# Expected shortfall assessment in commodity (L)ETF portfolios with semi-nonparametric specifications

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## Abstract

This paper studies the risk assessment of semi-nonparametric (SNP) distributions for leveraged exchange trade funds, (L)ETFs. We applied the SNP model with dynamic conditional correlations (DCC) and EGARCH innovations, and implement recent techniques to backtest Expected Shortfall (ES) to portfolios formed by bivariate combinations of major (L)ETFs on metal (Gold and Silver) and energy (Oil and Gas) commodities. Results support that multivariate SNP-DCC model outperforms the Gaussian-DCC and provides accurate risk measures for commodity (L)ETFs.

*Keywords:* Gram-Charlier; DCC, Expected shortfall; Backtesting; Commodity ETF.

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## 1. Introduction

Since 2000s investment in commodities has rapidly grown and this growth has been benefited from the creation of exchange-traded funds (ETFs) (Yan and Garcia, 2017), which allow to trade on commodities as if they were stocks. ETFs are employed to track the price of a reference asset, whereas the aim of leveraged exchanged-traded funds (LETFs) is to amplify the returns of the benchmark asset. So far, the leverage ratio is available up to (minus and plus) three times the target asset returns. However, LETFs and inverse ETFs have been criticized from a regulatory point of view (e.g. Financial Stability Board and Bank for International Settlements) due to its systemic risk and this is maybe explained by the timing and their rebalancing activities (Shum et al., 2016).

For an excellent and recent review of the literature regarding ETFs (also LETFs) and financial markets, see Ben-David et al. (2017). The authors find that ETFs add (nonfundamental) volatility into market prices and have effects on the correlation structure of returns. Based on the results of other studies, it is also found that LETFs injected market volatility in the subprime crisis. Several events where ETFs have had impact on market instabilities are also documented. In particular, on June 20, 2013 most of the prices in emerging markets plunged and the prices of ETFs declined. Another event on August 24, 2015, the high volatility of almost half of the US equity markets was due to ETF trading. On that day, 11 ETFs were interrupted more than 10 times. This was a consequence of a decision of the Security Exchange Commission (SEC), which, after the Flash Crash of 2010, has the power to interrupt trading of securities, including ETFs, if the prices show extreme volatility fluctuations.

In addition, commodities exhibit higher volatilities than equity assets because, among other properties, they cannot be easily stored. In spite of these facts, little work has devoted to risk quantification of commodity (L)ETFs, and this work contributes to the literature in order to prevent future market crashes based on the adequate quantification of market risks when trading (L)ETFs.

In a recent paper, Del Brio et al. (2018) investigate the risk assessment for individual commodity ETFs under different density specifications. The authors apply traditional backtesting methods for Value-at-Risk (VaR) and a recent proposal for Expected Shortfall (ES). Nevertheless, investment strategies usually involve portfolio decisions and thus the extension of these results to the multivariate framework is worthwhile. This paper covers this gap by comparing ES backtesting results for commodity ETF and LETF portfolios. To this end, we extend the study in Del Brio et al. (2011), by applying semi-nonparametric (SNP) dynamic conditional correlation (DCC) model in a risk measure framework. To the best of our knowledge, this is the first work that employs the SNP-DCC model to provide ES measures for portfolio risk. The SNP-DCC model encompasses the Gaussian-DCC (Engle, 2002) by accounting for skewness and kurtosis, as well as further high-order moments. This issue is extremely important in commodity assets, which not only exhibit high volatility but also feature skewness, leptokurtosis and extreme events. For instance, in a recent paper Fernandez-Perez et al. (2018) find a negative relationship between skewness of commodity futures returns and expected returns.

Most of the literature concerning commodity portfolios has been based on the investment (asset allocation) point of view (Daskalaki and Skiadopoulos, 2011; You and Daigler, 2013; Levine et al., 2016). With the introduction of sophisticated multivariate models, especially multivariate GARCH models, there have been broad applications in risk management and quantification. Multivariate GARCH models have been employed not only to analyze hedging in commodities (Coakley, 2008; Fernandez, 2008; Aroui, et al., 2015; Zhang and Choudry, 2015; Abul Basher and Sadorsky, 2016; Charalampous and Madlener, 2016; Ulusoy and Onbirler, 2017), and interactions between assets and markets (see e.g. Manera et al., 2013; Mensi et al., 2014; Gardebroek et al., 2016; Kang et al., 2017; Roy and Roy, 2017; Sanjuan-Lopez and Dawson, 2017, for commodity market applications) but also to estimate portfolio VaR (Berens et al., 2015; Huang et al., 2016; Amendola and Candila, 2017; Kole et al., 2017; Scheffer and Weiss, 2017).

On the other hand, LETFs have been recently the focus of study in the financial literature. For instance, Leung et al. (2017) analyze the implied volatility surfaces of LETFs; Giannetti (2017) proposes a panel data study to investigate the dynamics of LETFs returns; March-Dallas et al. (2018) examine the differences of liquidity factors between (unlevered) ETFs and LETFs; Jiang and Peterburgsky (2017) analyze investment strategies involving LETFs; Tang et al. (2014) examine the tracking performance of international LETFs; and Charupat and Miu (2013) analyze the pricing efficiency of LETFs. Nevertheless, few papers analyze Commodity LETFs (Guo and Leung 2015; Leung and Ward 2015).

For portfolio commodity risk quantification, White and Dawson (2005) show that multivariate GARCH models outperform RiskMetrics model to estimate VaR for commodities of UK arable farms. Moreover, Zolotko and Okhrin (2014) introduce a family of dynamic conditional correlation models based on hierarchical Archimedean copulae (HAC-DCC) models and found that certain of this type of models produce accurate VaR estimates by modelling commodity forward curves. In a similar study, Aepli et al. (2017) apply DCC model to multivariate elliptical copulas and show that the static Clayton copula, followed by the dynamic Clayton model are more suitable for estimating risk applied to commodity futures portfolio.

On the other hand, Zou et al. (2015) show that entropy-optimized bivariate empirical mode decomposition (BEMD)-based model outperforms multivariate exponential weighted moving average (MEWMA) and DCC-GARCH model for estimating portfolio VaR in the electricity markets. By also employing BEMD model, He et al. (2016) analyze the precious metal markets to estimate portfolio VaR. The authors show that the proposed model improves portfolio risk forecasting performance.

Lu et al. (2014) combine copula (t-Copula, Gaussian copula and Symmetric Joe-Clayton copula) with GARCH-type models to estimate VaR of an equally weighted portfolio formed by crude oil futures and natural gas futures. The results show that t-Copula performs well and skewed-t has a better fit than normal and Student-t for individual assets. Another study involving copulas is conducted by Ghorbel and Trabelsi (2014). The authors examine the relation between WTI crude oil, natural gas and heating oil markets. The results show that GARCH-t, conditional EVT and FIGARCH extreme value copula methods produce acceptable VaR estimates. In a related study, regarding risk-adjusted returns of carbon assets, Wen et al. (2017) employ static and generalized autoregressive score dynamic copulas to model the performance of two portfolio strategies involving energy commodity futures and carbon assets. Other type of copula, named R-vine model is employed by Koliai (2016) to compare the accuracy of VaR estimates with DCC-GARCH model for commodity assets (Brent, gold, copper, wheat, and corn). The author found that copula-based models seem more efficient than DCC models.

Our SNP-DCC model is related to the use of copulas (see Del Brio et al., 2014 for a related SNP copula) but inherits the flexibility of the SNP approach, which is the basis of commodity ETFs risk measures accuracy. Furthermore, it also presents a clear advantage with respect to other multivariate estimation approaches, which is the fact that can be consistently estimated in two steps as explained in Section 3.

All in all, the paper presents contributions in three different directions: Firstly, it presents a general model, the SNP-DCC with EGARCH innovations, which have not used before for portfolio risk management. We show that this model is very tractable and provides very accurate results. Secondly, we applied recently techniques for backtesting ES and, particularly, for computing the ES with GC densities. Finally, our application focuses on portfolio (L)ETFs, which has scarcely been studied in the literature and never with a (multivariate) SNP approach.

The remainder of this paper is organized as follows: Section 2 introduces the discussion on risk measures and backtesting, Section 3 describes the risk models and VaR and ES methodologies, Section 4 presents the data and the empirical performance of the models for forecasting ES with a sample of three different commodity LETFs and Section 5 includes further results that allow a direct comparison to the individual asset backtesting procedures in Del Brio et al. (2018). Finally, Section 6 summarizes the main conclusions of the paper.

## 2. Risk quantification and backtesting

In the last decades, VaR and ES (conditional VaR) have been the two standard risk measures for market risk. As is well-known, VaR is defined as the maximum loss given a certain confidence level (usually 99%) for market risk and time horizon (1 or 10 days). Formally, if $X$ represents the returns, VaR is computed as

$$\mathrm{VaR}_{\alpha}(X) = F_X^{-1}(1-\alpha) = \inf(x \in \mathbb{R}: \mathbb{P}(X \leq x) \geq 1-\alpha), \qquad (1)$$

where $\alpha = 0.01$ (i.e. for computing 99%-VaR), and $F_X^{-1}$ is the quantile function. Though VaR has been the standard risk measure in the financial industry for three decades, VaR is not a coherent risk measure (Artzner et al., 1999) since it might violate the subadditivity axiom. Consequently, the VaR of a portfolio formed by two assets can be higher than the sum of the individual VaR of both assets. This might happen when (i) the return distribution is seriously heavy-tailed (ii) the return distribution is highly skewed, and/or (iii) there is special dependence structure of the asset returns (McNeil et al., 2005). On the other hand, ES is proven to be a coherent risk measure, and can be defined as the expected loss given that losses have exceeded VaR,

$$\mathrm{ES}_{\alpha}(X) = \frac{1}{\alpha}\int_{0}^{\alpha} \mathrm{VaR}_{u}(X)du, \tag{2}$$

and for a continuous cumulative distribution function (cdf) of returns it holds that

$$\mathrm{ES}_{\alpha}(X) = \mathbb{E}[X|X < \mathrm{VaR}_{\alpha}(X)]. \tag{3}$$

Therefore, for a given $\alpha$, ES is higher than (or equal to) VaR. After the global financial crisis, the Basel Committee of Banking Supervision (BCBS) decided to switch from 99%-VaR to 97.5%-ES for regulatory capital market requirements, since VaR seems not to be able to adequately capture tail risk and, according to the Committee (BCBS, 2012), ES provides more accurate risk measures for periods of financial distress (BCBS, 2016). For the Gaussian distribution 97.5%-ES is very close to 99%-VaR, however, under heavy-tailed distributions these two measures are far from being equivalent. As stylized facts of financial asset returns reveal that empirical return distributions exhibit heavier tails than the Gaussian, then, with the new regulation an increasing regulatory capital for market risk is expected.

Despite ES is a coherent risk measure, under general conditions, ES does not satisfy the elicitability property, whereas VaR does. Elicitability is a desirable property in order to make and evaluate point forecasts (Gneiting, 2011). That is, if $y$ is a prediction computed on the basis of the statistic $\hat{y}$, such statistic $\hat{y}$ is elicitable if it solves

$$\hat{y}(F) = \underset{y}{\operatorname{argmin}}\, \mathbb{E}_F\left[S_{\hat{y}}(y, X)\right], \tag{4}$$

for some scoring (loss) function $S_{\hat{y}}(y, X)$, and $X$ being a random variable with distribution $F$. For more details, see for instance Gneiting (2011), Ziegel (2016), and Bellini and Bignozzi (2013), and the references therein. However, backtesting VaR is straightforward, since a well-known scoring function for this risk measure is given by (Gneiting, 2011; Fissler and Ziegel, 2016)

$$S_V(v, x) = \left(I_{\{x \le v\}} - \alpha\right)\left(G(v) - G(x)\right), \tag{5}$$

where $G$ is a strictly increasing function, $x$ represents the observed return and $v$ the estimated VaR.

The purpose of backtesting is to validate a certain model for internal approach to quantify risk. The traditional procedure for VaR is to estimate (forecast) this risk measure, based on past returns. Then, the forecasted VaR is compared to the realized return, and if a negative return is less than the estimated VaR for a given date, it is considered as an exception or violation. The model performs relatively well if the number of exceptions is nearly around the $\alpha$ level times the backtesting period, named the expected number of violations. If the number of exceptions for a certain model is significantly less (greater) than the expected number of violations, then the model overestimate (underestimate) risk. According to this procedure, the binomial test provides a natural way for VaR backtesting assessment. See Christoffersen (1998) for description of traditional tests for VaR.

Though Gneiting (2011) show that ES itself is not elicitable, Fissler et al. (2016) prove that ES is jointly elicitable with VaR and propose as appropriate scoring function

$$S_{V,E}(v, e, x) = (I_{x \le v} - \alpha)\left(G_1(v) - G_1(x)\right)$$

$$+\frac{1}{\alpha}G_2(e)I_{x\leq v}(v-x)+G_2(e)(e-v)-\mathcal{G}_2(e), \qquad (6)$$

where $x$ represents the observed return, $v$ and $e$ the estimated VaR and ES respectively. Furthermore, $\mathcal{G}_2' = G_2$, $G_1$ and $G_2$ are continuous, differentiable and strictly increasing functions, see Fissler and Ziegel (2016) for more details. In this paper, we employ[1] $G_1(x) = x$ and $G_2(x) = \exp(x)$. For such scoring function, the relative performance of the models can be assessed by implementing Diebold-Mariano (DM) test. The test statistic (DM) is calculated as

$$DM = \frac{\bar{d}}{s.e.(d)}, \qquad (7)$$

where $\bar{d}$ denotes the mean of $d_t$ which is

$$d_t = S_{V,E}^{(i)}\left(v_t^{(i)}, e_t^{(i)}, x_t\right) - S_{V,E}^{(j)}\left(v_t^{(j)}, e_t^{(j)}, x_t\right), \qquad (8)$$

where $S_{V,E}^{(i)}$ and $S_{V,E}^{(j)}$ denote the scores obtained from VaR and ES models $(i)$ and $(j)$ respectively, $s.e.(d)$ denotes the standard error of the statistics and requires the implementation of a heteroskedasticity and autocorrelation consistent (HAC) variance estimator (e.g., Newey-West estimator). The null hypothesis is $\mathbb{E}[d_t] = 0$, meaning that both models present equal predictive accuracy and the composite alternative is $H_1^{(i)}: \mathbb{E}[d_t] < 0$ and $H_1^{(j)}: \mathbb{E}[d_t] > 0$. Since under null hypothesis (and "fairly weak" conditions) the limiting distribution of DM statistic is Gaussian, the null hypothesis is rejected when $|\mathrm{DM}| > 1.96$ at 5% significance level, indicating the outperformance of model $(i)$ for large negative values of the statistic and the opposite signaling that model $(j)$ is better.

[1] Another variant is to employ $G_2(x) = \frac{\exp(x)}{1+\exp(x)}$, thus $\mathcal{G}_2(x) = \log(\exp(x)+1)$, as suggested by Fissler et al. (2016). We also implement this function as a check on the robustness of the test.

Another test for ES is based on the violation residuals which is calculated as

$$K_{t+1} = \left(\frac{L_{t+1} - ES_{t+1}^{\alpha}}{ES_{t+1}^{\alpha} - \mu_{t+1}}\right) I_{\{L_{t+1} > VaR_{t+1}^{\alpha}\}}, \tag{9}$$

where $L_{t+1}$ is the actual loss, $ES_{t+1}^{\alpha}$ is the estimated ES – which is expressed for a portfolio in equation (12) – and $\mu_{t+1}$ represents the conditional mean of the model. The indicator function $I_{\{L_{t+1} > VaR_{t+1}^{\alpha}\}}$ takes value 1 when the actual loss has exceeded the estimated VaR, and 0 otherwise. Then, the null hypothesis of zero mean violation residuals may be tested by a simple t-test on this variable (McNeil et al., 2015),

$$t - stat = \frac{\bar{K}}{s/\sqrt{T}}, \tag{10}$$

where $\bar{K}$ is the sample mean of the violation residuals of size $T$, and $s$ denotes its standard deviation.

Further methods have been proposed to test ES. For instance, Acerbi and Székely (2014; 2017) suggest several (non-parametric) tests to validate ES estimates. Costanzino and Curran (2015) introduce a coverage test for ES, whereas Du and Escanciano (2016) propose to backtest ES based on cumulative violations. Based on the approximation of ES by a weighted sum of VaRs, Kratz et al. (2016) propose a multinomial test for different VaR levels as an implicit method to backtest ES.

## 3. Models and Methodology

The Gram-Charlier (GC) distribution has recently attracted the attention of risk management literature (León and Moreno, 2017; Zoia et al., 2018; Del Brio et al., 2018). Particularly, the latter has proved the GC to be an accurate distribution to measure ETFs risk according to both VaR and ES. In this paper we extend these analyses to portfolios of leveraged ETFs by modelling the multivariate GC (henceforth, named as multivariate

SNP distribution) and studying its relative performance for backtesting VaR and ES to different competing models (Gaussian or Historical Simulation).

For both the VaR and ES backtesting procedures the predicted risk measures for the portfolio return $R_p$ at the time horizon $t$+1 and with confidence level $1-\alpha$ is given by

$$\mathrm{VaR}_{p,t+1}^{1-\alpha} = \hat{\mu}_{p,t+1} + \hat{\sigma}_{p,t+1}\hat{q}_{\alpha}\left(z_{p,t+1}\right), \tag{11}$$

$$\mathrm{ES}_{p,t+1}^{1-\alpha} = \hat{\mu}_{p,t+1} + \hat{\sigma}_{p,t+1}\hat{S}_{\alpha}\left(z_{p,t+1}\right), \tag{12}$$

where $\hat{q}_{\alpha}$ and $\hat{S}_{\alpha}$ are estimations of VaR and ES given by equation (1) and (2), respectively, and forecasted conditional mean and variance of the portfolio are computed as in equations (13) and (14) below.

$$\hat{\mu}_{p,t+1} = \sum_{i=1}^{n} w_i \hat{\mu}_{i,t+1}, \tag{13}$$

$$\hat{\sigma}_{p,t+1}^{2} = \sum_{i=1}^{n} w_i w_j \hat{\sigma}_{ij,t+1}, \tag{14}$$

which are obtained through the forecasted mean for every asset $i$ ($\hat{\mu}_{i,t+1}$) and the forecasted covariance of every couple of assets $i$ and $j$

$$\hat{\sigma}_{ij,t+1} = \hat{\rho}_{ij,t+1}\hat{\sigma}_{i,t+1}\hat{\sigma}_{j,t+1}, \tag{15}$$

and considering the weight of every asset $i$, denoted by $w_i$ and satisfying $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Note that $\hat{\rho}_{ij,t+1}$ accounts for the estimated conditional correlation coefficient between assets $i$ and $j$ and then $\hat{\sigma}_{ij,t+1} = \hat{\sigma}_{i,t+1}^{2}$ when $i = j$.

The model may be estimated with ease, since loglikelihood functions of both Gaussian-DCC and SNP-DCC models can be split in two terms, volatility and correlation, which allows to consistently estimate the density parameters in two steps: Firstly, conditional means and variances are estimated independently for every variable and, secondly, conditional correlations are estimated in the standardized (zero mean and unit variance)

Gaussian and SNP distributions. This second step involves the jointly estimation of both skewness and kurtosis for the SNP distributions.

*First stage: Volatility part. EGARCH model*

We assume that the marginal distribution of every individual asset follows the ARMA(1,1)-EGARCH(1,1) model in equations (16)-(19) with either Gaussian or GC innovations with $H$ distribution.

$$R_t = \mu_t + \sigma_t Z_t, \tag{16}$$

$$\mu_t = \varpi + \delta\mu_{t-1} + \theta\varepsilon_{t-1} + \varepsilon_t\ , \tag{17}$$

$$\log \sigma_t^2 = \omega + \alpha(|Z_{t-1}| - \mathbb{E}[|Z_{t-1}|]) + \gamma Z_{t-1} + \beta \log \sigma_{t-1}^2\ , \tag{18}$$

$$Z_t = \varepsilon_t/\sigma_t,\ \ Z_t \sim H(0,1), \tag{19}$$

where $\varpi$ and $\omega$ are the intercepts of the conditional mean and variance models, $\delta$ and $\theta$ the parameters of AR(1) and MA(1) structures of the conditional mean, and $\alpha$ and $\gamma$ the parameters associated with the size and sing effects of the EGARCH model (respectively).

The probability density functions (pdf) for $H$ are described below.

(i) Gaussian pdf:

$$\phi(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{{z_t}^2}{2}}. \tag{20}$$

(ii) GC Type A pdf:

$$f(z_t, \boldsymbol{d}) = (1 + \textstyle\sum_{s=3}^{S} d_s H_s(z_t))\phi(z_t), \tag{21}$$

where $\phi(z_t)$ denotes the standard normal pdf in equation (20), $\boldsymbol{d}' = (d_1, \dots, d_S) \in \mathbb{R}^S$ is a vector of parameters such that $f(z_t, \boldsymbol{d}) \geq 0$ and $H_s$ is the Hermite polynomial (HP) of order $s$, which is defined in terms of the $s^{\text{th}}$ order derivative of $\phi(z_t)$ as

$$\frac{d^s \phi(z_t)}{d{z_t}^s} = (-1)^s H_s(z_t)\phi(z_t). \tag{22}$$

In particular, the first four HP are: $H_1(z_t) = z_t$, $H_2(z_t) = {z_t}^2 - 1$, $H_3(z_t) = {z_t}^3 - 3z_t$, $H_4(z_t) = {z_t}^4 - 6{z_t}^2 + 3$. These polynomials form an orthonormal basis, thus satisfying the orthogonality property,

$$\int H_s(z_t) H_j(z_t) \phi(z_t)\, dz_t = 0 \ \ \forall s \neq j\ , \tag{23}$$

which is the basis of the characterization of GC series as a pdf (i.e. the GC density integrates to one) and its parameters in terms of density moments. For instance, even non-central moments depend on the even $d_s$ parameters (e.g. $d_2$ accounts for the variance, $d_4$ for the excess kurtosis and the rest of the even parameters capture higher-order moments) and skewness is captured by the odd parameters (particularly, $d_3$).

The loglikelihood function of the GC incorporating conditional EGARCH variances is given by (after dropping out some constants)

$$LogL = -\frac{1}{2}\log\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\frac{\varepsilon_t^2}{\sigma_t^2} + \log\left[1 + \sum_{s=2}^{S} d_s\, H_s\left(\frac{\varepsilon_t}{\sigma_t}\right)\right], \tag{24}$$

where $\varepsilon_t = \sigma_t Z_t$, $\sigma_t^2$ is described in equation (18) and $\mathbb{E}[|Z_t|] = \sqrt{\frac{2}{\pi}}(1 - d_4)$. See Appendix C in Del Brio et al. (2018) for derivation of expected value of the GC-innovations in an EGARCH model. The Gaussian case is obtained as a particular nested distribution by setting $d_s = 0$, $\forall$s.

In addition, an estimation for the conditional correlation $\hat{\rho}_{ij,t+1}$ is necessary to find the predicted volatility of portfolio returns. To this end, standard Dynamic Conditional Correlation (Gaussian-DCC) (Engle, 2002) and semi-nonparametric DCC (SNP-DCC) (Del Brio et al., 2011) models are estimated in the second step. Next subsection reviews these models.

*Second stage: Correlation part. The Gaussian-DCC and SNP-DCC models*

The multivariate SNP of vector $\boldsymbol{z}_t = (z_{1t}, z_{2t}, \dots, z_{nt})' \in \mathbb{R}^n$ with $z_{\mathrm{it}} \sim GC(0,1)$ – i.e. distributed as in equation (21) – and conditional correlation matrix $\boldsymbol{R}_t$ (with general element $\{\rho_{ij}\}$) is characterized in terms of the following pdf:

$$\boldsymbol{f}_{SNP}(\boldsymbol{z}_t|\Omega_{t-1}) = (2\pi)^{-\frac{n}{2}}|\boldsymbol{R}_t|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\boldsymbol{z}_t'\boldsymbol{R}_t^{-1}\boldsymbol{z}_t\right\}\left[\sum_{i=1}^{n}\psi_i(x_{it})\right]\frac{1}{n}, \tag{25}$$

where

$$\psi_i(x_{it}) = 1 + d_{3i}(x_{it}^3 - 3x_{it}) + d_{4i}\left(x_{it}^4 - 6x_{it}^2 + 3\right), \tag{26}$$

$$\boldsymbol{x}_t = (x_{1t}, x_{2t}, \dots, x_{nt})' = \boldsymbol{R}_t^{-1/2}\boldsymbol{z}_t. \tag{27}$$

It is noteworthy that equation (26) corresponds to the case where only skewness ($d_{3i}$) and kurtosis ($d_{4i}$) are considered and that the multivariate SNP distribution becomes the multivariate Gaussian when $d_{3i} = d_{4i} = 0$ and thus Gaussian-DCC is a particular case of SNP-DCC. Furthermore, the transformation in equation (27) is not unique, although this fact does not impact the estimates of the conditional correlations. For example, for the bivariate case ($n = 2$) and the eigenvalue decomposition this transformation yields

$$x_{1t} = \frac{1}{2}\left(\frac{1}{\sqrt{1+\rho_{12,t}}} + \frac{1}{\sqrt{1-\rho_{12,t}}}\right)z_{1t} + \frac{1}{2}\left(\frac{1}{\sqrt{1+\rho_{12,t}}} - \frac{1}{\sqrt{1-\rho_{12,t}}}\right)z_{2t} \tag{28}$$

$$x_{2t} = \frac{1}{2}\left(\frac{1}{\sqrt{1+\rho_{12,t}}} - \frac{1}{\sqrt{1-\rho_{12,t}}}\right)z_{1t} + \frac{1}{2}\left(\frac{1}{\sqrt{1+\rho_{12,t}}} + \frac{1}{\sqrt{1-\rho_{12,t}}}\right)z_{2t} \tag{29}$$

Then, the DCC model employed in this article can be formulated as:

$$\boldsymbol{r}_t = \boldsymbol{\mu}_t(\boldsymbol{\phi}) + \boldsymbol{\varepsilon}_t \tag{30}$$

$$\boldsymbol{\varepsilon}_t \sim \boldsymbol{GC}(\boldsymbol{0}, \boldsymbol{D_t R_t D_t}) \tag{31}$$

$$\log\boldsymbol{D}_t^2 = diag\{\omega_i\} + diag\{\alpha_i\} \circ (|\boldsymbol{\varepsilon}_{t-1}| - \mathbb{E}[|\boldsymbol{\varepsilon}_{t-1}|]) + diag\{\gamma_i\} \circ \boldsymbol{\varepsilon}_{t-1} + diag\{\beta_i\} \circ \log\boldsymbol{D}_{t-1}^2 \tag{32}$$

$$\boldsymbol{z}_t = \boldsymbol{D}_t^{-1}\boldsymbol{\varepsilon}_t \tag{33}$$

$$\boldsymbol{Q}_t = \boldsymbol{S} \circ (\boldsymbol{\iota\iota}' - \boldsymbol{A} - \boldsymbol{B}) + \boldsymbol{A} \circ \boldsymbol{z}_{t-1}\boldsymbol{z}_{t-1}' + \boldsymbol{B} \circ \boldsymbol{Q}_{t-1} \tag{34}$$

$$\boldsymbol{R}_t = diag\{\boldsymbol{Q}_t\}^{-1/2}\boldsymbol{Q}_t diag\{\boldsymbol{Q}_t\}^{-1/2} \tag{35}$$

where $\boldsymbol{D}_t^2$ is the diagonal matrix of conditional variances with EGARCH dynamics and $\boldsymbol{Q}_t$ the conditional covariance matrix of the DCC type – i.e. $\boldsymbol{\iota}$ is a vector of ones, the symbol $\circ$ represents the element-by-element multiplication operator (Hadamard product), matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{\iota\iota}' - \boldsymbol{A} - \boldsymbol{B}$ are positive definite matrices and $\boldsymbol{S}$ is the unconditional correlation matrix of $\boldsymbol{z}_t$. Note that, to the best of our knowledge, this paper presents the first application of the SNP-DCC model with EGARCH innovations.

In particular, the loglikelihood function in the second stage for SNP-DCC becomes (after deleting unnecessary constants)

$$LogL(SNP) = -\frac{1}{2}\log|\boldsymbol{R}_t| - \frac{1}{2}\boldsymbol{z}_t'\boldsymbol{R}_t^{-1}\boldsymbol{z}_t + \sum_{t=1}^{T}\log\{\sum_{i=1}^{n}\psi_i(\boldsymbol{x}_{it})\}. \tag{36}$$

Once the model is estimated, VaR and ES measures are computed for the fitted portfolio density. It is also noteworthy that the VaR computation in equation (1) involves the estimation of the $\alpha$-quantile of the portfolio distribution, $\hat{q}_\alpha(z_{p,t+1})$, and given this value the ES of the portfolio GC density can be easily obtained through the following expression:

$$\hat{S}_\alpha(z_{p,t+1}) = \phi\left(\hat{q}_\alpha(z_{p,t+1})\right)\left[1 + \sum_{s=3}^{S} d_s \left[H_s\left(\hat{q}_\alpha(z_{p,t+1})\right) + sH_{s-2}\left(\hat{q}_\alpha(z_{p,t+1})\right)\right]\right], \quad (37)$$

where $\phi$ is the pdf of standard Gaussian and $H_s$ the Hermite polynomial of *s-th* order. In order to compare the performance of SNP-DCC and Gaussian-DCC models, the traditional Historical Simulation method will also be employed.

*Historical Simulation approach*

The Historical Simulation technique assumes that future portfolio returns can be approximated by the empirical distribution of previous returns. Therefore, the 97.5% VaR is calculated as the 2.5% percentile of the past portfolio returns. On the other hand, 97.5% ES is estimated as the expected value of the returns given that the returns are less than 2.5% percentile of historical returns. Though Historical Simulation is one of the most popular methods to forecast VaR at commercial banks (Perignon and Smith, 2010), several studies have shown that this method could lead inconsistent risk estimates (Pritsker, 2006; Escanciano and Pei, 2012).

## 4. Data and Application

Three different Commodity LETFs[2] are considered for the empirical application, which are the VelocityShares 3X Long Natural Gas ETN (LGas), the ProShares Ultra Bloomberg Crude Oil Exchange Traded Fund (LOil), and the VelocityShares 3X Long Silver ETN (LSilver). For backtesting analysis, according to recent regulation (BCBS, 2016), 12 months are employed to test the risk measures. Then, three years are considered

[2] The reason to choose these three leveraged ETFs is because they are the largest commodity LETFs by total assets for 2018 according to the ETF Database (ETFdb.com). More details are found in Appendix A.

in our analysis: 2015, 2016 and 2017 to perform the tests.[3] Table 1 shows the descriptive statistics of each leveraged ETF for the abovementioned dates.

[INSERT TABLE 1]

High volatility is presented in Commodity LETFs returns, especially for LGas, which annual volatility ranged between 101% (2017) and 123% (2016). Though LOil exhibits the minimum annual standard deviation (47% in 2017), it is higher than average annual volatility of stock returns. For instance, the average of annual volatility of S&P500 is 16% between 1966 and 2015, and it is 25% in the period of 2006 and 2010.[4] These large values of volatility may be explained by the leverage multiplier and the volatility of the underlying asset (Cheng and Madhavan, 2009).

We form three equally weighted portfolios with the individual Leveraged Commodity ETFs: Portfolio A (LGas and LOil), Portfolio B (LGas and LSilver) and Portfolio C (LOil and LSilver) and its descriptive statistics is shown in Table 2.

[INSERT TABLE 2]

Similar characteristics of individual LETFs are presented for LETF Portfolio returns. The minimum annual volatility is presented for Portfolio C in 2017 (37%) and Portfolio A has the maximum annual standard deviation (81%) in 2016. These figures are higher than volatilities of (unlevered) ETFs portfolios in the period of 2007-2016 (see Table 7). Both individual and portfolio LETFs returns exhibit identical empirical characteristics of

[3] Important events related to (L)ETFs that affected financial markets have occurred in the three analyzed periods. In 2017, A LETF was blamed for highly fluctuations in gold stock prices from Toronto to Sidney. In September 2016 the Bank of Japan hit a record in ETF (tracking the Nikkei 225) purchase (before May 2018) and then diminished its stock purchases. The stock market crash in August 24, 2015 was, in part, caused by ETFs trading. Source: Financial Times and Bloomberg news.
[4] Source: McKinsey Corporate Performance Analytics.

financial returns: mean returns are close to zero, negative skewed (most of them) and leptokurtic distributions.

*Results of VaR tests*

The traditional and very well-known coverage test for 97.5%-VaR is applied to the three equally-weighted LETFs portfolios and shown in Table 3. For each year (2015, 2016 and 2017) a two-year length rolling window[5] was used for parameter estimation (e.g., in-sample period covers around the 500 observations previous to the forecasting date) and backtesting (out-of-sample) period spanned through all trading dates (around 250 days) of every subsequent natural year, therefore, the number of expected exceptions being around 6.

[INSERT TABLE 3]

The results show that Gaussian model assumption does not perform well in all cases, while Historical Simulation and SNP-DCC work relatively well for all portfolios. All models present good results for independence test (i.e. VaR violations seem to be independently distributed) with the exception of the Gaussian-DCC model for Portfolio C in 2017, as shown in Table 4.

[INSERT TABLE 4]

*Results of ES tests*

The mean of violation residuals test show that Gaussian-DCC model performs poorly for all portfolios. On the other hand, SNP-DCC model works well in all cases, whereas Historical Simulation does not perform well in one case (Portfolio C in 2015), and it was

[5] The rolling window procedure results in higher forecast accuracy than other (recursive) backtesting procedures and its use seem to be analytically convenient in economic time series (Giacomini and White, 2006).

not possible to compute the test in one case (Portfolio C in 2017). The results are shown in Table 5.

[INSERT TABLE 5]

The relative comparison tests show that Gaussian-DCC model is outperformed by SNP-DCC model and Historical Simulation method for all cases. In just one case (Portfolio C in 2017) the SNP-DCC model's performance is surpassed by the Historical Simulation technique. The abovementioned results are displayed on Table 6.

[INSERT TABLE 6]

As a robustness check, Table 7 shows that similar results are obtained when $G_2(x) = \frac{\exp(x)}{1+\exp(x)}$ is assumed in the score function – see equation (6).

[INSERT TABLE 7]

An illustration of the 97.5%-ES estimated by the three analyzed methods for Portfolio A (LGas and LSilver) is provided in Figure 1. It is noteworthy that Gaussian-DCC underestimates risk, but also that the dynamics of the return instability is not accurately captured by Historical Simulation. On the contrary, the SNP-DCC model provides accurate risk dynamics measures.

[INSERT FIGURE 1]

## 5. Further Results

One contribution of this paper is the extension of procedures for backtesting ES provided in Del Brio et al. (2018) to portfolios of (multivariate) SNP distributions. Therefore, in this section we perform a counterpart portfolio application for equally weighted portfolios and DCC-type dependencies with exactly the same dataset, which allows a direct comparison of the results. Particularly, we consider three bivariate ($n$ = 2) and equally weighted ($w_i = 0.5, \forall i = 1,2$) portfolios: Portfolio A, formed by Gold and Silver ETFs; Portfolio B, formed by Gold and Oil ETFs; and Portfolio C, formed by Silver and Oil ETFs. The data covers a sample of daily prices from January 2007 to January 2016 obtained from Bloomberg (see Appendix B for more details). Table 8 displays the main descriptive statistics for the portfolio returns, which are characterized by the same stylized facts than the individual financial assets, i.e. daily median returns are approximately zero, and they exhibit negative skewness and fat tails.

[INSERT TABLE 8]

Table 9 shows the estimated parameters for Gaussian-DCC (Panel A) and SNP-DCC (Panel B). The estimation period comprises a sample of daily data from December 2008 to January 2016. The first two rows present the estimations for the DCC part, and results support the conditional correlation model. It must be noted that our application incorporates an AR(1)-EGARCH(1,1). As expected, the parameters of the volatility part are similar to the estimations found in Table 2 for the univariate application of Del Brio et al. (2018). Furthermore, the parameters of the GC density for both dimensions are significant reflecting the outperformance of the SNP-DCC model.

[INSERT TABLE 9]

Table 10 presents the results of the t-test for ES at 97.5%. The backtesting was is carried out with rolling windows on the sample (December 2008 to January 2016) keeping a total of 1771 days for backtesting. The model is good enough if the null hypothesis cannot be rejected. The SNP-DCC and Historical Simulation perform adequately in all cases, whereas Gaussian-DCC fails in the three analyzed portfolios.

[INSERT TABLE 10]

The results of relative performance test for ES are shown in Table 11. A pairwise Diebold Mariano test less than −1.96 indicates that SNP-DCC is preferred to Gaussian-DCC at 5% of significance. Moreover, Gaussian-DCC is outperformed by Historical Simulation in all cases and SNP-DCC works better than Historical Simulation for Gold-Oil portfolio. The results show the outperformance of SNP-DCC for the analyzed portfolios.

[INSERT TABLE 11]

Finally, Figure 2 presents the comparison of 97.5%-ES estimated by Gaussian-DCC and SNP-DCC for the conditional correlations of the three analyzed portfolios. This figure illustrates the underperformance of the Gaussian-DCC for capturing extreme values and the more accuracy of the SNP-DCC for this purpose.

[INSERT FIGURE 2]

From the multivariate perspective used for analyzing portfolios, different results from calculating ES at 97.5% under a Gaussian or SNP distribution can be observed. The ratio SNP-ES/Gaussian-ES for Portfolio B (Gold-Oil) and C (Silver-Oil) is 1.66, and for Portfolio A (Gold-Silver) is 1.91. This evidence implies that risk is underestimated more than a half (for Portfolio A) if a financial institution employs the Gaussian model to buffer capital against potential losses in commodity ETF markets. Therefore, the SNP-DCC model should be considered as an interesting tool for risk management.

## 6. Conclusions

The recent change proposed by the Basel Committee of Banking Supervision of replacing VaR by ES for market risk poses a challenge to academics and financial industry regarding the adequate manner to validate ES estimates. Unlike the VaR, this risk measure is not elicitable and this finding generated a discussion around whether ES can be backtestable or not. In a recent study, Fissler et al. (2016) show that ES is jointly elicitable with VaR, and then relative comparison of models for risk assessment is possible. However, how to backtest ES appropriately and the best models to perform for highly volatile financial instruments are still open questions. Our paper sheds some light on this issue.

In particular, we employ the result of Fissler et al. (2016) and t-test for the sample mean of violation residuals to validate ES calculations for commodity (L)ETF portfolios. To this end, we employ Gaussian-DCC and SNP-DCC to model conditional correlation and EGARCH with both Gaussian and GC innovations to model volatilities. In addition, the results are compared with the Historical Simulation technique. The so-obtained portfolio volatilities are used to compute conditional risk measures. The results show that for portfolios of commodity (L)ETFs, the SNP-DCC and Historical Simulation are preferred to the Gaussian-DCC according to t-test and relative performance tests. Nevertheless, Historical Simulation fails to capture the dynamics of risk forecasting instability.

Therefore, the SNP-DCC results in an accurate tool for dealing with (L)ETFs risk management, which represents an important contribution from the financial stability point of view, since LETFs may induce systemic instability according to independent reports and research studies. For example, Ben-David et al. (2017) states that “The concerns raised by academics and regulators about the risks that these classes of investors may create during events of market turbulence deserve additional investigation…As a result of the financial crisis of 2008,…both investors and policymakers have raised concerns about the fragility of the ETF market…Our hope is that the academic research about ETFs is useful in quantifying the systemic risks that these investment vehicle pose and that it can potentially help address them.” In line with these arguments, recently, some ETFs and LETFs have implied high risks and led to extreme losses, e.g. the crash of XIV ETN (tracking the inverse return of VIX).

Our study sheds light on how to tackle the risk quantification of LETFs portfolios with relatively simple SNP techniques. This approach provides more accurate results than simpler models and admits two-step estimation, which eases the model implementation.

Future research will be focused on the comparison of the results obtained by other methods proposed in the literature to test ES (Acerbi and Székely, 2014, 2017; Costanzino and Curran, 2015; Du and Escanciano, 2016; Kratz et al., 2016). Another future research can be devoted to analyzing different SNP models for portfolio returns such as copulas (Del Brio et al., 2014) or general moment expansions (Ñíguez and Perote, 2016).

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**Table 1. Descriptive statistics of commodity LETF individual assets**

| 2015 | LGas | LOil | LSilver |
|---|---|---|---|
| Mean | -0.8041 | -0.5438 | -0.2531 |
| Median | -0.3656 | -0.5627 | 0.0000 |
| Standard deviation | 7.0421 (111.34) | 5.1142 (80.86) | 4.4622 (70.55) |
| Min | -21.2258 | -14.6758 | -20.1460 |
| Max | 28.3829 | 16.5692 | 14.3673 |
| Skewness | 0.3136 | 0.1058 | -0.2414 |
| Excess Kurtosis | 1.3870 | 0.5233 | 2.3265 |
| **2016** | **LGas** | **LOil** | **LSilver** |
| Mean | -0.1066 | -0.0272 | 0.0586 |
| Median | 0.0000 | 0.0000 | 0.3252 |
| Standard deviation | 7.8342 (123.87) | 5.0963 (80.58) | 4.6244 (73.12) |
| Min | -26.1561 | -16.6153 | -21.7001 |
| Max | 22.0816 | 16.3731 | 13.4774 |
| Skewness | -0.2512 | 0.1185 | -0.6255 |
| Excess Kurtosis | 0.5116 | 0.6625 | 2.9202 |
| **2017** | **LGas** | **LOil** | **LSilver** |
| Mean | -0.7109 | 0.0013 | 0.0060 |
| Median | -0.2178 | 0.2106 | 0.0000 |
| Standard deviation | 6.3991 (101.18) | 2.9969 (47.38) | 3.1602 (49.97) |
| Min | -38.5000 | -10.7960 | -11.5596 |
| Max | 18.3822 | 6.7139 | 7.9292 |
| Skewness | -1.1784 | -0.8137 | -0.1989 |
| Excess Kurtosis | 5.6874 | 1.5538 | 0.8332 |

Annual volatility in parentheses.

**Table 2. Descriptive statistics of commodity LETFs Portfolios**

| **2015** | **Portfolio A (LGas and LOil)** | **Portfolio B (LGas and LSilver)** | **Portfolio C (LOil and LSilver)** |
|---|---|---|---|
| Mean | -0.6740 | -0.5286 | −0.3985 |
| Median | -0.2192 | -0.1819 | -0.4992 |
| Standard deviation | 4.6844 (74.07) | 4.3284 (68.44) | 3.7870 (59.88) |
| Min | -15.6076 | -17.4032 | -11.3274 |
| Max | 17.6238 | 12.6855 | 11.9131 |
| Skewness | -0.0832 | -0.1225 | 0.2738 |
| Excess Kurtosis | 0.8034 | 0.3647 | 0.5204 |
| **2016** | **Portfolio A (LGas and LOil)** | **Portfolio B (LGas and LSilver)** | **Portfolio C (LOil and LSilver)** |
| Mean | -0.0669 | -0.0240 | 0.0157 |
| Median | 0.0157 | -0.0474 | 0.000 |
| Standard deviation | 5.1078 (80.76) | 4.5734 (72.31) | 3.6366 (57.50) |
| Min | -18.3554 | -13.4635 | -13.6611 |
| Max | 12.1445 | 15.9078 | 11.6113 |
| Skewness | -0.3608 | -0.1622 | -0.1095 |
| Excess Kurtosis | 0.5048 | 0.5971 | 0.8636 |
| **2017** | **Portfolio A (LGas and LOil)** | **Portfolio B (LGas and LSilver)** | **Portfolio C (LOil and LSilver)** |
| Mean | -0.3548 | -0.3524 | 0.0037 |
| Median | 0.0000 | -0.1931 | 0.0000 |
| Standard deviation | 3.6870 (58.30) | 3.6595 (57.86) | 2.3244 (36.75) |
| Min | -21.5271 | -15.9903 | -7.8276 |
| Max | 9.6873 | 10.3028 | 6.4194 |
| Skewness | -1.0291 | -0.4985 | -0.3869 |
| Excess Kurtosis | 4.3003 | 1.9033 | 0.7188 |

Annual volatility in parentheses.

## Table 3. Backtesting 97.5%-VaR for Commodity LETFs returns

| | 2015 | | |
|---|---|---|---|
| 250 days | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 8 (0.565) | 4 (0.286) | 12 (0.050) |
| Gaussian-DCC | 101 (0.000) | 94 (0.000) | 92 (0.000) |
| SNP-DCC | 7 (0.844) | 3 (0.121) | 3 (0.121) |
| | 2016 | | |
| 250 days | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 7 (0.844) | 11 (0.101) | 8 (0.558) |
| Gaussian-DCC | 90 (0.000) | 70 (0.000) | 64 (0.000) |
| SNP-DCC | 4 (0.289) | 3 (0.121) | 3 (0.121) |
| | 2017 | | |
| 250 days | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 4 (0.289) | 3 (0.122) | 1 (0.007) |
| Gaussian-DCC | 73 (0.000) | 80 (0.000) | 54 (0.000) |
| SNP-DCC | 2 (0.037) | 2 (0.037) | 3 (0.121) |

P-values for the Bernoulli Coverage test in parentheses. Expected number of exceptions are 6.

## Table 4. Independence Test for 97.5%-VaR for Commodity LETFs returns

| | 2015 | | |
|---|---|---|---|
| | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 0.510 (0.475) | 0.125 (0.723) | 1.166 (0.280) |
| Gaussian-DCC | 0.786 (0.375) | 0.090 (0.764) | 0.034 (0.854) |
| SNP-DCC | 0.389 (0.533) | 0.070 (0.791) | 0.070 (0.791) |
| | 2016 | | |
| | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 1.912 (0.167) | 0.508 (0.476) | 0.512 (0.474) |
| Gaussian-DCC | 3.358 (0.067) | 0.181 (0.670) | 0.036 (0.848) |
| SNP-DCC | 0.125 (0.723) | 0.070 (0.791) | 0.070 (0.791) |
| | 2017 | | |
| | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | 0.094 (0.759) | 0.047 (0.828) | 0.008 (0.930) |
| Gaussian-DCC | 0.542 (0.461) | 1.702 (0.192) | 4.219 (0.040) |
| SNP-DCC | 0.015 (0.901) | 0.015 (0.901) | 0.031 (0.860) |

P-values for the Independence Coverage test in parentheses.

### Table 5. T-test for 97.5%-ES for Commodity ETFs returns

| **Mean of violation residual is zero** | | | |
|---|---|---|---|
| 2015 | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | **0.478** (0.647) | **-0.967** (0.405) | -3.123 (0.009) |
| Gaussian-DCC | 11.67 (0.000) | 11.358 (0.000) | 10.652 (0.000) |
| SNP-DCC | **0.967** (0.371) | **1.257** (0.428) | **-0.887** (0.469) |
| 2016 | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | **0.276** (0.792) | **-1.436** (0.181) | **-0.846** (0.425) |
| Gaussian-DCC | 9.015 (0.000) | 8.283 (0.000) | 7.303 (0.000) |
| SNP-DCC | **-0.095** (0.930) | **-2.372** (0.141) | **0.308** (0.787) |
| 2017 | *Portfolio A* | *Portfolio B* | *Portfolio C* |
| Historical Simulation | **0.356** (0.745) | **1.363** (0.306) | NA* |
| Gaussian-DCC | 7.307 (0.000) | 7.942 (0.000) | 5.906 (0.000) |
| SNP-DCC | **0.557** (0.676) | **0.491** (0.710) | -**6.676**** (0.094) |

P-values for the t-test test in parentheses. *There was just one value to perform the test. ** t-test with one degree of freedom

**Table 6. Pairwise Diebold-Mariano test for 97.5%-ES (2015-2017)**

| **Model A →** / **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| **2015 Portfolio A:** | | |
| Historical Simulation | 6.192 (0.999) | -1.285 (0.099) |
| Gaussian-DCC | | **-5.748** (0.000) |
| **Portfolio B:** | | |
| Historical Simulation | 5.836 (0.999) | -0.341 (0.367) |
| Gaussian-DCC | | **-4.583** (0.843) |
| **Portfolio C:** | | |
| Historical Simulation | 5.960 (0.999) | -1.298 (0.098) |
| Gaussian-DCC | | **-5.170** (0.000) |

| **Model A →** / **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| **2016 Portfolio A:** | | |
| Historical Simulation | 4.555 (0.999) | -1.498 (0.068) |
| Gaussian-DCC | | **-4.503** (0.000) |
| **Portfolio B:** | | |
| Historical Simulation | 4.634 (0.999) | -1.541 (0.062) |
| Gaussian-DCC | | **-4.676** (0.000) |
| **Portfolio C:** | | |
| Historical Simulation | 3.622 (0.999) | 0.283 (0.611) |
| Gaussian-DCC | | **-4.041** (0.000) |

| **Model A →** / **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| **2017 Portfolio A:** | | |
| Historical Simulation | 3.855 (0.999) | 0.897 (0.814) |
| Gaussian-DCC | | **-3.010** (0.001) |
| **Portfolio B:** | | |
| Historical Simulation | 4.355 (0.999) | -0.769 (0.221) |
| Gaussian-DCC | | **-3.521** (0.000) |
| **Portfolio C:** | | |
| Historical Simulation | 3.644 (0.999) | 5.284 (0.999) |
| Gaussian-DCC | | **-3.330** (0.000) |

The table shows the Diebold-Mariano statistics for different models. Bold figures indicate Model A is preferred than Model B, figures in red indicate the opposite. Otherwise both models present the same performance. P-values in parentheses.

**Table 7. Pairwise Diebold-Mariano test for 97.5%-ES (2015-2017)**

**2015 Portfolio A:**

| **Model A →** **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| Historical Simulation | 5.777 (0.999) | -1.284 (0.100) |
| Gaussian-DCC | | **-5.772** (0.000) |
| **Portfolio B:** | | |
| Historical Simulation | 4.723 (0.999) | -0.341 (0.367) |
| Gaussian-DCC | | **-4.707** (0.000) |
| **Portfolio C:** | | |
| Historical Simulation | 5.783 (0.999) | -1.303 (0.097) |
| Gaussian-DCC | | **-5.772** (0.000) |

**2016 Portfolio A:**

| **Model A →** **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| Historical Simulation | 4.166 (0.999) | -1.498 (0.067) |
| Gaussian-DCC | | **-4.155** (0.000) |
| **Portfolio B:** | | |
| Historical Simulation | 4.553 (0.999) | -1.541 (0.062) |
| Gaussian-DCC | | **-4.611** (0.000) |
| **Portfolio C:** | | |
| Historical Simulation | 3.469 (0.999) | 0.270 (0.606) |
| Gaussian-DCC | | **-3.468** (0.000) |

**2017 Portfolio A:**

| **Model A →** **Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| Historical Simulation | 3.277 (0.999) | 0.896 (0.814) |
| Gaussian-DCC | | **-3.216** (0.000) |
| **Portfolio B:** | | |
| Historical Simulation | 4.180 (0.999) | -0.770 (0.221) |
| Gaussian-DCC | | **-4.174** (0.000) |
| **Portfolio C:** | | |
| Historical Simulation | 3.636 (0.999) | 5.284 (0.999) |
| Gaussian-DCC | | **-3.634** (0.000) |

The table shows the Diebold-Mariano statistics for different models. Bold figures indicate Model A is preferred than Model B, figures in red indicate the opposite. Otherwise both models present the same performance. P-values in parentheses.

**Table 8. Descriptive statistics of commodity ETF Portfolios**

| Portfolios | A: Gold-Silver | B: Gold-Oil | C: Silver-Oil |
|---|---|---|---|
| Mean | 0.0144 | −0.0243 | −0.343 |
| Median | 0.0592 | 0.0013 | −0.0061 |
| Standard deviation | 1.6434 (25.98) | 1.4132 (22.34) | 1.8221 (28.81) |
| Min | −13.7779 | −7.0676 | −12.1956 |
| Max | 12.0853 | 7.4417 | 8.3828 |
| Skewness | −0.7347 | −0.1067 | −0.4720 |
| Excess Kurtosis | 6.7643 | 2.4751 | 3.4321 |

Annual volatility in parentheses.

## Table 9. Estimates of DCC models

### Panel A: Gaussian-DCC

| Portfolios | A: Gold-Silver | B: Gold-Oil | C: Silver-Oil |
|---|---|---|---|
| $A$-DCC | 0.041 (0.000) | 0.044 (0.000) | 0.024 (0.002) |
| $B$-$DCC$ | 0.937 (0.000) | 0.909 (0.000) | 0.950 (0.000) |
| $\phi_1$ | 0.021 (0.338) | 0.021 (0.338) | -0.015 (0.686) |
| $\omega_1$ | 0.014 (0.002) | 0.014 (0.002) | 0.051 (0.000) |
| $\alpha_1$ | -0.012 (0.504) | -0.012 (0.504) | -0.016 (0.345) |
| $\delta_1$ | 0.980 (0.000) | 0.980 (0.000) | 0.972 (0.000) |
| $\beta_1$ | 0.146 (0.000) | 0.146 (0.000) | 0.201 (0.023) |
| $\phi_2$ | -0.015 (0.686) | -0.042 (0.191) | -0.042 (0.191) |
| $\omega_2$ | 0.051 (0.000) | 0.013 (0.000) | 0.013 (0.000) |
| $\alpha_2$ | -0.016 (0.345) | -0.052 (0.000) | -0.052 (0.000) |
| $\beta_2$ | 0.972 (0.000) | 0.991 (0.000) | 0.991 (0.000) |
| $\gamma_2$ | 0.201 (0.023) | 0.102 (0.000) | 0.102 (0.000) |

### Panel B: SNP-DCC

| Portfolios | A: Gold-Silver | B: Gold-Oil | C: Silver-Oil |
|---|---|---|---|
| $A$-DCC | 0.030 (0.000) | 0.039 (0.000) | 0.025 (0.001) |
| $B$-DCC | 0.942 (0.000) | 0.911 (0.000) | 0.945 (0.000) |
| $\phi_1$ | 0.003 (0.000) | 0.003 (0.000) | -0.003 (0.925) |
| $\omega_1$ | 0.016 (0.000) | 0.016 (0.000) | 0.061 (0.000) |
| $\alpha_1$ | -0.004 (0.621) | -0.004 (0.621) | -0.003 (0.848) |
| $\beta_1$ | 0.980 (0.000) | 0.980 (0.000) | 0.963 (0.000) |
| $\gamma_1$ | 0.053 (0.000) | 0.053 (0.000) | 0.055 (0.000) |
| $\phi_2$ | -0.003 (0.925) | -0.036 (0.490) | -0.036 (0.490) |
| $\omega_2$ | 0.061 (0.000) | 0.010 (0.000) | 0.010 (0.000) |
| $\alpha_2$ | -0.003 (0.848) | -0.055 (0.000) | -0.055 (0.000) |

| | | | |
|---|---|---|---|
| $\beta_2$ | 0.963 (0.000) | 0.990 (0.000) | 0.990 (0.000) |
| $\gamma_2$ | 0.055 (0.000) | 0.022 (0.000) | 0.022 (0.000) |
| $d_{31}$ | -0.008 (0.700) | -0.067 (0.003) | -0.073 (0.001) |
| $d_{41}$ | 0.101 (0.000) | 0.099 (0.000) | 0.116 (0.000) |
| $d_{32}$ | -0.059 (0.005) | -0.048 (0.010) | -0.052 (0.007) |
| $d_{42}$ | 0.121 (0.000) | 0.034 (0.000) | 0.032 (0.000) |

*A-DCC* and *B-DCC* are the parameters of the DCC model; $\phi_i$ is the parameter of the AR(1) model; $\omega_i$, $\alpha_i$, $\gamma_i$ and $\beta_i$ are the parameters of the EGARCH(1,1) model; $d_{3i}$ and $d_{4i}$ are the parameters of the GC model. P-values in parentheses.

## Table 10. T-test for 97.5%-ES for commodity ETFs portfolio returns

**Mean of violation residual is zero**

Data: December 2008 – January 2016 (1771 days for backtesting)

| Portfolios | *A: Gold-Silver* | *B: Gold-Oil* | *C: Silver-Oil* |
|---|---|---|---|
| Historical Simulation | 0.196 (**0.845**) | -0.589 (**0.559**) | 0.184 (**0.855**) |
| Gaussian-DCC | 6.944 (0.000) | 4.810 (0.000) | 4.713 (0.000) |
| SNP-DCC | 1.938 **(0.061)** | 1.664 **(0.110)** | 1.699 **(0.100)** |

P-values for the t-test test in parentheses.

## Table 11. Pairwise Diebold Mariano test for 97.5%-ES

| **Model A →**<br>**Model B ↓** | **Gaussian-DCC** | **SNP-DCC** |
|---|---|---|
| *A: Gold-Silver* | | |
| Historical Simulation | 4.327 (0.999) | 1.068 (0.857) |
| Gaussian-DCC | | **-5.540** (0.000) |
| *B: Gold-Oil* | | |
| Historical Simulation | 4.264 (0.999) | **-1.702** (0.044) |
| Gaussian-DCC | | **-3.532** (0.000) |
| *C: Silver-Oil* | | |
| Historical Simulation | 3.561 (0.999) | 0.271 (0.607) |
| Gaussian-DCC | | **-3.584** (0.000) |

The table shows the Diebold-Mariano statistics for different models. Bold figures indicate Model A is preferred than Model B, figures in red indicate the opposite. Otherwise both models present the same performance. P-values in parentheses.

**Figure 1. 97.5%-ES for Portfolio of commodity LETFs (LGas and LSilver) in 2015**

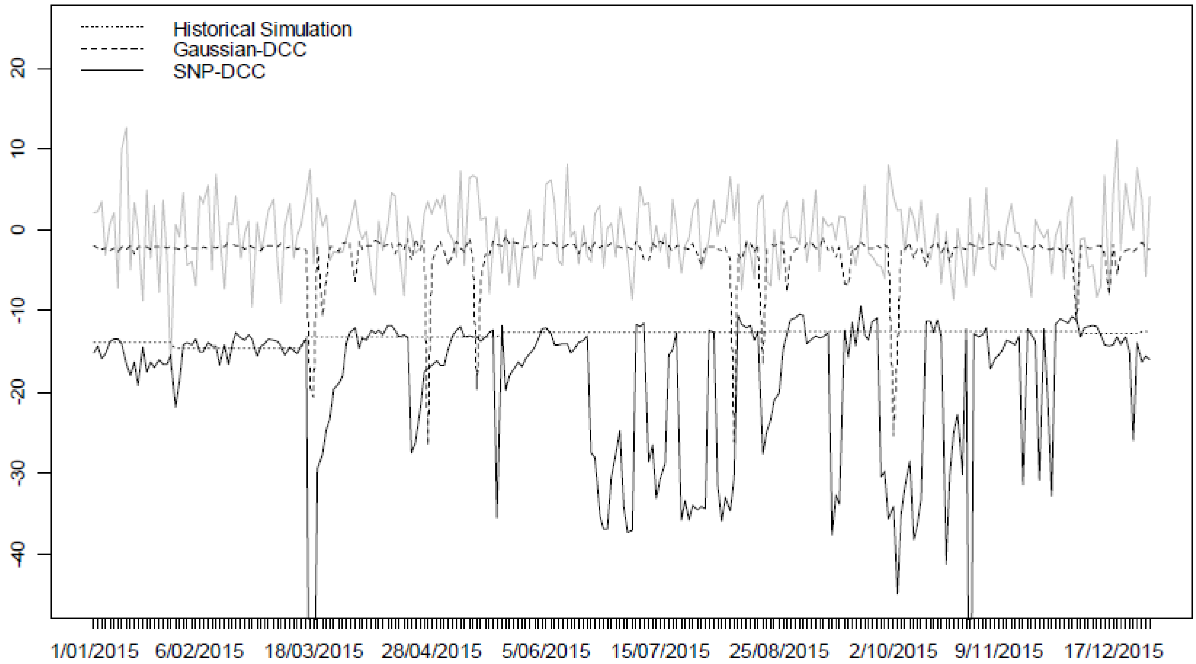
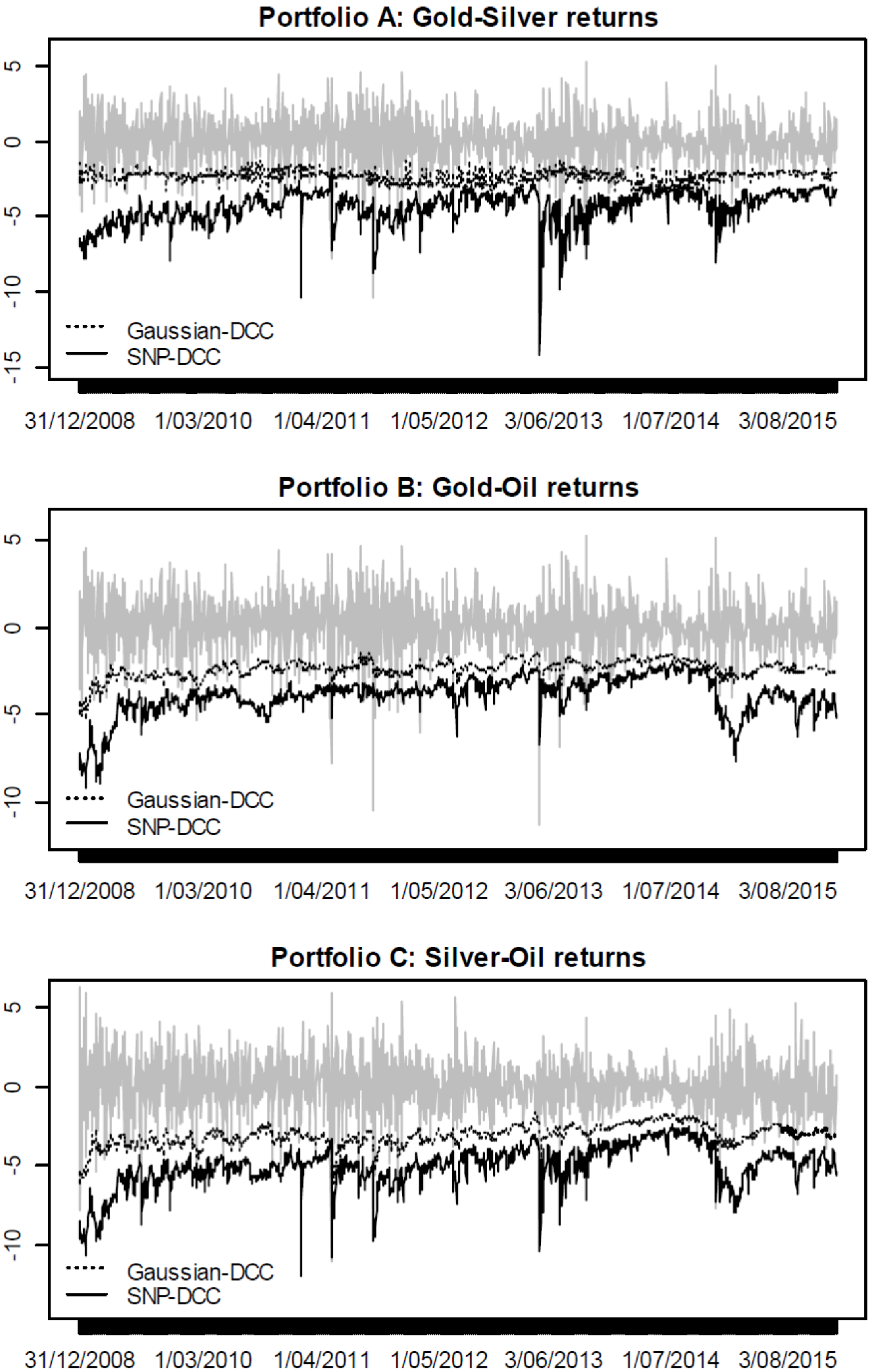


**Figure 2. 97.5%-ES for portfolio of commodity ETFs**



## Appendix A. Data description

| LETF Commodity | Ticker | Description |
|---|---|---|
| LGas | UGAZ | VelocityShares Daily 3x Long Natural Gas ETN is an exchange-traded note issued in the USA. The Note will provide investors with a cash payment at the scheduled maturity or early redemption based on the performance of the underlying index, S&P GSCI Natural Gas Index ER. |
| LSilver | USLV | VelocityShares 3x Long Silver ETN is an exchange-traded note issued in the USA. The Note will provide investors with a cash payment at the scheduled maturity or early redemption based on the performance of the underlying index, the S&P GSCI Silver Index Excess Return. |
| LOil | UCO | ProShares Ultra Bloomberg Crude Oil is an exchange-traded fund incorporated in the USA. The Fund will seek daily investment results that correspond to twice (200%) the daily performance of its corresponding benchmark, the Bloomberg Crude Oil Sub-Index. |

Source: Bloomberg LP.

## Appendix B. Data description

| ETF Commodity | Ticker | Description |
|---|---|---|
| Gold | GLD | SPDR Gold Shares is an investment fund incorporated in the USA. The investment objective of the Trust is for the Shares to reflect the performance of the price of gold bullion, less the Trust's expenses. The Trust holds gold and is expected from time to time to issue Baskets in exchange for deposits of gold and to distribute gold in connection with redemptions of Baskets. |
| Silver | SLV | iShares Silver Trust is a trust formed to invest in silver. The assets of the trust consist primarily of silver held by the custodian on behalf of the trust. The objective of the trust is for the shares to reflect the price of silver owned by the trust, less the trust's expenses and liabilities. |
| Oil | USO | United States Oil Fund LP is a Delaware limited partnership incorporated in the USA. The Fund's objective is to have changes in percentage terms of its unit's net asset value reflect the changes of the price of WTI Crude Oil delivered to Cushing, Oklahoma, as measured by changes in percentage terms of the price of the WTI Crude Oil futures contract on the NYMEX. |

Source: Bloomberg LP.