Tail dependence structure of metal commodity futures in London Metal Exchange

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Abstract

Since the 2008 financial crisis, academics and practitioners have paid more attention to the dependence structures among futures contracts in futures market. We use the vine-copula approach to study the dependence structures among major metal commodity futures in the London Metal Exchange, with a focus on analyzing the change after the crisis. We find that the core of metal futures moves from copper to zinc after the crisis. The risk diversification benefit among metal futures is shown to diminish. However, the dependence structure between core futures and the futures who exhibits the highest (lowest) concordance with core futures remains unchanged after the crisis.

Keywords: metal commodity futures; r-vine copula; financial crisis;

JEL classification: C58, G01; L61

1 Introduction

The dependence structure between different futures contracts in the commodity futures market reduces the effectiveness of hedging strategies and increases the difficulty of portfolio management. Therefore, academics and practitioners pay much more attention to the dependence structure between futures contracts in the commodity futures market. The metal futures market is an important part of the commodity futures market. This is due to the fact that the hedging capacity of precious metals caused by their low correlation with equity markets makes them even more attractive (Hillier et al., 2006). Additionally, the value of metal futures contracts is relatively stable compared with other commodity futures. Hence, metal futures are widely included in asset portfolios in order to enhance portfolio performance and obtain diversification benefits. The dependence between metal futures contracts therefore raises further concern.

In this paper, we use the vine-copula approach to study the dependence structures among major futures contracts in the London Metal Exchange (LME) and their changes after the crisis. We select five major futures contracts in the LME, namely aluminum, copper, nickel, zinc, and lead. The sample dates from 03 December 1993 to 29 September 2017. We find that the core metal commodity futures - the metal commodity futures with the strongest dependence with the other four futures - moves from copper to zinc after the crisis. The diversification benefit among major metal commodity futures was also shown to diminish. However, the dependence structures between core futures and the futures who exhibits the highest concordance with core futures remains unchanged after the crisis. Same evidence can also be found for the futures who exhibits the lowest concordance with core futures.

This study contributes to the literature in three ways: first, we investigate the dependence structures among different metal commodity futures in the LME, with a focus on analyzing the change after the crisis, which to the best of our knowledge has rarely been studied; second, we find that the change of dependence structures among these metal commodity futures in the LME indicates a reduction in their diversification benefit, which generates valuable insights for portfolio management; third, we further show that the metal commodity futures which has the strongest dependence with the remaining futures moves from copper to zinc after the crisis.

In the literature, the evidence of less diversification benefits has been confirmed in many other types of markets; for example, Silvennoinena and Thorp (2013) suggest that diversification benefits to investors across the equity, bond and stock markets were significantly reduced during the crisis period they studied. The evidence from Lu et al. (2013) indicates that diversification benefits between the US REIT (Real Estate Investment Trust) market and twelve international REIT markets eroded considerably during turbulent market conditions. Gerlach et al. (2014) examined the impact that the 1997 Asian financial crisis has had upon the integration and dynamic links between a number of Asia-Pacific real estate markets; The results show that the diversification benefits in the Asia-Pacific region are actually less than those suggested by an analysis incorrectly ignoring the crisis.

Many studies have focused on the dependence and volatility across spot markets or metal futures markets. Sensoy (2013) used the DCC (dynamic conditional correlation) model to examine volatility shift contagion effects in the returns of four major precious metals (i.e. gold, silver, platinum, and palladium) and found no significant effect on the volatility levels of gold and silver during the turbulent year of 2008. Using daily data over the period 1992 to 1998, Ciner (2001) reported that gold and silver futures contracts traded in Japan were not cointegrated. Xu and Fung (2005) found evidence of strong volatility feedback between these precious metals across both markets. Choi and Hammoudeh (2010) used the dynamic conditional correlation model to identify increasing correlations among all the considered spot commodity returns (i.e., Brent oil, WTI oil, copper, gold and silver) over recent years.

Due to the sophisticated dependence structures among financial asset returns, the sample linear dependence cannot fully capture these dependence structures. Hence, the linear correlation coefficient cannot precisely measure the non-linear dependence structures as well as tail-dependence structures among financial asset returns.

In order to remedy the defect of the linear correlation coefficient, the copula approach is widely used in risk management and option pricing. The copula function captures the dependence more completely. In addition, based on the copula function, many new measures can be defined to determine the dependence structure, which to some extent expands the limitations of existing measures.

Embrechts (1999) introduced the copula approach to the field of finance to study the dependence problem. Longin and Solnik (2001) studied the dependence structure across international stock markets using the Gumbel copula. Patton (2002) used the copula approach to examine the economic and statistical significance of asymmetries of stock returns for asset allocation decisions in an out-of-sample setting. Ang and Chen (2002), Poon et al. (2003) and Hartmann et al. (2004) found that a multivariate normal distribution could not fully capture the tail dependence among financial asset returns, especially the lower tail dependence. Junker and May (2005) modified the t-copula and Clayton-copula to investigate portfolio dependence. Nelsen (2006) systematically introduces the bivariate copula and its properties. Patton (2006) used the copula approach to analyze the asymmetric dependence structure of the foreign exchange market. Ning (2010) examined the dependence structure between the stock market and foreign exchange market in developed countries and found that there exists a significant symmetric tail dependence between them, though this dependence weakened after the establishment of the European Union. Zhang (2014) employed the copula approach to investigate the dependence structure between the sovereign debt markets of major European countries and further calculated the potential probabilities of future sovereign debt crises of these markets. Huang and Ning (2017) investigated the inter- and intra-continental dependence of developed stock markets before and after the financial crisis of 2008 to identify whether inter-continental and intra-continental diversification potential is disappearing.

However, the dependence among high-dimensional random variables cannot be captured by a bivariate copula function, therefore, the multivariate copula was introduced to solve high-dimensional cases. The multivariate copula uses a specific multivariate copula family to capture the dependence among high-dimensional random variables. Unfortunately, the choice of multivariate copulas is rather limited in contrast to the bivariate case, where a rich variety of different copula types exhibiting flexible and complex dependence patterns exists (Brechmann and Czado, 2013). Multivariate copula equips different pairs of random variables with the same dependence structure. Hence, a single multivariate copula is still unable to fully and completely capture the sophisticated dependence among high-dimensional random variables. For example, multivariate Archimedean copulas only allow exchangeable structures with a narrower range of negative dependences in a higher dimension (McNeil and Neslehova, 2009). Demarta and McNeil (2007) presented multivariate skewed t-copula, which model well, but are computationally intensive. In summary, the defects of multivariate copula in capturing high dimensional dependence are inflexible and insufficient.

The vine-copula approach can remedy the defects of multivariate copulas. This approach has been widely used to capture high dimensional dependence. Schlüter (2009) compared the different copula estimation methods used in vine-copula. Low et al. (2013) applied the vine-copula approach to portfolio management. Weiss and Supper (2013) used the vine-copula approach to predict VaR. So and Yeung (2014) used the dynamic GARCH-vine-copula model to study the time-varying dependence structures among five stocks on the Hong Kong stock market. Abbara (2014) used the vine-copula model to study the dependence and contagion of the stock market. Markwat (2014) applied the vine-copula approach to investigate the probability of a global stock market crash. Zhang (2014) used vine-copula to study the dependence between European sovereign debt markets and predict the probability of a sovereign debt crisis. Nagler and Czado (2016) applied non-parametric estimation to the vine-copula model and reported that it can overcome the "curse of dimensionality". Pircalabu and Jung (2017) utilized the vine-copula model to examine the dependence between wind power production and electricity prices and discussed its implications for the pricing and risk distributions associated with contracts that are exposed to joint price and volumetric risk. Shahzad et al. (2018) investigate the downside and upside spillover effects, systemic and tail dependence risks of the DJ World Islamic (DJWI) and DJ World Islamic Financial (DJWIF) indices, and of Islamic equity indices from Japan, USA and the UK. In this paper, they employ a robust modeling framework consisting of various models, such as delta conditional VaR (ΔCoVaR), vine-copula to study the problem they concern.

Few studies in metal commodity futures have used the R-vine copula approach. However, it is a suitable method because different dependence structures between each pair of metal commodity futures can be captured while modelling the high dimensional dependence among these futures. This present study is one of the first to employ the R-vine copula model to investigate the dependence structures among major futures contracts in the European metal futures market, with a focus on analyzing the change after the 2008 financial crisis. Our study expands the literature on metal commodity futures, while our findings may help improve the performance of hedging strategies, thus being valuable for portfolio managers in optimizing their asset allocation and diversifying risk.

This paper is organized as follows: Section 2 describes the methodology, Section 3 addresses the data description and empirical analysis, while finally, Section 4 presets the conclusion.

2 Methodology

2.1 Vine-copula approach

A copula is a multidimensional joint distribution function whose marginal is uniformly distributed on [0,1]. The capacity of a copula to capture dependence structures is endowed by Sklar's theorem. However, a single copula function can only characterize well the dependence structure in the two-dimension case. When the dimension is higher than two, a multivariate copula forces the same dependence structure for each pair of random variables. This defect greatly restricts the capacity of multivariate copula to capture multidimensional dependence structures. Hence, the multivariate copula approach is not widely used.

The vine-copula approach was first proposed by Aas et al. (2009) based on the work of Bedford and Cooke (2002). This approach uses the pair-copula construction (PCC) (Aas et al., 2009) to capture high dimensional dependence by building a multi-level tree structure. Specifically, this approach decomposes a multivariate density function into the product of the marginal density function and a series of unconditional or conditional pair-copulas. Furthermore, various kinds of pair-copula families can be chosen to model the dependence of each pair of random variables. Therefore, the vine-copula approach is more flexible and efficient than the multivariate copula approach in terms of the measurement of dependence.

The construction of vine-copula models is diverse due to different types of connections between nodes and edges and different progressive relationships of tree structures. Bedford and Cooke (2002) proposed a graphical construction approach: the regular vine (R-vine). In this approach, the tree structure of each level is different. The nodes in each tree are connected through the edges, and each node comes from a specific edge in the previous tree. Two nodes in each tree are only connected by an edge if they share a common node in the previous tree (proximity condition). Paired random variables corresponding to each edge are characterized by a pair-copula.

Using the vine-copula approach to decompose n-dimensional random vectors $X_n = (\mathrm{X}_1, \mathrm{X}_2, \dots, \mathrm{X}_\mathrm{n})$ will generate n-1 tree structures and n(n-1) pairs of random variables which need to be characterized by pair-copula functions. If $f(x_1, x_2, \dots, x_n)$ denotes the joint density function of this random vector, the R-vine decomposition of the joint density function is as follows:

$$f(x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i) \prod_{i=1}^{n-1} \prod_{e \in E_i} c_{j_e, k_e | d_e} \{F(j_e | d_e), F(k_e | d_e); d_e\}$$

where $f(x_k), k = 1,2, \dots, n$ denotes the marginal density of X_n, $c_{j_e, k_e | d_e}(\cdot,\cdot)$ represents the pair-copula density corresponding to edge e which connects nodes j and k in the ith tree, E_i is a set that consists of all edges in the ith tree. $d_e = \mathrm{A}_j \cap A_k$, where A_j and $A_k$are two sets of nodes in the first tree that are reachable by nodes

j and k, $j_e = \mathrm{A}_j - d_e$, $k_e = \mathrm{A}_k - d_e$. The second product symbol takes all n-1 trees, while the third product symbol takes all n-j pair-copula functions in the ith tree.

Each bivariate copula in vine-copula depends on two conditional cumulative distribution functions (CDF): $F(j_e|d_e)$, $F(k_e|d_e)$ and conditioning variables in d_e. The number of variables in d_e increases with the tree level goes higher, which makes each bivariate copula a $|d_e| + 2$ dimensional function to be estimated. Haff et al (2010) suggest that it may still be possible to estimate a bivariate copula that depends additionally on the single conditioning variable, using some sort of smoothing technique. However, at higher levels, this becomes very difficult in a parametric setting, and impossible in a non-parametric one. In order to make inference to be fast, flexible and robust, one must assume that each bivariate copula in vine copula construction is independent of the conditioning variables. i.e.

$$c_{j_e,k_e|d_e}\{F(j_e|d_e), F(k_e|d_e); d_e\} = c_{j_e,k_e|d_e}\{F(j_e|d_e), F(k_e|d_e)\}$$

This equation is so-called simplifying assumption.

Whether simplifying assumption can be relax depends on the given dataset and the tree structure of vine copula. Recently, Kurz and Spanhel (2017) proposed a new statistical method called constant conditional correlation (CCC) test to check whether the simplifying assumption is suitable for each conditional bivariate copula in a vine copula. The idea of this test is that if a conditional copula can be represented by an unconditional copula then the correlation coefficient corresponding to conditional copula is a constant with respect to conditioning variables. Based on this fact, the null hypothesis of this method is:

$$\rho_{\Omega_1} = \cdots = \rho_{\Omega_L}$$

Where $\rho_{\Omega_i} := Corr\left(U_{j_e|D_e}, U_{k_e|D_e} | \mathbf{U}_{D_e} \in \Omega_i\right)$, $U_{j_e|D_e}$ and $U_{k_e|D_e}$ are conditioned variables in conditional copula. $\mathbf{U}_{D_e}$ is conditioning variables set. The support Ω_0 of $\mathbf{U}_{D_e}$ is divided by a partition $\Gamma := \{\Omega_1, \ldots, \Omega_L\}$, Ω_i is an element in the partition Γ.

Kurz and Spanhel (2017) further derive a test statistic to test the null hypothesis by using the asymptotic normality of the vector consist of estimated conditional correlations: $\hat{\rho}_{\Omega_i}^{(n)}$. This novel testing procedure can mitigate the curse of dimensions due to the discretizing of the conditioning variable set and the penalty incorporated in the test statistic.

2.2 Model selection and estimation

Financial data are commonly found to exhibit conditional heteroscedasticity, skewness, and leptokurtosis in logarithmic returns (Siburg et al., 2015). The generalized autoregressive conditional heteroscedastic (GARCH) model proposed by Bollerslev (1986) is widely used to model time series data with conditional heteroscedasticity. Further, compared with the standard normal distribution, using the student-t distribution as the conditional innovation distribution in the GARCH model can better capture the skewness and leptokurtosis of financial time series data (Bollerslev, 1987). In addition, most theoretical results for copulas only hold for independent identical distributed (i.i.d.) samples, and financial data are usually filtered by GARCH models to yield approximately i.i.d. samples of standardized residuals (Siburg et al., 2015). The approximately i.i.d. sample is necessary for the inference for a specific pair-copula decomposition (Aas et al., 2009). Besides, using GARCH filters to account for the time-varying volatility in financial returns is indeed necessary when later estimating a copula model (Garmann and Grundke, 2013). Based on the studies above, we employ the GARCH-t model to fit the marginal distribution of the return series considered in this paper.

The following three steps are necessary for fitting an R-vine copula specification to a given high-dimensional data set:

1) Select a specific R-vine copula structure.

2) Select the appropriate pair-copula family for each pair in the selected R-vine-copula structure.

3) Estimate the parameter(s) for each copula.

The ideal way is to repeat step (2) and (3) for every possible R-vine copula structure. However, n-dimensional random variables may have $\binom{n}{2} \times (n-2)! \times 2^{\binom{n-2}{2}}$ possible R-vine copula structures (Morales-Napoles et al., 2010). Compared to the number of dimensions, the rapidly expanding number of R-vine copula structures make the ideal method inefficient.

Therefore, in this paper the sequential estimation method proposed by Dißmann et al. (2013) is applied to fit the R-vine copula to the data set considered. Dißmann et al. claimed that it is more important to model the dependence structure between random variables that have high dependencies correctly; furthermore, the copula families specified in the first tree of the R-vine often have the greatest influence on the model fit. Joe et al. (2010) also stated that in order for a vine-copula to have dependence for all bivariate margins, it is sufficient for the bivariate copulas in the first tree to display dependence. Thus, the stronger dependence the first tree structure can capture, the more independent the transform variables in the subsequent tree structures are. Whether the first tree structure can fully capture the strongest dependence structure among variables is of vital importance. Based on these pieces of evidence, this method starts from the first tree structure and repeats steps (1), (2) and (3) for each tree structure sequentially, and employs the MST (maximum spanning tree) method based on some dependence measure as the selection criteria for the first tree and subsequent tree structures. In other words, the tree structure that solves the following optimization problem is selected:

$$\max_{t_i \in T_i} \sum_{\substack{edges\ e=\{i,j\} in \\ spanning\ tree\ t_i}} \left|\delta_{i,\mathrm{j}}\right| \quad , \mathrm{i} \neq \mathrm{j}$$

where T_i is a collection of all possible tree structures in the ith tree. t_i denotes a specific ith tree structure, e is an arbitrary edge in tree t_i , $\delta_{\mathrm{i,j}}$ is the dependence measure's value between a pair of random variables corresponding to edge e. Since dependence measures that are negative in value indicate negative dependence among random variables, the dependence measure requires that the absolute value is taken so as to take negative dependence into account (Brechmann and Czado, 2013). $\text{Kendall's}\,\tau$ is a type of dependence measure which is especially useful when combining different (non-Gaussian) copula families, since the assumed distribution cannot affect the measurement of its dependence (Dißmann et al., 2013). Hence, we use $\text{Kendall's}\,\tau$ as the dependence measure in the MST method.

After the tree structure selection, we need to select an appropriate pair-copula family for each pair of random variables in this tree. Two approaches are widely used as selection criteria for pair-copulas: the copula goodness-of-fit test and the AIC criteria. Manner (2007) found that compared with the copula goodness-of-fit test, the AIC criteria is more reliable in the context of pair-copula selection. Therefore, we use the AIC criteria as the pair-copula selection criterion.

Finally, we apply the maximum likelihood estimation to estimate the parameters of each specified pair-copula in this tree. After parameter(s) estimation, the transformed variables which are used as input parameters for the next trees are obtained by using the $h(\cdot)$ function (Aas et al., 2009; Dißmann et al., 2013). The treewise selection and estimation procedure described here gives sequential estimates of pair-copula parameters, which are quite quickly obtained and can be used as starting values for a full maximum likelihood estimation (Aas et al., 2009; Hobæk Haff, 2010). Since most of the pair-copula families can model the independence well, this method also reduces the difficulty of fitting high order tree structures. In addition, sequential estimation can minimize rounding errors caused by high-order tree structures (Brechmann and Czado, 2013).

3 Data and empirical study

3.1 Data

This paper examines the log-returns of weekly settlements of five major metal commodity futures, namely aluminum, copper, nickel, zinc, lead, in the European metal futures market. These metal futures have large trading volumes and their underlying metal commodities are widely used for industrial purposes. The sample comes from the London Commodity Exchange (LME). The LME is the world's largest trading center for non-precious metals and thus can be regarded as a useful starting point to investigate the dependence among metal commodity futures. The sample starts from 03 December 1993 and goes to 29 September 2017.

The sample period covers the global financial crisis in 2008, allowing us to study the dependence structures among these five major metal commodity futures, with a focus on analyzing the change after the crisis. Thus, we separate the sample into two periods: 03 December 1993 to 29 December 2006 is Period 1 and represents the period before the crisis; 05 January 2007 to 29 September 2017 is Period 2 and represents the period during and after the crisis. Table 1 and Table 2 summarize the descriptive statistics for the weekly log-returns of the five metal commodity futures in both periods.

Table 1 and Table 2 show that the log-returns of all five metal commodity futures exhibit negative skewness and leptokurtosis in both periods. Correspondingly, the Jarque-Bera normality test of each metal commodity future confirms this fact.

Table 1 The descriptive statistics for weekly log-returns of five metal commodity futures in period 1

	n	Min	Max	Mean	Std.Dev.	Skew	Kurtosis	Jarque–Bera
Al	682	-13.48	8.33	0.14	2.55	-0.18	1.54	0.0000
CO	682	-13.89	11.02	0.2	3.08	-0.28	1.63	0.0000
NIC	682	-19.97	14.75	0.29	4.37	-0.02	1.41	0.0000
ZINC	682	-14.09	16.09	0.22	3.21	0.02	2.63	0.0000
LEAD	682	-15	11.54	0.2	3.37	-0.23	1.83	0.0000

In addition, the minimum log-return of each metal commodity future in Period 2 is generally smaller than those in Period 1, while the maximum log-return of each metal commodity future in Period 2 is generally greater than those in Period 1. This evidence tentatively implies that the extent of the European metal futures market shocked by the "tail events" increases after the crisis.

Table2 The descriptive statistics for weekly log-returns of five metal commodity futures in period 2

	n	Min	Max	Mean	Std.Dev.	Skew	Kurtosis	Jarque–Bera
Al	560	-16.49	9.57	-0.04	3.13	-0.27	1.69	0.0000
CO	560	-24.45	13.48	0.02	3.92	-0.93	5.96	0.0000
NIC	560	-22.31	32.01	-0.21	5.23	0.21	3.29	0.0000
ZINC	560	-17.75	13.53	-0.04	4.4	-0.26	1.43	0.0000
LEAD	560	-19.09	23.35	0.07	5.19	-0.06	2.29	0.0000

3.2 Marginal distributions modelling

The ADF test shows that all five log-return sequences are stationary. The LM test (Engle, 1982) indicates the heteroscedasticity of all five log-return sequences in both periods. Following Fantazzini (2009) and Siburg et al. (2015), we employ the standard GARCH model with t-distributed innovations to fit the marginal distributions.

The i.i.d sample is necessary and essential for the copula parameter(s) estimation. A GARCH-filter with well identified orders can remove the heteroscedasticity in the sample and obtain the approximately i.i.d residuals for the following copula estimation. Based on this evidence, we thus employ the GARCH(1,1)-t model to filter the return sequences:

$$R_{\mathrm{t,j}} = \mu_j + \sigma_{t,j} Z_{t,j} \tag{1}$$

$$\sigma_{t,j}^2 = \omega_j + \alpha_{1j}\left(R_{t-1,j} - \mu_j\right) + \beta_{1,j}\sigma_{t-1,j}^2 \qquad j = 1, \dots, d \qquad t = 1, \dots, n_i \qquad (2)$$

where $Z_{t,j} \sim t(0,1,v)$. $R_{\mathrm{t,j}}$ denotes the log-return of variety j in time t, d denotes the number of commodity futures studied, n_i $(i = 1, 2)$ denotes the sample sizes of Period 1 and Period 2. The conditions of coefficients that ensure positive volatility and the existence of second moments are respectively: $\alpha_1, \beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$

The parameter estimates and statistical tests for the GARCH (1,1)-t-filter in both periods are listed in Table 3 and Table 4 respectively. In both periods, all the coefficients of the five log-return sequences satisfy the condition, $\alpha_1, \beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$, which ensures the positive conditional volatility and confirms the existence of a second moment of a GARCH –t model.

The Ljung-Box tests applied to each marginal model strongly reject the null hypothesis of autocorrelation at lag 1, 2, and 5 at the 5% significance level in both periods, which indicates no autocorrelation in standardized residuals of marginal models.

The LM tests applied to each marginal model strongly reject the null hypothesis at lag 3, 5, and 7 at the 5% significance level in both periods, which implies no homoscedasticity of standardized residuals of marginal models.

The sign bias test (SB test) can diagnose the existence of an asymmetry response of the volatility to innovations (Engle et al., 1993). The positive and negative SB test accept the null hypothesis of no asymmetry volatility at the 5% significance level in both periods. Thus, the asymmetric GARCH model (EGARCH, TGARCH) is unnecessary for marginal modelling.

To sum up, the GARCH (1,1)-t-filter is an adequate marginal model for the five time series of log-returns of metal commodity futures.

Table 3 Parameter estimates and statistic tests for GARCH(1,1)-t in period 1

Parameter	AL		CO		NIC		ZINC		LEAD	
	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
μ	0.0801	0.3642	0.1068	0.3221	0.1892	0.2156	0.0464	0.6336	0.0791	0.4905
ω	0.3061	0.0390	0.5107	0.0401	0.9608	0.1242	0.1533	0.1187	0.0557	0.4567
α_1	0.1006	0.0007	0.1063	0.0006	0.0671	0.0062	0.0911	0.0000	0.0294	0.0064
β_1	0.8517	0.0000	0.8387	0.0000	0.8847	0.0000	0.8966	0.0000	0.9677	0.0000
shape	28.7366	0.2868	34.9707	0.4082	6.8541	0.0002	10.0189	0.0068	6.9692	0.0001
AIC	4.6202		4.9788		5.7177		4.9148		5.1631	
Logli.	-1570.4890		-1692.7730		-1944.7390		-1670.9340		-1755.6010	
LB test:										
lags	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
1	0.1077	0.7428	2.7980	0.0944	1.8040	0.1792	0.5603	0.4542	3.6700	0.0554
2	0.1095	0.9131	2.9300	0.1466	1.8050	0.2974	0.5628	0.6648	3.7120	0.0901
5	0.3265	0.9809	3.0660	0.3952	3.4450	0.3318	1.4888	0.7427	4.0100	0.2529
LM test:										
lags	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
3	1.7360	0.1877	0.2323	0.6298	0.4530	0.5009	0.8039	0.3699	2.1990	0.1381
5	3.3840	0.2388	0.7395	0.8114	0.6461	0.8398	1.5360	0.5829	3.5660	0.2176
7	4.3750	0.2957	0.8403	0.9381	0.8999	0.9293	3.5326	0.4190	6.2150	0.1274
SB test										
	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
Negative	0.5082	0.6115	0.0878	0.9301	0.8725	0.3833	0.7885	0.4307	1.0450	0.2962
Positive	0.4403	0.6599	1.9054	0.0572	1.8119	0.0704	0.1049	0.9165	1.9240	0.0547

Table 4 Parameter estimates and statistic tests for GARCH(1,1)-t in period 2

Parameter	AL		CO		NIC		ZINC		LEAD	
	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
μ	-0.0056	0.9620	-0.0018	0.9877	-0.1908	0.2889	0.0666	0.6470	0.0260	0.8622
ω	0.0883	0.2329	0.1707	0.1598	0.2447	0.1836	0.0684	0.2356	0.0891	0.2851
α_1	0.0413	0.0052	0.0804	0.0019	0.0537	0.0053	0.0435	0.0002	0.0613	0.0011
β_1	0.9487	0.0000	0.9072	0.0000	0.9350	0.0000	0.9517	0.0000	0.9353	0.0000
shape	18.0344	0.1174	6.7676	0.0001	15.6411	0.0664	12.6012	0.0379	8.6643	0.0029
AIC	5.0187		5.2606		5.9466		5.6054		5.8042	
Logli.	-1400.2380		-1467.9810		-1660.0420		-1564.5230		-1620.1750	
LB test:										
lags	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
1	0.9917	0.3193	1.5280	0.2163	0.1734	0.6771	0.0560	0.8129	0.0505	0.8222
2	1.2326	0.4286	2.6630	0.1732	1.1299	0.4579	0.0789	0.9347	0.2814	0.8062
5	2.1187	0.5907	4.3010	0.2188	1.8704	0.6494	1.1198	0.8320	0.8448	0.8937
LM test:										
lags	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
3	0.0001	0.9919	0.1880	0.6645	0.7427	0.3888	0.0001	0.9930	0.5976	0.4395
5	1.0629	0.7144	1.7740	0.5234	2.8733	0.3087	0.5730	0.8619	0.6490	0.8389
7	2.9505	0.5230	6.1150	0.1337	3.5533	0.4156	3.4770	0.4282	3.0689	0.5006
SB test:										
	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value	Est./Stat.	p-value
Negative	0.4982	0.6185	0.8268	0.4087	0.2143	0.8304	0.0615	0.9510	0.5724	0.5673
Positive	1.5045	0.1330	1.4006	0.1619	1.4120	0.1585	0.5411	0.5887	0.1375	0.8907

4 Empirical results

We use the vine-copula approach to study the dependence structures among major metal commodity futures in the London Metal Exchange, with a focus on analyzing the change after the financial crisis.

There is always some uncertainty in the innovation distribution when the sample size is large (Dißmann et al., 2013). In order to overcome this issue, first, we transform the standardized residuals obtained from the GARCH-filter into marginally uniform data by using the empirical probability integral transformation (PIT). Then we apply the vine copula approach to investigate the dependence structures among transformed data, and use CCC method to test whether simplifying assumption holds for each conditional bivariate copula in vine copula.

The CCC test is applied to R-vine copula we modeled in both periods. The results show that the total number of rejections in all three high level tree structures is zero at 5% significant level in both periods, which indicates that the simplifying assumption cannot be rejected at 5% significant level in both periods. The dependence structure modeled by the R-vine copula in both periods is listed in Table 5. Figure 1 shows the first tree structure of the R-vine copula in Period 1 and Figure 2 shows the first tree structure of the R-vine copula in Period 2.

4.1 The dependence structure before the 2008 financial crisis

Copper and aluminum have strong correlation and concordance, and the t-copula between them implies the existence of a symmetrical tail dependence. The Clayton-copula and indices of tail dependence between copper and lead indicate the existence of weak lower tail dependence. Copper has no tail dependence with zinc and nickel.

Excluding copper, aluminum exhibits a slight concordance with the other three metal commodity futures. However, the value of the indices of lower tail dependence of aluminum and nickel ($\lambda_L = 0.2$) and the Gumbel copula (see Appendix B) between them together indicate high lower tail dependence between the two metal commodity futures. The value of the indices of lower tail dependence of aluminum and lead ($\lambda_L = 0.01, \lambda_U = 0.01$) and the t-copula between them together show a symmetrical tail dependence between the two metal commodity futures. The value of the tail dependence indices of aluminum and zinc are both equal to zero, which indicates no tail dependence.

Nickel has weak concordance with zinc and lead. Nickel and zinc have no tail dependence. The Gumbel copula between nickel and lead implies lower tail dependence between the two metal commodity futures. Zinc and lead have strong concordance, and the Gumbel copula between this pair indicates lower tail dependence between the two metal commodity futures.

The first tree structure of an R-vine copula can capture the strongest dependence structure between random variables to the maximum extent. Figure 1 shows that there are three edges connected to copper while only four edges exist in the first tree structure; thus, copper has the strongest dependence with the other four futures before the crisis.

4.2 The dependence structure after the 2008 financial crisis

The concordance levels of aluminum with zinc, copper, nickel and lead are shown to decrease after the crisis. There is no tail dependence between aluminum and the remaining three metal commodity futures except zinc.

Zinc has strong concordance with aluminum, copper and lead, while weak concordance was shown between zinc and nickel. The BB1 copula (see Appendix B) between zinc and aluminum implies strong asymmetric tail dependence. Furthermore, the lower tail dependence is relatively strong. The copulas for the zinc-copper pair and zinc-lead pair are both t-copula, which indicates that zinc has a symmetrical tail dependence with these two metal commodity futures. Moreover, the large value of the tail dependence indices of these two pairs further implies a high degree of symmetrical tail dependence. The Gumbel copula between zinc and nickel implies lower tail dependence between the two metal commodity futures. Copper has no tail dependence but weak concordance with nickel and lead.

Nickel has weak concordance with lead, and the t-copula between this pair implies symmetrical tail dependence between the two metal commodity futures. Further, the indices of the tail dependence of this pair indicate a low degree of tail dependence.

Figure 2 shows that there are three edges connected to zinc while only four edges exist in the first tree structure; thus, zinc has the strongest dependence with the other four futures after the financial crisis.

4.3 Comparison between the dependence structure before and after the 2008 financial crisis

First, we compare the dependence structures of each metal commodity futures before and after the crisis. Kendall′s τ between aluminum and the other metal commodity futures ranges in [0.07,0.46] before the crisis, which decreases to [0.03,0.41] afterwards. Additionally, the concordance between aluminum and the other metal commodity futures generally decreases. In particular, the metal commodity futures which have strongest concordance with aluminum change from copper to zinc after the crisis. Before the crisis, aluminum has a symmetrical tail dependence with copper and lead, but afterwards, the tail dependence disappears. In contrast, aluminum and zinc exhibit asymmetric tail dependence after the crisis, moreover, the lower tail dependence is stronger after the crisis.

The concordance between copper and aluminum decreases after the crisis, but the concordance between copper and the other three metal commodity futures generally increases. Before the crisis, copper has no tail dependence with nickel and zinc.

However, an asymmetric tail dependence appears in this pair after the crisis. Furthermore, the lower tail dependence of this pair is stronger after the crisis. Similarly, after the crisis, copper and zinc exhibit symmetric tail dependence, and the lower tail dependence between this pair disappears. The change of tail dependence between copper and aluminum has been discussed in this paper.

After the crisis, the concordances of nickel-aluminum, nickel-zinc and nickel-lead generally decrease, while the concordance between nickel and copper increases. The dependence structure between nickel and zinc changes from no tail dependence to lower tail dependence after the crisis. The dependence structure between nickel and lead changes from lower tail dependence to symmetric tail dependence after the crisis. The change of the tail dependences of nickel-copper and nickel-aluminum have been discussed in this paper.

Kendall's τ between zinc and the other metal commodity futures ranges in [0.13, 0.42] before the crisis which increases to [0.17, 0.51] after the crisis, and the concordance between zinc and the other metal commodity futures generally increases. The dependence structure between zinc and aluminum changes from lower tail dependence to symmetric tail dependence after the crisis. The change in tail dependence between zinc and the other three metal commodity futures has been discussed in this paper.

The concordance between lead-zinc and lead-copper increases after the crisis, and the concordance between lead and zinc increases significantly. In contrast, the concordance between lead-aluminum and lead-nickel decreases. The change in the dependence between lead and the remaining four metal commodity futures has been discussed in this paper.

By analyzing the first level tree structure formed by five metal commodity futures before and after the crisis, we find that as shown in figure1 and figure2, the first level tree structure does not change after the crisis. Before the crisis, copper exhibits strongest dependence with the other four metal commodity futures. After the crisis, zinc shows the strongest dependence with peripheral futures. Kendall′s τ of the first tree ranges in [0.35, 0.46] before the crisis, which increases to [0.37, 0.48] after the crisis. The increase in Kendall′s τ means that the concordance among the five metal commodity futures becomes stronger, together with the change of core metal futures—

—the metal commodity futures which has strongest dependence with the remaining four futures.

In addition, the range of the indices of upper tail dependence of the first tree structure increases from [0, 0.12] to [0.19, 0.33］ after the crisis. Similarly, the range of the indices of lower tail dependence of the first tree structure shifts from [0.12, 0.43] to [0.21, 0.4] after the crisis. The degree of concordance and the tail dependence become higher after the crisis, which implies the disappearance of diversification benefit among the major metal commodity futures.

Before the crisis, metal commodity futures are ranked according to the degree of concordance with core futures contract, the ranking order from high to low is as follows: aluminum, zinc, nickel and lead. After the crisis, this order changes to: lead, copper, aluminum and nickel. By analyzing the differences in tail dependence of the metal commodity futures in these two descending orders, we find that t copula captures the dependence between first-place futures and core futures contract before and after the crisis. This evidence indicates that the symmetric tail dependence between the first-place futures contract and the core futures contract does not change after the crisis.

Similarly, clayton copula captures the dependence between the last-place futures and the core futures contract before the crisis. After the crisis, this dependence is captured by sGumbel copula. Since both clayton copula and sGumbel copula display lower tail dependence, this result implies that the lower tail dependence between the last-place futures contract and the core futures contract does not change after the crisis. Therefore, the tail dependence structure between the first-place futures contract and the core futures contract is not affected by the crisis, as also found for the last-place contract. Before the crisis, the second and third place futures contracts (zinc, nickel) do not display any tail dependence with the core futures contract; however, after the crisis, copper and nickel exhibit symmetric tail dependence and asymmetric lower tail dependence with the core futures contract, respectively. This change implies that the tail dependence between the second-place futures contract and core futures contract becomes stronger after the crisis. The same results are shown for the third-place futures contract.

Table 5 The R-vine copula in period 1 and period 2

		Period1				**Period 2**			
Tree-1:	Margins	**2,1**	**2,3**	**4,2**	**5,4**	**4,1**	**2,3**	**4,2**	**5,4**
	Family	t	N	F	SG	SBB1	SBB1	t	t
	θ_1	0.66	0.51	4.42	1.54	0.29	0.3	0.68	0.71
	θ_2	12.45	0	0	0	1.47	1.38	8.64	5.5
	τ	0.46	0.34	0.42	0.35	0.41	0.37	0.48	0.51
	λ_U	0.12	0	0	0	0.19	0.19	0.21	0.33
	λ_L	0.12	0	0	0.43	0.4	0.35	0.21	0.33
Tree-2:	Margins	**3,1|2**	**4,3|2**	**5,2|4**		**2,1;4**	**4,3;2**	**5,2;4**	
	Family	SG	N	C		F	SG	N	
	θ_1	1.18	0.29	0.18		1.83	1.21	0.24	
	θ_2	0	0	0		0	0	0	
	τ	0.15	0.18	0.08		0.2	0.17	0.15	
	λ_U	0	0	0		0	0	0	
	λ_L	0.2	0	0.02		0	0.23	0	
Tree-3:	Margins	**4,1|3,2**	**5,3|4,**			**3,1;2,4**	**5,3;4,2**		
	Family	N	SG			N	t		
	θ_1	0.21	1.1			0.22	0.06		
	θ_2	0	0			0	7.95		
	τ	0.13	0.09			0.14	0.04		
	λ_U	0	0			0	0.02		
	λ_L	0	0.12			0	0.02		
Tree-4:	Margins	**5,1|4,3,2**				**5,1;3,2**			
	Family	t				SC			
	θ_1	0.1				0.07			
	θ_2	12.24				0			
	τ	0.07				0.03			
	λ_U	0.01				0			
	λ_L	0.01				0			

*1:AL，2:CO，3:NIC，4:ZINC，5:LEAD

*t: Student-t copula, N: Gaussian copula, F: Frank copula, C: Clayton copula, SBB1:sBB1 copula, SG: sGumbel copula, SC: sClayton copula.

Figure 1 the first tree structure of R-vine copula in period 1

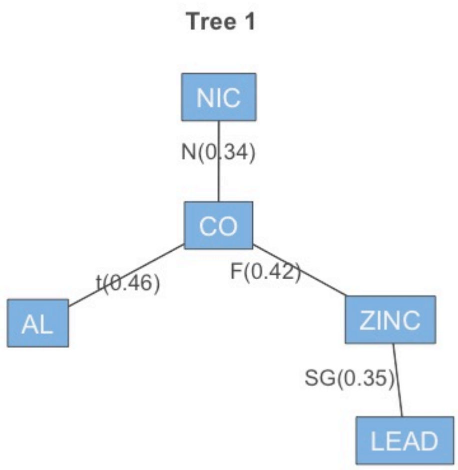
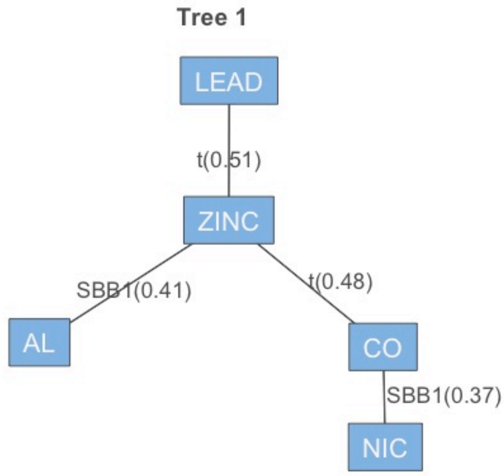


Figure 2 the first tree structure of R-vine copula in period 2



5 Conclusion

To sum up, the core futures contract moves from copper to zinc after the crisis. The diversification benefit among the major metal commodity futures has diminished. However, the first level tree structure formed by five metal commodity futures remains the same after the crisis. Ranking the remaining four futures by concordance with core futures contract from high to low, we find that the dependence structure between the first-place futures and the core futures contract remains unchanged after the crisis. Same result can be observed for the last-place futures.

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Figures

Figure 1. The first tree structure of the R-vine copula in Period 1.

Figure 2. The first tree structure of the R-vine copula in Period 2.

Tables

Table 1 The descriptive statistics for weekly log-returns of the five metal commodity futures in Period 1.

Table2 The descriptive statistics for weekly log-returns of the five metal commodity futures in Period 2.

Table 3 Parameter estimates and statistical tests for GARCH(1,1)-t in Period 1.

Table 4 Parameter estimates and statistical tests for GARCH(1,1)-t in Period 2.

Table 5 The R-vine copula in Period 1 and Period 2.

Appendix A. Measure of dependence

Many kinds of dependence measures can be defined based on the copula theory. This paper refers to two dependence measures: Kendall's τ and the indices of tail dependence.

A.1.Kendall's τ

Kendall's τ measures the concordance between two random variables. The higher the concordance between two random variables, the stronger the dependence. In the discrete case, given two random vectors with the same joint distribution and copula function (X_1, Y_1) and (X_2, Y_2), the vectors are said to be concordant if $X_1 > X_2$ whenever $Y_1 > Y_2$, and $X_1 < X_2$, whenever $Y_1 < Y_2$; the vectors are said to be discordant in the opposite case. Kendall's τ measures the difference between the probability of concordance and of the discordance between two independent random vectors.

Definition: Kendall's τ for two random variables X_1 and X_2 with copula $C(u, v)$, is:

$$\tau = 4 \int_0^1 \int_0^1 C(u, v)\, dC(u, v) - 1$$

A.1.The indices of tail dependence

The indices of tail dependence measure the dependence in a tail, or extreme values of two random variables. In particular, there are two kinds of indices: the indices of upper tail dependence and the indices of lower tail dependence.

Definition: X and Y are two continuous random variables, with distribution functions $F_x(.), F_y(.)$. If C(.,.) denotes the copula for X and Y, then:

$$\lambda_L = \lim_{u\to 0} \Pr\left(F_x(x) \le u | F_y(.) \le u\right) = \lim_{u\to 0} \frac{C(u,u)}{u}$$

$$\lambda_U = \lim_{u\to 1} \Pr\left(F_x(x) \ge u | F_y(.) \ge u\right) = \lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u}$$

Where λ_L and λ_U represent the indices of lower tail dependence and upper tail dependence respectively. λ_L, $\lambda_U \in (0,1)$. If λ_L and λ_U take positive values, it means that there exists tail dependence between the two random variables. The definition of these two measures is independent from the marginal distribution $F_x(.)$, and $F_y(.)$ and only relates to the copula C(.,.).

Appendix B. Bivariate copula families

The vast majority of studies have confirmed the existence of extreme and asymmetric volatility in various financial asset markets. Therefore, we apply several bivariate copulas with different tail dependence structures to fully capture the tail dependence between the variables considered in this paper.

B.1. Elliptical copula family

The Gaussian copula function is as follows:

$$C^{Gaussian}(u, v; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v); \rho)$$

Where $\rho \in (-1,1)$ denotes the linear correlation coefficient between two random variables. Φ_ρ is the bivariate normal distribution function. Φ^{-1} is the inverse of the univariate normal distribution function. The Gaussian copula has no tail dependence.

The bivariate student-t copula is as follows:

$$C_{v}^{student-t}(u, v; \rho) = t_{\rho\, v}(t^{-1}(u), t^{-1}(v); \rho)$$

Where $\rho \in (-1,1)$ denotes the linear correlation coefficient between two random variables. $t_{\rho\, v}$ is the bivariate t-distribution function with linear correlation coefficient ρ and the degree of freedom v. t^{-1} is the inverse of the univariate t-distribution function. The t-copula exhibits symmetrical tail dependence.

B.2. Archimedean copula family

The bivariate Archimedean copula function is:

$$\mathrm{C}(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2))$$

Where $\varphi\colon [0,1] \to [0, \infty]$ is a continuous strictly decreasing convex such that $\varphi(1) = 0$ and $\varphi^{[-1]}$ is the pseudo-inverse φ as follows:

$$\varphi^{[-1]} = \begin{cases} \varphi^{-1}(t), & 0 \leq t \leq \varphi(0) \\ 0, & \varphi(0) \leq t \leq \infty \end{cases}$$

Table 6 presents some properties of Clayton, Gumbel, Frank, and BB1 copulas that belong to the bivariate Archimedean copula families.

Table 6 Properties of bivariate copula families

Name	Function	Para.range	Kendall's τ	Tail.dep.(l.u.)
Frank	$-\log\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\theta \in \Re$	$1-\frac{4}{\theta}+\frac{4D_1(\theta)^*}{\theta}$	$(0,0)$
Gumbel	$(-\log t)^{\theta}$	$\theta \geq 1$	$1-\frac{1}{\theta}$	$(2,2-2^{\frac{1}{\theta}})$
BB1	$(t^{-\theta}-1)^{-\delta}$	$\theta > 0, \delta \geq 1$	$1-\frac{2}{\delta(\theta+2)}$	$(2^{-\frac{1}{\theta\delta}}, 2-2^{\frac{1}{\theta}})$
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta > 0$	$\frac{\theta}{(\theta+2)}$	$(2^{-\frac{1}{\theta}})$

$^*D_1(\theta) = \int_0^{\theta} \frac{c/\theta}{exp(x)-1} dx$ is Debye function.

The table above shows that the Frank-copula has no tail dependence, the Gumbel-copula exhibits asymmetrical lower tail dependence, and the BB1 copula has asymmetrical upper dependence.

B.3. Survival copula

A survival copula is a special rotated-copula function. The rotation of the copula greatly enriches the types of copulas and enables them to better capture the dependence.

A copula function which is rotated 180 degrees is called the survival copula of the original：

$$C_{180}(u_1, u_2) = u_1 + u_2 - 1 + C(1 - u_1, 1 - u_2)$$

In contrast, the Gumbel copula has an asymmetrical upper tail dependence, while the BB1 copula exhibits an asymmetrical lower tail dependence.

Footnotes

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