The Endogenous Price under Perfect Liquidity

BOGDAN NEGREA*

Abstract

The midquote conjecture states that the fundamental value of a security equals the midpoint of the bid-ask spread. I develop a model which provides theoretical foundation to endogenously estimate the fundamental value. The model derives the endogenous underlying value of stocks as a weighted average of the bid and ask prices. The weights are functions of the price volatility and the risk-free interest rate. As a consequence, I revisit the liquidity-adjusted CAPM and I derive a new estimator of the bid-ask spread.

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KEYWORDS: Liquidity, Bid-Ask Spread, Volatility, Winner's Curse, Adverse Selection.

*Department of Money and Banking, Bucharest University of Economic Studies, 5-7 Moxa St, 010961, Bucharest, Romania. E-mail: bogdan.negrea@fin.ase.ro

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I. Introduction

The fundamental value of a security is a core but also a hard to assess concept in finance.[1] The fundamental value is typically unobservable. Beginning with the seminal paper of Demsetz (1968), the midpoint of the bid-ask spread is commonly used as the heuristic estimator of the fundamental value of stocks. This midquote conjecture has multiple applications in market microstructure theory (e.g. liquidity measurement, price discovery). In liquidity measurement, the implicit trading cost borne by a trader for the immediacy of her buy or sell market order is measured by the deviation of the trading price from the fundamental value. The use of the midpoint as a proxy for the fundamental value facilitated the construction of the well known illiquidity measures such as the relative quoted spread, the effective spread, and the realized spread.[2]

This paper provides theoretical foundation for the estimation of the fundamental value based on an endogenous model. I refer to this estimation of the security's fundamental value as the endogenous underlying value of the security.[3] The model derives the endogenous underlying value of the security in two steps: firstly, it evaluates the implicit trading cost function associated to a limit order submitted on a continuous order-driven market, and, secondly, it estimates the fundamental value at equilibrium under a zero trading cost endogenous condition. Therefore, I state that the security's endogenous underlying value is a weighted average of the best bid and ask prices. The weights depend on the volatility of the security and on the risk-free interest rate. The midpoint becomes only a particular case of the endogenous underlying value. The endogenous underlying value of the stock explains the asymmetrical price pressure on buy and sell trades. This value can be closer to the bid price or to the ask price. It has major implications on liquidity measurement, price discovery or asset pricing as an alternative to the traditional use of the midpoint.

The starting point of the model is that limit orders submissions generate a winner's curse phenomenon and an adverse selection effect. Biais, Glosten, and Spatt (2005) argue that the limit order traders behave as the bidders in an auction. The winner's curse and the adverse selection risks occur because the specified limit price is fixed over time and the evolution of the market price may lead to over/underestimation, giving a profit opportunity to a more

[1] The fundamental value is also called the consensus value or the true value.

[2] The Rule 605 of the SEC (The Securities and Exchange Commission) requires the monthly reporting of the average effective spread and the average realized spread.

[3] In accordance with Foucault (1999), the underlying value denotes the expected stock's fundamental value conditional on public information.

informed trader at the expense of the uninformed liquidity supplier. In continuous trading, the informed trader identifies in the order book those pending limit orders, the execution of which will create profit opportunity (i.e., the "picking off" risk). The commitment to trade at a fixed limit price makes the liquidity supplier vulnerable to all these risks.

On the other hand, Copeland and Galai (1983) state that the commitment to trade at a fixed price is equivalent to deliberately offering an option to other traders on the market. I overlap the gain associated to the strategic game of the informed trader on the payoff of this option. As the option replicates the trading mechanism generated by the winner's curse and the adverse selection, I name it replicating option. The pricing of this option gives the implicit trading cost function associated to the limit orders. Thus, the model shows that the trading cost function of a buy limit order is given by the price of a perpetual American barrier put option, while the trading cost function of a sell limit order is defined by the price of a perpetual American barrier call option. At equilibrium, these cost functions provide the endogenous condition to obtain the estimate of the fundamental value.

The theoretical models of limit order market developed by Seppi (1997), Parlour and Seppi (2003), and Goettler, Parlour, and Rajan (2005) take into account that limit orders are a source of "picking off" risk. According to Foucault (1999), the limit order execution is uncertain and price fluctuations are likely to induce the "picking off" risk generating a winner's curse effect. Foucault's dynamic model shows that this risk depends on the volatility of the stock and on the structure of the orders flow. The structure of the orders flow depends on the traders' decision to trade immediately by placing market orders or to expect for a better execution price by placing limit orders. Foucault, Kadan, and Kandel (2005) identify two types of traders, patient traders and impatient traders, and show that, at equilibrium, patient traders become liquidity suppliers for impatient traders. They find that traders submit aggressive limit orders when the spread is large and the proportion of patient traders is high.

The paper is organized as follows. Section II describes the model setup. Section III presents the derivation of the function of the trading cost induced by the winner's curse and adverse selection effects associated to the placement of a limit order. The probability that the winner's curse and adverse selection effects will not occur is also derived. Section IV provides the endogenous underlying value of the security and its implications on illiquidity measures. In this section, I revise traditional illiquidity measures such as the relative quoted spread, the effective spread and the realized spread. Section V exhibits the revisited Liquidity-adjusted CAPM and illiquidity risk premium. Section VI provides a new estimator of the bid-ask spread. Empirical investigation is provided in Section VII. Section VIII concludes.

II. The Model Setup

A. The Market and the Traders

Consider a continuous limit order market. There are two types of orders on this market: market orders and limit orders. These orders are executed according to price and time priority. The buy orders at higher prices and the sell orders at lower prices have priority. If the orders are at the equal prices, time priority is applied.

There are two main categories of traders on the market: liquidity traders and liquidity suppliers. Liquidity traders submit market orders and they are willing to pay a cost to obtain immediacy for their urgent consumption needs. Liquidity suppliers submit limit orders and they are willing to provide liquidity at their own prices to the liquidity traders. Liquidity traders are either informed or uninformed traders. Uninformed traders have access to public information only. Informed traders also hold private information which gives them the opportunity to estimate the future price of the security more precisely than the uninformed traders. There are also noise traders on the market, among both liquidity traders and liquidity suppliers. Noise traders conceal the presence of informed traders on the market.

The liquidity supplier who submits a limit order is facing the winner's curse risk, adverse selection risk and non-execution risk. On one hand, when trading with the informed trader the liquidity supplier will pay an implicit trading cost associated to these risks. On the other hand, the liquidity supplier will obtain a liquidity premium from the liquidity trader which compensates for these risks. The limit order trader recoups the loss borne in trades with informed liquidity traders by the gain obtained in trades with uninformed liquidity traders.

The trading price or the observed price P_t is the execution price of the limit order. The limit price becomes the trading price when the limit order is executed. The trading price becomes either the highest bid limit price or the lowest ask limit price as the buy and sell limit orders are executed.

B. The Cash-in-the-Market Strategy of the Informed Trader

Let $V > 0$ be the fundamental or the true value of the asset. Let Ω_t be the set of all informations about the asset value at time t. As in the Glosten and Milgrom (1985) model, the information set Ω_t contains news and order flows. The information set Ω_t can be decomposed into a set of public information available to all traders Ω_t^p and a set of private information available only to informed traders Ω_t^i, $\Omega_t = \{\Omega_t^i \cup \Omega_t^p\}$ and $\{\Omega_t^i \cap \Omega_t^p\} = \emptyset$.

Let K_b be the bid limit price of a buy limit order and let K_a be the ask limit price of a sell limit order. The limit prices are set in accordance with all public available information regarding the security's value, Ω_t^p. Conditional on their heterogeneous beliefs, liquidity suppliers submit limit orders with different limit prices. All limit order traders believe that their bid limit prices are lower than the security's underlying value and their ask limit prices are higher than the security's underlying value.

The informed trader holds the best possible estimate X_t of the fundamental value, where

$$X_t = E\left[V|\Omega_t\right]. \tag{1}$$

According to the efficient market hypothesis, the trading price incorporates instantaneously all available information on the market being at any time the best possible estimate of the fundamental value. Due to microstructure frictions, the trading price is likely to deviate from the efficient price. The trading price at time t is defined as

$$P_t = X_t + \xi_t, \tag{2}$$

where ξ_t denotes the deviation from the efficient price. As the deviation is temporary $E\left[\xi_\tau|\Omega_t\right] = 0$, which implies $E\left[P_\tau|\Omega_t\right] = E\left[X_\tau|\Omega_t\right]$, $t < \tau$. Because the efficient price is a martingale conditional on Ω_t, $X_t = E\left[X_\tau|\Omega_t\right]$, it results

$$X_t = E\left[P_\tau|\Omega_t\right]. \tag{3}$$

The random time τ is defined as the time when there is no more informational advantage and trades take place at the fair price given all available information. At this time, the price incorporates the information initially known by the informed trader. In other words, the informed trader expects that the future trading price reaches X_t.

The informed trader's objective is to cash-in-the-market her private information as quickly as possible. This strategy depends on the speed with at which the information is incorporated in the price. The informed trader is the only one who knows X_t. The informed trader will exploit her superior information if the limit order trader is mispricing the stock. Thus, if $X_t < K_b$, the informed trader will sell the stock, while if $X_t > K_a$, the informed trader will buy the stock. The informed trader arrives to cash-in-the-market her competitive advantage when the stock price will incorporate the information and will reach the expected level X_t. Actually, insofar as limit order traders update their beliefs, the price will be continuously adjusted incorporating information.

The limit order trader does not observe whether the informed trader's expectation X_t is above the ask limit price K_a, below the bid limit price K_b or within these two bounds. As opposed, the informed trader can observe the relation between X_t and the limit prices.

Insofar as the profit opportunity occurs, the limit order is picked off by a marketable order initiated by the informed trader. The overpricing or the underpricing of the security will be optimally exploited by contrary orders only when the price has incorporated the private information. Henceforth, the optimal payoff of the informed trader's cash-in-the-market strategy is defined by

$$\text{Payoff} = \begin{cases} X_t - K_a \text{ if she buys at } K_a \text{ and she sells at } X_t > K_a \\ K_b - X_t \text{ if she sells at } K_b \text{ and she buys at } X_t < K_b \end{cases}.$$

When $X_t < K_b$ the trading price may reach X_t if and only if the buy limit order with the bid limit price K_b has been entirely executed. Similarly, when $X_t > K_a$ the trading price may reach X_t if and only if the sell limit order with the ask limit price K_a has been entirely executed. The target value X_t is a post-trade price. This one can occur only after the execution of the limit order.

C. The Replicating Option

The informed trader's strategy to cash-in-the-market her private information can be replicated by means of an option mechanism. Copeland and Galai (1983) state that the commitment to trade the security at a fixed price is equivalent to offering an option to other traders. Following this statement, I associate the submission of a limit order to writing an American option which has the strike price equal to the limit price. This option can be also considered as being perpetual (i.e. its maturity is infinite) because the limit order can stay indefinitely unexecuted in the limit order book. This option is free because the limit order trader encashes no payment for writing the option. Pricing the option is equivalent to estimating the implicit trading cost borne by the liquidity supplier at the order submission.

In informed traders' presence, the perpetual American option can be redefined as a replicating option of the trading mechanism by matching its strictly positive payoff to the gain associated to the cash-in-the-market strategy of the informed trader. The cash-in-the-market strategy of the informed trader is an optimal exercise strategy. Thus, the replicating option will be also optimally exercised. The replicating option's payoff corresponds to a strategic game between traders because the resulting payoff comes from sequences of their choices.[4] This payoff occurs at a random time.

The replicating option is also a barrier option. Actually, the option can be exercised anytime until its infinite maturity, but only at the moment when the trading price hits an established barrier level given by X_t. If the stock price does not reach X_t, the option will

[4]See Ziegler (2004) for a game theory analysis of perpetual options.

not be exercised, its payoff will be zero and the winner's curse and the adverse selection will not occur. The replicating option is not exercised when the limit order stays unexecuted in the order book. Only when the cash-in-the-market strategy of the informed trader succeeds, it will lead to a trading cost for the uninformed liquidity supplier.

It is unrealistic to assume that the trading price could instantaneously become X_t without reaching any limit price of limit orders before that. For liquidity reasons, the informed trader picks-off strategically and quickly any limit order which under/overestimates the security before its eventual cancelling.[5] The trading price reaches at least one ask limit K_a or one bid limit K_b before incorporating private information and becoming X_t. Therefore, the value of the replicating option defines the trading cost associated to non-cancelled limit orders picked off by informed traders.

The replicating option is a perpetual American barrier call option when the liquidity supplier submits a sell limit order. If the liquidity supplier submits a buy limit order, the replicating option will become a perpetual American barrier put option.

D. The Valuation Model

The function of the trading cost associated to a limit order at its submission time is generated by a simple pricing model of the replicating option based on risk-neutral valuation technique. Thus, the stock trading price is assumed to follow a risk-neutral stochastic process.

Assumption 1. *In a risk-neutral world, the trading price process* $(P_T, T \geq t)$ *is defined as a geometric Brownian motion*

$$dP_T = rP_T dT + \sigma P_T d\widetilde{W}_T, \tag{4}$$

where the risk-free interest rate r *and the volatility of the security* σ *are supposed to be constant over time.* $\widetilde{W}_T$ *is a standard Brownian motion under a risk-neutral probability measure* Q*. The initial point in time is* t *and the process is initialized at point* $A_t > 0$ *(the best ask) or* $B_t > 0$ *(the best bid) depending on whether a sell trade or a buy trade takes place.*

The option is optimally exercised by its owner when the trading price hits her estimation of the true value. Because X_t is publicly unknown, it can be equal to any positive real number $\overline{X}$. Thus, the pricing model evaluates the call replicating option as a function of the variable $\overline{X}$ when $\overline{X} > K_a$ and the put replicating option as a function of the variable $\overline{X}$ when $\overline{X} < K_b$.

[5] Biais, Hilton, Mazurier, and Pouget (2005) show that the miscalibrated traders with overconfidence in judgement tend to overestimate the precision of their information becoming vulnerable to the winner's curse trap.

In other words, the replicating option is evaluated when the stock price reaches the level $\overline{X}$. The call option payoff is $\overline{X} - K_a$ with $\overline{X} > K_a$, while the payoff $K_b - \overline{X}$ with $\overline{X} < K_b$ corresponds to the put option. These payoffs occur at a random time τ, which can be modelled as a stopping time.

Assumption 2. *Let τ be a hitting time which is defined as the first time when the trading price process P_T hits the level $\overline{X}$*

$$\tau = \min \left\{ T \geq t; P_T = \overline{X} \right\},$$

where $\overline{X}$ is a real positive number.

Using the risk-neutral valuation, the trading cost function of a buy limit order is the price of a perpetual American barrier put option defined by

$$\Pi_t^b \left(\overline{X} \right) = E_Q \left[e^{-r(\tau - t)} \max \left(K_b - \overline{X}, 0 \right) \right] = \max \left(K_b - \overline{X}, 0 \right) E_Q \left[e^{-r(\tau - t)} \right]. \tag{5}$$

Similarly, the trading cost function of the sell limit order is the price of a perpetual American barrier call option given by

$$\Pi_t^a \left(\overline{X} \right) = E_Q \left[e^{-r(\tau - t)} \max \left(\overline{X} - K_a, 0 \right) \right] = \max \left(\overline{X} - K_a, 0 \right) E_Q \left[e^{-r(\tau - t)} \right]. \tag{6}$$

When the bid barrier $\overline{X} < K_b$ is reached, the payoff of the put replicating option is achieved. Similarly, if the ask barrier $\overline{X} > K_a$ is reached, the payoff of the call replicating option is attained. Two situations are possible. If the replicating option is never exercised ($\tau = \infty$), then $e^{-r(\tau - t)} = 0$ and the trading cost is zero (the level $\overline{X}$ is never reached by the trading price). If the replicating option is exercised ($\tau < \infty$), the trading cost functions are strictly positive.

III. The Trading Cost Function

A. The Probability of the Winner's Curse and Adverse Selection Effects

If the trading price does not hit $\overline{X}$, the winner's curse and adverse selection effects will not occur and there will be no trading cost associated to the limit order. The following proposition defines the probabilities that the trading price will not reach a level $\overline{X}$ lower or equal to the bid limit price and a level $\overline{X}$ higher or equal to the ask limit price.

Proposition 1. *Let K_b be the limit price of a buy limit order. At the order submission time t, if $\min \left(\overline{X}, K_b \right) < B_t$, the probability that the trading price will not hit the level $\overline{X} \leq K_b$ is* *given by*

$$Q_b(\tau=\infty)=\begin{cases}1-\left[\frac{\min\left(\overline{X},K_b\right)}{P_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } r>\frac{1}{2}\sigma^2\\ 0 & \textit{otherwise}\end{cases}. \tag{7}$$

If $B_t<\min\left(\overline{X},K_b\right)<A_t$, *the probability is given by*

$$Q_b(\tau=\infty)=\begin{cases}1-\left[\frac{\min\left(\overline{X},K_b\right)}{B_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } P_t=B_t \textit{ and } r<\frac{1}{2}\sigma^2\\ 1-\left[\frac{\min\left(\overline{X},K_b\right)}{A_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } P_t=A_t \textit{ and } r>\frac{1}{2}\sigma^2\\ 0 & \textit{otherwise}\end{cases}. \tag{8}$$

Let K_a *be the limit price of a sell limit order. At the order submission time* t, *if* $\max\left(\overline{X},K_a\right)>A_t$, *the probability that the trading price will not hit the level* $\overline{X}\geq K_a$ *is defined by*

$$Q_a(\tau=\infty)=\begin{cases}1-\left[\frac{\max\left(\overline{X},K_a\right)}{P_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } r<\frac{1}{2}\sigma^2\\ 0 & \textit{otherwise}\end{cases}. \tag{9}$$

If $B_t<\max\left(\overline{X},K_a\right)<A_t$, *the probability is defined by*

$$Q_a(\tau=\infty)=\begin{cases}1-\left[\frac{\max\left(\overline{X},K_a\right)}{A_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } P_t=A_t \textit{ and } r>\frac{1}{2}\sigma^2\\ 1-\left[\frac{\max\left(\overline{X},K_a\right)}{B_t}\right]^{\frac{2r}{\sigma^2}-1} & \textit{if } P_t=B_t \textit{ and } r<\frac{1}{2}\sigma^2\\ 0 & \textit{otherwise}\end{cases}. \tag{10}$$

Proof. The proof is provided in Appendix A. □

If $\min\left(\overline{X},K_b\right)=K_b$ and $\max\left(\overline{X},K_a\right)=K_a$, $Q_b(\tau=\infty)$ and $Q_a(\tau=\infty)$ give the probabilities that the buy and sell limit orders will not be executed. If $\min\left(\overline{X},K_b\right)=\overline{X}$ and $\max\left(\overline{X},K_a\right)=\overline{X}$, $Q_b(\tau=\infty)$ and $Q_a(\tau=\infty)$ give the probabilities that the winner's curse and the adverse selection will not occur. There are non-zero probabilities that the winner's curse and adverse selection effects will not occur. These probabilities have different values depending on the sign of the drift $(r-\sigma^2/2)$ of the stock's log-price risk-neutral stochastic process. Thus, if the value $\overline{X}$ is outside the bid-ask spread, a non-zero probability $Q_a(\tau=\infty)$ will be induced by a negative drift, while a non-zero probability $Q_b(\tau=\infty)$ will be given by a positive drift. If the value $\overline{X}$ is inside the bid-ask spread, a non-zero probability will result depending on the drift's sign but also on the initial trading price which can be either the best bid price or the best ask price.

B. *The Trading Cost Function of a Buy Limit Order*

The implicit trading cost of a buy limit order occurs when the bid limit price K_b is higher than $\overline{X}$. There are two possibilities to submit a buy limit order: (i) the limit price K_b is framed between the best bid price B_t and the best ask price A_t and (ii) the limit price K_b is lower than the best bid price B_t. In the first case, the put replicating option is in-the-money, while, in the second one, the option is out-of-the-money. At the moment when the buy limit order is submitted, the trading cost function is given by relation (5). In this relation, the expected discount factor is the Laplace transform for first passage time of drifted Brownian motion. Based on this Laplace transform, the trading cost function is defined by the following proposition.

Proposition 2. *At the order submission time t, the trading cost of a buy limit order with the limit price K_b is a function of $\overline{X}$. When the initial price is the best bid ($P_t = B_t$), the trading cost function is*

$$\Pi_t^b\left(\overline{X}\right) = \begin{cases} \max\left(K_b - \overline{X}, 0\right)\frac{B_t}{\overline{X}} & \text{if } \overline{X} \geq B_t \\ \max\left(K_b - \overline{X}, 0\right)\left(\frac{B_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} & \text{if } \overline{X} < B_t \end{cases}. \tag{11}$$

When the initial price is the best ask ($P_t = A_t$), the trading cost function is

$$\Pi_t^b\left(\overline{X}\right) = \max\left(K_b - \overline{X}, 0\right)\left(\frac{A_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} \quad \text{if } \overline{X} < A_t. \tag{12}$$

Proof. The proof is provided in Appendix B. □

The trading cost depends on three parameters: the best bid price or the best ask price, the risk-free interest rate and the stock volatility. If the volatility increases, the trading cost will also increase, as $\partial\Pi_t^b/\partial\sigma > 0$. If the risk-free interest rate increases, the trading cost will decrease, as $\partial\Pi_t^b/\partial r < 0$. The trading cost will be a decreasing and convex function of the bid price (i.e., $\partial\Pi_t^b/\partial B_t < 0$ and $\partial^2\Pi_t^b/\partial B_t^2 > 0$) if $\overline{X} < B_t$, and an increasing and concave function of the bid price (i.e., $\partial\Pi_t^b/\partial B_t > 0$ and $\partial^2\Pi_t^b/\partial B_t^2 < 0$) if $\overline{X} \geq B_t$. The trading cost is a decreasing and convex function of the ask price, as $\partial\Pi_t^b/\partial A_t < 0$ and $\partial^2\Pi_t^b/\partial A_t^2 > 0$.

C. *The Trading Cost Function of a Sell Limit Order*

The trader placing a sell limit order pays an implicit trading cost when the ask limit price K_a is lower than $\overline{X}$. The trader can choose to set the limit price K_a (i) between the best bid price B_t and the best ask price A_t or (ii) at higher level than the best ask price A_t. When the limit price is inside the bid-ask spread the call replicating option is in-the-money, while the option is out-of-the-money when the limit price is outside the bid-ask spread. Using the relation (6) and the theorem of Laplace transform for first passage time of drifting Brownian motion, the following proposition results.

Proposition 3. *At the order submission time t, the trading cost of a sell limit order with limit price K_a is a function of $\overline{X}$. When the initial price is the best ask ($P_t = A_t$), the trading cost function is*

$$\Pi_t^a\left(\overline{X}\right) = \begin{cases} \max\left(\overline{X} - K_a, 0\right)\frac{A_t}{\overline{X}} & \text{if } \overline{X} > A_t \\ \max\left(\overline{X} - K_a, 0\right)\left(\frac{A_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} & \text{if } \overline{X} \le A_t \end{cases}. \tag{13}$$

When the initial price is the best bid ($P_t = B_t$), the trading cost function is

$$\Pi_t^a\left(\overline{X}\right) = \max\left(\overline{X} - K_a, 0\right)\frac{B_t}{\overline{X}} \qquad \text{if } \overline{X} > B_t. \tag{14}$$

Proof. The proof is similar to that of the Proposition 2. □

The trading cost of the sell limit orders is an increasing and concave function of the best ask price (i.e., $\partial\Pi_t^a/\partial A_t > 0$ and $\partial^2\Pi_t^b/\partial A_t^2 < 0$) when $\overline{X} > A_t$, and a decreasing function of the best ask price (i.e., $\partial\Pi_t^a/\partial A_t < 0$) when $\overline{X} \le A_t$. The trading cost is a convex function of the ask price when $\overline{X} \le A_t$. The increase of the bid price leads to the increase of the trading cost, as $\partial\Pi_t^a/\partial B_t > 0$. The trading cost is an increasing function of the volatility, as $\partial\Pi_t^a/\partial\sigma > 0$, and a decreasing function of the risk-free interest rate, as $\partial\Pi_t^a/\partial r < 0$.

IV. The Endogenous Underlying Value of Stocks

A. The Equilibrium

Beginning with the seminal papers of Demsetz (1968) and Roll (1984), a large market microstructure literature heuristically considers that the midpoint of the bid-ask spread $M_t = (A_t + B_t)/2$ is the best estimate of the security's fundamental value. I adopt the definition of the equilibrium as in Demsetz (1968) and Copeland and Galai (1983). According to this definition, the equilibrium price is obtained in a world where there is no supply or demand for immediacy (i.e., there is no market order) and there are only equally well informed traders.

The trading cost functions of buy and sell limit orders can be represented as a bottom straddle strategy by using in-the-money put and call replicating options. Actually, the current bid and ask prices frame the perfect liquidity price. If the limit prices of the two opposite orders tend to a common value, K_a, $K_b \rightarrow L_t$, they may be directly compensated without being traded through order markets, creating perfect liquidity conditions. The perfect liquidity price is defined by L_t.

The informed trader's estimate of the fundamental value X_t can be straddled or not by the current bid and ask prices. Firstly, consider $\overline{X} > B_t$, $\overline{X} < A_t$ and K_a, $K_b \rightarrow L_t$. When X_t is inside the bid-ask spread, the implicit trading costs of buy and sell limit orders at the initial prices ($P_t = B_t$ and $P_t = A_t$) can be rewritten as

$$\begin{aligned} C_{in}^{B}\left(\overline{X}, L_t\right) &= \left|\overline{X} - L_t\right| \frac{B_t}{\overline{X}}, && (15) \\ C_{in}^{A}\left(\overline{X}, L_t\right) &= \left|\overline{X} - L_t\right| \left(\frac{A_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}}. && (16) \end{aligned}$$

Secondly, consider $\overline{X} < B_t$, $\overline{X} > A_t$ and K_a, $K_b \rightarrow L_t$. When X_t is outside the current bid-ask spread, the implicit trading costs at the initial bid and ask prices can be redefined as

$$\begin{aligned} C_{out}^{B}\left(\overline{X}, L_t\right) &= \max\left(L_t - \overline{X}, 0\right) \left(\frac{B_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} + \max\left(\overline{X} - L_t, 0\right) \frac{B_t}{\overline{X}}, && (17) \\ C_{out}^{A}\left(\overline{X}, L_t\right) &= \max\left(L_t - \overline{X}, 0\right) \left(\frac{A_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} + \max\left(\overline{X} - L_t, 0\right) \frac{A_t}{\overline{X}}. && (18) \end{aligned}$$

Let Φ_t be the estimate of the fundamental value given that all traders are equally well informed.

Condition 1. *In informational efficient markets, the underlying value Φ_t is the efficient price that cancels all implicit trading costs associated to limit orders. The value X_t equals the underlying value Φ_t if private information vanishes and all traders possess symmetric information. In equilibrium, the endogenous condition of the zero trading cost is*

$$\begin{cases} \lim_{\overline{X} \rightarrow \Phi_t} C_{in}^{A}\left(\overline{X}, L_t\right) = \lim_{\overline{X} \rightarrow \Phi_t} C_{in}^{B}\left(\overline{X}, L_t\right) = 0 \\ \lim_{\overline{X} \rightarrow \Phi_t} C_{out}^{A}\left(\overline{X}, L_t\right) = \lim_{\overline{X} \rightarrow \Phi_t} C_{out}^{B}\left(\overline{X}, L_t\right) = 0 \end{cases}. \tag{19}$$

Based on the above condition, the endogenous underlying value Φ_t is given by proposition 4. The zero trading cost condition implies that the underlying value Φ_t equals the price under perfect liquidity L_t. This is in accordance with Demsetz's definition of the equilibrium.

Proposition 4. *Let A_t and B_t be the best ask and the best bid prices of a stock. Let r be the risk-free interest rate and let σ be the volatility of the trading price. Then, the endogenous* *underlying value of the stock is defined by*

$$\Phi_t = A_t^{\gamma} B_t^{1-\gamma}, \tag{20}$$

where $\gamma = r / \left(r + \frac{1}{2}\sigma^2\right)$ *and* $0 < \gamma < 1$.[6,7]

Proof. The proof is provided in Appendix C. □

The proposition states that, at equilibrium, the fundamental value should be straddled by the current ask and bid prices. The endogenous underlying value of stocks is also an average, as it is the midquote ($A_t > \Phi_t > B_t$). The difference is that this estimate of the fundamental value is endogenously obtained and it is defined as a weighted average of the ask and bid prices. The weights are given by a coefficient that depends on the risk-free interest rate and on the volatility of the security. If the expected risk-neutral log-return is negative ($r - \sigma^2/2 < 0$), Φ_t will be closer to the bid price, and if the expected risk-neutral log-return is positive ($r - \sigma^2/2 > 0$), Φ_t will be closer to the ask price. If the return drift is null ($r - \sigma^2/2 = 0$), the endogenous underlying value will be the geometric mean of the ask and bid prices, $\Phi_t = \sqrt{A_t B_t}$.

The midquote becomes a particular case of the endogenous underlying value. The midquote is exogenous and it is based on the assumption that the buy and sell trades have symmetric impact on market liquidity. Nevertheless, the endogenous underlying value of stocks captures the asymmetric effects of buy and sell trades on the market liquidity. The endogenous estimate of the fundamental value can be closer to the bid price or to the ask price, signaling that the market is adjusted downward or upward.

B. Endogenous Illiquidity Measures

Illiquidity measures are extremely important in practice as market illiquidity generates a cost defined as the gap between the trading price and the proxy of the fundamental value. The simplest illiquidity measures that gauge this implicit trading cost are the quoted spread, the effective spread, and the realized spread. The endogenous underlying value of the stock can be used in redefining these widespread illiquidity measures.

The relative quoted spread is defined as the bid-ask spread divided to the midquote. This illiquidity measure, commonly used in practice, may be redefined by dividing the bid-ask spread to the underlying value Φ. Considering q_A and q_B the depths of the best ask and

[6] The expression of the endogenous underlying value Φ is similar to the Cobb-Douglas production function with constant returns to scale. Thus, γ and $1-\gamma$ can be interpreted as elasticities of the ask and bid prices.

[7] If the security is a dividend-paying stock and d is the dividend yield, the parameter γ can be rewritten as $\gamma = (r-d) / \left(r - d + \frac{1}{2}\sigma^2\right)$.

the best bid, for small size trades (i.e., $q_t \leq \min(q_A, q_B)$),

$$S_q = \frac{A_t - B_t}{\Phi_t} = \left(\frac{A_t}{B_t}\right)^{1-\gamma} - \left(\frac{B_t}{A_t}\right)^{\gamma}. \tag{21}$$

For large size trades ($q_t > \min(q_A, q_B)$), the illiquidity measure is given by the weighted relative quoted spread

$$\overline{S}_q = \frac{\overline{A}(q_t) - \overline{B}(q_t)}{\Phi_t}, \tag{22}$$

where $\overline{A}(q_t)$ is the average execution price for a buy market order of size q_t and $\overline{B}(q_t)$ is the average execution price for a sell market order of size q_t.

The relative effective spread is commonly defined as the difference between the trading price and the midquote divided to the same midquote. Similarly, the illiquidity measure may be redefined based on the underlying value Φ, by replacing the midpoint, as follows

$$S_e = \frac{|P_t - \Phi_t|}{\Phi_t}. \tag{23}$$

The difference between the trading price and the perfect liquidity price defines the impact cost. This impact cost is symmetrical for buy and sell orders when using the midpoint, while it becomes asymmetrical if the endogenous underlying value is applied. An asymmetrical impact cost of traded buy/sell orders is naturally more accurate because orders placed on the market have different effects on ask and bid prices inducing an increase or a decrease in the market depending on their size. That explains the price pressure for buy or sell trades. If the trading price is the best ask price quoted the instant before trading ($P_t = A_t > \Phi_t$), then the relative effective spread at the ask price is defined by

$$S_e^A = \frac{A_t - \Phi_t}{\Phi_t} = \left(\frac{A_t}{B_t}\right)^{1-\gamma} - 1. \tag{24}$$

If the trading price is the best bid price quoted just before transaction ($P_t = B_t < \Phi_t$), then the relative effective spread at the bid price is defined by

$$S_e^B = \frac{\Phi_t - B_t}{\Phi_t} = 1 - \left(\frac{B_t}{A_t}\right)^{\gamma}. \tag{25}$$

The sum of the two measures of relative effective spreads is equal to the relative quoted spread measure. An increase in the volatility of the security leads to a higher effective spread when the transaction is carried out at the ask price and to a lower effective spread when the transaction is done at the bid price (the derivative of each effective spread with respect to the volatility is positive and negative, respectively). In other words, high volatility leads to an increase in consumed liquidity when the security is to be sold and to a decrease in consumed liquidity when the security is to be bought. Therefore, there are opposite effects of volatility on the liquidity of buy and sell orders. The risk-free interest rate has a reverse effect than the volatility. Increasing the risk-free interest rate decreases the effective spread (increases liquidity) when the transaction takes place at the ask price and increases the effective spread (decreases liquidity) when the transaction takes place at the bid price. The differently exerted pressure on the bid prices and on the ask prices can be interpreted as a signal of the market increase or decrease.

The realized spread is defined as the difference between the trading price and the fundamental value estimate at time $t + \Delta t$ after the transaction

$$S_r = \left| P_t - \Phi_{t+\Delta t} \right|. \tag{26}$$

The relative realized spread at the ask price is defined by

$$S_r^A = \frac{A_t - \Phi_{t+\Delta t}}{\Phi_t} = \left(\frac{A_t}{B_t} \right)^{1-\gamma} - \frac{\Phi_{t+\Delta t}}{\Phi_t}, \tag{27}$$

while the relative realized spread at the bid price is given by

$$S_r^B = \frac{\Phi_{t+\Delta t} - B_t}{\Phi_t} = \frac{\Phi_{t+\Delta t}}{\Phi_t} - \left(\frac{B_t}{A_t} \right)^{\gamma}. \tag{28}$$

V. Revisited Liquidity-Adjusted CAPM

Amihud and Mendelson (1986) and Acharya and Pedersen (2005) show that trading costs diminish investors' return. In consequence, rational investors will demand a compensation for the risk of obtaining a lower return due to the lack of liquidity on the market. Liquidity varies over time, and its fluctuations add an additional risk, called liquidity risk, to the underlying risk of the asset. As liquidity risk is non-diversifiable, the return expected by investors must also contain an additional illiquidity premium in excess over the risk premium. Amihud and Mendelson (1986) studied the impact of liquidity on asset prices and expected returns and pointed out that ask prices include a premium for immediate buy, while bid prices include a compensation for immediate sale. The sum between the premium and the compensation represents the bid-ask spread. The model developed in the previous section shows that due to asymmetric information the value of the premium is not necessarily equal to the value of the compensation, the endogenous underlying value of the security being asymmetrically positioned between the two, as opposed to the midpoint. Henceforth, I assume that the endogenous underlying value Φ equals the fundamental value of the security.

The net return obtained by investors over a holding period $t+h$ is computed by taking into account the buy price at t (ask) and the sell price at $t+h$ (bid) which include the premium and the compensation

$$B_{t+h} = A_t e^{R_{t+h} h}. \tag{29}$$

The net return is the required rate of return which gives the minimum price accepted by investors. By normalizing the holding period to $h=1$, the net return is defined as

$$R_{t+1} = \ln \frac{B_{t+1}}{A_t}. \tag{30}$$

The future fundamental value over h holding periods is obtained by continuously compounding the fundamental value of the security at the required rate of return R^{ϕ}

$$\Phi_{t+h} = \Phi_t e^{R^{\phi}_{t+h} h}. \tag{31}$$

The fundamental value at time t is correlated with the trading cost. The lower the trading cost, the higher the value of the asset. Over a single holding period, the fundamental value's return may be decomposed into the net return R and the returns on spread R^s

$$R^{\phi}_{t+1} = \ln \frac{\Phi_{t+1}}{\Phi_t} = R_{t+1} + \gamma_{t+1} R^s_{t+1} + (1-\gamma_t) R^s_t, \tag{32}$$

where the return on spread at time $t+1$ is $R^s_{t+1} = \ln (A_{t+1}/B_{t+1}) > 0$ and the return on spread at time t is $R^s_t = \ln (A_t/B_t) > 0$. Thus, if the weighted sum of the returns on spread would be added to the effective or net return, the result would be the gross return (i.e., the fundamental value's return). At time t, the relative illiquidity cost C^l (i.e., the difference between the gross return and the net return) under information asymmetry is defined by

$$C^l_t = R^{\phi}_t - R_t = \gamma_t R^s_t + \left(1-\gamma_{t-1}\right) R^s_{t-1} > 0. \tag{33}$$

Therefore, the higher the bid-ask spread when buying and/or when clearing the position on the security, the higher the illiquidity cost borne by the investor. The fundamental value's return is greater than the net return because the difference compensates investors for the transaction costs.[8]

Several empirical studies, beginning with Chordia, Roll, and Subrahmanyam (2000),

[8] See Foucault, Pagano and Röell (2013).

Hasbrouck and Seppi (2001), show that illiquidity measures are usually positively correlated with returns, the co-movement phenomenon being called "commonality" in liquidity. The co-movements in liquidity prove that the liquidity risk of a security cannot be diminished through diversification and, therefore, contributes to its systematic risk. The fact that the fundamental value estimator and the illiquidity cost are endogenously obtained can be exploited by adjusting the CAPM so that it takes into account the impact of liquidity shortage on the expected return. Acharya and Pedersen (2005) proposed such an extension, known as the Liquidity-adjusted CAPM (LCAPM). The Acharya-Pedersen model defines the net return as a difference between the gross return and the illiquidity cost. Acharya and Pedersen (2005) claim that in two economies, with and without frictions, it is possible to obtain the expected return at equilibrium and, more than that, the prices at equilibrium are the same in both economies. Therefore, the translation of the CAPM equation from one economy to another, leads to obtaining LCAPM for gross returns. For each stock i, the relation between the gross and the net return is endogenously obtained ($R_i = R_i^{\phi} - C_i^l$). By using this in translating the CAPM equation, it results

$$E\left[R_i^{\phi} - C_i^l\right] = r + \beta_i \left(E\left[R_M - r\right]\right), \tag{34}$$

where the β_i coefficient may be written

$$\beta_i = \frac{COV\left(R_i, R_M\right)}{VAR\left(R_M\right)} = \underbrace{\frac{COV\left(R_i^{\phi}, R_M\right)}{VAR\left(R_M\right)}}_{\beta_i^{\phi}} - \underbrace{\frac{COV\left(C_i^l, R_M\right)}{VAR\left(R_M\right)}}_{\beta_i^l}. \tag{35}$$

Considering the decomposition of the β_i coefficient into underlying beta (β_i^{ϕ}) and illiquidity beta (β_i^l), the expected gross return is given by the following revisited LCAPM equation

$$E\left[R_i^{\phi}\right] = r + \beta_i^{\phi}\left(E\left[R_M - r\right]\right) + E\left[C_i^l\right] - \beta_i^l\left(E\left[R_M - r\right]\right), \tag{36}$$

where the required total risk premium is composed of the illiquidity risk premium defined by

$$IP_i = E\left[C_i^l\right] - \beta_i^l\left(E\left[R_M - r\right]\right), \tag{37}$$

and the underlying risk premium given by

$$UP_i = \beta_i^{\phi}\left(E\left[R_M - r\right]\right). \tag{38}$$

Unlike Acharya and Pedersen (2005), the market premium is set to be unchanged. The market portfolio should be the benchmark accepted by all investors in order to assess the average risk-aversion on the market. Even if the stock market indices, generally used as proxies for the market portfolio, are investable and traceable through investment vehicles such as index funds or exchange-trading funds, they do not effectively trade on the market and are not effectively subjected to liquidity costs. The CAPM is a normative model of asset pricing under risk conditions, which only implies the existence of one optimal portfolio on the efficient frontier. The market portfolio should be observable for all investors and it represents the complete benchmark portfolio in investment strategies. If a proxy for the market portfolio is the benchmark accepted by all investors, then the illiquidity premium should be determined with respect to this proxy. Moreover, this is consistent with another important assumption of the CAPM model: the homogeneity of investors' expectations. Even if this assumption is not realistic, Sharpe (1964) argues that not the realism of an assumption is important but the acceptability of its implications by the market participants.

Another difference is that our illiquidity measure is not exogenously derived (Acharya and Pedersen (2005) use the measure of Amihud (2002), as the measure of illiquidity cost), but endogenously, based on the definition of the endogenous underlying value. Testing the revisited LCAPM relation is easier with gross returns computed based on the endogenous underlying value.[9]

VI. A New Bid-Ask Spread Measure

Illiquidity measures, such as the quoted spread, the effective spread or the realized spread, require data on best bid and best ask prices. Because such data is not always available the spread between these quotes needs to be estimated. Beginning with Roll (1984), a large branch of literature is focused on estimating the bid-ask spread of financial securities. Roll's bid-ask spread estimator is related to the serial covariance of price changes. Roll (1984) associates the midquote to the fundamental value which follows a random walk. The extensions of the model, developed, among others, by Stoll (1989), George, Kaul, and Nimalendran (1991), Huang and Stoll (1997), Stoll (2000), Hasbrouck (2009) are also based on similar hypothesis. Some authors make different assumptions on the dynamic of the stock returns or about an unbalanced orders flow.

Corwin and Schultz (2012) propose a bid-ask spread estimator based on the definition of the variance of stock prices given by Parkinson (1980) and Garman and Klass (1980), taking into consideration the highest and the lowest price registered in a trading day. They

[9]Foucault, Pagano, and Röell (2013) derive the liquidity-adjusted CAPM for gross returns based on midquotes. The model testing is relying on midquotes, rather than on trading prices.

also assume that the midquote follows a diffusion process.

The new endogenously obtained estimator of the fundamental value is useful in estimating the bid-ask spread. I set up the hypothesis that the bid-ask spread ($S_t = A_t - B_t$) does not change over two consecutive trading periods, otherwise used, by Roll (1984) or Corwin and Schultz (2012) for their bid-ask spread estimation. Taking into account that the trading price may become an ask or a bid price depending on whether the trade is corresponding to a buy or to a sell order, the following approximation of the return on spread (R_t^s) is appropriate

$$R_t^s \simeq \frac{S_t}{P_t}, \tag{39}$$

because $(A_t - B_t)/B_t \simeq \ln(A_t/B_t)$ and $(A_t - B_t)/A_t \simeq -\ln(B_t/A_t)$. The net return is approximated to the effective return which is continuously compounded over a single holding period based on trading prices, $R_t \simeq \ln(P_t/P_{t-1})$. Based on these approximations and on the hypothesis that the spread is constant over two consecutive trading periods ($S = S_t = S_{t-1} > 0$), the equation (33) that defines the strictly positive difference between the gross return and the net return becomes

$$\left|R_t^{\phi} - R_t\right| = \left(\frac{\gamma_t}{P_t} + \frac{1-\gamma_{t-1}}{P_{t-1}}\right) S = \Psi_t S, \tag{40}$$

where $\Psi_t = \gamma_t/P_t + \left(1-\gamma_{t-1}\right)/P_{t-1} > 0$.

The next proposition defines the bid-ask spread measure based on the above relation and the assumption that the fundamental value follows a random walk and its expected return is zero.

Proposition 5. *Let P_t be the trading price of the security. Let r_t be the risk-free interest rate and let σ_t be the volatility of the trading price. Then, the bid-ask spread of the security can be measured by*

$$\widehat{S} = \left|\frac{\overline{R_t\Psi_t}}{\overline{\Psi_t^2}}\right|, \tag{41}$$

where $R_t = \ln(P_t/P_{t-1})$ and $\Psi_t = \gamma_t/P_t + \left(1-\gamma_{t-1}\right)/P_{t-1}$ with $\gamma_t = r_t/\left(r_t + \frac{1}{2}\sigma_t^2\right)$.[10]

Proof. The proof is provided in Appendix D. □

The new bid-ask spread estimator given by (41) is easy to compute. This bid-ask spread estimator depends only on three parameters: the trading price, its volatility and the risk-free interest rate. As opposed to the Roll's family of estimators, the new bid-ask spread measure

[10]The expressions $\overline{R_t\Psi_t}$ and $\overline{\Psi_t^2}$ refer to the sample average of $R_t\Psi_t$ and Ψ_t^2, respectively.

does not admit complex values. Also, as opposed to the measure proposed by Corwin and Schultz, the new bid-ask spread measure is more realistic by not admitting negative values.

VII. Empirical Investigation

A. Data and Descriptive Statistics

This section reports several empirical results on liquidity measurement and the impact of liquidity on stock prices. More concrete, the empirical results from the revisited Liquidity-adjusted CAPM, as well as bid-ask spreads estimations, are presented in the following.

The database comprises financial daily data collected from two markets that use limit orders, NYSE and Euronext Paris. The sample consists of the most traded 30 corporations on each market, included in the structure of the market indexes DJIA and CAC40. Actually, all corporations composing the American index DJIA, have been encompassed in our database. The period under analysis covers approximately one year between May 2, 2016 and June 1, 2017. On the American market, the US 10-Year Treasury yield is used as a proxy for the risk-free interest rate. On the European market, the Germany 10-Year Treasury yield is used as a proxy for the risk-free interest rate. The database covers daily opening and closing prices, highest and lowest prices, ask and bid quotes at the closing trading day. For both markets, the same number of observations is used in estimations.

As the trading cost model is based on the hypothesis that the prices are observed continuously and considering that the geometric Brownian motion characterizes the stochastic dynamic of stocks prices, the variances (σ^2) of the prices are computed for all corporations included in the sample, according to the methodology indicated by Parkinson (1980) and Corwin and Schultz (1980). In order to determine the time series of variances, the variance is estimated in rolling window using a length $N = 60$ observations, as follows

$$\widehat{\sigma}_t^2 = \frac{1}{\theta}\left\{\frac{1}{N}\sum_{i=t-N+1}^{t}\left[\ln\left(\frac{P_i^H}{P_i^L}\right)\right]^2\right\}, \tag{42}$$

where P_i^H is the highest price in day i, P_i^L is the lowest price in the same trading day i, and $\theta = 4\ln 2$ is a correction factor. Furthermore, the variances are annualized and the values of the parameters γ and the endogenous values Φ are computed.

- *Please insert Table* I *about here* -

Table I reports the descriptive statistics of the prices and parameters of the model. Panel A presents the statistics for the series of mean values of prices and parameters of all corporations. On the other hand, panel B reports statistics for the series of last values of variables for each corporation (i.e., values recorded in the last trading day of the analyzed period). It can be observed that both the bid-ask spread and the volatility are higher on Euronext Paris, than on NYSE. On the other hand the γ parameter is higher than 0.5 on NYSE, and lower than 0.5 on Euronext Paris. This result is correlated with the difference between the endogenous underlying value Φ and the midquote M, which is very small and positive on NYSE and negative on Euronext Paris. This is explainable, as the high volatility and the low interest rate on Euronext Paris lead to endogenous underlying values which are closer to the bid prices, while the situation is reverse on NYSE, where the endogenous underlying values are closer to the ask prices. In consequence, the endogenous underlying value in relation to the bid and ask prices may offer valuable information about the liquidity impact on the evolution of the stock market. Unlike the midquote, the endogenous underlying value explains how liquidity conditions on the market exercise more pressure on the buy price than on the sell price or vice versa.

B. *Illiquidity Premium*

One of the implications of determining an endogenous underlying value of the stock is that it leads to an illiquidity premium under the terms of assets pricing models. Therefore, for each stock i, the following models are estimated

$$\begin{cases} R_{it}^{\phi} - C_{it}^{l} = \alpha_i + \beta_i R_{Mt} + \epsilon_{it} \\ R_{it}^{\phi} = \alpha_i^{\phi} + \beta_i^{\phi} R_{Mt} + \epsilon_{it}^{\phi} \\ R_{it}^{\phi} - C_{it}^{l} - r_t = \alpha_i^{a} + \beta_i^{a} \left(R_{Mt} - r_t\right) + \epsilon_{it}^{a} \end{cases}, \tag{43}$$

where R_{Mt} is the market return in day t, computed based on daily closing values of the market index. According to the definitions, the underlying returns R_{it}^{ϕ} and the illiquidity costs C_{it}^{l} are computed based on daily best bid and best ask quotations recorded at the closing of the market and on daily values of the volatility and of the risk-free interest rate. For each stock i, the β_i^l coefficient is computed as the difference between beta coefficients β_i^{ϕ} and β_i.

- Please insert Table II about here -

Table II reports the estimated results for the most liquid 30 stocks traded on NYSE and on Euronext Paris, respectively. For all stocks on both markets the beta coefficients, β_i^{ϕ} and β_i, are significantly different from zero. According to t-statistic, two stocks listed on NYSE and only one stock listed on Euronext Paris are characterized by a Jensen's alpha different from zero.

The illiquidity premiums are computed based on the estimated coefficients. Illiquidity premiums are significantly lower for the stocks trading on NYSE than for the stocks trading on Euronext Paris. On the French market, the investors demand for a higher liquidity risk premium. Moreover, the results also show that the weights of illiquidity premiums in total risk premiums are higher for stocks trading on Euronext Paris, than on NYSE. On Euronext Paris, for several stocks, the weights of illiquidity premiums in total risk premiums are higher than the weights of underlying risk premiums in total risk premiums. On NYSE, this happens only for a single stock, while for the rest of the stocks the weights of illiquidity premiums in total risk premiums are always bellow half. On average, the values of the expected gross returns are higher on Euronext Paris than on NYSE, and so are the empirical mean returns (measured based on trading prices). The underlying risk premiums are significantly lower for the stocks trading on NYSE than for the stocks trading on Euronext Paris. Only few French stocks have the underlying risk premium below the average of the risk premiums recorded on NYSE.

A positive illiquidity beta coefficient (β_i^l) requires a lower expected gross return because the illiquidity premium becomes implicitly smaller. A negative β_i^l coefficient induces the opposite effect. Regardless of whether the β_i^l coefficients are positive or negative, they are very small and closer to zero for all the stocks analyzed on both markets. Generally, on both markets, the co-movements between the market excess returns and the illiquidity of individual stocks are quite weak. On Euronext Paris, these co-movements are higher than on NYSE, but the differences are very small. Actually, on both markets, the illiquidity premium is composed of the expected illiquidity cost in a very high proportion.

C. Bid-Ask Spread Estimation

Based on Proposition 5, the bid-ask spread ($\widehat{S}$) is estimated for each stock traded on NYSE and on Euronext Paris, as follows

$$\widehat{S}_t = \left| \frac{\sum_{i=t-N+1}^{t} R_i \Psi_i}{\sum_{i=t-N+1}^{t} \Psi_i^2} \right|, \tag{44}$$

where N is the number of observations. The number of observations is set to be equal to the number of observations used in estimating the volatility ($N = 60$). The moment t is the last trading day of the analyzed period. The bid-ask spread estimator is computed using daily continuously compounded returns, daily risk-free interest rates and daily stock price volatilities. Once the estimated value of the bid-ask spread ($\widehat{S}_t$) is known, it is used in estimating the quoted spread ($\widehat{S}_{qt}$) and the effective spread ($\widehat{S}_{et}$). Thus, based on relation

(21) and using the approximation given in (39), it follows that

$$\widehat{S}_{qt} = e^{-\gamma \frac{\widehat{S}_t}{P_t}} \left(e^{\frac{\widehat{S}_t}{P_t}} - 1 \right). \tag{45}$$

Based on relation (24), the effective spread estimator at ask price is given by

$$\widehat{S}_{et}^{A} = e^{(1-\gamma)\frac{\widehat{S}_t}{P_t}} - 1. \tag{46}$$

Similarly, based on relation (25), the estimator of the effective spread at bid price is defined by

$$\widehat{S}_{et}^{B} = 1 - e^{-\gamma \frac{\widehat{S}_t}{P_t}}. \tag{47}$$

The values presented in table III show that on Euronext Paris the average of relative effective spreads and relative quoted spreads is higher than on NYSE for the stocks included in the database while the average of the estimated bid-ask spreads is approximately the same on both markets. On NYSE, the proportion of the effective spread at the bid price in the quoted spread was on average 76.96 %, meanwhile on Euronext Paris the proportion was on average 19.24 %. The proportion of the effective spread at the ask price in the quoted spread equaled on average 23.03 % on NYSE and 80.75 % on Euronext Paris. Estimates show that on both stock exchanges, the transactions corresponding to buy and sell orders have asymmetric effects on the bid-ask spread. They are not equidistantly reflected in the formation of the bid-ask spread.

- *Please insert Table* III *about here* -

On the French market, the buy and sell orders are unbalanced in relation to the bid-ask spread on the market. The endogenous underlying value of stocks is much closer to the bid price than to the ask price. Thus, the liquidity is much more consumed by buy market orders than by sell market orders. The increasing difference between the best ask price and the endogenous underlying value and the decreasing difference between the endogenous underlying value and the best bid price indicate a larger illiquidity cost from dried up liquidity for buy trades than for sell trades. Impatient sellers obtain a trading price much closer to the fundamental value than impatient buyers. As counterpart, patient buyers obtain a trading price much closer to the fundamental value than patient sellers.

Therefore, if the fundamental value is closer to the bid price, the market price will be downward adjusted which, for a seller is a signal to submit a market order to get immediacy, while, for a buyer is a signal to place a limit order in order to obtain a better price. These impatient market exit behavior and patient market enter behavior on the French market are also explained by a negative expected risk-neutral return ($r - \sigma^2/2 < 0$).

Investors' behavior on the American market is opposite to investors' behavior on the French market. The endogenous underlying value of stocks is much closer to the ask price than to the bid price. Therefore, the market price is upward adjusted. Sellers become patient submitting limit orders, while buyers become impatient submitting market orders. These impatient market enter behavior and patient market exit behavior occur when the expected continuously compounded return is positive ($r - \sigma^2/2 > 0$). In consequence, on both markets, the pressure to buy and sell securities is disproportionately reflected in the bid-ask spread in relation to the endogenous underlying value of stocks.

VIII. Conclusion

I derive a model for estimating the fundamental value of the stock. This estimate is endogenously derived based on the valuation of the trading cost function associated to a limit order. The trading cost is caused by the winner's curse phenomenon and the adverse selection effect. The formation mechanism of the trading cost associated to a buy limit order is described by a perpetual American barrier put option, meanwhile the formation mechanism of the trading cost associated to a sell limit order is described by a perpetual American barrier call option.

The endogenous underlying value of the stock is deduced as a weighted average of the ask and bid prices. The weights are functions of the security's price volatility and the risk-free interest rate. The endogenous underlying value is an alternative to the use of the mid-quote price in constructing various illiquidity measures. Thus, immediate implications on liquidity measuring are disclosed. First, I redefine the relative quoted spread, the relative effective spread and the realized spread. Second, through the revisited liquidity-adjusted CAPM, I deduce the illiquidity premium by using the gross yield based on the estimator of the fundamental value. Third, based on the endogenous relationship between net and gross yields, I propose a new estimator of the bid-ask spread.

By analyzing the estimated bid-ask spread, it follows that on the French stock exchange, as opposed to the American stock exchange, there is a different aggregate strategy of investors to place limit orders and to provide liquidity for buy trades against sell trades. The empirical results indicate an impatient market enter behavior and a patient market exit behavior on the American market, as opposed to an impatient market exit behavior and a patient market enter behavior on the French market.

APPENDIX

A Proof of Proposition 1

The proof is based on the following theorem which gives the probability of the first passage time.

Theorem 1. *Let $\widetilde{W}_t$ be a Brownian motion under the probability measure Q. Let μ be a real number. Define the drifted Brownian motion $Y_t = \mu t + \widetilde{W}_t$ and the hitting time $\tau = \min\{t \geq 0;\ Y_t = k\}$, where k is a positive real number. Then,*

$$Q(\tau \geq t) = N\left(\frac{k-\mu t}{\sqrt{t}}\right) - e^{2\mu q} N\left(\frac{-k-\mu t}{\sqrt{t}}\right), \tag{A.1}$$

where $N(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du$ is the standard cumulative normal distribution.

Dana and Jeanblanc (2007) give an explicit proof of this theorem. According to the model assumptions, the initial time of the price stochastic process is t and the initial price can be the best ask A_t or the best bid B_t. The limit order is submitted instantaneously with the transaction at the initial price. Let us consider that the trading price follows a geometric Brownian motion (4), under the initial price $P_t = B_t$. According to Itô's Lemma, for $T \geq t$, it results the drifted Brownian motion

$$\frac{1}{\sigma} \ln \frac{P_T}{B_t} = \frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right)(T-t) + \left(\widetilde{W}_T - \widetilde{W}_t\right). \tag{A.2}$$

If $P_\tau = \overline{X} > B_t$, then $\ln\left(\overline{X}/B_t\right) > 0$. The parameters μ and k are given by

$$\begin{cases} k = \frac{1}{\sigma} \ln \frac{\overline{X}}{B_t} > 0 \\ \mu = \frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right) \end{cases}. \tag{A.3}$$

By replacing the above expressions of μ and k in relation (A.1),

$$Q(\tau \geq T) = N(-z_1) - \left(\frac{\overline{X}}{B_t}\right)^{\frac{2r}{\sigma^2}-1} N(-z_2), \tag{A.4}$$

where the term z_1 is given by $z_1 = \left[\ln\left(B_t/\overline{X}\right) + (r - \sigma^2/2)(T-t)\right]/\sigma\sqrt{(T-t)}$ and the term z_2 is given by $z_2 = \left[\ln\left(\overline{X}/B_t\right) + (r-\sigma^2/2)(T-t)\right]/\sigma\sqrt{(T-t)}$.

If $P_\tau = \overline{X} < B_t$, then $\ln\left(\overline{X}/B_t\right) < 0$. Now, the parameter k is defined by

$$k = -\frac{1}{\sigma}\ln\frac{\overline{X}}{B_t} > 0. \tag{A.5}$$

By rearranging the terms in drifted Brownian motion, it results

$$-\frac{1}{\sigma}\ln\frac{\overline{X}}{B_t} = -\frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right)(T-t) - \left(\widetilde{W}_T - \widetilde{W}_t\right). \tag{A.6}$$

In order to apply the above theorem, the drift is identified as follows:

$$\mu = -\frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right). \tag{A.7}$$

Because $-\widetilde{W}_t$ and $\widetilde{W}_t$ are Brownian motions defined under the same probability measure Q, by replacing the above definitions of μ and k in relation (A.1),

$$Q(\tau \geq T) = N(z_1) - \left(\frac{\overline{X}}{B_t}\right)^{\frac{2r}{\sigma^2}-1} N(z_2). \tag{A.8}$$

If $P_\tau = \overline{X} = B_t$, then $k = 0$. By replacing this value in (A.1), $Q(\tau \geq T) = 0$. Thus, at the order submission time, the probability that the bid trading price hits $\overline{X}$ for the first time under initial condition $P_t = B_t$ is defined by

$$Q(\tau \geq T) = \begin{cases} N(-z_1) - \left(\frac{\overline{X}}{B_t}\right)^{\frac{2r}{\sigma^2}-1} N(-z_2) & \text{if } \overline{X} > B_t \\ 0 & \text{if } \overline{X} = B_t \\ N(z_1) - \left(\frac{\overline{X}}{B_t}\right)^{\frac{2r}{\sigma^2}-1} N(z_2) & \text{if } \overline{X} < B_t \end{cases}. \tag{A.9}$$

Under initial condition $P_t = A_t$, the probability of the first passage time is similarly derived. At the limit order submission time t, the probability that the ask trading price hits $\overline{X}$ for the first time is given by

$$Q(\tau \geq T) = \begin{cases} N(-z_3) - \left(\frac{\overline{X}}{A_t}\right)^{\frac{2r}{\sigma^2}-1} N(-z_4) & \text{if } \overline{X} > A_t \\ 0 & \text{if } \overline{X} = A_t \\ N(z_3) - \left(\frac{\overline{X}}{A_t}\right)^{\frac{2r}{\sigma^2}-1} N(z_4) & \text{if } \overline{X} < A_t \end{cases}, \tag{A.10}$$

where the term z_3 is defined by $z_3 = \left[\ln\left(A_t/\overline{X}\right) + \left(r - \sigma^2/2\right)(T-t)\right]/\sigma\sqrt{T-t}$ and the term z_4 is defined by $z_4 = \left[\ln\left(\overline{X}/A_t\right) + \left(r - \sigma^2/2\right)(T-t)\right]/\sigma\sqrt{T-t}$.

Based on relations (A.9) and (A.10), the hitting time probability $Q_b(\tau \geq T)$ is defined considering all $\overline{X} \leq K_b < A_t$, while the hitting time probability $Q_a(\tau \geq T)$ is defined considering all $\overline{X} \geq K_a > B_t$. By letting $T \to \infty$ and taking into account the function $sgn\left(r - \sigma^2/2\right)$ and the functions $\min\left(\overline{X}, K_b\right)$ and $\max\left(\overline{X}, K_a\right)$, the probabilities $Q_b(\tau = \infty)$ and $Q_a(\tau = \infty)$ that the trading price will not hit the level $\overline{X}$ are derived.

B Proof of Proposition 2

The proof is based on the following theorem of Laplace transform for the first passage time.

Theorem 2. *Let $\widetilde{W}_t$ be a Brownian motion under the probability measure Q. Let μ be a real number. Define the drifted Brownian motion $Y_t = \mu t + \widetilde{W}_t$ and the hitting time $\tau = \min\{t \geq 0; Y_t = k\}$, where k is a positive real number. Then,*

$$E_Q\left[e^{-\lambda\tau}\right] = e^{-k\left(-\mu+\sqrt{\mu^2+2\lambda}\right)}, \tag{B.1}$$

where λ is a real number.

Shreve (2004) and Dana and Jeanblanc (2007) give an explicit proof of this theorem. Let us consider that the trading price follows a geometric Brownian motion (4) with the initial time t, under the initial condition $P_t = B_t$. The buy limit order is submitted simultaneously with the transaction at the initial price. If $P_\tau = \overline{X} > B_t$, then $\ln\left(\overline{X}/B_t\right) > 0$. The positive real parameter k and the drift μ are given by

$$\begin{cases} k = \frac{1}{\sigma}\ln\frac{\overline{X}}{B_t} > 0 \\ \mu = \frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right) \end{cases}. \tag{B.2}$$

By applying the theorem, when $\lambda = r$, it results

$$E_Q\left[e^{-r(\tau-t)}\right] = e^{-k\left(-\mu+\sqrt{\mu^2+2r}\right)} = \frac{B_t}{\overline{X}_b}. \tag{B.3}$$

If $P_\tau = \overline{X} < B_t$, then $\ln\left(\overline{X}/B_t\right) < 0$. Now, k and μ are defined by

$$\begin{cases} k = -\frac{1}{\sigma}\ln\frac{\overline{X}}{B_t} > 0 \\ \mu = -\frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right) \end{cases}. \tag{B.4}$$

Because $-\widetilde{W}_t$ and $\widetilde{W}_t$ are Brownian motions defined under the same probability measure Q, by applying the theorem, when $\lambda = r$, the expected discount factor is defined as

$$E_Q\left[e^{-r(\tau-t)}\right] = e^{-k\left(-\mu+\sqrt{\mu^2+2r}\right)} = \left(\frac{B_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}}. \tag{B.5}$$

Under the $\overline{X} = B_t$ condition ($k = 0$), by applying the Laplace transform theorem for the first passage time, $E_Q\left[e^{-r(\tau-t)}\right] = 1$. Thus, the expected discount factor is defined by

$$E_Q\left[e^{-r(\tau-t)}\right] = \begin{cases} \frac{B_t}{\overline{X}} & \text{if } \overline{X} \geq B_t \\ \left(\frac{B_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}} & \text{if } \overline{X} < B_t \end{cases}. \tag{B.6}$$

Under the initial condition $P_t = A_t$, if $P_\tau = \overline{X} < A_t$, then $\ln\left(\overline{X}/A_t\right) < 0$.[11] Now, k and μ are defined by

$$\begin{cases} k = -\frac{1}{\sigma}\ln\frac{\overline{X}}{A_t} > 0 \\ \mu = -\frac{1}{\sigma}\left(r - \frac{1}{2}\sigma^2\right) \end{cases}. \tag{B.7}$$

Henceforth, the expected discount factor is defined as

$$E_Q\left[e^{-r(\tau-t)}\right] = e^{-k\left(-\mu+\sqrt{\mu^2+2r}\right)} = \left(\frac{A_t}{\overline{X}}\right)^{-\frac{2r}{\sigma^2}}. \tag{B.8}$$

In relation (5), the expected discount factor is replaced with the above results, in compliance with the ordering relationship between $\overline{X}$, B_t, A_t and K_b. and the trading cost function is obtained.

C Proof of Proposition 4

From the first condition of the endogenous zero trading cost (19), by adding and subtracting the two limits, the following system in Φ_t and L_t results

$$\begin{cases} |\Phi_t - L_t|\,\frac{B_t}{\Phi_t} - |\Phi_t - L_t|\left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}} = 0 \\ |\Phi_t - L_t|\,\frac{B_t}{\Phi_t} + |\Phi_t - L_t|\left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}} = 0 \end{cases}. \tag{C.1}$$

The system can be rewritten as

$$\begin{cases} |\Phi_t - L_t|\left[\frac{B_t}{\Phi_t} - \left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}}\right] = 0 \\ |\Phi_t - L_t|\left[\frac{B_t}{\Phi_t} + \left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}}\right] = 0 \end{cases}. \tag{C.2}$$

[11] Because $\overline{X} < K_b$, the case $P_\tau = \overline{X} > A_t$ is impossible as $K_b < A_t$.

The first equation from the condition (C.2) leads to the following system of equations satisfied by Φ_t

$$\begin{cases} \Phi_t - L_t = 0 \\ \frac{B_t}{\Phi_t} - \left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}} = 0 \end{cases}. \tag{C.3}$$

The system of equations has following solutions

$$\begin{cases} \Phi_t = L_t \\ \Phi_t = A_t^{\gamma} B_t^{1-\gamma} \end{cases}, \tag{C.4}$$

where $\gamma = r/\left(r + \sigma^2/2\right)$. The second equation from the condition (C.2) only implies that

$$\Phi_t - L_t = 0 \tag{C.5}$$

because

$$\frac{B_t}{\Phi_t} + \left(\frac{A_t}{\Phi_t}\right)^{-\frac{2r}{\sigma^2}} > 0. \tag{C.6}$$

The above equation (C.5) should be satisfied by both solutions (C.4). When $\Phi_t = L_t$ the equation is obviously verified. When $\Phi_t = A^{\gamma} B^{1-\gamma}$, it results that $L_t = A_t^{\gamma} B_t^{1-\gamma}$.

Thus, the endogenous underlying value of stocks is given by

$$\Phi_t = L_t = A_t^{\gamma} B_t^{1-\gamma}, \tag{C.7}$$

and the final result (20) is derived.

From the second condition of the endogenous zero trading cost (19) it simply results that $\Phi_t = L_t$.

D Proof of Proposition 5

The log endogenous underlying value of stocks is supposed to follow a random walk

$$R_t^{\phi} = \ln \Phi_t - \ln \Phi_{t-1} = \varepsilon_t, \tag{D.1}$$

where $E\left[\varepsilon_t\right] = 0$ and $E\left[\varepsilon_t \varepsilon_s\right] = 0$ for all $t \neq s$. The equation (40) can be written as a system of linear regressions

$$\begin{cases} R_t = \Psi_t S + \varepsilon_t \\ R_t = -\Psi_t S + \varepsilon_t \end{cases}. \tag{D.2}$$

Using ordinary least squares estimation, $\widehat{S}$ is given by

$$\widehat{S} = \pm\frac{\overline{R_t\Psi_t}}{\overline{\Psi_t^2}}. \tag{D.3}$$

In consequence, the positive solution is of interest ($\widehat{S} > 0$)

$$\widehat{S} = \begin{cases} \frac{\overline{R_t\Psi_t}}{\overline{\Psi_t^2}} & \text{if } \overline{R_t\Psi_t} > 0 \\ -\frac{\overline{R_t\Psi_t}}{\overline{\Psi_t^2}} & \text{if } \overline{R_t\Psi_t} < 0 \end{cases} = \left|\frac{\overline{R_t\Psi_t}}{\overline{\Psi_t^2}}\right|. \tag{D.4}$$

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TABLE I: Descriptive Statistics on Prices and Parameters

	NYSE				Euronext Paris			
	γ	σ^2	$\Phi - M$	*Spread*	γ	σ^2	$\Phi - M$	*Spread*
Panel A. Statistics on Mean Values of Variables								
Mean	0.73	0.01	0.00	0.01	0.15	0.03	-0.01	0.05
25^{th} Perc.	0.68	0.01	0.00	0.01	0.10	0.02	-0.02	0.01
Median	0.73	0.01	0.00	0.01	0.15	0.03	-0.01	0.03
75^{th} Perc.	0.77	0.02	0.00	0.01	0.18	0.04	-0.00	0.07
St. Dev.	0.06	0.00	0.00	0.00	0.04	0.01	0.01	0.05
Panel B. Statistics on Last Trading Day Values of Variables								
Mean	0.77	0.01	0.00	0.01	0.19	0.02	-0.01	0.05
25^{th} Perc.	0.73	0.00	0.00	0.01	0.14	0.01	-0.01	0.01
Median	0.79	0.01	0.00	0.01	0.19	0.02	-0.01	0.05
75^{th} Perc.	0.83	0.01	0.00	0.02	0.24	0.03	-0.00	0.09
St. Dev.	0.07	0.00	0.00	0.02	0.06	0.01	0.01	0.05

The table reports descriptive statistics for several variables: the gamma parameter, the annualized variance, the difference between the endogenous underlying value and the midquote, and the observed bid-ask spread at the closing day. The variables are daily computed for all 30 component stocks of DJIA and for 30 component stocks of CAC40.

TABLE II: Revisited Liquidity-adjusted CAPM Estimations

NYSE					Euronext Paris				
Co.	β_i^ϕ (t-stat)	$E\left[R_i^\phi\right]$ %	IP_i %	UP_i %	Co.	β_i^ϕ (t-stat)	$E\left[R_i^\phi\right]$ %	IP_i %	UP_i %
KO	0.23	0.05	0.02	0.01	RI	0.44	0.14	0.09	0.04
	(2.06)		[55.51]	[44.49]		(5.00)		[66.91]	[33.09]
PG	0.27	0.04	0.01	0.02	VIE	0.45	0.13	0.08	0.04
	(2.05)		[34.84]	[65.16]		(2.96)		[63.75]	[36.25]
VZ	0.34	0.05	0.02	0.02	OR	0.52	0.12	0.06	0.05
	(2.07)		[42.26]	[57.74]		(6.29)		[53.62]	[46.38]
WMT	0.36	0.05	0.01	0.03	PUB	0.56	0.14	0.08	0.06
	(2.47)		[34.34]	[65.66]		(4.76)		[59.06]	[40.94]
MCD	0.47	0.05	0.01	0.04	SW	0.57	0.17	0.11	0.06
	(3.94)		[21.49]	[78.51]		(6.23)		[65.52]	[46.11]
JNJ	0.50	0.06	0.01	0.04	EI	0.68	0.15	0.08	0.07
	(4.15)		[22.03]	[77.97]		(4.37)		[53.89]	[46.11]
DIS	0.62	0.07	0.01	0.05	CA	0.70	0.12	0.04	0.07
	(5.59)		[17.65]	[82.35]		(6.17)		[39.29]	[60.71]
TRV	0.64	0.07	0.01	0.05	CAP	0.71	0.14	0.06	0.07
	(5.04)		[19.89]	[80.11]		(5.74)		[46.34]	[53.66]
NKE	0.72	0.08	0.01	0.06	ML	0.75	0.19	0.10	0.08
	(3.89)		[23.99]	[76.01]		(8.24)		[57.44]	[42.56]
AAPL	0.73	0.08	0.01	0.06	ORA	0.77	0.13	0.05	0.08
	(4.83)		[19.78]	[80.22]		(8.02)		[39.35]	[60.65]
CSCO	0.76	0.10	0.03	0.06	KER	0.79	0.19	0.10	0.08
	(4.77)		[34.74]	[65.26]		(5.87)		[55.48]	[44.52]
MMM	0.79	0.08	0.01	0.06	SAN	0.81	0.12	0.03	0.08
	(8.61)		[13.80]	[86.20]		(8.03)		[31.29]	[68.71]
MRK	0.81	0.09	0.01	0.07	ENGI	0.86	0.16	0.07	0.09
	(4.74)		[19.20]	[80.80]		(6.17)		[43.23]	[56.77]
MSFT	0.86	0.10	0.02	0.07	FP	0.88	0.12	0.03	0.09
	(6.52)		[22.79]	[77.21]		(11.42)		[26.73]	[73.27]
UTX	0.87	0.09	0.01	0.07	AC	0.89	0.18	0.08	0.09
	(8.73)		[14.13]	[85.87]		(7.73)		[46.88]	[53.12]
INTC	0.88	0.11	0.03	0.07	RNO	0.89	0.14	0.04	0.09
	(5.55)		[29.87]	[70.13]		(6.46)		[34.14]	[65.86]
PFE	0.88	0.11	0.03	0.07	AI	0.89	0.17	0.07	0.09
	(5.76)		[29.49]	[70.51]		(12.27)		[43.93]	[56.07]
V	0.89	0.09	0.01	0.07	MC	0.91	0.17	0.07	0.09
	(6.56)		[15.49]	[84.51]		(10.25)		[42.61]	[57.39]

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HD	0.89	0.09	0.01	0.07	STM	0.96	0.24	0.14	0.10
	(6.96)		[13.49]	[86.51]		(4.61)		[58.08]	[41.92]
CVX	0.93	0.09	0.01	0.08	EN	1.01	0.18	0.07	0.10
	(6.79)		[13.00]	[87.00]		(9.54)		[40.78]	[59.22]
XOM	0.93	0.10	0.01	0.08	VIV	1.03	0.18	0.07	0.11
	(7.77)		[14.99]	[85.01]		(7.08)		[39.01]	[60.99]
GE	0.94	0.12	0.03	0.08	AIR	1.06	0.16	0.05	0.11
	(7.91)		[30.21]	[69.79]		(9.45)		[30.42]	[69.58]
UNH	0.97	0.10	0.01	0.08	FR	1.14	0.21	0.08	0.12
	(5.87)		[13.62]	[86.38]		(9.83)		[42.04]	[57.96]
IBM	1.06	0.10	0.01	0.09	LHN	1.16	0.27	0.14	0.12
	(8.12)		[10.54]	[89.46]		(8.71)		[54.37]	[45.63]
DWDP	1.18	0.12	0.01	0.10	UG	1.19	0.20	0.07	0.12
	(7.89)		[13.58]	[86.42]		(8.95)		[37.05]	[62.95]
BA	1.26	0.12	0.01	0.10	SGO	1.29	0.19	0.05	0.13
	(9.70)		[9.61]	[90.39]		(15.44)		[28.62]	[71.38]
AXP	1.29	0.13	0.01	0.11	CS	1.53	0.21	0.05	0.16
	(7.46)		[11.54]	[88.46]		(14.10)		[24.13]	[75.87]
JPM	1.85	0.17	0.01	0.15	ACA	1.72	0.24	0.06	0.18
	(14.21)		[8.29]	[91.71]		(13.19)		[25.41]	[74.59]
CAT	2.00	0.19	0.01	0.17	BNP	1.93	0.25	0.05	0.20
	(10.57)		[7.92]	[92.08]		(16.56)		[19.66]	[80.34]
GS	2.22	0.21	0.02	0.19	GLE	2.03	0.26	0.04	0.21
	(13.14)		[10.24]	[89.76]		(14.46)		[18.46]	[81.54]

All corporations are identified by their market symbol. The table exhibits the underlying beta coefficient. The t-statistics are reported in parentheses. The critical values at 1% and 5% are 2.6063 and 1.9746, respectively. The expected gross return, the illiquidity premium and the underlying risk premium are also reported. The weight of the illiquidity premium (%) and the weight of the underlying risk premium (%) in total risk premium are reported between square brackets.

TABLE III: Bid-Ask Spread Estimates

NYSE					Euronext Paris				
Co.	$\widehat{S}_t$	95% CI	$\widehat{S}_{et}^A$	$\widehat{S}_{et}^B$	Co.	$\widehat{S}_t$	95% CI	$\widehat{S}_{et}^A$	$\widehat{S}_{et}^B$
KO	0.06	[0.01; 0.10]	0.02	0.12	RI	0.22	[0.01; 0.43]	0.14	0.06
	(0.02)		{15.50}	{84.50}		(0.10)		{69.65}	{30.35}
PG	0.03	[0.00; 0.15]	0.00	0.03	VIE	0.05	[0.00; 0.11]	0.29	0.05
	(0.06)		{14.74}	{85.26}		(0.02)		{84.00}	{16.00}
VZ	0.04	[0.00; 0.15]	0.02	0.07	OR	0.25	[0.00; 0.56]	0.10	0.04
	(0.05)		{23.09}	{76.91}		(0.15)		{71.53}	{28.47}
WMT	0.15	[0.02; 0.29]	0.05	0.17	PUB	0.10	[0.00; 0.27]	0.12	0.03
	(0.06)		{24.55}	{75.45}		(0.08)		{76.80}	{23.20}
MCD	0.37	[0.11; 0.64]	0.05	0.24	SW	0.29	[0.05; 0.54]	0.19	0.08
	(0.13)		{17.57}	{82.43}		(0.12)		{69.38}	{30.62}
JNJ	0.07	[0.00; 0.30]	0.01	0.05	EI	0.18	[0.00; 0.48]	0.14	0.03
	(0.11)		{19.04}	{80.96}		(0.15)		{82.10}	{17.90}
DIS	0.04	[0.00; 0.24]	0.00	0.03	CA	0.00	[0.00; 0.07]	0.02	0.00
	(0.09)		{17.65}	{82.35}		(0.03)		{79.16}	{20.84}
TRV	0.05	[0.00; 0.23]	0.00	0.03	CAP	0.18	[0.00; 0.45]	0.17	0.04
	(0.08)		{18.15}	{81.85}		(0.13)		{78.90}	{21.10}
NKE	0.07	[0.00; 0.25]	0.04	0.08	ML	0.15	[0.00; 0.46]	0.11	0.03
	(0.09)		{33.31}	{66.69}		(0.15)		{77.50}	{22.50}
AAPL	0.22	[0.00; 0.55]	0.02	0.13	ORA	0.01	[0.00; 0.06]	0.10	0.03
	(0.16)		{17.69}	{82.31}		(0.02)		{77.00}	{23.00}
CSCO	0.04	[0.00; 0.14]	0.02	0.09	KER	1.04	[0.05; 2.03]	0.34	0.10
	(0.05)		{19.87}	{80.13}		(0.49)		{76.53}	{23.47}
MMM	0.24	[0.00; 0.52]	0.01	0.10	SAN	0.09	[0.00; 0.28]	0.09	0.02
	(0.13)		{15.34}	{84.66}		(0.09)		{79.16}	{20.84}
MRK	0.01	[0.00; 0.12]	0.00	0.01	ENGI	0.01	[0.00; 0.05]	0.09	0.02
	(0.05)		{29.42}	{70.58}		(0.02)		{81.26}	{18.74}
MSFT	0.09	[0.00; 0.21]	0.02	0.11	FP	0.01	[0.00; 0.12]	0.01	0.00
	(0.06)		{20.35}	{79.65}		(0.05)		{73.66}	{26.34}
UTX	0.15	[0.00; 0.33]	0.02	0.11	RNO	0.01	[0.00; 0.30]	0.01	0.00
	(0.08)		{18.98}	{81.02}		(0.14)		{88.47}	{11.53}
INTC	0.00	[0.00; 0.08]	0.00	0.01	AI	0.11	[0.00; 0.36]	0.09	0.03
	(0.03)		{27.57}	{72.43}		(0.12)		{75.52}	{24.48}
PFE	0.02	[0.00; 0.06]	0.01	0.04	AC	0.09	[0.00; 0.22]	0.21	0.05
	(0.02)		{27.26}	{72.74}		(0.06)		{81.09}	{18.91}
V	0.10	[0.00; 0.23]	0.02	0.09	MC	0.62	[0.06; 1.18]	0.24	0.07
	(0.06)		{17.99}	{82.01}		(0.28)		{75.65}	{24.35}

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HD	0.15	[0.00; 0.40]	0.02	0.08	STM	0.00	[0.00; 0.07]	0.02	0.00
	(0.12)		{19.99}	{80.01}		(0.03)		{91.74}	{8.26}
CVX	0.12	[0.00; 0.38]	0.02	0.08	EN	0.02	[0.00; 0.14]	0.04	0.01
	(0.13)		{22.36}	{77.64}		(0.06)		{80.68}	{19.32}
XOM	0.03	[0.00; 0.18]	0.00	0.02	VIV	0.05	[0.00; 0.12]	0.26	0.06
	(0.07)		{22.74}	{77.26}		(0.03)		{81.18}	{18.82}
GE	0.03	[0.00; 0.10]	0.02	0.10	AIR	0.07	[0.00; 0.29]	0.08	0.01
	(0.03)		{18.63}	{81.37}		(0.10)		{81.68}	{18.32}
UNH	0.17	[0.00; 0.52]	0.02	0.07	FR	0.07	[0.00; 0.29]	0.10	0.02
	(0.17)		{26.97}	{73.03}		(0.11)		{83.63}	{16.37}
IBM	0.45	[0.05; 0.84]	0.05	0.19	LHN	0.01	[0.00; 0.22]	0.02	0.00
	(0.19)		{21.38}	{78.62}		(0.10)		{83.55}	{16.45}
DWDP	0.01	[0.00; 0.24]	0.00	0.00	UG	0.02	[0.00; 0.09]	0.12	0.01
	(0.11)		{30.00}	{70.00}		(0.03)		{91.04}	{8.96}
BA	0.08	[0.00; 0.49]	0.01	0.03	SGO	0.08	[0.00; 0.25]	0.15	0.04
	(0.20)		{25.22}	{74.78}		(0.08)		{78.56}	{21.44}
AXP	0.02	[0.00; 0.23]	0.00	0.02	CS	0.00	[0.00; 0.08]	0.02	0.00
	(0.10)		{25.21}	{74.79}		(0.04)		{86.17}	{13.83}
JPM	0.13	[0.00; 0.38]	0.04	0.10	ACA	0.03	[0.00; 0.09]	0.22	0.03
	(0.12)		{27.53}	{72.47}		(0.03)		{88.05}	{11.95}
CAT	0.15	[0.00; 0.54]	0.06	0.09	BNP	0.08	[0.00; 0.36]	0.13	0.01
	(0.19)		{37.94}	{62.06}		(0.14)		{88.66}	{11.34}
GS	0.56	[0.00; 1.43]	0.07	0.14	GLE	0.04	[0.00; 0.29]	0.09	0.00
	(0.43)		{35.12}	{64.88}		(0.12)		{90.25}	{9.75}

The table reports the estimation outputs of bid-ask spread, the effective spread at ask price (%) and the effective spread at bid price (%) for 30 stocks traded on NYSE and for 30 stocks traded on Euronext Paris. All corporations are identified by their market symbol. The standard error of the bid-ask spread estimate is reported between round brackets. The weights (%) of estimated effective spreads in estimated quoted spread are reported between braces. The 95% confidence interval for the estimated bid-ask spread is reported between square brackets.