The Debt Tax Shield, Economic Growth and Inequality*

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Abstract

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We study the general-equilibrium implications of the corporate debt tax shield in a growth economy that taxes household income and firm profits and redistributes tax revenues in an attempt to harmonize the lifetime consumption opportunities among households that differ in their endowments. Our model shows that in general equilibrium the tax shield's reduction in the corporate after-tax borrowing rate is counteracted by an increase in the pre-tax rate. Our model further predicts the debt tax shield to lead to a higher growth rate of the economy and an increase in the degree of disparity in households' lifetime consumption opportunities.

Key Words: debt tax shield, macroeconomic growth, redistributive tax system
JEL Classification Codes: E21, E23, G11, H23, H31, H32

1 Introduction

Departing from the pioneering work of Modigliani and Miller (1958), a huge literature investigates corporate capital structure decisions. Especially the tax-deductibility of corporate interest expenses – the corporate debt tax shield – has caught a lot of attention in both the theoretical and empirical literature. This literature demonstrates that the debt tax shield heavily affects corporate capital structure decisions. However, the macroeconomic implications of the debt tax shield, which our work focuses on, have been largely overlooked so far.

We set up a general-equilibrium model with a representative firm, households that differ by their initial endowments, and a government that taxes household income as well as firm profits and redistributes tax revenues in an attempt to reduce disparities in lifetime consumption opportunities among households. Households earn income by investing into risky corporate equity, risk-free corporate debt, and risk-free bonds traded among the households.

Irrespective of whether a removal of an existing debt tax shield is investigated ceteris paribus or in a tax-neutral way that adjusts the corporate tax rate in such a way that the government's expected tax revenue is unaffected, the removal decreases the risk-free rate. This decrease is affected through both changes in the supply of and demand for corporate debt. When a debt tax shield is removed, a lower degree of corporate leverage is optimal, thus decreasing the supply of securitized corporate debt. Simultaneously, the removal of the debt tax shield makes investments into equity less attractive causing an increase in the demand for investments into corporate debt. To raise the desired amount of debt, the corporation can thus offer a lower interest rate. This reduction in the risk-free rate partly offsets the tax shield removal's increase in the after-tax corporate borrowing rate, but does not completely eliminate it.

The decrease in the risk-free rate leads to further macroeconomic effects. Specifically, it increases the price of future over present consumption, which leads to lower savings and thus, in the end, to a lower growth rate of the economy. Simultaneously, the removal of the debt tax shield leads to a lower degree of disparity in households' lifetime consumption opportunities. A high growth rate of the economy and a reduction of the disparities in lifetime consumption opportunities among households trade off against each other.

An endogenous determination of general equilibrium effects from corporate capital structure is important for a better understanding of the implications of the debt tax shield for the risk-free rate and other macroeconomic variables. If the tax burden on firm profits paid out to households as interest is lower than that paid out as dividend, there is a tax advantage to debt financing, and the firm operates with leverage; otherwise the firm remains unlevered.

Whether such a tax advantage exists depends, among other things, on whether the debt tax shield applies. We model the economy using a representative firm and limit the degree of leverage such that the payout to shareholders is non-negative in all states of the world. This is in accordance with the original work of Modigliani and Miller (1958) and numerous subsequent papers. Although a single firm may be subject to default risk, a default of the representative firm would be unreasonable.

Our work contributes to two important lines of literature. It complements the literature dealing with the macroeconomic implications of taxes. It is a well-known fact that it is generally not optimal to tax accumulating production factors, because this discourages savings, slows down factor accumulation and thus, ultimately, innovation (Mukherjee, Singh, and Zaldokas, 2016) and economic growth (e.g., Chamley, 1986; Diamond, 1975; Eaton and Rosen, 1980; Varian, 1980; Judd, 1985; Gravelle and Kotlikoff, 1995; Jones, Manuelli, and Rossi, 1997).[1] Simultaneously, Hackbarth, Miao, and Morellec (2006) and Chen (2010), among others, demonstrate that macroeconomic conditions affect corporate capital structure decisions. However, the reverse channel, i.e., how the debt tax shield and its effect on corporate capital structure decisions affect macroeconomic conditions, such as the risk-free rate, the growth rate of the economy, or the allocation of resources among households, has received surprisingly little attention so far. Our work investigates these questions in a model with non-uniform taxation of capital income.

Our work further contributes to a growing literature on the implications of the debt tax shield. The debt tax shield has recently caught renewed interest in both theoretical and empirical work. Empirical work estimates that the debt tax shield accounts for about 10% of corporate values (e.g., Graham, 2000; Kemsley and Nissim, 2002; vanBinsbergen, Graham, and Yang, 2010), depicts the evolution of corporate leverage ratios over time (DeAngelo and Roll, 2015; Graham, Leary, and Roberts, 2015) as well as over the business cycle (Korajczyk and Levy, 2003; Halling, Yu, and Zechner, 2016), and documents that taxes in general, and the debt tax shield in particular, significantly affect corporate capital structure decisions (e.g., MacKie-Mason, 1990; Graham, 1996, 1999; Gordon and Lee, 2001; Hovakimian, Opler, and Sheridan, 2001; Bell and Jenkinson, 2002; Graham and Lucker, 2006; Becker, Jacob, and Jacob, 2013; Longstaff and Strebulaev, 2014; Devereux, Maffini, and Xing, 2015; Doidge and Dyck, 2015; Faccio and Xu, 2015; Heider and Ljungqvist, 2015; Faulkender and Smith, 2016; Ljungqvist, Zhang, and Zuo, 2017). Schepens (2016) argues that this makes tax shields a valuable tool for policy makers. However, all these papers focus on the impact of the debt

[1] Boskin (1978), Blanchard and Perotti (2002), Romer and Romer (2010), and Cloyne (2013) provide supporting empirical evidence. Optimal redistribution is studied, among others, in Golosov, Troshkin, and Tsyvinski (2016).

tax shield for corporate valuation and capital structure decisions, but do not investigate the broader macroeconomic implications of the debt tax shield, which is the focus of our work.

Theoretical work, including Miles and Ezzell (1980) and Cooper and Nyborg (2006), has so far primarily focused on the valuation of the debt tax shield. A notable exception is the work of Fischer and Jensen (2018) that investigates how the debt tax shield affects households' consumption-investment strategies via the government's budget constraint. However, their work builds on an endowment economy model. In their framework, the growth rate of the economy is thus exogenously given and the risk-free rate is unaffected by whether a debt tax shield applies, which renders an investigation of the broader macroeconomic implications impossible. Our work instead builds on a production economy and contributes to what Fama (2011) calls one of the big open challenges in financial economics: understanding the implications of corporate taxation. Our model predicts that the debt tax shield significantly affects macroeconomic variables, such as the risk-free rate, inequality, and the growth rate of the economy.

Intuitively, the government could try to increase the growth rate of the economy by investing into the production process itself. However, we show that in our model households can and will undo any effect that such government policy may be intended to have. Hence, there is no scope for such fiscal interventions in our model.

This paper proceeds as follows. Section 2 outlines our model. In section 3, we present its analytical solution and discuss our model's predictions. In section 4, we illustrate the quantitative implications of the debt tax shield for the risk-free rate, economic growth, and inequality. Section 5 concludes. The Online Appendix (currently at the end of this file) provides proofs of our theorems.

2 The debt tax shield in a production economy

2.1 The economy

We consider an economy populated by n households and a representative firm that makes up the production sector. The firm has a risky one-period production technology. That is, it only generates an output in the next period. This output can either be consumed or be reinvested in preparation for consumption in the subsequent periods. The output produced and available at time t depends on the evolution of the economy. It is given by $G_t I_{t-1}^a$, where G_t is the gross growth factor per unit of investment made at time $t-1$ and I_{t-1}^a denotes the aggregate investment in the production technology. For simplicity, we assume that the growth rates are independent and identically distributed copies of a discrete random variable G with M possible realizations G_m, where $G_1 > G_2 > \cdots > G_M > 1$. In the sequel we assume that these M realizations have equal probabilities $1/M$.[2]

Our production technology model is a discrete time version of the classical Cox-Ingersoll-Ross model (Cox, Ingersoll, and Ross, 1985) without intertemporal uncertainty about the production technology. The production technology can, for example, be thought of as farmers growing a perishable output, such as corn, with an identical distribution of the outcome from year to year. The next year's harvest depends on how much of this year's harvest is used for replanting and on the exogenously given realization of the growth rate of the output process, that may, e.g., reflect different weather conditions.

2.2 Corporate leverage

To run the production technology, the firm issues equity and corporate debt that the households can invest into. The aggregate investment, I_t^a, made at time t is financed by the aggregate amount of equity invested, E_t^a, and the aggregate amount of corporate debt, δ_t^a, outstanding from time t to $t+1$. The supply of corporate bonds is usually determined by the individual firm's optimal capital structure decision, and many models provide endogenous mechanisms that bound the degree of leverage in case there is a tax advantage to debt. We follow this tradition and only allow the representative firm to issue bonds up to a limit for which there is no risk of bankruptcy. We follow one of the standard assumptions in the literature and assume that the firm operates with a constant leverage ratio L. This is consistent with our i.i.d. assumption on the dynamics of the production technology. Apart from this constraint on the relation between E_t^a and δ_t^a, the supply of aggregate investment opportunities is perfectly elastic. The use of a constant leverage ratio is often referred to as the Miles-Ezzell assumption (Miles and Ezzell, 1980).

If the total tax burden on firm profits paid out to households as interest on corporate debt is lower than that paid out as dividend to equity holders, i.e., if there is a tax advantage of debt financing, the firm operates with a positive leverage. Otherwise, the firm remains unlevered. Since the total tax burden on firm profits paid out to households as interest depends on whether the debt tax shield applies, corporate leverage should be affected by whether the debt tax shield applies.

[2]The assumption of equal probabilities is solely made to ease notation. Our results can be generalized to allow for unequal probabilities with similar qualitative conclusions, although with a significantly blown-up amount of notation.

2.3 Traded assets

Households can trade three assets. First, households can trade a locally risk-free one-period bond paying a pre-tax return of r_t from time t to $t+1$. We denote household j's position in that asset by $\beta_{t,j}$. This asset comes in zero net supply. That is, if some households want to hold a long position in that asset, the market equilibrium has to bring about an interest rate that makes other households willing to issue such an asset. Second, households can invest into the firm's equity that entitles them to the firm's payout in proportion to their share of equity. We denote household j's investment into the firm's equity from time t to $t+1$ by $E_{t,j}$ and its share of equity by $\alpha_{t,j} = \frac{E_{t,j}}{E_t^a}$. Third, households can invest into one-period corporate bonds, issued by the firm. We denote household j's position in corporate bonds from time t to $t+1$ by $\delta_{t,j}$. Since the firm only issues bonds up to a limit where the net return on equity is non-negative, corporate bonds are default-free, and therefore perfect substitutes for the risk-free bond traded among households. Hence, they bear the same yield. Household j's initial endowment is denoted by $W_{0,j} > 0$ and its share of the total initial endowment $W_0^a = \sum_{j=1}^{n} W_{0,j}$ is denoted by $\alpha_{0-,j} = \frac{W_{0,j}}{W_0^a} > 0$.

2.4 The redistributive tax system

Throughout the last centuries, most industrial nations around the world implemented tax-financed social insurance and income support programs for poorer households to reduce disparities in lifetime consumption opportunities. Romer and Romer (2016) provide an overview of changes in social security benefits in the United States.

We consider a government that wants to reduce the disparity in lifetime consumption opportunities across households that differ by their initial financial endowments. To attain this goal, the government taxes corporate profits at rate τ_C, households' profits from investments into firm equity at rate τ_E, and households' interest income at rate τ_B. The government implements a linear redistributive tax system from which each household receives an identical share of tax revenues. That is, poorer households pay less in taxes than they receive in transfer income. These households are therefore net recipients of transfer income. Richer households, on the other hand, pay more in taxes than they receive in transfers. That is, after accounting for redistributions, the tax system is progressive. So-called linear redistributive tax systems that allocate equal shares of tax revenues to its citizens are commonly used in the public finance literature. Their use ranges back to the work of Romer (1975) and Meltzer and Richard (1981) and has later been used, among others, in Alesina and Angeletos (2005), Sialm (2006), Fischer and Jensen (2015), and Pástor and Veronesi (2016, 2017, 2018).

The redistribution mechanism implies that the government neither builds up wealth nor debt. Within the time horizon of our model, any government debt must be settled through tax payments by the households.[3] Consequently, government debt would never be considered net wealth by the households, cf. also the reasoning in Barro (1974). We provide a formal verification that there is no room for an active fiscal policy in our model in section 3.3.

2.5 The debt tax shield

Debt tax shields for corporate interest expenses exist in many countries to avoid a double-taxation of interest at both the company level and the level of the final recipient of the interest payment.[4] Whether a debt tax shield exists or not directly affects corporate capital structure decisions, because the debt tax shield reduces the after-tax cost of debt, thus making debt-financed investments more desirable.[5] The debt tax shield also directly affects the payout, P_t, to equity holders at time t:

$$P_t = I_{t-1}^a \left(1 + g_t \left(1 - \tau_C\right)\right) - \delta_{t-1}^a \widehat{R}_{t-1}, \tag{1}$$

in which $g_t = G_t - 1$ is the net growth rate of investments into the firm's production technology, $\widehat{R}_{t-1} = 1 + \widehat{r}_{t-1} = 1 + r_{t-1}\left(1 - \widehat{\tau}_C\right)$ is the firm's gross after-tax risk-free rate after accounting for whether the debt tax shield exists or not, and

$$\widehat{\tau}_C = \begin{cases} \tau_C & \text{with the debt tax shield} \\ 0 & \text{without the debt tax shield} \end{cases} \tag{2}$$

is the tax rate applicable to the firm's interest payments.[6] With the constant leverage ratio, $L = \delta_{t-1}^a / E_{t-1}^a$, the payout to shareholders from Equation (1) can be rewritten as

$$P_t = E_{t-1}^a \left[1 + g_t \left(1 - \tau_C\right) + L \left(g_t \left(1 - \tau_C\right) - \widehat{r}_{t-1}\right)\right]. \tag{3}$$

[3] We disregard the possibility that the government can embark on a Ponzi scheme and can ignore its long-run budget constraint.

[4] He and Matvos (2016) even argue that from a macroeconomic perspective it can be optimal to subsidize debt in industries with socially wasteful competition to prepone firm exits.

[5] We assume throughout that tax rates are such that the representative firm operates with corporate debt when the debt tax shield applies. Otherwise, the firm would never operate with debt and whether the debt tax shield applies or not would not have any consequences.

[6] We do not explicitly regard the case, in which interest expenses are deductible, but the tax compensation for deductions is lower than the tax paid on corporate profits ($0 < \widehat{\tau}_C < \tau_C$), throughout. Our model can be readily generalized to this case. We neither consider limitations on the amount of interest expenses that can deducted, such as, e.g., the recent ATA directive (European Commission, 2016) from the European Union.

When the debt tax shield applies, the firm faces lower debt servicing costs, implying a higher amount remaining for its shareholders, which in turn affects the households' demand for equity.

2.6 The household optimization problem

Each household maximizes its present discounted utility from consumption over an N-period investment horizon subject to its intertemporal budget constraint. Our results readily generalize to an infinite investment horizon by letting N go to infinity. We consider the case with a finite N important to highlight the evolution of the endogenous variables over time. By letting N go to infinity, our results readily generalize to the infinite-horizon case. Households have a common utility discount factor ρ and a time-additive constant relative risk aversion (CRRA) utility function with risk aversion parameter $\gamma \geq 0$. That is, the utility from a consumption of C is given by:

$$U(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \ln(C) & \text{if } \gamma = 1. \end{cases} \tag{4}$$

The evolution of household j's wealth after accounting for taxes consists of three components. First, the household receives the payout from its equity investments. After taxation on the household level, this leaves the household with an income of

$$\alpha_{t-1,j}\left(\left(P_t - E^a_{t-1}\right)(1-\tau_E) + E^a_{t-1}\right). \tag{5}$$

Second, the household receives income from its holdings of the risk-free asset and corporate debt of

$$(\beta_{t-1,j} + \delta_{t-1,j})\,\widetilde{R}_{t-1}, \tag{6}$$

in which

$$\widetilde{R}_{t-1} = 1 + r_{t-1}(1-\tau_B) \tag{7}$$

is the gross risk-free rate from time $t-1$ to t, after taxation on the household level. Third, the household receives transfer income, the level of which depends on the government's tax revenues that in turn consist of three components. First, the government generates a tax revenue of $\tau_E\left(P_t - E^a_{t-1}\right)$ by taxing gains from equity investments. Second, the taxation of interest on the household level provides a tax revenue of $\tau_B r_{t-1}\delta^a_{t-1}$. Finally, the government taxes the firm profit, Ω_t:

$$\Omega_t = \begin{cases} I^a_{t-1} g_t - r_{t-1}\delta^a_{t-1} & \text{with the debt tax shield} \\ I^a_{t-1} g_t & \text{without the debt tax shield} \end{cases} \tag{8}$$

at the corporate tax rate τ_C. Total tax revenues are thus given by

$$\tau_E \left(P_t - E^a_{t-1}\right) + \tau_B r_{t-1}\delta^a_{t-1} + \tau_C \Omega_t, \tag{9}$$

of which each household receives an equal share. From Equations (8) and (9), tax revenues are lower with the debt tax shield than without. The evolution of household j's wealth is given by:

$$\begin{aligned} W_{t,j} = & \alpha_{t-1,j}\left(\left(P_t - E^a_{t-1}\right)(1-\tau_E) + E^a_{t-1}\right) + \left(\beta_{t-1,j} + \delta_{t-1,j}\right)\widetilde{R}_{t-1} + \\ & \frac{1}{n}\left(\tau_E\left(P_t - E^a_{t-1}\right) + \left(\tau_B - \widehat{\tau}_C\right) r_{t-1}\delta^a_{t-1} + \tau_C I^a_{t-1} g_t\right). \end{aligned} \tag{10}$$

We denote the effective rate of (double) taxation that the equity return is subject to by $\widetilde{\tau}$:

$$\widetilde{\tau} \equiv \tau_C + \tau_E(1-\tau_C) = \tau_E + \tau_C(1-\tau_E) \;\Leftrightarrow\; 1 - \widetilde{\tau} \equiv (1-\tau_E)(1-\tau_C). \tag{11}$$

Table 1 summarizes the notation used in this paper.
Household j's optimization problem is then given by:

$$\max_{\{C_{t,j}, E_{t,j}, \delta_{t,j}, \beta_{t,j}\}_{t=0}^{t=N}} U\left(C_{0,j}\right) + \sum_{t=1}^{N} \rho^t \mathbb{E}_0\left[U\left(C_{t,j}\right)\right]$$

$$\text{s.t. } W_{t,j} = C_{t,j} + E_{t,j} + \beta_{t,j} + \delta_{t,j}, \quad t = 0,1,2,\ldots,N \tag{12}$$

$$E_{N,j} = \delta_{N,t} = \beta_{N,j} = 0. \tag{13}$$

The households' preference structure ensures that despite that financial markets are incomplete in the sense that a complete set of state-contingent claims cannot be constructed from the given stock and bond market, it is effectively complete in the sense that the conditions for an unconstrained Pareto optimal allocation are satisfied.

Having presented our model, we next turn to its closed-form solution and show how the debt tax shield affects the economy.

Table 1
Definition of variables

Variable	Description
ρ	The households' common utility discount factor
γ	The households' common degree of relative risk aversion
$\alpha_{0-,j}$	Household j's initial endowment
$\alpha_{t,j}$	Household j's share of equity investments in the production process from time t to time $t+1$
$\beta_{t,j}$	Number of risk-free assets, issued by households, that is held by household j from time t to $t+1$
$\delta_{t,j}$	Number corporate bonds held by household j from time t to $t+1$
$E_{t,j}$	Household j's equity investment from time t to time $t+1$
δ_t^a	Number of corporate bonds outstanding from time t to $t+1$
E_t^a	Aggregate equity investment from time t to time $t+1$
I_t^a	Total investment in production process from time t to $t+1$, $I_t^a = E_t^a + \delta_t^a$
L	Firm's constant leverage ratio: $L = \delta_t^a / E_t^a$
$C_{t,j}$	Household j's consumption at time t
C_t^a	Aggregate consumption at time t
τ_E	Tax rate applicable to household income from equity
τ_B	Tax rate applicable to household income from bonds
τ_C	Corporate tax rate
$\widehat{\tau}_C$	Corporate tax rate, applicable to a firm's interest payments
$\widetilde{\tau}$	Total tax rate applicable to a household's equity income: $\widetilde{\tau} = \tau_C + \tau_E(1-\tau_C)$
$\widehat{\tau}$	Tax rate measuring the loss in tax revenues from corporate and equity taxation per unit of equity replaced with debt: $= \widehat{\tau}_C + \tau_E(1-\widehat{\tau}_C)$
ξ, ψ	The relative tax disadvantage of using equity:$=(1-\tau_E)(1-\widehat{\tau}_C)/(1-\tau_B)$
R_t	Gross risk-free rate before taxes from time t to $t+1$
r_t	Net risk-free rate before taxes from time t to $t+1$: $r_t = R_t - 1$
$\widetilde{R}_t$	Gross risk-free rate after taxes on household level from time t to $t+1$
$\widehat{R}_t$	Gross risk-free rate after taxes on corporate level from time t to $t+1$
O_t	Output at time t
Ω_t	Taxable corporate income at time t
P_t	Payout from the firm to equity holders at time t
G_t	Gross growth factor of output O from time $t-1$ to t, $G_t = O_t/O_{t-1}$
G	Version of the independent stochastic gross growth factors G_t
$\{G_j\}_{j=1}^{j=M}$	Outcomes of G: G_1,G_2,$\ldots$,G_M
g_t	Net growth factor of output O from time $t-1$ to t, $g_t = G_t - 1$
$W_{t,j}$	Household j's wealth level at time t, before consumption and investment
n	Number of households in the economy
N	Length of investment horizon in periods

3 Implications of the debt tax shield

In this section, we present the general-equilibrium solution to our model introduced in section 2 in closed form. We impose the following upper bound on the degree of corporate levering:

$$L \leq \frac{g_M}{(\bar{g} - g_M)} \frac{1 - \tau_B}{(1 - \widehat{\tau}_C)(1 - \tau_E)}, \tag{14}$$

in which $g_M = G_M - 1$ is the lowest possible net growth rate of the production technology, and $\bar{g}$ is the expected value of the net growth rate g under the risk-neutral measure. To ensure a positive upper bound on the level of corporate leverage, we assume $g_M > 0$. We provide a formal derivation in the proof of Theorem 1 that this constraint not only ensures the solvency of the firm in all states of the world, but simultaneously also guarantees a non-negative tax base. In the proof of Theorem 1, we further show that the risk-neutral measure is independent of time, tax rates, and whether the debt tax shield applies or not. It is given by

$$q_m = \frac{G_m^{-\gamma}}{\sum_{k=1}^{M} G_k^{-\gamma}}. \tag{15}$$

3.1 Macroeconomic effects

We begin the presentation of our results by turning to the implications of the debt tax shield for the risk-free rate and growth of the economy in Theorem 1. We apply the following measures of the tax burden on equity relative to debt financing (cf. Miller (1977)):

$$\xi \equiv \frac{(1 - \tau_E)(1 - \tau_C)}{1 - \tau_B} = \frac{1 - \widetilde{\tau}}{1 - \tau_B}, \quad \psi \equiv \frac{(1 - \tau_E)(1 - \widehat{\tau}_C)}{1 - \tau_B} = \frac{1 - \widehat{\tau}}{1 - \tau_B}. \tag{16}$$

When the debt tax shield applies, $\xi = \psi$. If the tax shield does not apply, $\psi \equiv \frac{1-\tau_E}{1-\tau_B}$ and ψ is the relevant measure of the tax burden on equity relative to debt financing.

Theorem 1. *For the risk-free rate and the rate of economic growth it holds that:*

1. *The risk-free rate r is constant and given by*

$$r = \frac{\bar{g}\,(1 - \tau_C)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}(1 - \widehat{\tau}_C)} = \frac{\bar{g}\xi(1 + L)}{1 + L\psi}, \tag{17}$$

If there exists a tax advantage to debt, the interest rate is increasing in the degree of leverage L. Provided that $L_{nTS} \leq L_{TS}$, it ceteris paribus holds that

(a) The risk-free rate is higher when the debt tax shield applies, i.e., $r_{TS} > r_{nTS}$, in

which r_{TS} (r_{nTS}) denotes the risk-free rate when the debt tax shield applies (when it does not apply).

(b) *Despite the increasing effect of the debt tax shield on the pre-tax risk-free rate, the corporate after-tax borrowing rate remains lower with the tax shield, i.e., $r_{TS}\left(1-\tau_C^{TS}\right) < r_{nTS}$, in which τ_C^{TS} denotes the corporate tax rate when the debt tax shield applies.*

2. *If an existing debt tax shield is removed in a tax-revenue-neutral way, i.e., such that the corporate tax rate is adjusted in a way that expected tax revenues remain constant, Equation* (17) *remains valid. If the firm simultaneously maximizes corporate valuation and thus operates with the maximum possible degree of leverage from Equation* (14)*, items 1a and 1b remain valid.*

3. *Aggregate consumption, C_t^a, and total investments, I_t^a, into the real investment opportunity are given by*

$$C_t^a = (1-F_t)\, W_t^a, \quad I_t^a = F_t W_t^a, \tag{18}$$

in which F_t is the fraction of total output, $W_t^a = I_{t-1}^a G_t$, that is invested into the real investment opportunity as either equity or debt, and I_{t-1}^a can be expressed as

$$I_{t-1}^a = W_0^a \cdot \left(\prod_{i=1}^{t-1} G_i\right) \cdot \left(\prod_{i=0}^{t-1} F_i\right) = W_0^a \cdot \left(\prod_{i=1}^{t-1} G_i\right) \cdot \frac{1-H^{N+1-t}}{1-H^{N+1}}, \tag{19}$$

in which

$$H = \left(\frac{\rho}{M}\sum_{m=1}^{M} G_m^{-\gamma}\right)^{-\frac{1}{\gamma}} \widetilde{R}^{-\frac{1}{\gamma}}. \tag{20}$$

F_t is state independent, decreasing over time, and can be expressed in explicit form as:

$$F_t = \begin{cases} \frac{1-H^{N-t}}{1-H^{N-t+1}} & \textit{for } H \neq 1 \\ \frac{N-t}{N-t+1} & \textit{for } H = 1. \end{cases} \tag{21}$$

For $N \to \infty$ it holds that:

$$\lim_{N\to\infty} F_t = \begin{cases} 1 & \textit{for } H \leq 1 \\ \frac{1}{H} & \textit{for } H > 1. \end{cases} \tag{22}$$

When the debt tax shield applies, the share of wealth invested, F_t, as well as the the utility from aggregate consumption is higher.

4. The growth rate of consumption is the same for all households and follows the i.i.d. process with distribution:

$$\frac{C^a_{t+1}}{C^a_t} = \frac{C_{t+1,j}}{C_{t,j}} = \frac{1}{H}G. \tag{23}$$

Proof *The proof of Theorem 1 is provided in Online Appendix A.*

Theorem 1 reveals that the debt tax shield affect macroeconomic variables, such as the risk-free rate, aggregate investments, and economic growth even when the corporate tax rate after the removal of an existing debt tax shield is adjusted in a manner that keeps the expected tax revenue constant.

In partial equilibrium with a given pre-tax risk-free corporate borrowing rate r_{PE}, the removal of a debt tax shield increase the after-tax borrowing rate from $r_{PE}\,(1-\tau_C)$ to r_{PE}. Theorem 1 item 1 documents that in general equilibrium, this increase is counteracted by a decrease in the pre-tax risk-free rate. The decrease in the pre-tax risk-free rate is driven through both demand- and supply-side effects. First, the removal of the debt tax shield increases the after-tax cost of corporate debt and causes corporations to aim for a lower degree of corporate leverage (Equation (14)). The firm thus decreases the supply of corporate debt. Second, the increased after-tax cost of debt leaves less profits to distribute to equity holders, thus rendering equity investments less attractive. As a consequence, investors want to partly substitute equity for debt and demand more debt. To raise the desired amount of debt, the firm can thus offer a lower coupon rate. That is, market-clearing is archived with a lower pre-tax risk-free rate after the removal of a debt tax shield. This is so, even if the debt tax shield is removed in a tax-revenue neutral way that keeps the government's expected tax revenues constant. That is, Theorem 1 item 1 shows that even a tax-neutral removal of a debt tax shield has macroeconomic implications.

From Equation (16), it holds that $r\,(1-\tau_B) = \bar{g}\,(1-\widetilde{\tau})$ when the firm does not lever up. In equilibrium, the return on the risk-free asset after taxes on the household level then corresponds to the risk-neutral expected after-tax return on equity. With a tax advantage to debt, the firm levers up. From Equation (17), it then holds that $r\,(1-\tau_B) > \bar{g}\,(1-\widetilde{\tau})$. Hence, the risk-free rate after tax exceeds the expected growth rate of the economy under the risk-neutral measure after taxation on the corporate level under the risk-neutral measure.[7] That is, the debt tax shield has an effect that extends way beyond determining whether the firm operates with leverage or not. In particular, in general equilibrium it increases the risk-free rate, thus partly undoing the reduction of the corporate after-tax rate.

Item 3 reveals two important properties about the fraction F_t, of aggregate wealth in-

[7] Since households can only invest into the risk-free asset and levered firms, but not into the production technology itself, this it not a violation of the definition of the risk-neutral measure. As a matter of fact, it holds that $r(1-\tau_B) = \bar{g}(1+L)(1-\widetilde{\tau}) - rL(1-\widehat{\tau})$.

vested. First, F_t is state independent, reflecting the i.i.d. growth rates of the production process. Second, F_t is positively related to the interest rate. From Equation (20), an increase in the interest rate decreases H, the certainty equivalent of an additional marginal unit of investment. From Equations (21), the decrease in H decreases F_t. Economic growth thus increases in the risk-free rate, reflecting that a higher risk-free rate decreases the price of future relative to present consumption. That is, the households partly substitute present with future consumption.

In partial equilibrium, a change in the interest rate also implies a wealth effect from which households have an incentive to increase present consumption. Intuitively, when the interest rate increases, households know that they will be richer tomorrow. Hence, they can afford a higher present level of consumption. Whether the wealth or the substitution effect dominates depends, in partial equilibrium, on the elasticity of intertemporal substitution, which, with CRRA preferences, is the inverse of the degree of risk aversion, γ. Our results in general equilibrium, however, show that irrespective of households' degree of risk aversion, an increase in the risk-free rate increases household savings. This is so, because unlike in partial equilibrium, in our general equilibrium model, next period's wealth in the economy is endogenously determined, and the first-order effect of an increase in the risk-free rate is that it leads to a different distribution of future output to debt and equity holders. Hence, unlike in partial equilibrium, a higher risk-free rate does not in itself generate future wealth in the economy, which is why savings increase in the risk-free rate irrespective of the degree of risk aversion.

F_t is higher when the debt tax shield applies. Again, the channel driving this result is the impact of the debt tax shield on the risk-free rate. In the presence of the debt tax shield, the aggregate share of wealth consumed is lower and the share invested is higher. The higher share of wealth invested ultimately leads to more factor accumulation and thus, a higher growth rate of the economy. The higher growth rate of the economy leads to a higher welfare level from aggregate consumption, i.e., to a higher welfare for a representative investor.[8]

In addition to the well-documented effects of the debt tax shield on corporate financial structure, our model shows that in equilibrium, the debt tax shield also has important macroeconomic effects. In particular, the debt tax shield increases the risk-free rate, the growth rate of the economy, and aggregate welfare. Our model predicts a positive relationship between the risk-free rate and economic growth. This result is in direct contrast to the popular view that investment increases when the risk-free rate decreases. In our model,

[8] The expression for utility from aggregate consumption and the proof of this statement is given in Online Appendix A.

economic growth increases with the risk-free rate, because a high risk-free rate decreases the price of future versus present consumption and thus increases household savings, which in turn has a positive impact on economic growth.

From items 1 and 3, the two objectives of attaining a high rate of growth of the economy and a reduction in the disparities in lifetime consumption opportunities among households trade off against each other. To reduce disparities in lifetime consumption opportunities, the government has to increase transfer payments to poorer households. That is, it has to increase tax revenues by increasing tax rates or remove an existing debt tax shield. From Equation (17), these policies lead to a reduction in the risk-free rate. The reduction in the risk-free rate in turn reduces F_t, the share of wealth invested, by increasing the parameter H in Equation (20). A reduction in disparities in lifetime consumption opportunities among households thus comes at the cost of reducing economic growth. In section 4.3 we investigate in more detail how reductions in lifetime consumption opportunities trade off against macroeconomic growth. In particular, we document how this tradeoff is quantitatively affected by the removal of an existing debt tax shield.

From item 4, the growth rate of consumption is identical among households. Households establish a linear risk sharing rule via their trading of financial assets. The attempt to establish such a linear sharing rule has important implications for households' consumption-investment strategies and the effectiveness of fiscal policy that we turn to in sections 3.2 and 3.3.

3.2 Households' consumption-investment policies

Having derived closed-form solutions for the risk-free rate and economic growth, we next show how the debt tax shield affects individual households' consumption and investment strategies. Our key findings are summarized in Theorem 2:

Theorem 2. *For household j's consumption and investment policies, it holds that:*

1. *The allocation of macroeconomic risk is in accordance with a linear sharing rule relative to the distribution of wealth after taxes. Household j's position in the risk-free asset from time t to $t+1$ is proportional to the aggregate investment, I_t^a, and given by:*

$$\frac{\beta_{t,j} + \delta_{t,j}}{I_t^a} = \alpha_{t,j} \frac{L}{1+L} + \frac{1}{\widetilde{\widetilde{R}}} \cdot \left(\alpha_{t,j} - \frac{1}{n}\right) \left(\frac{L}{1+L} r \left(\tau_B - \widehat{\tau}\right) - \widetilde{\tau}\right). \tag{24}$$

For $N \to \infty$, the position in the risk-free asset is given by the expression in Equation (24) *with $\alpha_{t,j}$ substituted by the the limiting value of the equity position, α_j, from Equation* (30)*.*

2. *Household j's consumption share, $\omega_j \equiv C_{0,j}/C_0^a$, is constant over time and fulfills:*

$$\omega_j - \frac{1}{n} = D\left(\alpha_{0-,j} - \frac{1}{n}\right), \tag{25}$$

where

$$D = \frac{H - Y}{H^{N+1} - Y^{N+1}} \frac{H^{N+1} - 1}{H - 1}, \quad Y = \frac{\bar{G}}{\widetilde{R}}. \tag{26}$$

D is a measure of the degree of disparity in lifetime consumption opportunities. It holds that

$$\lim_{N \to \infty} D = \frac{H - Y}{H - 1}. \tag{27}$$

3. *Household j's equity share, $\alpha_{t,j}$, is given by*

$$\alpha_{t,j} = \frac{1}{n} + \frac{\left(\omega_j - \frac{1}{n}\right)}{1 - \widetilde{\tau}} Z_t \tag{28}$$

$$Z_t = \frac{H^{N-t} - Y^{N-t}}{H - Y} \frac{(H - 1)}{H^{N-t} - 1}. \tag{29}$$

Poorer households' equity shares increase over time and richer households' decrease. It holds that $D = 1/Z_{-1}$. For $N \to \infty$, the equity share is a constant:

$$\alpha_j = \frac{1}{n} + \left(\alpha_{0-,j} - \frac{1}{n}\right) \frac{1}{1 - \widetilde{\tau}}. \tag{30}$$

Proof *A detailed proof for all items of Theorem 2 is given in Online Appendix B.*

Theorem 2 shows that the debt tax shield not only affects macroeconomic variables, but also the degree of harmonization in lifetime consumption opportunities as well as individual households' consumption and investment strategies.

Theorem 2, item 1 reveals how households choose their exposure to the risk-free asset. From Equation (24), household j's position in the risk-free asset consists of two terms. The first term, $\alpha_{t,j} \frac{L}{1+L}$, is zero, when the firm operates without debt and is proportional to the share of corporate debt in the firm's total capital otherwise. When the firm operates with corporate debt, its shareholders have an implicit short position in the risk-free asset. For a household j that holds a share of $\alpha_{t,j}$ of firm equity, the implicit short position in the risk-free asset is $\alpha_{t,j} \frac{L}{1+L} I_t^a$. Hence, household j needs a position of $\alpha_{t,j} \frac{L}{1+L} I_t^a$ in the risk-free asset to undo this implicit short position. This hedging demand simultaneously ensures that aggregate demand for the risk-free asset meets aggregate supply.

In the second term, $\frac{1}{\widetilde{R}}\left(\alpha_{t,j} - \frac{1}{n}\right)\left(\frac{L}{1+L} r\left(\tau_B - \widehat{\tau}\right) - \widetilde{\tau}\right)$, the factor $\alpha_{t,j} - \frac{1}{n}$ is negative for poorer households with below-average equity holdings, $\alpha_{t,j} < \frac{1}{n}$. When the firm operates without leverage, the second term is proportional to $\widetilde{\tau}$, the effective tax rate applicable to returns from investments into firm equity. From Equation (A.29), transfer income is then proportional to the tax revenues from taxing firm profits at rate $\widetilde{\tau}$. Given that returns to equity are subject to macroeconomic risk, the transfer income of poorer households with below-average equity holdings is subject to macroeconomic risk; i.e., these households have implicit long positions in firm equity, which they react to by decreasing their investments into firm equity and increasing their investments into the risk-free asset.

When the firm operates with corporate leverage, the term $\frac{L}{1+L} r \left(\tau_B - \widehat{\tau}\right)$ may become nonzero. It measures how the government's tax revenues are affected by corporate debt. For every unit of corporate debt that replaces firm equity, the government collects an additional tax revenue of $r\tau_B$ from the taxation of the return on the risk-free asset, but loses a tax revenue of $r\widehat{\tau}$ from the taxation of the replaced firm equity. When $\tau_B < \widehat{\tau}$, tax revenues decrease with corporate levering. Poorer households with below-average equity exposures react to this implied reduction in their risk-free transfer income by increasing their exposure to the risk-free asset. With tax-neutrality between corporate debt and equity, i.e., when the Miller (1977) conditions hold and $\tau_B = \widehat{\tau}$, tax revenues are independent of the level of corporate levering and households do not have to adjust their portfolio positions to changes in their transfer income. Because $\widehat{\tau} \geq \widetilde{\tau}$, there exists a tax advantage to equity for $\tau_B > \widehat{\tau}$. The firm then operates without corporate leverage, and $L = 0$. In sum, the term $\frac{L}{1+L} r \left(\tau_B - \widehat{\tau}\right)$ is only nonzero when $\tau_B < \widehat{\tau}$.

Theorem 2, item 2 reveals that the linear sharing rule from item 1 implies households attain time- and state-independent constant consumption shares. From Equation (25), household j's deviation from an equal consumption share is proportional to its initial deviation from the average initial endowment. The proportionality factor D can be interpreted as an inequality measure and take values between 0 and 1. A value of $D = 1$ represents no reduction in the disparity in lifetime consumption opportunities where each households' consumption share corresponds to its initial endowment. It corresponds to a world with no taxation at any level and, consequently, no redistribution. $D = 0$ implies the largest possible degree in the reduction of disparity in lifetime consumption opportunities where all households, irrespective of their initial endowments, attain the same consumption share. Such a perfect harmonization of lifetime consumption opportunities is only achievable with an infinite investment horizon and tax rates of 100%. A zero value of D is therefore only a theoretical lower bound.

From Theorem 1, we know that the two goals of achieving a high level of macroeconomic growth and a reduction in the disparities in lifetime consumption opportunities among households trade off against each other. High levels of the growth scaling parameter $1/H$ tend to occur simultaneously with high levels of the disparity measure D. We quantify this trade-off in more detail in section 4.3.

Theorem 2, item 3 shows that with a finite investment horizon, households' equity exposures are converging towards each other over time. Poorer households' equity exposures increase and richer households' decrease, reflecting that poorer households need to build up savings to finance their consumption share.

3.3 Fiscal policy

Our results in Theorem 1 show that removing an existing debt tax shield decreases aggregate production and thus the growth rate of the economy. In this section, we show why the government cannot reestablish the same level of aggregate production as in the presence of the debt tax shield via an active fiscal policy. Intuitively, the government could try to compensate for the decrease in aggregate production by investing in the production technology itself, thus financing this investment by issuing government debt. Government bonds and privately issued bonds are both risk-free assets and are therefore perfect substitutes carrying the same interest rate. To avoid lengthy notation, we assume that government bonds are single-period bonds. Hence, we do not need to introduce further variables into the model. Instead, we require that $\sum_{j=1}^{n} \beta_{t,j} = \beta_t^a$, where β_t^a is the total amount of government bonds outstanding from time t to $t+1$. That is, in equilibrium the government bonds issued must be held by the households. If the government invests into firm equity, household j's evolution of wealth is given by:

$$
\begin{aligned}
W_{t,j} = &\left(\left(E_{t-1,j} - \frac{E_{t-1}^a}{n}\right)(1-\tau_E) + \frac{E_{t-1}^a}{n}\right)\left((1+L)\left(1+g_t\left(1-\tau_C\right)\right) - L\widehat{R}_{t-1}\right) + \\
&\tau_E\left(E_{t-1,j} - \frac{E_{t-1}^a}{n}\right) + \frac{\tau_C}{n}E_{t-1}^a\left(1+L\right)g_t + \left(\beta_{t-1,j} + \delta_{t-1,j}\right)\widetilde{R}_{t-1} + \\
&\frac{E_{t-1}^a}{n}Lr_{t-1}\left(\tau_B - \widehat{\tau}_C\right) + \frac{1}{n}\beta_{t-1}^a\left(G - \widetilde{R}_{t-1}\right).
\end{aligned} \tag{31}
$$

Compared to the evolution of wealth without government debt from Equation (10), Equation (31) contains the additional term $\frac{1}{n}\beta_{t-1}^a\left(G - \widetilde{R}_{t-1}\right)$ that, ceteris paribus, leads to a higher effective exposure to the real investment and a lower exposure to the risk-free asset for every household j.

When the government purchases firm equity, from Equation (31), every household j's exposure to firm equity increases by $\frac{\beta_{t-1}^a}{n}$, implying an increase in every household's exposure to macroeconomic risk. Households react to this increase by reducing their equity holdings by $\frac{\beta_{t-1}^a}{n}$ units each. As a consequence, the aggregate demand for firm equity is unchanged, and the corporate leverage ratio is not altered. The government's intervention simultaneously implies a reduction in every household j's effective exposure to the risk-free asset by $\frac{\beta^a_{t-1}}{n}$ units. To undo this government-intervention-implied reduction, every household increases its exposure to the risk-free asset by $\frac{\beta^a_{t-1}}{n}$ units by purchasing government bonds. As a result of the households' adjustments to their trading strategies, the budget equation is fulfilled, households do not alter their consumption plans, they reestablish the linear sharing rule, markets clear, corporate leverage does not change, and the level of real investment remains unchanged. Hence, there is no room for an active fiscal policy.

If the government decides to invest into the firm via corporate debt, household j's budget constraint from Equation (10) is not affected. For every unit of corporate interest income the government earns, it has to pay exactly the same amount to the government debt holders. The firm reacts to the deviation from its optimal leverage ratio by decreasing its amount of corporate bonds outstanding to households by β^a_{t-1} units. In sum, each household reduces its corporate bond holdings by $\frac{\beta^a_{t-1}}{n}$ units and increases its government bond holdings by $\frac{\beta^a_{t-1}}{n}$ units. As a result of the firm's and the households' adjustments to their trading strategies, the budget equation is fulfilled, households do not alter their consumption plans, they reestablish the linear sharing rule, markets clear, corporate leverage does not change, and the level of real investment remains unchanged. In other words, there is perfect crowding out of the government intervention and no room for an active fiscal policy in our model. This model prediction is in line with the empirical evidence in Graham, Leary, and Roberts (2014) and Demirici, Huang, and Sialm (2019) that government debt crowds out corporate debt. Again, there is no room for an active fiscal policy in our model.

To conclude, markets are effectively complete and households have rational expectations in our model. Hence, households and firms can reach the same Pareto optimal solution with the desired linear sharing rule and corporate leverage L, both before and after a fiscal intervention. Hence, the government intervention can and will be undone by the supply and demand decisions of the firm and the households.

4 Quantitative effects

In this section we illustrate the quantitative implications of corporate leverage and the debt tax shield for the risk-free rate, the growth rate of the economy, and household consumption. We want to illustrate both immediate and long-term consequences of the debt tax shield. We therefore choose an investment horizon of $N = 100$ periods and assume one period to correspond to one year. The degree of risk aversion and the households' time preference parameter are set to $\gamma = 1$ and $\rho = 0.98$, respectively, which is in the range of values typically considered in the literature. The tax rates are set to $\tau_E = 20\%$, $\tau_B = 39.6\%$, and $\tau_C = 35\%$, the top tax rates for US households and corporations prior to the recent tax-cuts.[9]

For simplicity, we focus on a setting with $M = 2$ possible realizations throughout our numerical analysis. We set the mean of the growth rate of our real investment opportunity to 3.1%, corresponding to the average real post-war GDP growth in the US. The standard deviation of the real investment opportunity's growth rate is chosen to attain a level of corporate levering that is in line with the historical empirical evidence. More specifically, we set the standard deviation to 1.8%, implying a maximum debt-to-capital ratio of 46%, cf. (14). This value is in the range of historical ratios reported by Graham, Leary, and Roberts (2015) and used in our numerical examples throughout.[10]

From Theorem 1, the debt tax shield increases the risk-free rate, increases economic growth, and alters the intertemporal allocation of consumption. In this section we quantify the order of magnitude of these effects. We begin the discussion of our results with the risk-free rate that drives both economic growth (Equations (20) and (21)) and the strength of the harmonization of lifetime consumption opportunities (Equations (25) and (26)).

4.1 Risk-free rate

In our base case parameter setting, the risk-free rate is 2.8% when the debt tax shield applies and 2.6% when it does not apply. That is, the debt tax shield increases the risk-free rate by seven percent or 20 basis points. From Theorem 1, the risk-free rate is key in understanding the general-equilibrium implications of the debt tax shield, because it is the only endogenous variable that affects other macroeconomic variables, such as the growth rate of the economy and inequality. We next turn to a demonstration of how the level of the corporate tax burden quantitatively affects the impact of the debt tax shield on the risk-free rate.

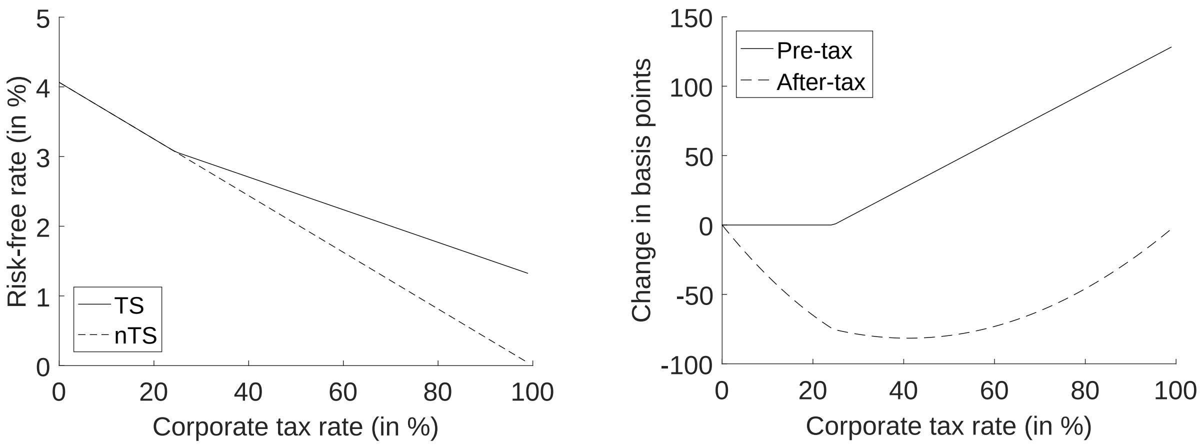
We first show how the corporate tax rate affects the risk-free rate. The left panel of Figure 1 depicts the risk-free rate as a function of the corporate tax rate (τ_C) in a setting with the debt tax shield (solid line) and without (dashed line), where the corporate tax rate varies between $\tau_C = 0\%$ and $\tau_C = 99\%$.[11] The right panel depicts the change in the pre-tax risk-free rate r (solid line) and the after-tax corporate borrowing rate $\widehat{r}$ (dashed line) due to the debt tax shield in basis points.

[9]We also explored the robustness of our results to other choices of γ and ρ as well as other tax rates and tax-revenue-neutral removals of the debt tax shield from Theorem 1, item 2. These changes only affect our results quantitatively, but not qualitatively, and are therefore not presented here.

[10]We also explored other choices of the distribution of the growth rate of the real investment opportunity. Given that these changes only affect our results quantitatively, but not qualitatively, they are not reported here.

[11]We also explored varying τ_E and τ_B, which resulted in similar results. The results mainly channel themselves through the variation of the parameters ξ and ψ.

Figure 1
The risk-free rate



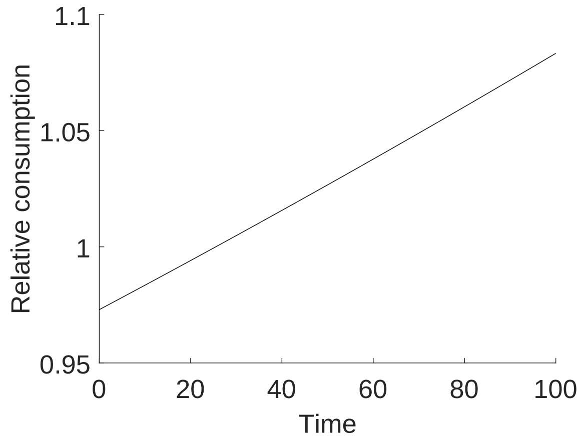
This figure depicts in the left panel the risk-free rate in a setting with the debt tax shield (solid line, TS) and without the debt tax shield (dashed line, nTS) as a function of the corporate tax rate. The right panel depicts the change in the pre-tax risk-free rate r (solid line) and the after-tax corporate borrowing rate $\widehat{r}$ (dashed line) due to the debt tax shield in basis points.

Confirming our results from Theorem 1, it shows that the risk-free rate decreases in the level of the corporate tax rate. For corporate tax rates below 24.5%, the firm operates without corporate debt; hence, whether the debt tax shield applies or not does not have an effect.

For levels of the corporate tax rate exceeding 24.5%, it becomes optimal to operate with corporate debt when the tax shield applies, and it remains optimal to be unlevered without the debt tax shield, which results in the linear relationship between the corporate tax rate and the risk-free rate when the debt tax shield does not apply. With the debt tax shield, there is a kink at a corporate tax rate of 24.5%. The debt tax shield reduces the after-tax cost of corporate debt, which makes investments into equity more desirable. The order of magnitude increases with the level of the corporate tax rate. To nevertheless find investors that are willing to hold corporate debt, the firm has to offer a higher risk-free rate when the debt tax shield applies. For example, when corporate taxes do not apply, i.e., for $\tau_C = 0\%$, the risk-free rate is 4.1%. It decreases to 2.5% with the debt tax shield and 2.1% without the debt tax shield for a corporate tax rate of $\tau_C = 50\%$.

From the right panel of Figure 1, with a tax advantage to corporate debt, the pre-tax risk-free rate increases linearly in the level of the corporate tax rate. This increase in the pre-tax risk-free rate partly offsets, but does not completely eliminate the decrease in the

Figure 2
Impact of debt tax shield on consumption



This figure depicts the evolution of consumption over time in a setting with the debt tax shield relative to a setting without.

after-tax corporate borrowing rate stemming from the debt tax shield.

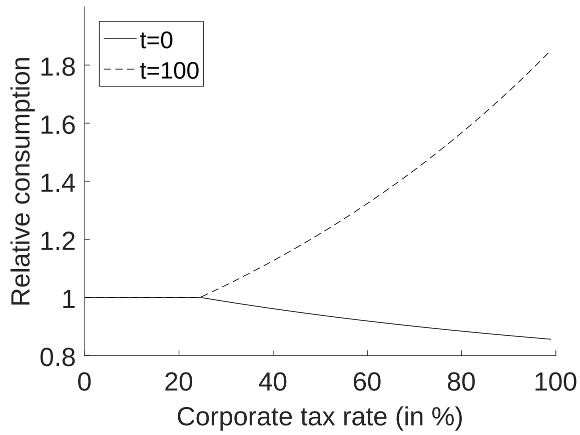
4.2 Macroeconomic growth and consumption

Having depicted how the debt tax shield affects the risk-free rate, we next ask how it affects aggregate consumption and wealth in the economy. From Theorem 1, we know that a higher risk-free rate increases savings. The higher savings rate alters the intertemporal allocation of consumption and increases the growth rate of the economy. We quantify these effects in Figure 2, which depicts the evolution of consumption over time in a setting with the debt tax shield relative to a setting without.

In our base case parameter setting aggregate consumption at time $t = 0$ is 2.7% lower when the debt tax shield applies (Figure 2). That is, the debt tax shield significantly reduces immediate consumption. In the long run, however, the higher savings rate causes a wealth effect that results in the consumption level being higher from time $t = 20$ onwards. At time $t = 100$, consumption in the setting with the debt tax shield is 8.3% higher than in the setting without.

Figure 3 depicts consumption in a setting with debt tax shield relative to a setting without as a function of the corporate tax rate. The solid lines show results at time $t = 0$, the dashed lines at time $t = 100$. Consistent with our results for the risk-free rate, consumption is identical with and without the debt tax shield for corporate tax rates below 24.5%. For

Figure 3
Impact of corporate tax rate on consumption



This figure depicts consumption in a setting with debt tax shield relative to a setting without debt tax shield (Relative consumption) as a function of the corporate tax rate. The solid lines show results at time $t = 0$, the dashed lines at time $t = 100$.

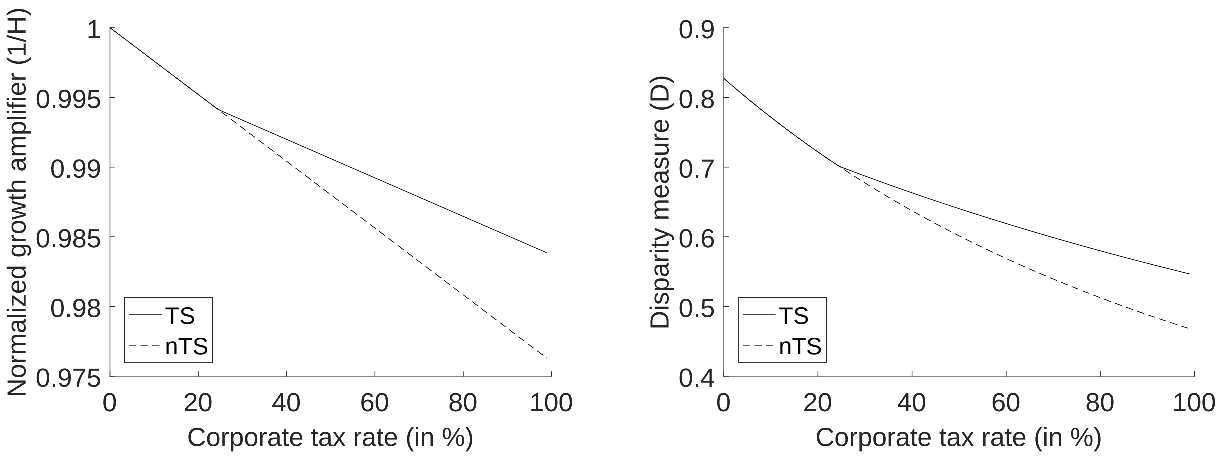
rates exceeding 24.5%, the amount spent on consumption at time $t = 0$ is lower when the debt tax shield applies and the order of magnitude of this effect amplifies in the level of the corporate tax rate. For example, for a corporate tax rate of $\tau_C = 35\%$, consumption at time $t = 0$ is 2.7% lower with the debt tax shield as also shown in Figure 2. The order of magnitude of this effect increases to 6.1% for a corporate tax rate of $\tau_C = 50\%$.

At time $t = 100$, however, relative consumption levels dramatically increase in the level of the corporate tax rate, reflecting that the effect of more households saving in the presence of the debt tax shield amplifies when the level of the corporate tax rate increases. For example, for a corporate tax rate of $\tau_C = 35\%$, consumption at time $t = 100$ is 8.3% higher with the debt tax shield. The order of magnitude of this effect increases to 22.0% for a corporate tax rate of $\tau_C = 50\%$.

4.3 Tradeoff between growth and inequality

We know from Theorem 1 that the growth rate of aggregate consumption and the disparity in lifetime consumption opportunities among households trade off against each other. In this section, we quantify this tradeoff. From Equation (23) the growth rate of consumption is proportional to $1/H$. We therefore interpret $1/H$ as a scaling factor and a measure for economic growth. From Equation (25), the deviation of each households' consumption share from an equal consumption share is the household's deviation from the average initial

Figure 4
Tradeoff between growth and inequality



This figure depicts the impact of the corporate tax rate on growth (left panel) and inequality (right panel). The growth amplifier $1/H$ is normalized to 1 for a corporate tax rate of $\tau_C = 0\%$. The solid lines show results with the debt tax shield (TS), the dashed lines with no tax shield (nTS).

endowment times D. From Theorem 2, D can therefore be interpreted as a disparity measure. It can take values between 0 and 1. 1 represents the highest level of disparity, where poorer households' consumption shares correspond to their initial endowments. This situation occurs without taxation and redistribution, i.e., for tax rates of $\tau_B = \tau_C = \tau_E = 0\%$. 0 represents the theoretically lowest possible level of inequality, where all households are endowed with equal consumption shares. In reality, an inequality measure of 0 should not be a reasonable objective from a policy makers perspective, because it comes at the cost of removing households' incentives to invest.[12]

Figure 4 depicts how the corporate tax rate affects economic growth (normalized growth amplifier ($1/H$), left panel) and the disparity measure (D, right panel). We normalize the growth multiplier to 1 for a corporate tax rate of $\tau_C = 0\%$, thus allowing us to easily measure reductions in the annual gross growth rate, G, of the economy relative to a setting with a corporate tax rate of $\tau_C = 0\%$. The solid lines show results when the debt tax shield applies, the dashed lines, when it does not.

Consistent with Theorem 1, Figure 4 shows that higher levels of the corporate tax rate decrease both economic growth and inequality. These effects are amplified in the absence of a debt tax shield. That is, the debt tax shield significantly affects the tradeoff between

[12]Fischer and Jensen (2015) rationalize the linear taxation and redistribution scheme as the solution to an optimization problem with a government objective function for reducing disparity in consumption opportunities combined with friction cost of collecting taxes.

economic growth and inequality. For example, in our base case parameter setting with the debt tax shield and a corporate tax rate of $\tau_C = 35\%$, the annual gross growth rate of the economy is 0.7% lower than with a corporate tax rate of $\tau_C = 0\%$, and the inequality measure is reduced from 0.827 to 0.675, implying that the consumption share of a poorer household with an initial endowment of 20% of the average initial endowment increases from 25.2% to 29.8% when the debt tax shield applies. When the debt tax shield does not apply, the annual gross growth rate of consumption is 0.8% lower, and the inequality measure is 0.657, implying the consumption share of the poorer household increases to 30.3%.

For higher tax rates, these effects are further amplified. For example, for a corporate tax rate of $\tau_C = 50\%$, the annual growth rate of the economy is 0.9% lower than with a corporate tax rate of $\tau_C = 0\%$ and the inequality measure is reduced to 0.640, implying that the consumption share of the poorer household increases to 30.8% when the debt tax shield applies. When the debt tax shield does not apply, the annual gross growth rate of consumption is 1.2% lower, and the inequality measure is 0.397, implying that the consumption share of the poorer household increases to 32.0%.

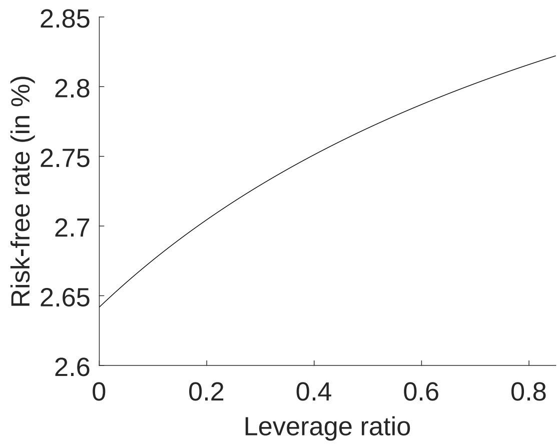
4.4 Impact of corporate leverage

Having discussed how the risk-free rate, macroeconomic growth, and inequality is affected by the debt tax shield when the firm chooses the highest degree of leverage that ensures solvency of the firm, in this section, we ask how other choices of corporate leverage, which may, e.g., be caused by frictions, affect our results. For that purpose, we depict how the firm's choice of the degree of corporate leverage affects these variables. In our base case parameter setting, it is optimal to operate with corporate leverage when the debt tax shield applies and without when it does not. We therefore focus our analysis in this section on the case where the debt tax shield applies. We begin by plotting the risk-free rate as a function of the degree of corporate leverage in Figure 5.

From Theorem 1, corporate leverage increases the risk-free rate when there is a tax advantage to debt, reflecting that levering makes equity investments more attractive, thus requiring a compensation for debt holders. Figure 5 quantifies this relationship in our base case parameter setting. For a leverage ratio of $L = 0$, the risk-free rate is about 2.64%. It increases by 6.8 percent, corresponding to 18 basis points, to 2.82% for $L = 0.8$. These effects are already economically significant in our base case parameter setting. They are even larger in the classical tax system, where $\tau_E = \tau_B$, such as in several European tax laws, including Germany, Italy, Spain, Austria, the Netherlands, Sweden, and Denmark.

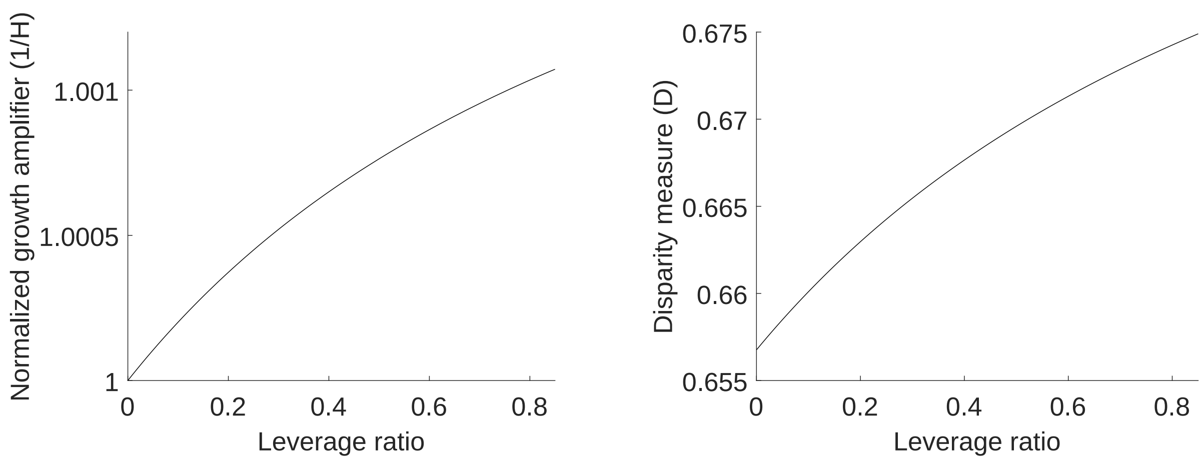
Having depicted the quantitative effect of corporate leverage on the risk-free rate in Figure

Figure 5
Impact of corporate leverage on risk-free rate



This figure depicts the risk-free rate in a setting with the debt tax shield as a function of the degree of corporate leverage.

Figure 6
Leverage, growth, and inequality



This figure depicts the impact of the degree of corporate leverage on growth (left panel) and inequality (right panel). The growth amplifier $1/H$ is normalized to 1 for a degree of corporate leverage of $L = 0$.

5, we next turn to illustrating the impact of corporate leverage on macroeconomic growth (left panel) and disparities in consumption opportunities among households (right panel). From Figure 6 corporate levering affects both macroeconomic growth and the disparity in lifetime consumption opportunities among households. From Equation (20), the growth amplifier $1/H$ increases in the after-tax interest rate on household level, $\widetilde{r}$. When there is a tax advantage to debt, an increase in the leverage ratio makes both equity investments and – via the implied increase in the risk-free rate – investments into corporate debt more attractive. Hence, the price of future over present consumption decreases and households invest more, which in turn leads to higher macroeconomic growth. In our base case parameter setting, an increase of corporate leverage from $L = 0$ to $L = 0.8$ leads to an increase in the growth rate of the economy of about 10 basis points per year.

From the right panel of Figure 6, the level of disparity in households' lifetime consumption opportunities increases in the degree of corporate leverage. That is, when the firm levers up more, poorer households' lifetime consumption opportunities are smaller. With the tax advantage to debt, an increase in corporate leverage implies lower tax revenues to the government and thus lower transfer income. As a direct consequence, disparities in lifetime consumption opportunities are harmonized less when the firm operates with more corporate leverage. Without corporate levering the disparity measure is $D = 0.657$ and a poorer household with an initial endowment of 20% of the average initial endowment has a consumption share of 23.7%. When $L = 0.8$, the disparity measure increases to $D = 0.674$ and this share decreases to 23.0%.

Overall, our results in this section depict quantitatively important effects of the debt tax shield on the intertemporal allocation of resources, the risk-free rate, macroeconomic growth, and inequality in lifetime consumption opportunities among households.

5 Conclusion

This paper studies the implications of the debt tax shield in a growth economy that taxes household income and redistributes tax revenues in an attempt to harmonize lifetime consumption opportunities among households. Our general-equilibrium model predicts the debt tax shield to increase the risk-free rate. This increase partly offsets the reduction in the corporate after-tax risk-free rate implied by the debt tax shield, but does not fully eliminate it.

The increase in the risk-free rate leads to a higher growth rate of the economy, and the debt tax shield increases the degree of disparity in households' lifetime consumption opportunities. The debt tax shield thus contributes to a higher macroeconomic growth rate at the expense of a higher degree of inequality among households in the economy. The goals of achieving a high growth rate of the economy and a low degree of inequality trade off against each other.

References

Alesina, A., and G.-M. Angeletos, 2005, "Fairness and Redistribution," *American Economic Review*, 95(4), 960–980.

Barro, R., 1974, "Are Government Bonds Net Wealth?" *Journal of Political Economy*, 82(6), 1095–1117.

Becker, B., M. Jacob, and M. Jacob, 2013, "Payout Taxes and the Allocation of Investment," *Journal of Financial Economics*, 107(1), 1–24.

Bell, L., and T. Jenkinson, 2002, "New Evidence of the Impact of Dividend Taxation and on the Identity of the Marginal Investor," *Journal of Finance*, 57(3), 1321–1346.

Blanchard, O., and R. Perotti, 2002, "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output," *Quarterly Journal of Economics*, 117(4), 1329–1368.

Boskin, M. J., 1978, "Taxation, Saving, and the Rate of Interest," *Journal of Political Economy*, 86(2), 3–27.

Chamley, C., 1986, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54(3), 607–622.

Chen, H., 2010, "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," *Journal of Finance*, 65(6), 2171–2212.

Cloyne, J., 2013, "Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom," *American Economic Review*, 103(4), 1507–1528.

Cooper, I. A., and K. G. Nyborg, 2006, "The Value of Tax Shields IS Equal to the Present Value of Tax Shields," *Journal of Financial Economics*, 81(1), 215–225.

Cox, J. C., J. E. Ingersoll, and S. A. Ross, 1985, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, 53(2), 363–384.

DeAngelo, H., and R. Roll, 2015, "How Stable are Corporate Capital Structures?" *Journal of Finance*, 70(1), 373–418.

Demirici, I., J. Huang, and C. Sialm, 2019, "Government Debt and Corporate Leverage: International Evidence," *Journal of Financial Economics*, forthcoming.

Devereux, M. P., G. Maffini, and J. Xing, 2015, "Corporate Tax Incentives and Capital Structure: Empirical Evidence from UK Tax Returns," working paper, Oxford University.

Diamond, P., 1975, "A Many-Person Ramsey Rule," *Journal of Public Economics*, 4(4), 335–342.

Doidge, C., and A. Dyck, 2015, "Taxes and Corporate Policies: Evidence from a Quasi Natural Experiment," *Journal of Finance*, 70(1), 45–89.

Eaton, J., and H. Rosen, 1980, "Taxation, Human Capital, and Uncertainty," *American Economic Review*, 70(4), 705–715.

European Commission, 2016, "Council Directive (EU) 2016/1164," (ATA directive), 12 July 2016.

Faccio, M., and J. Xu, 2015, "Taxes and Capital Structure," *Journal of Financial and Quantitative Analysis*, 50(3), 1–24.

Fama, E. F., 2011, "My Life in Finance," *Annual Review of Financial Economics*, 3(1), 1–15.

Faulkender, M., and J. Smith, 2016, "Taxes and Leverage at Multinational Corporations," *Journal of Financial Economics*, 122(1), 1–20.

Fischer, M., and B. A. Jensen, 2015, "Taxation, Transfer Income and Stock Market Participation," *Review of Finance*, 19(2), 823–863.

Fischer, M., and B. A. Jensen, 2018, "The Debt Tax Shield in General Equilibrium," working paper, Copenhagen Business School.

Golosov, M., M. Troshkin, and A. Tsyvinski, 2016, "Redistribution and Social Insurance," *American Economic Review*, 106(2), 359–386.

Gordon, R. H., and Y. Lee, 2001, "Do Taxes Affect Corporate Debt Policy? Evidence from U.S. Corporate Tax Return Data," *Journal of Public Economics*, 82(2), 195–224.

Graham, J. R., 1996, "Debt and the Marginal Tax Rate," *Journal of Financial Economics*, 41(1), 41–73.

Graham, J. R., 1999, "Do Personal Taxes Affect Corporate Financing Decisions?" *Journal of Public Economics*, 73(2), 147–185.

Graham, J. R., 2000, "How Big are the Tax Benefits of Debt?" *Journal of Finance*, 55(5), 1901–1941.

Graham, J. R., M. T. Leary, and M. R. Roberts, 2014, "How does Government Borrowing Affect Corporate Financing and Investment?" working paper, NBER Working Paper No. 20581.

Graham, J. R., M. T. Leary, and M. R. Roberts, 2015, "A Century of Capital Structure: The Leveraging of Corporate America," *Journal of Financial Economics*, 118(3), 658–683.

Graham, J. R., and A. L. Lucker, 2006, "Tax Shelters and Corporate Debt Policy," *Journal of Financial Economics*, 81(3), 563–594.

Gravelle, J. G., and L. J. Kotlikoff, 1995, "Corporate Taxation and the Efficiency Gains of the 1986 Tax Reform Act," *Economic Theory*, 6(1), 51–81.

Hackbarth, D., J. Miao, and E. Morellec, 2006, "Capital Structure, Credit Risk, and Macroeconomic Conditions," *Journal of Financial Economics*, 82(3), 519–550.

Halling, M., J. Yu, and J. Zechner, 2016, "Leverage Dynamics over the Business Cycle," *Journal of Financial Economics*, 122(1), 21–41.

He, Z., and G. Matvos, 2016, "Debt and Creative Destrubtion: Why Could Subsidizing Corporate Debt Be Optimal?" *Management Science*, 62(2), 303–325.

Heider, F., and A. Ljungqvist, 2015, "As Certain as Debt and Taxes: Estimating the Tax Sensitivity of Leverage from State Tax Changes," *Journal of Financial Economics*, 118(3), 684–712.

Hovakimian, A., T. Opler, and T. Sheridan, 2001, "The Debt-Equity Choice," *Journal of Financial and Quantitative Analysis*, 36(1), 1–24.

Jones, L. E., R. E. Manuelli, and P. E. Rossi, 1997, "On the Optimal Taxation of Capital Income," *Journal of Economic Theory*, 73(1), 93–117.

Judd, K. L., 1985, "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28(1), 59–83.

Kemsley, D., and D. Nissim, 2002, "Valuation of the Debt Tax Shield," *Journal of Finance*, 57(5), 2045–2073.

Korajczyk, R., and A. Levy, 2003, "Capital Structure Choice: Macroeconomic Conditions and Financial Constraints," *Journal of Financial Economics*, 68(1), 75–109.

Ljungqvist, A., L. Zhang, and L. Zuo, 2017, "Sharing Risk with the Government: On the Causal Effects of Taxes on Corporate Risk-Taking," *Journal of Accounting Research*, 55(3), 669–707.

Longstaff, F. A., and I. A. Strebulaev, 2014, "Corporate Taxes and Capital Structure: A Long-Term Historical Perspective," working paper, NBER Working Paper No. 20372.

MacKie-Mason, J., 1990, "Do Taxes Affect Corporate Financing Decisions?" *Journal of Finance*, 45(5), 1471–1493.

Meltzer, A. H., and S. F. Richard, 1981, "A Rational Theory of the Size of Government," *Journal of Politial Economy*, 89(5), 914–927.

Miles, J. A., and J. R. Ezzell, 1980, "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis*, 15(3), 719–730.

Miller, M. H., 1977, "Debt and Taxes," *Journal of Finance*, 32(2), 261–275.

Modigliani, F., and M. H. Miller, 1958, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, 48(3), 261–297.

Mukherjee, A., M. Singh, and A. Zaldokas, 2016, "Do Corporate Taxes Hinder Innovation?" *Journal of Financial Economics*, 124(1), 195–221.

Pástor, L., and P. Veronesi, 2016, "Income Inequality and Asset Prices under Redistributive Taxation," *Journal of Monetary Economics*, 81(1), 1–20.

Pástor, L., and P. Veronesi, 2017, "Political Cycles and Stock Returns," working paper, NBER Working Paper No. 23184.

Pástor, L., and P. Veronesi, 2018, "Inequality Aversion, Populism, and the Backslash Against Globalization," working paper, NBER Working Paper No. 24900.

Romer, C. D., and D. H. Romer, 2010, "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," *American Economic Review*, 100(3), 763–801.

Romer, C. D., and D. H. Romer, 2016, “Transfer Payments and the Macroeconomy: the Effects of Social Security Benefit Changes, 1952-1991,” *American Economic Journal: Macroeconomics*, 8(4), 1–42.

Romer, T., 1975, “Individual Welfare, Majority Voting, and the Properties of a Linear Income Tax,” *Journal of Public Economics*, 4(2), 163–185.

Schepens, G., 2016, “Taxes and Bank Capital Structure,” *Journal of Financial Economics*, 120(3), 585–560.

Sialm, C., 2006, “Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium,” *Journal of Economic Dynamics and Control*, 30(3), 511–540.

vanBinsbergen, J. H., J. R. Graham, and J. Yang, 2010, “The Cost of Debt,” *Journal of Finance*, 65(6), 2089–2136.

Varian, H. R., 1980, “Redistributive Taxation as Social Insurance,” *Journal of Public Economics*, 14(1), 49–68.

Online Appendix

A Proof of Theorem 1:

We express the equity variables as $E_t^a = \frac{1}{1+L} I_t^a$ and $E_{t,j} = \alpha_{t,j} E_t^a = \alpha_{t,j} \frac{1}{1+L} I_t^a$. Using Equations (3) and (8), we can rewrite Equation (10) as

$$\begin{aligned} W_{t,j} = & \left[\left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) (1 - \tau_E) + \frac{E_{t-1}^a}{n} \right] \left[(1+L)(1+g_t(1-\tau_C)) - L\widehat{R}_{t-1} \right] + \\ & \tau_E \left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) + \frac{\tau_C}{n} E_{t-1}^a (1+L) g_t + (\beta_{t-1,j} + \delta_{t-1,j}) \widetilde{R}_{t-1} + \\ & \frac{E_{t-1}^a}{n} L r_{t-1} (\tau_B - \widehat{\tau}_C) \end{aligned} \tag{A.1}$$

Recalling that $\alpha_{N,j} = \beta_{N,j} = \delta_{N,j} = 0$, each household's Lagrangian can be written as

$$\begin{aligned} L = & \sum_{t=0}^{N} \rho^t \mathbb{E}_0 \left[\frac{C_{t,j}^{1-\gamma}}{1-\gamma} \right] - \lambda_{0,j} \left[C_{0,j} - W_{0,j} + E_{0,j} + \beta_{0,j} + \delta_{0,j} \right] - \\ & \sum_{t=1}^{N} < \lambda_{t,j}, C_{t,j} + E_{t,j} + (\beta_{t,j} + \delta_{t,j}) > + \\ & \sum_{t=1}^{N} < \lambda_{t,j}, \left(\left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) (1-\tau_E) + \frac{E_{t-1}^a}{n} \right) \left((1+L)(1+g_t(1-\tau_C)) - L\widehat{R}_{t-1} \right) > + \\ & \sum_{t=1}^{N} < \lambda_{t,j}, \tau_E \left(E_{t-1,j} - \frac{E_{t-1}^a}{n} \right) > + \\ & \sum_{t=1}^{N} < \lambda_{t,j}, \frac{\tau_C}{n} E_{t-1}^a (1+L) g_t + (\beta_{t-1,j} + \delta_{t-1,j}) \widetilde{R}_{t-1} + \frac{E_{t-1}^a}{n} L r_{t-1} (\tau_B - \widehat{\tau}_C) > \end{aligned} \tag{A.2}$$

where $< .,. >$ is the scalar product over the attainable states.
The first-order condition with respect to $C_{t,j}$ is

$$\left(\frac{\rho}{M} \right)^t C_{t,j}^{-\gamma} = \lambda_{t,j}. \tag{A.3}$$

The first-order condition with respect to $E_{t,j}$ is

$$\lambda_{t,j} = \sum_{m=1}^{M} \lambda_{t+1,j}^m \left[\left((1+L)(1+g_m(1-\tau_C)) - L\widehat{R}_t \right) (1-\tau_E) + \tau_E \right]. \tag{A.4}$$

The first-order condition with respect to $\beta_{t,j} + \delta_{t,j}$ is

$$\lambda_{t,j} = \widetilde{R}_t \sum_{m=1}^{M} \lambda_{t+1,j}^{m}. \tag{A.5}$$

To continue, observe that the first-order conditions are homogeneous in the following sense: If a given solution, $(\{C_{t,j}\}_{t=0}^{t=N}; \{E_{t,j}\}_{t=0}^{t=N}; \{\beta_{t,j}+\delta_{t,j}\}_{t=0}^{t=N-1}; \{\lambda_{t,j}\}_{t=0}^{t=N})$, satisfies the conditions for given levels of wealth $W_{0,j}$, $j = 1,2,\ldots,n$, then

$$\left(x\{C_{t,j}\}_{t=0}^{t=N}; x\{E_{t,j}\}_{t=0}^{t=N}; x\{\beta_{t,j} + \delta_{t,j}\}_{t=0}^{t=N-1}; x^{-\gamma}\{\lambda_{t,j}\}_{t=0}^{t=N}\right) \tag{A.6}$$

also satisfies the conditions for the wealth levels $xW_{0,j}$, $j = 1,2,\ldots,n$. This proportionality property tells us that households in an optimal solution settle on a linear sharing rule for aggregate consumption and linear asset demand functions. From Equation (A.3) it follows that

$$\frac{\lambda_{t+1,j}}{\lambda_{t,j}} = \frac{\rho}{M}\left(\frac{C_{t+1,j}}{C_{t,j}}\right)^{-\gamma} = \frac{\rho}{M}\left(\frac{C_{t+1}^{a}}{C_{t}^{a}}\right)^{-\gamma}. \tag{A.7}$$

This is also the pricing kernel from where the risk-neutral measure Q is readily derived by normalization. This is done by multiplying through with $\widetilde{R}_t$, as already shown in Equation (A.5):

$$q_{m,t} = \widetilde{R}_t \frac{\lambda_{t+1,j}}{\lambda_{t,j}} = \widetilde{R}_t \frac{\rho}{M}\left(\frac{C_{t+1,j}}{C_{t,j}}\right)^{-\gamma} = \widetilde{R}_t \frac{\rho}{M}\left(\frac{C_{t+1}^{a}}{C_{t}^{a}}\right)^{-\gamma}. \tag{A.8}$$

Proof of item 1

From the evolution of aggregate wealth in the economy, it holds that

$$W_t^a = \sum_{j=1}^{n} W_{t,j} = I_{t-1}^a G_t, \tag{A.9}$$

$$C_t^a = \sum_{j=1}^{n} C_{t,j} = (1 - F_t)\, W_t^a = (1 - F_t)\, I_{t-1}^a G_t, \tag{A.10}$$

and

$$I_t^a = F_t W_t^a. \tag{A.11}$$

The aggregate amount invested into equity and corporate debt is given by $E_t^a = \frac{1}{1+L} I_t^a$ and $\delta_t^a = \frac{L}{1+L} I_t^a$, respectively. From Equation (A.10), it follows that

$$\frac{C_{t+1}^a}{C_t^a} = \frac{1 - F_{t+1}}{1 - F_t} \frac{I_t^a G_{t+1}}{I_{t-1}^a G_t} = \frac{1 - F_{t+1}}{1 - F_t} F_t G_{t+1}. \tag{A.12}$$

Since the term in front of G_{t+1} is state independent, the risk-neutral martingale measure is:

$$q_m = \frac{G_m^{-\gamma}}{\sum_{k=1}^{M} G_k^{-\gamma}}. \tag{A.13}$$

From Equations (A.4), (A.5), and (A.8) we have:

$$\widetilde{R}_t = \mathbb{E}_t^Q \left[\left((1+L)\left(1+g\left(1-\tau_C\right)\right) - L\widehat{R}_t \right) (1-\tau_E) + \tau_E \right] \quad \Leftrightarrow \tag{A.14}$$

$$r_t = \frac{\mathbb{E}_t^Q\left[g\right](1-\tau_C)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}\left(1-\widehat{\tau}_C\right)}. \tag{A.15}$$

From Equation (A.15), the risk-free rate is time independent. Hence, we can drop the index t, use $\bar{g} = \mathbb{E}^Q\left[g\right]$ and rewrite the risk-free rate as:

$$r = \frac{\bar{g}\left(1-\tau_C\right)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E} + \frac{L}{1+L}\left(1-\widehat{\tau}_C\right)} = \frac{\bar{g}\xi(1+L)}{1+L\psi}. \tag{A.16}$$

This is an increasing function of ψ whenever $\psi < 1$.

We now turn to showing that the before-tax risk-free rate is higher when the debt tax shield applies. More formally, when r_{TS} denotes the risk-free rate when the debt tax shield applies and r_{nTS} when it does not apply, we have to show that $r_{TS} - r_{nTS} > 0$. When L_{TS} and L_{nTS} denote the leverage ratios with and without debt tax shield,[13] showing this inequality is equivalent to showing that

$$r_{TS} - r_{nTS} = \frac{\bar{g}\xi\left(1+L_{TS}\right)}{1+L_{TS}\xi} - \frac{\bar{g}\xi\left(1+L_{nTS}\right)}{1+L_{nTS}\bar{\psi}} > 0 \;\Leftrightarrow\; \frac{1+L_{TS}}{1+L_{TS}\xi} - \frac{1+L_{nTS}}{1+L_{nTS}\bar{\psi}} > 0. \tag{A.17}$$

Here $\bar{\psi} = \frac{1-\tau_E}{1-\tau_B}$ denotes the value of ψ in the absence of the debt tax shield. Due to the assumed tax advantage to debt with the debt tax shield, it holds that $\xi < 1$. Two cases must be considered: 1) $\xi < 1 < \bar{\psi}$ and 2) $\xi < \bar{\psi} < 1$.

We begin by considering the first case with $\xi < 1 < \bar{\psi}$. In that case, there is a tax advantage to debt with the debt tax shield, but not in its absence. Hence, $L_{nTS} = 0$ and Equation (A.17) simplifies to

$$\frac{1+L_{TS}}{1+L_{TS}\xi} - 1 > 0, \tag{A.18}$$

which is trivially fulfilled.

We next look at the second case with $\xi < \bar{\psi} < 1$. In that case, using $\frac{\xi}{1-\tau_C} = \bar{\psi}$, the

[13] We do not explicitly model any mechanism at this stage for choosing the optimal leverage in these two cases, but it is reasonable to assume that the optimal leverage is higher in the case with the debt tax shield than without.

inequality in (A.17) can be rewritten as follows:

$$\begin{aligned}(1+L_{TS})\left(1+L_{nTS}\frac{\xi}{1-\tau_C}\right) &> (1+L_{nTS})(1+L_{TS}\xi) \quad \Leftrightarrow \\ L_{TS}(1-\tau_C)(1-\xi) &> L_{nTS}(1-\tau_C-\xi) - L_{nTS}L_{TS}\xi\tau_C,\end{aligned} \tag{A.19}$$

which is fulfilled for $L_{nTS} \leq L_{TS}$. We have thus shown that the condition from inequality (A.17) holds for both cases. Hence, it holds that $r_{TS} - r_{nTS} > 0$.

We next show that even though the debt tax shield increases the pre-tax risk-free rate, the corporate after-tax risk-free rate with the debt tax shield is still lower than without. Hence, the increase in the pre-tax risk-free rate only leads to a smaller reduction of the after-tax risk-free rate with the debt tax shield, but not to a higher after-tax risk-free rate. More technically, we want show that

$$r_{TS}(1-\tau_C) - r_{nTS} < 0. \tag{A.20}$$

Inserting the values for r_{TS} and r_{nTS}, respectively, the inequality in (A.20) can be rewritten as:

$$\begin{aligned}&\frac{\bar{g}\xi(1+L_{TS})(1-\tau_C)}{1+L_{TS}\xi} - \frac{\bar{g}\xi(1+L_{nTS})}{1+L_{nTS}\bar{\psi}} < 0 \;\Leftrightarrow \\ &\frac{(1+L_{TS})(1-\tau_C)}{1+L_{TS}\xi} - \frac{1+L_{nTS}}{1+L_{nTS}\bar{\psi}} < 0.\end{aligned} \tag{A.21}$$

For the case with $L_{nTS} = 0$ this reduces to:

$$(1+L_{TS})(1-\tau_C) < 1+L_{TS}\xi \;\Leftrightarrow\; 1+L_{TS} < \frac{1}{1-\tau_C} + L_{TS}\frac{1-\tau_E}{1-\tau_B}. \tag{A.22}$$

As $L_{nTS} = 0$ and there is no tax advantage to debt when the debt tax shield does not apply, $1-\tau_E > 1-\tau_B$. Hence, the inequality in (A.22) is clearly satisfied.

When there is a tax advantage to debt in case the debt tax shield does not apply, we have $1-\tau_E < 1-\tau_B$ and $\bar{\psi} < 1$. In that case, the fraction $(1+L_{nTS})/(1+L_{nTS}\bar{\psi})$ is an increasing function of L_{nTS}. It is then sufficient to prove the inequality in (A.22) for the maximum possible value of L_{nTS}, $L_{nTS} = L_{TS}$. For that case, the inequality becomes:

$$\begin{aligned}&r_{TS}(1-\tau_C) - r_{nTS} < 0 \;\Leftrightarrow\; \frac{(1+L_{TS})(1-\tau_C)}{1+L_{TS}\xi} < \frac{1+L_{TS}}{1+L_{TS}\bar{\psi}} \;\Leftrightarrow \\ &(1+L_{TS})(1-\tau_C)\left(1+L_{TS}\bar{\psi}\right) < (1+L_{TS})(1+L_{TS}\xi) \;\Leftrightarrow \\ &(1+L_{TS})(1-\tau_C) < (1+L_{TS}).\end{aligned} \tag{A.23}$$

This is also clearly satisfied, which proves the inequality in (A.20) in all cases.

Maximum level of corporate leverage

From Equation (17), the risk-free rate depends on the leverage ratio, L. The realized return on the real investment and the expected return under the risk-neutral measure, cf. Equation (3), can then be written as shown in Equations (A.24) and (A.25), respectively:

$$\frac{P_t}{E_{t-1}^a} = 1 + g\left(1-\tau_C\right) + L\left(g\left(1-\tau_C\right) - r\left(1-\widehat{\tau}_C\right)\right) \tag{A.24}$$

$$\mathbb{E}^Q\left[\frac{P_t}{E_{t-1}^a}\right] = 1 + \bar{g}\left(1-\tau_C\right) + L\left(\bar{g}\left(1-\tau_C\right) - r\left(1-\widehat{\tau}_C\right)\right). \tag{A.25}$$

The right hand sides are both time independent and scale invariant. Hence, it is a time-consistent assumption to assume a constant leverage ratio.

The maximum possible degree of corporate leverage that ensures a non-negative return on equity has to fulfill the constraint

$$g\left(1-\tau_C\right) + L\left(g\left(1-\tau_C\right) - r\left(1-\widehat{\tau}_C\right)\right) \geq 0. \tag{A.26}$$

This inequality has to hold for all possible values of g. In particular, it has to hold for the lowest possible g, g_M, where the risk of violating the constraint is highest. Hence,

$$\begin{aligned} & g_M\left(1-\tau_C\right) + L\left(g_M\left(1-\tau_C\right) - r\left(1-\widehat{\tau}_C\right)\right) \geq 0 \;\Leftrightarrow \\ & \left(1+L\right)g_M\left(1-\tau_C\right) \geq L\frac{\bar{g}\xi(1+L)}{1+L\psi}\left(1-\widehat{\tau}_C\right) \quad \Leftrightarrow \\ & g_M \geq L\frac{\bar{g}\psi}{1+L\psi} \quad \Leftrightarrow \quad L \leq \frac{g_M}{\left(\bar{g}-g_M\right)\psi}, \end{aligned} \tag{A.27}$$

which verifies the condition stated in Equation (14).

Proof of item 2

In explicit form, the total tax revenue TTR_t at time t is given by

$$TTR_t = \tau_E\left(P_t - E_{t-1}^a\right) + (\tau_B - \widehat{\tau}_C)r_{t-1}\delta_{t-1}^a + \tau_C I_{t-1}^a g_t. \tag{A.28}$$

Inserting the terms from Equations (3) and (17), it can be written as

$$TTR_t = I_{t-1}^a \cdot \left[\widetilde{\tau} g_t - \bar{g}\frac{L\xi}{1+L\psi}\left(\widehat{\tau} - \tau_B\right)\right]. \tag{A.29}$$

Identical expected tax revenues before and after the removal of an existing debt tax shield then require that

$$\widetilde{\tau}_{TS}\mathbb{E}\left[g_t\right] - \bar{g}\frac{L_{TS}\xi_{TS}}{1+L_{TS}\psi_{TS}}\left(\widetilde{\tau}_{TS} - \tau_B\right) = \widetilde{\tau}_{nTS}\mathbb{E}\left[g_t\right] - \bar{g}\frac{L_{nTS}\xi_{nTS}}{1+L_{nTS}\psi_{nTS}}\left(\tau_E - \tau_B\right), \tag{A.30}$$

in which $\widetilde{\tau}_{TS}$, L_{TS}, ψ_{TS}, and ξ_{TS} denote the expressions of $\widetilde{\tau}$, L, ψ, and ξ when the debt tax shield applies, and $\widetilde{\tau}_{nTS}$, L_{nTS}, ψ_{nTS}, and ξ_{nTS} when it does not apply. Similarly, we use the notation τ_C^{TS} and τ_C^{nTS} for the corporate tax rate in the presence of the debt tax shield as well as the corporate tax rate after the removal of the debt tax shield that leads to an identical tax revenue, respectively.

When a tax advantage to debt applies, the firm chooses the maximum degree of corporate leverage, implies that the inequality in Equation (14) is fulfilled with equality and

$$L = \frac{g_M}{\left(\bar{g} - g_M\right)}\frac{1}{\psi} \quad \Rightarrow \quad L\psi = \frac{g_M}{\bar{g} - g_M}, \quad 1 + L\psi = \frac{\bar{g}}{\bar{g} - g_M}. \tag{A.31}$$

When the debt tax shield applies, $\xi_{TS} = \psi_{TS}$, whereas $\xi_{nTS} = (1 - \tau_C^{nTS})\psi_{nTS}$ if the debt tax shield does not apply. When the firm operates with leverage when the debt tax shield applies ($L_{TS} > 0$), depending on the parameter values of the model, it may be optimal to operate with or without leverage when the debt tax shield does not apply. We treat these two cases consecutively in our proof.

Case $L_{nTS} > 0$

We first turn to the case with $L_{nTS} > 0$. To keep expected tax revenues constant after abandoning an existing debt tax shield, it has to hold that

$$\begin{aligned}
\widetilde{\tau}_{TS}\mathbb{E}\left[g\right] - g_M\left(\widetilde{\tau}_{TS} - \tau_B\right) &= \widetilde{\tau}_{nTS}\mathbb{E}\left[g\right] - g_M\left(\tau_E - \tau_B\right)\left(1 - \tau_C^{nTS}\right) \quad \Leftrightarrow \\
\left(\widetilde{\tau}_{TS} - \widetilde{\tau}_{nTS}\right)\left(\mathbb{E}\left[g\right] - g_M\right) &= \left(\widetilde{\tau}_{nTS} - \tau_B - \left(\tau_E - \tau_B\right)\left(1 - \tau_C^{nTS}\right)\right) g_M \quad \Leftrightarrow \\
\left(\tau_C^{TS} - \tau_C^{nTS}\right)\left(\mathbb{E}\left[g\right] - g_M\right) &= g_M \tau_C^{nTS}\frac{1-\tau_B}{1-\tau_E} \quad \Leftrightarrow \\
\tau_C^{nTS} &= \tau_C^{TS}\frac{1}{\frac{g_M}{\mathbb{E}[g]-g_M}\frac{1-\tau_B}{1-\tau_E} + 1}.
\end{aligned} \tag{A.32}$$

From Equation (A.32), $\tau_C^{TS} > \tau_C^{nTS}$, i.e., when abandoning an existing debt tax shield, the government can decrease the corporate tax rate, while keeping its expected tax revenues constant.

Inserting the maximum degree of leverage from Equation (A.31) into the expression for the risk-free rate of interest (Equation (17)), it holds that

$$\begin{aligned} r_{TS} &= \bar{g}\xi_{TS} + g_M(1-\xi_{TS}) \\ r_{nTS} &= \bar{g}\xi_{nTS} + g_M(1-\psi_{nTS})(1-\tau_C^{nTS}) \\ r_{TS} - r_{nTS} &= \bar{g}\left(\xi_{TS} - \xi_{nTS}\right) + g_M\left[(1-\xi_{TS}) - (1-\psi_{nTS})(1-\tau_C^{nTS})\right], \end{aligned} \tag{A.33}$$

in which r_{TS} and r_{nTS} denote the risk-free rates when the debt tax shield applies and after its removal, respectively. We now plug the definitions of ξ_{TS}, ξ_{nTS}, and ψ_{nTS} in and multiply through by the term $(1-\tau_B)/(1-\tau_E)$ and arrive at:

$$\frac{1-\tau_B}{1-\tau_E}\left(r_{TS} - r_{nTS}\right) = \left(\bar{g} - g_M\right)\left(\tau_C^{nTS} - \tau_C^{TS}\right) + g_M\frac{1-\tau_B}{1-\tau_E}\tau_C^{nTS}. \tag{A.34}$$

Since $\mathbb{E}\left[g\right] > \bar{g} > g_M$ and $\tau_C^{TS} > \tau_C^{nTS}$, it holds with Equation (A.32) that

$$\frac{1-\tau_B}{1-\tau_E}\left(r_{TS} - r_{nTS}\right) > \left(\mathbb{E}\left[g\right] - g_M\right)\left(\tau_C^{nTS} - \tau_C^{TS}\right) + g_M\frac{1-\tau_B}{1-\tau_E}\tau_C^{nTS} = 0, \tag{A.35}$$

which verifies the claim that $r_{TS} - r_{nTS} > 0$, i.e., that the risk-free rate is higher when the debt tax shield applies.

As for the the claim that the after-tax corporate borrowing rate is smaller when the debt tax shield applies, we note that

$$r_{TS}(1-\tau_C^{TS}) - r_{nTS} < 0 \;\Leftrightarrow \tag{A.36}$$

$$\bar{g}\xi_{TS}(1-\tau_C^{TS}) + g_M(1-\xi_{TS})(1-\tau_C^{TS}) < \bar{g}\xi_{nTS} + g_M(1-\psi_{nTS})(1-\tau_C^{nTS}) \quad \Leftrightarrow \tag{A.37}$$

$$\bar{g}\left(\xi_{TS}(1-\tau_C^{TS}) - \xi_{nTS}\right) < g_M\left((1-\psi_{nTS})(1-\tau_C^{nTS}) - (1-\xi_{TS})(1-\tau_C^{TS})\right) \;\Leftrightarrow \tag{A.38}$$

$$\left(\bar{g} - g_M\right)\left((1-\tau_C^{TS})^2 - (1-\tau_C^{nTS})\right) < g_M\left(\tau_C^{TS} - \tau_C^{nTS}\right)\frac{1-\tau_B}{1-\tau_E}. \tag{A.39}$$

Since $\tau_C^{TS} > \tau_C^{nTS}$, the left handside of Equation (A.39) is negative, whereas the right handside is positive, which verifies that $r_{TS}\left(1-\tau_C^{TS}\right) - r_{nTS} > 0$, i.e., that the after-tax corporate borrowing rate is smaller when the debt tax shield applies.

Case $L_{nTS} = 0$

For $L_{nTS} = 0$, tax-revenue-neutrality with and without debt tax shield requires that

$$\widetilde{\tau}_{TS}\mathbb{E}\left[g\right] - \bar{g}\frac{L_{TS}\xi_{TS}}{1+L_{TS}\xi_{TS}}\left(\widetilde{\tau}_{TS} - \tau_B\right) = \widetilde{\tau}_{nTS}\mathbb{E}\left[g\right] \tag{A.40}$$

$$\Leftrightarrow \widetilde{\tau}_{TS}\mathbb{E}\left[g\right] - g_M\left(\widetilde{\tau}_{TS} - \tau_B\right) = \widetilde{\tau}_{nTS}\mathbb{E}\left[g\right]. \tag{A.41}$$

Since $\widetilde{\tau}_{TS} - \tau_B > 0$ (tax advantage to debt with tax shield), it follows that $\widetilde{\tau}_{TS} > \widetilde{\tau}_{nTS}$ and thus $\tau_C^{TS} > \tau_C^{nTS}$, i.e., the corporate tax rate is higher when the debt tax shield applies than when it does not apply. In explicit form, the relation between τ_C^{nTS} and τ_C^{TS} can be obtained by manipulating Equation (A.41):

$$\tau_C^{nTS} = \tau_C^{TS} \frac{\mathbb{E}[g] - g_M}{\mathbb{E}[g]} - \frac{g_M}{\mathbb{E}[g]} \frac{\tau_E - \tau_B}{1 - \tau_E}. \tag{A.42}$$

For the proof that the risk-free rate is higher when the debt tax shield applies, we know from Equation (17) that $r_{nTS} = \bar{g}\xi_{nTS}$. Hence,

$$r_{TS} - r_{nTS} = \bar{g}\left(\xi_{TS} - \xi_{nTS}\right) + g_M\left(1 - \xi_{TS}\right). \tag{A.43}$$

Plugging the definitions of ξ_{TS} and ξ_{nTS} in and multiplying through by $\frac{1}{\bar{g}}\frac{1-\tau_B}{1-\tau_E}$, it holds that

$$\frac{1}{\bar{g}}\frac{1-\tau_B}{1-\tau_E}\left(r_{TS} - r_{nTS}\right) = \tau_C^{nTS} - \tau_C^{TS}\frac{\bar{g} - g_M}{\bar{g}} + \frac{g_M}{\bar{g}}\frac{\tau_E - \tau_B}{1 - \tau_E}. \tag{A.44}$$

With $\mathbb{E}[g] > \bar{g}$ and Equation (A.42), it follows that the right handside of Equation (A.44) takes a positive value and thus $r_{TS} - r_{nTS} > 0$.

For the proof that the after-tax corporate borrowing rate is smaller when the debt tax shield applies, it holds that

$$r_{TS}(1 - \tau_C^{TS}) - r_{nTS} < 0 \;\Leftrightarrow\; \bar{g}\xi_{TS}(1 - \tau_C^{TS}) + g_M(1 - \xi_{TS})(1 - \tau_C^{TS}) < \bar{g}\xi_{nTS} \tag{A.45}$$

Since $\xi_{TS} \in (0,1)$ and $\bar{g} > g_M$, the l.h.s. of this inequality is bounded from above by $\bar{g}(1-\tau_C^{TS})$. Hence, a sufficient condition for the inequality in (A.45) to be satisfied is

$$1 - \tau_C^{TS} < \xi_{nTS} = \frac{(1 - \tau_C^{nTS})(1 - \tau_E)}{1 - \tau_B} \;\Leftrightarrow\; \frac{1 - \tau_C^{TS}}{1 - \tau_C^{nTS}} < \frac{1 - \tau_E}{1 - \tau_B}. \tag{A.46}$$

The l.h.s. of (A.46) does not exceed one, whereas the r.h.s. by assumption is larger than one, which verifies that $r_{TS}\left(1 - \tau_C^{TS}\right) - r_{nTS} < 0$, i.e., that the after-tax corporate borrowing rate is smaller when the debt tax shield applies. In total, this completes the proof of item 2.

Proof of item 3

From Equation (A.4), we get:

$$1=\sum_{m=1}^{M}\frac{\lambda_{t+1,j}^{m}}{\lambda_{t,j}}\left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}_t\right)\left(1-\tau_E\right)+\tau_E\right]. \tag{A.47}$$

Using Equation (A.7), this can be rewritten as

$$1=\frac{\rho}{M}\sum_{m=1}^{M}\frac{\left(\left(1-F_{t+1}\right)I_t^aG_m\right)^{-\gamma}}{\left(C_t^a\right)^{-\gamma}}\cdot\left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}_t\right)\left(1-\tau_E\right)+\tau_E\right]. \tag{A.48}$$

From here we can isolate C_t^a:

$$\begin{aligned}\left(C_t^a\right)^{-\gamma}&=\frac{\rho}{M}\left(\left(1-F_{t+1}\right)I_t^a\right)^{-\gamma}\cdot\\&\quad\sum_{m=1}^{M}G_m^{-\gamma}\left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}_t\right)\left(1-\tau_E\right)+\tau_E\right]\\C_t^a&\equiv I_t^a\left(1-F_{t+1}\right)\cdot H,\end{aligned} \tag{A.49}$$

where

$$H=\left(\frac{\rho}{M}\right)^{-\frac{1}{\gamma}}\left(\sum_{m=1}^{M}G_m^{-\gamma}\left(\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}_t\right)\left(1-\tau_E\right)+\tau_E\right)\right)^{-\frac{1}{\gamma}}. \tag{A.50}$$

This can be rewritten as

$$\begin{aligned}H^{-\gamma}&=\frac{\rho}{M}\sum_{m=1}^{M}G_m^{-\gamma}\left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}\right)\left(1-\tau_E\right)+\tau_E\right]\\&=\frac{\rho}{M}\sum_{k=1}^{M}G_k^{-\gamma}\sum_{m=1}^{M}\frac{G_m^{-\gamma}}{\sum_{k=1}^{M}G_k^{-\gamma}}\left[\left((1+L)\left(1+g_m\left(1-\tau_C\right)\right)-L\widehat{R}\right)\left(1-\tau_E\right)+\tau_E\right]\\&=\frac{\rho}{M}\sum_{k=1}^{M}G_k^{-\gamma}\mathbb{E}^Q\left[\left((1+L)\left(1+g\left(1-\tau_C\right)\right)-L\widehat{R}\right)\left(1-\tau_E\right)+\tau_E\right].\end{aligned} \tag{A.51}$$

Using Equation (A.14), Equation (A.51) becomes:

$$H^{-\gamma}=\frac{\rho}{M}\sum_{m=1}^{M}G_m^{-\gamma}\widetilde{R}, \tag{A.52}$$

and thus

$$H = \left(\frac{\rho}{M} \sum_{m=1}^{M} G_m^{-\gamma} \right)^{-\frac{1}{\gamma}} \widetilde{R}^{-\frac{1}{\gamma}}. \tag{A.53}$$

It further holds that

$$W_t^a = C_t^a + I_t^a = I_t^a \left(1 - F_{t+1}\right) H + I_t^a = I_t^a \left(\left(1 - F_{t+1}\right) H + 1\right) \;\Rightarrow \tag{A.54}$$

$$I_t^a = \frac{W_t^a}{1 + \left(1 - F_{t+1}\right) H}, \tag{A.55}$$

and, consequently, that F_t follows the backward difference equation

$$F_t = \frac{1}{1 + (1 - F_{t+1})H}, \tag{A.56}$$

with boundary condition $F_N = 0$. The solution to Equation (A.56) is:

$$F_t = \begin{cases} \frac{1-H^{N-t}}{1-H^{N-t+1}} & \text{for } H \neq 1 \\ \frac{N-t}{N-t+1} & \text{for } H = 1. \end{cases} \tag{A.57}$$

The fact that F_t decreases over time can be shown directly – for the cases $H < 1$ and $H > 1$, respectively, or by backwards induction, using that that $0 \leq F_t < 1$:

$$F_t = \frac{1}{1 + (1 - F_{t+1})H} > F_{t+1} \;\Leftrightarrow\; 1 > F_{t+1} + (1 - F_{t+1})F_{t+1}H \;\Leftrightarrow\; 1 > F_{t+1}H. \tag{A.58}$$

Since $F_N = 0$ this inequality in trivially true for N. Assume that it is true for N,$N-1,\ldots,t+1$. Then:

$$\begin{aligned} & F_{t+1}H < 1 \;\Leftrightarrow\; (1 - F_{t+1})H > H - 1 \;\Leftrightarrow\; 1 + (1 - F_{t+1})H > H \;\Leftrightarrow \\ & F_t H = \frac{H}{1 + (1 - F_{t+1})H} < 1, \end{aligned} \tag{A.59}$$

which verifies the claim that F_t decreases over time. The limiting behavior for $N \to \infty$, is trivial for $H \leq 1$. For $H > 1$ we have

$$F_t = \frac{1 - H^{N-t}}{1 - H^{N-t+1}} = \frac{H^{-(N-t)} - 1}{H^{-(N-t)} - H} \underset{N\to\infty}{\longrightarrow} \frac{1}{H}, \tag{A.60}$$

which verifies Equation (22).

The debt tax shield increases economic growth

From Equation (20), H is proportional to $\widetilde{R}^{-\frac{1}{\gamma}}$. Using Equation (A.56) we can show by induction that F_t increases when the debt tax shield is introduced. For $t = N-1$ it holds that

$$F_{N-1} = \frac{1}{1+H}. \tag{A.61}$$

We know that H is lower with the debt tax shield than without. Hence, F_{N-1} is larger with the debt tax shield than without. For the next step, we assume that for $t < N-1$, F_{t+1} is higher when the debt tax shield applies. At time t it holds that

$$F_t = \frac{1}{1+(1-F_{t+1})\,H}. \tag{A.62}$$

H is lower and (by the induction assumption) F_{t+1} is higher with the debt tax shield than without. Hence, the denominator in Equation (A.62) decreases and, consequently, F_t increases as a result of the debt tax shield.

Utility from aggregate consumption is higher with debt tax shield

From Equation (23) (to be proven below) we have that $C^a_{t+1} = C^a_t\,(G_{t+1}/H)$. The entire consumption path can then be expressed in terms of the initial consumption C^a_0:

$$C^a_t = C^a_0 \prod_{j=1}^{t} \frac{G_j}{H} = W^a_0(1-F_0) \prod_{j=1}^{t} \frac{G_j}{H}. \tag{A.63}$$

Hence, by the i.i.d. assumption of G_j we have:

$$\begin{aligned}
\frac{1}{1-\gamma} \sum_{t=0}^{N} \rho^t \mathbb{E}_0\left(C^a_t\right)^{1-\gamma} &= \frac{1}{1-\gamma} \sum_{t=0}^{N} \rho^t \left(W^a_0(1-F_0)\right)^{1-\gamma} \mathbb{E}_0\left(\prod_{j=0}^{t}\left[\frac{G_j}{H}\right]^{1-\gamma}\right) \\
&= \frac{1}{1-\gamma} \sum_{t=0}^{N} \rho^t \left(W^a_0(1-F_0)\right)^{1-\gamma} \prod_{j=0}^{t} \mathbb{E}_0\left(\left[\frac{G_j}{H}\right]^{1-\gamma}\right) \\
&= \frac{1}{1-\gamma} \left(W^a_0(1-F_0)\right)^{1-\gamma} \sum_{t=0}^{N} \left(\frac{\rho\mathbb{E}[G^{1-\gamma}]}{H^{1-\gamma}}\right)^t.
\end{aligned} \tag{A.64}$$

From the solution for F_t we get:

$$1 - F_0 = 1 - \frac{1-H^N}{1-H^{N+1}} = \frac{1-H^{N+1}-(1-H^N)}{1-H^{N+1}} = H^N \frac{1-H}{1-H^{N+1}} = \frac{1}{\sum_{j=0}^{N} H^{-j}}, \tag{A.65}$$

which can be inserted into Equation (A.64):

$$\frac{1}{1-\gamma}\sum_{t=0}^{N}\rho^t\mathbb{E}_0\left(C_t^a\right)^{1-\gamma}=\frac{1}{1-\gamma}\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t H^{-t(1-\gamma)}. \quad \text{(A.66)}$$

Differentiation after H in the expected utility produces two terms:

$$\begin{aligned}&\frac{\partial}{\partial H}\left(\frac{1}{1-\gamma}\sum_{t=0}^{N}\rho^t\mathbb{E}_0\left(C_t^a\right)^{1-\gamma}\right)=\\ &\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-2}\sum_{j=0}^{N}jH^{-j-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t H^{-t(1-\gamma)}- \\ &\left(W_0^a\right)^{1-\gamma}\left(\sum_{j=0}^{N}H^{-j}\right)^{\gamma-1}\sum_{t=0}^{N}\left(\rho\mathbb{E}[G^{1-\gamma}]\right)^t tH^{-t(1-\gamma)-1}.\end{aligned} \quad \text{(A.67)}$$

The sign of this derivative is negative if

$$\sum_{j=0}^{N}j\frac{H^{-j}}{\sum_{j=0}^{N}H^{-j}}-\sum_{t=0}^{N}t\frac{\left(\frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]}\right)^{-t}}{\sum_{t=0}^{N}\left(\frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]}\right)^{-t}} \quad \text{(A.68)}$$

is negative. The expression in Equation (A.68) is the difference between two "duration measures" of an annuity; one with the discount factor H and the other with the discount factor $H^{1-\gamma}/\rho\mathbb{E}[G^{1-\gamma}]$. The duration of an annuity is a decreasing function of the discount factor, so the claim that the expected utility of the aggregate consumption stream varies inversely with H is proven if $H > H^{1-\gamma}/\rho\mathbb{E}[G^{1-\gamma}]$; a condition which can be reformulated as shown in Equation (A.69):

$$H>\frac{H^{1-\gamma}}{\rho\mathbb{E}[G^{1-\gamma}]}\;\Leftrightarrow\;\rho\mathbb{E}[G^{1-\gamma}]>H^{-\gamma}=\rho\mathbb{E}_0\left[G^{-\gamma}\right]\widetilde{R}\;\Leftrightarrow\;\widetilde{R}<\bar{G}. \quad \text{(A.69)}$$

With Equation (17) this statement is equivalent to

$$\frac{\bar{g}\left(1-\tau_C\right)\left(1-\tau_B\right)}{\frac{1}{1+L}\frac{1-\tau_B}{1-\tau_E}+\frac{L}{1+L}\left(1-\widehat{\tau}_C\right)}<\bar{g} \quad \text{(A.70)}$$

$$\Leftrightarrow\left(1-\tau_C\right)\left(1-\tau_B\right)\left(1-\tau_E\right)<\frac{1}{1+L}\left(1-\tau_B\right)+\frac{L}{1+L}\left(1-\widehat{\tau}_C\right)\left(1-\tau_E\right), \quad \text{(A.71)}$$

which is obviously true. Hence, the expected utility from consumption increases when H decreases. From the proof of item 3, we know that H decreases when the interest rate increases, including the case where the debt tax shield applies versus the case where it does not apply. Hence, the claim in item 2 is proven.

Proof of item 4

From Equation (A.12) it holds that

$$\frac{C^a_{t+1}}{C^a_t} = \frac{1-F_{t+1}}{1-F_t} F_t G = \frac{(1-F_{t+1})H}{\frac{1}{F_t}-1}\frac{G}{H} = \frac{G}{H}, \tag{A.72}$$

where the last equality is a simple rewriting of Equation (A.56). Because of the linear sharing rule property, this is also the growth rate of consumption on the individual household level:

$$\frac{C^a_{t+1}}{C^a_t} = \frac{C_{t+1,j}}{C_{t,j}} = \frac{1}{H}G. \tag{A.73}$$

which verifies Equation (23).

B Proof of Theorem 2

Proof of item 1

The evolution of wealth from Equation (10) can be rewritten as:

$$\begin{aligned} W_{t,j} =& I^a_{t-1}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1}{n}\right)G_t + I^a_{t-1}\left(\alpha_{t-1,j}-\frac{1}{n}\right)(\widetilde{\tau}-\tau_E) - \\ & I^a_{t-1}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\tau_E)+\frac{1}{n}\right)\frac{L}{1+L}\widehat{R} + \frac{1}{1+L}I^a_{t-1}\tau_E\left(\alpha_{t-1,j}-\frac{1}{n}\right) + \\ & \left(\beta_{t-1,j}+\delta_{t-1,j}\right)\widetilde{R} + \frac{I^a_{t-1}}{n}\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}_C\right). \end{aligned} \tag{A.74}$$

Because investors aim at a linear risk-sharing rule, the bond position has to remove the (predictable) terms not related to G_t. Hence:

$$\begin{aligned} \beta_{t-1,j}+\delta_{t-1,j} =& \frac{I^a_{t-1}}{\widetilde{R}}\Bigg[\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\tau_E)+\frac{1}{n}\right)\frac{L}{1+L}\widehat{R} - \\ & \left(\alpha_{t-1,j}-\frac{1}{n}\right)(\widetilde{\tau}-\tau_E) - \frac{1}{1+L}\tau_E\left(\alpha_{t-1,j}-\frac{1}{n}\right) - \\ & \frac{1}{n}\frac{L}{1+L}r\left(\tau_B-\widehat{\tau}_C\right)\Bigg]. \end{aligned} \tag{A.75}$$

Collecting and reorganizing terms results in the expression in Equation (24).

Proof of items 2 and 3

With the bond position from Equation (24), Equation (A.74) becomes:

$$W_{t,j} = I_{t-1}^{a}\left(\left(\alpha_{t-1,j}-\frac{1}{n}\right)(1-\widetilde{\tau})+\frac{1}{n}\right)G_t = I_{t-1}^{a}\left(\widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau})+\frac{1}{n}\right)G_t, \tag{A.76}$$

where we for shorter hand notation have redefined the equity position as the deviation from an equal share:

$$\widetilde{\alpha}_{t,j} = \alpha_{t,j} - \frac{1}{n}. \tag{A.77}$$

Household j's consumption is then given by inserting the expressions for the equity and the bond position, $E_{t,j} = \frac{1}{1+L} I_t^a \alpha_{t,j}$, and Equation (24):

$$\begin{aligned} C_{t,j} =& W_{t,j} - E_{t,j} - (\beta_{t,j} + \delta_{t,j}) \\ =& I_{t-1}^{a}\left(\widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau})+\frac{1}{n}\right)G_t - \frac{1}{1+L} I_t^a \alpha_{t,j} - \\ & \alpha_{t,j} I_t^a \frac{L}{1+L} - \frac{1}{\widetilde{R}} I_t^a \widetilde{\alpha}_{t,j}\left(\frac{L}{1+L} r\left(\tau_B - \widehat{\tau}\right) - \widetilde{\tau}\right) \\ =& I_{t-1}^{a}\widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau})G_t - I_t^a \widetilde{\alpha}_{t,j} Y(1-\widetilde{\tau}) + \frac{1}{n}\left(I_{t-1}^a G_t - I_t^a\right), \end{aligned} \tag{A.78}$$

in which

$$Y = \frac{(\widetilde{R}-\widetilde{\tau}) + \frac{L}{1+L} r\left(\tau_B - \widehat{\tau}\right)}{(1-\widetilde{\tau})\widetilde{R}}. \tag{A.79}$$

After insertion of the interest rate r from Equation (17) and some manipulations, this reduces to $Y = \frac{\bar{G}}{\widetilde{R}}$. From Equation (A.78), the consumption-wealth ratio follows:

$$\frac{C_{t,j}}{I_{t-1}^a G_t} = \widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau}) + \frac{1-F_t}{n} - F_t \widetilde{\alpha}_{t,j} Y (1-\widetilde{\tau}). \tag{A.80}$$

Because of the linear risk sharing rule, every individual household j's consumption share is constant. We denote it by $\omega_j \equiv \frac{C_{t,j}}{I_{t-1}^a G_t (1-F_t)}$. It thus holds that

$$\left(\omega_j - \frac{1}{n}\right)(1-F_t) = \widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau}) - \widetilde{\alpha}_{t,j}(1-\widetilde{\tau}) F_t Y. \tag{A.81}$$

and use this to rewrite (A.81) as

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_t) = \widetilde{\alpha}_{t-1,j}(1-\widetilde{\tau}) - F_t Y \widetilde{\alpha}_{t,j}(1-\widetilde{\tau}). \tag{A.82}$$

At time $t = N$, it holds that

$$\left(\omega_j - \frac{1}{n}\right) = \widetilde{\alpha}_{N-1,j}(1-\widetilde{\tau}). \tag{A.83}$$

Working backwards, it holds at time $t = N-1$ that

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_{N-1}) = \widetilde{\alpha}_{N-2,j}(1-\widetilde{\tau}) - F_{N-1} Y \widetilde{\alpha}_{N-1,j}(1-\widetilde{\tau}) \quad \Leftrightarrow \tag{A.84}$$

$$\widetilde{\alpha}_{N-1,j}(1 - F_{N-1}) = \widetilde{\alpha}_{N-2,j} - F_{N-1} Y \widetilde{\alpha}_{N-1,j} \quad \Leftrightarrow \tag{A.85}$$

$$\widetilde{\alpha}_{N-2,j} = \widetilde{\alpha}_{N-1,j}\left(1 - F_{N-1} + F_{N-1}Y\right). \tag{A.86}$$

Similarly, at time $t = N-2$ it holds that

$$\left(\omega_j - \frac{1}{n}\right)(1 - F_{N-2}) = \widetilde{\alpha}_{N-3,j}(1-\widetilde{\tau}) - F_{N-2} \widetilde{\alpha}_{N-2} Y (1-\widetilde{\tau}) \quad \Leftrightarrow \tag{A.87}$$

$$\widetilde{\alpha}_{N-1,j}(1 - F_{N-2}) = \widetilde{\alpha}_{N-3,j} - F_{N-2} \widetilde{\alpha}_{N-2} Y. \tag{A.88}$$

Inserting $\widetilde{\alpha}_{N-2}$ from Equation (A.86), we arrive at

$$\widetilde{\alpha}_{N-3,j} = \widetilde{\alpha}_{N-1,j}\left(1 - F_{N-2} + F_{N-2}Y\left(1 - F_{N-1} + F_{N-1}Y\right)\right). \tag{A.89}$$

More generally, $\widetilde{\alpha}_{t,j}$ satisfies the backwards difference equations

$$\widetilde{\alpha}_{t,j}(1-\widetilde{\tau}) = \left(\omega_j - \frac{1}{n}\right) Z_t, \tag{A.90}$$

in which

$$Z_t = 1 - F_{t+1} + F_{t+1} Y Z_{t+1}. \tag{A.91}$$

With the terminal condition $Z_N = 0$, we have an explicit solution for Z_t in Equation (A.91):

$$Z_t = \frac{H^{N-t} - Y^{N-t}}{H - Y} \frac{(H-1)}{H^{N-t} - 1} = \frac{\sum_{k=0}^{N-t-1}\left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t-1}\left(\frac{1}{H}\right)^k}. \tag{A.92}$$

To find an explicit solution for consumption and investments, it is important to disentangle the relationship between ω_j and $\alpha_{t,j}$. From Equation (A.75) combined with Equation (A.78) we have:

$$C_{0,j} = W_{0,j} - \frac{1}{n} I_0^a - I_0^a \widetilde{\alpha}_{0,j} Y (1 - \widetilde{\tau}). \tag{A.93}$$

With

$$C_0^a = W_0^a (1 - F_0), \tag{A.94}$$

it then holds that

$$\omega_j = \frac{C_{0,j}}{C_0^a} = \frac{W_{0,j}}{W_0^a (1 - F_0)} - \frac{W_0^a F_0}{n W_0^a (1 - F_0)} - \frac{W_0^a F_0}{W_0^a (1 - F_0)} \widetilde{\alpha}_{0,j} Y (1 - \widetilde{\tau}) \tag{A.95}$$

$$= \frac{W_{0,j}}{W_0^a (1 - F_0)} - \frac{F_0}{1 - F_0} \left(\widetilde{\alpha}_{0,j} Y (1 - \widetilde{\tau}) + \frac{1}{n} \right). \tag{A.96}$$

Plugging (A.90) in, gives

$$\omega_j (1 - F_0) = \frac{W_{0,j}}{W_0^a} - F_0 \left(\left(\omega_j - \frac{1}{n} \right) Z_0 Y + \frac{1}{n} \right) \quad \Leftrightarrow \tag{A.97}$$

$$\left(\omega_j - \frac{1}{n} \right) (1 - F_0 + F_0 Y Z_0) = \frac{W_{0,j}}{W_0^a} - \frac{1}{n} \quad \Leftrightarrow \tag{A.98}$$

$$\omega_j - \frac{1}{n} = \frac{\frac{W_{0,j}}{W_0^a} - \frac{1}{n}}{1 - F_0 + F_0 Y Z_0} = \frac{\frac{W_{0,j}}{W_0^a} - \frac{1}{n}}{Z_{-1}}. \tag{A.99}$$

With $D = 1/Z_{-1}$ and because $\frac{W_{0,j}}{W_0^a} = \alpha_{0-,j}$, this can be written as $\omega_j - \frac{1}{n} = D \left(\alpha_{0-,j} - \frac{1}{n} \right)$.

To complete the proof, we must show that Z_t is a decreasing sequence in $[1,\infty)$. It is easily seen that $Z_{N-1} = 1$. For the remainder, we show that $Y > 1$, because given this, it follows from Equation (29) that Z_t is decreasing. It holds that

$$Z_t = \frac{\sum_{k=0}^{N-t-1} \left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t-1} \left(\frac{1}{H}\right)^k} \quad \text{and} \quad Z_{t-1} = \frac{\sum_{k=0}^{N-t} \left(\frac{Y}{H}\right)^k}{\sum_{k=0}^{N-t} \left(\frac{1}{H}\right)^k}. \tag{A.100}$$

Hence,

$$Z_{t-1} - Z_t = \frac{\sum_{k=0}^{N-t} \left(\frac{Y}{H}\right)^k \sum_{p=0}^{N-t-1} \left(\frac{1}{H}\right)^p - \sum_{k=0}^{N-t-1} \left(\frac{Y}{H}\right)^k \sum_{p=0}^{N-t} \left(\frac{1}{H}\right)^p}{\sum_{k=0}^{N-t} \left(\frac{1}{H}\right)^k \sum_{k=0}^{N-t-1} \left(\frac{1}{H}\right)^k}. \tag{A.101}$$

To prove that this difference is positive, we ignore the denominator, that is trivially positive, and look at the numerator in the following. It holds that

$$\sum_{k=0}^{N-t}\sum_{p=0}^{N-t-1} Y^k H^{-(k+p)} - \sum_{k=0}^{N-t-1}\sum_{p=0}^{N-t} Y^k H^{-(k+p)} = \tag{A.102}$$

$$\sum_{k=0}^{N-t-1}\sum_{p=0}^{N-t} Y^p H^{-(k+p)} - \sum_{k=0}^{N-t-1}\sum_{p=0}^{N-t} Y^k H^{-(k+p)} = \tag{A.103}$$

$$\sum_{k=0}^{N-t-1}\sum_{p=0}^{N-t-1} H^{-(k+p)}\left(Y^p - Y^k\right) + \sum_{k=0}^{N-t-1} H^{-(k+N-t)}\left(Y^{N-t} - Y^k\right) = \tag{A.104}$$

$$\sum_{k=0}^{N-t-1} H^{-(k+N-t)}\left(Y^{N-t} - Y^k\right) \quad \text{(by symmetry)}. \tag{A.105}$$

When $Y > 1$, this expression is clearly positive. So it remains to verify that $Y > 1$. This follows in a straightforward manner from Equation (17):

$$Y > 1 \quad \Leftrightarrow \quad \bar{G} > \widetilde{R} \quad \Leftrightarrow \quad \bar{g} > \widetilde{r} \quad \Leftrightarrow \; 1 > \frac{\xi(1+L)}{1+L\psi} \quad \Leftrightarrow \quad 1 + L\psi > \xi(1+L), \tag{A.106}$$

since $\xi < 1$ and $\xi \leq \psi$. Hence, it follows that $Z_t < Z_{t-1}$ and, with $Z_N = 1$, that Z_t is a decreasing sequence in $[1,\infty)$. Hence, poorer households' equity shares increase over time and richer households' decrease. For $N \to \infty$, the equity share converges to

$$\alpha_j = \frac{1}{n} + \left(\alpha_{0-,j} - \frac{1}{n}\right)\frac{1}{1-\widetilde{\tau}}, \tag{A.107}$$

which completes the proof of items 2 and 3.