology that nests many alternative portfolio construction rules.

Asset pricing tests narrow down the number of commodity factors that are priced among commodity-sorted portfolios. Szymanowska et al. (2014) find evidence supporting the pricing of the basis in the cross-section of commodity returns while Yang (2013) provides evidence in support of the average commodity factor (an equally weighted portfolio of all commodities) as an additional factor. Bakshi, Gao and Rossi (2017) provide evidence for a three-factor model that includes commodity momentum in addition to the basis[4] and the average commodity factor while Boons and Prado (2017) finds evidence of the pricing of basis-momentum (measured as the difference in momentum signals of first and second nearby futures contracts). According to Bakshi, Gao and Rossi (2017) the basis factor provides to investors compensation for the low returns of the factor during periods of high global volatility. The momentum factor on the other hand tend to do well when aggregate speculative activity increases. The basis-momentum factor proposed by Boons and Prado (2017) cannot be explained by the classical theories of

[3] See Miffre (2016) for a comprehensive review of the literature of the performance of various investment strategies in commodity futures markets.
[4] Bakshi, Gao and Rossi (2017) use the term carry factor.

storage (Kaldor, 1939), backwardation (Keynes, 1930) or hedging pressure (Cootner, 1960, 1967). Instead the authors suggest that the basis-momentum factor premium is compensation for volatility risk.

While capturing commodity risk premia requires the construction of passive portfolios with the desired exposure to commodity factors, timing commodity returns presupposes the ability to predict commodity returns and risk and calls for the design of dynamic trading strategies that rotate between the factors. Hong and Yogo (2012) provide evidence on the predictability of individual commodity futures using the short-term interest and the term premium, financial variables used in the stock and bond forecasting literature. They also show that commodity specific variables like aggregate open interest, the basis and commodity market imbalance (the ratio of short-long divided by short-long positions of commercial traders) predict individual commodity returns even after controlling for short term interest rates, the default premium and proxies for economic activity (Chicago Fed National Activity Index).[5] Interestingly, commodity specific variables also predict equity and bond prices.

In an out-of-sample study of individual commodity and a basis-based commodity portfolio predictability, Ahmed and Tsvetanov (2016) find weak evidence that conditional and unconditional forecasts of the average commodity portfolio and the basis factor, predict future commodity returns. Commodity return forecasts generate no economic gain to investors who use the predictions to build commodity timing strategies. Ahmed and Tsvetanov (2016), using prediction model forecasts as inputs in an asset allocation framework, also find no support for the hypothesis that commodities provide diversification benefits to investors who are invested in traditional stock/bond portfolios. This evidence is consistent with the conclusions in Daskalaki and Skiadopoulos (2011) that commodities add little value to traditional stock/bond portfolios. Gao and Nardari (2016) in contrast, using a forecast combination approach to predict equity, bond and commodity returns and the dynamic conditional correlation model of Engle (2002) to predict risk find that the addition of commodities to the traditional stock-bond-cash asset mix improves utility. The evidence on the predictability of commodity returns are as controversial as the evidence on the predictability in equity markets.

[5] Chen, Rogoff and Rossi (2010) show that "commodity currencies" predict the price of the commodity produced by the countries of these currencies. Bork, Kaltwasser and Sercu (2014) argue that the results are not robust to variations in the test design and the use of average rather than end of period prices of the commodity indexes used.

Our contribution in this study is fourfold. First, based on the framework of factor investing, we create a well-diversified portfolio of commodity factors. To address the issue of estimation risk, we use alternative portfolio construction methodologies in the factor combination. Consistent with the current practice in benchmark creation, we create portfolios without short positions in individual commodities but we also consider long-short versions that allow for short positions especially since shorting is inexpensive and straight forward in the commodities futures market. Second, we use recently developed statistical methodologies to choose the appropriate factors to be included in the portfolio. The proliferation of commodity factors that explain commodity returns and provide better performance compared to passive benchmarks raises the risk of data dredging i.e. choosing factors "…that come close to spanning the ex post mean-variance-efficiency (MVE) tangency portfolio of a particular period" (Fama and French 2017, page 24). Like equities, the number of candidate commodity factors is large and increasing. Following Fama and French's (2017) advice we limit the number of factors and models and consider factors for which there is theoretical justifications and evidence of cross sectional pricing. We use the testing methodology proposed by Barillas and Shanken (2017) and applied in Fama and French (2017) and the methodology developed by Harvey and Liu (2017) to test whether the factors proposed in the literature are real risk factors. Based on the evidence and theoretical justification provided by Yang (2013), Szymanowska et al. (2014), Bakshi, Gao and Rossi (2017) and Boons and Prado (2017) we test whether the average commodity portfolio and the basis, momentum and basis-momentum factors are real commodity risk factors.

Third, we compare the performance of the commodity portfolio to existing commodity benchmarks and in particular the S&P GSCI which represents the leading fully collateralized investable index and is the preferred benchmark for the majority of professionally managed portfolios. Fourth, we add to the existing literature on the predictability of individual commodities by providing evidence on the predictability of commodity factor-based portfolios. To assess the economic benefits of risk and returns predictability we create dynamic investment strategies based on prediction signals and measure the improvement in performance compared to passive investment strategies.

Our study supports the following conclusions. First, the spanning regressions of Barillas and Shanken (2017) and Fama and French (2017) and the methodology developed by Harvey and Liu (2017) identify the equally weighted portfolio of all commodities, and portfolios based on the basis, momentum and basis-momentum as risk factors for the commodities market. The evidence is consistent with a four-factor pricing model for commodities which nests the one-factor model of Szymanowska et al. (2014), the two-factor model of Yang (2013) and the three-factor model of Bakshi, Gao and Rossi (2017). Second, an equally weighted commodity factor portfolio combining the low basis, high momentum and the high basis-momentum factor portfolios, achieves over the period 1975-2015 a Sharpe ratio of 0.68 that represents a major improvement compared with the return to risk offered by the S&P GSCI (0.03) and an equally weighted portfolio of all commodities (0.28). The improvement in return-to-risk is significantly better when short positions are allowed in the construction of the commodity factor portfolios (Sharpe ratio 1.02). Using mean-variance, minimum variance, maximum diversification or risk parity weights makes little differences in performance compared to equal weights.

Third, the factor-based portfolio represents a dramatic improvement compared with the S&P GSCI, the benchmark used by most institutional investors, ETFs, ETNs and mutual funds. In particular, over the 1975-2015 period the S&P GSCI achieved an annual excess return of 0.63% compared with an annual excess return of 13.05% of a an equally weighted long-only commodity factor portfolio. The significant outperformance has been achieved with much lower volatility (16.12% vs.19.48%) and is robust across sub-periods, the business cycle and volatility states. The evidence suggests that the S&P GSCI is unlikely to be on the mean-variance efficient frontier and that switching to the factor-based commodity benchmark increases the return to risk from investing in commodities significantly.

Finally, we build dynamic factor portfolio timing strategies based on predictions of factor returns and volatility. Volatility timing is profitable, producing statistically significant alphas for the average commodity portfolio as well as the long-only versions of the momentum, basis and basis-momentum factor portfolios. Volatility timing for the long-only versions of the commodity factor portfolios works because the bulk of the return of the momentum, basis and basis-momentum portfolios is due to the average commodity portfolio, for which volatility timing is profitable. We find strong evidence suggesting that volatility timing works out-of-sample for the long-short commodity momentum premium, consistent with the findings of the success of volatility based timing for equity momentum reported in Barroso and Santa-Clara (2015) but adds little value to passive investments in the long-short basis or basis-momentum factor premiums.

We use different approaches to predict commodity factor portfolio returns and find little evidence to suggest that return forecasting adds value once volatility timing has been implemented. The failure of return forecasting to add value applies to both long-short and long-only versions of the commodity factor portfolios with the exception of the S&P GSCI.[6] The evidence is robust across the business cycle and volatility states and consistent with the results reported in Ahmed and Tsvetanov (2016).

Our findings have important implications for commodity portfolio management. A multifactor commodity portfolio combining the high momentum, the low basis and the high basis-momentum commodity portfolios is significantly better to the widely used S&P GSCI benchmark. The commodity factor portfolio outperforms the S&P GSCI consistently across sub-periods, the business cycle and volatility regimes. The difference in performance is statistically significant and unlikely to be the result of chance. The Harvey and Liu (2017) testing methodology suggests that the S&P GSCI is not a risk factor. The implication from this finding is that investors should replace the S&P GSCI with the better diversified and performing portfolio of commodity factors.

Our results also suggest that the conclusions from papers like Daskalaki and Skiadopoulos (2011) and Ahmed and Tsvetanov (2016) suggesting that commodities do not add value to traditional stock/bond/cash portfolios should be revisited in light of the evidence presented in this paper suggesting that a passive multifactor portfolio is significantly better than the S&P GSCI or the average commodity portfolio of individual commodities used in previous studies to assess the role of commodities in asset allocation. Finally, the evidence on commodity factor portfolio timing suggests that volatility timing might prove to be beneficial to long-only portfolios and the commodity momentum factor. However, once volatility timing has been applied, commodity factor portfolio return forecasting has no value in timing commodity factor portfolios.

The rest of the paper is organized as follows. In Section 2 we describe the data. In Section 3 we discuss the return and risk characteristics of commodities and the appropriate factors to be included in the commodity portfolio. Section 4 examines the benefits from a diversified portfolio of commodity factor premia. Section 5 examines the performance of dynamic tactical commodity allocation based on the predictability of commodity return and volatility timing. Finally, Section 6 concludes.

[6] The evidence on the predictability of the S&P GSCI reported in this paper is consistent with the findings in Gao and Nardari (2016).

2. Data and Variables

In this Section we discuss the data we use in our empirical analysis.

2.1 Commodity futures data

We base our analysis on monthly data covering the period January 1975 to December 2015. The commodity monthly futures returns are constructed from end-of-day settlement prices sourced from Bloomberg. Our dataset consists of 32 commodity futures contracts covering five major sectors, namely, energy, grains and oilseeds, livestock, metals and softs**.** Table 1 tabulates the 32 commodities grouped by category, the exchange on which they are traded, the corresponding Bloomberg ticker symbol, the year of the first recorded observation, the delivery months and the Commodity Futures Trading Commission (CTFC) code. The dataset is comparable with the dataset used by Gorton, Hayashi and Rouwenhorst (2012), Hong and Yogo (2012), Szymanowska et al. (2014) and Bakshi, Gao and Rossi (2017).

We calculate monthly futures returns in excess of the risk-free rate $\left(R_j\right)$ for each commodity j as $R_{j,t+1}^{T_n} = \dfrac{F_{j,t+1}^{T_n} - F_{j,t}^{T_n}}{F_{j,t}^{T_n}}$ where $F_{j,t+1}^{T_n}$ is the futures price at the end of month t for the contract of commodity j with delivery month $t+T_n$. We consider the first nearby futures contracts $\left(n=1\right)$ and exclude future contracts with less than one month to maturity, in which case futures traders need to take a physical delivery of the underlying commodity (Hong and Yogo, 2012). Hence, the monthly futures returns are calculated based on a roll-over strategy where an investor maintains a long position in the futures contract on commodity *j* and expiration in month $t+T_1$ and rolls-over on the last trading day of the month before delivery.

Table 2 reports the summary statistics of the 32 commodities over the period January 1975 to December 2015. Table 2 shows that investment in individual commodities is unattractive; 25 out of 32 commodities have Sharpe ratios below 0.25, consistent with findings by Bakshi, Gao and Rossi (2017, Table Internet-II). The absolute first-order autocorrelation for 26 out of 32 commodities is below 0.1, indicating that most commodity future returns are serially uncorrelated. Most of the commodities have a positive skewness. Finally, 21 of 32 commodities are in contango on average.[7] In general, the magnitudes shown in Table 2 are consistent with

[7] Positive basis denotes that the commodity market is in contango (upward sloping yield curve); negative basis means that the commodity market is in backwardation (downward sloping yield curve).

the evidence reported in Erb and Harvey (2006, Table 4), Gorton, Hayashi, and Rouwenhorst (2013, Table I) and Bakshi, Gao and Rossi (2017, Table Internet-II).

2.2 Commodity factor portfolios

We construct long-only and long-short commodity factor portfolios. We focus on three commodity sorting characteristics, i.e. momentum (Fuertes, Miffre and Fernandez-Perez, 2015, Bakshi, Gao and Rossi, 2017, Boons and Prado, 2017), basis (Szymanowska et al., 2014, Gorton, Hayashi and Rouwenhorst, 2012, Yang, 2013, Fuertes, Miffre and Fernandez-Perez, 2015, Bakshi, Gao and Rossi, 2017, Boons and Prado 2017) and basis-momentum (Boons and Prado 2017).

We define momentum for each commodity j as the cumulative excess futures returns from the prior 12 months, i.e. $Momentum_t^j = \prod_{s=t-11}^{t}\left(1+R_{j,t,s}^{T_1}\right)-1$, where $R_{j,t}^{T_1}$ denotes the future returns of the nearby contracts of commodity j. The basis for each commodity j is defined as $Basis_t^j = \left(\frac{F_{j,t}^{T_2}-F_{j,t}^{T_1}}{F_{j,t}^{T_1}}\right)^{\frac{1}{(T_2-T_1)}}$, where $F_{j,t}^{T_1}$ and $F_{j,t}^{T_2}$ are the futures prices of the nearby and next-to-nearby contracts, respectively. Finally, the basis-momentum is defined as the difference between momentum in a first- and second-nearby futures strategy, i.e. $BasisMomentum_t^j = \prod_{s=t-11}^{t}\left(1+R_{j,t,s}^{T_1}\right)-\prod_{s=t-11}^{t}\left(1+R_{j,t,s}^{T_2}\right)$, where $R_{j,t}^{T_1}$ and $R_{j,t}^{T_2}$ stand for the future returns of the nearby and next-to-nearby contracts of commodity j, respectively.

To construct the commodity factor portfolios we sort at the end of each month the future returns of the 32 commodities based on their sorting characteristics and then calculate the equally weighted return of the top 30 percent and bottom 30 percent of the commodities. Finally, we calculate the return of the average commodity portfolio as the equally weighted return of the 32 commodity future contracts, rebalanced monthly. Note that at the beginning of our sample (January 1975) 14 commodity futures are available. The complete set of 32 commodity futures is available from May 2005 until the end of our sample.

Table 3 presents the number of months in which a commodity enters in the long and short legs of the momentum, basis and basis-momentum portfolios. Softs, i.e. orange juice, coffee and cocoa appear most of the times both in the long and short legs of the momentum portfolio; live cattle, sugar and orange juice appear most of the times in both components of basis portfolio;

natural gas, live cattle and cotton appear most of the times in both legs of basis-momentum strategy. Momentum, basis and basis-momentum strategies load on different commodities. For instance, live cattle appears 191 times in the long component of the momentum portfolio and 227 times in the long component of basis portfolio. Our results are consistent with the findings by Bakshi, Gao and Rossi (2017, Table 13).

3. "Efficient" benchmarks for commodity portfolios

3.1 The return and risk of commodity portfolios

The S&P Goldman Sachs Commodity Index (S&P GSCI) is a buy and hold world production-based index, with a large weight in the energy sector (approximately 70%). It is one of the most popular commodity benchmarks used by institutional investors and can be traded via over-the-counter swap agreements, exchange-traded funds (ETF) and exchange-traded notes (ETN) (Stoll and Whaley, 2010). The S&P GSCI consists of 24 deep and liquid individual commodity futures indices. These include six energy related commodities (crude oil, Brent crude oil, heating oil, gasoil, natural gas and unleaded gasoline), seven metals (gold, silver, copper, aluminium, zinc, nickel and lead), and eleven agricultural commodities (corn, soybeans, wheat (CBOT), wheat (Kansas), sugar, coffee, cocoa, cotton, lean hogs, live cattle and feeder cattle). Geman (2009) and Erb and Harvey (2006) provide a detailed description of the S&P GSCI commodity index.[8]

Table 4 presents descriptive statistics of the commodity benchmarks (Panel A), commodity long-only factor portfolios (Panel B) and commodity long-short factor portfolios (Panel C) over the full sample period January 1975 – December 2015. Performance statistics over the sub-sample periods January 1975 - June 1995 and July 1995 – December 2015 are presented in Table A1 in the Appendix A. Figure 1 presents the Sharpe ratios of the commodity benchmarks and long-short commodity factors in NBER recession and expansion periods as well as in low and high volatility periods. For the full descriptive statistics for all commodities considered in this study in the NBER recession and expansion periods, and in low and high volatility periods, refer to Tables A2 and A3 in Appendix A, respectively. Mean, standard deviation, skewness and kurtosis are annualised (Cumming et al., 2014).

Table 4 shows, that over the period 1975-2015, the Goldman Sachs Commodity Index (S&P GSCI) and the average commodity market factor (AVG) had average excess returns of 0.63%

[8] More information on the S&P GSCI Methodology can be found at http://eu.spindices.com/documents/methodologies/methodology-sp-gsci.pdf.

and 3.64% per annum, respectively. The volatility of the S&P GSCI (19.48%) is significantly higher than the volatility of the average commodity market factor (13.00%) and reflects the overweighting of energy in the S&P GSCI (the standard deviation of the S&P GSCI Light Energy, which invests less in energy is 14% per annum).

The long-only high momentum commodity portfolio exhibits the highest realized excess return (12.80%) followed by the high basis-momentum (11.44%) and low basis (9.60%) commodity portfolios. High returns are associated with higher risk (standard deviation): the high momentum commodity portfolio exhibits also the highest volatility (20.33%), followed by the high basis-momentum (17.57%) and low basis (17.06%) commodity portfolios. These results are in line with the studies of Gorton and Rouwenhorst (2006) and Erb and Harvey (2006). The long-short commodity momentum exhibits the highest realized excess return (16.61%) followed by the basis-momentum (13.39%) and basis (13.37%) factors. The long-short momentum exhibits also the highest volatility (22.10%), followed by the basis (18.24%) and basis-momentum (17.98%). The profitability of the long-short momentum, basis and basis-momentum strategies is attributed to both long and short components.

Sharpe ratio comparisons show that the S&P GSCI (0.032) offers a less attractive return to risk trade-off than the average commodity portfolio (0.280). The long-only commodity factor portfolios exhibit higher Sharpe ratios than either the S&P GSCI or the average commodity portfolio. The high basis-momentum commodity portfolio achieved a Sharpe ratio of 0.651, the high momentum commodity portfolio a Sharpe ratio of 0.630 and the low basis commodity portfolio a Sharpe ratio of 0.563 all statistics measured over the 1975-2015 period. The long/short version of the commodity factor portfolios achieve higher returns but also higher volatility. As a result, the return to risk trade-off offered by commodity portfolios which allow short positions is slightly better than long-only commodity factor portfolios.

Sub-period results presented in Table A1 in Appendix A are consistent with results based using the full sample. Long-only commodity factor portfolios experience positive returns and lower volatility in periods of economic expansion and negative returns and higher volatility during recessions. The results in Table A2 (in Appendix A) show that the S&P GSCI had a Sharpe ratio of 0.187 (-0.610) in expansion (recession) periods. Positive Sharpe ratios during expansions and negative Sharpe ratios during recessions is also the characteristic of the average commodity portfolio, the high momentum, the low basis and the high basis-momentum commodity portfolios. These results suggest that commodities offer a risk premium as compensation for the negative performance of commodities during recessions. The return and risk of long/short versions of the commodity factors is also different during economic expansion/recessions. The commodity risk premia tend to be lower in recessions than expansions. We find very similar performance across periods of low volatility versus periods of high volatility; the monthly return of each commodity factor is classified in the high (low) volatility period when its monthly volatility is above (below) its average volatility over the full sample period (see Table A3, Appendix A). Figure 1 compares the Sharpe ratios of the commodity factor premiums across expansions and recessions and low and high volatility periods. The return to risk tends to be low (negative in the case of the S&P GSCI and the average commodity portfolio) in recessions and high risk and positive in periods of economic expansion and low volatility. Overall, the empirical evidence suggests that commodity returns perform well in expansions and low volatility periods, and poorly in recessions and high volatility periods.

3.2 Choosing priced commodity factors

The results in Table 4 and Figure 1 confirm evidence in the literature suggesting that commodity factor-based portfolios offer a superior risk-return trade-off compared to the widely used in practice S&P GSCI benchmark. Factor-based portfolios outperform also an equally weighted portfolio of the 32 commodities we examine in this study. The average commodity portfolio has been used in many academic studies as a proxy of the "market" portfolio for commodities and as a superior alternative to the S&P GSCI. In this Section we apply the research methodologies of Harvey and Liu (2017) and Barillas and Shanken (2017) and Fama and French (2017) to test whether the S&P GSCI, the average commodity portfolio and the basis, momentum and basis-momentum factors are priced in the cross-section of commodity returns. In the presence of multiple priced commodity risk premia an investor in the commodity "market" portfolio should also consider exposure to non-market risk premia. If commodity factor premia are uncorrelated, investing in a portfolio of commodity risk premia should provide considerable efficiency gains compared to the benchmark commodity market portfolio.

To limit the effects of data dredging we restrict the number of tested factors to those for which there is a theoretical motivation and has been found to be priced in previous cross-sectional tests. For equities, Fama and French (2017), argue that theory should be used to avoid data dredging and limit the number of factors and models considered. Following this advice we restrict the choice of candidate factors, to the factors proposed by Yang (2013, average commodity and basis factors), Szymanowska et al (2014, basis factor), Bakshi, Gao and Rossi (2017, average commodity, the basis and momentum factors) and Boons and Prado (2017, average commodity and the basis-momentum factors) to describe the cross-section of commodity returns. Our list of candidate factors excludes commodity volatility, open interest, hedging pressure, industrial production, US TED spread or inflation, factors that did not have any impact on the cross-section of commodity returns in previous research (Szymansowska et al., 2014, Bakshi, Gao and Rossi, 2017).

The methodology developed in Harvey and Liu (2017) identifies from among a number of candidate factors those that are priced, addresses data mining directly, takes into account the cross-correlation between factors and allows for general distributional assumptions and more specifically non-normality. The Harvey Liu (2017) methodology which can be applied using either portfolios or individual securities as test assets has been designed to answer the following question: given a benchmark and an alternative factor model, what is the incremental contribution of the alternative model? Barillas and Shanken (2017) and Fama and French (2017) use an alternative testing methodology to assess the benefits from adding a factor to a factor model. The methodology involves running a spanning regression of a candidate factor on a model's other factors. A non-zero intercept indicates that the factor makes a marginal contribution to the factor model and helps explain average returns. The GRS (Gibbons, Ross and Shanken, 1989) test of competing models tests whether a new factor improves the mean-variance efficiency of a portfolio constructed from existing factors.

3.2.1 The Harvey and Liu (2017) Method

Harvey and Liu (2017) utilize multiple hypothesis testing and a bootstrapping technique to identify the factors that can explain the cross-section of expected commodity returns. The test consists of estimating two factor models: the baseline model and an augmented model that includes an additional factor relative to the baseline model. According to Harvey and Liu (2017) p. 18 "a risk factor is considered useful if, relative to the baseline model, the inclusion of the risk factor in the baseline model helps reduce the magnitude of the cross section of intercepts under the baseline model". Two test-statistics are used to evaluate the statistical significance in explaining the cross-section of commodity expected returns between the baseline and the augmented regression model. The first test-statistic calculates the difference (in percentage) in the mean absolute intercepts of the baseline regression $\left(\left|a_i^b\right|\right)$ and the augmented regression $\left(\left|a_i^g\right|\right)$, scaled by the standard error of the absolute intercept of the baseline regression $\left(s_i^b\right)$, defined as follows: $SI_{ew}^{m} = \dfrac{\frac{1}{N}\sum_{i=1}^{N}\left(\left|a_i^g\right| - \left|a_i^b\right|\right)/s_i^b}{\frac{1}{N}\sum_{i=1}^{N}\left|a_i^b\right|/s_i^b}$. To take into account possible outliers in the cross-section of commodity returns Harvey and Liu (2017) use a second test-statistic, as a robustness measure, and calculate the difference (in percentage) in the median intercepts of the baseline regression $\left(\left|a_i^b\right|\right)$ and the augmented regression $\left(\left|a_i^g\right|\right)$, scaled by the standard error of the absolute intercept of the baseline regression $\left(s_i^b\right)$,defined as follows:

$$SI_{ew}^{med} = \frac{median\left(\left\{\frac{\left|a_i^g\right|}{s_i^b}\right\}_{i=1}^{N}\right) - median\left(\left\{\frac{\left|a_i^b\right|}{s_i^b}\right\}_{i=1}^{N}\right)}{median\left(\left\{\frac{\left|a_i^b\right|}{s_i^b}\right\}_{i=1}^{N}\right)}.$$

Table 5 presents the (i) SI_{ew}^{m} and SI_{ew}^{med}, (ii) the bootstrapped 5th percentile on the distribution of SI_{ew}^{m} and SI_{ew}^{med} for each individual commodity risk factor with the corresponding p-values[9] under the null hypothesis that the commodity risk factor individually has no ability to explain the cross-section of test assets returns (single hypothesis testing) and (iii) the bootstrapped 5th percentile on the distribution of the minimum SI_{ew}^{m} and SI_{ew}^{med} amongst the commodity risk factors with the corresponding p-values[10] under the null hypothesis that the commodity risk factor individually has no ability to explain the cross-section of test assets returns (multiple hypothesis testing).

Panel A of Table 5 tabulates the results when the 32 individual commodities of Table 1 are the test assets. We start our analysis by testing whether any of the five commodity risk factors, namely the S&P GSCI and the average commodity factor premia, as well as the long-short momentum, long-short basis and long-short basis-momentum, can explain the cross-section of expected individual commodity returns. We find that the average commodity factor is the best among the factors, since it reduces the mean (median) scaled absolute intercept by 30.9% (36.5%), higher than what the remaining factors do. The bootstrapped 5th percentile of SI_{ew}^{m}

[9] P-values are obtained by evaluating the realised test-statistics for each individual commodity risk factors against the corresponding test-statistics based on their empirical distribution from bootstrapping.

[10] P-values are obtained by evaluating the realised test-statistics for each individual commodity risk factor against the empirical distribution of the minimum test-statistic across the individual test statistics of the individual commodity risk factors that arise from bootstrapping.

$\left(SI_{ew}^{med}\right)$ for the average commodity factor is -0.276 (-0.332), a reduction in the mean (median) scaled intercept of 27.6% and 33.2% respectively. The actual factor reduces the mean (median) scaled intercept by more than the 5th percentile, which entails statistical significance with a p-value equal to 0.084 (0.018) (see Panel A.1). With respect to the multiple hypothesis test, the bootstrapped 5th percentile of SI_{ew}^{m} $\left(SI_{ew}^{med}\right)$ is -0.290 and statistical significant with a multiple testing p-value equal to 0.005 (0.018). Overall, the average commodity factor is the most important among the candidate factors and is statistical significant at 5% level with respect to the single and multiple hypothesis tests. We repeat the analysis by including the average commodity factor into the baseline model and we find that the second most dominant factor is the long-short basis-momentum factor with a multiple testing p-value equal to 0.000 based on SI_{ew}^{m} (Panel A.2). Then, we include the long-short basis-momentum factor into the baseline model and find that the third most important factor is the long-short basis, which performs better than long-short momentum; however, none of the long-short basis, long-short momentum and S&P GSCI is significant under the multiple hypothesis testing on SI_{ew}^{m} (p-value=0.309, see Panel A.3). When employing the test-statistic SI_{ew}^{med}, none of the factors is able to explain the cross-section of individual commodities, in addition to the average commodity factor.

Panel B of Table 5 tabulates the results when commodity portfolios are considered for test assets. In particular, we use the nine low, medium and high commodity factor portfolios. The long-short commodity momentum factor is the best among the factors, reducing the mean (median) scaled absolute intercept by 11.7% (18.9%), higher than the remaining factors. The bootstrapped 5th percentile of SI_{ew}^{m} $\left(SI_{ew}^{med}\right)$ for the long-short commodity momentum shows that the reduction in the mean (median) scaled intercept is 14.4% (14.9%), at the 5th percentile. The actual factor reduces the mean (median) scaled intercept by more than the 5th percentile with p-values equal to 0.000 (0.006) (see Panel B.1). With respect to the multiple hypothesis test, the bootstrapped 5th percentile of SI_{ew}^{m} $\left(SI_{ew}^{med}\right)$ is -0.250 and statistically significant with a multiple testing p-value equal to 0.004 (0.040). Overall, the long-short commodity momentum factor is the most important among the candidate factors and is statistical significant at 5% level with respect to the single and multiple hypothesis tests. We repeat our analysis by including the long-short commodity momentum factor into the baseline model and we find that the second most dominant factor is the average commodity factor with a multiple testing p-value equal to 0.002 based on SI_{ew}^{med} (Panel B.2).We repeat the analysis by including the average commodity factor into the baseline model and we find that the third most dominant factor is the long-short basis-momentum factor with a multiple testing p-value equal to 0.000 (0.001) based on SI_{ew}^{m} $\left(SI_{ew}^{med}\right)$ (Panel B.3). Then, we include the long-short basis-momentum factor into the baseline model and find that the fourth most important factor is the long-short basis with a multiple testing p-value equal to 0.000 based on SI_{ew}^{m} (Panel B.4). When we include the long-short basis into the baseline model, S&P GSCI is not significant under the multiple hypothesis testing on SI_{ew}^{m} (p-value=0.309, see Panel B.5). When employing the test-statistic SI_{ew}^{med}, neither S&P GSCI nor long-short basis is able to explain the cross-section of commodity portfolios.

Our results are sensitive to the use of individual commodities or commodity portfolios as test assets. There is no consensus in the prior academic asset pricing literature on equities whether individual stocks or equity portfolios should be used as test assets. A number of academic studies argue that individual stocks are very noisy to be considered as test assets (Black, Jensen and Scholes, 1972, Fama and MacBeth, 1973). Other studies argue that the portfolios might create bias and inefficiency in the asset pricing tests when served as test assets (Avramov and Chordia, 2006, Ang, Liu and Schwarz, 2016 and Lewen, Nagel and Shanken, 2010). Further, Harvey and Liu (2017) argue that the use of individual stocks as test assets minimise the data snooping bias that arises from portfolio-based asset pricing tests (Lo and MacKinlay, 1990). For more information see the discussion in Harvey and Liu (2017).

Using individual commodities as testing assets we find that average commodity portfolio is the most dominant commodity risk factor. The two-factor model comprised of the average commodity factor and the long-short basis-momentum can explain the cross section of individual commodities. Using commodity portfolios as test assets we find that a four-factor model comprised of the average commodity factor, the long-short momentum, the long-short basis and the long-short basis momentum can explain the cross section of commodity portfolios.

3.2.2 Spanning Tests

Barillas and Shanken (2017) and Fama and French (2017) use spanning regressions to find which commodity risk factors are significant in explaining the time variation of expected commodity returns. A risk factor is considered useful if, when regressed on the other factors, produces intercepts which are non-zero. The GRS statistic of Gibbons, Ross and Shanken

(1989) is used to test whether a factor or factors enhance a model's ability to explain expected returns. Table 6 presents results from a time-series regression over the period 1975-2015 in which the dependent variable is the return of the candidate commodity risk factor and the independent variables are the returns of the competing model commodity risk factors.

Panel A of Table 6 shows that the intercept in the spanning regression for the long-short momentum is 0.70% per month (t-stat = 2.828), for the long-short basis is 0.50% (t-stat= 2.128) and for the long-short basis-momentum is 0.60% (t-stat=2.680). Overall, we find that (a) the returns of the average commodity, long-short basis and long-short basis-momentum do not span the return of the long-short momentum factor, (b) the returns of the average commodity factor, long-short momentum and long-short basis-momentum do not span the return of the long-short basis factor and (c) the returns of the average commodity, long-short momentum and long-short basis factors do not span the long-short basis-momentum factors.

Panel B of Table 6 tabulates the GRS statistic (Gibbons, Ross, and Shanken, 1989) which tests whether multiple factors jointly provide additional explanation to a baseline model. We choose between the following models:

a) The three (the average commodity, basis and momentum) and four (average commodity, basis, momentum and basis-momentum) factor models against the single market factor (the average commodity) model.
b) The three (average commodity, basis and momentum) and four (average commodity, basis, momentum and basis-momentum) factor models against the single basis factor model of Szymanowska et al. (2014).
c) The three (average commodity, basis and momentum) factor model against the two (average commodity and basis) factor model of Yang (2013).
d) The four (average commodity, basis, momentum and basis-momentum) factor model against the two (the average commodity and basis-momentum) factor model of Boons and Prado (2017) and
e) The four (average commodity, basis, momentum and basis-momentum) factor model against the three (average commodity, basis and momentum) factor model of Bakshi, Gao and Rossi (2017)

The GRS test on the intercepts from the spanning regressions of long-short basis and long-short momentum on the average commodity factor rejects the null hypothesis that the intercepts are jointly zero with a p-value equal to zero (p-value=0.000). We find similar results when we jointly test the intercepts from the spanning regressions of long-short basis, long-short momentum and long-short basis momentum on the average commodity factor. GRS tests of a two and three factor model against the basis model of Szymanowska et al. (2014) suggests that the addition of the average commodity, momentum and basis-momentum factors adds to the explanatory model of the base model. Based on the estimated GRS statistics the two factor models of Yang (2013) and Boons and Prado (2017) are inferior to models that add the momentum and basis-momentum and the basis and momentum factors respectively. Finally, the non-zero intercept of the spanning regression with the basis-momentum as the LHS variable, suggests that basis-momentum has marginal explanatory power for commodity returns over and above the explanatory power of the other factors.

3.3 Is the S&P GSCI an "efficient" portfolio?

The S&P GSCI is the industry-standard benchmark for commodities investing. The index has been "designed to reflect the relative significance of each of the constituent commodities to the world economy, while preserving the tradability of the index by limiting eligible contracts to those with adequate liquidity".[11] While a capitalization weighted portfolio of all equities is consistent with the equilibrium world of the CAPM, the production weights used for the S&P GSCI cannot be justified similarly. That leaves open the question of what is an appropriate proxy of the "market" commodities portfolio.

The average arithmetic excess return of S&P GSCI over the 1975-2015 period was 0.63%, its volatility 19.48% implying a Sharpe ratio of just 0.032. In contrast, a much better diversified portfolio of equally weighted commodities achieved an average excess return of 3.64%, volatility 13% and a Sharpe ratio of 0.28. The return to risk trade-off of the S&P GSCI is clearly inferior to the average commodity portfolio and the high momentum, low basis and high basis-momentum commodity factor portfolios. Using the Harvey and Liu (2017) methodology, we find that the average commodity factor is considered the best among the candidate commodity risk factors in explaining the cross-section of individual commodity returns. In contrast, the S&P GSCI though is found to be statistical insignificant with a p-value = 0.419 for SI_{ew}^{m} and p-value = 0.473 for SI_{ew}^{med} (see Panel A.1 of Table 5). The evidence suggests that the S&P GSCI is unlikely to be a portfolio on the efficient frontier.

[11] See S&P GSCI Methodology, http://us.spindices.com/documents/methodologies/methodology-sp-gsci.pdf

4. Multifactor commodity portfolios: the benefits from diversification

Evidence based on historical returns suggests that exposure to the basis, momentum and basis-momentum factors has been rewarded with positive risk premiums. Spanning tests also suggest that the three non-market commodity premia represent independent and non-redundant sources of return available to commodity investors. In this Section we examine the benefits from a diversified portfolio of factor premia. To create the combined factor commodity portfolio, we use mean-variance optimization with expected return and variance-covariance based on historical data. To assess the robustness of the mean-variance based portfolios to estimation error we also use equal (EW), inverse variance (IV), minimum variance (MinVar) and maximum diversification portfolio (MDP) weights.[12]

Panel A in Table 7 presents the performance of commodity factor portfolios created using different portfolio construction rules. Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha, Appraisal ratio, Turnover and breakeven transaction costs are annualised. Alpha is estimated based on the time-series regression of the combined commodity portfolio $\left(R_t^{comb}\right)$ on the average commodity factor (AVG), i.e. $R_t^{comb} = a + \beta AVG_t + \varepsilon_t$. We test the hypothesis that the Sharpe ratios of the combined portfolio and the average commodity factor are equal using the methodology of Ledoit and Wolf (2008) with 5000 bootstrap resamples and a block size equal to b = 5. The appraisal ratio is defined as the alpha $\left(a\right)$ divided by the standard error of the regression $\left(\sigma_\varepsilon\right)$, i.e. $\frac{a}{\sigma_\varepsilon}$. Turnover is calculated as $12*\frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{N}\left(\left|w_{j,t+1}-w_{j,t+}\right|\right)$, where $w_{j,t+1}$ is the weight of portfolio j at time $t+1$ and $w_{j,t+}$ is the portfolio weight before the rebalancing at time $t+1$. Finally, the break-even transaction costs are defined as the fixed transaction cost that makes the alpha of the combined commodity factor portfolio against the average commodity portfolio equal to zero. Break-even transaction cost is calculated as the ratio of alpha divided by the turnover of the combined commodity factor portfolio, $\frac{a}{Turnover_m}$.

[12] See Appendix B for calculation details. The alternative weighting methodologies considered here are consistent with mean-variance optimization under specific assumptions about expected returns and risk (see Hallerbach, 2015).

Over the July 1986-December 2015 period, a mean-variance based factor portfolio achieved an annual excess return of 13.09% with a standard deviation of 16.59%. Over the same period the average commodity portfolio had an annual excess return of 5.35% with 12.27% standard deviation. The Sharpe ratio of a mean-variance based commodity factor portfolio is almost double the return to risk offered by the average commodity portfolio (0.789 versus 0.436). The difference in Sharpe ratios is statistically significant at the 1% level of significance. Using the average commodity portfolio as proxy for the commodity "market" portfolio, the mean-variance-based commodity factor portfolio has an annual alpha of 6.89% that is statistically different from zero and an appraisal ratio of 0.808. The combination of the low basis, high momentum and high basis-momentum factor portfolios is clearly better than the equally weighted portfolio of individual commodities.

Alternative portfolio construction rules produce commodity factor portfolios with very similar performance. The Sharpe ratios using alternative weighting schemes range between 0.810 (equally weighted) and 0.792 (minimum variance) and are statistically significantly different from the Sharpe ratio of the average commodity portfolio. Alphas and appraisal ratios using the average commodity portfolio as the benchmark, are very similar to the alpha and appraisal ratio of the mean-variance based commodity factor portfolio.

The annual turnover required to create the commodity factor portfolios are given in column 6 of panel A in Table 7. Annual turnover is significant and highest for the mean-variance based commodity factor portfolio (669.9% per annum) and lowest for the equally weighted commodity factor portfolio. In panel B of Table 7 we report performance statistics when we use the buy/hold cost mitigation strategy used by Novy-Marx and Velikov (2015) to reduce turnover. According to the buy/hold rule, a commodity futures contract remains in a factor portfolio until it falls out of the medium portfolio.

Application of the cost mitigation strategy is very effective in reducing turnover without a significant deterioration in performance. Turnover is reduced on average by approximately 60% to an average, across all portfolio construction rules, of 200% per annum. Annual excess returns and standard deviations are reduced for all commodity factor portfolio combinations but the reduction in Sharpe ratios is much smaller. Alphas are also lower but after adjusting for risk, the appraisal ratios are slightly better. Finally, the break-even transaction cost, the cost that makes a portfolio's alpha zero, improves significantly from 150 basis points to 212 basis points on average. A commodity factor portfolio, constructed under the turnover constraints usually imposed by institutional investors, remains significantly better than either the S&P GSCI or the average commodity portfolio. Its performance is also better than equities or bonds (see panel C of Table 7).

5. Timing commodity factor portfolios

Evidence on the predictability of commodity returns in Hong and Yogo (2012), Ahmed and Tsvetanov (2016) and Gao and Nardari (2016) suggests that commodity returns are time varying and predictable from macroeconomic and commodity specific variables. In the next Section we use recently developed forecasting models to predict the excess return of commodity portfolios. In Section 5.2 we use predicted returns and volatility timing to build dynamic tactical commodity allocation strategies and examine and compare their performance against passive commodity strategies.

5.1 Commodity factor return prediction models

Based on previous research on the predictability of commodity returns we consider three economic predictor variables (short rate, yield spread, default return spread) and three commodity-specific predictor variables (commodity basis, commodity market interest and commodity return) that have been found in the literature on commodity return predictability to predict commodity market returns. Short term rate, yield spread, commodity basis, commodity market interest and lagged commodity market return have been found statistically significant predictor variables on commodity market returns (see Table 6 in Hong and Yogo, 2012).

The *short rate* is defined as the monthly yield on the one-month T-bill. The *yield spread* is defined as the difference between Moody's Aaa corporate bond yield and the short rate. The *default return spread* is defined as the difference between long-term corporate bond and long-term government bond returns. To construct the *commodity basis* we follow Hong and Yogo (2012); first, we calculate the basis for each individual commodity j, then we compute the sector basis based on the median basis within sector[13] and finally we compute the equally weighted average of sector basis across the five sectors. To construct the *commodity market interest* we follow Hong and Yogo (2012); first, we sum the total number of futures (outstanding or traded) across all commodities in each of the five sectors to get the dollar open interest within each sector. Then, we compute the monthly growth rates of the sector open

[13] We use the median basis and not mean (average) basis, since the former is less sensitive to outliers (Hong and Yogo, 2012).

interest and the aggregate growth rate of open interest as an equally weighted average of the growth rate for each of the five sectors. Finally, we smooth these monthly growth rate series by taking a 12-month geometric average. The final predictor variable, the *lagged commodity return* is defined as the 1-month lagged commodity return.

Short term rate, yield spread and default return spread are constructed by Goyal and Welch (2008) and are available from the authors' website.[14] Data on open interest have been sourced from the Commitment of Traders reports issued by the Commodity Futures Trading Commission (CFTC). CFTC data are available electronically since January 1986. For the period that spans January 1975 to December 1985 we collect the data from Yogo's web page.[15] The CFTC data for Brent Crude Oil and Gasoil are sourced from the Intercontinental Exchange (ICE) website.[16]

Table A4 of the Appendix A tabulates descriptive statistics for the predictor variables for the 1975 to 2015 period. The commodity market interest, the yield spread and the short rate are highly persistent with a first order autocorrelation above 0.90; commodity basis exhibits a lower first-order autocorrelation (0.73). We document a very low correlation (below 20%) between the state variables; only the yield spread and the short rate exhibit a correlation of 88%. Our findings are of the same magnitude and in line with Hong and Yogo (2012).

We employ four forecasting models, namely, the historical average, the forecast combination (pooled average) (Rapach, Strauss, and Zhou, 2010), the diffusion indices (Ludvigson and Ng, 2007) and the multiple regression. A detailed description of the forecasting models we use can be found in Rapach and Zhou (2013) and Appendix C. We use ten years of data as the initial in-sample period to generate out-of-sample forecasts for the period July 1986 to December 2015. Following the literature we generate forecasts using a recursive (i.e. expanding) window.[17]

Table A5 in Appendix A reports out-of-sample forecasting statistics R^2_{OS} (Campbell and Thompson, 2008) and *MSFE -adjusted* (Clark and West, 2007) for the six individual predictor variables (Panel A) and the four forecasting methods based on multiple predictor variables

[14] Welch's website: http://www.ivo-welch.info/professional/, Goyal's website: http://www.hec.unil.ch/agoyal/
[15] Yogo's website: https://sites.google.com/site/motohiroyogo/home/research.
[16] ICE's website: https://www.theice.com/marketdata/reports/122

[17] See Neely et al (2012), Gao and Nardari (2017), Rapach and Zhou (2013), among others. Hansen and Timmermann (2012) show that out-of-sample tests of predictive ability have had better size properties when the forecast evaluation period is a relatively large proportion of the available sample.

(Panel B). The pooled average and diffusion index models have positive R_{OS}^2 statistics for forecasting the one-month excess return on the S&P GSCI, the average commodity portfolio and the high momentum and low basis commodity portfolios. In addition, pooled average and diffusion index forecast outperforms the historical average in terms of MSFE for the S&P GSCI, the average commodity portfolio and the long-only basis factor. The R_{OS}^2 statistic is positive and statistical significant for the multiple regression model when forecasting the one-month excess return on S&P GSCI, the average commodity portfolio and the long-only commodity basis factor. On the other hand, the pooled average, diffusion index and multiple regression forecasts for the one-month returns on long-short commodity factor premia underperform the historical average in terms of negative R_{OS}^2 and MSFE.

5.2 Return and variance timing

If commodity returns and risks are time varying, a mean-variance investor would practice tactical timing holding a position in the commodity portfolio that differs from the long-term allocation based on long term forecasts of risk and return.

The optimization problem faced by a mean-variance investor when the excess return, $\hat{r}_{t+1}^{j}$ and variance $\hat{\sigma}_{t+1}^{2}$ of the commodity portfolio are time varying is: $\max_{w_t}\left[w_t\hat{r}_{t+1}^{j}+r_f+\frac{\gamma}{2}w_t^2\hat{\sigma}_{t+1}^{2}\right]$ where w_t is the weight of the commodity portfolio ($\left(1-w_t\right)$ the weight in the risk-free asset)), γ is the investor's risk aversion and r_f is the risk-free rate. The optimal investment in the commodity portfolio is given by $w_t=\frac{1}{\gamma}\frac{\hat{r}_{t+1}^{j}}{\hat{\sigma}_{t+1}^{2}}$. An investor with no ability to forecast the time varying portfolio commodity excess return will use instead the long-term expected excess return $\hat{r}_{t+1}^{j}=\bar{r}$, in which case the weight in the commodity portfolio is given by $w_t=\frac{1}{\gamma}\frac{\bar{r}}{\hat{\sigma}_{t+1}^{2}}$ and denoting $\frac{\bar{r}}{\gamma}=c$, $w_t=\frac{c}{\hat{\sigma}_{t+1}^{2}}$. Volatility timing is the optimal asset allocation decision for a mean-variance optimizing investor who can forecast volatility but not expected returns.

To investigate whether (a) variance timing and (b) variance and return timing simultaneously add value in a commodity factor portfolio we construct two portfolios with the following excess returns:

(i) the excess return of the *variance-managed commodity portfolio* $\left(f_{t+1}^{\sigma^2} \right)$ defined as:

$$f_{t+1}^{\sigma^2} = \frac{c}{\hat{\sigma}_t^2} f_{t+1}$$

where f_{t+1} the excess return of the unmanaged commodity portfolio and $\hat{\sigma}_{t,f}^2$ is the conditional variance of the commodity factor portfolio; c is a constant and chosen so that managed commodity portfolio has the same unconditional volatility (standard deviation) as the unmanaged commodity portfolio (Muir and Moreira, 2017). The choice of a particular volatility target will affect the return, volatility and alpha of the volatility managed portfolio but will not affect portfolio performance measures such as the Sharpe ratio or the appraisal ratio.

(ii) the excess return of the *combined return-forecast and variance-managed portfolio* $\left(f_{j,t+1}^{\sigma^2,r} \right)$ is defined as:

$$f_{j,t+1}^{\sigma^2,r} = \frac{1}{\gamma} \frac{\hat{r}_{t+1}^{j}}{\hat{\sigma}_t^2} f_{t+1}$$

where f_{t+1} the unmanaged commodity portfolio; $\hat{r}_{t+1}^{j}$ is the forecast excess return one month ahead, $j = histavg$ stand for the historical average, $j = poolavg$ stands for the pooled average method, $j = DI$ stands for the diffusion index method and $j = MULT$ stands for the multiple regression method; $\hat{\sigma}_t^2$ stands the conditional variance of the unmanaged commodity portfolio. The conditional variance of the unmanaged commodity portfolio $\left(\hat{\sigma}_t^2 \right)$ is based on the daily returns of commodity portfolio in the previous month.

Table 8 tabulates the results for the variance-managed commodity portfolios $\left(f_{t+1}^{\sigma^2} \right)$ (variance timing) and Table 9 the results for the combined return-forecast and variance-managed portfolios $\left(f_{t+1}^{\sigma^2,r} \right)$ (variance and return timing). Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha, Turnover, Appraisal ratio and breakeven transaction costs are annualised. Alpha and beta are estimated based on the time-series regression of the managed commodity portfolio $\left(f_{t+1}^{m} \right)$ on the commodity portfolio $\left(f_{t+1} \right)$, i.e. $f_{t+1}^{m} = a + \beta f_{t+1} + \varepsilon_{t+1}$, where $m = \sigma^2$ for the variance-managed portfolio, $m = r$ for the return-forecast based commodity portfolio and $m = \sigma^2, r$ for the combined return-forecast and variance-managed portfolio.

Positive alpha (a) suggests that the managed commodity portfolios $\left(f_{t+1}^{m}\right)$ expand the mean-variance efficient frontier and increase the Sharpe Ratio compared to the passive commodity portfolios $\left(f_{t+1}\right)$. We test the hypothesis that the Sharpe ratios of two portfolios are equal following the method by Ledoit and Wolf (2008) with 5000 bootstrap resamples and a block size equal to b = 5. The appraisal ratio is defined as the alpha (a) divided by the standard error of the regression (σ_{ε}), i.e. $\frac{a}{\sigma_{\varepsilon}}$. The turnover is calculated as $12*\frac{1}{T-1}\sum_{t=1}^{T-1}\sum_{j=1}^{N}\left(\left|w_{j,t+1}-w_{j,t+}\right|\right)$, where $w_{j,t+1}$ is the weight of portfolio j at time $t+1$ and $w_{j,t+}$ is the portfolio weight before the rebalancing at time $t+1$. Finally, the break-even transaction cost is defined as the fixed transaction cost that makes the timing alpha (a) zero and is defines as alpha divided by the turnover of the managed portfolio, $\frac{a}{Turnover_m}$.

Variance timing the average commodity portfolio, over the January 1975 to December 2015 period, increases the Sharpe ratio of the timing strategy from 0.297 to 0.475. The timing alpha is positive (3.26% per annum) and statistically significantly different from zero. The timing strategy generates annual turnover of 612% which combined with transaction costs of 53 basis points will make the timing alpha zero. Investors who can transact at 6.3 (small trades) or 25.8 (large trades) basis points will find the strategy profitable. There is little evidence that variance timing will be beneficial to investors who hold the S&P GSCI portfolio.

Variance timing is beneficial for investors who invest in the low basis, high momentum and high basis-momentum commodity factor portfolios (Table 8, panel B). The variance timing strategies have higher Sharpe ratios, albeit not statistically different to the passive benchmarks, and positive and economically and statistically significant (at the 5% level) alphas. Variance timing almost doubles the turnover of the commodity factor portfolios and as a result the break-even transaction costs range between 70 (High basis-momentum) and 82 (low basis) basis points. Compared with the transaction cost estimates in Marshall et al. (2012)[18] variance timing the commodity factor portfolios provides significant after cost outperformance.

[18] Marshall et al. (2012) estimate, depending on different dollar value trade size buckets, half spreads between 3.1 to 4.4. Investors who require immediate execution, small trades cost on average 6.3 basis points while large trades cost on average 25.8 basis points.

Variance timing the long/short momentum commodity portfolio (Table 8, panel C) produces a better Sharpe ratio and an economically (6.93% per annum) and statistically significant timing alpha. Despite the high turnover (636% per annum) the break-even transaction cost suggests that the strategy will remain, after costs, profitable for most investors. The evidence on the success of variance timing of the long/short commodity momentum portfolio is consistent with the evidence on the success of variance timing of equity momentum reported in Barroso and Santa Clara (2015), Daniel and Moskowitz (2016) and Moreira and Muir (2017). For the other two long/short commodity portfolios variance timing is not profitable producing small positive alphas with high turnover and hence low break-even costs.

Table 9, panel A shows performance statistics for timing strategies that incorporate return and variance timing. Return predictions are based on the historical average, the pooled model, the diffusion index model and the multiple regression model. For the average commodity portfolio line one shows performance statistics for the unmanaged commodity portfolio, line two for the variance timing strategy and lines three to six for the return and variance timing strategy. Variance timing improves the Sharpe ratio of the average commodity portfolio and produces a positive alpha. Incorporating return forecasts in the timing process produces little improvement to the benefits generated by variance timing. Timing the S&P GSCI is not profitable except when return forecasts from the multiple regression model are used as the basis for timing.

Table 9, panel B presents the performance of timing strategies for the high momentum, low basis and the high basis-momentum long-only commodity portfolios. Consistent with the evidence in Table 8, variance timing improves Sharpe ratios and generate positive alphas. However, when return forecasts are also used in the timing strategy, there is no improvement to the performance generated by variance timing alone. For long-only commodity factor portfolios variance timing work but return timing does not.

The results in panel C of Table 9 suggest that, with the exception of variance timing for the momentum premium, timing strategies based on variance and return forecasts provide little benefit to investment in unmanaged commodity portfolios.

Variance forecasts based on last month's variance generate significant turnover in all commodity portfolios. Less volatile variance forecasts will generate less turnover but could be detrimental to the timing strategy's performance. To assess the robustness of the timing performance based on last month's variance as a predictor of next month's variance we also calculated variance based on the last six-month daily commodity portfolio returns (six-moth variance) and use it as a predictor of future variance. Performance statistics are reported in Table 10.

Using the six-month variance as predictor of future variance to time commodity portfolio returns reduces marginally the Sharpe ratio and alphas of the timed commodity portfolios. The managed high momentum commodity portfolio has an alpha of 5.14% per annum and annual turnover of 159.2%. Variance timing the low basis and high basis-momentum commodity portfolios produces annual alphas of 2.29% and 3.55% and annual turnover of 135.39% and 132.2% respectively. As expected, using a much smoother predictor for future variance reduces considerably (by more than 50%) the turnover of the timing strategies and as a result increases the break-even transaction cost required to make the alphas zero. For example, the break-even transaction cost for the average commodity portfolio increases from 46.157 to 122.634 basis points. Significant increases in breakeven transactions costs are observed for the high momentum, low basis and high basis-momentum commodity portfolios (from 62.160 to 323.076, 89.202 to 168.236 and 91.514 to 777.382 basis points respectively). Variance timing the average commodity portfolio, the S&P GSCI and the long-only factor based commodity portfolios using as variance predictor the variance based on the last six-month daily commodity return provides significant value added within the turnover limits currently stipulated in institutional investor mandates.

Table 10, panel C shows that for long/short commodity portfolios, variance timing works for the momentum premium, generating an annual alpha of 5.61%, but less so for the basis-momentum and basis premiums. These results are consistent with the evidence in Table 9 where we used the one-month variance for variance timing.

Finally, timing strategies that use the expected excess commodity portfolio returns generated by the four prediction models presented in Section 5.1 and forecasts of future variance based on the six-month variance increases the alpha of the unmanaged commodity portfolio strategies compared to variance timing only strategies, but the high turnover generated results in little improvement and in many cases deterioration of the break-even transaction cost statistic. The only significant exception is timing the S&P GSCI using the pool, diffusion and multiple regression prediction models for commodity returns.

6. Conclusions

We use a factor-based approach to combine commodity factor portfolios with exposure to commodity factor momentum, the basis and the basis-momentum. These factors were found to jointly explain best the cross-section of commodity returns. Irrespective of the portfolio construction methodology used to create the multifactor commodity portfolio, we find significant improvements in the return to risk trade-off offered by commodity portfolios benchmarked on the S&P GSCI and the average commodity portfolio. We find strong evidence to suggest that the S&P GSCI benchmark is probably an inefficient portfolio, inferior to the average commodity portfolio and the multifactor commodity portfolio.

We find strong evidence in favour of variance timing commodity portfolios. Increasing investments in the commodity portfolio when future variance is expected to be low and decreasing the investment weight to commodity portfolios when future variance is high, improves the unmanaged portfolios Sharpe ratios and generates positive and significant alphas against the average commodity portfolio. Variance timing strategies based on smoother forecasts of variance generate turnover within acceptable institutional investor limits.

We predict commodity portfolio returns using state-of-the art forecasting methodologies and construct dynamic commodity allocation strategies combining expected returns with volatility timing. Our findings are disappointing for the majority of the studied commodity portfolio dynamic strategies. There is little value added from return and risk forecasting to a timing strategy that is based only on variance timing.

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Table 1. Commodity Futures Data

This Table lists 32 commodities and tabulates the categories they belong, the exchange on which they are traded, the Bloomberg ticker symbol, the year of the first recorded observation, the delivery months and code in the Commitment of Traders reports issued by the Commodity Futures Trading Commission (CFTC). The commodity futures contracts are traded on the Chicago Board of Trade (CBOT), the Chicago Mercantile Exchange (CME), the New York Commodities Exchange (COMEX), the Intercontinental Exchange (ICE), the London Metal Exchange (LME) and the New York Mercantile Exchange (NYMEX).

Category	Commodity futures	Exchange	Ticker	Start	Delivery Months	CFTC Code
Energy	Brent Crude Oil	ICE	CO	1988:07	1 : 12	ICE website
	Gasoil	ICE	QS	1989:09	1 : 12	ICE website
	Gasoline	NYMEX	HU/XB	1986:12	1 : 12	111659
	Heating Oil	NYMEX	HO	1986:08	1 : 12	22651
	Natural Gas	NYMEX	NG	1990:05	1 : 12	23651
	WTI Crude Oil	NYMEX	CL	1983:04	1 : 12	67651
Grains & Oilseeds	Corn	CBOT	C	1959:08	3, 5, 7, 9, 12	002601, 002602
	Rough Rice	CBOT	RR	1989:11	1, 3, 5, 7, 9, 11	039601, 039781
	Soybean Meal	CBOT	SM	1959:08	1, 3, 5, 7, 8, 9, 10, 12	026603
	Soybean Oil	CBOT	BO	1959:08	1, 3, 5, 7, 8, 9, 10, 12	007601
	Soybeans	CBOT	S	1959:08	1, 3, 5, 7, 8, 9, 11	005601, 005602
	Wheat	CBOT	W	1959:08	3, 5, 7, 9, 12	001601, 001602
Livestock	Feeder Cattle	CME	FC	1971:12	1, 3, 4, 5, 8, 9, 10, 11	061641
	Lean Hogs	CME	LH	1987:01	2, 4, 6, 7, 8, 10, 12	054641, 054642
	Live Cattle	CME	LC	1964:12	2, 4, 6, 8, 10, 12	057642
Metals	Aluminium	LME	LA	1998:01	1 : 12	085691, 085692
	Copper	LME	LP	1998:01	1 : 12	085691, 085692
	Gold	COMEX	GC	1975:01	2, 4, 6, 8, 10, 12	088691
	Lead	LME	LL	1998:01	1 : 12	NA
	Nickel	LME	LN	1998:01	1 : 12	NA
	Palladium	COMEX	PA	1987:01	3, 6, 9, 12	075651
	Platinum	COMEX	PL	1987:01	1, 4, 7, 10	076651
	Silver	COMEX	SI	1975:01	1, 3, 5, 7, 9, 12	084691
	Tin	LME	LT	1998:01	1 : 12	NA
	Zinc	LME	LX	1998:01	1 : 12	NA
Softs	Sugar	ICE	SB	1962:01	3, 5, 7, 10	080732
	Orange Juice	ICE	JO	1967:03	1, 3, 5, 7, 9, 11	040701
	Lumber	CME	LB	1987:01	1, 3, 5, 7, 9, 11	058641, 058643
	Ethanol	CME	DL	2005:05	1 : 12	025601
	Cotton	ICE	CT	1959:08	3, 5, 7, 10, 12	033661
	Coffee	ICE	KC	1972:01	3, 5, 7, 9, 12	083731
	Cocoa	ICE	CC	1959:08	3, 5, 7, 9, 12	073732

Table 2. Summary Statistics of Commodities

This Table reports summary statistics of the 32 commodity futures returns in excess of the risk-free rate for the period 1975:01 to 2015:12. N denotes the number of observations, Mean is the average return, SD is the standard deviation, Skew denotes the skewness, Kurt is the kurtosis, SR is the Sharpe Ratio, AR(1) is autocorrelation of first order. The last column presents the average basis for each commodity. Mean, SD, Skew, Kurt and SR are annualised. For the annualized skewness and kurtosis, we follow Cumming et al (2014).

Category	Commodity futures	N	Mean	SD	Skew	Kurt	SR	AR(1)	Basis (mean)
Energy	Brent Crude Oil	331	11.49%	32.70%	0.129	3.272	0.351	0.230	-0.002
	Gasoil	318	11.41%	32.86%	0.117	3.144	0.347	0.163	-0.002
	Gasoline	353	21.80%	36.96%	0.175	3.289	0.590	0.106	-0.009
	Heating Oil	353	14.74%	36.24%	0.313	3.422	0.407	0.047	-0.003
	Natural Gas	308	-5.51%	49.26%	0.198	3.169	-0.112	0.047	0.019
	WTI Crude Oil	393	7.76%	33.07%	0.111	3.225	0.235	0.186	-0.001
Grains& Oilseeds	Corn	492	-2.30%	26.64%	0.184	3.313	-0.086	0.017	0.019
	Rough Rice	314	-5.94%	26.94%	0.255	3.405	-0.220	-0.010	0.020
	Soybean Meal	492	10.58%	34.38%	0.087	3.113	0.308	0.018	-0.001
	Soybean Oil	492	5.59%	31.82%	0.150	3.213	0.176	-0.022	0.006
	Soybeans	492	5.71%	28.68%	0.003	3.103	0.199	-0.006	0.005
	Wheat	492	-2.00%	27.98%	0.143	3.159	-0.072	-0.019	0.018
Livestock	Feeder Cattle	492	3.13%	16.73%	-0.072	3.142	0.187	-0.022	0.000
	Lean Hogs	348	2.03%	25.31%	0.015	3.085	0.080	-0.079	0.022
	Live Cattle	492	4.57%	17.77%	-0.010	3.106	0.257	0.018	-0.003
Metals	Aluminium	216	-3.24%	19.89%	0.063	3.025	-0.163	0.078	0.004
	Copper	216	9.28%	26.29%	-0.048	3.255	0.353	0.181	0.000
	Gold	492	1.00%	19.32%	0.134	3.280	0.052	-0.005	0.009
	Lead	216	9.31%	30.06%	0.002	3.079	0.310	0.055	0.001
	Nickel	216	9.54%	36.11%	0.075	3.021	0.264	0.079	-0.001
	Palladium	348	8.49%	31.78%	0.122	3.303	0.267	0.012	0.002
	Platinum	348	3.56%	20.88%	-0.129	3.242	0.171	0.040	0.000
	Silver	492	2.69%	34.01%	0.403	4.085	0.079	0.075	0.009
	Tin	216	9.30%	25.33%	0.133	3.092	0.367	0.077	-0.001
	Zinc	216	0.81%	27.18%	-0.011	3.139	0.030	-0.012	0.004
Softs	Sugar	492	-3.67%	39.92%	0.303	3.322	-0.092	0.150	0.018
	Orange Juice	492	4.53%	31.90%	0.484	3.773	0.142	-0.081	0.007
	Lumber	348	-3.54%	30.83%	0.148	3.158	-0.115	0.025	0.019
	Ethanol	128	45.71%	41.35%	0.287	3.205	1.106	0.097	-0.023
	Cotton	492	4.23%	26.79%	0.106	3.116	0.158	0.047	0.008
	Coffee	492	6.41%	37.79%	0.336	3.259	0.170	-0.022	0.006
	Cocoa	492	8.15%	31.94%	0.181	3.089	0.255	-0.083	0.007

Table 3. Membership in the long and short components of the momentum, basis and basis-momentum portfolios

This Table reports the memberships of the long and short components of the momentum, basis and basis-momentum strategies. Membership is defined as the number of months the commodity has entered the long and short components of the momentum, basis and basis-momentum factor portfolios. The long and short components are based on the 30% top and 30 % bottom portfolios for the three commodity factor strategies.

		Momentum		Basis		Basis-Momentum	
Category	**Commodity futures**	**Long component**	**Short component**	**Long component**	**Short component**	**Long component**	**Short component**
Energy	Brent Crude Oil	141	76	154	34	123	70
	Gasoil	114	82	126	32	150	52
	Gasoline	173	55	188	69	214	77
	Heating Oil	133	86	117	74	177	98
	Natural Gas	82	162	115	150	149	128
	WTI Crude Oil	144	115	168	60	88	186
Grains	Corn	82	190	61	321	93	154
	Rough Rice	46	131	34	231	39	188
	Soybean Meal	174	100	182	84	222	120
	Soybean Oil	102	160	65	113	79	52
	Soybeans	117	105	99	92	134	70
	Wheat	84	187	82	307	85	257
Livestock	Feeder Cattle	122	80	220	83	143	161
	Lean Hogs	99	113	147	161	105	195
	Live Cattle	133	77	227	157	167	192
Metals	Aluminium	18	58	22	30	11	58
	Copper	53	22	101	1	33	28
	Gold	106	117	47	30	53	19
	Lead	72	48	72	20	41	78
	Nickel	82	71	77	1	30	11
	Palladium	139	114	74	29	24	169
	Platinum	60	80	104	21	43	95
	Silver	117	160	65	70	73	41
	Tin	62	34	108	2	53	20
	Zinc	47	61	36	22	23	30
Softs	Sugar	149	224	174	228	137	282
	Orange Juice	165	167	167	194	181	194
	Lumber	78	140	111	188	80	187
	Ethanol	90	2	94	16	104	10
	Cotton	128	160	136	225	167	183
	Coffee	169	192	142	263	183	102
	Cocoa	166	193	130	192	231	78

Table 4. Descriptive Statistics over the full sample period: January 1975- December 2015

This Table presents the descriptive statistics for the period 1975:01 to 2015:12 of the commodity benchmarks, i.e. S&P GSCI, Average commodity market factor based on the individual commodities (AVG) and S&P GSCI Light Energy (Panel A), the low, medium, high and long-short commodity momentum (Panel B), the low, medium, high and long-short commodity basis (Panel C), the low, medium, high and long-short commodity basis-momentum (Panel D). The low and high commodity portfolio returns are returns of equally weighted commodity portfolios of the bottom 30 percent and top 30 percent of the 32 commodities we have in our sample. The mean, standard deviation (SD), Skewness, Kurtosis, Sharpe Ratio (SR) and Turnover are annualized.

	N	Mean	SD	Skewness	Kurtosis	SR	Max Drawdown	Turnover
Panel A. Commodity Benchmarks								
S&P GSCI	492	0.63%	19.48%	-0.064	3.159	0.032	79.67%	
AVG	492	3.64%	13.00%	-0.132	3.185	0.280	54.82%	0.679
S&P GSCI Light Energy	492	-0.48%	14.85%	-0.189	3.220	-0.032	67.27%	
Panel B. Commodity Momentum								
Low Momentum	492	-3.81%	16.75%	0.118	3.178	-0.228	91.85%	2.672
Medium Momentum	492	2.39%	13.74%	-0.062	3.183	0.174	51.60%	3.484
High Momentum	492	12.80%	20.33%	0.013	3.237	0.630	49.94%	2.735
Long-Short Momentum (High-Low)	492	16.61%	22.10%	0.089	3.149	0.752	43.47%	5.407
Panel C. Commodity Basis								
Low Basis	492	9.60%	17.06%	-0.056	3.148	0.563	48.66%	4.053
Medium Basis	492	4.33%	15.86%	-0.059	3.171	0.273	62.29%	4.389
High Basis	492	-3.79%	15.94%	0.024	3.141	-0.238	89.22%	4.197
Long-Short Basis (Low-High)	492	13.39%	18.24%	-0.020	3.062	0.734	49.62%	8.250
Panel E. Commodity Basis-Momentum								
Low Basis-Momentum	492	-1.93%	15.65%	0.016	3.161	-0.123	76.25%	2.656
Medium Basis-Momentum	492	2.03%	15.86%	0.052	3.231	0.128	67.03%	3.096
High Basis-Momentum	492	11.44%	17.57%	0.068	3.192	0.651	50.63%	2.531
Long-Short Basis-Momentum (High-Low)	492	13.37%	17.98%	0.000	3.171	0.744	36.33%	5.187

Table 5. Cross-Sectional tests

This Table presents the two metrics developed by Harvey and Liu (2017) SI_{ew}^{m} and SI_{ew}^{median} which measure the difference in equally weighted scaled mean/median absolute regression intercepts between the baseline model and the augmented model. The candidate factors are the average commodity factor based on individual commodities (AVG), S&P GSCI, long-short momentum, long-short basis and long-short basis-momentum. As for tests assets we consider the 32 individual commodities (Panel A) and the 9 commodity portfolio factors, i.e. low, medium and high portfolios (Panel B). The two metrics SI_{ew}^{m} and SI_{ew}^{median} are defined in Section 3.2. The period spans January 1975 to December 2015.

Panel A: Individual Commodities as Test Assets

Panel A.1: Baseline = No Factor

	single test			**single test**		
Factor	SI_{ew}^{m}	***5th percentile***	***p-value***	SI_{ew}^{median}	***5th percentile***	***p-value***
AVG	**-0.309**	-0.276	0.084	**-0.365**	-0.332	0.018
S&P GSCI	-0.252	-0.223	0.419	-0.249	-0.235	0.473
Momentum	0.024	-0.041	0.000	0.035	-0.076	0.030
Basis	0.041	-0.083	0.000	-0.038	-0.119	0.000
Basis-mom	-0.006	-0.047	0.000	-0.001	-0.084	0.001
	Multiple test			**Multiple test**		
	min	-0.290	**0.050**		-0.335	**0.018**

Panel A.2: Baseline = AVG

	single test			**single test**		
Factor	SI_{ew}^{m}	***5th percentile***	***p-value***	SI_{ew}^{median}	***5th percentile***	***p-value***
AVG						
S&P GSCI	0.044	-0.107	1.000	0.014	-0.194	0.988
Momentum	0.019	-0.027	0.045	0.082	-0.086	0.993
Basis	0.014	-0.048	0.001	0.004	-0.099	0.562
Basis-mom	**-0.034**	-0.032	0.017	-0.079	-0.101	0.911
	Multiple test			**Multiple test**		
	min	-0.060	**0.000**	min	-0.119	0.896

Panel A.3: Baseline = AVG + BASIS-MOM

	single test			**single test**		
Factor	SI_{ew}^{m}	***5th percentile***	***p-value***	SI_{ew}^{median}	***5th percentile***	***p-value***
AVG						
S&P GSCI	0.015	-0.105	1.000			
Momentum	-0.008	-0.026	0.503			
Basis	0.006	-0.035	0.061			
Basis-mom						
	Multiple test			**Multiple test**		
	min	-0.034	0.309			

Table 5 (Cont'd)

Panel B: Commodity Portfolios as Test Assets

Panel B.1 : Baseline = No Factor

	single test			single test		
Factor	SI^{m}_{ew}	***5th percentile***	***p-value***	SI^{median}_{ew}	***5th percentile***	***p-value***
AVG	0.115	-0.235	0.163	0.228	-0.422	0.991
S&P GSCI	-0.006	-0.185	0.380	-0.043	-0.333	0.731
Momentum	**-0.117**	-0.144	0.000	**-0.189**	-0.149	0.006
Basis	-0.044	-0.130	0.000	-0.140	-0.209	0.001
Basis-mom	0.145	-0.125	0.000	0.280	-0.134	0.029
	Multiple test			**Multiple test**		
	min	-0.250	**0.004**	min	-0.154	**0.040**

Panel B. 2 : Baseline = MOM

	single test			single test		
Factor	SI^{m}_{ew}	***5th percentile***	***p-value***	SI^{median}_{ew}	***5th percentile***	***p-value***
AVG	**-0.376**	-0.436	0.104	**-0.553**	-0.564	0.000
S&P GSCI	-0.326	-0.364	0.879	-0.467	-0.529	0.856
Momentum						
Basis	0.013	-0.162	0.000	0.011	-0.142	0.004
Basis-mom	-0.005	-0.179	0.000	-0.009	-0.171	0.007
	Multiple test			**Multiple test**		
	min	-0.465	**0.096**	min	-0.582	**0.002**

Panel B.3 : Baseline = MOM + AVG

	single test			single test		
Factor	SI^{m}_{ew}	***5th percentile***	***p-value***	SI^{median}_{ew}	***5th percentile***	***p-value***
AVG						
S&P GSCI	0.045	-0.034	0.971	0.033	-0.141	0.999
Momentum						
Basis	0.001	-0.190	0.000	0.004	-0.131	0.042
Basis-mom	**-0.029**	-0.185	0.000	**-0.057**	-0.146	0.000
	Multiple test			**Multiple test**		
	min	-0.120	**0.000**	min	-0.142	**0.001**

Panel B.4: Baseline = MOM + AVG +BASIS-MOM

	single test			single test		
Factor	SI^{m}_{ew}	***5th percentile***	***p-value***	SI^{median}_{ew}	***5th percentile***	***p-value***
AVG						
S&P GSCI	-0.044	-0.100	0.988	-0.049	-0.198	0.995
Momentum						
Basis	**-0.080**	-0.219	0.000	-0.213	-0.214	0.926
Basis-mom						
	Multiple test			**Multiple test**		
	min	-0.125	**0.000**	min	-0.221	**0.001**

Table 5 (Cont'd)

Panel B: Commodity Portfolios as Test Assets (Cont'd)

Panel B.5: Baseline = MOM + AVG +BASIS-MOM +BASIS

	single test			single test		
Factor	SI_{ew}^{m}	***5th percentile***	***p-value***	SI_{ew}^{median}	***5th percentile***	***p-value***
AVG						
S&P GSCI	-0.024	-0.129	0.982	0.005	-0.230	0.763
Momentum						
Basis						
Basis-mom						
Multiple test				**Multiple test**		
			0.004			**0.040**

Table 6. Time Series Tests

This Table presents the spanning regressions (Panel A) and the GRS statistic of Gibbons, Ross, and Shanken (1989) (Panel B) over the sample period from January 1975 to December 2015. In Panel B the first column is the baseline model, the second column is the sets of additional factors. We consider four baseline models; (a) a model that includes only the average commodity market factor (AVG), (b) the one factor model which includes the basis commodity factor (Szymanowska et al., 2014), (c) the two-factor model, which includes the average commodity (AVG) and the basis factors proposed (Yang, 2013) and (d) the two-factor model, which includes the average commodity (AVG)) and the basis-momentum factors (Boons and Prado, 2017). Momentum, Basis and Basis-Momentum, are the long-short commodity momentum, basis and basis-momentum portfolios, respectively.Int. denotes the intercept of the time series regression, $R^2 adj.$ denotes the adjusted R^2 of the regression, and se denotes the standard error of the time series regressions. Newey-West (1987) t-statistics are in parenthesis.

Panel A. Spanning Regressions

	Int.	AVG	Momentum	Basis	Basis-Momentum	$R^2 adj.$	se
Momentum	0.007	0.199		0.323	0.195	14.17%	0.059
	(2.828)	(1.906)		(3.453)	(2.267)		
Basis	0.005	0.021	0.215		0.256	16.23%	0.048
	(2.128)	(0.293)	(3.451)		(4.013)		
Basis-Momentum	0.006	0.094	0.131	0.258		12.83%	0.048
	(2.680)	(1.249)	(2.372)	(3.736)			

Panel B. Multi-factor tests

RHS returns (Base model)	LHS returns	GRS	p-value
Average commodity portfolio	Basis, Momentum	15.703	0.000
Average commodity portfolio	Basis, Momentum, Basis-Momentum	13.076	0.000
Basis (Szymanowska et al., 2014)	Average commodity, Momentum	6.127	0.002
Basis (Szymanowska et al.., 2014))	Average commodity, Momentum, Basis-Momentum	6.609	0.000
Average commodity and basis (Yang, 2013)	Momentum, Basis-Momentum	9.347	0.000
Average commodity, basis-momentum (Boons and Prado, 2017)	Basis, Momentum	9.256	0.000

Table 7. Combined Long-only Commodity Portfolios

This Table tabulates the results for the combined commodity long-only portfolios (Panel A) and for the combined commodity long-only portfolios under Turnover (TO) mitigation techniques (Panel B). We consider different portfolio construction techniques, i.e. equal (EW), inverse variance (IV), minimum variance (MinVar), maximum diversification portfolio (MDP) and Mean-Variance (MV, $\gamma = 5$) weighting schemes. Panel C presents the average commodity factor (AVG) and the S&P GSCI as for commodity benchmarks, MSCI US and MSCI World equity indices as for equity benchmarks and the US Government Bond Index, as for bond benchmark. Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha (vs. the average commodity factor (AVG)), Turnover, Appraisal ratio and breakeven transaction costs are annualised. We use ten years of data as the initial in-sample period. The forecast evaluation period spans July 1986 to December 2015. We generate forecasts using an expanding window approach. We test the hypothesis that the Sharpe ratios of the combined commodity long-only portfolio and the average commodity factor (AVG) are equal following Ledoit and Wolf (2008). We use Newey-West (1987) standard errors for the statistical significance of alpha. * denotes significance at 10% level, ** denotes significance at 5% level and *** denotes significance at 1% level.

	Mean	SD	SR	alpha	Appraisal ratio	Turnover	Break Even(bps)
Panel A. Combined Commodity Long-only Portfolios							
EW	13.05%	16.12%	0.810***	6.89%***	0.889	3.879	177.643
IV	12.86%	15.97%	0.806***	6.76%***	0.882	3.981	169.682
MinVar	12.57%	15.86%	0.792***	6.61%***	0.822	4.586	144.118
MDP	12.91%	16.03%	0.805***	6.77%***	0.882	4.066	166.635
MV	13.09%	16.59%	0.789***	6.89%***	0.808	6.699	102.837
Panel B. Commodity Combined Long-only Portfolios under TO mitigation techniques							
EW	9.80%	13.98%	0.701***	4.03%***	0.905	1.893	212.859
IV	9.82%	13.96%	0.703***	4.05%***	0.913	1.920	210.838
MinVar	9.96%	13.96%	0.713***	4.19%***	0.943	2.167	193.337
MDP	9.83%	13.98%	0.703***	4.06%***	0.910	1.890	214.736
MV	10.51%	14.57%	0.721***	4.77%***	0.764	2.026	235.470
Panel C. Equity, Bonds and Commodity Benchmarks							
AVG	5.35%	12.27%	0.436	-	-	0.680	-
S&P GSCI	2.14%	20.55%	0.104	-	-	-	-
MSCI US Equity Index	7.34%	15.28%	0.480	-	-	-	-
MSCI World Equity Index	5.65%	15.33%	0.369	-	-	-	-
US Government Bond Index	3.06%	7.26%	0.422	-	-	-	-

Table 8. Variance Managed commodity portfolios

This Table tabulates the results for the 1-month variance-managed commodity portfolio f^{σ^2}. We consider the unmanaged commodity benchmark portfolios, i.e. average commodity factor (AVG) and S&PGSCI (Panel A), the unmanaged long-only commodity factor portfolios (Panel B), and the unmanaged long-short commodity factor portfolios (Panel C). BASIS stands for Basis commodity portfolio MOM stands for Momentum commodity portfolio and BASIS-MOM stands for Basis-Momentum commodity portfolio. $j = histavg$ stands for the historical average, $j = poolavg$ stands for the pooled average method, $j = DI$ stands for the diffusion index method and $j = MULT$ stands for the multiple regression method. Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha (vs. the unamanaged commodity portfolio), beta, Turnover, Appraisal ratio and breakeven transaction costs are annualised. The evaluation period spans from January 1975 to December 2015. We test the hypothesis that the Sharpe ratios of the variance managed portfolio $\left(f^{\sigma^2}\right)$ and its unmanaged portfolio $\left(f\right)$ are equal following Ledoit and Wolf (2008). We use Newey-West (1987) standard errors for the statistical significance of alpha. * denotes significance at 10% level, ** denotes significance at 5% level and *** denotes significance at 1% level.

	Mean	SD	SR	alpha	beta	Appraisal ratio	Turnover	Break Even(bps)
Panel A. Commodity Benchmark Portfolios								
$f \equiv AVG$	3.85%	12.94%	0.297	-	-	-	0.679	-
f^{σ^2}	6.15%	12.94%	0.475	3.26%**	0.751	0.381	6.121	53.220
$f \equiv SPGSCI$	1.02%	19.33%	0.053	-	-	-	-	-
f^{σ^2}	1.58%	19.33%	0.082	0.87%	0.691	0.062	6.177	14.108
Panel B. Long-only Commodity Factors								
$f \equiv MOM$	13.01%	20.30%	0.641	-	-	-	2.735	-
f^{σ^2}	14.62%	20.30%	0.720	5.21%**	0.723	0.371	6.738	77.275
$f \equiv BASIS$	9.84%	17.00%	0.579	-	-	-	4.053	-
f^{σ^2}	12.19%	17.00%	0.717	4.66%**	0.765	0.425	5.667	82.143
$f \equiv BASIS - MOM$	11.46%	17.59%	0.651	-	-	-	2.531	-
f^{σ^2}	12.82%	17.59%	0.728	3.89%**	0.779	0.326	5.548	70.060
Panel C. Long-short Commodity Factors								
$f \equiv MOM$	16.63%	22.12%	0.752	-	-	-	5.404	-
f^{σ^2}	19.67%	22.12%	0.889	6.93%***	0.766	0.487	6.366	108.819
$f \equiv BASIS$	13.35%	18.26%	0.731	-	-	-	8.251	-
f^{σ^2}	11.28%	18.26%	0.618	0.21%	0.829	0.021	5.688	3.746
$f \equiv BASIS - MOM$	12.97%	17.81%	0.728	-	-	-	5.187	-
f^{σ^2}	12.50%	17.81%	0.702	1.61%	0.840	0.147	5.306	30.336

Table 9. Managed commodity portfolios

This Table tabulates the results for the (a) 1-month variance-managed commodity portfolio f^{σ^2} and (b) combined return-forecast and 1-month variance-managed portfolio. $f_j^{\sigma^2,r}$. We consider the unmanaged commodity benchmark portfolios, i.e. average commodity factor (AVG) and S&P GSCI (Panel A), the unmanaged long-only commodity factor portfolios (Panel B), and the unmanaged long-short commodity factor portfolios (Panel C). MOM stands for Momentum) and BASIS-MOM stands for Basis-Momentum. $j = histavg$ stands for the historical average, $j = poolavg$ stands for the pooled average method, $j = DI$ stands for the diffusion index method and $j = MULT$ stands for the multiple regression method. Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha (vs. the unmanaged commodity portfolio), beta, Turnover, Appraisal ratio and breakeven transaction costs are annualised. We use ten years of data as the initial in-sample period. The forecast evaluation period spans July 1986 to December 2015. We generate forecasts using an expanding window approach. We test the hypothesis that the Sharpe ratios of the f^{σ^2} or $f_j^{\sigma^2,r}$ and its 'original' portfolio (f) are equal following Ledoit and Wolf (2008). We use Newey-West (1987) standard errors for the statistical significance of alpha.* denotes significance at 10% level, ** denotes significance at 5% level and *** denotes significance at 1% level.

Panel A. Commodity Benchmark Portfolios

	Mean	SD	SR	alpha	beta	Appraisal ratio	Turnover	Break Even(bps)
$f \equiv AVG$	5.35%	12.27%	0.436	-	-	-	0.680	-
f^{σ^2}	6.83%	12.27%	0.557	2.92%	0.733	0.349	6.316	46.157
$f_{histavg}^{\sigma^2,r}$	5.32%	12.27%	0.434	1.33%	0.748	0.163	6.043	21.938
$f_{poolavg}^{\sigma^2,r}$	6.67%	12.27%	0.544	2.85%	0.689	0.332	7.077	40.221
$f_{DI}^{\sigma^2,r}$	7.19%	12.27%	0.586	3.71%	0.621	0.398	7.913	46.938
$f_{MULT}^{\sigma^2,r}$	6.77%	12.27%	0.552	3.53%	0.559	0.361	8.425	41.918
$f \equiv SPGSCI$	2.14%	20.55%	0.104	-	-	-	-	-
f^{σ^2}	3.18%	20.55%	0.155	1.71%	0.687	0.114	6.345	26.881
$f_{histavg}^{\sigma^2,r}$	-2.56%	20.55%	-0.125	-3.82%	0.588	-0.230	6.521	-58.589
$f_{poolavg}^{\sigma^2,r}$	3.73%	20.55%	0.182	2.57%	0.643	0.149	8.710	29.553
$f_{DI}^{\sigma^2,r}$	2.52%	20.55%	0.123	1.70%	0.382	0.090	9.665	17.639
$f_{MULT}^{\sigma^2,r}$	8.75%	20.55%	0.426	8.08%**	0.314	0.414	10.714	75.436

Table 9 (Cont'd)								
Panel B. Long-only Commodity Factors								
	Mean	**SD**	**SR**	**alpha**	**beta**	**Appraisal ratio**	**Turnover**	**Break Even(bps)**
$f \equiv MOM$	13.66%	18.99%	0.719	-	-	-	2.775	-
f^{σ^2}	13.93%	18.99%	0.734	4.14%	0.717	0.313	6.665	62.160
$f_{histavg}^{\sigma^2,r}$	13.47%	18.99%	0.709	3.60%	0.723	0.274	6.691	53.772
$f_{poolavg}^{\sigma^2,r}$	14.29%	18.99%	0.752	4.53%	0.713	0.341	7.085	63.911
$f_{DI}^{\sigma^2,r}$	14.38%	18.99%	0.757	5.45%*	0.652	0.380	8.222	66.342
$f_{MULT}^{\sigma^2,r}$	13.96%	18.99%	0.735	4.69%	0.678	0.337	8.688	53.987
$f \equiv BASIS$	12.11%	16.66%	0.727	-	-	-	4.087	-
f^{σ^2}	14.04%	16.66%	0.843	4.81%**	0.762	0.446	5.393	89.202
$f_{histavg}^{\sigma^2,r}$	12.47%	16.66%	0.749	3.05%	0.778	0.292	5.297	57.653
$f_{poolavg}^{\sigma^2,r}$	13.53%	16.66%	0.812	4.70%*	0.728	0.413	5.540	84.885
$f_{DI}^{\sigma^2,r}$	13.34%	16.66%	0.801	5.53%**	0.643	0.435	6.001	92.217
$f_{MULT}^{\sigma^2,r}$	13.60%	16.66%	0.817	6.15%**	0.614	0.469	6.856	89.734
$f \equiv BASIS - MOM$	13.39%	16.82%	0.796	-	-	-	2.412	-
f^{σ^2}	14.82%	16.82%	0.881	4.78%*	0.750	0.429	5.227	91.514
$f_{histavg}^{\sigma^2,r}$	14.30%	16.82%	0.850	3.94%	0.774	0.369	5.393	73.026
$f_{poolavg}^{\sigma^2,r}$	14.49%	16.82%	0.861	4.63%*	0.730	0.403	5.424	85.283
$f_{DI}^{\sigma^2,r}$	13.71%	16.82%	0.815	4.95%*	0.648	0.387	5.878	84.300
$f_{MULT}^{\sigma^2,r}$	13.82%	16.82%	0.821	5.14%*	0.643	0.399	6.670	77.018

Table 9 (Cont'd)

Panel C. Long-short Commodity Factors

	Mean	SD	SR	alpha	beta	Appraisal ratio	Turnover	Break Even(bps)
$f \equiv MOM$	15.20%	20.59%	0.738	-	-	-	5.505	-
f^{σ^2}	18.54%	20.59%	0.901*	7.07%***	0.755	0.523	12.908	54.777
$f_{histavg}^{\sigma^2,r}$	18.17%	20.59%	0.882	6.59%***	0.762	0.494	12.965	50.815
$f_{poolavg}^{\sigma^2,r}$	17.79%	20.59%	0.864	6.10%***	0.770	0.466	13.442	45.405
$f_{DI}^{\sigma^2,r}$	14.09%	20.59%	0.684	3.66%	0.688	0.245	17.736	20.623
$f_{MULT}^{\sigma^2,r}$	13.49%	20.59%	0.655	3.36%	0.667	0.220	18.759	17.934
$f \equiv BASIS$	13.26%	16.11%	0.823	-	-	-	8.092	-
f^{σ^2}	11.32%	16.11%	0.703	-0.15%	0.865	-0.019	10.945	-1.371
$f_{histavg}^{\sigma^2,r}$	11.09%	16.11%	0.689	-0.36%	0.863	-0.044	11.102	-3.209
$f_{poolavg}^{\sigma^2,r}$	10.71%	16.11%	0.665	-0.43%	0.838	-0.049	11.489	-3.738
$f_{DI}^{\sigma^2,r}$	7.01%	16.11%	0.435***	-2.82%	0.739	-0.261	13.722	-20.559
$f_{MULT}^{\sigma^2,r}$	8.10%	16.11%	0.503*	0.82%	0.547	0.061	14.760	5.554
$f \equiv BASIS - MOM$	12.96%	16.03%	0.808	-	-	-	5.006	-
f^{σ^2}	11.04%	16.03%	0.689	-0.08%	0.859	-0.010	10.212	-0.799
$f_{histavg}^{\sigma^2,r}$	10.63%	16.03%	0.663	-0.57%	0.864	-0.071	10.267	-5.587
$f_{poolavg}^{\sigma^2,r}$	10.46%	16.03%	0.652	-0.98%	0.869	-0.123	10.361	-9.417
$f_{DI}^{\sigma^2,r}$	9.72%	16.03%	0.607*	-1.69%	0.868	-0.211	10.161	-16.641
$f_{MULT}^{\sigma^2,r}$	8.60%	16.03%	0.537**	-2.55%	0.848	-0.298	11.799	-21.589

Table 10. Managed commodity portfolios (6-month average variance)

This Table tabulates the results for the (a) 6-month average variance-managed commodity portfolio $f^{\sigma^2_{6m}}$ and (b) combined return-forecast and 6-month average variance-managed portfolio. $f^{\sigma^2_{6m},r}$. We consider the 'original' commodity benchmark portfolios, i.e. average commodity factor (AVG) and S&PGSCI (Panel A), the 'original' long-only commodity factor portfolios (Panel B), and the 'original' long-short commodity factor portfolios (Panel C). BASIS stands for the basis commodity portfolio, MOM stands for Momentum commodity portfolio and BASIS-MOM stands for Basis-Momentum commodity portfolio. $j = histavg$ stands for the historical average, $j = poolavg$ stands for the pooled average method, $j = DI$ stands for the diffusion index method and $j = MULT$ stands for the multiple regression method. Average return (Mean), standard deviation (SD), Sharpe Ratio (SR), alpha (vs. the 'original' commodity portfolio), beta, Turnover, Appraisal ratio and breakeven transaction costs are annualised. The forecast evaluation period spans December 1986 to December 2015 (349 observations). We generate forecasts using an expanding window approach. We test the hypothesis that the Sharpe ratios of the $f^{\sigma^2_{6m}}$ or $f^{\sigma^2_{6m},r}$ and its 'original' portfolio (f) are equal following Ledoit and Wolf (2008). We use Newey-West (1987) standard errors for the statistical significance of alpha.* denotes significance at 10% level, ** denotes significance at 5% level and *** denotes significance at 1% level.

Panel A. Commodity Benchmark Portfolios

	Mean	SD	SR	alpha	beta	Appraisal ratio	Turnover	Break Even(bps)
$f \equiv AVG$	5.19%	12.33%	0.421	-	-	-	0.679	-
$f^{\sigma^2_{6m}}$	5.78%	12.33%	0.469	1.63%	0.800	0.220	1.326	122.634
$f^{\sigma^2_{6m},r}_{histavg}$	4.30%	12.33%	0.349	0.05%	0.820	0.007	1.382	3.323
$f^{\sigma^2_{6m},r}_{poolavg}$	6.16%	12.33%	0.500	2.17%	0.770	0.275	3.018	71.761
$f^{\sigma^2_{6m},r}_{DI}$	7.26%	12.33%	0.589	3.76%	0.675	0.413	4.953	75.928
$f^{\sigma^2_{6m},r}_{MULT}$	7.08%	12.33%	0.575	3.88%	0.616	0.400	5.334	72.774
$f \equiv SPGSCI$	1.87%	20.63%	0.091	-	-	-	-	-
$f^{\sigma^2_{6m}}$	2.18%	20.63%	0.106	0.74%	0.769	0.056	1.606	46.083
$f^{\sigma^2_{6m},r}_{histavg}$	-3.50%	20.63%	-0.170*	-4.79%	0.689	-0.320	2.021	-237.058
$f^{\sigma^2_{6m},r}_{poolavg}$	5.41%	20.63%	0.262	4.31%	0.591	0.259	6.312	68.249
$f^{\sigma^2_{6m},r}_{DI}$	6.00%	20.63%	0.291	5.30%	0.371	0.277	6.828	77.652
$f^{\sigma^2_{6m},r}_{MULT}$	10.51%	20.63%	0.509*	10.03%**	0.258	0.503	9.562	104.858

Table 10 (Cont'd)								
Panel B. Long-only Commodity Factors								
	Mean	**SD**	**SR**	**alpha**	**beta**	**Appraisal ratio**	**Turnover**	**Break Even(bps)**
$f \equiv MOM$	13.48%	19.07%	0.707	-	-	-	2.779	-
$f^{\sigma^2_{6m}}$	15.77%	19.07%	0.827	5.14%*	0.788	0.438	1.592	323.076
$f^{\sigma^2_{6m},r}_{histavg}$	15.25%	19.07%	0.799	4.51%*	0.797	0.391	1.588	283.873
$f^{\sigma^2_{6m},r}_{poolavg}$	15.66%	19.07%	0.821	5.16%*	0.779	0.431	2.308	223.628
$f^{\sigma^2_{6m},r}_{DI}$	14.90%	19.07%	0.781	5.63%*	0.688	0.407	4.271	131.793
$f^{\sigma^2_{6m},r}_{MULT}$	15.00%	19.07%	0.787	5.13%*	0.732	0.395	5.013	102.424
$f \equiv BASIS$	11.99%	16.67%	0.719	-	-	-	4.052	-
$f^{\sigma^2_{6m}}$	12.65%	16.67%	0.759	2.29%	0.864	0.273	1.359	168.236
$f^{\sigma^2_{6m},r}_{histavg}$	10.89%	16.67%	0.653	0.36%	0.879	0.045	1.328	27.031
$f^{\sigma^2_{6m},r}_{poolavg}$	13.05%	16.67%	0.783	3.11%	0.829	0.334	2.708	114.821
$f^{\sigma^2_{6m},r}_{DI}$	13.53%	16.67%	0.812	5.06%	0.707	0.429	4.228	119.591
$f^{\sigma^2_{6m},r}_{MULT}$	14.78%	16.67%	0.886	6.80%*	0.666	0.546	5.992	113.410
$f \equiv BASIS - MOM$	13.12%	16.85%	0.778	-	-	-	2.422	-
$f^{\sigma^2_{6m}}$	15.19%	16.85%	0.901	3.55%*	0.887	0.457	1.322	777.382
$f^{\sigma^2_{6m},r}_{histavg}$	14.51%	16.85%	0.861	2.75%	0.897	0.369	1.303	211.157
$f^{\sigma^2_{6m},r}_{poolavg}$	14.99%	16.85%	0.889	3.58%	0.869	0.430	2.184	164.074
$f^{\sigma^2_{6m},r}_{DI}$	14.30%	16.85%	0.849	4.29%	0.763	0.394	3.745	114.652
$f^{\sigma^2_{6m},r}_{MULT}$	14.95%	16.85%	0.887	4.78%	0.775	0.449	4.858	98.407

Table 10 (Cont'd)

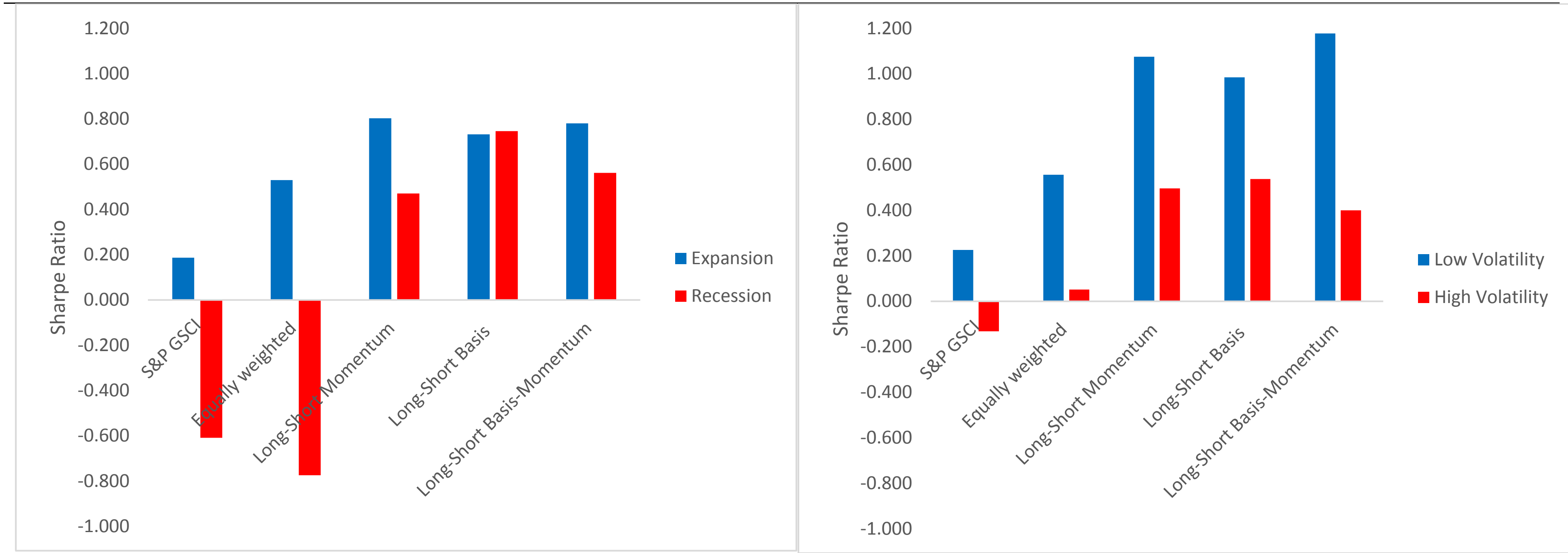
Panel C. Long-short Commodity Factors

	Mean	SD	SR	alpha	beta	Appraisal ratio	Turnover	Break Even(bps)
$f \equiv MOM$	15.30%	20.64%	0.742	-	-	-	5.510	-
$f^{\sigma^2_{6m}}$	19.17%	20.64%	0.929***	5.61%***	0.887	0.587	2.863	195.812
$f^{\sigma^2_{6m},r}_{histavg}$	18.75%	20.64%	0.909**	5.08%***	0.893	0.547	2.845	178.711
$f^{\sigma^2_{6m},r}_{poolavg}$	17.85%	20.64%	0.865	4.19%**	0.892	0.450	4.336	96.656
$f^{\sigma^2_{6m},r}_{DI}$	12.58%	20.64%	0.610	0.81%	0.769	0.062	11.710	6.939
$f^{\sigma^2_{6m},r}_{MULT}$	12.16%	20.64%	0.589	0.45%	0.765	0.034	12.888	3.477
$f \equiv BASIS$	13.20%	16.01%	0.825	-	-	-	8.049	-
$f^{\sigma^2_{6m}}$	12.69%	16.01%	0.793	0.39%	0.931	0.068	2.372	16.643
$f^{\sigma^2_{6m},r}_{histavg}$	12.39%	16.01%	0.774	0.24%	0.920	0.039	2.357	10.369
$f^{\sigma^2_{6m},r}_{poolavg}$	12.62%	16.01%	0.789	0.85%	0.892	0.117	4.169	20.325
$f^{\sigma^2_{6m},r}_{DI}$	9.21%	16.01%	0.575**	-0.82%	0.759	-0.078	9.023	-9.035
$f^{\sigma^2_{6m},r}_{MULT}$	9.81%	16.01%	0.613	2.98%	0.517	0.218	11.067	26.929
$f \equiv BASIS - MOM$	12.76%	15.94%	0.800	-	-	-	5.035	-
$f^{\sigma^2_{6m}}$	13.19%	15.94%	0.827	1.35%	0.928	0.227	2.353	57.268
$f^{\sigma^2_{6m},r}_{histavg}$	12.77%	15.94%	0.801	0.84%	0.935	0.148	2.352	35.571
$f^{\sigma^2_{6m},r}_{poolavg}$	12.39%	15.94%	0.777	0.43%	0.937	0.077	2.649	16.184
$f^{\sigma^2_{6m},r}_{DI}$	11.68%	15.94%	0.733	-0.28%	0.938	-0.051	2.759	-10.152
$f^{\sigma^2_{6m},r}_{MULT}$	9.78%	15.94%	0.613***	-1.47%	0.882	-0.196	7.130	-20.631

Figure 1

Sharpe Ratio: January 1975 – December 2015

NBER Expansion vs. NBER Recession Periods	Low Volatility vs. High Volatility Periods



Appendix A. Additional Tables

Table A1. Descriptive Statistics: Sub-period Analysis

This Table presents the descriptive statistics for the sub-periods 1975:01 to 1995:06 (Panel A) and 1995:07 to 2015:12 (Panel B) of the commodities. As for commodities we consider the (a) commodity benchmarks S&P GSCI, Average commodity market factor based on the individual commodities (AVG) and S&P GSCI Light Energy, (b) the low, medium, high and long-short commodity momentum, (c) the low, medium, high and long-short commodity basis, and finally (d) the low, medium, high and long-short commodity basis-momentum. The low and high commodity portfolio returns are returns of equally weighted commodity portfolios of the bottom 30 percent and top 30 percent of the 32 commodities we have in our sample. The mean, standard deviation (SD), Skewness, Kurtosis and Sharpe Ratio (SR) are annualized.

	N	Mean	SD	Skewness	Kurtosis	SR	Max Drawdown
Panel A. Sub-period: January 1975- June 1995							
Commodity Benchmarks							
S&P GSCI	246	1.23%	15.88%	0.048	3.236	0.078	49.42%
AVG	246	2.17%	12.37%	-0.053	3.064	0.176	54.82%
S&P GSCI Light Energy	246	0.94%	13.53%	-0.166	3.165	0.070	51.45%
Commodity Momentum							
Low Momentum	246	-5.89%	16.62%	0.146	3.193	-0.354	82.62%
Medium Momentum	246	0.79%	13.35%	-0.044	3.115	0.059	51.60%
High Momentum	246	12.77%	21.50%	0.122	3.248	0.594	49.94%
Long-Short Momentum (High-Low)	246	18.66%	24.07%	0.151	3.143	0.776	43.47%
Commodity Basis							
Low Basis	246	6.33%	16.02%	0.009	3.173	0.395	46.69%
Medium Basis	246	3.68%	15.73%	0.013	3.124	0.234	62.29%
High Basis	246	-5.02%	17.40%	0.073	3.138	-0.289	80.94%
Long-Short Basis (Low-High)	246	11.35%	20.23%	-0.056	3.050	0.561	49.62%
Commodity Basis-Momentum							
Low Basis-Momentum	246	-3.19%	16.89%	0.087	3.179	-0.189	74.91%
Medium Basis-Momentum	246	-0.10%	15.82%	0.209	3.278	-0.006	55.81%
High Basis-Momentum	246	11.11%	17.71%	0.252	3.255	0.627	50.27%
Long-Short Basis-Momentum (High-Low)	246	14.30%	20.62%	-0.004	3.155	0.694	36.33%
Panel B. Sub period: July 1995 - December 2015							
Commodity Benchmarks							
S&P GSCI	246	0.02%	22.54%	-0.097	3.089	0.001	79.67%
Equally weighted	246	5.10%	13.61%	-0.196	3.272	0.375	46.01%
S&P GSCI Light Energy	246	-1.90%	16.07%	-0.195	3.230	-0.118	67.27%
Commodity Momentum							
Low Momentum	246	-1.73%	16.89%	0.092	3.171	-0.103	61.28%
Medium Momentum	246	3.99%	14.13%	-0.081	3.242	0.282	49.53%
High Momentum	246	12.83%	19.12%	-0.142	3.211	0.671	47.30%
Long-Short Momentum (High-Low)	246	14.56%	19.97%	-0.033	3.119	0.729	32.66%
Commodity Basis							
Low Basis	246	12.86%	18.01%	-0.111	3.136	0.714	48.66%
Medium Basis	246	4.99%	16.01%	-0.128	3.222	0.311	52.44%
High Basis	246	-2.57%	14.35%	-0.055	3.116	-0.179	62.75%
Long-Short Basis (Low-High)	246	15.43%	16.03%	0.072	3.037	0.963	20.88%
Commodity Basis-Momentum							
Low Basis-Momentum	246	-0.66%	14.32%	-0.093	3.111	-0.046	50.87%
Medium Basis-Momentum	246	4.15%	15.90%	-0.102	3.207	0.261	46.01%
High Basis-Momentum	246	11.78%	17.47%	-0.122	3.135	0.674	50.63%
Long-Short Basis-Momentum (High-Low)	246	12.44%	14.92%	-0.002	3.065	0.834	24.46%

Table A2. Descriptive Statistics: NBER Recession and NBER Expansion Periods

This table presents the descriptive statistics for the NBER Recession period (Panel A) and NBER expansion period (Panel B) of the commodities. As for commodities we consider the (a) commodity benchmarks S&P GSCI, average commodity factor (AVG) and S&P GSCI Light Energy, (b) the low, medium, high and long-short commodity momentum, (c) the low, medium, high and long-short commodity basis, and finally (d) the low, medium, high and long-short commodity basis-momentum. The low and high commodity portfolio returns are returns of equally weighted commodity portfolios of the bottom 30 percent and top 30 percent of the 32 commodities we have in our sample. The mean, standard deviation (SD), Skewness, Kurtosis and Sharpe Ratio (SR) are annualized.

	N	Mean	SD	Skewness	Kurtosis	SR
Panel A. NBER Recession Period						
Commodity Benchmarks						
S&P GSCI	59	-18.57%	30.44%	0.012	3.108	-0.610
AVG	59	-15.09%	19.47%	-0.072	3.124	-0.775
S&P GSCI Light Energy	59	-17.60%	23.72%	-0.162	3.122	-0.742
Commodity Momentum						
Low Momentum	59	-21.48%	19.81%	0.150	3.210	-1.085
Medium Momentum	59	-15.36%	20.78%	0.129	3.111	-0.739
High Momentum	59	-8.74%	30.16%	0.055	3.224	-0.290
Long-Short Momentum	59	12.75%	27.11%	0.362	3.326	0.470
Commodity Basis						
Low Basis	59	-3.37%	23.20%	-0.164	3.117	-0.145
Medium Basis	59	-21.32%	23.42%	-0.003	3.114	-0.910
High Basis	59	-18.17%	20.44%	0.102	3.056	-0.889
Long-Short Basis	59	14.80%	19.86%	-0.012	2.967	0.745
Commodity Basis-Momentum						
Low Basis-Momentum	59	-21.04%	22.61%	0.005	3.017	-0.931
Medium Basis-Momentum	59	-14.97%	23.09%	0.139	3.144	-0.648
High Basis-Momentum	59	-8.13%	24.11%	0.017	3.193	-0.337
Long-Short Basis-Momentum	59	12.91%	23.01%	0.014	3.126	0.561
Panel B. NBER Expansion Period						
Commodity Benchmarks						
S&P GSCI	437	3.24%	17.38%	-0.033	3.038	0.187
AVG	437	6.19%	11.69%	-0.065	3.080	0.530
S&P GSCI Light Energy	437	1.86%	13.08%	-0.086	3.044	0.142
Commodity Momentum						
Low Momentum	437	-1.40%	16.19%	0.132	3.173	-0.087
Medium Momentum	437	4.81%	12.35%	-0.086	3.147	0.389
High Momentum	437	15.73%	18.48%	0.055	3.125	0.851
Long-Short Momentum	437	17.14%	21.36%	0.020	3.089	0.802
Commodity Basis						
Low Basis	437	11.36%	16.01%	0.019	3.102	0.710
Medium Basis	437	7.83%	14.27%	0.017	3.090	0.549
High Basis	437	-1.83%	15.16%	0.025	3.163	-0.121
Long-Short Basis	437	13.20%	18.03%	-0.022	3.081	0.732
Commodity Basis-Momentum						
Low Basis-Momentum	437	0.68%	14.31%	0.090	3.174	0.047
Medium Basis-Momentum	437	4.34%	14.51%	0.066	3.216	0.299
High Basis-Momentum	437	14.11%	16.37%	0.137	3.132	0.862
Long-Short Basis-Momentum	437	13.44%	17.22%	-0.004	3.168	0.780

Table A3. Descriptive Statistics: Low and High Volatility periods

This table presents the descriptive statistics for the low volatile period (Panel A) and high volatile period (Panel B) of the commodities. As for commodities we consider the (a) commodity benchmarks S&P GSCI, average commodity factor (AVG) and S&P GSCI Light Energy, (b) the low, medium, high and long-short commodity momentum, (c) the low, medium, high and long-short commodity basis, and finally (d) the low, medium, high and long-short commodity basis-momentum. The low and high commodity portfolio returns are returns of equally weighted commodity portfolios of the bottom 30 percent and top 30 percent of the 32 commodities we have in our sample. The mean, standard deviation (SD), Skewness, Kurtosis and Sharpe Ratio (SR) are annualized.

	N	Mean	SD	Skewness	Kurtosis	SR
Panel A. Low Volatility period						
Commodity Benchmarks						
S&P GSCI	297	3.24%	14.37%	-0.060	3.031	0.226
AVG	288	5.63%	10.13%	-0.077	3.033	0.556
Commodity Momentum						
Low Momentum	291	-4.70%	13.11%	0.018	3.065	-0.358
Medium Momentum	299	2.96%	10.02%	-0.078	3.043	0.296
High Momentum	295	15.38%	14.29%	0.105	3.040	1.076
Long-Short Momentum	273	19.49%	18.12%	0.068	3.052	1.075
Commodity Basis						
Low Basis	293	11.61%	13.30%	0.064	3.052	0.873
Medium Basis	297	5.05%	11.84%	-0.028	3.080	0.426
High Basis	293	-5.38%	11.99%	-0.018	3.050	-0.448
Long-Short Basis	285	14.41%	14.64%	-0.002	3.031	0.985
Commodity Basis-Momentum						
Low Basis-Momentum	286	0.09%	12.36%	-0.048	3.013	0.007
Medium Basis-Momentum	297	2.09%	11.38%	0.002	3.039	0.183
High Basis-Momentum	285	13.88%	13.81%	0.147	3.097	1.005
Long-Short Basis-Momentum	292	16.33%	13.86%	0.011	3.030	1.178
Panel B. High Volatility period						
Commodity Benchmarks						
S&P GSCI	195	-3.36%	25.36%	-0.030	3.067	-0.132
AVG	204	0.83%	16.20%	-0.113	3.122	0.051
Commodity Momentum						
Low Momentum	201	-2.53%	20.96%	0.133	3.109	-0.121
Medium Momentum	193	1.51%	18.08%	-0.041	3.080	0.083
High Momentum	197	8.94%	26.95%	0.016	3.113	0.332
Long-Short Momentum	219	13.03%	26.23%	0.111	3.124	0.497
Commodity Basis						
Low Basis	199	6.64%	21.43%	-0.071	3.079	0.310
Medium Basis	195	3.24%	20.54%	-0.052	3.075	0.158
High Basis	199	-1.46%	20.42%	0.015	3.058	-0.072
Long-Short Basis	207	11.99%	22.31%	-0.017	3.013	0.537
Commodity Basis-Momentum						
Low Basis-Momentum	206	-4.73%	19.31%	0.059	3.120	-0.245
Medium Basis-Momentum	195	1.93%	20.95%	0.058	3.116	0.092
High Basis-Momentum	207	8.09%	21.71%	0.058	3.128	0.372
Long-Short Basis-Momentum	200	9.06%	22.68%	0.026	3.107	0.399

Table A4. Descriptive Statistics of the state variables

This Table presents the descriptive statistics (Panel A) and the correlation matrix (Panel B) for the period 1975:01 to 2015:12 of the predictor (state) variables, i.e. market interest, basis, default return spread, yield spread and short rate. The mean, standard deviation (SD), Skewness and Kurtosis are annualized. AR (1) denotes the first-order autocorrelation.

Panel A. Summary Statistics

	N	**Mean**	**SD**	**Skewness**	**Kurtosis**	**AR(1)**
Market Interest	480	1.22%	1.29%	0.230	3.693	0.928
basis	492	0.31%	0.53%	-0.070	3.203	0.728
Default return spread (dfr)	492	0.07%	5.05%	-0.156	3.729	-0.035
yield spread	492	7.40%	2.42%	0.660	3.029	0.995
short rate	492	4.72%	1.00%	0.170	3.029	0.975

Panel B. Correlation Matrix

	Market Interest	***basis***	***dfr***	***yield spread***	***short rate***
Market Interest	1				
basis	-0.016	1			
dfr	-0.026	0.018	1		
yield spread	0.182	-0.015	-0.006	1	
short rate	0.206	0.074	-0.048	0.876	1

Table A5. Commodities Out-of-Sample Forecasting Statistics, January 1975 – December 2015

This Table presents the out-of-sample forecasting statistics R^2_{OS} and mean squared forecast error (MSFE) for the 6 individual predictor variables on the commodity risk premia (Panel A) and forecasting methods based on multiple predictor variables on the commodity risk premia (Panel B). R^2_{OS} measures the percent reduction in mean squared forecast error (MSFE) for the predictive regression forecast based on the predictor variable or forecasting method compared to the historical average benchmark forecast. Brackets show the p-values for the Clark and West (2007) MSFE-adjusted statistic under the null hypothesis that the historical average *MSFE* is less than or equal to the predictive regression or forecasting method *MSFE* against the alternative that the historical average MSFE is greater than the predictive regression or forecasting method MSFE, i.e. H_0 $R^2_{OS} \leq 0$ ≤ 0 against : $R^2_{OS} > 0$.

Panel A. Forecasts based on individual variables

	S&P GSCI	AVG	High Momentum	Low Basis	High Basis Momentum	Long Short Momentum	Long Short Basis	Long Short Basis Momentum
short rate	0.16%	64.30%	-0.25%	-0.53%	-0.01%	-0.95%	-3.56%	-0.33%
	[0.643]	[0.391]	[0.574]	[0.811]	[0.635]	[0.883]	[0.695]	[0.778]
yield spread	0.93%	13.54%	-0.26%	-0.61%	-0.34%	-0.34%	-1.91%	-0.87%
	[0.135]	[0.537]	[0.878]	[0.973]	[0.687]	[0.620]	[0.385]	[0.394]
default return spread	2.00%	8.49%	0.14%	2.69%	-0.09%	-1.89%	0.66%	-0.68%
	[0.085]	[0.086]	[0.277]	[0.039]	[0.426]	[0.538]	[0.115]	[0.401]
commodity basis	-1.22%	90.04%	-2.74%	-1.32%	-2.73%	-0.55%	-0.78%	-0.36%
	[0.900]	[0.405]	[0.965]	[0.274]	[0.708]	[0.358]	[0.707]	[0.171]
open interest	-0.79%	80.60%	-1.50%	-1.20%	-1.97%	0.00%	-2.86%	-0.31%
	[0.806]	[0.561]	[0.629]	[0.315]	[0.982]	[0.584]	[0.761]	[0.520]
commodity return	2.19%	11.67%	0.12%	2.63%	0.16%	-1.93%	-2.46%	-0.63%
	[0.117]	[0.372]	[0.192]	[0.034]	[0.219]	[0.922]	[0.951]	[0.381]

Panel B. Forecasts based on multiple variables

	S&P GSCI	AVG	High Momentum	Low Basis	High Basis Momentum	Long Short Momentum	Long Short Basis	Long Short Basis Momentum
Historical Avg	-	-	-	-	-	-	-	-
	-	-	-	-	-	-	-	-
Pooled Avg	1.53%	1.31%	0.29%	1.88%	0.41%	-0.34%	-0.33%	-0.40%
	[0.032]	[0.017]	[0.240]	[0.004]	[0.212]	[0.544]	[0.952]	[0.166
Diffusion Index	2.00%	1.00%	-0.24%	2.31%	-0.71%	-3.33%	-5.44%	-0.82%
	[0.173]	[0.045]	[0.183]	[0.016]	[0.256]	[0.628]	[0.151]	[0.060]
Multiple regression	3.05%	-0.29%	-1.99%	2.60%	-2.30%	-4.70%	-6.23%	-3.44%
	[0.070]	[0.030]	[0.144]	[0.008]	[0.245]	[0.674]	[0.929]	[0.076]

Appendix B. Portfolio Construction Techniques

The equally weighted commodity factor portfolio invests proportionally in each of the three commodity factors and since it does not use estimates of return or risk, is by definition free of estimation risk. EW will be mean-variance optimal when commodity factor expected returns, variances, and correlations are the same.

The inverse variance (IV) portfolio rule (Kirby and Ostdiek, 2012) depends only on variance and assumes that the correlation between the factors is zero. IV weights are calculated according to the following equation:

$$w_{it} = \frac{\left(1 / \sigma_{it}^2 \right)^h}{\sum_{i=1}^{N} \left(1 / \sigma_{it}^2 \right)^h}, i = 1, 2, 3$$

where σ_{it}^2 the estimated variance of commodity i.

MinVar rule is the short sale-constrained minimum-variance portfolio of commodity factors. The weights of the minimum- variance portfolio are defined in the following equation:

$$\min_{w} w^T \Sigma w, s.t. \mathbf{1}^T w = 1 \quad \text{and } w_i \geq 0, i = 1, 2, 3$$

MinVar portfolios will be mean-variance optimal under the assumption that expected returns are equal.

The maximum diversification portfolio (MDP) has been proposed by Choueifaty and Coignard (2008) and maximizes the diversification ratio defined as

$$\frac{w^T \sigma}{\sqrt{w^T \Sigma w}}, \text{and } w_i \geq 0, i = 1, 2, 3$$

where $\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$. The numerator is equal to portfolio volatility ignoring correlations while the denominator is the portfolio volatility that takes into account correlation (diversification). The MDP portfolio will be optimal if the assets included in the portfolio have the same Sharpe ratio. For the inverse variance, the minimum variance and the maximum diversification portfolios the estimation of variances and covariances are based on an expanding window approach with an initial sample of 10 years.

Appendix C. Forecasting Methodologies and Forecastability Measures

The first forecasting model is the historical average.

Second, we employ the forecast combination method, namely pooled average model. We use the following linear model:

$$r_{i,t+1} = a_i + \beta_i x_{i,t} + \varepsilon_{i,t+1}, \qquad \text{where} \qquad i = 1, 2, \ldots K$$

where r_{t+1} is excess return at time $t+1$, $x_{i,t}$ is the i th predictor at time t, and K is the total number of predictive variables. We produce the next period out-of-sample individual forecasts $\hat{r}_{i,t+1}$ using the available information up to time t as follows:

$$\hat{r}_{i,t+1} = \hat{a}_{i,t} + \hat{\beta}_{i,t} x_{i,t}$$

Where $\hat{a}_{i,t}$ and $\hat{\beta}_{i,t}$ are the estimates of a_i and β_i, respectively. Finally, we combine the individual forecasts $\hat{r}_{i,t+1}$ with equal weights and the combination forecast $\hat{r}_{t+1}^{poolavg}$ is given by:

$$\hat{r}_{t+1}^{poolavg} = \frac{1}{K} \sum_{i=1}^{K} \hat{r}_{i,t+1}$$

We use the six predictive variables $(K = 6)$ for each asset and detail them in Section 5.1.

The third forecasting model is based on the diffusion index approach. Our latent factor model is defined as follows:

$$r_{t+1} = a + \sum_{i=1}^{K} \beta_i l_{i,t} + \varepsilon_t$$

We estimate the latent factors f based on a principal component analysis (Bai, 2003; Stock and Watson, 2006). The diffusion index forecast $\hat{r}_t^{DI}$ is given by

$$\hat{r}_{t+1}^{DI} = \hat{a} + \sum_{i=1}^{K} \hat{\beta}_i \hat{l}_{i,t}$$

where $\hat{a}$ and $\hat{\beta}$ are the OLS estimates of a and β and $\hat{l}$ is the principal component estimate of l using the conditional information up to time t.

Finally, we use the multiple regression forecasting model based on our six predictor variables $(K = 6)$ presented in Section 2.3. Our multiple regression model is defined as follows:

$$r_{t+1} = a + \sum_{i=1}^{K} \beta_i x_{i,t} + \varepsilon_t$$

where r_{t+1} is excess return at time $t+1$, $x_{i,t-1}$ is the i th predictor at time t, and K is the total number of predictive variables. The multiple regression forecast $\hat{r}_{t+1}^{MULT}$ is given by

$$\hat{r}_{t+1}^{MULT} = \hat{a} + \sum_{i=1}^{K} \hat{\beta}_i x_{i,t}$$

where $\hat{a}$ and $\hat{\beta}$ are the OLS estimates of a and β using data available up to the time t.

We employ two measures to assess the forecastability of commodity returns out-of-sample. The first measure is the out-of-sample R^2 (Campbell and Thompson, 2008), denoted by R_{OS}^2, defined as

$$R_{OS}^2 = 1 - \frac{MSFE_i}{MSFE_0}$$

where $MSFE_i = \frac{1}{T-n_1} \sum_{s=1}^{T-n_1} \left(r_{n_1+s} - \hat{r}_{i,n_1+s} \right)^2$ is the mean squared forecast error for the predictive regression forecast i over the forecast evaluation period, $MSFE_0 = \frac{1}{T-n_1} \sum_{s=1}^{T-n_1} \left(r_{n_1+s} - \overline{r}_{n_1+s} \right)^2$ is the mean squared forecast error of historical average benchmark forecast, where $\overline{r}_{t+1}$ denotes the average expected return defined as $\overline{r}_{t+1} = \frac{1}{t} \sum_{s=1}^{t} r_t$, n_1 stands for the initial in-sample estimation period, T denotes the number full-sample observations. Positive values R_{OS}^2 indicates that the predictive regression-forecasting model outperforms the historical average model in terms of $MSFE$ (i.e. $MSFE_0 < MSFE_i$).

The second out-of-sample measure is the $MSFE$ *-adjusted* test of Clark and West (2007), which tests the null hypothesis that the historical average $MSFE_0$ is less than or equal to the predictive regression $MSFE_i$ against the one-sided alternative hypothesis that the historical average $MSFE_0$ is greater than the predictive regression $MSFE_i$; this corresponds to $H_0 : R^2_{OS} \leq 0$ against $H_0 : R^2_{OS} > 0$. To this end, Clark and West (2007) define first:

$$\hat{f}_{i,n_1+s} = \hat{u}_{0,n_1+s} - \left[\hat{u}_{i,n_1+s} - \left(\overline{r}_{n_1+s} - \hat{r}_{i,n_1+s} \right)^2 \right]$$

and then regress $\hat{f}_{i,n_1+s}$ on a constant for $s = 1, 2, \ldots, T - n_1$. The $MSFE$ *-adjusted* is the t-statistic corresponding to the constant.

The out-of-sample forecast based on an individual predictor variable is given by

$$\hat{r}_{i,t+1} = \hat{a}_{i,t} + \hat{\beta}_{i,t} x_{i,t}$$

where $\hat{a}_{i,t}$ and $\hat{\beta}_{i,t}$ are the OLS estimates from regressing $\{\hat{r}_S\}_2^T$ on a constant and $\{x_{i,s}\}_1^{T-1}$