Optimal financing of highly innovative projects under double moral hazard*

Gino Loyola† Yolanda Portilla‡

January 2, 2019

Abstract

A model is presented for analyzing the investor-entrepreneur relationship in the financing of highly innovative projects under double moral hazard. It is shown that a broad family of financing contracts are optimal, a conclusion that fits well with the mixed security structures observed in the real world such as convertible preferred equity, warrants and call options. Schemes that reward risk (and failure) are also desirable, as extreme returns can be good indicators of highly innovative investment projects. Numerical simulations show that the conditions for credit rationing emerge when straight debt is used, stressing the welfare-improving role played by hybrid securities. The simulation results replicate several stylized facts of innovative firms, suggesting that the proposed approach is a suitable starting point for modelling venture capital financing.

Keywords: reward for failure; optimal financial contract; entrepreneurship financing; convertible assets; non-monotone likelihood ratio property

JEL Code: D86, G11, G20, G30, J33, M52

*We thanks the comments from Gabriel Natividad, Michael Tröege and seminar participants at LACEA 2018, ESCP-ReFix, ESAN, and UDEP.

†Department of Management Control and Information Systems, University of Chile, Diagonal Paraguay 257, Of. 1903, 8330015, Santiago, Chile. E-mail: gloyola@fen.uchile.cl, Tel: +56 (2) 22 978 37 68.

‡School of Economics and Business, Universidad Nacional Andrés Bello, Fernández Concha 700, 7591538, Santiago, Chile. E-mail: yolanda.portilla@unab.cl, Tel: +56 (2) 22 661 81 65.

1 Introduction

Investment projects and the schemes for financing them in innovative high-tech industries exhibit a number of distinguishing features. First, their returns profiles tend to be characterized by (i) abnormally high but highly improbable success returns, and (ii) a high probability of failure.[1] Second, small high-growth firms in these industries display lower levels of financial leverage than large firms in less innovative industries,[2] often adopting hybrid financing structures that combine debt and equity with preferential seniority and conversion options. This is especially so in the U.S.[3] Third, investors financing small innovative businesses frequently supply the entrepreneur not only funds but also effort in the form of advice, management, networks and monitoring.[4] Fourth, and finally, high-growth innovative firms appear to face greater barriers in obtaining external financing, which suggests that credit rationing in this class of firms may be relatively more prevalent.[5]

The present paper attempts to account for these features and stylized facts. We propose a financial contracting model that formalizes the relationship between an investor and an entrepreneur on investment projects of the type just described. Our setting includes two assumptions not traditionally made in contract theory. First, to model highly innovative projects we posit a non-monotone statistical relationship between (verifiable) returns and (unverifiable) innovation, thus ruling out the conventional monotone likelihood ratio property (MLRP). Secondly, to allow the investor to influence innovation decisions we assume a two-sided moral hazard setup, meaning that both the principal and the agent must have an incentive to attain an optimal joint innovation level. Within this framework we then fully characterize the optimal financing schemes for highly innovative investment projects.

Our results show that the two-sided moral hazard nature of the proposed model implies that the first-best and second-best contracts generate a balanced solution, which in turn means there must be positive levels of innovation on the part of *both* the investor and the entrepreneur. In a symmetric information environment, the efficient innovation level can be implemented with either a full-insurance or a full-franchise

[1] Empirical evidence suggests that the proportion of innovative investment projects that is successful is typically no greater than 20% (Bergemann and Hege, 1998; Sahlman, 1990; Gorman and Sahlman, 1989).

[2] See Chang and Song (2014), Frank and Goyal (2003), Barclay et al. (1995), and Long and Malitz (1985).

[3] See Kaplan and Strömberg (2003), Kaplan et al. (2012), Bengtsson and Sensoy (2011), Sahlman (1990), and Trester (1998).

[4] See Kaplan and Strömberg (2004), Cumming and Johan (2007), Sapienza et al. (1996), Gorman and Sahlman (1989), and Bengtsson and Sensoy (2011).

[5] See Chang and Song (2014), and Brown (1997).

contract. Interestingly, however, both contract types must impose a penalty, in the former case on the entrepreneur and in the latter case on the investor.

We also show that under asymmetric information, a large family of financing rules can achieve the optimal (second-best) innovation level, including strictly increasing, strictly decreasing, and a wide variety of non-monotone schemes. However, because of the double moral hazard problem (reminiscent of team moral hazard), there is no financing rule in this environment that can implement first-best innovation levels.

The fact that under asymmetric information there can be a broad array of non-monotone optimal contracts is a consequence of the highly innovative nature of the type of businesses we are considering. This is so because in this class of projects, it is not the shape (average slope) of the contract that matters but rather the relationship between the rewards to returns at opposite extremes. Technically, this phenomenon occurs because in the absence of MLRP regarding observed returns and unobserved innovation decisions, failure as well as success can be a good signal of a truly innovative project. We thus show that a salient class of optimal non-monotone financing schemes consists of those that reward extreme returns and punish, in relative terms, moderate returns such as those for U-shaped and J-shaped contracts. These risk-reward schemes also have a failure reward property, which although counter-intuitive, can be associated with hybrid contracts used in practice by entrepreneurial businesses to finance highly innovative projects.

In the case of a strictly decreasing financing rule, the rule's optimality is a consequence of both the absence of MLRP and the double moral hazard setup. In this setting a scheme may reward extreme results through an optimal combination of incentives for both the investor and the entrepreneur. A decreasing scheme can then work if the relative reward given to the investor (entrepreneur) exceeds the relative penalty imposed on the entrepreneur (investor) when high (low) returns are observed. In other words, since the ultimate goal of the financial rule is joint innovation, a decreasing scheme can be optimal if it properly counterbalances the opposing incentives facing the two parties in the relationship.

We further demonstrate that the features of various of these theoretical optimal contracts are consistent with non-linear and hybrid financing schemes combining equity and other securities that are used in practice to fund high-return, high-risk investment projects. In particular, we explore the implementation of two optimal contracts. The first one is a strictly increasing scheme implemented through convertible preferred equity and a sequence of warrants while the second is a risk-reward scheme (the U-shaped contract) implemented through equity plus either call options or a dilution process in favour of the investor. In the latter case, we analyze the possibility that the investor can infuse new capital into the project when returns are sufficiently low and thus cross-subsidize incomes from success and failure scenarios. All of these optimal hybrid structures highlight the importance of combining both inside and outside equity, a property that ensures, under double moral hazard, the creation of high-powered incentives for both the investor and the entrepreneur to engage in innovation.

Also to be studied are two classes of suboptimal contracts that are relevant in the context of our model: straight debt and full franchise. In the straight debt case, our analysis suggests that it induces a corner solution in which only the entrepreneur engages in innovation. Under double moral hazard, the outcome is joint innovation and an expected surplus lower than the levels obtained with an optimal contract. This result follows from the inability of the pure debt scheme to replicate a crucial property of our optimal hybrid security, namely, providing equity to the investor and the entrepreneur and thus giving both parties high-powered incentives to undertake innovation. Interestingly, this theoretical finding is consistent with the above-cited evidence that leverage is less frequent in high-tech innovative industries.

A comparative statics analysis performed on the straight debt contract reveals that under this scheme, *credit rationing* may occur with profitable investment projects that have either (i) a sufficiently high probability of moderate returns, or (ii) an excessively high probability of failure. This implies that if our optimal mixed securities were used to finance investment projects with either of these two characteristics, they would be particularly welfare-enhancing. The reason is that despite being profitable, these projects may not be able to secure funding under conventional debt schemes such as bank loans or bond issues. This implication of the model is also supported by the evidence mentioned at the beginning of the paper, since more innovative firms seem to be particularly financially constrained.

In the full-franchise case, although the extant literature has shown that under risk-neutrality and single moral hazard this type of contract achieves the first-best effort level, we prove that under double moral hazard not only does this no longer hold but also that the contract is not even second best. This is so because the full-franchise scheme performs similarly to the debt contract (a corner solution) in the sense that the entrepreneur alone is strongly motivated to innovate.

An important contribution of this study is the characterization of innovation as an endogenous variable that depends on the primitive characteristics of the investment projects such as the parameters of the returns space and the returns probability distribution. To illustrate this, we conduct a number of comparative static analyses on these parameters and the effects of changes in them on the optimal contracts. Three classes of results emerge from these numerical simulations. First, the optimal (first-best and second-best) innovation levels and expected surplus increase if either (i) the probability of moderate returns decreases, i.e., what we will call probability extremism increases, or (ii) the level of abnormally good returns increases, i.e., returns are more skewed towards success. By contrast, optimal innovation and surplus decrease if the probability of low returns increases i.e., the returns are more skewed towards failure.

The second set of results suggests which types of projects incur higher costs due to asymmetric information by computing the innovation and surplus gaps between the first-best and second-best solutions. Finally, the results on the returns distribution parameters allow us to perform two additional analyses: (i) the a priori classification of a project according to a three-class typology often used in the venture capital industry to predict the performance of innovative investment prospects; and (ii) an examination of the impact of technological shocks on the innovation levels and the social value created by highly innovative investment projects.

The rest of this paper is organized as follows. Section 2 reviews the most closely related works in the existing literature, identifying by way of comparisons with them the contributions of the present study. Section 3 proposes a model of the financing of highly innovative investment projects under double moral hazard. Section 4 fully characterizes the family of optimal financial contracts (first-best and second-best), with special emphasis on those that involve a risk reward feature and thus a reward for failure. Section 5 performs various comparative statics analyses on optimal innovation and surplus and their relationship with different parameters of the project returns distribution. A similar analysis is also applied to the U-shaped contract, a specific case of the optimal risk-reward schemes. Section 6 explores and compares different alternatives for implementing the optimal financing scheme via straight debt, franchise contracts, and hybrid securities structures such as convertible preferred equity, warrants and call options. Finally, Section 7 presents our main conclusions. Most of the proofs are set out in the Appendix.

2 Related literature

The present paper is related to previous works on optimal financial arrangements under a double-sided moral hazard environment, especially those devoted to venture capital financing. One of the main goals of this literature is to show and justify the optimality of mixed securities for financing this class of entrepreneurial business. In this vein, Casamatta (2003) posits a single-stage financing in which effort and advice are substitutes and outside financial investment is endogenous, showing that it is optimal for the venture capitalist (entrepreneur) to be given preferred stocks when the level of outside financing is sufficiently high (low).[6] Repullo and Suarez (2004) assume a two-stage financing setting (start-up and expansion stages) in which effort and advice are complements, concluding that a set of standard non-linear claims (warrants) are optimal when a project's interim profitability is not verifiable. Schmidt (2003) adopts an incomplete contract approach where the efforts of the entrepreneur and the investor are sequential decisions and can be either substitutes or complements, finding that convertible debt outperforms any standard debt-equity contract since it induces first-best decisions in every state of the world. Wang and Zhou (2004) and Wang (2009) develop a two-period analysis in which entrepreneur effort and outside investment resemble a double moral hazard environment. Whereas the first paper demonstrates that equity sharing and staged (instead of upfront) financing work complementarily to achieve approximately the first-best solutions for high-potential ventures, the second one shows that with staged financing, certain types of convertibles can be fully efficient. Inderst and Müller (2004) propose an equilibrium model in which investor and entrepreneur efforts can be either substitutes or complements, showing that a combination of debt, inside and outside equity optimally balances the incentives of both agents when their bargaining powers depend ultimately on primitive market characteristics.

A number of theoretical works examining double-moral hazard in the investor-entrepreneur relationship, though broadly related to the present paper, do not have the same ultimate purpose. Some are focussed on the entrepreneur's choice between a venture capitalist and other financiers such as an angel investor (Fairchild, 2011) or a bank (de Bettignies and Brander, 2007) while others are concerned with the optimal portfolio of start-up firms in venture capital finance when managerial advice is scarce (Kanniainen and Keuschnigg, 2003, and 2004). But with different goals in mind, they all assume that the venture capitalist provides equity finance, and are therefore unable to characterize non-standard financial contracts.

The present paper also has connections with theoretical works that, although considering only single moral hazard or none at all, do attempt to provide explanations for the use of hybrid securities structures to finance innovative firms.[7] This literature argues that non-standard claims are optimal if they either (i) balances the venture capitalist's incentive to intervene and the entrepreneur's desire for control (Marx, 1998), (ii) reduce the entrepreneur's incentives to focus on short-term success and window-

[6] Vergara, Bonilla and Sepúlveda (2016) generalize some of Casamatta's results by assuming complementarity between the two partners' efforts. The authors' numerical simulations (perhaps influenced by their assumption that the partners' parameters are symmetric) suggest that a fifty-fifty combination of inside and outside equity is optimal no matter what the degree of effort complementarity might be.

[7] An exception is the one-sided moral hazard approach of Elitzur and Gavious (2003), whose results suggest that a straight debt contract is optimal in a multi-period setup.

dressing (Cornelli and Yosha, 2003), (iii) mitigate the distributional conflicts associated with a future sale of a company (Berglöf, 1994), or (iv) allow the entrepreneur to resolve the tension between two different dimensions of moral hazard, namely, effort and excessive risk-taking (Biais and Casamatta, 1999).

It should be noted, however, that unlike the present paper, none of the above cited works assuming either one-sided or two-sided moral hazard consider the returns distribution in modelling what it really means for a firm to be highly innovative. All of them make the classic MLRP assumption that the actions of the entrepreneur and the investor influence monotonically and positively the returns distribution, without linking innovative efforts to greater risk, heavier distribution tails or any other statistical moment apart from the expected return.

Finally, our paper is related to works that determine theoretically the conditions for credit rationing in the case of start-up and highly innovative firms. Biais and Casamatta (1999) attribute the lack of financing for profitable investment projects to both insufficient internal cash flow and the tension between the entrepreneur's incentives to put in effort and take risks. Cornelli and Yosha (2003) argue that short-term manipulation by the entrepreneur can render otherwise profitable projects unworthy of funding by the venture capitalist under a staged financing scheme. By contrast, under the model we develop here, credit rationing arises from the characteristics of the returns distribution for highly innovative projects, and specifically when either their failure rates are sufficiently high or the probability they will generate extreme returns is sufficiently low.

3 The Model

Consider the agency relationship between an investor (the principal, *she*) and an entrepreneur (the agent, *he*) who are partners in a project with an initial investment I normalized to zero. The entrepreneur has no initial wealth so the entire investment is financed by the investor. Each of them must choose their own level of innovation in a simultaneous-move game. The entrepreneur's innovation level is denoted by $a \in [0,1]$ and the investor's by $p \in [0,1]$. Whereas a can be interpreted as innovation in the venture's operational processes, p can be thought of as innovation or advice regarding the management side. We define $e = a + p$ as the *joint* innovation level of the two partners' agency relationship such that $e \in [0,1]$.[8]

[8] We are thus assuming that a and p are substitutes, as in Casamatta (2003), but other functional forms for joint innovation are also possible. In general terms, we can define $e = f(a,p)$ such that both parties' innovation levels are productive and mutually complementary (see Kim and Wang, 1998; Repullo and Suarez, 2004).

3.1 Innovation and return

Our model assumes that although each partner's innovation is private information and thus not verifiable by the other, the project return is verifiable by both. We then let $x_i^{(a,p)}$ be the return on a project with an innovation pair (a,p) in state of nature i. Also, let $\pi_i^{(a,p)}$ be the conditional probability of observing return $x_i^{(a,p)}$ such that $\pi_i^{(a,p)} > 0$ for all $i = 1,..,n$ and for all $(a,p) \neq (0,0)$. The sum of the conditional probabilities must satisfy $\sum_{i=1}^{n} \pi_i^{(a,p)} = 1$ for all (a,p). These assumptions imply that for any given observed x_i, no innovation pair (a,p) can be ruled out a priori except $(0,0)$.

For simplicity's sake we assume there are only three possible states of nature ($i = 1,2,3$) such that $x_i^{(a,p)} \in X = \{x_1, x_2, x_3\}$ for all (a,p), and $x_1 < x_2 < x_3$. Specifically, we posit the following formulations for returns and their conditional distribution.

Assumption 1 (A1). The returns space for $x_i^{(a,p)}$ is

$$X = \{1-\sigma, 1, k(1+\sigma)\},$$

where $\sigma > 0$ and $k > 1$, and their probability distribution is described by[9]

$$\pi_i^{(a,p)} = \begin{cases} m\gamma(a+p) & \text{if } x_i = 1-\sigma \quad (i=1) \\ -\gamma(1+m)(a+p)+1 & \text{if } x_i = 1 \quad (i=2) \\ \gamma(a+p) & \text{if } x_i = k(1+\sigma) \quad (i=3) \end{cases},$$

where $m > 1$ and $\gamma \in \left(0, \frac{1}{1+m}\right)$.

This assumption merits a number of comments.

Positive skewness. (A1) captures the idea that the entrepreneur faces an investment project whose returns distribution exhibits positive skewness.[10] This is consistent with the empirical evidence on highly innovative ventures. The skewness is the consequence of assumptions on the nature of parameters k and m. Regarding the first parameter, by assuming $k > 1$ we can formalize the idea that the success return $x_3 = k(1+\sigma)$ is potentially abnormally good because it deviates more from the intermediate return $x_2 = 1$ than the failure return $x_1 = 1-\sigma$. In statistical terms, this assumption ensures

[9] Biais and Casamatta (1999) use a distribution function similar to the one presented here, but with binary effort and risk choices. Their model does not study investment projects with both high returns and high risk, however. The distribution functions in Hermalin and Weisbach (2005) and Loyola and Portilla (2014) are also similar, but the first is formulated in the context of corporate governance and the second in the context of fund management compensation. None of these frameworks consider a double moral hazard setup.

[10] In one of the extensions of their initial coin offering (ICO) setup, Chod and Lyandres (2018) also consider a right-skewed payoff distribution. However, they do not study the conventional moral hazard problem (either single or double) in the investor-entrepreneur relationship, but a particular agency problem arisen from the ICO financing scheme.

that the right tail of the returns distribution is longer than its left tail. As for the m parameter, by assuming $m > 1$ we formalize the idea that failure is much more likely than an exceptional but unlikely success. Statistically, this assumption ensures that the mass of the distribution is more concentrated on the left side of the distribution than the right side.

To illustrate how these two parameters characterize the asymmetry of the returns distribution, we take the intermediate return $x_2 = 1$ as a reference point. Then, if k and m were 1, the right and left sides of the distribution would be equally long and fat, and the distribution would be symmetric. We thus say that while the parameter k measures the skew of the project's returns distribution towards success, that is, its *success returns skew*, the parameter m measures the skew of the project's probability towards failure, that is, its *failure probability skew.*

Type of projects. Assumption (A1) is also consistent with the empirical evidence, which suggests that innovative projects can be classified in terms of three different profiles: (i) poor projects, (ii) the living dead, and (iii) high flyers (Sahlman, 1990; Gorman and Sahlman, 1989). For instance, according to the sample studied by Salhman (1990), poor projects (35% of the sample) suffered a total loss or could not repay the initial investment. The living dead (50% of the sample) showed moderate profitability, but venture capitalists generally did not invest additional resources or effort in them. Lastly, high flyers (the remaining 15%) exhibited outstanding profitability, with a return more than five times their initial investment.

The values taken by some parameters of the models ex ante suggest whether a project will ex post end up in one of these three profiles. In particular, for a given pair of innovation (a, p), a high flyer should exhibit a priori a larger k while a poor project would display a larger m. A living dead project should a priori show a smaller γ since this term parameterizes positively (negatively) the probability of extreme (moderate) returns. We will therefore say that γ measures the project's *level of probability extremism.*

Technological shocks. The way we model the return distribution also allows us to examine the consequences of a change in the profitability of investments caused by real or perceived exogenous technological shocks. For example, the increase of k could be associated with a technological bubble like the Internet boom that occurred at the turn of the 21st century and an increase of m can then be associated with the bubble's later burst. Also, since episodes of boom and bust are accompanied by an increase in volatility, we can study this class of effects through the parameter σ, which we consider as the measure of the project's *level of returns extremism.*

Innovation and risk. The parameter definitions guarantee that $\partial\pi_i^{(a,p)}/\partial a > 0$ and

$\partial\pi_i^{(a,p)}/\partial p > 0$ for $i = 1$ and $i = 3$, but $\partial\pi_i^{(a,p)}/\partial a < 0$ and $\partial\pi_i^{(a,p)}/\partial p < 0$ for $i = 2$. Thus, a higher level of either a or p increases (decreases) the probability of extreme (moderate) events. This property is consistent with the idea that innovation parameterizes the project's risk, and is also consistent with the fact that real-world high-tech projects involve a high degree of uncertainty, whether technological, business/market-related or regulatory, especially in their early stages (Metric and Yasuda, 2010).

In statistical terms, this characteristic implies that MLRP is not satisfied. To illustrate this phenomenon, let us define the likelihood ratio associated with innovation a as

$$LR_i^a = \frac{\partial\pi_i^{(a,p)}/\partial a}{\pi_i^{(a,p)}}, \text{ for } i = 1, 2, 3,$$

from which it is easily verified that $LR_1^a > LR_2^a$ as

$$LR_2^a = \frac{1}{e - (\gamma(1+m))^{-1}} < 0 < \frac{1}{e} = LR_1^a, \tag{3.1}$$

even though $x_2 > x_1$.[11]

Recall that LR_i^a indicates how good is the return x_i as a signal of the fact that the entrepreneur selected an innovation level a. In other words, LR_i^a reflects how *informative* is the project's (verifiable) return x_i with respect to an (unverifiable) innovation decision. The larger is LR_i^a, the more likely it is that the entrepreneur chose an innovation level a. Thus, if this is the level the investor desires, the financial contract should compensate the entrepreneur more whenever return x_i is observed. But note that such a provision does not guarantee the optimal financing scheme will be increasing in the returns on the project. That property will only hold if the likelihood ratio is monotonically increasing in x_i, that is, if the MLRP condition is met. As will be established later, the non-satisfaction of this property in our model is crucial to our main results.

3.2 Preferences

The entrepreneur's preferences are assumed to be described by the following (ex post) additively separable risk-neutral utility function:

$$U(w, a) = w - \frac{\theta_1 a^2}{2},$$

where w is the entrepreneur's share of the return generated by the project. The second term of the function is the disutility the entrepreneur experiences by choosing a positive innovation level, with $\theta_1 > 0$ representing a coefficient of his aversion to innovation. Whereas the financing contract must satisfy a limited liability constraint on the entrepreneur side so that $w \geq 0$, there is no such constraint on the investor side.

[11] The inequalities in expression (3.1) hold because $e \in [0, 1]$ and $\gamma \in \left(0, \frac{1}{1+m}\right)$ by assumption (A1).

The investor is also risk neutral and her preferences (before the initial investment I) are represented by the (ex post) utility function

$$B(x, w, p) = x - w - \frac{\theta_2 p^2}{2},$$

where $x - w$ is her share of the project's return and the last term is the disutility to her of engaging in innovation, $\theta_2 > 0$ then being her coefficient of innovation aversion.

Since the optimal financing scheme may be contingent on returns, for simplicity we define $w_i \equiv w(x_i)$, the entrepreneur's share of the return if x_i is observed for all $i = 1, 2, 3$.

The investor's opportunity cost is the riskless interest rate r so the optimal contract must satisfy a gross expected payoff to her that is equal to or greater than $(1 + r)I$. Because of our normalization of I, however, this payoff is reduced to zero. Since all the bargaining power is in the hands of the investor, the optimal contract must offer the entrepreneur at least his reservation utility $\underline{U} > 0$. This utility represents the fact that in general the entrepreneur gives up an alternative wage when deciding to undertake a venture project.

To compare the optimal schemes under symmetric and asymmetric information, we define the investor's optimal expected payoff as[12]

$$EB^j = \sum_{i=1}^{3} \pi_i^{\left(a^j, p^j\right)} \left(x_i - w_i^j\right) - \frac{\theta_2 (p^j)^2}{2} \tag{3.2}$$

where j indicates the type of sharing rule: a first-best contract $(j = FB)$ and a second-best contract $(j = *)$. As for the expected surplus generated by sharing rule j, we define it as

$$S^j = \sum_{i=1}^{3} \pi_i^{\left(a^j, p^j\right)} x_i - \frac{\theta_1 (a^j)^2}{2} - \frac{\theta_2 (p^j)^2}{2}. \tag{3.3}$$

3.3 Technical assumptions

To obtain certain of our results we adopt two additional assumptions regarding the model parameters, as explained below.

Definition 1. Let Ψ be the project's *expected marginal return on innovation*, defined as

$$\Psi \equiv \frac{\partial E(x_i^{(a,p)})}{\partial a} = \frac{\partial E(x_i^{(a,p)})}{\partial p}.$$

[12]Because we normalized I to zero, this is indeed an expected payoff before and after initial investment. The same applies to the expected surplus defined below.

Thus, Ψ represents the change in the project's expected return when innovation a or p increases. In our model this term is given by

$$\begin{aligned}\Psi &= m\gamma(x_1 - x_2) + \gamma(x_3 - x_2) \\ &= \gamma(\sigma(k-m) + (k-1)).\end{aligned} \tag{3.4}$$

Assumption 2 (A2). The expected marginal return on innovation is such that

$$0 < \Psi < \frac{\theta_1\theta_2}{\theta_1 + \theta_2}.$$

The implications of these two bounds on Ψ are the following. The lower bound $\Psi > 0$ implies that the effect of innovation on a higher expected return conditional on success, represented by the positive term $\gamma(x_3 - x_2)$, outweighs the effect of a lower expected return conditional on failure, expressed by the negative term $m\gamma(x_1 - x_2)$. This is equivalent to assuming that an increase in innovation a or p shifts the returns distribution towards more profitable riskier ventures. The upper bound $\Psi < \frac{\theta_1\theta_2}{\theta_1+\theta_2}$ ensures that the first-best joint innovation level e is strictly smaller than 1, thus avoiding a corner solution (see Proposition 1 in Section 4).

The other additional assumption relates to the entrepreneur's reservation payoff.

Assumption 3 (A3). The entrepreneur's reservation utility is such that

$$\Psi^2 \frac{2\theta_1^2 + \theta_2^2}{2\theta_1(\theta_1 + \theta_2)^2} < \underline{U} < 1 + \Psi^2 \frac{\theta_1^2 + \theta_2^2 + \theta_1\theta_2}{2\theta_1\theta_2(\theta_1 + \theta_2)}.$$

Setting these bounds on $\underline{U}$ is not just a simplifying assumption since it guarantees two important results in the context of our analysis. First, the lower bound ensures that under asymmetric information, the optimal w_2 will be strictly positive (see Proposition 2 in Section 4), which rules out the possibility of a debt-like scheme as one of the optimal arrangements (see Corollary 1, case (c1) in Section 4). Second, the upper bound guarantees that under symmetric and asymmetric information, the optimal contract is such that the investor's participation constraint is satisfied, thus preventing the existence of a credit rationing equilibrium.

4 Results

In this section we characterize the optimal innovation pair and its associated financing rule under both symmetric and asymmetric information. We then focus our analysis on sharing rules that reward risk-taking and therefore failure.

4.1 First-Best Solution

Under symmetric information the optimal sharing rule must solve the following program:[13]

$$\max_{\{w_i\}_{i=1}^3,\ a,p\in[0,1]} \sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{\theta_2 p^2}{2} \tag{4.1}$$

subject to

$$\sum_{i=1}^{3} \pi_i^{(a,p)} w_i - \frac{\theta_1 a^2}{2} \geq \underline{U} \tag{4.2}$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{\theta_2 p^2}{2} \geq 0 \tag{4.3}$$

$$w_i \geq 0 \text{ for all } i \tag{4.4}$$

$$a + p \leq 1, \tag{4.5}$$

where (4.2) and (4.4) are the entrepreneur's participation and limited liability constraints, (4.3) is the investor's participation constraint and (4.5) is the constraint ensuring joint innovation is not greater than 1.

Once this program is solved, we can establish the following result.

Proposition 1. *The first-best incentive scheme is such that the optimal innovation levels are*

$$\begin{aligned} a^{FB} &= \frac{\Psi}{\theta_1} \\ p^{FB} &= \frac{\Psi}{\theta_2} \\ e^{FB} &= \Psi \frac{\theta_1 + \theta_2}{\theta_1 \theta_2}. \end{aligned}$$

Two possible sharing rules for implementing this scheme are as follows:
(i) A full-insurance contract that specifies a fixed compensation for the entrepreneur given by

$$w^{FB} = \underline{U} + \frac{\Psi^2}{2\theta_1}$$

if $a = a^{FB}$, and a penalization otherwise.
(ii) A full-franchise contract that specifies a fixed payment by the entrepreneur to the

[13]Since the initial investment I is normalized to zero, it is not included in the computation of the investor's expected payoff. Note, however, that because in our fixed-investment model I is an exogenous variable, even if this term were positive it would be excluded from the investor's objective function in all the optimization programs posited in this paper.

investor given by[14]

$$F^{FB} = 1 - \underline{U} + \Psi^2 \frac{2\theta_1 + \theta_2}{2\theta_1\theta_2}$$

if $p = p^{FB}$*, and a penalization otherwise.*

Proof. See the Appendix.

Thus, under symmetric information there is a balanced solution in terms of innovation in that both a and p are strictly positive due to assumption (A2). Since we have assumed that these two innovations are perfect substitutes and have the same social marginal expected benefit Ψ, the difference between individual innovations are explained solely by the two θ cost coefficients.

4.2 Second-Best Solution

Under asymmetric information, the optimal sharing rule must solve the following program:

$$\max_{\{w_i\}_{i=1}^3,\ a,p\in[0,1]} \sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{\theta_2 p^2}{2} \tag{4.6}$$

subject to

$$\sum_{i=1}^{3} \pi_i^{(a,p)} w_i - \frac{\theta_1 a^2}{2} \geq \underline{U} \tag{4.7}$$

$$\sum_{i=1}^{3} \pi_i^{(a,p)} (x_i - w_i) - \frac{\theta_2 p^2}{2} \geq 0 \tag{4.8}$$

$$a \in \arg\max_{\tilde{a}\in[0,1]} \sum_{i=1}^{3} \pi_i^{(\tilde{a},p)} w_i - \frac{\theta_1 \tilde{a}^2}{2} \tag{4.9}$$

$$p \in \arg\max_{\tilde{p}\in[0,1]} \sum_{i=1}^{3} \pi_i^{(a,\tilde{p})} (x_i - w_i) - \frac{\theta_2 \tilde{p}^2}{2} \tag{4.10}$$

$$w_i \geq 0 \text{ for all } i \tag{4.11}$$

$$a + p \leq 1, \tag{4.12}$$

where (4.7), (4.9), and (4.11) represent the entrepreneur's participation, incentive compatibility and limited liability constraints, and (4.8) and (4.10) are the investor's participation and incentive compatibility constraints.

Once this program is solved, we can establish the following additional result.

Proposition 2. *The second-best incentive scheme is such that the optimal innovation*

[14] The upper bound imposed by assumption (A3) on the entrepreneur's reservation utility implies that F^{FB} is always positive.

levels are

$$\begin{aligned} a^* &= \frac{\Psi}{\theta_1}\frac{\theta_2}{\theta_1+\theta_2} \\ p^* &= \frac{\Psi}{\theta_2}\frac{\theta_1}{\theta_1+\theta_2} \\ e^* &= \frac{\Psi}{\theta_1\theta_2}\frac{\theta_1^2+\theta_2^2}{\theta_1+\theta_2}, \end{aligned}$$

and the scheme can be implemented by a sharing rule characterized by the following set:

$$\Omega^* = \left\{ (w_1^*, w_2^*, w_3^*) : w_1^* \in \left[0, \frac{A}{m}\right], w_2^* = \underline{U} - B, w_3^* \in [0, A] \text{ and } w_3^* = A - m w_1^* \right\},$$

where

$$\begin{aligned} A &\equiv \Psi\frac{\theta_2}{\gamma(\theta_1+\theta_2)} + (1+m)\left(\underline{U} - \Psi^2\frac{2\theta_1^2+\theta_2^2}{2\theta_1(\theta_1+\theta_2)^2}\right), \\ B &\equiv \Psi^2\frac{2\theta_1^2+\theta_2^2}{2\theta_1(\theta_1+\theta_2)^2}. \end{aligned}$$

Proof. See the Appendix.

In terms of innovation, the optimal solution under asymmetric information is also balanced since both a^* and p^* are strictly positive. However, the optimal values are smaller than those under full information, which implies that despite risk-neutrality, there is no financial contract under asymmetric information that can achieve first-best levels of individual and joint innovation. Note also that unlike the symmetric information case, under asymmetric information each player's optimal innovation level depends positively on his/her counterpart's cost coefficient θ, which reflects a *substitution* effect between the two individual innovation efforts.[15]

In terms of sharing rules, the set Ω^* implies that there are multiple financial contracts for implementing the second-best innovation levels. This multiplicity of contracts is illustrated in Fig. 1, which clearly shows that the optimal contract allows a wide range of financing rules including strictly increasing, non-monotone and even strictly decreasing schemes. A formal characterization of this multiplicity of contracts is given in the next statement.

⟨Insert Fig. 1 here⟩

Corollary 1. *The second-best financing rule is such that all of the following classes of contracts are optimal:*
(a) Strictly increasing contracts: $w_3^* > w_2^* > w_1^*$.

[15] A more detailed discussion of the properties of the optimal innovation levels is presented in Section 5.

(b) Strictly decreasing contracts: $w_1^* > w_2^* > w_3^*$.

(c) Non-monotone contracts:

(c1) Bonus plus fixed compensation: $w_3^* > w_2^* = w_1^*$.

(c2) J-shaped contract: $w_3^* > w_1^* > w_2^*$.

(c3) U-shaped contract: $w_3^* = w_1^* > w_2^*$.

(c4) G-shaped contract: $w_1^* > w_3^* > w_2^*$.

(c5) L-shaped contract: $w_1^* > w_3^* = w_2^*$,

where $w_2^* > 0$.

Proof. The multiplicity of contracts follows directly from Proposition 2 and Fig. 1. Moreover, the lower bound on $\underline{U}$ imposed by assumption (A3) ensures that $w_2^* > 0$.

Thus, in the absence of MLRP, what measures the optimal contract's *incentive power* is not its *shape* or its *average slope* but rather the relationship of rewards paid to the extreme returns as given by the equation $w_3^* = A - mw_1^*$ in Proposition 2 (and illustrated in Fig. 1). Indeed, this means that it is irrelevant whether the average slope of the entire sharing rule increases or decreases (even if it changes from positive to negative). All of this makes sense because, according assumption (A1), whereas extreme returns (either high or low) are more indicative of a higher innovation level, moderate returns are indicative of a lower level. One consequence of this last phenomenon is that a scheme rewarding moderate returns (an inverse-U-shaped contract) is excluded from the wide range of optimal financing rules characterized in Corollary 1.

4.3 Strictly decreasing and risk-reward contracts

Two schemes in this broad family of second-best contracts merit further analysis: strictly decreasing and risk-reward. In the counter-intuitive strictly decreasing case, the optimality of this contract can be explained by the absence of MLRP and the double moral hazard environment. In such a setting, the incentives are provided by the particular combination of reward and punishment applied to the agent *and* the principal, since both of them contribute innovation to an entrepreneurial business venture. Thus, although in relative terms a decreasing contract $w(x)$ rewards the entrepreneur for failure and punishes him for success, the complementary optimal scheme $x - w(x)$ confronts the investor with the opposite pair of incentives. In the end, therefore, it is the interplay of the two schemes that induces the optimal individual and joint innovation levels. Under our model formulation, it is the optimal combination of incentives facing the two parties that induces a balanced innovation solution.[16]

[16] Kim and Wang (1998) also characterize the optimal financial contracts under double moral hazard, but they assume MLRP and agent risk-aversion. They find that one of the optimal contracts exhibits a less severe non-monotonicity than our strictly decreasing scheme, but in their setup this is due to

The risk-reward scheme, on the other hand, is a contract that in relative terms rewards extreme returns and punishes moderate results. Under this class of financing rules $w_1^* > w_2^*$, hence they promise a larger share to the entrepreneur when the observed return is low (x_1) than when it is moderate (x_2). Thus, we cannot rule out the possibility that a scheme with a *reward-for-failure* property will be an optimal arrangement for financing highly innovative ventures. Examples of this class of schemes are the J-shaped and U-shaped contracts identified in Corollary 1.

Although these contracts are counter-intuitive, in the context of entrepreneurial business financing they have an appealing economic rationale. In the real world, higher innovation levels usually mean greater expected returns but also greater risk and failure rates. Thus, we may reasonably expect that (ex post) low returns are often a better signal that a relatively innovative project was chosen than (ex post) moderate returns. It may therefore be optimal for the investor to punish the entrepreneur (in relative terms) when the returns are moderate and reward him when they are extreme (high or low). This in turn implies that it may be efficient to reward failure to the extent that low returns are sufficiently suggestive of a high degree of effort and creativity supplied by the entrepreneur in developing investment projects with attractive risk-return profiles.[17]

Our analysis suggests that this reward-for-failure property may explain some non-monotone characteristics of schemes commonly adopted to fund highly innovative firms. This issue is explored in Section 6, which focuses on the implementation of the optimal sharing rule through real-world financing structures that mix debt, equity and conversion clauses.

4.4 First-best vs. second-best contracts

We end this section by summarizing our main results in terms of innovation levels, the investor's expected payoff and expected surplus. We also compare the optimal schemes under symmetric and asymmetric information in terms of these three dimensions.

Corollary 2. *(i) Innovation levels under symmetric information are strictly greater than those induced under asymmetric information:*

$$\begin{aligned} a^{FB} &> a^* \\ p^{FB} &> p^* \\ e^{FB} &> e^*. \end{aligned}$$

the absence of a limited liability condition on the principal side.

[17] A similar notion of 'reward for failure' is discussed by Manso (2011) in the context of an exploration technological innovation, and Loyola and Portilla (2014) in the context of top executive compensation.

(ii) The investor's expected payoff under symmetric information is strictly greater than that obtained under asymmetric information:

$$EB^{FB} = 1 - \underline{U} + \Psi^2 \frac{\theta_1 + \theta_2}{2\theta_1\theta_2} > 1 - \underline{U} + \Psi^2 \frac{\theta_1^2 + \theta_2^2 + \theta_1\theta_2}{2\theta_1\theta_2(\theta_1 + \theta_2)} = EB^*.$$

(iii) The expected surplus generated under symmetric information is strictly greater than that generated under asymmetric information:

$$S^{FB} = EB^{FB} + \underline{U} > EB^* + \underline{U} = S^*.$$

Proof. See the Appendix.

It is well known that under double moral hazard and MLRP, there is no incentive scheme that induces the first-best effort level even when the agent is risk- neutral (Hölmstrom, 1982; Kim and Wang, 1998). Thus, the present work extends this result to a model with double moral hazard and *no* MLRP, as established in point (i) of Corollary 2.

5 Comparative statics

In this section we present the results of comparative statics analysis applied first to various parameters in optimal contracts and then to an optimal risk-reward scheme.

5.1 Comparative statics on optimal contracts

We begin by analyzing the properties of the optimal innovation and surplus levels with respect to changes in the innovation cost parameters.

Corollary 3. *(i) The first-best innovation levels for both the entrepreneur and the investor are decreasing in his/her own innovation cost parameter.*
(ii) The first-best joint innovation and surplus levels are decreasing in the two cost parameters.
(iii) The second-best innovation levels for both the entrepreneur and the investor are decreasing in his/her own innovation cost parameter (direct effect) but increasing in the cost parameter of his/her counterpart (cross effect).
(iv) The second-best joint innovation level is decreasing in a given innovation cost parameter if its direct effect dominates in absolute value terms its cross effect.
(v) The second-best surplus is decreasing in the two cost parameters.

Proof. See the Appendix.

Thus, under symmetric information, both actors' innovation levels depend only on their own individual cost parameters (direct effect) while under asymmetric information they also depend positively on the cost parameter of their counterpart (cross effect). This cross effect reflects substitution between the two innovation levels at the optimal solution and stems from our assumption of an additive formulation for joint innovation e. The latter can therefore be either increasing or decreasing in each θ, depending on the relative magnitudes of the direct and cross effects. The expected surplus, however, is always decreasing under asymmetric information in each θ, suggesting that even when an increase in one of these parameters produces a greater joint innovation level and thus a greater expected project return, it also generates an increase in innovation costs that in the end will outweigh the expected return.

We now present our comparative statics analysis of four parameters in the returns distribution. The results of the analysis may be summarized as follows.

Corollary 4. *The first-best and second-best levels of innovation and surplus all have the following characteristics:*
(i) Increasing in γ.
(ii) Increasing in k.
(iii) Decreasing in m.
(iv) Increasing (decreasing) in σ *as long as* $k > m$ *(*$k < m$*).*

Proof. See the Appendix.

Our model allows us to examine how innovation behaves in the face of changes in a project's primitives, that is, the returns space and returns distribution parameters. Thus, it can be applied to the analysis of the effects of technological shocks. For example, the Corollary 4 results indicate that the sign of the effect on optimal innovation and surplus levels of increasing positive skew in a project's returns distribution will depend on this greater skew's source. If it is an increase in a project's real or perceived success return (an increase in k) generated, for instance, by a positive technological change, the effect on the project's social value will be positive. An example of this change would be the Internet bubble at the turn of the present century. If, on the other hand, the source is an increase in real or perceived project failure rates (an increase in m) brought about, for example, by the later burst of the Internet bubble, the ultimate effect on social value will be negative.

Our framework also enables us to predict the effects of greater volatility in the returns on highly innovative projects caused, for instance, by a bubble-burst cycle such as the one just mentioned. Corollary 4 suggests that the effect of a high degree of returns extremism as represented by an increase in the σ parameter will depend on the phase of the cycle. During the bubble, the model predicts that an increase in σ will have a positive effect on project innovation levels and social value given that it is likely $k > m$. By contrast, once the bubble bursts, the model predicts that an increase of σ

will have a negative effect on innovation and social value since the relation between k and m is more likely to be the other way around.

Further results can be derived regarding the *gap* between the first-best and second-best contracts, which can be thought of as a measure of the cost of asymmetric information.

Corollary 5. *The gap between first-best and second-best innovation levels as well as the gap between first-best and second-best expected surplus all have the following characteristics:*
(i) Increasing in γ.
(ii) Increasing in k.
(iii) Decreasing in m.
(iv) Increasing (decreasing) in σ *as long as* $k > m$ $(k < m)$.
(v) Decreasing in θ_1 *and* θ_2.

Proof. See the Appendix.

Thus, in the case of highly innovative projects, the welfare cost of asymmetric information will increase with success returns skew and decrease with failure probability skew.

With the results of Corollaries 4 and 5 we can predict the performance of projects that have been identified ex ante as fitting one of the three empirical profiles described in Subsection 3.1. Projects with an ex ante profile resembling that of a living dead (i.e., with a low value of γ) should exhibit low optimal innovation levels and innovation social values. Due to asymmetric information, however, the welfare cost of such projects should also be low. Ventures more reminiscent of a poor project (i.e., with a high value of m) would according to our analysis yield low levels of both optimal innovation and the corresponding social value. However, they would also involve low social cost, due once again to asymmetric information. Lastly, projects with an ex ante profile more like that of a high flyer (i.e., with a high value of k) should experience a high level of both optimal innovation and social value. The welfare costs attributable to asymmetric information for this class of projects should also be high.

5.2 Comparative statics on a risk-reward scheme

Our second comparative statics analysis considers the U-shaped contract, a specific risk-reward type of optimal financing rule. Since $w_3^* = w_1^* = w^* > w_2^*$, we can define Δw^*, the *optimal risk reward,* as follows

$$\Delta w^* \equiv w^* - w_2^*,$$

such that Δw^* represents a reward for either success or failure. This risk reward measures the power of the incentives to the entrepreneur created by the U-shaped contract for engaging in innovation. It can be shown that

$$\Delta w^* = \frac{\theta_2}{\theta_1 + \theta_2} \frac{\sigma(k-m) + (k-1)}{2(1+m)},$$

from which it is simple to confirm that

$$\begin{aligned} \frac{\partial \Delta w^*}{\partial m} &= -\frac{\theta_2}{\theta_1 + \theta_2} \frac{\sigma(k+1) + k - 1}{2(m+1)^2} < 0, \\ \frac{\partial \Delta w^*}{\partial k} &= \frac{\theta_2}{\theta_1 + \theta_2} \frac{\sigma + 1}{2m+2} > 0, \end{aligned}$$

and if it is the case that $k > m$,

$$\frac{\partial \Delta w^*}{\partial \sigma} = \frac{\theta_2}{\theta_1 + \theta_2} \frac{k-m}{2m+2} > 0.$$

Note that the sign of these partial derivatives is the same as those established by Corollary 4 in the comparative statics analysis of innovation and surplus levels with optimal contracts. The effects of changes in the m, k and σ parameters on the optimal risk reward Δw^* can thus be interpreted quite intuitively. These results imply that whenever exogenous changes in either the returns space or the returns distribution generate higher levels of optimal innovation, the incentives of a U-shaped financing contract for the entrepreneur should be more powerful.

Note also that

$$\frac{\partial \Delta w^*}{\partial \theta_1} = -\frac{\theta_2}{(\theta_1 + \theta_2)^2} \frac{\sigma(k-m) + (k-1)}{2(1+m)} < 0$$

and

$$\frac{\partial \Delta w^*}{\partial \theta_2} = \frac{\theta_1}{(\theta_1 + \theta_2)^2} \frac{\sigma(k-m) + (k-1)}{2(1+m)} > 0,$$

which implies that the incentives created by a U-shaped contract will be lower-powered the more innovation-averse is the entrepreneur (a standard result in the literature in the case of effort) but higher-powered the more innovation-averse is the investor due to the substitution effect discussed earlier.

6 Implementation

In this section we present two sets of implementation analyses, one for optimal financing schemes and the other for suboptimal schemes.

6.1 Optimal financing rules

Our first set of analyses concerns the implementation of certain of the optimal financing rules characterized in Corollary 1 for three classes of hybrid securities: (i) convertible preferred equity with optimal dividend, (ii) a sequence of warrants, and (iii) an initial equity stake with either short positions in call options or dilution in favour of the investor.

All three examples highlight the idea that the optimal financing rule must give some level of preferred inside equity to the entrepreneur and some level of common outside equity to the investor. In a double moral hazard setup, a proper combination of these two claims will create incentives that are sufficiently high-powered to induce both parties to undertake a positive level of innovation and thus achieve a balanced solution.

6.1.1 Convertible preferred equity

For this class of securities, we focus on the optimal strictly increasing contract $w_3^* > w_2^* > w_1^*$ in which[18]

$$w_1^* = 0, \tag{6.1}$$

$$w_2^* = \underline{U} - B, \tag{6.2}$$

$$w_3^* = A. \tag{6.3}$$

Following Marx (1998), we consider a financing contract in which the investor receives an ex post payoff described by

$$x - w(x) = \begin{cases} x & \text{if } x < d \\ d + \beta(x - d) & \text{if } x \geq d \end{cases}$$

where $d > 0$ represents a fixed dividend payment. The contract can be interpreted as a convertible preferred stock whose holder has seniority over a preferred dividend d and converts this equity into common stock at a rate β if the threshold d is exceeded by the project's return.

In the context of our model, we must look for the optimal pair (d^*, β^*) that allows us to implement the contract (6.1)-(6.3) so that it satisfies

$$(1 - \beta^*)(x_2 - d) = w_2^*, \tag{6.4}$$

$$(1 - \beta^*)(x_3 - d) = w_3^*, \tag{6.5}$$

[18] This contract is obtained by substituting $w_1^* = 0$ into Proposition 2.

as long as $d \in (x_1, x_2)$. Upon combining equations (6.2), (6.3), (6.4) and (6.5), and recalling the values for x_2 and x_3 in Assumption 1, we have

$$d^* = \frac{A - k(1+\sigma)(\underline{U} - B)}{A - (\underline{U} - B)}, \tag{6.6}$$

and

$$\beta^* = 1 - \frac{\underline{U} - B}{1 - d^*}. \tag{6.7}$$

From (6.6), it can be shown that whereas it is indeed the case that $d^* < x_2$ (since $k(1+\sigma) > 1$), it is also the case that $d^* > x_1$ only if

$$\frac{k(1+\sigma) - (1-\sigma)}{\sigma} < \frac{A}{\underline{U} - B}. \tag{6.8}$$

To illustrate this type of implementation, we simulate the results obtained for the parameter values $\theta_1 = \theta_2 = 1$, $\gamma = 0.35$, $\sigma = 0.5$, $k = 1.6$, $m = 1.5$ and $\underline{U} = 0.15$, which generates the return space $X = \{0.5, 1, 2.4\}$.[19] The optimal financing rule is then given by $w_1^* = 0$, $w_2^* = 0.13059$ and $w_3^* = 0.65148$, which can be implemented by preferred equity with dividend $d^* = 0.59886$ and a conversion rate of $\beta^* = 0.67445$. Thus, the investor's payoff profile under this convertible security is given by

$$x - w^* = \begin{cases} x & \text{if } x < 0.59886 \\ 0.59886 + 0.67445(x - 0.59886) & \text{if } x \geq 0.59886 \end{cases},$$

which is displayed in Fig. 2.

⟨Insert Fig. 2 here⟩

6.1.2 Sequence of warrants

As in the previous example of convertible preferred stock, our analysis for a sequence of warrants is centred on the optimal strictly increasing contract represented by equations (6.1)-(6.3). However, inspired in this case by the analysis of Repullo and Suarez (2004) we now consider a financing contract that includes an investor's initial outside equity and various thresholds for the project's return at which the investor's share changes. This contract can be viewed as preferred equity plus a set of warrants that yield higher payoffs to the investor as long as the firm value hits higher and higher strike prices.[20]

In more concrete terms, the preferred equity gives the investor priority on dividend $d = d_1$, and given two strike prices (d_1, d_2) such that $d_1 < d_2$, the investor also holds a long position in two call options over additional equity. Then, if the project's return $x \in (d_1, d_2]$, the investor has the option to buy an additional equity stake β_1 at a

[19] Note that all of these parameters values satisfy assumptions (A1)-(A3).

[20] Since our framework is a one-stage financing setup, this contract can also be interpreted as a convertible bond with a coupon equal to the preferred dividend.

unitary exercise of d_1. Alternatively, if the return is sufficiently high so that $x > d_2$, the investor can then buy an extra equity stake β_2 at a unitary exercise price of d_2.

Thus, the investor's ex post payoff is represented by the structure[21]

$$x - w(x) = \begin{cases} x & \text{if } x \leq d_1 \\ d_1 + \beta_1 \max\{x - d_1, 0\} & \text{if } d_1 < x \leq d_2 \\ d_1 + \beta_2 \max\{x - d_2, 0\} & \text{if } x > d_2 \end{cases}.$$

Letting the strike prices be set exogenously at $d_1 = x_1$ and $d_2 = x_2$, we must then determine the optimal pair (β_1^*, β_2^*) that implements the contract described by (6.1)-(6.3) so that it satisfies

$$(1 - \beta_1^*)(x_2 - x_1) = w_2^*, \tag{6.9}$$

$$(x_3 - x_1) + \beta_2^*(x_2 - x_3) = w_3^*. \tag{6.10}$$

Upon combining equations (6.2), (6.3), (6.9) and (6.10), and recalling the values of x_i's in Assumption 1, we obtain

$$\beta_1^* = 1 - \frac{\underline{U} - B}{\sigma},$$

$$\beta_2^* = 1 + \frac{A - \sigma}{1 - k(1 + \sigma)}.$$

Illustrating this implementation with a numerical example using the same parameter values employed for the convertible preferred equity case, we get $\beta_1^* = 0.73882$ and $\beta_2^* = 0.8918$. Thus, the investor's payoff profile under this warrant-based structure is

$$x - w(x) = \begin{cases} x & \text{if } x \leq 0.5 \\ 0.5 + 0.73882(x - 0.5) & \text{if } 0.5 < x \leq 1 \\ 0.5 + 0.8918(x - 1) & \text{if } x > 1 \end{cases}$$

which is shown in Fig. 3.

⟨Insert Fig. 3 here⟩

6.1.3 Equity, call options, and dilution

Finally, we analyze a mixed financing structure that combines outside/inside equity with either short positions held by the investor in call options or an equity dilution process. This hybrid arrangement enables the investor to potentially infuse new funds if low returns are observed, thereby permitting a cross-subsidization between incomes from good and bad states of nature.

[21] Note that conditional on one of the two warrants being active, the call option involved in each of the contracts will always be 'in the money'.

To this end, we examine how to implement the class of optimal risk-reward schemes analyzed in subsections 4.3 and 5.2. For simplicity, we focus on the U-shaped contract in which $w_1^* = w_3^* = w^*$ and whose specific sharing rule, obtained from Proposition 2, is

$$w^* = \frac{A}{1+m}, \tag{6.11}$$

and

$$w_2^* = \underline{U} - B. \tag{6.12}$$

We further simplify by supposing that $x_1 > 0$ and normalizing the number of initial shares to 1. Two cases then arise, depending on the possibility of the investor infusing new funds.

Case 1: $w^* > x_1$. The U-shaped contract is implemented by a combination of a full-outside investor equity stake and a set of call options over different equity stakes depending on the project's returns. Following is a description of this mixed security structure.

Corollary 6 (Case 1). *Assume that $w^* > x_1$. The U-shaped contract is implemented by giving the investor an initial full equity stake and the investor (entrepreneur) a short (long) position in a set of equity call options triggered in accordance with the following scheme:*
(i) If $x \leq x_1$, a call option over an equity stake $(1 + \delta_1)$ with a zero strike price is active. For this option to be exercised, there must first have been an infusion by the investor of new funds in an amount equal to $\delta_1 x_1$ and a new share issue of $\delta_1 > 0$ such that

$$\delta_1 = \frac{A}{(1+m)(1-\sigma)} - 1.$$

(ii) If $x_1 < x \leq x_2$, a call option on an equity stake $(1 + \delta_2)$ with a zero strike price is active, where $\delta_2 > -1$ such that

$$\delta_2 = \underline{U} - B - 1.$$

(iii) If $x > x_2$, a call option over an equity stake $(1 + \delta_3)$ with a zero strike price is active, where $\delta_3 > -1$ such that

$$\delta_3 = \frac{(1+\delta_1)(1-\sigma)}{k(1+\sigma)} - 1.$$

Proof. See the Appendix.

This implies that the investor's ex post payoff is represented by the following structure:

$$x - w(x) = \begin{cases} x - (1+\delta_1)\max\{x,0\} & \text{if } x \leq x_1 \\ x - (1+\delta_2)\max\{x,0\} & \text{if } x_1 < x \leq x_2 \\ x - (1+\delta_3)\max\{x,0\} & \text{if } x > x_2 \end{cases}, \tag{6.13}$$

which reflects the difference between the full-initial-equity payoff and the losses on the investor's short position in each call option.

Therefore, if in the worst state of nature the lowest project return x_1 is sufficiently low (i.e., $x_1 < w^*$), this mixed financing structure allows the investor to infuse new resources. This result is the consequence of the reward-for-failure feature of the optimal risk reward scheme. Notice, however, that since the investor's expected payoff with the optimal contract must be positive (since the investor's participation constraint at the optimal solution is satisfied with inequality), at least one of the ex post payoffs in the other states of nature must be positive. By expression (6.13), this implies that either δ_2 or δ_3 (or both) must be strictly negative, which ensures that the investor enjoys a stake in the success returns of the project, and thus a combination of outside and inside equity must be adopted.

The foregoing analysis implies that in this case the risk-reward sharing rule allows the investor to cross-subsidize from high-profitability to low-profitability states, thereby avoiding credit rationing for projects that are ex ante profitable.[22]

We illustrate this phenomenon with a numerical example based on the same parameter values adopted so far. The optimal financing rule is then given by $w^* = 0.7106$ and $w_2^* = 0.58059$, which can be implemented by the mixed security structure of Corollary 5 with equity stakes $\delta_1^* = 0.4212$, $-\delta_2^* = 0.41941$ and $-\delta_3^* = 0.70392$. The investor's payoff profile under this scheme is[23]

$$x - w(x) = \begin{cases} x - 1.4212\max\{x,0\} & \text{if } x \leq 0.5 \\ 0.41941x & \text{if } 0.5 < x \leq 1 \\ 0.70392x & \text{if } x > 1 \end{cases},$$

which is displayed in Fig. 4.

⟨Insert Fig. 4 here⟩

Case 2: $w^* \leq x_1$. The U-shaped contract can be implemented by an initial profit-sharing rule that gives the investor an equity stake of $(1-\alpha)$ in the project's return, meaning that an initial combination of outside and inside equity is optimal. A subsequent issue of an amount ϕ_i of shares to the investor is considered when a return x_i

[22] A similar property emerges from the multiple-stage finance setting of Repullo and Suarez (2004).
[23] In this example the two last call options are always 'in the money'.

$(i = 2, 3)$ is observed. This additional equity dilutes the participation of the entrepreneur to a fraction $\frac{\alpha}{1+\phi_i}$ of the project's outcome. The specific values of these stakes and shares are characterized as follows.

Corollary 7 (Case 2). *Assume that $w^* \leq x_1$. A U-shaped financing rule can be implemented by giving the investor an initial equity stake $(1-\alpha) \in [0,1)$ such that*

$$\alpha = \frac{A}{(1+m)(1-\sigma)},$$

and by issuing new shares to her according to the following scheme:
(i) If $x_i = x_2$, the quantity of these additional shares is:

$$\phi_2 = \frac{\alpha - \underline{U} + B}{\underline{U} - B}.$$

(ii) If $x_i = x_3$, the quantity of these additional shares is:

$$\phi_3 = \frac{k(1+\sigma) + \sigma - 1}{1-\sigma}.$$

Proof. See the Appendix.

6.2 Suboptimal financing rules

We now examine the implementation of two notable suboptimal financing schemes: the straight debt and full-franchise contracts.

6.2.1 Debt contract

The pure debt contract is a particularly interesting scheme since it represents the sharing rule applied in traditional sources of financing such as a bank loan or a bond issue. In contrast to the framework adopted here, these conventional financial contracts generally involve a *single* moral hazard environment as neither the bank nor the bond-holders contribute managerial effort to, or advise the entrepreneur on, the process of selecting and/or managing innovative investment projects. To illustrate the extent to which optimal hybrid securities perform better than straight debt contracts under a double moral hazard setup, we present in what follows some numerical comparisons.

The debt scheme we will consider has a coupon $D = x_1$ such that the investor's ex post payoff (before disutility of innovation) is[24]

$$x - w(x) = \begin{cases} x & \text{if } x \leq x_1 \\ x_1 & \text{if } x > x_1 \end{cases}.$$

[24] We also explored a pure debt contract with a coupon $D = x_2$, but the results indicated that it performed much worse.

In the context of our three-outcome model, this implies that $w_1^d = 0$, $w_2^d = \sigma$ and $w_3^d = k(1+\sigma) - (1-\sigma)$, where superscript d refers to the debt contract. Upon substituting this scheme into the program in Subsection 4.2, we obtain that the innovation levels satisfying the incentive compatibility constraints are $a^d = \frac{\Psi}{\theta_1}$, $p^d = 0$ and $e^d = \frac{\Psi}{\theta_1}$. It can then be easily shown that the joint innovation level generated by the debt contract e^d will be larger than that of the second-best solution e^* as long as $\theta_1 < \theta_2$. Note that this condition represents a plausible situation as it means the entrepreneur is marginally more efficient at innovating than the investor.

The debt scheme solution delivers an expected investor's payoff of

$$EB^d = 1 - \sigma, \tag{6.14}$$

and an expected surplus of

$$S^d = 1 + \frac{\Psi^2}{2\theta_1}. \tag{6.15}$$

Note that this contract is admissible if, in addition, it satisfies a participation constraint for each actor. The entrepreneur's participation constraint was given earlier as (4.7) and is equivalent to the condition

$$\sigma \geq \underline{U} - \frac{\Psi^2}{2\theta_1}. \tag{6.16}$$

The investor's participation constraint (4.8) is equivalent to the condition

$$\sigma \leq 1. \tag{6.17}$$

Therefore, if either (6.16) or (6.17) is not satisfied, there is no financing through debt and the resulting equilibrium will involve *credit rationing.*

Observe also that it follows from Corollary 2 and equation (6.15) that the straight debt scheme cannot implement the second-best solution since

$$S^* > S^d.$$

Importantly, this result holds irrespective of the comparison between θ_1 and θ_2, most notably when $\theta_1 < \theta_2$, in which case, as noted earlier, the debt structure induces a level of joint innovation larger than that of the second-best contract. This occurs because, in terms of individual innovation levels, the debt scheme has a *corner* solution different from the balanced solution achieved by the optimal contract.

Thus, a pure debt scheme cannot provide an optimal solution to the double moral hazard problem. The reason for this suboptimality is that it does not create the same high-powered innovation incentives for the principal as for the agent. Under this scheme the returns profile for the entrepreneur's claims (inside common equity) is contingent and variable, strongly motivating him to innovate, whereas the returns on the investor's claims (straight debt) are fixed and riskless, a significantly weaker motivation. This stands in contrast with our optimal hybrid financial structure, which always considers some combination of inside and outside equity (see the three structures studied in Subsection 6.1) and thus gives both sides of the contract the required level of incentives.[25]

This theoretical result is consistent with three stylized facts regarding the financing schemes adopted in most innovative industries. First, in young, high-tech, high-growth firms, especially when they are funded either by venture capitalists investing in the US or more experienced ones investing elsewhere, financial structures are commonly hybrids consisting of debt and equity combinations rather than polar solutions of either one alone (Kaplan et al., 2007; Sahlman, 1990; Trester, 1998). Second, the investor frequently also provides the entrepreneur advice, management and/or active monitoring (Bengtsson and Sensoy, 2011; Kaplan and Strömberg, 2004; Sapienza et al., 1996), which as noted above, is not the case with a conventional debt scheme. Third, the suboptimality of straight debt established here theoretically is consistent with empirical evidence suggesting counter-intuitively that the pecking order theory may not apply in the case of firms with large information asymmetries such as those that are highly innovative. What this evidence shows is that leverage in high-tech industries is smaller than that observed in their low-tech counterparts (Chang and Song, 2014; Frank and Goyal, 2003; Barclay et al., 1995) and that it decreases with R&D expenditures (Long and Malitz, 1985).

To illustrate the suboptimality of the debt scheme, we conducted numerical simulations of the impact of changes in the probability extremism level parameter γ and the failure probability skew parameter m on the expected surplus.[26] In the case of γ, we assumed the same constellation of parameters as in Subsection 6.1 except that in this case $\underline{U} = 0.51$, yielding an expected surplus of

$$S^d = 1 + 0.21125\gamma^2,$$

as long as (6.16) is satisfied and thus there is no credit rationing, which is guaranteed by the condition[27]

$$\gamma \geq 0.2176.$$

[25] A result reminiscent of this mix between inside and outside equity is obtained by Casamatta (2003), but in a setting where outside investment arises endogenously.

[26] Simulation results for other returns distribution parameters are available from the authors upon request.

[27] Since $\sigma = 0.5$, condition (6.17) is satisfied and thereby the investor's participation constraint as well.

A graphical comparison of the straight debt scheme with the second-best solution is displayed in Fig. 5, in which we observe two phenomena. First, with a debt scheme there is credit rationing for investment projects that have a sufficiently low probability extremism level (the dashed segment of the green curve), or what amounts to the same thing, a probability mass sufficiently concentrated on moderate returns. In terms of the three-profile project typology discussed in previous sections, this result implies that conventional debt structures might not fund highly innovative projects if, despite yielding a positive expected surplus, they have an ex ante profile more like a living dead. This highlights the fact that mixed securities can be especially welfare-enhancing when used to finance profitable projects with such characteristics. In particular, note that in the numerical example illustrated in Fig.5, the expected surplus for the projects subject to credit rationing is not given by the dashed segment of the green curve but zero. Thus, the gap between S^* and S^d is larger than 1 for these projects.

Interestingly, this credit rationing result is consistent with empirical evidence showing that relative to other types of businesses, small, high-growth, highly innovative firms seem to be more financially constrained (Chang and Song, 2014; Brown, 1997), and that consequently, the latter make intensive use of private equity and other equity vehicles rather than bank loans or publicly traded bonds.

The second phenomenon observable in Fig. 5 is that the expected surplus gap between the second-best and straight debt contracts increases with γ. This means that those projects fundable through debt (the solid segment of the green curve) whose returns distributions most diverge from a living dead profile would generate a larger social welfare increase if they were funded instead by a hybrid security structure.

⟨Insert Fig. 5 here⟩

Now turning to the simulations of changes in m, we used the same parameters values and intervals as in Subsection 6.1 except that $\underline{U} = 0.53$. The expected surplus from the pure debt contract then becomes

$$S^d = 1 + \frac{1}{2}(0.49 - 0.175m)^2,$$

as long as there is no credit rationing, which by (6.16) is equivalent to the condition

$$m \leq 1.4003.$$

A comparison of the reactions of the second-best and the straight debt contracts to changes in m is shown in Fig. 6. Two results in particular are worthy of note. The first one is that a debt scheme can involve credit rationing for projects that generate a positive expected surplus but exhibit a large probability of failure (the dashed segment of the green curve). This implies that from a social standpoint, mixed security structures will show their greatest advantage over debt when financing profitable ventures with an ex ante profile similar to a poor project. This is so because under traditional debt schemes, such projects may not be funded at all.[28]

The other notable result illustrated in Fig. 6 is that the expected surplus gap between the second-best and straight debt contracts decreases with m. Thus, among the projects that may be financed with debt (the solid segment of the green curve), those whose returns distribution have relatively less asymmetric tails would generate a larger social welfare increase if they were funded using a mixed security structure instead.

⟨Insert Fig. 6 here⟩

6.2.2 Full-franchise contract

The franchise contract is an especially interesting case because under *single* moral hazard, this scheme induces the first-best effort when the agent is risk neutral, as is the case in our model. The previous literature has, however, established that under *double* moral hazard and MLRP, this fixed rent contract is just a second-best option unable to implement the first-best solution (Kim and Wang, 1998). We will show that in a double-moral hazard setup with *no* MLRP, not only does this result continue to hold but the franchise contract is not even a second-best solution.

Consider the contract

$$w(x) = x - F^f$$

for all x, where F^f is a fixed payment by the entrepreneur to the investor. Substituting this into the program in Section 4.2 yields the innovation levels $a^f = \frac{\Psi}{\theta_1}$, $p^f = 0$ and $e^f = \frac{\Psi}{\theta_1}$ and a fixed payment given by

$$F^f = 1 + \frac{\Psi^2}{2\theta_1} - \underline{U}.$$

Thus, under asymmetric information the investor's expected payoff is

$$EB^f = 1 + \frac{\Psi^2}{2\theta_1} - \underline{U}, \tag{6.18}$$

and the expected surplus is

$$S^f = EB^f + \underline{U}. \tag{6.19}$$

Comparing this surplus with the surplus generated by the second-best solution (Corollary 2) reveals that

$$S^* > S^f,$$

[28] Recall that the expected surplus for projects subject to credit rationing is zero.

which implies that the franchise scheme is not even a second-best solution, despite inducing a level of joint innovation that may actually be higher than that of the second-best contract.[29] As in the straight debt case, this occurs because the full-franchise scheme leads to a corner solution in which only the entrepreneur innovates whereas the second-best financing rule induces a balanced solution under which the investor innovates as well. This in turn is due to the fact that the full-franchise contract only provides high-powered incentives to the entrepreneur as a residual claimant, not to the investor whose claim is to a fixed and riskless payment.

7 Concluding Remarks

This article characterizes the optimal contract for the financing of a highly innovative project investment when both the investor and the entrepreneur undertake innovation under moral hazard. The framework developed for the purpose generates results that contribute to the previous literature in three main ways.

The first contribution arises from the framework's treatment of innovation as an endogenous variable that is shown to depend at the optimum on the primitive characteristics of a returns distribution relating non-monotonically observed profitability to unobserved innovation decisions. These primitives include the degree of the distribution's skew towards success and failure and the extent to which returns and probabilities concentrate around extreme results. This approach allows us to predict the expected social value generated by an investment project as a function of how much its primitives approximate ex ante one of the three typical profiles based on existing evidence from highly innovative ventures in the real world. The proposed methodology also provides a rich analytical setup for explaining and predicting the impact of technological shocks (real or perceived) on levels of innovation in high-tech sectors.

The second contribution is the development of a novel explanation for the frequent use of hybrid securities in financing high-tech firms. The explanation is founded on two interconnected elements: (i) the non-monotone statistical relationship between innovation and returns (i.e., the relaxation of the MLRP assumption) that potentially characterizes highly innovative ventures, and (ii) the need to provide high-powered incentives for both parties to the contract because of the double moral hazard. This leads to the conclusion, first, that to incentivize innovation, what matters is to satisfy a given relationship between the rewards to returns at the opposite extremes, and second, that an optimal financial arrangement must always combine both outside and inside equity.

[29] Like the pure debt scheme, the franchise contract dominates the second-best solution in terms of joint innovation as long as $\theta_1 < \theta_2$.

The third contribution is an alternative explanation for the credit rationing that in practice affects highly innovative firms more severely than others. While the existing literature has emphasized the role played by insufficient collateral, our explanation relies rather on the primitives of the projects innovative firms undertake. Specifically, we establish that ventures which are ex ante profitable but have excessively high rates of failure or too much probability mass concentrated on moderate returns might not be funded if conventional debt schemes are used.

In light of these three contributions, we argue that the proposed model constitutes a novel and advantageous starting point for analyzing venture capital financing. This is particularly so in that the formulation captures two features of this class of financial vehicles: (i) the financing of highly innovative entrepreneurial firms with abnormally large, but unlikely, success returns, and (ii) the venture capitalist's provision not only of funds but also of effort in the form of management, advice or monitoring.

As for future research, the framework set out here could be expanded in various directions to more fully account for venture capital financing. A natural extension would be to incorporate staged financing given that according to the empirical evidence, funding conditional on milestones is typical for this type of investment. Also, a more general functional form for joint innovation could be assumed in which investor and entrepreneur innovation levels are not necessarily substitutes (they could, for example, be assumed to be complementary). This might alter certain of our results, especially those regarding the suboptimality of a straight debt scheme. A further extension would be to conjecture that the effects of the venture capitalist's actions on the returns distribution are different from those effects due to the entrepreneur's actions. This points to yet another possibility for broadening the proposed approach, which is to assume that whereas the entrepreneur's actions can be interpreted as innovation leading to more profitable but riskier ventures, the venture capitalist's actions can be considered as managerial effort and advice leading to more profitable ventures *without* any increases in risk (and therefore an improvement in a first-order stochastic dominance sense). Finally, regarding our credit rationing result, it would be interesting to analyze how our explanation based on the primitives of projects undertaken by highly innovative firms interacts with more conventional explanations based on the insufficient collateral typically exhibited by such businesses.

8 Appendix

Proof of Proposition 1. We begin by characterizing first-best innovation levels. From the problem posed in Section 4.1, it can easily be shown that these innovation levels must maximize the social value of the project, and thus, they must solve the following program[30]:

$$\max_{a,p\in[0,1]} \sum_{i=1}^{3} \pi_i^{(a,p)} x_i - \frac{\theta_1 a^2}{2} - \frac{\theta_2 p^2}{2}$$

$$\text{subject to}$$

$$a + p \leq 1.$$

The FOC's of this problem imply that[31]

$$a^{FB} = \frac{\Psi}{\theta_1} \tag{8.1}$$

$$p^{FB} = \frac{\Psi}{\theta_2}, \tag{8.2}$$

where Ψ is defined by equation (3.4). Hence, the optimal joint innovation level is given by

$$e^{FB} = \Psi \frac{\theta_1 + \theta_2}{\theta_1 \theta_2}.$$

Note that assumption (A2) guarantees that $\Psi \in (0, \frac{\theta_1 \theta_2}{\theta_1 + \theta_2})$, and therefore that a^{FB} and p^{FB} are interior solutions. This ensures in turn that joint innovation e^{FB} is an interior solution as well.

Next, we show that the two following optimal schemes achieve these first-best innovation levels using the investor's program described by equations (4.1)-(4.5).

(i) A full-insurance contract for the entrepreneur implies that $w_i = w$ for all i. Substituting this financing rule into his binding participation constraint (4.2) yields

$$w(a) = \underline{U} + \frac{\theta_1 a^2}{2}.$$

This is substituted into the investor's objective function (4.1) and the FOC's are then derived, from which it is easily proved that the contract implements the first-best innovation levels described by equations (8.1) and (8.2). Thus, the optimal full-insurance financing rule can be written as

$$w^{FB}(a) = \begin{cases} \underline{U} + \frac{\Psi^2}{2\theta_1} & \text{if } a = \frac{\Psi}{\theta_1} \\ \underline{U} - \varepsilon & \text{otherwise} \end{cases}$$

where $\varepsilon > 0$ is a penalization.

(ii) A full-franchise contract for the entrepreneur implies that $w_i = x_i - F$ for all i. Substituting this financing rule into the binding entrepreneur's participation constraint

[30] Kim and Wang (1998), in the context of a double moral-hazard setup, discuss the equivalence between a program like that posed in Section 4.1 and the program solved in this proof.

[31] The SOC's for a maximum are also satisfied.

yields

$$F^{FB}(a,p) = \sum_{i=1}^{3} \pi_i^{(a,p)} x_i - \underline{U} - \frac{\theta_1 a^2}{2}.$$

This is substituted into the investor's objective function and the FOC's are then derived with respect to a and p, from which it is easily proved that the contract implements the first-best innovation levels of equations (8.1) and (8.2). Hence, the optimal franchise financing rule is described by the following fixed payment from the entrepreneur to the investor:

$$F^{FB}(p) = \begin{cases} 1 + \Psi^2 \frac{2\theta_1+\theta_2}{2\theta_1\theta_2} - \underline{U} & \text{if } p = \frac{\Psi}{\theta_2} \\ \tau & \text{otherwise} \end{cases}$$

where $\tau < 0$ is a penalization.

Finally, note that the assumptions regarding the bounds of Ψ and $\underline{U}$ ensure that $F^{FB} > 0$ when $p = \frac{\Psi}{\theta_2}$ and also that the investor's participation constraint (4.3) is satisfied. Furthermore, it is satisfied with inequality because of the upper bound imposed by assumption (A3) on the reservation utility. To check this, we compute the optimal investor's expected payoff by equation (3.2) such that

$$EB^{FB} = 1 + \Psi^2 \frac{\theta_1 + \theta_2}{2\theta_1\theta_2} - \underline{U} > 0,$$

which completes the proof. □

Proof of Proposition 2. According to the first-order approach (Rogerson, 1985), the FOC of the problem that solves the optimal innovation level for the entrepreneur can be substituted for the constraint (4.9) in the optimal sharing rule program.[32] This FOC is given by

$$m\gamma w_1 - \gamma (1+m) w_2 + \gamma w_3 - \theta_1 a = 0. \tag{8.3}$$

Moreover, it is easily verified that at the optimal contract the entrepreneur's participation constraint is binding and thus

$$(a+p)\gamma \left[m w_1 - (1+m) w_2 + w_3\right] + w_2 - \frac{\theta_1 a^2}{2} = \underline{U}. \tag{8.4}$$

Combining (8.3) and (8.4), we obtain

$$w_2(a,p) = \underline{U} - \frac{\theta_1 a^2}{2} - \theta_1 a p. \tag{8.5}$$

Then, substituting (8.5) into (8.3),

$$w_3(a,p,w_1) = \frac{\gamma (1+m) \left(\underline{U} - \frac{\theta_1 a^2}{2} - \theta_1 a p\right) - m\gamma w_1 + \theta_1 a}{\gamma}. \tag{8.6}$$

[32] The first-order approach can be applied given the concavity of π_i with respect to a and p.

Applying now the first-order approach to the investor's innovation problem, from the first order condition of (4.10) we get

$$m\gamma\left(x_1 - w_1\right) - \gamma\left(1+m\right)\left(x_2 - w_2\right) + \gamma\left(x_3 - w_3\right) - \theta_2 p = 0. \tag{8.7}$$

Upon substituting (8.3) into (8.7) and recalling the definition of Ψ given by equation (3.4), we obtain

$$p(a) = \frac{\Psi - \theta_1 a}{\theta_2}. \tag{8.8}$$

Substituting (8.8) into (8.5), we obtain

$$w_2(a) = \underline{U} + \frac{\theta_1(2\theta_1 - \theta_2)a^2}{2\theta_2} - \frac{\theta_1 \Psi a}{\theta_2}, \tag{8.9}$$

Similarly, substituting (8.8) into (8.6) yields

$$w_3(a, w_1) = \frac{\theta_1 a}{\gamma} - mw_1 + (1+m)\left(\underline{U} + \frac{\theta_1(2\theta_1 - \theta_2)a^2}{2\theta_2} - \frac{\theta_1 \Psi a}{\theta_2}\right). \tag{8.10}$$

Upon plugging (8.8), (8.9) and (8.10) into the investor's objective function, we get the program

$$\max_{a\in(0,1]} -\frac{1}{2\theta_2}\left(\theta_1^2 a^2 + \theta_1\theta_2 a^2 - 2\theta_2\Psi a - \Psi^2 - 2\theta_2 + 2\underline{U}\theta_2\right).$$

The FOC with respect to a yields

$$a^* = \frac{\Psi}{\theta_1}\frac{\theta_2}{(\theta_1+\theta_2)}.$$

Using this result in (8.8), (8.9), and (8.10), we finally obtain

$$\begin{aligned} p^* &= \frac{\Psi}{\theta_2}\frac{\theta_1}{(\theta_1+\theta_2)}, \\ e^* &= \frac{\Psi}{\theta_1\theta_2}\frac{\theta_1^2+\theta_2^2}{(\theta_1+\theta_2)}, \\ w_2^* &= \underline{U} - \Psi^2\frac{2\theta_1^2+\theta_2^2}{2\theta_1(\theta_1+\theta_2)^2}, \end{aligned}$$

and

$$w_3^* = A - mw_1^*, \tag{8.11}$$

where

$$A \equiv \Psi\frac{\theta_2}{\gamma(\theta_1+\theta_2)} + (1+m)\left(\underline{U} - \Psi^2\frac{2\theta_1^2+\theta_2^2}{2\theta_1(\theta_1+\theta_2)^2}\right).$$

We can now easily verify that given assumption (A3), both sharing w_2^* and term A are positive. Applying the limited liability constraints for w_1^* and w_3^* to equation (8.11) implies that

$$w_1^* \in \left[0, \frac{A}{m}\right],$$

and

$$w_3^* \in [0, A] \, .$$

Finally, we confirm that the investor's participation constraint (4.8) is satisfied. This is done by computing the optimal investor's expected payoff according to equation (3.2) such that

$$EB^* = 1 + \Psi^2 \frac{\theta_1^2 + \theta_2^2 + \theta_1\theta_2}{2\theta_1\theta_2(\theta_1 + \theta_2)} - \underline{U} > 0,$$

where the inequality holds because of the upper bound imposed by assumption (A3) over $\underline{U}$. □

Proof of Corollary 2. **(i)** From propositions 1 and 2 it is easily seen that

$$\begin{aligned} a^* &= a^{FB} \frac{\theta_2}{\theta_1 + \theta_2} < a^{FB}, \\ p^* &= p^{FB} \frac{\theta_1}{\theta_1 + \theta_2} < p^{FB}, \end{aligned}$$

since a^{FB},$p^{FB} > 0$ (because assumption (A2) guarantees that $\Psi > 0$) and $\theta_1, \theta_2 > 0$. Hence, it follows directly that $e^* < e^{FB}$.

(ii) Substituting the optimal contract $(a, p, \{w_i\}_{i=1}^3)$ under symmetric information (Proposition 1) and asymmetric information (Proposition 2) into equation (3.2) yields the respective values of EB^{FB} and EB^* given in the corollary. Applying some basic algebraic manipulations, it can be shown that the inequality in favour of EB^{FB} holds because $\Psi > 0$ and $\theta_1, \theta_2 > 0$ by assumption.

(iii) Substituting the optimal innovation pair (a, p) under symmetric information (Proposition 1) and asymmetric information (Proposition 2) into equation (3.3) yields the respective values of surpluses S^{FB} and S^* shown in the corollary. The inequality in favour of S^{FB} is a direct consequence of part (ii) of this corollary. □

Proof of Corollary 3. It can be easily shown that

(i)

$$\begin{aligned} \frac{\partial a^{FB}}{\partial \theta_1} &= -\frac{\Psi}{\theta_1^2} < 0, \\ \frac{\partial p^{FB}}{\partial \theta_2} &= -\frac{\Psi}{\theta_2^2} < 0. \end{aligned}$$

(ii)

$$\begin{aligned} \frac{\partial e^{FB}}{\partial \theta_i} &= -\frac{\Psi}{\theta_i^2} < 0, \\ \frac{\partial S^{FB}}{\partial \theta_i} &= -\frac{1}{2}\frac{\Psi^2}{\theta_i^2} < 0, \end{aligned}$$

for $i = 1, 2$.

(iii)

$$\begin{aligned}
\frac{\partial a^*}{\partial \theta_1} &= -\frac{\Psi}{\theta_1^2} \frac{\theta_2}{(\theta_1 + \theta_2)^2} (2\theta_1 + \theta_2) < 0, \\
\frac{\partial p^*}{\partial \theta_2} &= -\frac{\Psi}{\theta_2^2} \frac{\theta_1}{(\theta_1 + \theta_2)^2} (\theta_1 + 2\theta_2) < 0, \\
\frac{\partial a^*}{\partial \theta_2} &= \frac{\partial p^*}{\partial \theta_1} = \frac{\Psi}{(\theta_1 + \theta_2)^2} > 0.
\end{aligned}$$

(iv) From (iii), in the case of the entrepreneur's cost parameter we have

$$\frac{\partial e^*}{\partial \theta_1} = -\frac{\Psi}{\theta_1^2 (\theta_1 + \theta_2)^2} \left(-\theta_1^2 + 2\theta_1\theta_2 + \theta_2^2\right) < 0$$

if

$$-\frac{\partial a^*}{\partial \theta_1} > \frac{\partial p^*}{\partial \theta_1}.$$

Given the symmetry of the solutions, a similar result can be shown for the investor's cost parameter θ_2.

(v) Since by assumption, $\theta_1, \theta_2 > 0$ and $\Psi > 0$, we have

$$\frac{\partial S^*}{\partial \theta_i} = -\frac{1}{2} \frac{\Psi^2}{\theta_i^2} \frac{\theta_j}{(\theta_i + \theta_j)^2} (2\theta_i + \theta_j) < 0$$

for $i, j = 1, 2,\ i \neq j$. □

Proof of Corollary 4. Applying the chain rule to propositions 1 and 2 and Corollary 2 reveals that the sign of the effect of a marginal change in a given returns distribution parameter on optimal innovation and surplus levels is the same as the sign of the partial derivative of Ψ with respect to that parameter.[33] Thus, we may confine our analysis to this class of derivatives. The following results directly follow:

(i)

$$\frac{\partial \Psi}{\partial \gamma} = \frac{\Psi}{\gamma} > 0,$$

which is true by assumption (A2) and since $\gamma > 0$.

(ii)

$$\frac{\partial \Psi}{\partial k} = \gamma(1 + \sigma) > 0,$$

since $\gamma, \sigma > 0$.

(iii)

$$\frac{\partial \Psi}{\partial m} = -\gamma\sigma < 0.$$

(iv)

$$\frac{\partial \Psi}{\partial \sigma} = \gamma(k - m) \begin{cases} \geq 0 & \text{if } k \geq m \\ < 0 & \text{if } k < m \end{cases}.$$

[33] In the case of the surplus level this is true since assumption (A2) implies that $\Psi > 0$.

□

Proof of Corollary 5. From propositions 1 and 2 it is easily seen that the gap between first-best and second-best innovation for both the entrepreneur and the investor is $\frac{\Psi}{\theta_1+\theta_2}$. The corresponding gap for the expected surplus is $\frac{1}{2}\frac{\Psi^2}{\theta_1+\theta_2}$. Thus, the same results in terms of comparative statics for a, p, e and S proved in Corollary 4 can immediately be extended to these gaps. Also, simple inspection reveals that they are decreasing in the two innovation cost parameters. □

Proof of Corollary 6. By equations (6.11) and (6.12), the investor's ex post payoff under the U-shaped scheme is represented by the following structure:[34]

$$x_i - w_i^* = \begin{cases} 1-\sigma-\frac{A}{1+m} & \text{if } i=1 \\ 1-\underline{U}+B & \text{if } i=2 \\ k(1+\sigma)-\frac{A}{1+m} & \text{if } i=3 \end{cases}. \tag{8.12}$$

We must then show that the proposed mixed security scheme yields the same payoff structure (8.12) for each state of nature.

(i) When $i=1$, the investor's payoff is given by

$$x_1 - \max\{(1+\delta_1)x_1, 0\}, \tag{8.13}$$

where the first term is the payoff from the full equity stake and the second term is the payoff from a short position in a zero-strike-price call option over a stake $1+\delta_1$ in the equity. Since $\delta_1 > 0$ and $x_1 > 0$, payoff (8.13) is

$$-\delta_1 x_1,$$

which, given the formula for δ_1 and since $x_1 = 1-\sigma$ (see Assumption 1 in Section 3.1), becomes

$$1-\sigma-\frac{A}{1+m} = x_1 - w^* < 0,$$

where the inequality holds because by assumption in Case 1, $x_1 < w^*$. Since this payoff is negative, the investor must then infuse additional funds of an amount given by

$$\begin{aligned} -\left(1-\sigma-\frac{A}{1+m}\right) &= \left(\frac{A}{(1+m)(1-\sigma)}-1\right)(1-\sigma) \\ &= \delta_1 x_1. \end{aligned}$$

(ii) When $i=2$, adopting the same line of reasoning as in (i) and since $\delta_2 > -1$ and $x_2 = 1$, it follows that the investor's payoff is given by

$$\begin{aligned} x_2 - \max\{(1+\delta_2)x_2, 0\} &= 1-\underline{U}+B \\ &= x_2 - w_2^*. \end{aligned}$$

[34] This, of course, is before the innovation disutility and initial investment.

(iii) When $i = 3$, since $\delta_3 > -1$ and $x_3 = k(1+\sigma)$, it follows that the investor's payoff is given by

$$\begin{aligned} x_3 - \max\{(1+\delta_3)x_3, 0\} &= k(1+\sigma) - \frac{A}{1+m} \\ &= x_3 - w^*, \end{aligned}$$

which completes the proof. □

Proof of Corollary 7. As in the proof of Corollary 6, we must demonstrate that the proposed hybrid financing scheme yields an investor's ex post payoff structure identical to that described by (8.12).

(i) When $i = 1$, the investor's payoff is solely characterized by her equity stake

$$(1-\alpha)x_1.$$

Given the formula for α and since $x_1 = 1 - \sigma$ (Assumption 1), this becomes

$$1 - \sigma - \frac{A}{1+m} = x_1 - w^* \geq 0,$$

where the inequality holds because by assumption in Case 2, $x_1 \geq w^*$.

(ii) When $i = 2$, the investor's payoff is given by

$$\left(\frac{1-\alpha+\phi_2}{1+\phi_2}\right)x_2,$$

where the term in brackets represents the investor's total equity stake after an amount ϕ_2 of new shares are issued in her favour.[35] Given the formulae for α and ϕ_2, and since $x_2 = 1$ (Assumption 1), the investor's payoff is

$$1 - \underline{U} + B = x_2 - w_2^*.$$

(iii) When $i = 3$, by the same logic used for $i = 2$ the investor's payoff is described by

$$\begin{aligned} \left(\frac{1-\alpha+\phi_3}{1+\phi_3}\right)x_3 &= k(1+\sigma) - \frac{A}{1+m} \\ &= x_3 - w^*, \end{aligned}$$

where the first equality holds given the formulae for α and ϕ_3 and that from Assumption 1, $x_3 = k(1+\sigma)$. □

[35] Recall that the amount of initial shares is normalized to 1.

References

[1] Barclay, M., C.W. Smith and R.L. Watts, 1995, "The determinants of corporate leverage and dividend policies", *Journal of Applied Corporate Finance*, 7, 4-19.

[2] Bengtsson, O. and B.A. Sensoy, 2011, "Investor abilities and financial contracting: Evidence from venture capital", *Journal of Financial Intermediation*, 20, 477-502.

[3] Bergemann, D. and U. Hege, 1998, "Venture capital financing, moral hazard, and learning", *Journal of Banking and Finance*, 22, 703-735.

[4] Berglöf, E., 1994, "A control theory of venture capital finance", *Journal of Law, Economics & Organization*, 10(2), 247-267.

[5] Biais, B. and C. Casamatta, 1999, "Optimal Leverage and Aggregate Investment", *Journal of Finance*, 54(4), 1291-1323.

[6] Brown, W., 1997, "R&D intensity and finance: Are innovative firms financially constrained?", DP 271, London School of Economics, Financial Markets Group.

[7] Casamatta, C., 2003, "Financing and advising: Optimal financial contracts with venture capitalists", *Journal of Finance*, 58(5), 2059-2085.

[8] Chang, H. and F. Song, 2014, "Testing the pecking order theory with financial constraints", *Working Paper*, The University of Hong Kong.

[9] Chod, J. and E. Lyandres, 2018, "A Theory of ICOs: Diversification, Agency, and Information Asymmetry", *Working Paper*, Boston College and Boston University.

[10] Cornelli, F. and O. Yosha, 2003, "Stage financing and the role of convertible securities", *Review of Economic Studies*, 70, 1-32.

[11] Cumming, D.J. and S.A. Johan, 2007, "Advice and monitoring in venture capital finance", *Financial Markets and Portfolio Management*, 21, 3-43.

[12] de Bettignies, J-E. and J.A. Brander, 2007, "Financing entrepreneurship: Bank finance versus venture capital", *Journal of Business Venturing*, 22, 808-832.

[13] Elitzur, R. and A. Gavious, 2003, "A multi-period game theoretic model of venture capital investment", *European Journal of Operational Research*, 144(2), 440-453.

[14] Fairchild, R., 2011, "An entrepreneur's choice of venture capitalist or angel-financing: A behavioral game-theoretic approach", *Journal of Business Venturing*, 26, 359-374.

[15] Frank, M. and V. Goyal, 2003, "Testing the pecking order theory of capital structure", *Journal of Financial Economics*, 67, 217-248.

[16] Gorman, M. and W. Sahlman, 1989, "What do venture capitalists do", *Journal of Business Venturing*, 4, 231-248.

[17] Hermalin, B.E. and M.S. Weisbach, 2005, "Corporate reform and governance", *Working Paper*, University of California and University of Illinois.

[18] Hölmstrom, B., 1982, "Moral hazard in teams", *The Bell Journal of Economics*, 13(2), 324-340.

[19] Inderst, R. and H.M. Müller, 2004, "The effect of capital market characteristics on the value of startup firms", *Journal of Financial Economics*, 72, 319-356.

[20] Kanniainen, V. and C. Keuschnigg, 2003, "The optimal portfolio of start-up firms in venture capital finance", *Journal of Corporate Finance*, 9, 521-534.

[21] Kanniainen, V. and C. Keuschnigg, 2004, "Start-up investment with scarce venture capital support", *Journal of Banking & Finance*, 28, 1935-1959.

[22] Kaplan, S.N., Martel, F. and P. Strömberg, 2007, "How do legal differences and experience affect financial contracts?", *Journal of Financial Intermediation*, 16, 273-311.

[23] Kaplan, S.N. and P. Strömberg, 2003, "Financial contracting theory meets the real world: An empirical analysis of venture capital contracts", *The Review of Economic Studies*, 70, 281-315.

[24] Kaplan, S.N. and P. Strömberg, 2004, "Characteristics, contracts and actions: Evidence from venture capital analyses", *Journal of Finance*, 59(5), 2177-2210.

[25] Kim, S.K. and S. Wang, 1998, "Linear contracts and the double moral-hazard", *Journal of Economic Theory*, 82, 342-378.

[26] Long, M. and I. Malitz, 1985, "The investment-financing nexus: Some empirical evidence", *Midland Corporate Finance Journal*, 3, 53-59.

[27] Loyola, G. and Y. Portilla, 2014, "Reward for failure and executive compensation in institutional investors", *Finance Research Letters*, 11, 349-361.

[28] Manso, G., 2011, "Motivating Innovation", *Journal of Finance*, 66(5), 1823-1860.

[29] Marx, L., 1998, "Efficient venture capital financing combining debt and equity", *Review of Economic Design*, 3, 371-387.

[30] Metric, A. and A. Yasuda, 2010, *Venture capital and the finance of innovation* (2nd ed.). New York: John Wiley & Sons.

[31] Repullo, R. and J. Suarez, 2004, "Venture capital finance: a security design approach", *Review of Finance*, 8, 75-108.

[32] Rogerson, W.P., 1985, "The First-Order Approach to Principal-manager Problems", *Econometrica*, 53, 1357-1367.

[33] Sahlman, W., 1990, "The structure and governance of venture capital organizations", *Journal of Financial Economics*, 27, 473-521.

[34] Sapienza, H.J., Manigart, S. and W. Vermeir, 1996, "Venture capitalist governance and value added in four countries", *Journal of Business Venturing*, 11, 439-469.

[35] Schmidt, K.M.., 2003, "Convertible securities and venture capital finance", *Journal of Finance*, 58(3), 1139-1166.

[36] Trester, J.J., 1998, "Venture capital contracting under asymmetric information", *Journal of Banking and Finance*, 22(6-8), 675-699.

[37] Vergara, M., Bonilla, C., and J. Sepúlveda, 2016, "The complementarity effect: Effort and sharing in the entrepreneur and venture capital contract", *European Journal of Operational Research*, 254(3), 1017-1025.

[38] Wang, S., 2009, "Convertibles in sequential financing", *Review of Finance*, 13(4), 727-760.

[39] Wang, S. and H. Zhou, 2004, "Staged financing in venture capital: moral hazard and risks", *Journal of Corporate Finance*, 10(1), 131-155.

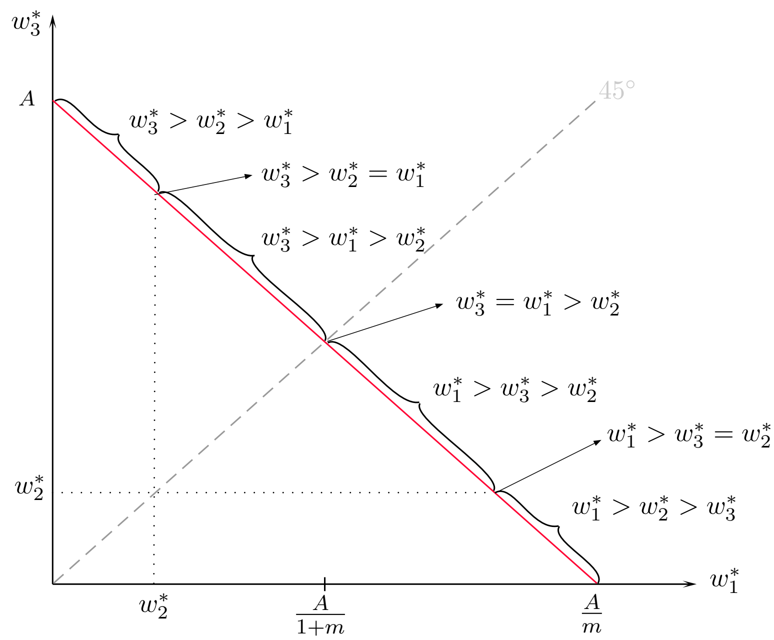

Figure 1. Optimal financing rules under asymmetric information. The red line contains all the optimal combinations (w_1^*, w_3^*) such that $w_3^* = A - mw_1^*$. Optimal $w_2^* = \underline{U} - \frac{3\Psi^2}{8} < \frac{A}{1+m}$ is also represented so that the three compensations can be compared.

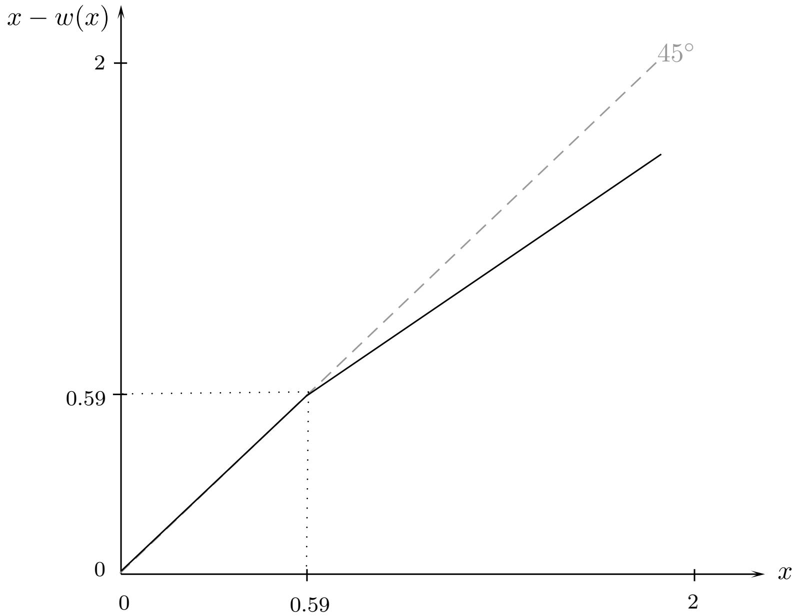

Figure 2. Convertible preferred equity with $d^* = 0.59886$ and $\beta^* = 0.67445$ assuming $\theta_1 = \theta_2 = 1$, $\sigma = 0.5$, $k = 1.6$, $m = 1.5$, $\gamma = 0.35$ and $\underline{U} = 0.15$.

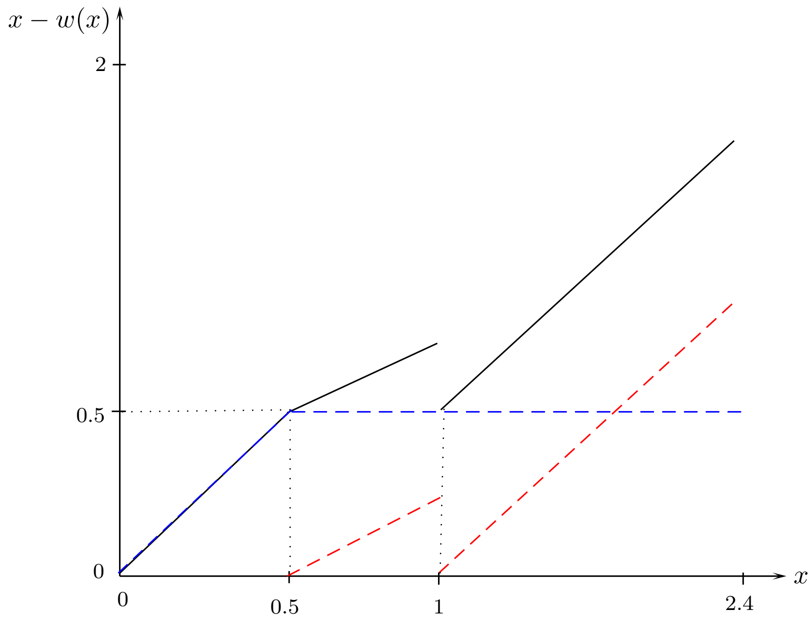

Figure 3. Preferred equity with dividend $d = x_1$, and a set of two warrants with strike price-equity stake pairs (d_1,β_1) and (d_2,β_2) assuming $\theta_1 = \theta_2 = 1$, $\sigma = 0.5$, $k = 1.6$, $m = 1.5$, $\underline{U} = 0.15$ and $\gamma = 0.35$. The investor's final payoff (black lines) is the result of summing the payoffs on the preferred equity (blue line) and the warrants (red lines).

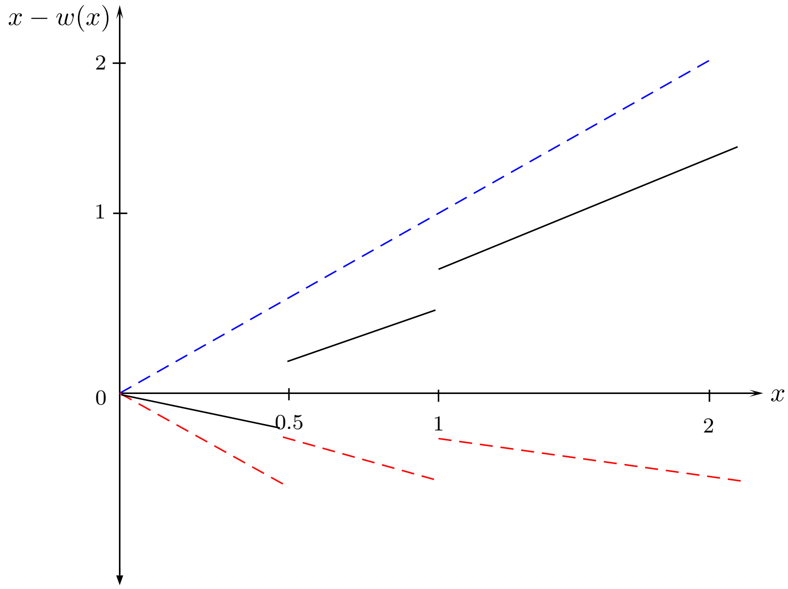

Figure 4. Initial full outside equity plus a set of call options with zero strike prices and equity stakes δ_i assuming $\theta_1 = \theta_2 = 1$, $\sigma = 0.5$, $k = 1.6$, $m = 1.5$, $\underline{U} = 0.6$ and $\gamma = 0.35$. The investor's final payoff (black lines) is the result of subtracting the payoff on her short position in call options (red lines) from the payoff on the full equity claim (blue line).

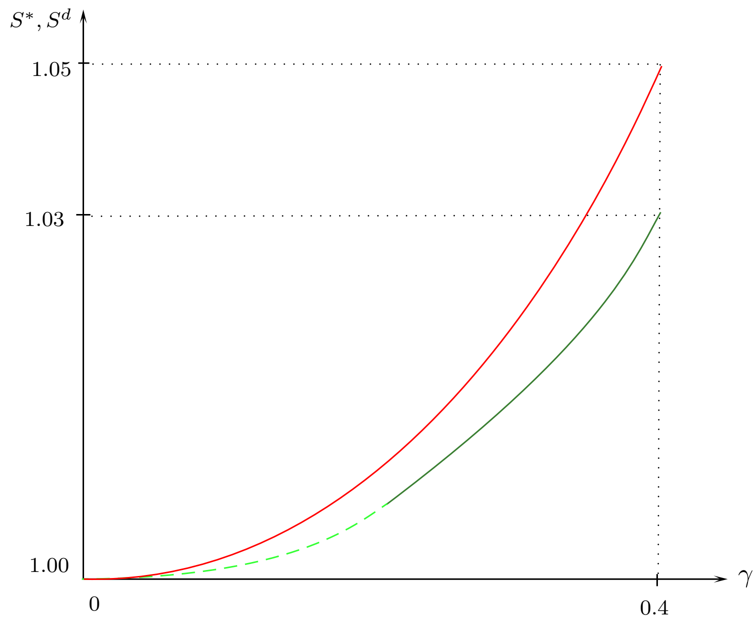

Figure 5. Expected surplus S as a function of γ for the second-best financing rule (red line) and for a pure debt contract under asymmetric information (green line). The dashed line indicates values of γ for which there is credit rationing under the debt scheme.

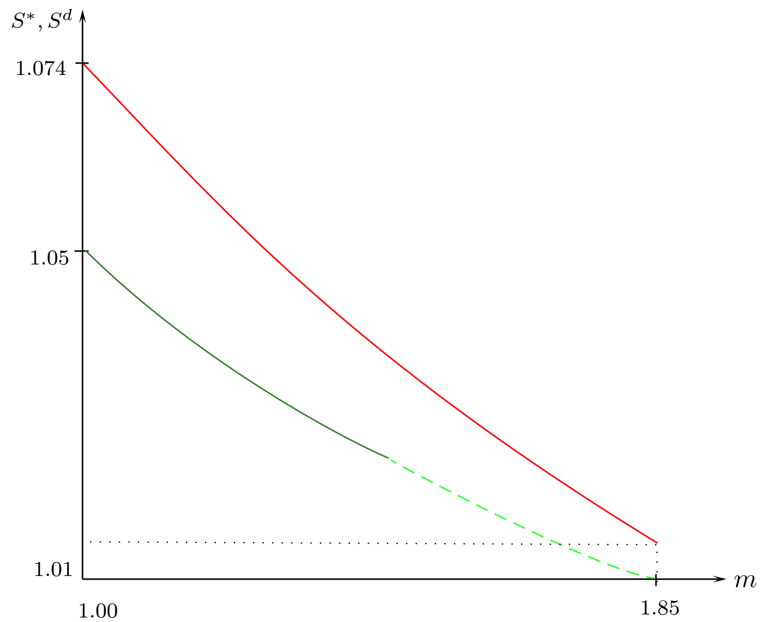

Figure 6. Expected surplus S as a function of m for the second-best financing rule (red line), and pure debt contract under asymmetric information (green line). The dashed line indicates values of m for which there is credit rationing under the debt scheme.