Asset Pricing in a Model with Executive Incentive Compensation Contracts

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Abstract

In this paper, we study asset prices in a two-agent general equilibrium production model with two key features: convex compensation contracts for the executive, running the firm, and limited stock market participation. In this model, executive compensation contracts approximate many high-profile CEO's remuneration packages, the bulk of which comes in the form of stock options and direct stock grants. The majority of the population do not participate in the stock market and smooth consumption by trading in the bond market, while executives also hold and trade stocks. The model is able to produce a substantial equity premium, relatively smooth interest rates and high volatility of returns on equity. In our model, the aggregate risk is concentrated among a small group of executives, who hold stocks. In turn, they demand a high rate of return for bearing the aggregate equity risk.

1. Introduction

Executive compensation in public companies has seen a rapid increase in the past two decades. With the current emphasis on the provision of incentives to top executives, the bulk of many high-profile CEO's compensation comes in the form of various options and option-like rewards.[1] These contracts are highly convex in various measures of firm performance such as dividends, overall profits or stock price. It means that for every dollar increase in stock price, dividends, or overall profits of the firm, the compensation of its top executives increases by more than one dollar, in many cases by several times more.

Executive compensation issues have typically been discussed in an atemporal partial equilibrium setting. This is due to the nature of dynamic agency theory, which has largely been developed in partial equilibrium or under the hypothesis of risk neutrality. Yet, while option-like, convex executive compensation contracts could be desirable from a company's perspective, by providing strong incentives to executives, they may also have unintended macroeconomic consequences. Indeed, Donaldson, Gershun and Giannoni (2013) show in a simple and traditional business cycle model that even a conservative degree of convexity (as compared to real-life and often much more lavish compensation packages) in the executive compensation contracts can easily produce macroeconomic instability or may cause the economy to be prone to self-fulfilling fluctuations. The mechanism that leads to such macroeconomic implications is robust so that similar instability can be found in a broad class of models.

In this paper, we examine asset pricing implications of convex executive compensation contracts within a general equilibrium dynamic production setting. Our model has two key features, which distinguish it from a canonical consumption-based asset pricing model: convex compensation contracts for executives and a limited participation in the stock market. A limited participation in the stock market is well-documented in the U.S. data. According to the Federal Reserve Bank of

[1] According to Hall and Murphy (2002), "in fiscal 1999, 94% of S&P 500 companies granted options to their top executives. More significantly, the grant-date value of stock options accounted for 47% of total pay for S&P 500 CEOs in 1999." For March 2007, Mercer Consulting estimates that equity related incentive pay represented, on average, 2/3 of total compensation for the top executives of the 100 largest U.S. based firms by sales. Fixed salary compensation represented only 19% (Mercer Consulting Company (2008)).

St. Louis, as of the end of 2015, the top 10% of US households owned more than 80% of stock wealth. In our model, only executives hold and trade stocks.

We focus on the impact of these two features on asset pricing. We ask if macroeconomic instability caused by managerial decisions and unrelated to the fundamentals of the economy in combination with incomplete markets helps explain high equity premium and other asset pricing phenomena, observed in the U.S. data. To ask this question in the context of our model is essentially to ask to what extent convex compensation contracts distort the alignment of the stochastic discount factor of the senior manager-stockholder vis-a-vis that of non-stockholders-workers. Consequently, if chief executives are given contracts such as those widely observed, what impact their choices have on asset prices relatively to the asset prices obtained in the standard consumption-based asset pricing model such as in Hansen (1985).

We find that even mildly convex compensation contracts result in difference between two stochastic discount factors strong enough to increase the expected return and volatility of equity securities. Both, the return on equity and its volatility, are greater in our model than in the standard model in which workers are also the managers of their firms and executive contracts are, therefore, irrelevant. We also show that the prices of risk-free securities remain largely unaffected by the convex contracting. As a result, the risk premium to stock ownership in our model increases to 2.4% from nearly zero risk premium, obtained using the standard model. The volatility of the stock returns is approximately 19% in our model.

We attribute favorable asset pricing implications of our model to the following mechanism: In equilibrium, both types of agents participate in the bond market. In our model, managers make interest payments to workers in a countercyclical fashion. These payments smooth consumption of workers, but amplify the volatility of consumption for executives-stockholders. The latter group also unable to smooth this risk by trading stocks because stock wealth is highly correlated with the income managers receive via their compensation packages. As a result, stockholders demand high premium for holding the aggregate risk, which is shifted to them.

Our results suggest that the use of convex CEO pay practices may contribute to the high stock return premium. There is at least some empirical evidence to support such a relationship. Using annual returns on the S&P 500 index, Fama and French (2002) estimated that the average real equity premium during the period, 1951 - 2000, was 7.43%. During the sub-period, 1951 - 1960 (their Table II), the real equity premium was 14. 27%. After the 1950 Tax Reform Act, firms began to widely use restricted executive stock options to compensate their top managers. According to Frydman and Saks (2008), in their sample of the largest 50 publicly-traded corporations almost no executives received stock options prior to 1950, but more than 18% of top officers were granted an option in 1951. Prior research on executive pay has found less frequent stock option use during the 1970s and the early 1980s (Hall and Liebman (1998), Jensen et al. (2004)). According to Fama and French (2002), the equity premium during those two decades was 2.42% and 4.28% respectively. During the sub-period of 1991 - 2000, the realized real equity premium was 12.54%. The increase in the equity premium during the nineteen nineties thus seems also to coincide with the unprecedented growth in performance-based incentive pay.

An outline of the paper is as follows. Section 2 describes the model and the convex, performance-based contracts. Section 3 explains the impact convex contracts have on the dynamics of the model. Section 4 details methods used in the numerical solution of the model; Section 5 assesses the extent to which high convexity, performance-based contracts influence the returns on financial assets in the model economy and attempts to explain the source of the increased equity premium. Section 6 explores the consistency of the model with the stylized real business cycle facts. Section 7 provides a review of the related literature and Section 8 concludes.

2. The Model and the Convex Executive Compensation Contract

Our model is a version of the model described in Donaldson, Gershun and Giannoni (2013). We study an economy with competitive markets and a neoclassical production technology. There are two types of agents. The first type of agents are workers. They do not participate in the stock market where claims to the firm’s future dividend stream are traded, but they can still smooth consumption intertemporally by trading risk-free bonds.

The second type of agents are self-interested executive managers (collectively referred to as “the manager”).[2] Under his compensation contract, the manager undertakes the firm’s investment and hiring decisions. The total population measure is 1+μ, of which a measure μ are managers and a measure one are shareholder-workers. We assume that the entire economy's output is produced by a continuum of identical firms indexed by $f \in [0, 1]$.

2.1 Workers and Firms

The representative worker’s objective is to maximize his expected lifetime utility over consumption and leisure by choosing the fraction of the time endowment, n_t^s, he agrees to work, and by selecting the amount of the risk free , z_{t+1}^{sb} he wishes to hold:

$$V^s(z_0^b) = \max_{\{z_{t+1}^b, n_t^s\}} E_0\left(\sum_{t=0}^{\infty} \beta^t u^s(c_t^s, 1-n_t^s)\right) \text{ where } \qquad 0 < \beta < 1 \qquad (1)$$

subject to:

$$\begin{aligned} &c_t^s + z_{t+1}^{sb} p_t^b \le z_t^b + w_t n_t^s \\ &0 \le n_t^s \le 1 \end{aligned} \qquad (2)$$

In equations (1) and (2), β is the subjective time discount factor, c_t^s denotes period t per-capita worker’s consumption; w_t is the period t competitive wage rate and $u^s(c_t^s, 1-n_t^s)$ is the worker’s period utility function. p_t^b denotes period t price of the risk free bond.

We note that these agents do not have information concerning the economy’s capital stock, total factor productivity, aggregate labor supplied, etc. In making these assertions, we use the standard interpretation that, individually, workers have a measure zero, and do not recognize they are identical to other workers.

[2] We abstract from both moral hazard and adverse selection considerations and consider a full-information equilibrium. In a full information equilibrium, why managers? Under one possible scenario, there is a fixed utility cost that each shareholder must independently bear to access information and coordinate with other shareholders in selecting the firm’s investment and hiring policies, a cost that can be avoided if those decisions are delegated to a single manager.

The representative worker's preference ordering is given by:

$$u^s(c_t^s, 1-n_t^s) = \frac{(c_t^s)^{1-\eta_s}}{1-\eta_s} - \frac{(n_t^s)^{1+\zeta}}{1+\zeta} \tag{3}$$

where η_s denotes the representative his relative risk-aversion (RRA) coefficient ($0 < \eta_s < \infty$) and ζ is the inverse of the Frisch elasticity of labor supply ($0 < \zeta < \infty$).

The necessary and sufficient first-order conditions for problem (1)-(2) are

$$n_t^s: \qquad (c_t^s)^{-\eta_s} w_t = B(n_t^s)^\zeta \tag{4}$$

$$z_t^{sb}: \qquad p_t^b = E_t[m_{t+1}^b] \tag{5}$$

where m_{t+1}^b is the pricing kernel for bonds, which we discuss in section 2.3.

On the production side, we postulate a single firm, which acts as our proxy for a continuum of identical firms. The firm is perfectly competitive and combines capital, k_t, and labor, n_t^f, to produce the consumption good via a standard constant returns to scale production function:

$$y_t = a_t k_t^{\alpha}(n_t^f)^{1-\alpha} \tag{6}$$

The current level of technology is a_t. We assume that the log of the latter follows an AR(1) process with the persistence coefficient $\rho \in (0,1)$:

$$\begin{aligned} \hat{a}_t &= \ln a_t = \rho \hat{a}_{t-1} + \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma_\varepsilon) \end{aligned}, \quad a_0 \text{ given.} \tag{7}$$

The evolution of the capital stock follows:

$$k_{t+1} = (1-\delta)k_t + i_t, \quad k_0 \text{ given} \tag{8}$$

where i_t is period t investment and δ, $0 < \delta < 1$, is the depreciation rate.

2.2. The Manager

Since workers lack full information by which to evaluate the manager's decisions, they endow him with a convex compensation contract. The main purpose of the contract is to align manager's interests with those of workers. The manager does not receive hourly wages and therefore the labor-leisure trade-off is irrelevant for him. The manager's contract has the general form, $g^m(h_t)$, where $h_t = h(\ )$ is the measure of the firm's performance observed by the shareholders. The workers do not, however, observe the individual arguments of $h(\ )$.

At the beginning of period t, the manager privately observes the realization of the productivity parameter a_t and a vector ν_t of exogenous sunspot shocks, which he believes are relevant to forecasting future events. Then, in light of his remuneration contract, he takes his own utility-maximizing decision $(c_t^m, i_t, n_t^f, z_{t+1}^e, z_{t+1}^{mb})$. Here, c_t^m represents the manager's period t consumption, n_t^f the available hours hired by the manager in period t and z_{t+1}^e, z_{t+1}^{mb} manager's asset holding of equity and risk free bond respectively.

In the absence of retained earnings, the manager's choice of $\left(i_t, n_t^f\right)$ yields dividends, d_t, where

$$d_t = y_t - n_t^f w_t - i_t - \mu c_t^m. \tag{9}$$

The manager's problem is thus:

$$V^m(k_0, w_0, a_0 . \nu_0) = \max_{\left\{c_t^m, i_t, n_t^f, z_{t+1}^e, z_{t+1}^{mb}\right\}} E_t\left(\sum_{t=0}^{\infty} \beta^t u^m(c_t^m)\right) \tag{10}$$

s.t.

$$c_t^m + z_{t+1}^e p_t^e + z_{t+1}^{mb} p_t^b \leq z_t^e (p_t^e + d_t) + z_t^{mb} + g^m(h_t) \tag{11}$$

$$h_t = h(i_t, n_t, k_t, d_t, w_t) \tag{12}$$

and equation (8).

The manager's period utility of consumption, $u^m(\ )$, is given by

$$u^m(c_t^m) = \frac{\left(c_t^m\right)^{1-\eta_m}}{1-\eta_m} \quad . \tag{13}$$

Under standard recursivity arguments, the necessary and sufficient first-order conditions for problem (10) – (12) are

$$n_t^f: \quad u_1^m(c_t^m)\, g_1^m(h_t)\, h_2(i_t, n_t^f, k_t, d_t, w_t) = 0 \tag{14}$$

$$i_t:$$

$$\begin{aligned} &-u_1^m(c_t^m)\, g_1^m(h_t)\, h_1(i_t, n_t^f, k_t, d_t, w_t) = \\ &= \beta E_t[u_1^m(c_{t+1}^m)\, g_1^m(h_{t+1})(h_3(i_{t+1}, n_{t+1}^f, k_{t+1}, d_{t+1}, w_{t+1}) - h_1(i_{t+1}, n_{t+1}^f, k_{t+1}, d_{t+1}, w_{t+1})(1-\delta)] \end{aligned} \tag{15}$$

$$z_t^{mb}: \quad p_t^b = E_t[m_{t+1}^b] \tag{16}$$

$$z_{t+1}^e: \quad p_t^e = E_t[m_{t+1}^m(p_{t+1}^e + d_{t+1})] \tag{17}$$

where $m_{t+j}^m = \beta^j \frac{(c_{t+j}^m)^{-\eta_m}}{(c_t^m)^{-\eta_m}}$ is manager's intertemporal marginal rate of substitution in consumption. By assumption, only manager participates in the equity market, which is, therefore, not complete.

2.3. Equilibrium

Equilibrium in this economy for a given managerial contract $g^m(h_t(\ ))$ is a triple of price functions $w_t = w(k_t, a_t, \nu_t)$, $p_t^e = p^e(k_t, a_t, \nu_t)$, $p_t^b = p^b(k_t, a_t, \nu_t)$, an aggregate investment function $i_t = i(k_t, a_t, \nu_t)$, and an employment function $n_t = n(k_t, a_t, \nu_t)$, such that:

1. The first-order conditions of the representative non-stockholder-worker (4) and (5), and of the stockholder-manager (14) -- (17) are satisfied together with the usual transversality condition: $\lim_{t\to\infty} \beta^t u_1^m\left(c_t, n_t\right) k_{t+1} = 0$ for given initial k_0.

2. The labor, goods and capital markets clear: $n_t^s = n_t^f = n_t$ and $y_t = i_t + c_t^s + \mu c_t^m$. Equilibrium in the financial market requires that stockholderss hold all outstanding equity shares, $z_t^e = 1$, and all other assets (one period bonds) are in zero net supply, $z_t^b = 0$.[3]

2.4. Convex Performance-Based Executive Contracts in General Equilibrium

We consider the family of contracts

$$g^m(h_t(\,)) = A + (h_t(\,))^{\theta} \tag{18}$$

with A ≥ 0, θ > 0 constants, and the measure of firm performance, $h_t(\,)$, described by

$$h_t(\,) = \varphi \kappa w_t n_t + \varphi d_t \tag{19}$$

In expression (19), φ and κ are constants satisfying 0<φ≤1, 0≤ κ ≤1. The expression $w_t n_t$ denotes the equilibrium aggregate wage bill. Parameter κ represents the relative compensation weight applied to the wage bill vis-à-vis the dividend, and φ the overall compensation scale parameter.

Danthine and Donaldson (2008) provide restrictions on the values of the parameters μ, A, θ, κ, and φ with which convex contracts of the form (18)-(19) become optimal within our model context. Optimal contracts ensure the first best allocation of resources. To achieve optimality, the degree of the contract convexity and the magnitude of the salary component, A, must be finely tuned. In this study, we focus on the asset pricing implications of convex contracts of the type (18)-(19), which may possess parameter choices that do not satisfy strict optimality. We justify this choice on the following grounds. Optimal contracting requires precise knowledge not only of the manager's and worker's RRA coefficients, but of the worker's elasticity of labor supply. These parameters are nearly impossible to estimate with high precision. But our main motivation arises from the observation that the large degree of

[3] In this economy, the manager possesses an information advantage, something that he may clearly use for his private advantage. Nevertheless, we eschew the expression "moral hazard" since this label more typically applies to circumstances where the manager's information advantage concerns some aspect of his own abilities or preferences.

convexity of actual performance-based contracts seems to far exceed the convexity of the contract, optimal in our model's context.

How large is the degree of convexity in the typical compensation contract? Using the aggregated compensation index of all CEOs included in the S&P 500, Jensen, Murphy, and Wruck (2004) estimate that an increase of 1% in the mean of the largest 500 firms' asset market values increases CEO compensation by 1.14% on average in the 1970-2003 sample (see their Table 3). Their Figure 1 suggests that this elasticity is much larger in the 1990-2000 period. While we will focus our analysis on moderate levels of contract convexity, it is important to note that this convexity can easily be very large when the compensation involves call options.

3. Performance-Based Contracts and the Dynamics of the Model

The model described in Section 2 can give rise to multiple equilibria solutions for a wide range of parameters related to managerial risk-aversion and contract design (i.e. η_m, θ and A)[4]. In this case, in addition to the fundamental technology shock, ε_t, the manager's personal rate of return on investment, i.e., the effective rate of return from the manager's point of view, plays an important role in his investment decisions. That rate of return represents not only the additional output generated by other unit of investment in physical capital, but also the additional compensation distributed to the manager as a result of that additional unit of output.

To illustrate the how the incentives contract influences manager's investment policy and affects the dynamics of the model, we examine the linearized Euler equation for the optimal intertemporal allocation of the manager's consumption, i.e. the linearized version of (17):

$$\hat{c}_t^m = E_t[\hat{c}_{t+1}^m] - \Omega^{-1} E_t[\hat{r}_{t+1}] \tag{20}$$

where

[4] For a detailed discussion of indeterminacy and instability in this model see Donaldson, Gershun and Giannoni (2013).

$$\Omega = \eta_m - \frac{\theta - 1}{\theta} \frac{1}{(1 - A/\bar{c}^m)(1 + \mu \varphi \theta (\varphi \kappa \overline{wn} + \varphi \bar{d})^{\theta - 1})} \tag{21}$$

In equation (21) Ω corresponds to the manager's RRA coefficient adjusted for features of the incentive contract.[5]

To emphasize the importance of contract convexity ($\theta > 1$) for indeterminacy, we examine the case in which $\theta = 1$, a linear contract, so that $\Omega^{-1} = \frac{1}{\eta_m}$. In these circumstances, equation (21) reduces to the same log-linearized version of Euler equation that we would obtain for the standard representative agent problem:

$$\hat{c}_t^m = E_t[\hat{c}_{t+1}^m] - \frac{1}{\eta_m} E_t[\hat{r}_{t+1}] \qquad (22).$$

In equation (22) date t consumption responds negatively to increases in the expected rate of return for given expected future consumption. The response coefficient is the elasticity of intertemporal substitution. We assume that the manager expects a very high rate of return on capital next period, $\hat{r}_{t+1}$. In this case, $\left(E_t \hat{c}_{t+1}^m - \hat{c}_t^m\right)$ must increase or $\hat{c}_t^m$ must get smaller, which can occur only if the agent saves more so that $\hat{c}_{t+1}^m$ simultaneously increases. But with the expectation of the higher return on capital, the next period's capital stock, $\hat{k}_{t+1}$, also increases. The increase in the capital stock causes the marginal product of capital to drop (e.g., $\hat{r}_{t+1}$ declines). Hence, expectations cannot be filled and there is no supportable equilibrium indeterminacy.

When the manager's contract has higher convexity $(\theta > 1)$, a given increase in the firm's output, generated by an additional unit of physical investment, results in a more than proportional increase in the manager's income. If convexity of the manager's contract is sufficiently larger than 1, Ω becomes negative. In this case, if the manager believes, unrelated to fundamentals, that his own personal return will be "high" next period, then in the interests of consumption

[5] Appendix with the derivation of equations (20) and (21) is available upon request. In log-linearized equations, a ^ above a variable represents its log-deviations from the steady state value, which we denote by the variable with an overhead bar, e.g., $\bar{c}^m$ indicates the steady state consumption of the manager.

smoothing, a perception of a high income next period will cause him to consume more today, and thus to reduce his investment today. The lower investment leads to a higher rate of return on capital, which confirms the manager's belief of a high personal rate of return. In general, the larger the convexity of the executive contract, the more likely that Ω is negative and indeterminacy will arise, so that business cycle fluctuations can result from self-fulfilling fluctuations in manager's expectations.

Equation (21) also shows that the lower the manager's RRA coefficient the more likely Ω is to be negative. This feature implies that the more risk averse the manager is, the more dramatic the incentives portion of his contract must be to make his expectations self-fulfilling. Similarly, for any convex contract ($\theta > 1$), the increase in the constant salary component of the executive contract, A, increases the likelihood Ω becoming negative. That is, the higher fixed salary component makes the manager's aggregate compensation less volatile, and reduces the magnitude of the incentives part of his contract necessary for indeterminacy.

4. Solving the Model and Computing Prices of Financial Assets

Here, we explore the economy of Section 2 to evaluate how performance-based compensation contracts and resulting managerial decisions affect prices of the equity and the risk free securities in our model. Our numerical work is guided by the following questions: (1) How do parameters of the managerial contract, in particular its degree of convexity and the relative importance of the incentives versus the salary, impact the expected return on and the volatility of the stock price? (2) Does this contract have a similar impact on the prices and volatility of the risk-free asset? (3) Does the introduction of the convex contracting compromise the ability of the model to replicate stylized business cycle facts?

We use numerical methods to obtain the solution to the recursive competitive equilibrium defined in Section 2.3 and approximate the equilibrium functions using quadratic approximation methods, developed by Schmitt-Grohe and Uribe (2004) and Sims (2000) and generate artificial time series accordingly. Because markets are incomplete, we solve for allocations and pricing functions simultaneously, following Krusell and Smith (1997) and Storesletten et al (2007)).

4.1 Calibration

We divide the model's parameters into two groups. Values for the first group are obtained using standard calibrations, while the second set of values are estimated using GMM techniques. The model is simulated at quarterly frequencies.

The calibrated parameters include the capital's share of output, α, chosen to equal 0.36, the quarterly capital depreciation rate is set at $\delta = 0.025$, the quarterly subjective discount factor, β, is fixed at 0.9953. Following the standard practice, the value of the persistence parameter is set at $\rho = 0.95$. The remaining calibrated parameters concern the non-stockholder-worker's utility representation. We choose his coefficient of intertemporal elasticity of substitution in consumption to be $\eta_s = 5$ and the inverse of the Frisch elasticity of labor supply is $\zeta = 3$. Following Hansen (1985) and many others we choose B= 2.86 so that the steady state value of labor, $\bar{n}$, is equal to one-third of the time endowment. All calibrated values are in line with empirical macro estimates and represent values commonly used in the literature, see for instance Boldrin, Christiano and Fisher (2001). Following the earlier justification, the measure of managers, μ, is established at 0.1.

When the model is driven by the technology shocks alone, we select the volatility, σ_{ε}, to match the empirical standard deviation of departures from trend output in the U.S. data (1.77%). The i.i.d. sunspot shock, if present, is distributed $N(0, \sigma_{\nu})$. As with shocks to productivity, we choose the standard deviation, σ_{ν}, to match the volatility of output, σ_{y}, observed in the data, a common practice in the literature on indeterminacy when the model's fluctuations result from sunspots shocks only.

Our focus is exclusively on executive incentive contacts of the form (17) in equilibrium. Following Jensen, Murphy and Wruck (2004), we choose the share of the manager's salary component to be one half of his steady state consumption level: $A/\bar{c}^{m} = 0.5$, the average share of fixed salary in the executive compensation contracts, currently prevalent in the U.S.

Commonly accepted values for the second group of parameters are unavailable in the literature. This subset of parameters includes the manager's elasticity of intertemporal substitution in consumption, η_m, the share of the aggregate wage bill in the manger's contract, φ, and the contract's convexity, θ. When the model economy incorporates technology shocks and the sunspot shocks simultaneously, there is no obvious way to estimate the individual variances of these shocks or their correlation. Following Jermann (1998) and others, we choose the parameter values of the second group in a way that maximizes the model's ability to replicate certain asset pricing moments. Let ϑ denote the set of the remaining model parameters to calibrate: $\vartheta = [\eta_m, \varphi, \theta, \sigma_\varepsilon, \sigma_\nu, \rho_{\varepsilon\nu}]'$, and g_T the set of data moments to match. In our case, g_T includes standard deviations of output, total consumption, investment, and labor, and contemporaneous correlations of consumption and labor with output. We calibrate ϑ using a minimum distance procedure that minimizes the following criterion:

$$J(\vartheta) = [g_T - f(\vartheta)]'\Sigma^{-1}[g_T - f(\vartheta)]$$

where $f(\vartheta)$ is the vector of moments implied by the model for a given realization of ϑ, Σ is a weighting matrix and the vector g_T contains the point estimates of the target moments, computed using the data. The matrix Σ is a diagonal matrix with the standard errors of the estimates in g_T on the main diagonal. Therefore, the calibration procedure minimizes a weighted average of the moment deviations. We evaluate the criterion $J(\vartheta)$ for the following grids of parameter values: $\eta_m \in [0; 1]$, $\varphi \in [0.01, 1]$, $\theta \in [0.5, 10]$, $\sigma_\varepsilon \in [0.005, 0.05]$, $\sigma_\nu \in [0.005, 0.1]$, and $\rho_{\varepsilon\nu} \in [-1, 1]$. The choice of grids for the first three parameters guarantees that the baseline model supports multiple equilibria. Table 1 summarizes results of the calibration procedure.

Table 1 -- Parameter choices for the Baseline Model

Parameter	Symbol	Value
Capital share of output	α	0.36
Subjective discount factor	β	0.9953
Capital depreciation rate	Ω	0.025

Workers' elasticity of intertemporal substitution in consumption	η_s	5
Inverse of the Frisch elasticity of labor supply	ζ	3
Persistence of the technology process	ρ	0.95
Fraction of the salary component in the managerial contract	$A/\overline{c}^m$	0.5
Manager's elasticity of intertemporal substitution in consumption	η_m	0.25
Percentage of the aggregate wage in the managerial contract	φ	0.01
Convexity of the managerial contract	θ	2
Standard deviation of the technology shock	σ_ε	0.0117
Standard deviation of the sunspot shock	σ_ν	0.085
Correlation coefficient between the technology shock and the sunspot	$\rho_{\varepsilon\nu}$	0

Given the other parameterized and estimated values in Table 1, the GMM estimate of $\theta = 2$ is conservative as compared to the elasticity of the manager's compensation with respect to firm value as estimated by Gabaix and Landier (2008). In particular, for the baseline parameterization of Table 1, we need to raise the overall contract convexity to about 5.5 to obtain a steady state elasticity of 1.14, the Gabaix and Landier's (2008) estimate.

Under the baseline parameterization, non-stockholders-workers are more risk averse than the manager. Accordingly, they would wish to discourage the manager from the level of risk taking to which he would naturally be inclined. In these circumstances results in Danthine and Donaldson (2008) suggest that the optimal contract from the shareholder's perspective should be concave and possibly even negatively sloped in the firm's free cash flow. Such contracts are not observed and would, indeed, be viewed as absurd in the current executive compensation environment. We thus elect to explore the consequences of contracts as they are, rather than as they should be, hence the indicated parameterization. As a comfort for the relevance of our explorations, we note that both the steady states and lifetime utilities of all the participants differ little under the optimal contract or the more realistic contract considered here.

4.2. Calculating the Price of the Risk Free Bond

To calculate the price of the bond, we adapt the method, developed in Krusell and Smith (1997) and Guvenen (2009). We solve for the bond allocations, by using an arbitrary value of the pricing kernel m_t^b. We then search for the value of the pricing kernel, $\widehat{m}_t$ such that the bond market clears, i.e. $z_t^b < \tau$, where the value of $\tau = 10^{-10}$. The period t price of the risk free bond is then determined by:

$$p_t^b = \widehat{m}_t \tag{23}$$

5. Quantitative Results

5.1. The Baseline Model

Table 2 presents asset-pricing statistics from the baseline model alongside the corresponding statistics from a standard dynamic stochastic general equilibrium (DSGE) model such as in Hansen (1985). For the legitimacy of comparison, we use the same parameter values for the Hansen's model as we do for our baseline model: the risk-aversion coefficient and disutility of work parameter of the representative agent are, respectively, $\eta = 5$ and $\zeta= 3$. Column three of Table 2 contains empirical counterparts for the model's statistics, which we obtain from the U.S. data. The baseline model supports sunspot equilibria. Consequently, there are two exogenous disturbances in the baseline model: the shock to the total factor productivity and the sunspot shock.

Results in Table 2 show that the baseline model is able to match the low average risk-free rate and the standard deviation of the return on the risk-free asset. The latter feature of the risk free rate is challenging to explain in a macro-asset pricing model. Consumption-based asset pricing models, which produced high equity premium and high volatility of the return on equity, also resulted in excessively volatile interest rates (see for example Jermann (1998) and Boldrin et al. (2001)).

Table 2: Asset-pricing statistics from the baseline model, standard DSGE model, and the data

Variable	Statistic	Baseline Model	Standard DSGE Model	Data
Risk-free rate	Mean	0.9	1.86	1.07
	Standard Deviation	3.58	0.51	2.3
	AR(1)	0.95	0.95	0.52
Return on equity	Mean	3.28	1.8607	7.7
	Standard Deviation	19.17	0.58	16.4
	AR(1)	-0.02	0.22	-0.06
Risk Premium	Mean	2.38	0.0007	6.63
	Covariance with non-stockholder -worker's consumption growth	0.48	0	NA

Note: We base the models' statistics on 500 samples of 200 observations each. In the baseline model, the manager's contract is given by equation (19); $\varphi = 0.01$; the share of the manager's fixed salary component is $A/\bar{c}^{m} = 0.5$; the convexity of the contract is $\theta = 2$. The manager's RRA coefficient is $\eta_m = 0.25$. The standard deviation of output (GDP) is 1.77% as in the U.S. data. We report financial statistics in annualized percentage terms. We obtain U.S. data on equity returns, Treasury bill returns, and consumer price index from the CRSP.

The standard deviation of the return on equity is 19.17%, relatively close to its empirical value of 16.4%. The model's equity premium is 2.38%, which falls short of 6.63% equity premium observed in the data. In replicating the equity premium, our model underperforms several other models, such as in studies by Jermann (1998), Boldrin et al. (2001), and Guvenen (2009). However, our value for the equity premium represents a significant improvement over the standard real business cycle model in which this statistic is almost zero.

The baseline model produces very persistent interest rates. The AR(1) coefficient for returns on the risk-free bond is 0.95, higher than in the data. The same parameter for the equity returns is -0.02, closely matching its empirical value of -0.06. In the standard RBC model, equity returns exhibit a counterfactually high positive correlation of 0.22.

5.2. The Effect of Different Shocks

To examine the explanatory contribution of the different shocks, we consider two settings. These settings are identically parameterized to the baseline model, but are either driven by the sunspot shock alone (Model 1) or by the technology shock alone (Model 2). In the cases we consider, we choose the standard deviation of shocks so that in both cases the standard deviation of output is 1.77%, as in U.S. data.

Table 3: Asset-pricing statistics from the models with different shocks

Variable	Statistic	Model 1	Model 2	Data
Risk-free rate	Mean	0.36	1.28	1.07
	Standard Deviation	4.21	1.08	2.3
	AR(1)	0.98	0.92	0.52
Return on equity	Mean	4.16	4.2	7.7
	Standard Deviation	25.8	10.68	16.4
	AR(1)	-0.007	-0.03	-0.06
Risk Premium	Mean	3.8	2.92	6.63
	Covariance with non-stockholder-worker's consumption growth	0.72	0.24	NA

Table 3 shows that the equity premium in Model 1 is 3.8%, higher than in the baseline case. The risk-free rate is 0.36%, lower than in the data (1.07%). The standard deviations of asset returns are somewhat exaggerated: 25.8% for the equity security and 4.21% for the risk-free bond.

In Model 2, the equity premium is slightly smaller at 2.92%, mostly due to the increase in the risk-free rate, 1.28%. The standard deviation of the return on equity is only 10.68%. Overall, we conclude that the asset-pricing performance of both models is comparable to the financial implications of the baseline model.

5.3. Determinate Model

We now examine a variant of the model with the lower contract convexity in which $\theta = 1.1$. With this level of contract convexity, the model has a unique solution. The only source of exogenous uncertainty is a technology shock, since introducing a sunspot shock in this case violates the transversality condition.

Table 4: Asset-pricing statistics from the determinate model

Variable	Statistic	Determinate Model	Data
Risk-free rate	Mean	1.76	1.07
	Standard Deviation	3.11	2.3
	AR(1)	0.83	0.52
Return on equity	Mean	1.91	7.7
	Standard Deviation	3.64	16.4
	AR(1)	0.36	-0.06
Risk Premium	Mean	0.13	6.63
	Covariance with non-stockholder-worker's consumption growth	0	NA

Note: We base the models' statistics on 500 samples of 200 observations each. In the Determinate Model, the manager's contract is given by equation (19); $\varphi = 0.01$; $\kappa = 0.1$; the share of the manager's fixed salary component is $A/\bar{c}^m = 0.5$; the convexity of the contract is $\theta = 1.1$. The manager's RRA coefficient is $\eta_m = 0.25$. The standard deviation of output (GDP) is 1.77%. We report financial statistics in annualized percentage terms. We obtain U.S. data on equity returns, Treasury bill returns, and consumer price index from the CRSP.

Table 4 shows that the model without indeterminacy performs significantly worse in replicating stylized asset-pricing facts: the equity premium is only 0.13% and the standard deviation of the return on equity is 3.64%. The AR(1) coefficient for equity returns is 0.36. These statistics contrast starkly with the observed empirical regularities. We experimented with various values for the convexity parameter, θ, and found that the financial implications are robust to changes in the convexity parameter per se as long as the model retains its ability to support sunspot equilibria. Therefore, in the context of the model in which the delegated manager is endowed with a convex executive contract, we conclude that self-fulfilling expectations and the resulting macro-economic volatility are essential for the good asset-pricing implications.

5.4. The Source of the Model's Equity Premium

There are two features of our model responsible for the substantial equity premium and other improvements in financial statistics. First, limited market participation disentangles pricing kernels used to find prices of stocks and risk free bonds from the growth rate of the aggregate consumption and leads to an asymmetry in consumption smoothing opportunities of non-stockholders-workers and stockholders-mangers. In equilibrium, stockholders make interest payments to workers, which allows the latter group to smooth uncertainty in their labor income. Since workers have higher risk aversion and can trade only in the bond market, their demand for bonds is stronger than that of managers. In addition, since there are more workers in our model (they are of measure one) than managers, the price of bonds and the risk free rate is mainly determined by the workers' marginal rate of substitution in consumption, which has low volatility. While bonds allow workers to smooth their consumption, the volatility of managerial consumption is actually amplified by bond trading.

The second reason for the increase in equity premium in our model is the high volatility of managerial consumption. Because of the structure of the manager's contract, his income largely depends on the dividend stream. It is well known that in real business cycle models, dividends are highly volatile. The high convexity of the managerial compensation contract, which leads to the presence of the sunspot equilibria, increases this volatility even further. Consequently, the volatility of the manager's consumption and his marginal rate of substitution is high. Moreover, manager's consumption is pro-cyclical since the contract is structured to reward the manager when his firm is doing well. As a result, managers are reluctant to own the shares of the firm they manage, since the share price is also pro-cyclical. Therefore, stockholders-managers demand a substantial premium for holding stocks.

When the degree of the managerial contract convexity is low and model does not result in sunspot equilibria, the volatility of the stockholder-manager's consumption relatively low and the equity premium, although still higher than in Hansen's model, goes down.

6. Robustness Checks: The Model's Macroeconomic Implications

It appears that the performance-based executive compensation contracts contribute to the higher return on equity and increase volatility of stock prices in the context of the DSGE model with incomplete markets. Overall, the baseline model's ability to replicated stylized statistics of asset returns in the U.S. data is significantly better than that of a standard DSGE model. But the popularity of the standard model mainly comes from its remarkably accurate representation of the stylized real business cycle facts. In order to compete with the standard DSGE model, our model with convex executive compensation contracts and limited market participation would need to remain consistent with the stylized features of the real business cycle. Tables 5 and 6 report macroeconomic statistics computed from artificial time series generated from simulations and detrended using the Hodrick-Prescott filter.

Table 5: Business cycle statistics from the baseline model, standard RBC model, and the data

Variable	Statistic	Baseline Model	Standard RBC Model	Data
Total consumption	Relative standard deviation	0.86	0.22	0.69
	Correlation with GDP	0.2	0.97	0.88
Employment	Relative standard deviation	1.12	0.06	0.99
	Correlation with GDP	0.08	-0.18	0.88
Investment	Relative standard deviation	3.62	2.82	2.93
	Correlation with GDP	0.83	1	0.8

Note: We base the models' statistics on 500 samples of 200 observations each. In the baseline model, the manager's contract is given by equation (19); $\varphi = 0.01$; the share of the manager's fixed salary component is $A/\bar{c}^m = 0.5$; the convexity of the contract is $\theta = 2$. The manager's RRA coefficient is $\eta_m = 0.25$. The standard deviation of output (GDP) is 1.77% as in the U.S. data. We compute U.S. business-cycle statistics by using Citibase quarterly data. The data range is 1947:Q1 to 2000:Q4

We now consider the macroeconomic implications of the baseline model. Table 5 shows that the baseline model with two types of shocks largely follows the basic stylized facts of the business cycle: investment is more volatile than output, which is in turn more volatile than consumption. The relative standard deviation of consumption is 0.86, compared to 0.69 in the data. The correlations of consumption and employment with output are too small, relative to the data, however.

Table 6 contains the key macro statistics derived from Models 1 and 2. On the macroeconomic side, in Model 1, sunspot shocks alone result in several data inconsistencies. First, relative standard deviations of consumption, employment, and especially investment, are higher than in the data. This fact is not entirely surprising, since sunspot shocks do not affect output directly, and thus must induce large responses in hours and investment in order to replicate the standard deviation of output at the empirically observed $\sigma_y = 1.77\%$.

Table 6: Business cycle statistics from the models with different shocks

Variable	Statistic	Model 1	Model 2	Data
Total consumption	Relative standard deviation	1.06	0.83	0.69
	Correlation with GDP	-0.97	0.98	0.88
Employment	Relative standard deviation	1.57	0.76	0.99
	Correlation with GDP	0.98	-1	0.88
Investment	Relative standard deviation	5.67	1.4	2.93
	Correlation with GDP	1	1	0.8

Note: We base the models' statistics on 500 samples of 200 observations each. In Models 1 and 2 the manager's contract is given by equation (19); $\varphi = 0.01$; the share of the manager's fixed salary component is $A/\bar{c}^m = 0.5$; the convexity of the contract is $\theta = 2$. The manager's RRA coefficient is $\eta_m = 0.25$. The standard deviation of output (GDP) is 1.77%. We compute U.S. business-cycle statistics by using Citibase quarterly data. The data range is 1947:Q1 to 2000:Q4

However, we are disturbed to find that consumption is countercyclical. Sunspot equilibria, per se, seem to have manifestations that violate the notion of consumption as a normal good, at least in this case. We note that a sunspot shock is essentially a shock to the rate of return on capital stock, and that it induces very large procyclical responses in investment without output being itself simultaneously increased. In equilibrium, consumption must be countercyclical. It does not appear that convex-contract-induced sunspot equilibria alone can satisfactorily replicate the stylized facts of the business cycle.

Turning to the labor and capital markets, Model 2 understates the relative standard deviations of employment and investment. More problematic is the fact that in Model 2, employment is strongly negatively correlated with output, but is highly procyclical in the data. The reason for this is as follows. We first observe that the manager's consumption is countercyclical because the incentive portion of the managerial contract depends on dividends, which are themselves negatively correlated with output. Ceteris paribus, a favorable shock to technology increases output and hence decreases the dividend and the manager's consumption. To smooth his own consumption, the risk-averse manager reduces the wage bill of his firm by hiring less labor.

7. Review of Related Literature

Numerous studies explore optimal contracting in partial equilibrium models. Bolton and Dewatripont (2005) extensively review this literature. Only a few recent studies address the issue of the imperfect corporate control and associated contracting arrangements in a general equilibrium framework. Dow et al. (2005) and Philippon (2006) present models in which managers have strong preferences for overinvestment. Shareholders use some of the firm's resources to hire auditors to constrain the manager's empire-building tendencies.

Albuquerque and Wang (2008) study the asset pricing implications of a model with two classes of shareholders. In their setting the controlling shareholders pursue private benefits at the expense of the outside shareholders. Various investor protections mitigate the agency conflict between the two groups of shareholders. Consistent with the data, they find that countries with weak investor protections have higher equity premiums, greater equity return volatility, higher interest rates, etc.

Danthine and Donaldson (2008) derive optimal executive contracts that lead to the first-best allocation of resources in a DSGE model with delegation in which the manager has standard preferences. Donaldson et al. (2013) show that convex contracts, which are generally typical of U.S. CEO pay-practices, may cause generic sunspot equilibria in otherwise standard DSGE modeling frameworks.

Our paper is also related to studies on asset pricing in incomplete markets such as Basak and Cuoco (1998), Storesletten et al. (2007), Guvenen (2009), and Gomes and Michaelides (2008), which examine the effect of restricted market participation and incomplete information on asset pricing. The mechanism for producing favorable asset pricing results in our model is somewhat similar to that in Guvenen (2009) in that the bond trade serves as an effective devise for consumption smoothing for non-stockholder-workers, which increasing the volatility of consumption for stockholders-managers. However, the reason for this amplification in our model mainly comes from the fact that the manager's income and his consumption are determined by the convex contract, which depends on the performance of his firm.

8. Conclusion

In this paper, we consider a production based macro-asset pricing model with two types of agents. They differ in the source of their income as well as in their ability to participate in financial markets. The majority of the population are workers and earn their income by supplying labor. The only financial security available to them is the risk free bond. The other group of agents are managers endowed by the compensation contract. They represent a small fraction of the population. In addition to participating in the bond market, they also trade stocks. We show that these self-interested, mildly risk-averse CEOs, when confronted with compensation contracts, which are moderately convex to the firm's stock price or free cash flow, may find it optimal to adopt investment and hiring policies that lead to equilibrium indeterminacy. As a result, the time path of the managerial consumption, at least with respect to its volatility, may bear little relation to fundamentals of the economy and to the aggregate consumption in particular. In this environment, the stock price is determined by the CEO's marginal rate of substitution in consumption, while the price of the risk free bond closely relates to the aggregate consumption growth.

Our findings suggest that the cross-sectional heterogeneity generated by the interaction of these two features increases equity premium to about one third of its empirical value, while attaining a low average risk-free rate, found in the U.S. data, as well as realistic standard deviations of asset returns. The two reasons for the higher equity premium are the limited consumption smoothing opportunities available to the majority of the population, who do not participate in the stock market, and the high correlation of the stock price with the marginal rate of substitution in the CEO's consumption. In our model, non-stockholder-workers (the majority) buy risk free bonds to smooth uncertainty in their labor income. The CEO-stockholders provide the supply of bonds and by doing so, assume the aggregate risk, which they smooth by participating in the stock market. On the other hand, since the stockholder-manager's consumption growth depends on the performance of his firm, it is highly correlated with the price of equity. As a result, shareholders require high return for holding stock, which leads to the high equity premium.

In addition to improving upon asset pricing implication of the traditional macro-asset pricing model, the model presented in this paper is broadly consistent with the statistical summary of the U.S. business cycle.

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