Can Investors Benefit from Hedge Fund Strategies? Utility-Based, Out-of-Sample Evidence*

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Abstract

We report systematic, out-of-sample evidence on the benefits to an already well-diversified investor that may derive from further diversification into various hedge fund strategies. We investigate dynamic strategic asset allocation decisions that take into account investors' preferences as well as return predictability. Our results suggest that not all hedge fund strategies benefit a long-term investor who is already well diversified across stocks, government and corporate bonds, and REITs. Only strategies whose payoffs are highly nonlinear (e.g., fixed income relative value and convertible arbitrage), and therefore not easily replicable, constitute viable options. Most of the realized economic value fails to result from a mean-variance type of improvement but comes instead from an improvement in realized higher-moment properties of optimal portfolios. Medium to highly risk-averse investors benefit the most from this alternative asset class.

Keywords: Strategic asset allocation, hedge fund strategies, predictive regressions, out-of-sample performance, certainty equivalent return.
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1 Introduction

A number of leading scholars have recently voiced the view that hedge funds do not—and could not—represent a separate, financially relevant asset class on their own.[1] Former hedge fund

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[1] For instance, John Cochrane has been quoted by Lim (2013) to have stated: "Hedge funds are not a new asset class. They trade in exactly the same securities you already own."

manager Simon Lack (2012) has pointedly written that "[i]f all the money that's ever been invested in hedge funds had been put in Treasury bills instead, the results would have been twice as good" (p. 1). The academic literature reflects this chasm. Ackermann, McEnally and Ravenscraft (1999), Brown, Goetzmann and Ibbotson (1999), and Liang (1999) showed that in the aggregate, hedge funds (henceforth, HFs) realize positive risk-adjusted performance, which is a condition to generate economic value in a mean-variance framework; however, Griffin and Xu (2009) find little evidence that HFs, on average, deliver abnormal performance. At the individual fund level, Chen and Liang (2007) show that HFs time the equity market and Kosowski, Naik and Teo (2007) show that abnormally high performance of top HFs cannot be explained by luck, even though Yet, Fung, Hsieh, Naik and Ramadorai (2008) find that only a quarter of all funds of HFs produce significantly positive alphas and Dichev and Yu (2011) report that the HF returns are not much higher than the risk-free rate once investor capital flows into and out of funds are taken into account.

In spite of the raging debate, investors kept pouring wealth into the HF industry with renewed vigor after the 2007-2009 Global Financial Crisis, and the assets under management by the overall industry are predicted to exceed USD3.3 trillion during 2018 (Hedge Fund Research, 2018).[2] Are investors just after a mirage? Are they just after the record performances allegedly achieved by a few lonely but famed HFs during the 1980s and 1990s, when the industry was nascent and many of the very HF "stars" were still small and riding green pastures, free of strategy over-crowding? To try and tackle these questions, our paper presents comprehensive, out-of-sample evidence on the potential benefits accrued to investors who diversify their portfolios of bonds, stocks, and publicly traded real estate to include HF strategies.

HFs are alternative investment vehicles that are subject to limited regulation and thus can take advantage of sophisticated strategies that rely on leverage, short-selling, and derivatives (see, e.g., Agarwal, Mullally and Naik, 2015, and Getmansky, Lee and Lo, 2015, for an introduction and references to seminal papers). Major investors in HFs include foundations, public and

[2] In 2017 alone, the hedge fund industry's AUM has increased by $10 billion and by the end of 2017 the total assets of hedge funds exceeded $3.2 trillion (Hedge Fund Research, 2018). For comparison, hedge funds assets were less than $40 billion in 1990 (see Agarwal, Mullally and Naik, 2015).

private pension funds, university endowments, and funds of HFs, but the ability of relatively small investors to expand their asset menus to include HF strategies has recently been facilitated by the advent of investable HF indices. Therefore whether or not HFs do create economic value (at least) in stylized portfolio choice problems and under fairly realistic assumptions seems to have become a pressing research question of general interest.

Although HFs tout their sophisticated strategies and promise to deliver superior returns that are largely immune to adverse developments in the financial markets, it remains important to provide systematic, consumer-optimization-based evidence on whether investors can actually reap risk-adjusted benefits from diversifying into this alternative asset class. In fact, a literature exists that has investigated the null hypothesis that HFs could not add significant (often risk-adjusted) economic value. In many respects, the seminal paper is Ackermann et al. (1999) which assessed the portfolio value of HFs using Elton, Gruber and Rentzler's (1987) mean-variance methodology for estimating the contribution of an alternative investment portfolio to an existing portfolio. They reported that the correlations between HF returns and eight international stock and bond indices were sufficiently low, and the Sharpe ratio of HF was sufficiently high to augment the overall Sharpe ratio. Similarly, Agarwal and Naik (2000) found that a portfolio comprising of passive asset classes and investing in mainly nondirectional HF strategies provided better ex-ante risk-expected-return tradeoff than just investing passively in a broad range of asset classes comprising of equities, bonds, currencies, and commodities. These conclusions were discussed by a number of other papers set up in a Markowitz's static mean-variance framework, wherein HFs are usually given high weights at the expense of bonds; among these papers are Amenc, El Bied and Martellini (2003), Terhaar, Staub and Singer (2003), and recently Mladina (2015). However, there are severe doubts as to whether a standard mean-variance framework and the Sharpe ratio as a leading performance index to rank funds may be suitable to HF strategies. Although inclusion of HFs leads to mean–variance improvement, Amin and Kat (2003a) have shown that including them in a given (not optimized) portfolio may frequently lead to lower skewness and higher kurtosis, which are then impossible to gauge in a two-moment set up. Cremers, Kritzman and Page (2005) have rejected the validity of mean-variance analysis for HFs due to the strong and statistically significant non-normalities of HFs and experimented instead with the maximization of the log utility of wealth (which turns out to give rise to maximum growth portfolio).[3] Recognizing the significant tail risk that HFs expose to, Agarwal and Naik (2004) have proposed to assess the economic value of HFs in a mean-conditional Value at-Risk (M-CVaR) framework.

In this paper, we also take steps from a need to go past the risk-return characterization of HFs, and we contribute to the literature relative to each of these studies along two or more of the following dimensions: (i) we perform a dynamic, long-horizon portfolio optimization that admits cash outflows (in the stylized form of consumption streams) under constant relative risk aversion preferences that do not only integrate mean and variance in expected utility optimization, but focus instead on the entire predictive density of future outcomes, à la Campbell, Chan and Viceira (2003), (ii) we measure the welfare benefits of HFs as an asset class relying on realized utility differential measures of risk-adjusted performance, (iii) we use a broad set of benchmark portfolios, and (iv) we conduct an out-of-sample (OOS) analysis.

Importantly, because we embrace a dynamic portfolio approach which estimates hedging demands and features long-horizon investors, in this paper we take into account the existence (if any) of linear predictability in the returns of the assets in the menu of choice. In this, we follow Bali, Brown and Caglayan (2012, 2014) and Wegener, von Nitzsch and Cengiz (2010) who have stressed that while HFs are not market-neutral, they are exposed to systematic, macroeconomic-type risks, such as the default premium and inflation shocks that predict performance. In fact, as emphasized by Amenc, El Bied and Martellini (2003), Avramov, Barras and Kosowski (2013), and Avramov, Kosowski, Naik and Teo (2011), HF returns are exposed to a large number of rewarded risk factors and, as such, we should expect them to be predictable because, as argued by Ferson and Harvey (1991) most of the predictability in financial returns can be attributed to predictable shifts in risks and the market wide reward for risks.[4] For

[3] Since the seminal work by Agarwal and Naik (2004), it is well understood that HFs may exhibit non-normal payoffs for reasons such as their use of options, or of option-like dynamic strategies. The payoffs on a large number of equity-oriented hedge strategies resemble those from writing put options.

[4] Amenc et al. (2003) find evidence of predictability in HF index returns using the (lagged) yield on 3-

instance, HFs rely heavily on leverage, which might be highly sensitive to business cycle conditions. To this purpose, we use simple but popular vector autoregressive (VAR) models, as in Campbell et al. (2003). The investor maximizes expected power utility defined over a monthly consumption stream over a long (in principle, infinite) investment horizon. It is important to produce utility-based evidence as even a superior risk-return trade-off of a HF strategy in a static perspective may not improve an investor's risk-adjusted expected performance in the light of the remaining assets in the menu of choice (Amin and Kat, 2003a).[5] With this goal in mind, we conduct a wide range of recursive OOS experiments and assess the realized performance of portfolios using two metrics, the certainty equivalent return (CER) and the Sharpe ratio. The CER, defined as the riskless return that an investor is willing to accept in order to forego a risky portfolio/strategy/asset menu, is the most appropriate measure for ranking alternative models because it is a function not only of the underlying return generating process but also of the investor's preferences. We also report the Sharpe ratio for completeness but note that it may lead to inaccurate rankings due to (spurious) serial correlation in HF returns, which can be attributed to return smoothing and the presence of illiquid securities in HF portfolios (see Getmansky, Lo and Makarov, 2004; Khandani and Lo, 2011).

Our analysis is performed in two steps. In the first step, we compute the optimal portfolio-consumption rules for an investor who diversifies across stocks, government bonds, corporate bonds, and REITs; we refer to this setup as the *baseline asset menu*. For each of three values of the relative risk-aversion coefficient (2, 5, and 10), we entertain a total of 64 VAR models, which correspond to all possible combinations that can be built assuming either one or two autoregressive lags, two different sample selection methods (i.e., rolling vs. expanding windows), and using up to four predictors (i.e., the default and term structure spreads, the 3-month short rate, and the dividend yield) which are widely used in the literature on return

month T-bills, the dividend yield, the default spread, the term spread, the US and world equity factors, and changes in a volume-weighted basket of currencies vs. the US dollar. Avramov et al. (2013) examine whether conditional strategies based on simple trading rules can successfully exploit predictability from the default spread, the dividend yield, the VIX index, and the net aggregate flows into the HF industry.

[5] Similarly, Bollen (2013) has suggested that while market-neutral (i.e., zero-R^2) hedge funds are characterized by high Sharpe ratios, they likely expose the investors to substantial downside risk.

predictability. Macroeconomic variables and uncertainty proxies such as these were recently shown to have explanatory power for HF returns (see Avramov et al., 2011; Bali et al., 2014), which is why we use these same predictors (in addition to HF strategy-specific predictors, following Fung and Hsieh, 2004) in our VAR models in the second step of the analysis.

The model yielding the highest CER within the baseline asset menu is expanded in a second step to one (out of ten) HF strategy at a time using Hedge Fund Research style indices; we refer to this environment as the *extended asset menu*. In each case, we also re-optimize the structure of the models to include HF strategy-specific predictors. Using the resulting realized OOS CER estimates, we evaluate whether extending the asset menu to include HF strategies is desirable to long-term, risk-averse investors who are already well diversified across a broad spectrum of classical asset classes. This approach also allows us to identify which hedge strategy, if any, provides the highest realized utility gains relative to the optimal baseline portfolio.

The key results of our analysis can be summarized as follows. In both the baseline and extended asset menus, the optimal portfolio weights are highly levered and unstable across time, particularly during the global financial crisis. We present evidence that the inclusion of HF strategies leads to a further increase in leverage obtained from shorting 1-month T-bills and that implied hedging demands for HF strategies tend to be negative because their returns show a high first-order serial correlation while they have positive coefficients on past lags of returns. As a result, long positions in HFs cannot be used to hedge intertemporal stochastic variations in investment opportunities. If investor were able to detect top performing models for the prediction of risk premia on the different asset classes, most HF strategies and, as a result, also the composite HFR index do outperform a classical asset menu on a risk-adjusted basis, even taking the resulting sample uncertainty into account. The strategies, whose payoffs are highly nonlinear and therefore not easily replicable in the baseline asset space, yield the highest utility gains. In particular, in our data, relative value strategies are the best viable option to be considered, while a few other strategies result in utility losses (significantly, funds of funds). Most of the OOS economic value fails to result from a mean-variance order improvement: in fact, when combined with classical assets, most (all) HF strategies yield realized mean returns

(Sharpe ratios) that are inferior to the benchmark portfolio. For instance, for an investor with a constant relative risk aversion coefficient of 5, while classical assets only lead to a mean return of 3.8% per month and a monthly Sharpe ratio of approximately 0.17, when the HFR composite index is used to expand the asset menu, the realized annual mean return declines to 0.8% and the Sharpe ratio is just positive. However, HF strategies grossly improve the higher-moment properties of the optimal portfolio: skewness increases from -0.87 to +0.80 and kurtosis stays essentially constant at just below 3. As a result, medium to highly risk-averse investors are found to benefit the most from diversifying into this alternative asset class.

However, under the more realistic assumption that an investor could not know in advance what the best performing model (in terms of realized CER) would have to be ex-post, so that we pick at random a median model, an investor would have not fared so well unless she had known—again, unrealistically—which specific hedge strategy to pick. Indeed, betting on the composite HFR index or on a fund-of-funds strategy leads to median realized CERs that are negative and therefore dominated by the simplest of the portfolio strategies: 100% in cash at all times. While the realized CERs of the median predictability model are promising for a few strategies because it is positive and exceeds the performance of the median model applied to a classical asset menu, other strategies lead to a non-positive CER. Depriving investors from the possibility to fine tune the predictability model hurts in particular the strategies that trade equities.

Our paper draws primarily on three strands of the literature. The first strand attempts to explain HF returns using style analysis, multifactor, and nonlinear models (see, e.g., Fung and Hsieh, 2002a, 2004; Hamza, Kooli and Roberge, 2006; Bali et al., 2012, 2014). A second strand of literature focuses on the performance evaluation and optimal portfolio decisions involving hedge strategies (see, e.g., Agarwal and Naik, 2004, Mladina, 2015, Panopoulou and Vrontos, 2015). Finally, there is extensive research on the underlying biases in the data on HF returns and the perils these would pose to a meaningful assessment of the risk-adjusted benefits (see, e.g. Agarwal, Fos and Jiang, 2013, Aiken, Clifford and Ellis, 2013). Our specific contribution is that we pursue a dynamic, optimizing consumption-portfolio approach that recognizes the existence of predictability in HF returns as well as in all other asset classes typically available to an investor. In doing so, we echo the recommendation by Amin and Kat (2003a) to distinguish between an analysis of "(…) whether in terms of risk and return hedge funds offer investors value for money." and an integrated portfolio view as "It is important to note from the outset, however, that strictly speaking this is a different question than whether hedge funds should be included in an investment portfolio. The fact that an investment offers a superior risk-return profile does not automatically mean investors should buy into it as it may not fit their preferences and/or fit in with other available alternatives." (p. 253).[6]

Hoevenaars, Molenaar, Schotman and Steenkamp (2008) have adapted Campbell et al.'s (2003) model to a long-term asset-liability investor who is diversified across several asset classes including HFs. Alternative assets are found to have a substantial impact on the portfolio rules. We extend their analysis in several ways. First, while Hoevenaars et al. model HF returns solely with the HFRI Fund of Funds Conservative Index, we use a wider spectrum spanning ten HF strategies. Second, we employ additional predictors specific to HF returns, such as the CBOE S&P 500 BuyWrite Index, in order to better capture the time variation in HFs returns. Third, we perform an OOS recursive back-test. Finally, we employ a utility-based metric (CER) to compare the benefits of diversifying into the HF strategies.

There is also one small literature that has investigated the effects of the (sizeable, between 3 and 4% on average, see Ibbotson, Chen, and Zhu, 2011; Jurek and Stafford, 2015) fees charged by HFs on typical inferences on their value to portfolio diversification. In fact, in our design, we have used HF returns net-of-fees and transaction costs along with returns on other, more classical asset classes that are treated in more heterogeneous ways: for instance, stock and bond returns are gross-of-fees, while REIT returns are net of management fees and costs internalized by the trusts. On the one hand, this does not appear to be a first-order concern just because stocks and bonds are now tradeable with cheap (in the limit, zero-fees) exchange traded funds (ETFs); while ETFs also exist in the case of REITs, most management costs and

[6] Amin and Kat (2003a) find that the majority of individual HFs as well as HF indices cannot in isolation produce efficient payoffs, but that they are able to do so when combined in a portfolio with the S&P 500 index, which suggests that a relatively well-developed portfolio approach to the problem is advisable.

fees are netted out of reported performances as these are already fund returns.[7] On the other hand, we then may accept the key findings of our paper as a lower bound to the portfolio value created by HFs: if HFs may create value in spite of the way transaction costs are accounted for—as our results seem to imply, in CER terms and subject to some caveats—and if we were to place all asset classes on a levelled playing field, then such value would be even higher. On the other hand, the literature (see, e.g., Greenwood and Scharfstein, 2013; Ibbotson et al., 2011) has emphasized that obtaining clean data on net-of-fees HF returns remains a chimera because many fees are privately negotiated and not reported, and the connection between gross and net returns is further complicated by the application of high-water marks.

The next section describes the research design that allows us to exploit the predictability in asset returns, determine optimal consumption-portfolio rules, and measure OOS performance. Section 3 describes the data on the baseline assets, HF indices, and predictor variables. Section 4 systematically selects the best-performing model from within the baseline asset menu. Section 5 computes optimal allocations with HF strategies and studies which strategies, if any, can improve the realized utility of the investor. Section 6 concludes.

2 Research design

We compute the optimal portfolio-consumption rules using the approximate discrete-time solution of Campbell et al. (2003) and perform recursive, realized OOS evaluations while adjusting for small-sample bias following Engsted and Pedersen (2012). The investor is assumed to have a five-year investment horizon. The return generating process has either a VAR or Gaussian IID structure, and model estimation and optimization take place within rolling

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[7] The comparison would become more difficult if we were to ask a question *different* from the one we ask in the paper, e.g., can investors internalize the HF payoffs and replicate themselves HF-style returns without trading HFs? Indeed, if we further add options (that have nonlinear payoffs) in the benchmark model while ignoring transaction costs in options trading, we may spursiously find a low investment value for HFs. But we do ask a differentthe following question here: can HFs create value in *otherwise traditional*, linear investment portfolios? The theorerical analysis in Lan, Wang and Yang (2013) predicts that in present value terms management fees will capture all the value created by hedge funds, so that HF may even be expected to create value only by providing diversification benefits to various risks of other asset classes. This has been recently confirmed by the empirical analysis in Jennings and Payne (2016).

and expanding window schemes. This section gives details on these methodologies.

2.1 *Predictability of asset returns*

2.1.1 Vector autoregressive models and bias correction

The dynamics of investment opportunities are described by a range of reduced-form, *p*th-order VAR(*p*) processes. All variables, including the predictors, are modeled as endogenous. A vector of state variables $\boldsymbol{z}_{t+1}$ is defined as

$$\boldsymbol{z}_{t+1} \equiv \begin{bmatrix} r_{1,t+1} \\ \boldsymbol{x}_{t+1} \\ \boldsymbol{y}_{t+1} \end{bmatrix}, \tag{1}$$

where $r_{1,t+1}$ is the log return on a benchmark short-term security, $\boldsymbol{x}_{t+1}$ is an (n - 1) vector of log excess returns on the risky asset, and $\boldsymbol{y}_{t+1}$ is an m vector of predictor variables. The stochastic evolution of $\boldsymbol{z}_{t+1}$ in a VAR(1) model is given by[8]

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1}, \tag{2}$$

where $\boldsymbol{\Phi}_0$ is the (n+m) vector of intercepts, $\boldsymbol{\Phi}_1$ is the (n+m)x(n+m) coefficients matrix, and $\boldsymbol{v}_{t+1}$ is a vector of Gaussian white noise processes distributed as

$$\boldsymbol{v}_{t+1} = \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \textstyle\sum_{\boldsymbol{v}}), \qquad \textstyle\sum_{\boldsymbol{v}} \equiv Var_t(\boldsymbol{v}_{\boldsymbol{t+1}}) = \begin{bmatrix} \sigma_1^2 & \boldsymbol{\sigma}'_{\boldsymbol{1x}} & \boldsymbol{\sigma}'_{\boldsymbol{1y}} \\ \boldsymbol{\sigma}_{\boldsymbol{1x}} & \boldsymbol{\Sigma}_{\boldsymbol{xx}} & \boldsymbol{\Sigma}'_{\boldsymbol{xy}} \\ \boldsymbol{\sigma}_{\boldsymbol{1y}} & \boldsymbol{\Sigma}_{\boldsymbol{xy}} & \boldsymbol{\Sigma}_{\boldsymbol{yy}} \end{bmatrix} \tag{3}$$

The shocks are zero mean, homoskedastic normal variables, which are contemporaneously correlated but IID over time. Normality is therefore induced in the unconditional distribution of $\boldsymbol{z}_t$, where the mean $\boldsymbol{\mu}_{\boldsymbol{z}}$ and the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{zz}}$ are described by

$$\boldsymbol{\mu}_{\boldsymbol{z}} = (\boldsymbol{I}_{(n+m)} - \boldsymbol{\Phi}_1)^{-1}\boldsymbol{\Phi}_0, \qquad vec(\textstyle\sum_{\boldsymbol{z}}) = (\boldsymbol{I}_{(n+m)^2} - \boldsymbol{\Phi}_1 \otimes \boldsymbol{\Phi}_1)^{-1} vec(\boldsymbol{\Sigma}_v). \tag{4}$$

Differently from Campbell et al. (2003), we take into account the instability of the VAR parameters and adjust the estimates for small-sample bias as in Engsted and Pedersen (2012). Using Pope's (1990) formula, Engsted and Pedersen quantify the bias in the estimate $\widehat{\boldsymbol{\Phi}}_1$ of the slope parameters of the VAR in (2) as

[8] All higher-order VAR can be re-written as a VAR(1) by way of a companion form representation (see e.g., Hamilton, 1994, p. 259).

$$\boldsymbol{Bias}_T = -\frac{\boldsymbol{b}}{T} + O\left(T^{-\frac{3}{2}}\right), \tag{5}$$

where T is the number of observations used in estimation and

$$\boldsymbol{b} = \boldsymbol{\Sigma}_v \left[\left(\boldsymbol{I}_{(n+m)} - \boldsymbol{\Phi}_1'\right)^{-1} + \boldsymbol{\Phi}_1'\left(\boldsymbol{I}_{(n+m)} - (\boldsymbol{\Phi}_1')^2\right)^{-1} + \sum_{i=1}^{n+m} \lambda_i \left(\boldsymbol{I}_{(n+m)} - \lambda_i \boldsymbol{\Phi}_1'\right)^{-1} \right] \boldsymbol{\Sigma}_z^{-1}, \tag{6}$$

$\boldsymbol{\Sigma}_v$ and $\boldsymbol{\Sigma}_z$ are defined in (3) and (4), respectively, and λ_i is the ith eigenvalue of $\boldsymbol{\Phi}_1$.[9]

Starting from the OLS $\widehat{\boldsymbol{\Phi}}_1$, the bias-correction procedure is implemented in four steps. First, as long as there are no unit roots in $\widehat{\boldsymbol{\Phi}}_1$, we compute the bias $\boldsymbol{B}_T$ by substituting $\widehat{\boldsymbol{\Phi}}_1$ for $\boldsymbol{\Phi}_1$ in (6). Second, we subtract the result from the OLS estimate to arrive at the bias-corrected $\widetilde{\boldsymbol{\Phi}}_1$. Third, we check whether the latter contains unit roots and, if so, find the maximum value $\kappa \in [0, 0.01, 0.02, \dots, 0.99]$ that multiplies $\boldsymbol{B}_T$ such that the bias-corrected $\widetilde{\boldsymbol{\Phi}}_1$ lies again in the stationarity region (see Kilian, 1998). Finally, we calculate the bias-adjusted estimate of the intercept $\boldsymbol{\Phi}_0$ by imposing that the unconditional mean vector of $\boldsymbol{z}_t$ coincides with its full-sample mean:[10]

$$\widetilde{\boldsymbol{\Phi}}_0 = \left(\boldsymbol{I}_{(n+m)} - \widetilde{\boldsymbol{\Phi}}_1\right) * \widehat{\boldsymbol{\mu}}_{\boldsymbol{z}}. \tag{7}$$

2.1.2 Gaussian IID model

To establish a benchmark against which to assess the VAR-based results, we also computed optimal portfolios on the basis of a Gaussian IID model in which returns evolve according to

$$\boldsymbol{x}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{v}_{t+1}, \qquad \boldsymbol{v}_{t+1} = \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \textstyle\sum_v). \tag{8}$$

This model implies no predictability and, equivalently, constant investment opportunities.

Under this dynamics for excess returns, the investor will choose the same portfolio allocation regardless of the investment horizon. Bias correction has no effect in this framework as both the bias-corrected and the unadjusted estimates of $\boldsymbol{\Phi}_0$ coincide.

[9] The rate of convergence of the error in (5) is equal to $T^{-3/2}$ and is comparable to that of either a bootstrap or Monte Carlo bias-adjustment simulation.

[10] Through a simulation study, Engsted and Pedersen (2012) find that the bias-correction procedure outlined above leads to an improvement upon the initial OLS estimates in terms of mean square error, variance and bias, and that the improvement is more significant as the samples are smaller.

2.2 *Portfolio selection*

2.2.1 Portfolio returns

As in Campbell et al. (2003), the investor can allocate her savings among *n* securities, with the resulting gross portfolio returns given by

$$R_{p,t+1} = \sum_{i=2}^{n} \alpha_{i,t}\left(R_{i,t+1} - R_{1,t+1}\right) + R_{1,t+1}, \tag{9}$$

where $R_{\mathrm{i,t+1}}$ is the gross return on the risky asset *i* which has been assigned a weight $\alpha_{i,t}$, and $R_{1,t+1}$ is the gross return on a benchmark, short-term security.[11] Campbell et al. convert (9) into logs using an approximation based on a first-order Taylor expansion:

$$r_{p,t+1} = r_{1,t+1} + \boldsymbol{\alpha}_t' \boldsymbol{x}_{t+1} + \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{\alpha}_t' (\boldsymbol{\sigma}_x^2 - \boldsymbol{\Sigma}_{xx} \boldsymbol{\alpha}_t), \tag{10}$$

where $\boldsymbol{\sigma}_x^2 \equiv diag(\boldsymbol{\Sigma}_{xx})$ is the (n-1) vector of variances of the log excess returns.

2.2.2 Preferences and optimal portfolio-consumption choice

In line with the portfolio choice literature (see Brandt, 2009), we assume that the investor maximizes time-separable, CRRA power utility preferences, here written in recursive form,

$$U\left(C_t, E_t(U_{t+1})\right) = \left[(1-\delta)C_t^{1-\gamma} + \delta\left(E_t\left(U_{t+1}^{1-\gamma}\right)\right)\right]^{\frac{1}{1-\gamma}}, \tag{11}$$

where δ is the discount factor and $\gamma > 0$ is the coefficient of relative risk aversion. γ also determines the investor's consumption substitution patterns across time as the constant elasticity of intertemporal substitution is simply $\psi = \gamma^{-1}$.[12]

As it is well-known (see Ang, 2014) power utility makes an investor's expected utility dependent on features of the entire distribution of the realized consumption/wealth process,

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[11] We express all returns in nominal terms because of a relatively low and constant realized inflation in our data ought to have been discounted in assets prices and it is unlikely to affect the key findings.

[12] It is well known that the Euler equation deriving from (11) for the CRRA case has been rejected in a large body of finance and economics research. For instance, Campbell et al. (2003) also use Epstein-Zin recursive preferences that separate relative risk aversion (γ) from elasticity of intertemporal substitution (ψ). However, it is well known that while γ mainly drives optimal portfolio allocations, ψ determines the optimal consumption-savings ratio. Because here we just focus on the realized portfolio performances, preliminary experiments on the benchmark asset menu revealed that ψ indeed exercised a rather modest effect on optimal recursive weights.

including moments of order higher than mean and variance, which resonates well with the great emphasis that has been placed on the (allegedly, poor) skewness, kurtosis, and left tail risk properties of HFs (see, e.g., Amin and Kat, 2003b; Agarwal, Ruenzi and Weigert, 2017). On each period t, the investor allocates her savings ($W_t - C_t$) across different financial assets, thus facing the intertemporal capital accumulation (budget) constraint:

$$W_{t+1} = (W_t - C_t)\, R_{p,t+1}. \tag{12}$$

Maximizing utility (11) subject to the budget constraint (12) yields the following set of Euler equations for each asset *i* as well as for the portfolio *p*:

$$\mathrm{E_t}\left[\left\{\delta\left(\frac{\mathrm{C_{t+1}}}{\mathrm{C_t}}\right)^{-\gamma}\right\}\mathrm{R_{i,t+1}}\right] = 1. \tag{13}$$

In a Gaussian IID framework, this first-order condition implies that the optimal portfolio rule is fully myopic and that the investor will consume the same percentage of wealth in each period. Under the VAR dynamics in (2), the portfolio weights solving the first-order condition is solved by Campbell et al. (2003) within an approximate analytical framework. The solution is based on a log-linearization around the ergodic mean of the log-consumption-wealth ratio and a second-order Taylor expansion around the conditional expected values of Δc_{t+1}, $r_{p,t+1}$, and $r_{i,t+1}$. This approach leads to an approximate linear portfolio policy that expresses the weights $\boldsymbol{\alpha_t}$ is a linear function of the VAR parameters:

$$\boldsymbol{\alpha_t} = \boldsymbol{A_0} + \boldsymbol{A_1 z_t}, \tag{14}$$

$$\boldsymbol{A_0} = \left(\frac{1}{\gamma}\right)\boldsymbol{\Sigma}_{xx}^{-1}\left(\boldsymbol{H}_x\boldsymbol{\Phi}_0 + \frac{1}{2}\boldsymbol{\sigma}_x^2 + (1-\gamma)\boldsymbol{\sigma}_{1x}\right) + \left(1 - \frac{1}{\gamma}\right)\boldsymbol{\Sigma}_{xx}^{-1}\left(\frac{-\boldsymbol{\Lambda_0}}{1-\psi}\right) \tag{15}$$

$\boldsymbol{H}_x$ is the selection matrix which retrieves the excess returns from $\boldsymbol{z_t}$, $\boldsymbol{A_0}$ is an (n-1) vector, $\boldsymbol{A_1}$ is an (n-1)x(n+m) matrix, and $\boldsymbol{\Lambda_0}$ and $\boldsymbol{\Lambda_1}$ are functions of the intercept and slope parameters of the VAR, the optimal consumption rule, and the investor's preferences.[13] The sum of the first terms in $\boldsymbol{A_0}$ and $\boldsymbol{A_1}$ reflects the myopic part of the total asset demand and is not affected by the values of the elasticity of intertemporal substitution. If the benchmark short-term security is risky (i.e., $\boldsymbol{\sigma}_{1x} \neq \boldsymbol{0}$), investors with $\gamma \neq 0$ will modify their portfolio by $(1-\gamma)\boldsymbol{\sigma}_{1x}$. The

[13] Detailed expressions can be found in the on-line appendix to Campbell et al. (2003, p. 5).

remaining terms in $\boldsymbol{A_0}$ and $\boldsymbol{A_1}$ are the intertemporal hedging demands. This component allows investors to hedge against stochastic investment opportunities shifts and is a consequence of return predictability.[14,15]

2.3 *Out-of-sample performance measurement*

Comprehensive out-of-sample (OOS) performance evaluation is not possible within Campbell et al.'s (2003) framework because the investor is assumed to have an infinite planning horizon. In our approximation, which is driven by the size of our OOS period (and, in turn, data availability) we study a 5-year investment horizon (H) and perform recursive OOS experiments using the optimal weights from (14). We take the period 2004:01–2014:12 as our reference OOS period in which we estimate realized performances recursively. Importantly, such period includes both pre- and post-Great Financial Crisis data. We assume that investors may choose to either follow a 5-year buy-and-hold strategy (optimal when investment opportunities are constant but always efficient because it lowers turnover) or rebalance their portfolio on a monthly basis, which is optimal under time-varying investment opportunities. If rebalancing is pursued, investors are given the possibility to fully exploit the predictability captured by the vector $\boldsymbol{z}_t$: the first investor allocates her wealth starting in 2004:01 according to the corresponding optimal weights and adjusts her exposure at the beginning of each of the next 60 months. The second investor acts in the same way as the first, although starting and ending a month later, etc. This is repeated until T – H. When the investor simply implements a buy-and-hold strategy, the weights computed at time t are held for H months before the optimal portfolio structure is re-estimated in the light of new data.

We assess the realized OOS performance at the end of the investment horizon for each investor using two metrics: the CER and the Sharpe ratio. The CER is the riskless return that makes adopting a portfolio rule as attractive as cashing in a safe return equal to the CER. A negative

[14] Under a Gaussian IID model, the hedging demand would be null as both $\boldsymbol{\Lambda_0}$ and $\boldsymbol{\Lambda_1}$ are matrices of zeros (since $\boldsymbol{\Phi}_1 = \boldsymbol{0}$), and thus $\boldsymbol{A_1} = \boldsymbol{0}$ and (17) reduces to $\boldsymbol{\alpha_t} = \boldsymbol{A_0}$.

[15] From the Euler equation, we derive the optimal policy $c_t - w_t = b_0 + \boldsymbol{B}_1'\boldsymbol{z}_t + \boldsymbol{z}_t'\boldsymbol{B}_2\boldsymbol{z}_t$. Campbell et al. (2003) develop an ad-hoc iterative procedure which starts with arbitrary values $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$, calculates $\boldsymbol{A_0}$ and $\boldsymbol{A_1}$, and then inputs the latter in an equation similar to (15)-(16) to get the new values of $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$. This process is repeated until convergence (see the appendix to Campbell et al., 2003, p. 12).

CER would signal the investor's willingness to pay to avoid a risky strategy:

$$\sum_{t=1}^{T-H} \beta^t E_t\left[\frac{\hat{C}_t^{1-\gamma}(\widehat{\boldsymbol{\alpha}}_t)}{1-\gamma}\right] = \sum_{t=1}^{T-H} \beta^t E_t\left[\frac{\tilde{C}_t^{1-\gamma}}{1-\gamma}\right], \tag{16}$$

where $\tilde{C}_t \equiv (1-\beta CER_H^{1-\gamma})/[1-\left(\beta CER_H^{1-\gamma}\right)^{(H-t+1)/\gamma}]$ is the consumption stream derived from a riskless strategy paying a monthly return of $\mathrm{CER_H}$, for the entire holding period H.

For completeness, we also evaluate model performance using a more conventional H-period Sharpe ratio, which is the ratio of the excess mean return to the standard deviation of the portfolio being evaluated:

$$SR_H \equiv \frac{r_{p,H} - r_{1,H}}{\sigma_{p,H}}, \tag{17}$$

where $r_{p,H}$ and $r_{1,H}$ are the cumulative returns (over H months) on the portfolio and on the benchmark, short-term security, respectively, while $\sigma_{p,H}$ is the volatility of cumulative portfolio excess returns. We deem the CER the most appropriate realized performance measure because it is a function of not only the underlying return generating process but also of the investor's preferences. Moreover, the Sharpe ratio may be biased by high serial correlation in HF returns due to illiquidity and returns smoothing (Getmansky et al., 2004; Khandani and Lo, 2011).

The OOS recursive design proceeds as follows. In a first step, we compute the optimal portfolio-consumption rules for an investor who has no access to HFs but is otherwise well diversified across stocks, long-term government bonds, corporate bonds, and real estate (i.e., the baseline asset menu). Assuming either one or two lags in the VAR models, using up to (i.e., also the realized OOS performance of lighter models that include less predictors is examined) four predictors among the default spread, the term structure spread, the 3-month nominal rate, and the dividend yield, and two different sample selection methods (rolling and expanding window), we estimate a total of 192 models. The first vector of portfolio weights in (14) is estimated using data for a 1994:01–2003:12 sample. For the next-period estimation, one additional set of monthly observations (referring to 2004:01) is added to the initial sample. This process is repeated recursively until the last available observation (2014:12) is included in the analysis. While under the expanding window scheme the sample size increases with each new estimation, in the rolling-window scheme, it is kept constant at 10 years (120 observations) by rolling the sample forward and discarding the oldest observations.[16]

The OOS experiment is repeated considering three alternative risk aversion coefficients (γ = 2, 5, 10). The VAR model that provides the investor with the highest CER is taken as optimal and assumed to be the one against which the same investor benchmarks possible modifications to her baseline asset menu. In a second step, we extend the best-performing model (as defined by the number of lags, estimation scheme, predictors included, etc.) by including one HF strategy at a time along with a set of predictor variables tailored to each strategy. This occurs separately for each of the three values of γ. We thus estimate 17 VAR models for each of the ten HF strategies, each of the three values of the coefficient of risk aversion and each of the two types of recursive OOS experiments performed (i.e., rolling and expanding windows); this yields a total of 1020 alternative VARs entertained.[17]

Finally, by comparing the CER obtained in this second stage with the CER of the initial best-performing VAR, we are able to give a data-driven answer to our research question — that is, whether or not extending strategic asset allocation to include HFs is desirable to a long-term investor who is already well diversified across a broad spectrum of both classical and alternative asset classes. Additionally, our research design allows us to answer the question of which HF strategy yields the highest utility gains.[18]

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[16] The use of 10 years of data in the rolling window scheme addresses the investors' need of protection against structural breaks in the underlying predictive relations (Stock and Watson, 1996). We have experimented with longer windows with qualitative similar results but a loss of OOS evidence as the recursive scheme implied that the first allocation that can be assessed is determined by how many initial observations are required. Shorter windows, and in particular the classical 5-year moving window are instead unfeasible because of the relatively large size of the estimated VAR(2) models.

[17] The vector $\boldsymbol{z}_t$ in (2) includes the baseline assets, the HF strategy under investigation, the best-performing predictors for the baseline menu, and up to four strategy-specific predictors, for a total $n + m$ that ranges between 5 and 13.The 17 VAR models cited in the text encompass the 16 combinations of the four predictor variables (including the case when only the baseline asset predictors are included, without any specific variable for the hedge strategy) and a pure AR process without additional predictors ($\boldsymbol{y}_{t+1}$=1 for all t).

[18] For comparison purposes, optimal allocation and realized performances are also reported for a Gaussian IID underlying return generating process (i.e., constant investment opportunities). Our CER spread estimates are of course model-driven, even though the combination of an extensive search over predictors and the addition of HF-specific predictors ought to guarantee some degree of robustness.

3 The data

This section summarizes the data on the baseline and extended asset menus and the corresponding predictors, and describes how our choice of HFR indices tries to minimize the effect of the biases prevalent in HFs data. Our choice of a January 1994 - December 2014 sample is driven by the availability and characteristics of the HFs data (described below).[19]

3.1 *Baseline asset menu*

We use the 1-month T-bill rate, the CRSP value-weighted equity index (inclusive of dividends), the CRSP/Ibbotson 10-year US government bond index, the FTSE NAREIT Composite Index, and the Barclays Long U.S. Corporate Total Return Index to proxy for our five baseline asset classes.[20] The Barclays long-term corporate bond index tracks the performance of US corporate bonds with maturities of 10 years or greater, and the FTSE NAREIT index is an indirect index built on all tax-qualified REITs.[21]

Consistent with the literature, we use four predictors to model the time variation in investment opportunities as defined by our baseline asset menu. In line with Avramov et al. (2013) and Campbell et al. (2003), we employ the dividend yield, whose forecasting ability with respect to equity returns has been demonstrated at least since Rozeff (1984) and Campbell and Shiller (1988). The predictive power of the dividend yield extends also to other asset classes, including corporate bonds (Fama and French, 1989) and REITs (Karolyi and Sanders, 1998; Fugazza et al., 2009). Second, following Fama (1981), we use the short-term riskless interest rate proxied by the 3-month Treasury constant maturity rate. Third, we rely on the term spread, which is calculated as the difference between the 10-year Treasury constant maturity rate and the corresponding 3-month rate. The predictive power of the term spread concerns not only excess

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[19] Our sample is sufficiently well-balanced as it encompasses major macroeconomic and idiosyncratic events that affected all asset classes under consideration (e.g., the 1997-1998 Asian crisis, the 1998 Russian default and LTCM fall, the technology bubble, the 2008-2009 financial crisis and the subsequent recovery, and the 2013 taper tantrum). Optimizing portfolio choices in such a context enables us to evaluate how well the models perform in bull and bear markets and whether they can adjust over time.

[20] The data on monthly asset returns and predictor variables are obtained from CRSP, Datastream, Bloomberg, the web site of NAREIT, and the Federal Reserve Bank of St. Louis' FRED.

[21] The use of FTSE NAREIT returns is in line with the literature on real estate predictability (see, e.g., Fugazza, et al. 2009), which opts for an indirect measures over direct-appraised and transactions data.

bond returns (Fama, 1990), but also the state of the economy at large and thus other asset returns (Campbell, 1987). Finally, we include the default spread computed as the yield differential between Moody's seasoned Baa and Aaa corporate bond portfolio rates. Keim and Stambaugh (1986) find that default spreads are able to predict corporate and government bonds as well as stock returns, while Fugazza et al. (2009) and Ling, Naranjo and Ryngaert (2000) point to the predictive power of both the term spread and the default spread for to REIT excess returns.

Panel A of Table 1 presents key descriptive statistics for the baseline menu asset returns and their predictors. The monthly (annualized) average excess log-returns in our sample are 0.53% (6.41%), 0.28% (3.37%), 0.39% (4.65%), and 0.61% (7.35%) for stocks, government bonds, corporate bonds, and REITs, respectively. Not surprisingly, higher monthly mean returns correspond to higher estimates of volatility: 4.50% (15.58%), 2.03% (7.04%), 2.62% (9.08%), and 5.68% (19.66%). REITs show the lowest unconditional monthly Sharpe ratio (0.11), corporate bonds the highest (0.15), with stocks and long-term Treasuries falling in between (0.12 and 0.14). The fact that bonds command higher Sharpe ratios than stocks and REITs is driven by the inclusion of the post-crisis, 2009-2014 period in our sample, with declining rates and FED-driven support to the bond markets. Kurtosis is well in excess of three for all returns, skewness is on average negative, and as a result, the Jarque-Bera test points to the rejection of normality for all asset classes and predictors.

3.2 *Extended asset menu*

Perhaps the most important issue endemic to HF data is the selection bias that stems from the lack of reporting standards and, consequently, from the discretionary decisions by HF managers as to whether to report the returns and to which databases (see, e.g., Fung et al., 2008, Akien et al., 2012). This leads to a limited compatibility among various hedge strategy indices, which is further exacerbated by the providers' disparate choices with respect to weighting and fund inclusion thresholds and characteristics (e.g., Titman and Tiu, 2010). Although Agarwal et al. (2013) suggest that the incentives underlying the choice of whether to submit data to an index provider may skew an index return either upward (i.e., when returns are *more* likely to be disclosed after a positive track record) or downward (i.e., when returns are *less* likely to be disclosed after a positive track record to preserve confidentiality or to avoid broadening the investor base), Aiken et al. (2012) compare reporting and non-reporting funds and conclude that the net selection bias is positive — i.e., it leads to an overestimation of HF returns.[22]

Because the literature has generally concluded that on a net basis, these biases may tilt upwards the recorded HF returns (and bias downward the estimable volatility, because of the smoothing effects of positively serially correlated returns) in commercial data bases, we use the HFRI style indices distributed by Hedge Fund Research (HFR) as proxies for HF strategies. HFRI data are (i) net-of-fees, (ii) available starting from 1990 on a monthly basis for most of the main strategies and sub-strategies, (iii) compiled using data on both surviving and non-surviving funds, (iv) encompass both closed and open funds, and (v) impose either a minimum threshold of $50 million of assets under management (AUM) or a track record of more than a year. Importantly, no backfilling bias plagues the HFRI indices. Moreover, HFR provides, within the limits of the AUM thresholds imposed, a rather comprehensive coverage of the HF universe. To some extent (see the discussion in Tuchschmid et al. 2010), HFRI indices are investable via synthetic replication products and there is some evidence that such products are traded over-the-counter. Our sample starts in in 1994 because from after that year the HFRI's survivorship bias is virtually non-existent, as the track record of non-surviving funds has been retained starting on that year (e.g., Liang 2000). The selection bias in HFRI is less severe than with other sources of hedge index returns and HFR makes documented efforts (e.g., by directly contacting the managers and investors) to minimize the liquidation bias.

In our empirical work, we focus on ten HF style indices that have been most frequently used in the literature (see, e.g., Fung and Hsieh, 2002a, Agarwal and Naik, 2004, Boyson, Stahel and Stulz, 2010, Panopoulou and Vrontos, 2015). Our dataset includes the two flagship indices

[22] Other biases that can significantly distort the true representation of hedge funds returns include backfiling (or instant history), survivorship, liquidation, and incubation biases, see Agarwal et al. (2015) for a discussion and review of the literature. However, Edelman, Fung and Hsieh (2013) have recently issues some re-assurances on the reliability of standard data sets as they find that the performance measures for mega hedge fund management companies that collectively manage over 50% of the industry's assets that do not report to commercial databases are similar to those of funds reporting.

(Fund Weighted Composite Index and Fund of Funds Composite Index), all four main strategies (Equity Hedge, Event Driven, Global Macro and Relative Value), and four sub-strategies—Equity Hedge Equity Market Neutral, Event Driven Merger Arbitrage, Event Driven Distressed/Restructuring and Relative Value Fixed Income Convertible Arbitrage.[23]

As hinted at in Section 2, we include in our analysis predictors that are tailored to each strategy in addition to the four predictors used for the benchmark asset menu (i.e., the dividend yield, the short-term bill rate, the default spread, and the term spread, which recent literature has shown to have forecasting power for HF excess returns as well, see Avramov et al., 2011, Bali et al., 2012, 2014). To model the time-varying risk premia in the excess returns on the two flagship HFRI indices we follow Fung and Hsieh (2004) and use the Fama-French size factor (SMB), the CBOE S&P 500 BuyWrite Index (henceforth BMX, consistent with the corresponding Bloomberg ticker), Carhart's momentum factor, and a commodity trend-following factor.[24] For the HFRI Macro strategy, we employ the SMB, the BMX and the commodity and currency trend-following factors. To model the Equity Hedge, Equity Market Neutral, Event Driven, Merger Arbitrage and fixed income Relative Value/Arbitrage excess returns we use the SMB, BMX, Fama-French value factor (HML) and the momentum factor (see, e.g., Fung and Hsieh, 2002a, Agarwal and Naik 2004, Wegener et al. 2010), except for the replacement of the momentum factor with a bond trend-following factor when predicting the Event Driven strategy. Distressed/ Restructuring and Fixed Income Convertible Arbitrage are found to be best forecast by the default spread (as in Bali et al. 2014), the SMB, and by bond and short-term interest rate trend-following factors (e.g., Fung and Hsieh, 2002b, and Hamza et al., 2006).

The descriptive statistics for the extended asset menu and the HF predictors are reported in Panel B of Table 1. Monthly average excess log-returns on the 10 HF strategies range from

[23] Definitions and methodologies of construction of each composite index and each style category can be found at https://www.hedgefundresearch.com/indices. HFRI Indices are investable via synthetic replication products and there is some evidence that such products are traded over-the-counter, see Tuchschmid, Wallerstein and Zaker (2010).

[24] BuyWrite is an option strategy combining a long position on the S&P 500 index with a short position on the near-term call on the same index. From the put-call parity, the strategy is equivalent to writing a put option on the S&P500 and investing the premium at the risk-free rate.

0.21% (funds of funds, FoF) to 0.57% (event driven). Equity hedge strategies have the highest monthly standard deviations (2.58%), while equity market neutral strategies have the lowest (0.85%). The merger arbitrage strategy registers the highest Shapre ratio (0.40) and FoF the lowest (0.13). Yet, for 9 of the 10 strategies/indices under investigation, the full-sample Sharpe ratio exceeds the highest Sharpe ratio for traditional assets (0.15). Also in panel B, excess returns are highly non-normal (consistently with well-known evidence, e.g., Mitchell and Pulvino, 2001) and are characterized by positive excess kurtosis and negative skewness (Anson, Ho and Silberstein, 2007). The returns on the four trend-following factors, the SMB factor, and the macro HF index have positive skewness. Interestingly, the short-term rate trend-following factor posted triple-digit monthly returns during the financial crisis, which shows it can capture flight-to-quality phenomena.

Table 2 reports pairwise linear correlations for portfolio return series under consideration. Excess returns are generally weakly correlated not only in the baseline asset menu but also in the extended menu, suggesting that the diversification into of HFs may improve the overall portfolio performance. However, we observe that event driven and equity hedge strategies' correlation with stocks exceeds 0.75, which may attenuate the potential benefits of extending the baseline asset menu, at least in their case.

4 Preliminary results for the baseline asset menu

4.1 *Linear predictability of returns*

Table 3 presents the parameter estimates, as well as the correlation matrix of the residuals for a full VAR(1) model estimated on the full, 1994:01–2014:12 sample. For every asset and predictor, the table reports the bias-adjusted estimates, the original OLS estimates, and the associated *t*-statistics. Two remarks are in order. First, small-sample bias is particularly severe for the dividend yield coefficient in the VAR equation for excess stock returns, where the corrected coefficient (0.917) is less than half the (biased) OLS estimate (1.890). Other parameter estimates are similarly affected, although the differences are not always statistically significant. With very few exceptions, the bias-adjusted estimates tend to be smaller in absolute value vs. the unadjusted ones. Second, and especially when bias adjustment is performed, the overall evidence of linear predictability is rather weak, as evidenced by the low R^2 for most of the equations for excess returns in the VAR (the expected exception is the short-term rate). In fact, the majority of the *t*-statistics lie inside the rejection region for the 5% significance level. Notable exceptions are excess stock returns, which seem to be well predicted by past values of the dividend yield and the term spread, and excess long-term government and corporate bond returns, which are positively driven by one lag of the term spread. These results are similar to the results in Campbell et al. (2003) and should be assessed in light of the burgeoning literature pointing to the disappearance of linear predictability in the returns of stocks and bonds (e.g., Pesaran and Timmermann 2002, Welch and Goyal, 2008). Finally, all four predictor variables can be approximately described by unit root AR(1) processes, even though as a whole the estimated VAR(1) model is stationary. The distortions implied by the high persistence of the predictors are partially mitigated by the bias-correction technique.

4.2 *Strategic asset allocation: total and hedging demands*

Table 4 presents sample means, standard deviations, and the 90% empirical ranges for the monthly recursive portfolio weights computed following the recursive scheme described in Section 3. These weights summarize how a risk-averse investor with intermediate risk aversion (γ = 5) should have optimally allocated her wealth across 1-month T-bill, stocks, long-term government and corporate bonds, and REITs between 2004 and 2014. Since it is unfeasible to report the statistics for all 192 VAR models estimated for this "reference" investor, we have selected the ten models that are found to yield the highest CERs (computed in the next section). Table 4 shows that average weights are highly levered; such leverage is obtained from shorting 1-month T-bills and government bonds, while the demand for corporate bonds is more heterogenous across different models, with long and massive average weights not impossible. The average optimal weight to be allocated to stocks and REITs is more stable and generally positive, especially for long-horizon investors. However, (average) optimal portfolio rules vary widely across model specifications. For instance, according to an expanding VAR(1) with the default spread, the short-term nominal rate and the dividend yield as predictors, a long-horizon investor should build a portfolio which, on average, is long 53% in stocks, 215% in long-term government bonds and 111% in REITs, and is short 163% in T-bills and 115% in corporate bonds. On the other hand, when in this specification the default spread is replaced by the term spread, the same investor is required to offset the average long positions in stocks (63%), corporate bonds (163%), and REITs (35%), partially financed by a short position in long-term government bonds (161%) and basically shunning any investment in 1-month T-bills. In fact, this last model will be shown to rank first in terms of realized OOS CER and produces less extreme weights in REITs and 1-months T-bills than a Gaussian IID model.

Average hedging demands (i.e., average differences between long- and short-horizon optimal portfolio weights that hedge portfolio performance against future changes in investment opportunities) are in general low for all asset classes. One aspect of our research design that contributes to this effect is the small-sample bias correction of the intercept and of the slope parameter estimates reported in Table 3. These parameters, together with the correlation matrix of the residuals terms, govern the relative speed of reversion to the mean by excess returns (see Barberis, 2000). For example, in the case of the full VAR(1) in Table 3, unreported results show that the hedging demand for stocks is drastically reduced after correcting for small-sample biases. Because the coefficient on the dividend yield in the excess stock return equation tends to be halved by the small-sample bias correction, a positive innovation in the dividend yield, which is induced by a negative innovation in stock returns (this is enforced by the negative sign of the correlation coefficient between residuals to the excess stock return and the dividend yield equations), translates into a reduced forecast of the equity return. Because such reduced forecast follows a negative excess return shock, this means that the long-run mean-reversion speed of the asset class is reduced. For the 192 VARs models entertained in our paper, average hedging demands are usually negative for 1-month T-bills and corporate bonds, and positive for government bonds and stocks. Interestingly, and consistent with Hoevenaars et al. (2008), in the case of REITs, the difference between total and myopic demands tends to be almost null, pointing to a flat term structure of risk. In the Gaussian IID models there is no predictability to be exploited and therefore no hedging demand is required.

Table 4 shows that recursive optimal weights change not only across models but also across time: both the reported standard deviations and 90% empirical ranges suggest that there are periods in which average leverage is magnified and the sign of the weights changes frequently. Such a pattern is in line with what has been systematically documented in studies on linear predictability (e.g., Brandt and Santa-Clara, 2006, and Fugazza et al., 2009, when real estate is included). In contrast, the Gaussian IID models produce the least volatile allocations despite the non-negligible implied leverage. This derives from the fact that time variation in weights derives in this case only from the updating of the sample estimates and not from the fact that the investment opportunities are time-varying.

Figure 1 facilitates our understanding by plotting the dynamics over the OOS 2004-2014 sample of the optimal weights (for H = 1 and 60 months) and hedging demands of the best performing VAR (the top row in each panel in Table 4), as well as for an IID myopic model. We derive three takeaways from Figure 1. First, while a somewhat erratic behavior tends to characterize the whole period, peaks and troughs are most visible during the 2008-2009 financial crisis. Interestingly, the best-fitting VAR(1) detects the upcoming collapse of the real estate sector and suggests to the investor to massively short REIT already by September 2008. Second, hedging demands are rather stable except for two spikes of opposite sign characterizing stocks and government bonds, occurring during the crisis. Third, as one would expect, IID myopic demands are very stable and relatively close to zero.

Much of our discussion so far has concerned the case of γ = 5, but the same qualitative insights also apply to the recursive OOS results obtained assuming either γ = 2 or γ = 10.[25] The most notable differences concern the fact that while the sign of the average total portfolio allocations to the five asset classes is preserved across the three values of γ, the size is directly proportional to the investor's risk tolerance ($1/\gamma$). More conservative investors (γ = 10) are generally less levered and tend to tilt their portfolios towards long-term government bonds while shunning stocks and REITs. Mildly risk-averse investors (γ = 2) instead attach large and

[25] These results are not tabulated here due to space considerations but are available in an Internet Appendix, or from the Authors upon request.

positive weights (usually above 100%) to these risky assets, also borrowing in the corporate credit market. Dispersion measures are also proportional to the investor's risk tolerance, suggesting bigger spikes in the time series of portfolio weights for less risk-averse investors.

4.3 *Realized portfolio performance and optimal allocation*

Table 5 presents the realized performance measures obtained in the recursive OOS experiment for an investor with intermediate risk aversion (γ = 5).[26] The top panel analyzes the recursive rebalancing case while the bottom panel pertains to the buy-and-hold strategy. The table reports the annualized mean, volatility, Sharpe ratio and CER of the portfolio rules implied by the VARs and the Gaussian models. The last two columns report the skewness and kurtosis of realized portfolio returns, thereby providing a more comprehensive view of the realized distribution of returns. The realized CER rankings in Table 5 are used to determine the best-performing model against which we benchmark the marginal contributions of HF strategies.

Table 5 shows that an investor who chooses to follow a simple buy-and-hold strategy attains, on average, lower CERs than an investor who rebalances on a monthly basis. In fact, only the two Gaussian IID models yield positive CERs (2.26% for the expanding and 2.14% for the rolling window) when a buy-and-hold strategy is pursued, while the best-performing VAR produces a whapping CER of –50.70%; in the latter case, the investor would rather pay a high premium than be forced to use this model in a long-run SAA. This is expected: a fixed proportions buy-and-hold strategy is indeed optimal only when investment opportunities are constant. This result is driven by the high negative skewness and high positive kurtosis of realized portfolio returns, which are fully taken into account by the investor's power utility function.

Whereas buy-and-hold strategies optimized on the ten best VARs result in average negative performances with limited volatility, monthly rebalancing enables the investor to substantially improve the CER, albeit at the expense of higher volatility as reflected in the wider 90%

[26] Tabulated results for γ = 2 and γ = 10 are in the Internet Appendix. Our findings are robust across different levels of the relative risk-aversion.

bootstrapped confidence intervals.[27] The top panel of Table 5 lists two VAR models that have positive CERs: an expanding window VAR(1) that includes the term spread, the short-term rate, and the dividend yield as predictors, and a rather similar VAR(1) in which the short-term rate is replaced by the default spread. The former model generates an annualized CER of 13.68%, whereas the latter produces a CER of 3.38%. These VAR models outperform the Gaussian IID benchmarks (estimated both under expanding and rolling window schemes), which rank third and fourth. Within a no-predictability framework, the investor would pay either 1.70% (rolling window scheme) or 0.79% (expanding) to allocate her wealth according to the rule in (14). These findings generally align with linear predictability OOS studies on US data (see, e.g., Brennan, Schwartz and Lagnado, 1997; Guidolin and Hyde, 2012). An in-depth analysis of the weights and implied portfolio returns from the best-performing VAR reveals that its high CER may stem from the stable increase in wealth and corresponding consumption flows produced by the optimal investment rule applied. Monthly negative returns tend to concentrate at the beginning of the OOS period and to be relatively small (when compared to other VARs) with a minimum value of –26% in May 2004. Interestingly, between July 2008 and August 2009, this strategy yields positive returns because it is able to profit from the collapse of the real estate sector by placing short bets on REITs. Overall, the leverage underlying this strategy does not generate too high a dispersion among returns as evident by the realized kurtosis of 2.70.[28]

In light of this evidence, we study next the benefits of diversification into HFs within the monthly rebalancing scheme. The best-performing model against which we benchmark the marginal contribution of the HF strategies as potential new asset classes are given by the expanding VAR(1) that includes the term spread, the short-term rate, and the dividend yield as predictors (for $\gamma = 2$ and $\gamma = 5$), and a leaner rolling VAR(2) with the dividend yield as the only

[27] The 95% confidence intervals are calculated by means of a block bootstrap technique with a block size equal to 12 monthly observations and 10,000 simulated paths.

[28] Especially in the case of $\gamma = 2$, we find that several models (among the 192 entertained) generate, at least on a single month, negative returns in excess of –100%, which implies total loss of the invested wealth and zero consumption in the following months. To guard against this scenario, we have modelled a stop loss at 0.01: that is, the investor can lose at most 99% of her wealth. In practical terms, the motivation behind this assumption can be related to Federal Reserve's Reg T. As expected, such mechanism contributes to the positive skewness reported in Table 5.

predictor for the highly risk-averse investor (γ = 10).

5 Main results: portfolio selection extended to hedge fund strategies

5.1 *Linear predictability of hedge fund returns*

Table 6 exemplifies our procedure by presenting parameter estimates and the residual correlation matrix for the best-performing VAR model for γ = 5 as specified in Table 3 now extended to include, for starters, excess returns on the relative value HF strategy (RVR).[29] The model is a VAR(1) estimated using an expanding window scheme on a sample up to December 2014 and includes six predictors: the term spread, the short-term rate, the dividend yield, the S&P 500 BuyWrite index returns, HML, and momentum. Table 6 reports the bias-adjusted estimates, the original OLS estimates, and the associated *t*-statistics. Bias adjustment plays an important role in this extended asset space, just as it does in the baseline menu. This is particularly evident from the dividend yield coefficients in the equations for the risky excess returns: for example, in the excess stock returns equation, the biased coefficient is three times the bias-corrected coefficient. We expect that such a reduction in value (relative to the biased estimates), when combined with the negative residual correlations between unexpected stock returns and unexpected changes in the dividend yield, would translate into a slower rate of mean-reversion which, in turn, is likely to command smaller hedging demands for this asset. As a result, correcting for the bias makes stocks less attractive for hedging intertemporal stochastic changes in their own future returns. While a similar effect is generated by bias-adjusting the dividend yield coefficient in the RVR_{t+1} equation, the opposite effect is obtained in the $Corp_{t+1}$ and $REIT_{t+1}$ equations, where the speed of mean-reversion grows. The unadjusted OLS coefficients for the term spread are also heavily biased, especially in the case of government and corporate bond excess returns. Focusing on the RVR_{t+1} equation, which also flaunts the highest R^2 among the risky assets equations, now also the lagged stategy excess return imply a positive and statistically significant coefficient. This finding may be explained by

[29] This procedure is repeated for each of the ten hedge fund strategies and the three values of the relative risk-aversion coefficient.

positive serial correlation stemming from exposures of the hedge strategy to securities that are not actively traded or with discontinuous market prices, as documented by a literature since at least Getmansky et al. (2004). Interestingly, the returns on the BuyWrite strategy seem to be explained, at least partially, by past relative value returns.

5.2 *Strategic asset allocation: total and hedging demands*

We now compute monthly recursive OOS portfolio weights for the extended asset menu. Similarly to Section 4.2, we discuss in some details the results for an investor wit intermediate risk aversion (γ = 5) who allocates her wealth across 1-month T-bills, stocks, long-term government and corporate bonds, REITs, and a HF strategy. To individually assess the economic value of each of the HF strategies, we include them in the asset menu one at the time, from a starting pool of ten strategies.[30] For each of the resulting ten portfolios, Tables 7 through 9, as well as additional tables in an Internet Appendix, report sample means, standard deviations, and the 90% realized range for monthly recursive OOS portfolio weights for the ten models that provide the investor with the highest CERs when the asset menu is expanded to each of the ten strategies, one at the time.

Comparing Tables 7-9 with Table 4, we note that the inclusion of HF strategies in the portfolios prompts an overall increase in optimal leverage as compared to the allocation under the baseline asset menu. However, this gearing affects only 1-month T-bills and the HF strategy: large short weights in the former are offset by large positive weights in the latter. The other four risky assets are affected only to a small degree relative to the baseline portfolio in Table 4, even though the extended portfolios are on average not as long and in some cases even short in stocks and REITs. While under the baseline asset menu government bonds are shorted to finance long positions in corporate bonds, the opposite happens with three out of the ten HF

[30] Tabulated results for investors with γ = 2 and γ = 10 are available from the authors upon request. Much of the discussion for γ = 5 applies to the cases of γ = 2 and γ = 10 with the same caveat: the average portfolio leverage and standard deviations turn out to be proportional to risk tolerance ($1/\gamma$). Interestingly, at least in the case of γ = 5 and 10, the best models give positive weight to hedge strategies across the board of our OOS period, which allows us to skim over the fact that a few strategies—barring their outright replica (see, e.g., O'Doherty et al., 2016)—would be hard to short because HFR indices are not (always, reliably, see Getmansky et al., 2015) traded as shortable exchange traded notes.

strategies investigated, i.e., in the case of fixed income relative value (Table 9), event driven, and distressed restructuring strategies. Mean hedging demands for HFs are usually negative because their returns show high first-order positive serial correlations: in other words, this alternative asset class cannot be used to hedge intertemporal stochastic variations in the very opportunities they offer. The resulting term structure of risk is positively sloped so that short-horizon investments in HFs may be perceived as less risky than long-horizon investments. Fund of funds, equity market neutral and merger arbitrage strategies are the only strategies showing positive, albeit modest, hedging demands.

In some additional detail, Table 7 shows the results for an investor who is allowed to trade the HFRI Fund Weighted Composite Index (FWC) along with the baseline menu of assets. This is of course key evidence, because it may be argued that FWC represents a weighted average return for the whole HF industry. In the long run, the optimal weights to this alternative strategy are large and positive, often in excess of 200% of the total wealth. Still, such allocation is inferior to the short-horizon weights due to a negative intertemporal hedging demand. Positions in the other five assets are relatively balanced, even though the investor needs to short T-bills and government bonds (occasionally also stocks) to finance positions in HFs exceeding 100%. Of course, HFs themselves may then provide (especially when their market beta is positive, as often found in the empirical literature, see e.g., Patton, 2008) exposures to equity and Treasury-bond risks, besides other types of tail risk-type and non-linear exposures, as documented by Agarwal, Arisoy and Naik (2018). For instance, based on the best-performing model (AR(1) with an expanding window), a long-horizon investor should build a portfolio which, on average, is long 280% in the HF strategy, 65% in REITs, 50% in corporate bonds and 59% in government bonds, and is short 230% in T-bills and 123% in stocks. Such an average allocation is rather stable over the entire OOS period with the exception of a few spikes during the financial crisis, see Figure 2. The negative weights of stocks most likely correspond to the net positive weights of the composite overall HF index, as recently argued by Ang (2014).

When fund of HFs (FFP) is the strategy selected to be tested in addition to the baseline asset portfolio, Table 8 shows that a long-horizon investor should, on average, refrain from allocating her wealth to funds of funds. Total portfolio demands for the other five assets are similar to the baseline case, although in this case leverage is obtained by shorting long-term government bonds rather than T-bills and corporate bonds. The best-performing model is still an expanding autoregressive model and it now requires the investor to buy corporate bonds (89%) and REITs (78%) by borrowing 1-month T-bills (23%), stocks (14%) and government bonds (27%). While the implied average leverage is low, portfolio weights change frequently during the OOS period (see Figure 3). The insight that funds of funds would fail to create positive economic value and as such should be avoided by rational, long-horizon investors echoes earlier findings by Amin and Kat (2003a) and Liang (2004).

Table 9 presents results for the fixed income relative value/arbitrage strategies (RVR). This strategy has been selected and reported in detail because it implies very large optimal weights to be assigned on average to the strategy, almost entirely financed by short positions in 1-month T-bills and to some extent corporate credit. Because a short-horizon investor ought to increase the weights to RVR by an additional 25% at least, this implies a negative hedging demand that is due to the high first-order serial correlation of RVR excess returns (0.50) which, combined with a positive slope coefficient in the VAR equation, translates current negative innovations in RVR into lower predicted returns. This effect is strong enough to dominate the positive hedging demands generated by the remaining six predictors. The best-performing VAR (which has six predictors, i.e., the term spread, the short-term rate, the dividend yield, the BMX, the HML and the momentum factors) is the top performer in terms of realized CER among not only the 17 models estimated for the RVR strategy, but also as compared with all models of all HFs investments. According to this rich VAR, a long-horizon investor should allocate 882% of her wealth to RVR, 114% to government bonds, –740% to 1-month T-bills, –9% to stocks, –118% to corporate bonds and –29% to real estate.

As expected, the inclusion of Global Macro HFs (MAC) reduces exposures to stocks, REITs, and corporate bonds at both long and short horizons, as this strategy tends to also use traditional asset classes to take positions supported by macro views.[31] Hedging demands are on average negative, indicating an increasing term structure of risk. Under the best-performing VAR, which includes the three baseline predictors and Fung and Hsieh's (2004) currency trend-following factor, the weights change rather erratically over time, although they fluctuate within relatively tight ranges. For an investor wishing to add equity hedge strategies (EQH) to her portfolio, the equity exposure is given by the sum of the weights to this alternative asset class and stocks. Allocation to stocks is, on average, almost negligible and is used primarily to compensate large negative positions in EQH during certain periods.

In the case of event driven strategies (EVD), the optimal investment in this alternative asset class are large and positive, often in excess of 100% of the total wealth, and short positions are frequently assumed in T-bills, stocks, corporate bonds, and REITs. In the case of merger arbitrage (MEA), the best-performing model is the no-predictability benchmark, which implies that both short- and long-horizon investors should follow the same portfolio rules. Optimizing investors go long in MEA, REITs and government bonds while shorting T-bills, stocks and corporate bonds. These allocations are the least volatile over the entire OOS period, as one would expect for a Gaussian IID model.

When distressed/restructuring strategies (DSE) are added to the baseline asset menu, the best-performing VAR is represented by an expanding AR(1) model, and the optimal portfolio weights call for going long in DSE, government bonds, and REITs; these positions are also financed by shorting T-bills, stocks, and corporate bonds. Diversifying into convertible arbitrage strategies (COA) results in relatively small portfolio leverage on average, although the weights as well as the level of leverage vary substantially over time. Relatively small average, albeit varying over time, leverage is also derived in the case of equity market neutral HFs (EMN). The best-performing model in this case recommends that a long-horizon investor builds a portfolio which, on average, is long 48% in EMN, 9% in REITs, 73% in stocks and 106% in corporate bonds, and short 15% in the risk-free asset and 121% in government bonds.

[31] Tabulated results and plots of optimal portfolio weights for Global Macro, Equity Hedge, Event Driven, Merger Arbitrage, Event Driven Distressed/Restructuring, fixed income Relative Value and Convertible Arbitrage and Equity Hedge Equity Market Neutral strategies are reported in an Internet Appendix.

5.3 *Realized portfolio performances and the economic value of hedge funds*

The key set of results concerns the realized OOS performance of the extended portfolios for each of the three levels of relative risk aversion. For each of the 10 HF strategies under analysis, we have analyzed a total of 18 models. These include the Gaussian IID model, the purely autoregressive model, and the 16 VARs which can be built for all possible combinations of the four new predictors tailored to each HF strategy. Through a comparison of the estimated CERs of these 180 models with the CER of 13.68% obtained from the best performing model applied to the benchmark asset menu, we are able to determine whether or not an allocation to HFs is attractive to a long-term investor who is already well diversified across a broad spectrum of classical asset classes. In the following, we do not formally test whether HF-strategies, as a whole, dominate traditional portfolios: this would be subject to a clear multiple, overlapping hypotheses testing problem best solved adopting model techniques from model confidence set estimation and reality check testing in econometrics. Instead, we ask a more modest question which, however, represents a necessary condition to the measurement of the economic value of HFs to investors: do HF strategies exist that can generate risk-adjusted performances in excess of more traditional strategies?

When the composite, value-weighted basket of HF strategies is added to the baseline menu of an intermediate risk-averse investor, the best-performing VAR is represented by an expanding autoregressive model that provides an annualized CER of 19.89% (Table 10). This is a very large estimate that exceeds the effective costs of investing in hedged funds that have been estimated in the literature (see Ibbotson et al., 2011, for a discussion and estimates). Skewness and kurtosis are equal to 0.64 and 3.05, respectively, while the annualized mean return and volatility are 8.48% and 164.75%. Because of the monthly rebalancing, even though HFs returns per se are characterized by a volatility that is inferior to equity markets, the resulting market timing strategy yields rather risky realized OOS performance, at least in terms of recorded second moment. Note, moreover, that because $8.48/(164.75/(132)^{1/2}) = 0.58$, this realized mean performance per se is not significant in a statistical sense; in fact, the empirical 90% confidence region for the mean includes zero. Yet, the lower bound of the 90% confidence region for the CER is a hefty 18.70%. This model leads to a low Sharpe ratio (0.04 vs 0.40 under the benchmark) but compensates a power utility investor with a positive skewness and zero excess kurtosis (vs. -0.87 skewness in the absence of HFs, see Table 5). In some ways, HFs make a "lottery ticket component" available to positively skew the realized performance while modestly inflating the realized tails; yet, there is a steep cost to be paid in terms of lost Sharpe ratio, differently from the classical sales pitch in favor of HFs. This result is consistent with Amin and Kat's (2003b), who, however, do not take predictability into account and fail to consider expected utility-maximizing portfolios.

In Table 10, there is a clear pattern among the 18 models entertained with reference to this HF style: leaner models provide the highest welfare measures. Three models produce CERs above the baseline 13.68%, suggesting that the investor can profit from investing in FWC, and this is in spite of the implied realized variance, which remains high because of the portfolio turnover. Yet, the median CER across all expanding VAR remains negative, -6.94% per year: picking at random some predictability model to be applied to an asset menu that includes a HF index will not bring a positive risk-adjusted performance to investors. Because this strategy is a weighted basket of a number of HF strategies, we can expect to find specific strategies in this index to give both higher and smaller CERs vs. this best-performing model for FWC.

The results in Table 11 suggest instead that a long-horizon investor should refrain from investing in funds of funds (FFP) strategies, in line with the bulk of the literature (see, e.g., Amin and Kat, 2003a). The highest CER (9.22%), obtained by an expanding AR(1) model, is approximately 450bp below the baseline threshold and characterized by an almost nil Sharpe ratio. The median strategy keeps delivering a negative CER (-6.23% per year). Notably, the table shows that adding funds-of-funds to the menu of choice stabilizes the performance (both in terms of realized variance and kurtosis) and induces some degree of positive skewness; however, presumably because of their double layer of fees, the realized mean is considerably penalized, to the point that realized Sharpe ratios and CERs decline vis-à-vis Table 5.[32] On the

[32] However, as γ increases, an investor will care less for the mean and more for the spread of the resulting distribution of realized portfolio returns (hence, wealth and consumption flows), so that that

contrary, and as an example of one specific type of strategy, making an investment in the RVR index provides substantial economic value to an investor regardless of the model selected, except for the no-predictability benchmark (Table 12). While the lowest-ranked VAR model yields a CER of 13.35%, the highest-ranked model provides a CER of 32.38% — the largest value among all HF strategies tested in this paper (which is why RVR is presented in detail) and largely exceeding the baseline asset menu. The best-performing model produces a mean annual return and volatility that are above the median values (15 and 35% per year), while the resulting Sharpe ratio (0.23) is below the median (0.37). Table 12 shows the usual mechanics through which a hedge strategy produces economic value: by reducing realized volatility and kurtosis and generating positive skewness in returns. The only difference is that RVR does that in a large enough magnitude to generate higher CER in net terms. In fact, the median across expanding VAR models in this case leads to an 18.1% CER, with 90% empirical confidence bounds of 15.5 and 24.4 percent per year, so that even the *median* lower bound outperforms the *best* investment scheme under the baseline asset menu.

Figures 4 and 5 collect the main results in Tables 5 and 10-12 concerning the comparison of economic value estimates obtained with and without HF strategies, and extend our presentation of results to all strategies.[33] Figure 4 compares the CER, mean returns, Sharpe ratios, skewness, and kurtosis of the top performing model for each hedge strategy, and plots them against the benchmark in Table 5. Figure 5 performs the same comparison with reference to median statistics for the expanding sample VAR models, selected because they represent the top performing model in Table 5. In the top portion of Figure 4 we see that—if investor were able to detect top-performing models for the prediction of risk premia on the different asset classes—most strategies and, as a result, also the composite HFR index would outperform a classical asset menu on a risk-adjusted basis; even taking the resulting sample uncertainty into

the distance between the top CERs of the baseline asset menu and those obtained including FFP strategies shrinks. We can speculate that for very high values of γ, FFP may stabilize performance so much that it generates positive economic value.

[33] Tabulated results for Global Macro, Equity Hedge, Event Driven, Merger Arbitrage, Event Driven Distressed/Restructuring, Relative Value Fixed Income Convertible Arbitrage and Equity Hedge Equity Market Neutral strategies are available in the Internet Appendix.

account, the only exceptions are the equity long-short, equity market neutral, and the fund-of-funds strategies. The fixed income convertible arbitrage and relative value strategies give particularly strong results. Generally, strategies with highly nonlinear payoff, such as RVR, MEA, DSE and COA, are advantegeous. Interestingly, most of the realized economic value fails to result from a mean-variance order improvement: in fact, when combined with classical assets, most (all) HF strategies yield realized mean returns (Sharpe ratios) that are inferior to the benchmark portfolio. For instance, while classical assets only lead to an annualized mean return of 3.8% per month and a monthly Sharpe ratio of approximately 0.17, when the HFR composite index becomes available (and as we saw in Table 7, it will be heavily demanded), the realized annual mean return declines to 0.8% and the Sharpe ratio is barely positive. However, HF strategies grossly improve the higher-moment properties of the optimal portfolio: skewness increases from -0.87 to +0.80 and kurtosis stays essentially constant at just below 3. It turns out that a long-run investor with $\gamma = 5$ cares enough for the shape of the entire density of realized performances to considerably tilt her allocation towards HFs because these buy positive skew and hence chances of high, right-tail performances without inflating the overall thickness of the tails of the distribution. Mechanically, this is possible only because the resulting portfolio weights in Table 7 become sufficiently extreme and time-varying to increase at the same time the resulting portfolio variance, which explains why the Sharpe ratio declines.

One tricky issue in this story is the difference between ex-ante moments (more generally, predictive density of realized consumption flows from the cumulative wealth process) and realized, ex-post moments from OOS backtesting. Although separate calculations confirm that a $\gamma = 5$ tilts her portfolio selection away from classical fixed income securities and towards hedge strategies in the way described to trade-off less mean, more variance, a lower Sharpe ratio, in exchange for higher skewness and even lower kurtosis on an ex-ante basis, this shift appears to occur on a more aggressive tone in terms of ex-post realized performance measure than an investor had budgeted for. In particular, the drastic reduction in Sharpe ratio deriving from the excess realized variance recorded by a dynamic portfolio that includes diversified hedge strategy positions seems to be also a product of VAR model specifications. In other words, an investor appears to achieve the desired positive skewness, but she also pays a price in terms of reduced Sharpe ratio that is ex-post excessive. Additionally, the decline in kurtosis that ex-ante an investor would be looking for, does not seem to fully materialize, even though in Figure 4 the realized kurtosis of most HF strategies is inferior to the benchmark.

Finally, Figure 4 shows that in terms of ex-post OOS CER, the two best strategies are fixed income relative value (RVR) and fixed convertible arbitrage (COA). They confirm the pattern that, especially ex-post, both strategies disappoint in the mean-variance space, as RVR leads to a sub-par monthly mean (approximately 2%) and both imply disappointing risk-reward ratios (0.06 and 0.10 on a monthly basis, respectively). Yet, they imply strong and precisely estimated ex-post realized skewness of almost 1, exceeding that of the composite index. By the same token, it is clear why funds-of-funds disappoint at least ex-post: they reduce mean returns and hence Sharpe ratios without bringing about much of an improvement in asymmetry.[34]

In unreported plots, we have found evidence similar to Figure 4 for $\gamma = 2$ and $\gamma = 10$.[35] For $\gamma = 2$, only three VARs out of 180 yield a positive CER when hedge strategies are made available. The two best-ranking models are obtained with the inclusion of RVR strategies, while the third is obtained with COA strategies. With a CER of 14.11%, the best-performing model is an expanding VAR(1) that includes the term spread, the short-term nominal rate, the dividend yield, and HML factor as predictors. RVR is also ex-post preferred to be included in the asset menu in the case of $\gamma = 10$: under a rolling window VAR(2) that bases its risk premia forecasts on the dividend yield, BMX returns and HML, a long-term investor would have achieved a realized OOS CER of 43.57%. In fact, in the case of $\gamma = 10$, RVR allows the investor to gain additional risk-adjusted returns, relative to the baseline asset menu, under almost all VARs used to capture time-varying investment opportunities. The same highly risk-averse investor can also improve her realized utility by investing in EVD, DSE, and COA strategies. Thus, there is clear evidence that the preference for highly nonlinear payoffs found in the case of $\gamma = 5$ is

[34] In the case of MEA strategies, the best model in Figure 4 is the no-predictability benchmark. More generally, the best model within each strategy for $\gamma = 5$ is represented in four cases by an autoregressive model (FWC, FFP, EVD, DSE) and otherwise it is an expanding VAR (COA, MAC, EQH, RVR, EMN).

[35] These tables are available upon request from the authors.

robust across different degrees of relative risk-aversion. Because both funds-of-funds strategies and the HFR composite index are weighted combinations of individual strategies, correspondingly we find that while for a γ = 2 investor they fail to generate any economic value, for γ = 10 they generate large and robust improvements in risk-adjusted performance. This finding is coherent with Agarwal and Naik's (2000) result that—within a portfolio comprising of passive asset classes and investments in nondirectional HF strategies—the relative importance of passive and alternative portfolios changes as one shifts down the risk-return trade-off towards the minimum variance portfolio as the weight of the equity class decreases, that of bonds and HF strategies increase; within HFs the weight of the directional strategies falls while that of the non-directional strategies rises.

However, before rushing to a conclusion that for intermediate and highly risk-averse investors HF strategies are appealing investment opportunities, Figure 5 (that refers again to a γ = 5 investor) offers a sobering view. Indeed, Figure 4 has been built under the unrealistic assumption that an investor would know in advance what the best performing model (in terms of realized CER) would have to be ex-post. In reality, this is hardly the case: while academics have been heatedly debating whether there is any exploitable predictability in financial returns, a fortiori we know much less about what model could represent the "right one" on an ex-ante basis. Figure 5 has the same structure as Figure 4 but it reports the realized OOS performance of the median prediction model for asset risk premia (including, as a special case, the IID no predictability model). Picking at random "some model" an investor would have not fared so well unless she had known—once more, rather unrealistically—which specific hedge strategy to pick. On the one hand, betting on the composite HFR index or on a fund-of-fund strategy leads to median realized CERs that are negative and therefore dominated by the simplest of the portfolio strategies: 100% in cash at all times. On the other hand, while the realized CERs of the median predictability model are rather promising for a few hedge strategies because positive and exceeding the performance of the median model applies to the classical asset menu (this is particularly obvious for RVR, MAC, MEA, DSE, and COV), other strategies lead to a non-positive CER (EQH and EMN). It seems that not fine-tuning the predictability model hurts in particular the strategies that simply trade equities. Interestingly, and contrary to the best models in Figure 4, in the case of the median predictability models, when the economic value created is positive, this comes almost entirely from an improvement in realized mean returns for approximately comparable realized volatility and therefore from higher realized Sharpe ratios vs. the benchmark; some additional contribution to high and positive CERs in this case comes from reductions in realized kurtosis vs. the benchmark, while the contribution of (non-negative) skewness to the realized utility of investors is limited at best.

6 Discussion and conclusions

We report systematic, out-of-sample evidence on the potential economic benefits of diversifying into various HF strategies accruing to a long-term, risk-averse investor, who is already well-diversified across stocks, REITs, and government and corporate debt. We have obtained the optimal weights and consumption rules using the approximate analytical solution of Campbell et al. (2003) while adjusting the underlying VAR estimates for small-sample biases following Engsted and Pedersen (2012). In a range of recursive OOS experiments, we have estimated the CERs for a range of models that can capture predictability in the risk premia of the asset classes under investigation to assess which HF strategies, if any, lead to an improved realized OOS utility relative to a baseline asset menu.

We found similar patterns with respect to optimal portfolio weights as the ones documented in earlier studies on the predictability of returns (e.g., Brandt and Santa-Clara, 2006, and Guidolin and Hyde, 2012): average portfolio leverage is high and tends to be magnified in periods of crisis, while weights fluctuate widely. The inclusion of HF strategies prompts a further increase in leverage through short positions in 1-month T-bills to finance relatively large and persistent long positions in hedge strategies. The small-sample bias correction has a sizable effect on total and hedging demands as well as on the speed of mean-reversion implied by the estimated VAR models. Hedging demands for HFs tend to be negative due to the high positive first-order serial correlation in the returns.

Our OOS experiments within the baseline asset space show that an investor who chooses to follow a simple buy-and-hold strategy achieves, on average, lower CERs than an investor who rebalances on a monthly basis. In the case of the extended menu, our OOS experiments show that not all HF strategies have the potential to benefit long-term investors. Only strategies whose payoffs are highly nonlinear (relative value, merger arbitrage, distressed restructuring, convertible arbitrage), and therefore not easily replicable (by going long or short in the original asset classes) yield the highest utility gains. Our key results are robust across different values of the coefficient of relative risk aversion, although the highest benefits from diversifying into HFs accrue to medium and highly risk-aversion investors. Interestingly, the key findings on the value of HFs stem not from the ability of hedge strategies to increase realized mean returns, lower volatility, and therefore improve realized Sharpe ratios (as often debated by financial commentators), but from the ability of hedge strategies—when combined within well-diversified portfolios of stocks, bonds, and REITs—to improve the higher-order moments of optimal portfolios (i.e., higher skewness and lower excess kurtosis). However, a portion of the findings that turn out to be encouraging for an assessment of HF performance critically hinges on the assumption that investors can accurately detect the best performing model for predictable risk premia. When we assess the OOS performance of the median model of predictable returns (or lack thereof), we find that only specific HF strategies may still generate economic value, while composite value-weighted portfolio or strategies (as well as funds-of-funds) fail to do so.

Our analysis has several limitations. This paper has ignored transaction costs. As Balduzzi and Lynch (1999) pointed out, their presence decreases the utility gain from exploiting asset return predictability in portfolio choice. In other words, the performance of all the optimized portfolios (both with and without HFs) may be overstated. Even though it is unclear whether such a bias may also result in an overstament of the economic value of HF strategies (as also whether the baseline strategic asset allocation is affected), it remains the case that it would be interesting—although computationally challenging, given the long-horizon, discrete-time nature of the massive sequence of recursive problems solved—to extend this paper to contrast the cases with and without transaction costs. Moreover, our key results do not seem to crucially hinge on the presence of predictability in asset returns, when the impact of transaction costs would be understandably higher. For instance, comparing Tables 5 and 10 (with a 5-year buy-and-hold horizon), we note that the best benchmark model with no HF investments is indeed a no-predictability model which gives an annualized mean return of 3.22%, with a standard deviation of 5.04%, to yield a Sharpe ratio of 0.78 and a CER of 2.26% affected by a large negative skewness and positive excess kurtosis; when the HFRI composite index is made available, the welfare gain is visible: the annualized mean is 9.37% with a standard deviation of 10.97%, to yield a similar Sharpe ratio of 0.80 but a much higher CER of 5.44% that benefits from an almost zero skewness and negative excess kurtosis. The results, at least in terms of realized CER and higher-order moments, are similar for the no-predictability recursive strategies involving HFs in Tables 12 (relative value) and for many other strategies (see the Appendix). Insofar as removing predictability minimizes transaction costs, it seems our results may be reasonably robust to their explicit consideration.

Apart from not taking into account transaction costs, we have not imposed any short-sale constraints. Such a choice may be problematic in the case of HFs because it is difficult to short this asset class, although in practice short positions can be obtained through synthetically tracking portfolios (see the discussion in Hamza et al., 2006). Therefore, our results should be construed as a necessary condition, rather than sufficient, in favor of HF investing; in this respect, the evidence crucially hinges on the best models of predictable asset returns being able to be discovered and tested. Our own assessment of the state-of-the-art in the empirical finance literature tends to be lukewarm at best (see the review in Rapach and Zhou, 2013). Moreover, recent empirical work by Joenvaara, Kosowski and Tolonen (2014) induces us to maintain a healthy degree of skepticism: when they account for the investment constraints faced by real-world HF investors, they report a reduction in average performance and in performance persistence (see also Kumar, 2015).

Our findings should encourage further analyses along several dimensions. First, distinguishing between bull and bear regimes may generate optimal portfolios which yield superior performances relative to simple VARs (see, e.g., Guidolin and Hyde, 2012; Tu, 2010). For instance, Avramov et al. (2011) note that in times of crisis, some HF strategies (e.g., global macro) perform better than others (e.g., equity long/short). The widespread evidence of regimes in investment opportunities may affect our results. Second, the precision of our estimates, and therefore the quality of our forecasts, might be further improved if the exposures to state variables were allowed to be time-varying in the spirit of Bollen and Whaley (2009). Finally, along the lines of recent work by Panopoulou and Vrontos (2015), given the long set of candidate predictors suggested by the literature, we could construct improved HF fund return predictions by carefully integrating the information content through combinations of forecasts.

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Table 1

Summary Statistics for Asset Returns and Predictor Variables of the Dataset

The table presents summary statistics for monthly returns on stocks, bonds, publicly traded real estated, and HFR hedge fund strategy returns. The sample is January 1994 - December 2014. We use four predictors to model the time variation in investment opportunities, the dividend yield, the short-term riskless interest rate proxied by the 3-month Treasury constant maturity rate, the term spread calculated as the difference between the 10-year Treasury constant maturity rate and the corresponding 3-month rate, and the default spread computed as the yield differential between Moody's seasoned Baa and Aaa corporate bond portfolio rates. In the case of hedge funds, we use HFRI style indices distributed by Hedge Fund Research (HFR).

	Mean	Median	Std. Dev.	Uncond. Sharpe ratio	Minimum	Maximum	Skewness	Kurtosis	JB Test
Panel A: Initial Asset Menu									
30-day T-bill	0.220	0.214	0.180	0.000	-0.003	0.536	0.031	1.374	27.79**
Excess stock return	0.534	1.321	4.496	0.119	-18.893	10.749	-0.930	4.722	67.49**
Excess long-term govt. bonds returns	0.281	0.303	2.031	0.138	-6.749	8.515	0.063	4.120	13.35**
Excess long-term corporate bonds returns	0.388	0.557	2.621	0.148	-11.973	13.203	-0.271	7.422	208.41**
Excess real estate returns	0.613	1.169	5.675	0.108	-36.521	24.726	-1.630	12.974	1,156.27**
Default_spread (Baa-Aaa rate)	0.964	0.860	0.442	-	0.550	3.380	2.979	14.158	1,680.11**
Riskless Term spread (10y-3m)	1.743	1.845	1.168	-	-0.700	3.690	-0.191	1.901	14.21**
Short-term nominal rate (3m)	2.763	2.810	2.236	-	0.010	6.360	0.013	1.334	29.14**
Div_yield	1.893	1.837	0.467	-	1.085	3.759	0.895	4.299	51.32**
Panel B: Hedge Funds									
HFRI Fund Weighted Hedge Fund excess return	0.463	0.602	1.967	0.236	-9.503	6.995	-0.816	6.303	142.46**
HFRI Fund Of Funds Composite excess return	0.212	0.401	1.655	0.128	-8.165	6.249	-0.938	7.524	251.89**
HFRI Equity Hedge excess return	0.564	0.685	2.575	0.219	-10.015	9.951	-0.409	5.268	61.06**
HFRI Event- Driven excess return	0.565	0.900	1.902	0.297	-9.723	4.632	-1.423	8.153	363.86**
HFRI Macro excess return	0.382	0.343	1.805	0.212	-6.826	6.221	0.076	4.241	16.40**
HFRI Relative Value excess return	0.436	0.573	1.203	0.362	-8.451	3.856	-3.013	21.133	3,833.54**
HFRI EH Equity Market Neutral excess return	0.227	0.273	0.849	0.267	-3.067	3.149	-0.517	5.246	64.19**
HFRI ED Merger Arbitrage excess return	0.384	0.537	0.972	0.395	-6.260	2.720	-1.943	11.924	994.79**
HFRI ED Distressed/Restructuring excess return	0.511	0.713	1.783	0.287	-9.284	5.399	-1.584	9.223	511.90**
HFRI RV Fixed Inc.- Conv.Arb. excess return	0.382	0.630	2.035	0.188	-17.526	9.296	-3.358	33.650	10,337.68**
CBOE S&P 500 Buywrite Index	0.272	0.510	1.388	-	-7.125	4.145	-1.479	8.211	376.93**
Ptfs Currency Lookback Straddle	-0.848	-5.220	19.252	-	-30.130	90.270	1.331	5.508	140.50**
Ptfs Commodity Lookback Straddle	-0.237	-2.895	14.210	-	-24.650	64.750	1.093	4.728	81.50**
Ptfs Bond Lookback Straddle	-1.559	-3.900	15.258	-	-26.630	68.860	1.361	5.477	142.21**
Ptfs Short Term Interest Rate Lookback Straddle	-0.386	-5.615	26.022	-	-34.640	221.920	4.274	30.425	8,664.24**
SMB	0.120	-0.100	3.402	-	-18.272	20.147	0.388	10.422	584.74**
HML	0.156	0.120	3.229	-	-14.053	13.024	-0.228	6.311	117.32**
Momentum	0.300	0.553	5.460	-	-42.434	16.873	-2.562	20.564	3,514.80**

* Significance at 5%

** Significance at 1%

Table 2

Correlation Matrix

The table presents estimate correlations and reported significance levels for monthly returns on stocks, bonds, publicly traded real estated, and HFR hedge fund strategy returns. The sample is January 1994 - December 2014. The estimated pairwise correlations also involve the four predictors described in Table 1.

Panel A: Initial Asset Menu

	Rf	Stocks	Gov.	Corp.	REITS	DY	Def.	Term.	Short
Stocks	-	1	-0.175*	0.243**	0.587**	-0.080	-0.119	0.001	-0.023
Gov.	-	-	1	0.673**	-0.003	0.051	0.061	-0.003	-0.043
Corp.	-	-	-	1	0.354**	0.039	0.101	0.069	-0.102
REITS	-	-	-	-	1	-0.143*	-0.138	0.042	-0.048
DY	-	-	-	-	-	1	0.453**	0.360**	-0.255**
Def.	-	-	-	-	-	-	1	0.327**	-0.487**
Term.	-	-	-	-	-	-	-	1	-0.769**
Short	-	-	-	-	-	-	-	-	1

* Significance at 5%

** Significance at 1%

Panel B: Initial Asset Menu vs. Hedge Funds

	FWC	FFP	EQH	EVD	MAC	RVR	EMN	MEA	DSE	COA	BMX	PtfsFX	PtfsCM	PtfsBD	PtfsIR	SMB	HML	Mom.
Stocks	0.823**	0.667**	0.831**	0.781**	0.359**	0.612**	0.325**	0.608**	0.633**	0.504**	0.869**	-0.198**	-0.175*	-0.252**	-0.285**	0.247**	-0.222**	-0.267**
Gov.	-0.194**	-0.154*	-0.206**	-0.237**	0.152*	-0.147*	-0.076	-0.147*	-0.247**	-0.091	-0.177**	0.119	0.081	0.233**	0.050	-0.195**	0.043	0.158*
Corp.	0.248**	0.244**	0.216**	0.248**	0.252**	0.358**	0.055	0.222**	0.221**	0.413**	0.257**	-0.091	-0.040	0.049	-0.229**	0.000	0.028	-0.128
REITS	0.464**	0.364**	0.465**	0.516**	0.142*	0.518**	0.176*	0.415**	0.481**	0.450**	0.583**	-0.153*	-0.157*	-0.150*	-0.199**	0.254**	0.275**	-0.318**
DY	-0.110	-0.179**	-0.139	-0.116	-0.072	-0.094	-0.157*	-0.042	-0.126	-0.041	-0.119	-0.016	-0.040	0.045	0.107	-0.045	-0.099	-0.147*
Def.	-0.093	-0.135	-0.125	-0.154*	-0.006	-0.076	-0.236**	-0.083	-0.166*	0.039	-0.144*	0.046	-0.035	0.028	0.121	0.064	-0.096	-0.227**
Term.	-0.024	-0.041	-0.083	-0.016	-0.036	-0.016	-0.148*	-0.151*	0.099	-0.027	-0.076	-0.028	-0.018	-0.019	-0.099	0.118	-0.036	-0.070
Short	0.048	0.035	0.101	0.047	0.045	0.001	0.175*	0.178**	-0.043	0.000	0.077	0.048	-0.017	0.034	0.133	-0.088	0.044	0.116

* Significance at 5%

** Significance at 1%

Table 2 (continued)

Correlation Matrix

Panel C: Hedge Funds

	FWC	FFP	EQH	EVD	MAC	RVR	EMN	MEA	DSE	COA	BMX	PtfsFX	PtfsCM	PtfsBD	PtfsIR	SMB	HML	Mom.
FWC	1	0.925**	0.965**	0.916**	0.621**	0.759**	0.477**	0.703**	0.804**	0.630**	0.680**	-0.133	-0.140	-0.260**	-0.353**	0.463**	-0.327**	-0.110
FFP	-	1	0.867**	0.851**	0.696**	0.753**	0.502**	0.628**	0.802**	0.633**	0.537**	-0.094	-0.085	-0.274**	-0.403**	0.395**	-0.262**	0.037
EQH	-	-	1	0.876**	0.537**	0.724**	0.507**	0.675**	0.750**	0.623**	0.696**	-0.152*	-0.153*	-0.240**	-0.354**	0.479**	-0.327**	-0.080
EVD	-	-	-	1	0.494**	0.817**	0.469**	0.772**	0.886**	0.683**	0.692**	-0.195**	-0.215**	-0.328**	-0.385**	0.428**	-0.145*	-0.202**
MAC	-	-	-	-	1	0.301**	0.322**	0.332**	0.400**	0.225**	0.228**	0.196**	0.181**	-0.032	-0.093	0.244**	-0.207**	0.120
RVR	-	-	-	-	-	1	0.439**	0.673**	0.833**	0.877**	0.602**	-0.291**	-0.253**	-0.354**	-0.448**	0.238**	-0.038	-0.205**
EMN	-	-	-	-	-	-	1	0.417**	0.446**	0.349**	0.257**	0.006	-0.081	-0.235**	-0.206**	0.143*	0.034	0.348**
MEA	-	-	-	-	-	-	-	1	0.603**	0.533**	0.627**	-0.106	-0.178**	-0.225**	-0.317**	0.289**	-0.076	-0.143*
DSE	-	-	-	-	-	-	-	-	1	0.710**	0.563**	-0.214**	-0.224**	-0.418**	-0.385**	0.355**	-0.039	-0.157*
COA	-	-	-	-	-	-	-	-	-	1	0.498**	-0.257**	-0.236**	-0.241**	-0.427**	0.172*	-0.010	-0.256**
BMX	-	-	-	-	-	-	-	-	-	-	1	-0.203**	-0.179**	-0.239**	-0.315**	0.130	-0.080	-0.272**
PtfsFX	-	-	-	-	-	-	-	-	-	-	-	1	0.353**	0.270**	0.256**	-0.017	0.008	0.119
PtfsCOM	-	-	-	-	-	-	-	-	-	-	-	-	1	0.190**	0.228**	-0.071	-0.032	0.189**
PtfsBD	-	-	-	-	-	-	-	-	-	-	-	-	-	1	0.211**	-0.077	-0.075	0.019
PtfsIR	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.107	-0.005	0.000
SMB	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.360**	0.054
HML	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	-0.153*
Mom.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1

* Significance at 5%
** Significance at 1%

Legend: DY (dividend yield), Def. (default spread), Term. (riskless term spread), Short (short-term 3-month nominal rate), FWC (HFRI Fund Weighted Composite Index), FFP (HFRI Fund of Funds Composite), EQH (HFRI Equity Hedge), EVD (HFRI Event Driven), MAC (HFRI Macro), RVR (HFRI Relative Value), EMN (HFRI EH Equity Market Neutral), MEA (HFRI ED Merger Arbitrage), DSE (HFRI ED Distressed/Restructuring), COA (HFRI RV Fixed Income Convertible Arbitrage), BMX (CBOE S&P 500 BuyWrite Index), PtfsFX (portfolio of lookback straddles on currency), PtfsCom (portfolio of lookback straddles on commodities), Ptfs BD (portfolio of lookback straddles on bonds), PtfsIR (portfolio of lookback straddles on interest rates), SMB (small-minus-big), HML (high-minus-low), Mom. (momentum),

Table 3

Full Sample (1994:01–2014:12) Estimates of Full VAR(1): Baseline Asset Menu

The table presents full-sample OLS estimate of a VAR(1) model

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1},$$

where $\boldsymbol{z}_{t+1}$ collects the short rate, benchmark excess returns, and the predictors, $\boldsymbol{\Phi}_0$ is the $(n+m)$ vector of intercepts, $\boldsymbol{\Phi}_1$ is the $(n+m)$x$(n+m)$ coefficient matrix, and $\boldsymbol{v}_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \sum_{\boldsymbol{v}})$. We take into account the instability in parameters and adjust the estimates for small-sample bias as in Engsted and Pedersen (2012), by using the formula $\widehat{\boldsymbol{\Phi}}_1$ as $\boldsymbol{Bias}_T = -\frac{\boldsymbol{b}}{T} + O\left(T^{-\frac{3}{2}}\right)$.

Dependent variable	Rf_t	$Stocks_t$	Gov $Treas_t$	$Corp_t$	$REITs_t$	Def_t	$Term_t$	Short $Rate_t$	DY_t	R^2
Rf_{t+1} - adj.	-0.068	0.000	-0.001	0.001	0.000	0.000	-0.005	0.083	-0.003	
Not adjusted	-0.082	0.000	-0.001	0.001	0.000	0.000	-0.005	0.084	-0.003	0.988
t-stat	(-1.29)	(-0.84)	(-1.04)	(0.67)	(0.93)	(0.04)	(-3.49)	(15.80)	(-0.75)	
$Stocks_{t+1}$ - adj.	-3.122	0.087	-0.033	0.191	-0.026	-1.905	-0.558	-0.203	0.917	
Not adjusted	-2.379	0.085	-0.036	0.189	-0.025	-1.987	-0.664	-0.215	1.890	0.057
t-stat	(-0.18)	(0.87)	(-0.11)	(0.60)	(-0.30)	(-1.80)	(-2.01)	(-0.20)	(2.77)	
Gov $Treas_{t+1}$ - adj.	-1.427	-0.079	-0.013	0.078	-0.063	-0.169	0.196	0.070	-0.210	
Not adjusted	-1.538	-0.080	-0.019	0.077	-0.062	-0.075	0.354	0.215	-0.262	0.096
t-stat	(-0.35)	(-2.20)	(-0.20)	(0.83)	(-1.75)	(-0.17)	(2.07)	(0.61)	(-0.83)	
$Corp_{t+1}$ - adj.	0.740	0.035	0.021	0.139	-0.079	0.432	0.319	-0.070	-0.317	
Not adjusted	0.653	0.037	0.030	0.121	-0.077	0.666	0.497	0.092	-0.151	0.064
t-stat	(0.12)	(0.80)	(0.16)	(0.63)	(-1.26)	(0.89)	(2.21)	(0.22)	(-0.41)	
$REITs_{t+1}$ - adj.	-12.826	0.257	0.186	0.339	-0.096	-1.496	0.407	0.847	-0.979	
Not adjusted	-12.550	0.270	0.185	0.343	-0.111	-1.432	0.365	0.904	-0.181	0.094
t-stat	(-0.93)	(1.87)	(0.33)	(0.61)	(-0.78)	(-0.66)	(0.87)	(0.83)	(-0.25)	
Def_{t+1} - adj.	0.389	-0.001	0.035	-0.031	-0.002	0.975	0.002	-0.033	-0.002	
Not adjusted	0.393	-0.001	0.035	-0.031	-0.002	0.961	0.001	-0.034	-0.004	0.959
t-stat	(1.66)	(-0.79)	(3.98)	(-3.30)	(-0.77)	(39.76)	(0.10)	(-1.92)	(-0.29)	
$Term_{t+1}$ - adj.	2.227	0.000	-0.024	-0.025	0.006	0.064	0.965	-0.190	0.005	
Not adjusted	2.186	0.000	-0.024	-0.025	0.006	0.063	0.945	-0.196	0.009	0.968
t-stat	(3.97)	(0.05)	(-2.04)	(-2.92)	(1.58)	(1.54)	(52.78)	(-4.51)	(0.29)	
Short $Rate_{t+1}$ - adj.	-2.128	0.001	-0.035	0.016	0.001	-0.058	0.021	1.179	0.036	
Not adjusted	-2.076	0.001	-0.034	0.016	0.001	-0.064	0.027	1.172	0.039	0.994
t-stat	(-4.08)	(0.32)	(-3.59)	(2.24)	(0.54)	(-2.38)	(1.94)	(29.82)	(1.74)	
DY_{t+1} - adj.	0.077	-0.002	-0.001	-0.004	0.000	0.017	0.003	-0.005	0.984	
Not adjusted	0.061	-0.002	-0.001	-0.004	0.000	0.018	0.005	-0.005	0.962	0.962
t-stat	(0.28)	(-1.04)	(-0.09)	(-0.51)	(0.07)	(0.55)	(0.80)	(-0.28)	(64.24)	

Correlation of residuals (bias-adjusted coefficients)

	Rf	Stock	Gov. Treas	Corp.	REITS	Def.	Term.	Short Rate	DY
Rf	1	-0.021	0.042	0.019	0.065	0.072	0.014	-0.017	0.013
Stock	-	1	-0.161	0.261	0.585	-0.222	0.062	0.072	-0.922
Gov. Treas	-	-	1	0.686	0.016	0.079	-0.547	-0.169	0.091
Corp.	-	-	-	1	0.353	-0.171	-0.432	0.034	-0.320
REITS	-	-	-	-	1	-0.233	-0.076	0.028	-0.670
Def.	-	-	-	-	-	1	-0.062	-0.075	0.245
Term.	-	-	-	-	-	-	1	-0.560	-0.054
Short Rate	-	-	-	-	-	-	-	1	-0.047
DY	-	-	-	-	-	-	-	-	1

Table 4

Summary Statistics for Monthly Realized, Recursive Optimal Portfolio Weights: Baseline Asset Menu (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Lags	Predictors included				Cash			Stocks			US Long-Term Treasuries			US Corporate Bonds			REITs		
			Def.	Term.	Short	DY	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
Sample mean of portofolio weights																					
1	Expanding VAR	1	N	Y	Y	Y	-0.791	-0.002	0.789	0.742	0.630	-0.112	-0.680	-1.612	-0.932	1.458	1.634	0.176	0.271	0.350	0.079
2	Expanding VAR	1	Y	Y	N	Y	-0.987	-0.717	0.270	1.417	1.295	-0.122	1.148	1.332	0.185	-1.270	-1.640	-0.370	0.693	0.730	0.037
3	Rolling N. IID	0	-	-	-	-	-1.350	-1.350	0.000	0.393	0.393	0.000	1.623	1.623	0.000	-0.090	-0.090	0.000	0.423	0.423	0.000
4	Expanding N. IID	0	-	-	-	-	-1.144	-1.144	0.000	0.431	0.431	0.000	1.428	1.428	0.000	-0.200	-0.200	0.000	0.485	0.485	0.000
5	Expanding VAR	1	N	N	Y	Y	-1.237	-0.711	0.526	0.308	0.280	-0.029	0.270	-0.377	-0.647	1.108	1.152	0.044	0.551	0.656	0.105
6	Expanding VAR	1	Y	N	Y	Y	-2.172	-1.630	0.542	0.629	0.527	-0.102	1.878	2.146	0.267	-0.383	-1.148	-0.765	1.048	1.105	0.058
7	Rolling VAR	2	Y	N	Y	Y	-7.430	-8.326	-0.896	3.600	4.099	0.498	8.824	9.978	1.154	-4.055	-4.946	-0.891	0.062	0.196	0.134
8	Rolling VAR	1	Y	Y	Y	Y	-3.732	-4.511	-0.780	3.244	3.966	0.722	4.382	5.753	1.371	-2.467	-3.888	-1.420	-0.426	-0.319	0.107
9	Rolling VAR	1	N	N	Y	Y	-2.926	-3.013	-0.087	0.585	0.901	0.316	2.457	2.005	-0.452	0.169	0.283	0.113	0.715	0.824	0.109
10	Expanding VAR	2	Y	N	Y	Y	-1.567	-1.681	-0.114	0.654	0.794	0.140	1.853	2.470	0.618	-1.061	-1.717	-0.656	1.122	1.134	0.012
Sample Standard Deviation of Portofolio Weights																					
1	Expanding VAR	1	N	Y	Y	Y	3.352	3.308	0.717	1.082	1.167	0.449	5.753	5.800	1.344	4.306	4.226	0.438	1.099	1.084	0.082
2	Expanding VAR	1	Y	Y	N	Y	4.185	4.116	0.260	1.480	1.593	0.329	7.245	7.884	1.112	5.027	5.796	1.267	1.099	1.060	0.089
3	Rolling N. IID	0	-	-	-	-	0.716	0.716	0.000	0.545	0.545	0.000	0.657	0.657	0.000	0.371	0.371	0.000	0.412	0.412	0.000
4	Expanding N. IID	0	-	-	-	-	0.237	0.237	0.000	0.093	0.093	0.000	0.500	0.500	0.000	0.404	0.404	0.000	0.310	0.310	0.000
5	Expanding VAR	1	N	N	Y	Y	3.740	3.746	0.854	0.878	1.197	0.526	6.095	5.860	0.642	4.013	3.725	0.623	1.160	1.147	0.093
6	Expanding VAR	1	Y	N	Y	Y	3.762	3.689	0.390	1.211	1.269	0.165	6.871	7.206	0.979	4.879	5.331	1.051	1.075	1.064	0.093
7	Rolling VAR	2	Y	N	Y	Y	10.747	10.205	2.187	5.724	5.625	1.423	15.182	15.417	1.530	9.960	10.342	1.387	3.309	3.167	0.425
8	Rolling VAR	1	Y	Y	Y	Y	7.174	7.304	1.891	3.269	3.548	0.754	9.554	10.896	2.808	5.914	7.472	2.356	2.116	2.241	0.422
9	Rolling VAR	1	N	N	Y	Y	3.458	3.536	1.158	1.712	2.050	0.874	5.864	5.818	0.673	4.061	4.029	0.395	1.573	1.463	0.217
10	Expanding VAR	2	Y	N	Y	Y	5.429	5.431	0.643	2.990	2.886	0.535	11.229	11.427	0.990	9.606	9.647	1.102	2.259	2.113	0.228
Empirical 90% Range																					
1	Expanding VAR	1	N	Y	Y	Y	9.697	10.224	1.753	3.604	3.936	0.750	18.267	18.613	2.446	12.138	12.533	1.320	3.470	3.435	0.202
2	Expanding VAR	1	Y	Y	N	Y	11.673	11.743	0.776	4.094	4.493	0.927	22.632	24.541	3.013	14.446	16.944	3.705	3.390	3.325	0.253
3	Rolling N. IID	0	-	-	-	-	0.716	0.716	0.000	0.545	0.545	0.000	0.657	0.657	0.000	0.371	0.371	0.000	0.412	0.412	0.000
4	Expanding N. IID	0	-	-	-	-	0.753	0.753	0.000	0.277	0.277	0.000	1.430	1.430	0.000	1.377	1.377	0.000	0.810	0.810	0.000
5	Expanding VAR	1	N	N	Y	Y	10.501	10.387	1.560	3.340	3.525	0.886	19.984	19.410	1.215	12.131	12.149	0.542	2.907	2.982	0.226
6	Expanding VAR	1	Y	N	Y	Y	11.905	12.403	1.183	3.824	4.137	0.519	21.583	22.681	3.163	15.817	16.551	3.123	2.975	2.989	0.237
7	Rolling VAR	2	Y	N	Y	Y	34.278	31.650	6.803	18.532	16.352	4.421	49.529	47.184	5.026	28.991	28.982	4.344	10.524	10.000	1.446
8	Rolling VAR	1	Y	Y	Y	Y	20.638	22.058	6.582	10.013	10.936	2.467	30.235	34.907	8.698	18.275	21.959	7.122	6.408	7.062	1.474
9	Rolling VAR	1	N	N	Y	Y	10.380	10.377	3.641	5.795	7.050	2.677	18.189	18.740	2.131	10.454	9.772	1.154	5.289	4.957	0.716
10	Expanding VAR	2	Y	N	Y	Y	15.239	15.189	1.950	7.691	7.097	1.723	34.317	35.014	2.929	34.908	34.235	3.309	7.367	6.760	0.689

Table 5

Top 10 Models Ranked According to Realized CER for Buy-and-Hold and Monthly Rebalancing Strategies: Baseline Asset Menu (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months).

CER rank	Model	Lags	Predictors included				*H*	Annualized mean			Annualized volatility			Sharpe ratio			Annualized CER			Skewness	Kurtosis
			Def.	Term.	Short	DY		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Volatility	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
Monthly rebalancing																					
1	Expanding VAR	1	N	Y	Y	Y	60	44.338	40.528	50.005	102.111	40.426	120.433	0.408	0.337	1.009	13.679	11.832	15.356	-0.865	2.696
2	Expanding VAR	1	Y	Y	N	Y	60	35.651	32.802	44.529	108.940	65.530	117.768	0.303	0.260	0.459	3.382	2.369	7.937	0.426	2.272
3	Rolling N. IID	0	-	-	-	-	60	5.420	-0.285	7.531	21.009	2.514	21.114	0.132	-0.109	0.326	1.703	-0.535	3.847	1.502	3.773
4	Expanding N. IID	0	-	-	-	-	60	3.792	-0.913	5.723	16.642	2.236	16.593	0.069	-0.379	0.315	0.787	-1.184	2.912	1.475	3.699
5	Expanding VAR	1	N	N	Y	Y	60	42.224	33.376	54.193	152.635	67.423	156.638	0.259	0.202	0.454	-0.879	-3.365	5.262	0.394	2.106
6	Expanding VAR	1	Y	N	Y	Y	60	35.214	34.731	42.253	86.689	48.583	88.438	0.376	0.314	0.655	-1.454	-1.530	37.838	-0.123	2.414
7	Rolling VAR	2	Y	N	Y	Y	60	27.598	19.704	40.799	140.452	70.247	192.890	0.178	0.162	0.257	-2.586	-2.850	6.330	1.700	5.419
8	Rolling VAR	1	Y	Y	Y	Y	60	39.169	23.320	56.136	213.128	80.771	247.562	0.171	0.135	0.265	-3.235	-3.630	19.852	1.090	2.759
9	Rolling VAR	1	N	N	Y	Y	60	12.358	-2.988	22.222	51.647	18.963	53.279	0.188	-0.145	0.382	-3.926	-6.658	6.033	0.569	1.721
10	Expanding VAR	2	Y	N	Y	Y	60	32.011	28.737	43.058	121.591	79.600	141.336	0.242	0.204	0.331	-4.661	-5.983	24.503	0.841	2.897
		Median Expanding VAR performance					60	-8.302	-6.415	-12.831	28.793	0.962	38.114	-0.380	-0.015	-0.304	-17.201	-14.209	-40.569	2.548	8.613
		Median Rolling VAR performance					60	-4.011	-0.923	-1.847	34.846	4.555	43.745	-0.191	0.078	-0.037	-16.834	-13.667	-31.673	1.677	4.533
Buy-and-hold																					
1	Expanding N. IID	0	-	-	-	-	60	3.221	3.244	4.194	5.041	1.815	5.346	0.636	0.782	1.780	2.257	2.333	4.016	-2.100	10.961
2	Rolling N. IID	0	-	-	-	-	60	2.710	2.623	3.322	4.152	1.911	4.261	0.649	0.776	1.365	2.137	2.271	3.186	-1.968	11.172
3	Rolling VAR	1	Y	N	N	Y	60	0.502	0.419	1.146	6.912	0.887	7.938	0.071	0.143	0.457	-50.698	-50.698	0.954	-4.854	30.257
4	Rolling VAR	1	Y	Y	N	Y	60	0.225	-0.055	0.848	8.932	1.535	11.903	0.024	-0.045	0.070	-52.377	-53.333	0.636	-3.883	19.052
5	Rolling VAR	1	Y	N	Y	Y	60	-1.667	-1.787	-1.021	9.636	6.428	10.066	-0.174	-0.280	-0.103	-52.377	-52.377	-4.965	-2.497	10.708
6	Rolling VAR	1	N	Y	N	Y	60	-1.334	-2.817	-0.012	11.714	4.604	15.152	-0.115	-0.615	-0.002	-52.377	-53.999	-1.707	-1.818	6.415
7	Rolling VAR	1	N	N	N	Y	60	-0.870	-0.866	0.385	10.812	2.254	12.044	-0.082	-0.390	0.031	-53.333	-53.999	0.075	-2.699	10.826
8	Expanding VAR	1	N	N	Y	Y	60	-3.133	-4.593	-2.655	13.386	11.802	14.048	-0.235	-0.390	-0.190	-53.333	-54.510	-50.698	-1.346	4.769
9	Rolling VAR	1	Y	N	N	N	60	-1.406	-2.029	0.126	13.017	10.383	13.210	-0.109	-0.197	0.008	-53.822	-54.370	-49.822	-1.718	6.443
10	Expanding VAR	1	Y	N	Y	Y	60	-3.289	-5.481	-2.212	13.753	11.230	16.832	-0.240	-0.489	-0.132	-53.999	-55.827	-50.698	-1.174	4.410
		Median Expanding VAR performance					60	-10.605	-12.438	-8.750	16.401	14.684	17.764	-0.616	-0.822	-0.462	-56.714	-57.271	-55.946	-0.032	2.483
		Median Rolling VAR performance					60	-6.244	-8.606	-3.869	15.130	12.168	16.547	-0.397	-0.739	-0.226	-55.433	-56.651	-53.999	-0.912	3.310

Table 6

Full Sample (1994:01–2014:12) Estimates of Best Performing Full VAR(1): Relative Value Hedge Funds

The table presents full-sample OLS estimate of a VAR(1) model

$$\boldsymbol{z}_{t+1} = \boldsymbol{\Phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{z}_t + \boldsymbol{v}_{t+1},$$

where $\boldsymbol{z}_{t+1}$ is extended to include one specific hedge fund strategy return and $\boldsymbol{v}_{t+1} \overset{i.i.d.}{\sim} \mathcal{N}(\boldsymbol{0}, \sum_v)$. We adjust the estimates for small-sample bias as in Engsted and Pedersen (2012), by using the formula $\widehat{\boldsymbol{\Phi}}_1$ as $\boldsymbol{Bias}_T = -\frac{\boldsymbol{b}}{T} + O\left(T^{-\frac{3}{2}}\right)$ where **b** is in the main text.

Dependent variable	Rf_t	$Stocks_t$	Gov $Treas_t$	$Corp_t$	$REIT_t$	Def_t	$Term_t$	Short $Rate_t$	DY_t	$BuyWrite_t$	HML_t	$Momentum_t$	R^2
Rf_{t+1} - adj.	-0.029	0.000	-0.002	0.001	0.000	0.002	-0.005	0.080	0.000	-0.001	0.001	0.001	
Not adjusted	-0.042	0.000	-0.002	0.001	0.000	0.002	-0.005	0.081	-0.001	-0.001	0.001	0.001	0.989
t-stat	(-0.76)	(0.28)	(-1.24)	(0.80)	(-0.01)	(1.58)	(-3.49)	(17.01)	(-0.15)	(-0.32)	(2.60)	(2.14)	
$Stocks_{t+1}$ - adj.	-5.600	0.097	0.133	0.016	0.015	0.466	-0.383	0.227	0.336	-0.283	-0.094	-0.004	
Not adjusted	-4.883	0.091	0.134	0.013	0.015	0.481	-0.446	0.242	1.160	-0.277	-0.090	-0.006	0.064
t-stat	(-0.38)	(0.53)	(0.44)	(0.04)	(0.16)	(0.88)	(-1.28)	(0.24)	(1.41)	(-0.59)	(-0.66)	(-0.08)	
Gov $Treas_{t+1}$ - adj.	-1.895	0.017	-0.124	0.174	-0.052	-0.274	0.219	0.146	-0.248	-0.263	0.025	0.018	
Not adjusted	-1.887	0.019	-0.133	0.175	-0.051	-0.284	0.329	0.257	-0.272	-0.268	0.024	0.019	0.119
t-stat	(-0.41)	(0.30)	(-1.24)	(1.70)	(-1.33)	(-1.57)	(1.98)	(0.69)	(-0.86)	(-1.45)	(0.52)	(0.79)	
$Corp_{t+1}$ - adj.	-0.211	0.142	-0.038	0.207	-0.088	-0.137	0.319	-0.013	-0.219	-0.394	0.028	-0.033	
Not adjusted	-0.197	0.149	-0.032	0.193	-0.087	-0.154	0.431	0.110	-0.023	-0.403	0.028	-0.034	0.074
t-stat	(-0.04)	(1.50)	(-0.19)	(1.10)	(-1.34)	(-0.53)	(2.00)	(0.24)	(-0.06)	(-1.30)	(0.52)	(-0.99)	
$REIT_{t+1}$ - adj.	-15.152	0.320	0.630	-0.023	-0.248	1.048	0.673	1.214	-1.446	0.000	0.312	-0.087	
Not adjusted	-14.691	0.330	0.627	-0.019	-0.262	1.047	0.648	1.263	-0.749	0.001	0.315	-0.088	0.139
t-stat	(-1.02)	(1.43)	(1.31)	(-0.04)	(-1.82)	(1.23)	(1.54)	(1.09)	(-0.68)	(0.00)	(2.14)	(-1.38)	
Def_{t+1} - adj.	4.391	0.011	-0.042	0.086	0.009	0.408	-0.032	-0.356	0.022	-0.071	-0.018	-0.011	
Not adjusted	4.458	0.014	-0.044	0.087	0.009	0.391	-0.041	-0.353	0.184	-0.073	-0.017	-0.012	0.261
t-stat	(1.60)	(0.48)	(-0.60)	(1.17)	(0.42)	(2.92)	(-0.61)	(-1.59)	(1.14)	(-0.64)	(-0.90)	(-0.82)	
$Term_{t+1}$ - adj.	2.272	-0.004	-0.018	-0.030	0.004	0.018	0.959	-0.201	0.023	0.005	-0.002	-0.003	
Not adjusted	2.233	-0.004	-0.018	-0.030	0.004	0.018	0.942	-0.206	0.025	0.005	-0.002	-0.003	0.968
t-stat	(3.91)	(-0.57)	(-1.44)	(-2.80)	(1.18)	(1.28)	(53.95)	(-4.66)	(0.86)	(0.20)	(-0.36)	(-1.07)	
Short $Rate_{t+1}$ - adj.	-2.131	-0.003	-0.030	0.010	0.004	0.014	0.025	1.185	0.017	0.004	-0.006	0.002	
Not adjusted	-2.092	-0.003	-0.030	0.010	0.004	0.015	0.032	1.180	0.019	0.004	-0.006	0.002	0.994
t-stat	(-4.19)	(-0.47)	(-2.91)	(1.43)	(1.57)	(1.10)	(2.32)	(31.28)	(0.95)	(0.17)	(-1.51)	(0.65)	
DY_{t+1} - adj.	0.099	-0.001	-0.008	0.002	0.001	-0.021	0.000	-0.009	0.989	0.003	0.000	0.001	
Not adjusted	0.083	-0.001	-0.008	0.002	0.001	-0.021	0.001	-0.010	0.971	0.003	0.000	0.001	0.963
t-stat	(0.36)	(-0.31)	(-1.05)	(0.31)	(0.37)	(-1.54)	(0.20)	(-0.52)	(46.16)	(0.29)	(0.07)	(0.92)	
$BuyWrite_{t+1}$ - adj.	-2.923	-0.016	0.085	-0.004	0.000	0.247	-0.096	0.235	-0.014	0.012	-0.046	-0.021	
Not adjusted	-2.738	-0.012	0.084	-0.005	0.000	0.252	-0.117	0.235	0.209	-0.005	-0.045	-0.022	0.076
t-stat	(-0.71)	(-0.23)	(1.02)	(-0.06)	(0.01)	(1.31)	(-1.11)	(0.77)	(0.88)	(-0.03)	(-1.15)	(-0.96)	
HML_{t+1} - adj.	-4.074	0.085	0.259	-0.148	-0.096	0.367	0.218	0.400	-0.736	0.283	0.180	-0.075	
Not adjusted	-4.382	0.083	0.258	-0.147	-0.097	0.368	0.223	0.437	-0.806	0.290	0.162	-0.075	0.086
t-stat	(-0.41)	(0.63)	(1.44)	(-0.86)	(-1.64)	(1.33)	(0.94)	(0.51)	(-1.24)	(0.86)	(1.30)	(-1.05)	
$Momentum_{t+1}$ - adj.	-12.076	-0.219	-1.083	0.733	0.040	-0.572	0.566	1.393	-2.712	-0.402	-0.225	0.055	
Not adjusted	-12.803	-0.231	-1.078	0.736	0.041	-0.576	0.668	1.448	-3.145	-0.389	-0.226	0.040	0.162
t-stat	(-0.89)	(-1.47)	(-1.86)	(1.58)	(0.45)	(-1.26)	(1.71)	(1.31)	(-2.78)	(-0.55)	(-1.38)	(0.34)	

Correlation of residuals (bias-adjusted coefficients)

	Rf_t	$Stocks_t$	Gov $Treas_t$	$Corp_t$	$REITs_t$	Def_t	$Term_t$	Short $Rate_t$	DY_t	$BuyWrite_t$	HML_t	$Momentum_t$
Rf_t	1	-0.021	0.042	0.031	0.050	-0.046	0.024	-0.020	0.017	-0.001	0.048	-0.131
$Stocks_t$	-	1	-0.152	0.250	0.608	0.633	0.040	0.079	-0.927	0.873	-0.244	-0.249
Gov $Treas_t$	-	-	1	0.683	0.039	-0.047	-0.542	-0.160	0.068	-0.141	0.103	0.105
$Corp_t$	-	-	-	1	0.356	0.436	-0.424	0.035	-0.325	0.281	0.037	-0.143
$REIT_t$	-	-	-	-	1	0.509	-0.101	0.050	-0.670	0.595	0.212	-0.308
Def_t	-	-	-	-	-	1	-0.066	0.187	-0.624	0.598	-0.130	-0.198
$Term_t$	-	-	-	-	-	-	1	-0.577	-0.031	0.057	-0.054	-0.144
Short $Rate_t$	-	-	-	-	-	-	-	1	-0.046	0.114	0.015	0.040
DY_t	-	-	-	-	-	-	-	-	1	-0.836	0.047	0.348
$BuyWrite_t$	-	-	-	-	-	-	-	-	-	1	-0.115	-0.274
HML_t	-	-	-	-	-	-	-	-	-	-	1	-0.141
$Momentum_t$	-	-	-	-	-	-	-	-	-	-	-	1

Table 7

Summary Statistics for Monthly Realized, Recursive Optimal Portfolio Weights: Asset Menu Including HFRI Fund Weighted Composite Index (FWC) (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			FWC		
		Lags	Term.	Short	DY	SMB	BMX	Mom.	PtfsCM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
													Sample mean of portofolio weights														
1	Expanding AR	1	-	-	-	-	-	-	-	-2.742	-2.303	0.439	-1.197	-1.233	-0.037	0.110	0.590	0.480	1.040	0.503	-0.537	0.638	0.646	0.008	3.150	2.797	-0.353
2	Expanding VAR	1	Y	Y	Y	N	N	N	N	-1.810	-1.286	0.525	-0.083	-0.227	-0.144	-0.451	-0.492	-0.040	0.931	0.865	-0.066	0.247	0.252	0.005	2.167	1.887	-0.280
3	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	-1.752	-1.218	0.534	0.118	-0.046	-0.163	-0.452	-0.487	-0.036	1.038	0.956	-0.082	0.151	0.154	0.003	1.896	1.641	-0.255
4	Expanding VAR	1	Y	Y	Y	N	N	N	Y	-1.905	-1.391	0.514	0.058	-0.098	-0.156	-0.455	-0.450	0.005	1.144	1.027	-0.118	0.169	0.178	0.009	1.988	1.734	-0.255
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	-2.073	-1.606	0.468	0.015	-0.159	-0.174	-0.435	-0.411	0.024	1.183	1.088	-0.095	0.148	0.162	0.014	2.163	1.926	-0.237
6	Expanding VAR	1	Y	Y	Y	N	Y	N	N	-2.010	-1.520	0.490	-0.174	-0.338	-0.164	-0.425	-0.453	-0.028	0.889	0.850	-0.039	0.222	0.231	0.009	2.498	2.230	-0.268
7	Expanding N. IID	0	-	-	-	-	-	-	-	-4.228	-4.228	0.000	-1.233	-1.233	0.000	2.171	2.171	0.000	-0.898	-0.898	0.000	0.418	0.418	0.000	4.770	4.770	0.000
8	Expanding VAR	1	Y	Y	Y	N	N	Y	N	-1.856	-1.268	0.587	-0.040	-0.193	-0.153	-0.459	-0.578	-0.120	0.870	0.843	-0.027	0.216	0.215	0.000	2.269	1.982	-0.287
9	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	-1.863	-1.341	0.522	0.039	-0.100	-0.139	-0.308	-0.396	-0.089	0.756	0.726	-0.030	0.119	0.122	0.004	2.257	1.988	-0.269
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	-1.855	-1.295	0.560	-0.022	-0.147	-0.125	-0.450	-0.615	-0.166	0.855	0.863	0.008	0.119	0.125	0.006	2.353	2.070	-0.283
													Sample Standard Deviation of Portofolio Weights														
1	Expanding AR	1	-	-	-	-	-	-	-	5.688	5.287	0.567	3.952	3.837	0.215	5.665	5.486	0.461	4.294	4.175	0.340	1.627	1.549	0.153	9.498	8.782	0.793
2	Expanding VAR	1	Y	Y	Y	N	N	N	N	4.399	4.200	0.695	3.438	3.343	0.423	4.558	4.469	1.130	3.669	3.582	0.438	1.005	0.966	0.114	8.121	7.491	0.759
3	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	4.484	4.260	0.686	4.025	3.866	0.414	4.916	4.778	1.037	4.404	4.261	0.442	1.433	1.364	0.137	8.757	8.088	0.820
4	Expanding VAR	1	Y	Y	Y	N	N	N	Y	4.502	4.267	0.712	3.902	3.790	0.409	5.104	4.986	1.112	4.423	4.285	0.460	1.338	1.285	0.123	8.523	7.859	0.800
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	5.084	4.870	0.725	3.866	3.757	0.397	5.276	5.191	1.125	4.586	4.432	0.474	1.319	1.272	0.120	8.670	8.010	0.808
6	Expanding VAR	1	Y	Y	Y	N	Y	N	N	4.817	4.650	0.701	3.353	3.258	0.411	4.569	4.526	1.138	3.683	3.593	0.445	0.992	0.958	0.110	8.076	7.447	0.768
7	Expanding N. IID	0	-	-	-	-	-	-	-	0.227	0.227	0.000	0.160	0.160	0.000	0.632	0.632	0.000	0.474	0.474	0.000	0.122	0.122	0.000	0.269	0.269	0.000
8	Expanding VAR	1	Y	Y	Y	N	N	Y	N	4.338	4.167	0.662	3.608	3.445	0.423	4.507	4.392	1.066	3.651	3.565	0.420	1.217	1.151	0.136	8.322	7.658	0.801
9	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	5.235	5.090	0.706	3.924	3.801	0.588	4.490	4.547	1.236	3.756	3.623	0.515	1.420	1.360	0.139	8.812	8.164	0.832
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	5.168	5.047	0.728	3.652	3.545	0.584	4.625	4.616	1.250	3.775	3.641	0.487	1.257	1.193	0.142	8.682	8.025	0.834
													Empirical 90% Range														
1	Expanding AR	1	-	-	-	-	-	-	-	12.184	10.708	1.628	11.039	10.615	0.677	17.353	16.867	1.141	12.968	12.466	0.955	3.532	3.519	0.434	29.159	26.856	2.522
2	Expanding VAR	1	Y	Y	Y	N	N	N	N	12.802	12.308	1.956	10.060	9.902	0.660	14.768	14.519	2.214	10.162	11.213	1.247	3.381	3.207	0.352	24.289	22.206	2.507
3	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	11.505	11.201	1.843	10.795	10.838	0.720	14.970	15.115	2.083	10.268	11.116	1.300	4.779	4.910	0.374	23.798	21.710	2.743
4	Expanding VAR	1	Y	Y	Y	N	N	N	Y	12.385	11.322	2.077	11.917	11.549	0.745	17.963	17.111	2.396	10.710	12.880	1.339	4.595	4.590	0.385	24.878	22.529	2.728
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	15.410	15.521	2.200	11.841	11.481	0.788	15.685	15.500	2.404	11.107	12.402	1.418	4.694	4.498	0.410	24.718	23.479	2.732
6	Expanding VAR	1	Y	Y	Y	N	Y	N	N	13.868	13.480	1.902	9.213	8.887	0.734	14.501	13.797	2.218	11.159	11.175	1.209	3.318	3.237	0.364	22.804	21.176	2.564
7	Expanding N. IID	0	-	-	-	-	-	-	-	0.692	0.692	0.000	0.510	0.510	0.000	1.748	1.748	0.000	1.555	1.555	0.000	0.395	0.395	0.000	0.824	0.824	0.000
8	Expanding VAR	1	Y	Y	Y	N	N	Y	N	11.302	11.355	1.887	9.068	9.099	0.683	14.653	14.198	2.212	11.119	11.169	1.233	3.779	3.586	0.359	21.657	20.331	2.515
9	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	13.353	12.951	2.079	10.588	10.582	0.792	14.267	14.882	2.330	11.454	11.266	1.378	4.865	4.825	0.373	22.994	20.819	2.618
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	13.218	12.447	2.106	9.584	9.991	0.701	14.433	14.019	2.229	11.200	11.262	1.302	4.336	4.044	0.382	23.615	21.325	2.695

Table 8

Summary Statistics for Monthly Realized, Recursive Optimal Portfolio Weights: Asset Menu Including HFRI Fund of Funds Composite (FFP) (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			FFP		
		Lags	Term.	Short	DY	SMB	BMX	Mom.	PtfsCM	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
															Sample mean of portofolio weights												
1	Expanding AR	1	-	-	-	-	-	-	-	-0.276	-0.234	0.042	-0.052	-0.142	-0.091	-0.424	-0.273	0.150	1.236	0.889	-0.347	0.699	0.776	0.077	-0.183	-0.015	0.168
2	Expanding VAR	1	Y	Y	Y	N	N	N	Y	-0.020	0.211	0.232	0.875	0.774	-0.101	-0.964	-1.573	-0.608	1.314	1.411	0.097	0.209	0.295	0.087	-0.413	-0.120	0.293
3	Expanding VAR	1	Y	Y	Y	N	Y	N	N	-0.048	0.199	0.247	0.726	0.628	-0.098	-0.800	-1.449	-0.648	1.053	1.219	0.166	0.231	0.313	0.082	-0.161	0.089	0.251
4	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	0.077	0.308	0.231	0.849	0.746	-0.103	-0.859	-1.415	-0.556	1.126	1.208	0.082	0.228	0.310	0.082	-0.422	-0.157	0.265
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	-0.044	0.159	0.226	0.864	0.748	-0.120	-1.038	-1.596	-0.630	1.386	1.473	0.133	0.164	0.270	0.089	-0.332	-0.054	0.302
6	Expanding VAR	1	Y	Y	Y	N	N	N	N	-0.044	0.253	0.285	0.766	0.650	-0.085	-0.812	-1.672	-0.683	1.032	1.405	0.148	0.288	0.350	0.082	-0.229	0.014	0.254
7	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	-0.055	0.150	0.206	0.851	0.731	-0.119	-0.817	-1.395	-0.578	1.199	1.311	0.112	0.197	0.284	0.087	-0.374	-0.082	0.292
8	Expanding N. IID	0	-	-	-	-	-	-	-	-2.195	-2.195	0.000	0.134	0.134	0.000	1.657	1.657	0.000	-0.446	-0.446	0.000	0.484	0.484	0.000	1.366	1.366	0.000
9	Expanding VAR	1	Y	Y	Y	N	N	Y	N	-0.010	0.291	0.302	0.806	0.727	-0.080	-0.829	-1.516	-0.688	0.983	1.120	0.138	0.232	0.315	0.083	-0.182	0.063	0.246
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	-0.215	0.101	0.315	0.825	0.726	-0.099	-0.828	-1.556	-0.728	1.070	1.240	0.169	0.217	0.301	0.084	-0.071	0.188	0.259
															Sample Standard Deviation of Portofolio Weights												
1	Expanding AR	1	-	-	-	-	-	-	-	7.961	7.045	1.116	3.494	3.273	0.282	5.628	5.281	0.904	4.052	4.008	0.509	1.700	1.575	0.226	12.129	10.615	1.599
2	Expanding VAR	1	Y	Y	Y	N	N	N	Y	6.642	5.962	1.007	3.504	3.340	0.405	4.881	4.694	1.531	4.168	4.105	0.651	1.294	1.220	0.171	10.832	9.554	1.395
3	Expanding VAR	1	Y	Y	Y	N	Y	N	N	6.808	6.206	0.962	3.022	2.871	0.404	4.692	4.678	1.570	3.683	3.723	0.633	0.993	0.934	0.168	10.561	9.323	1.339
4	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	6.558	5.904	1.003	3.645	3.447	0.388	4.849	4.628	1.418	4.215	4.158	0.608	1.460	1.358	0.189	10.981	9.710	1.420
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	6.953	6.299	1.010	3.484	3.313	0.393	5.209	5.145	1.569	4.500	4.449	0.671	1.285	1.229	0.172	10.995	9.707	1.410
6	Expanding VAR	1	Y	Y	Y	N	N	N	N	6.384	5.778	0.943	3.027	2.899	0.416	4.514	4.758	1.531	3.479	4.197	0.620	0.976	0.935	0.166	10.307	9.092	1.314
7	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	7.136	6.472	1.031	3.651	3.452	0.379	5.231	5.076	1.483	4.565	4.490	0.640	1.477	1.374	0.198	11.374	10.080	1.448
8	Expanding N. IID	0	-	-	-	-	-	-	-	0.261	0.261	0.000	0.101	0.101	0.000	0.551	0.551	0.000	0.444	0.444	0.000	0.269	0.269	0.000	0.269	0.269	0.000
9	Expanding VAR	1	Y	Y	Y	N	N	Y	N	6.476	5.877	0.964	3.263	3.077	0.390	4.475	4.284	1.435	3.539	3.573	0.581	1.258	1.153	0.190	10.714	9.480	1.375
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	7.144	6.505	1.048	3.256	3.075	0.377	4.704	4.599	1.477	3.725	3.755	0.600	1.296	1.176	0.205	11.123	9.863	1.404
															Empirical 90% Range												
1	Expanding AR	1	-	-	-	-	-	-	-	20.965	18.219	3.636	10.017	9.190	0.825	18.171	15.922	2.865	11.487	11.701	1.601	4.213	4.014	0.701	36.968	32.464	4.977
2	Expanding VAR	1	Y	Y	Y	N	N	N	Y	17.886	15.854	3.207	9.698	9.257	0.735	15.334	15.398	4.085	13.217	13.940	2.048	4.553	4.512	0.453	32.800	28.209	4.597
3	Expanding VAR	1	Y	Y	Y	N	Y	N	N	18.782	17.366	3.019	9.076	8.763	0.729	15.069	15.045	3.998	11.865	12.115	2.006	3.229	3.338	0.484	32.243	29.387	4.259
4	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	17.931	16.938	3.432	11.029	10.727	0.723	15.029	14.428	3.849	11.960	13.165	1.858	4.553	4.640	0.481	31.637	26.869	4.704
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	17.631	16.850	3.259	10.220	9.568	0.778	15.650	16.383	4.133	12.198	13.077	2.132	4.622	4.488	0.498	33.424	29.532	4.419
6	Expanding VAR	1	Y	Y	Y	N	N	N	N	17.706	15.704	2.850	8.862	8.457	0.709	14.830	14.647	3.982	11.084	13.452	1.909	3.269	3.279	0.488	29.970	26.686	4.360
7	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	16.546	15.769	3.510	11.027	10.832	0.798	15.505	15.915	3.923	12.529	12.586	1.964	4.914	4.863	0.511	31.816	27.317	4.791
8	Expanding N. IID	0	-	-	-	-	-	-	-	0.984	0.984	0.000	0.331	0.331	0.000	1.601	1.601	0.000	1.499	1.499	0.000	0.720	0.720	0.000	0.868	0.868	0.000
9	Expanding VAR	1	Y	Y	Y	N	N	Y	N	18.815	15.995	3.182	8.593	8.210	0.658	15.071	13.444	3.711	11.764	11.794	1.818	4.232	3.862	0.482	29.645	25.580	4.618
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	18.585	16.811	3.595	9.298	8.742	0.734	15.313	15.030	3.663	11.762	11.953	1.810	4.457	4.199	0.521	29.451	25.904	4.937

Table 9

Summary Statistics for Monthly Realized, Recursive Optimal Portfolio Weights: Asset Menu Including HFRI Relative Value (RVR) (γ = 5)

The tables shows sample means, standard deviations, and the lower and upper bounds of the 90% sample range of the recursive portfolio weights computed from a range of VAR models for predictable risk premia and of constant investment opportunities (IID) models. The table presents statistics for 1-m T-bill weights, long-term (infinite horizon) weights, and for their differences, the hedging demands.

CER rank	Model	Predictors included								Cash			Stock			US Long-Term Treasuries			US Corporate			REITs			RVR		
		Lags	Term.	Short	DY	SMB	BMX	HML	Mom.	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging	T=1	Long	Hedging
																Sample mean of portofolio weights											
1	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	-9.930	-7.403	2.527	-0.047	-0.086	-0.039	1.259	1.135	-0.123	-0.918	-1.181	-0.263	-0.191	-0.285	-0.095	10.826	8.821	-2.006
2	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	-10.013	-7.420	2.593	-0.073	-0.100	-0.026	1.270	1.129	-0.140	-1.033	-1.300	-0.267	-0.153	-0.255	-0.102	11.002	8.945	-2.057
3	Expanding VAR	1	Y	Y	Y	N	N	Y	N	-9.701	-7.039	2.607	-0.141	-0.194	-0.054	0.958	0.692	-0.185	-0.640	-0.785	-0.212	-0.104	-0.187	-0.098	10.629	8.512	-2.059
4	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	-9.396	-6.892	2.505	-0.104	-0.149	-0.045	0.931	0.731	-0.200	-0.591	-0.792	-0.202	-0.141	-0.231	-0.090	10.300	8.332	-1.968
5	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	-10.454	-7.838	2.560	-0.060	-0.114	-0.058	1.562	1.465	-0.019	-1.162	-1.386	-0.287	-0.159	-0.261	-0.112	11.274	9.134	-2.084
6	Expanding VAR	1	Y	Y	Y	N	N	N	Y	-10.243	-7.626	2.617	-0.075	-0.113	-0.038	1.315	1.193	-0.123	-1.025	-1.319	-0.294	-0.188	-0.306	-0.118	11.216	9.171	-2.044
7	Expanding AR	1	-	-	-	-	-	-	-	-9.379	-7.511	1.868	-0.775	-0.789	-0.014	1.766	1.913	0.146	-0.705	-1.108	-0.404	0.535	0.394	-0.140	9.557	8.101	-1.456
8	Expanding VAR	1	Y	Y	Y	N	N	N	N	-9.742	-7.138	2.604	-0.142	-0.185	-0.043	1.005	0.795	-0.210	-0.686	-0.913	-0.227	-0.114	-0.234	-0.120	10.678	8.674	-2.004
9	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	-10.347	-7.860	2.487	-0.038	-0.092	-0.054	1.522	1.506	-0.016	-1.108	-1.395	-0.287	-0.180	-0.285	-0.105	11.151	9.126	-2.025
10	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	-10.257	-7.642	2.615	-0.033	-0.077	-0.045	1.416	1.285	-0.131	-1.070	-1.347	-0.278	-0.166	-0.292	-0.125	11.110	9.073	-2.037
																Sample Standard Deviation of Portofolio Weights											
1	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	13.283	10.972	2.591	2.484	2.455	0.383	5.044	5.340	0.850	3.802	3.936	0.731	1.509	1.486	0.153	16.671	14.440	2.506
2	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	12.689	10.507	2.460	2.570	2.544	0.387	4.931	5.183	0.785	3.578	3.732	0.709	1.469	1.452	0.148	16.400	14.260	2.379
3	Expanding VAR	1	Y	Y	Y	N	N	Y	N	12.255	10.160	2.379	2.561	2.537	0.312	4.758	5.023	0.773	3.528	3.724	0.720	1.344	1.341	0.142	16.109	14.068	2.285
4	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	13.192	10.844	2.609	2.499	2.504	0.397	4.972	5.233	0.873	3.790	3.890	0.735	1.351	1.337	0.145	16.519	14.296	2.504
5	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	12.242	10.269	2.299	2.609	2.532	0.293	4.546	4.731	0.488	3.215	3.410	0.682	1.659	1.649	0.155	16.505	14.514	2.184
6	Expanding VAR	1	Y	Y	Y	N	N	N	Y	12.581	10.494	2.353	2.302	2.238	0.316	4.776	4.941	0.760	3.442	3.616	0.698	1.180	1.178	0.156	16.557	14.483	2.285
7	Expanding AR	1	-	-	-	-	-	-	-	17.036	14.383	2.784	2.620	2.576	0.165	5.319	5.327	0.453	3.806	3.908	0.532	1.386	1.411	0.252	21.300	18.839	2.580
8	Expanding VAR	1	Y	Y	Y	N	N	N	N	12.502	10.374	2.369	2.257	2.227	0.314	4.584	4.754	0.795	3.393	3.564	0.714	0.916	0.936	0.146	16.451	14.397	2.274
9	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	13.216	10.950	2.577	2.552	2.509	0.361	4.654	4.791	0.630	3.472	3.543	0.707	1.680	1.665	0.158	17.067	14.866	2.445
10	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	12.383	10.284	2.348	2.555	2.468	0.319	4.729	4.899	0.754	3.316	3.485	0.713	1.184	1.181	0.159	16.557	14.494	2.273
																Empirical 90% Range											
1	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	40.515	32.899	8.302	7.680	7.637	0.900	17.942	18.481	1.800	11.325	11.927	2.089	5.618	5.582	0.468	49.491	41.815	7.551
2	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	40.358	33.375	7.269	7.284	7.619	0.934	16.383	17.679	1.792	11.298	11.912	2.056	5.128	5.200	0.496	49.181	41.915	6.862
3	Expanding VAR	1	Y	Y	Y	N	N	Y	N	37.196	29.756	6.988	8.310	8.309	0.802	16.350	17.070	1.817	11.756	12.009	2.062	5.028	4.718	0.488	47.402	40.391	6.695
4	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	39.316	32.239	8.268	8.472	8.547	0.785	17.352	17.606	1.833	11.202	11.219	2.065	4.903	4.736	0.467	49.320	40.005	7.521
5	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	38.301	29.815	6.669	7.580	7.538	0.976	15.942	16.160	1.536	9.855	11.021	2.068	6.218	5.917	0.487	47.255	41.959	6.173
6	Expanding VAR	1	Y	Y	Y	N	N	N	Y	37.743	30.386	6.872	6.416	6.239	0.838	16.365	17.878	1.764	11.042	11.405	2.041	4.035	4.225	0.453	47.976	42.185	6.745
7	Expanding AR	1	-	-	-	-	-	-	-	39.572	33.330	6.338	6.579	6.457	0.508	17.113	16.915	1.272	11.562	11.232	1.367	3.253	3.689	0.779	45.579	39.520	5.978
8	Expanding VAR	1	Y	Y	Y	N	N	N	N	37.064	29.472	7.231	6.879	6.864	0.774	14.894	16.917	1.861	10.820	10.823	2.083	3.018	3.153	0.457	47.808	40.738	6.767
9	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	39.495	32.353	7.077	7.697	7.853	0.959	15.393	15.596	1.728	10.446	10.817	2.040	6.674	6.625	0.466	49.803	44.022	6.727
10	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	38.078	29.752	7.033	6.856	7.052	0.864	16.424	16.663	1.700	10.088	10.747	2.093	4.215	4.302	0.481	47.473	42.115	6.441

Table 10

Top 10 Models Ranked According to Realized CER: HFRI Fund Weighted Composite Index (FWC) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included							*H*	Annualized mean			Annualized volatility			Sharpe ratio			Annualized CER			Skewness	Kurtosis
			Term.	Short	DY	SMB	BMX	Mom	PtfsCM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Vol.	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding AR	1	-	-	-	-	-	-	-	60	8.482	-1.883	17.945	164.745	79.579	206.265	0.035	-0.008	0.193	**19.887**	**18.700**	**24.635**	**0.800**	2.874
2	Expanding VAR	1	Y	Y	Y	N	N	N	N	60	26.083	21.998	29.327	**38.792**	**27.272**	**43.558**	**0.604**	**0.577**	0.691	**15.477**	13.467	**17.856**	**0.248**	**1.860**
3	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	60	24.764	20.882	29.403	**45.519**	**28.578**	**53.955**	**0.486**	**0.505**	0.626	**13.871**	12.511	**16.792**	**0.899**	**2.618**
4	Expanding VAR	1	Y	Y	Y	N	N	N	Y	60	22.876	19.225	26.130	**32.409**	**25.487**	**35.490**	**0.624**	**0.618**	0.646	13.403	11.321	**16.631**	**0.160**	**1.622**
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	60	19.262	15.825	22.694	**28.687**	**21.880**	**32.520**	**0.579**	**0.561**	0.619	10.156	7.994	14.048	**0.111**	**1.806**
6	Expanding VAR	1	Y	Y	Y	N	Y	N	N	60	15.378	10.377	18.540	**26.510**	**17.788**	**29.916**	**0.481**	**0.462**	0.531	6.464	4.173	9.461	**0.178**	**1.749**
7	Expanding IID	0	-	-	-	-	-	-	-	60	9.368	5.561	11.079	**10.973**	**6.908**	**11.892**	**0.613**	**0.689**	0.798	5.438	3.422	7.300	**-0.162**	**1.902**
8	Expanding VAR	1	Y	Y	Y	N	N	Y	N	60	14.144	6.944	19.817	**36.029**	**19.553**	**42.640**	0.319	0.304	0.398	1.672	-0.863	6.920	**0.558**	**1.968**
9	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	60	8.507	3.188	11.790	**21.183**	**14.936**	**24.700**	0.277	0.182	0.409	0.484	-1.915	4.316	**0.551**	2.730
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	60	2.391	-4.333	5.409	**18.943**	**12.604**	**19.746**	-0.013	-0.295	0.234	-6.944	-9.312	-1.874	**0.640**	**2.492**
11	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	60	5.985	-11.973	16.734	**56.761**	**4.802**	**73.304**	0.059	-0.161	0.196	-10.710	-12.267	-8.558	**1.949**	4.145
12	Expanding VAR	1	Y	Y	Y	Y	N	Y	N	60	-1.386	-10.854	4.888	**24.406**	**8.298**	**31.238**	-0.165	-0.261	0.134	-12.357	-14.410	-7.694	**1.578**	3.554
13	Expanding VAR	1	Y	Y	Y	Y	Y	N	N	60	-1.704	-15.067	5.988	**31.204**	**4.583**	**38.743**	-0.139	-0.407	0.132	-13.906	-15.777	-10.375	**1.742**	3.467
14	Expanding VAR	1	Y	Y	Y	Y	N	N	N	60	-1.361	-14.981	6.356	**31.949**	**5.092**	**39.753**	-0.125	-0.365	0.137	-13.970	-15.859	-10.442	**1.722**	3.413
15	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	60	-2.591	-16.425	5.560	**30.102**	**4.796**	**38.348**	-0.174	-0.450	0.124	-15.264	-17.155	-11.765	**1.792**	3.633
16	Expanding VAR	1	Y	Y	Y	Y	Y	N	Y	60	-5.054	-19.916	3.198	**27.863**	**3.376**	**34.868**	-0.276	-0.754	0.078	-17.514	-19.293	-14.160	**1.793**	3.578
17	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	60	-8.178	-24.586	-0.009	**25.406**	**2.479**	**32.123**	-0.426	-1.234	-0.001	-21.355	-23.061	-18.090	**2.220**	5.602
18	Expanding VAR	1	Y	Y	Y	Y	Y	Y	N	60	-16.895	-30.547	-11.501	**14.492**	**1.855**	**18.457**	-1.348	-2.626	-0.535	-27.444	-29.049	-22.440	**2.758**	7.814
	Median Expanding VAR performance									60	**5.985**	**-4.333**	**6.356**	30.102	**12.604**	35.490	**0.111**	**-0.295**	**0.196**	-6.944	-9.312	-1.874	0.459	3.255

Table 11

Top 10 Models Ranked According to Realized CER: HFRI Fund of Funds Composite (FFP) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included							*H*	Annualized mean			Annualized volatility			Sharpe ratio			Annualized CER			Skewness	Kurtosis
			Term.	Short	DY	SMB	BMX	Mom	PtfsCM		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Vol	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding AR	1	-	-	-	-	-	-	-	60	10.362	24.138	-4.095	227.179	124.373	264.820	0.046	-0.016	0.194	9.228	7.276	**22.085**	**-0.376**	**2.400**
2	Expanding VAR	1	Y	Y	Y	N	N	N	Y	60	19.171	9.588	26.066	**62.416**	**23.897**	**74.639**	0.307	**0.349**	0.401	4.899	1.003	11.160	**0.734**	**2.148**
3	Expanding VAR	1	Y	Y	Y	N	Y	N	N	60	17.642	11.366	21.369	**38.580**	**25.730**	**47.703**	**0.457**	**0.441**	0.448	4.094	-0.559	15.191	**-0.310**	**2.042**
4	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	60	18.018	8.969	25.682	**60.911**	**25.889**	**75.587**	0.296	**0.340**	0.346	2.943	-1.127	10.797	**0.794**	**2.319**
5	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	60	16.279	9.121	22.144	**47.113**	**25.797**	**59.159**	0.345	**0.353**	0.374	2.722	-1.541	11.824	**0.528**	**2.181**
6	Expanding VAR	1	Y	Y	Y	N	N	N	N	60	16.092	9.515	20.303	**37.381**	**26.014**	**44.461**	**0.430**	**0.365**	0.456	1.565	-3.051	13.335	**-0.340**	**1.855**
7	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	60	14.173	5.069	21.462	**51.892**	**19.274**	**64.787**	0.273	0.262	0.331	-0.107	-4.106	7.339	**0.839**	**2.370**
8	Expanding IID	0	-	-	-	-	-	-	-	60	2.362	-2.652	3.919	**19.852**	**2.761**	**20.666**	0.118	-0.966	0.189	-1.114	-2.958	-0.234	**1.621**	4.030
9	Expanding VAR	1	Y	Y	Y	N	N	Y	N	60	12.994	5.695	18.225	**38.224**	**25.974**	**45.548**	0.340	0.219	0.400	-5.537	-9.765	9.479	**-0.176**	**1.938**
10	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	60	13.269	6.029	19.049	**40.255**	**27.532**	**48.281**	0.329	0.218	0.394	-6.229	-10.452	8.930	**-0.139**	**1.941**
11	Expanding VAR	1	Y	Y	Y	Y	Y	N	Y	60	9.145	-1.716	17.230	**48.402**	**14.962**	**57.404**	0.189	-0.116	0.300	-8.833	-12.754	-0.190	**0.803**	**2.097**
12	Expanding VAR	1	Y	Y	Y	Y	Y	N	N	60	7.294	0.011	12.205	**30.824**	**20.278**	**38.056**	0.236	0.000	0.320	-9.781	-13.910	2.688	**0.099**	**2.045**
13	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	60	11.474	-3.860	22.312	**69.910**	**11.873**	**82.108**	0.164	-0.326	0.272	-9.927	-13.336	-2.790	**1.013**	**2.293**
14	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	60	8.653	-5.355	17.594	**55.815**	**12.205**	**65.607**	0.155	-0.440	0.268	-11.340	-14.870	-4.845	**0.931**	**2.315**
15	Expanding VAR	1	Y	Y	Y	Y	N	N	N	60	5.495	-3.179	11.110	**32.475**	**17.841**	**39.629**	0.169	-0.179	0.280	-12.820	-16.837	-1.260	**0.422**	**2.217**
16	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	60	6.956	-9.463	17.499	**59.208**	**9.802**	**70.061**	0.117	-0.967	0.250	-16.325	-19.754	-9.153	**0.993**	**2.327**
17	Expanding VAR	1	Y	Y	Y	Y	Y	Y	N	60	3.831	-3.119	8.893	**26.699**	**17.981**	**33.110**	0.143	-0.174	0.268	-16.963	-20.798	-1.488	**-0.180**	**2.039**
18	Expanding VAR	1	Y	Y	Y	Y	N	Y	N	60	2.087	-5.722	7.694	**27.095**	**17.843**	**33.966**	0.076	-0.321	0.226	-18.912	-22.696	-4.930	**0.198**	**2.272**
			Median Expanding VAR performance							60	**11.474**	**5.069**	**18.225**	47.113	20.278	57.404	**0.236**	**0.000**	**0.320**	**-6.229**	**-10.452**	**7.339**	**0.422**	**2.181**

Table 12

Top 10 Models Ranked According to Realized CER: HFRI Fixed Income Relative Value/Arbitrage (RVR) (γ = 5)

The tables shows annualized sample means, standard deviations, the Sharpe ratio, and the Certainty Equivalent Return computed from the recursive portfolio weights under the ten top performing (in terms of CER) models for (predictable) risk premia, as defined by the VAR order (where a VAR(0) is equivalent to IID, constant investment opportunities), the selection of predictors, and whether the models are estimated either on an expanding or on a rolling 10-year window of data. The calculations are performed assuming the investor assess realized performance with a 5-year horizon (H = 60 months). In the table, we have boldfaced all performance statistics that turn out to be superior to those recorded in Table 5 for the baseline asset menu that excludes the hedge fund strategies.

CER rank	Model	Lags	Predictors included							*H*	Annualized mean			Annualized volatility			Sharpe ratio			Annualized CER			Skewness	Kurtosis
			Term	Short	DY	SMB	BMX	HML	Mom		Mean returns	90% Conf. Int. - LB	90% Conf. Int. - UB	Vol	90% Conf. Int. - LB	90% Conf. Int. - UB	Sharpe ratio	90% Conf. Int. - LB	90% Conf. Int. - UB	CER (% Ann.)	90% Conf. Int. - LB	90% Conf. Int. - UB		
1	Expanding VAR	1	Y	Y	Y	N	Y	Y	Y	60	24.288	22.394	26.913	**95.354**	69.090	111.942	0.227	0.175	0.238	**32.380**	**30.881**	**37.571**	**0.357**	**1.858**
2	Expanding VAR	1	Y	Y	Y	N	N	Y	Y	60	20.900	19.398	22.477	**47.130**	**35.824**	**62.543**	0.387	0.261	0.414	**28.902**	**26.241**	**33.586**	**0.119**	**2.637**
3	Expanding VAR	1	Y	Y	Y	N	N	Y	N	60	20.641	18.184	23.149	**68.101**	44.919	**86.684**	0.264	0.194	0.270	**27.001**	**24.824**	**30.947**	**0.604**	**2.258**
4	Expanding VAR	1	Y	Y	Y	N	Y	Y	N	60	19.195	16.160	21.997	**68.520**	42.094	**85.244**	0.242	0.188	0.256	**22.944**	**20.802**	**27.080**	**0.505**	**1.855**
5	Expanding VAR	1	Y	Y	Y	Y	N	Y	Y	60	17.528	15.841	19.133	**35.661**	**26.682**	**47.311**	0.417	0.294	0.439	**22.933**	**20.369**	**27.556**	**0.117**	**2.266**
6	Expanding VAR	1	Y	Y	Y	N	N	N	Y	60	17.486	15.239	18.783	**43.036**	**23.381**	**55.001**	0.345	0.248	0.434	**21.250**	**18.556**	**27.224**	**0.667**	3.327
7	Expanding AR	1	-	-	-	-	-	-	-	60	6.860	11.558	1.124	109.180	55.277	142.255	0.039	0.006	0.139	**19.978**	**17.620**	**24.380**	**0.512**	3.009
8	Expanding VAR	1	Y	Y	Y	N	N	N	N	60	16.452	13.776	18.233	**39.415**	**25.347**	**52.707**	0.350	0.252	0.362	**18.754**	**16.105**	**24.735**	**0.228**	**2.216**
9	Expanding VAR	1	Y	Y	Y	Y	Y	Y	Y	60	15.289	13.641	16.489	**25.117**	**16.699**	**34.137**	**0.504**	**0.351**	0.544	**18.160**	**15.517**	**24.767**	**-0.590**	**2.695**
10	Expanding VAR	1	Y	Y	Y	Y	N	N	Y	60	15.523	13.254	16.643	**32.371**	**20.447**	**40.717**	0.398	0.297	0.432	**17.679**	**15.013**	**24.289**	**0.209**	**2.634**
11	Expanding VAR	1	Y	Y	Y	Y	N	N	N	60	14.940	12.437	16.373	**30.427**	**22.178**	**39.089**	0.404	0.305	0.435	**16.672**	**14.112**	**22.212**	**-0.122**	**2.110**
12	Expanding VAR	1	Y	Y	Y	N	Y	N	N	60	15.236	12.785	17.046	**33.468**	**22.891**	**44.650**	0.376	0.278	0.380	**16.504**	**13.862**	**23.054**	**0.149**	**2.276**
13	Expanding VAR	1	Y	Y	Y	N	Y	N	Y	60	14.665	12.458	15.772	**30.140**	**16.380**	**38.728**	0.399	0.296	0.507	**15.899**	**13.279**	**22.903**	**0.421**	3.187
14	Expanding VAR	1	Y	Y	Y	Y	Y	N	N	60	14.179	11.651	15.623	**27.895**	**20.075**	**35.801**	0.414	0.317	0.441	**14.865**	**12.320**	**20.903**	**-0.235**	**2.034**
15	Expanding VAR	1	Y	Y	Y	Y	Y	N	Y	60	13.961	11.386	15.076	**34.471**	**14.287**	**43.830**	0.328	0.250	0.531	**14.327**	11.758	**21.058**	**1.185**	4.232
16	Expanding VAR	1	Y	Y	Y	Y	N	Y	N	60	12.633	10.335	13.910	**21.420**	**14.515**	**27.523**	0.467	**0.367**	0.475	**13.828**	11.415	**19.279**	**-0.238**	**1.895**
17	Expanding VAR	1	Y	Y	Y	Y	Y	Y	N	60	12.898	10.555	14.424	**23.495**	**16.358**	**30.987**	0.437	0.338	0.443	13.354	10.871	**19.461**	**-0.098**	**2.089**
18	Expanding IID	0	-	-	-	-	-	-	-	60	8.045	3.598	9.960	**30.959**	**3.216**	**39.788**	0.175	0.167	0.745	4.640	4.488	6.450	**1.556**	3.755
	Median Expanding VAR performance									60	**15.289**	**13.254**	**16.643**	**34.471**	22.891	44.650	**0.367**	**0.278**	**0.387**	**18.160**	**15.517**	**24.380**	**0.209**	**2.266**

Figure 1

Recursive Optimal Portfolio Weights: Baseline Asset Menu (γ = 5)

The plots refer to an Expanding VAR(1) model with the term spread, the short-term nominal rate, and the dividend yield included as predictors. The Gaussian IID myopic demand is also plotted as a benchmark.

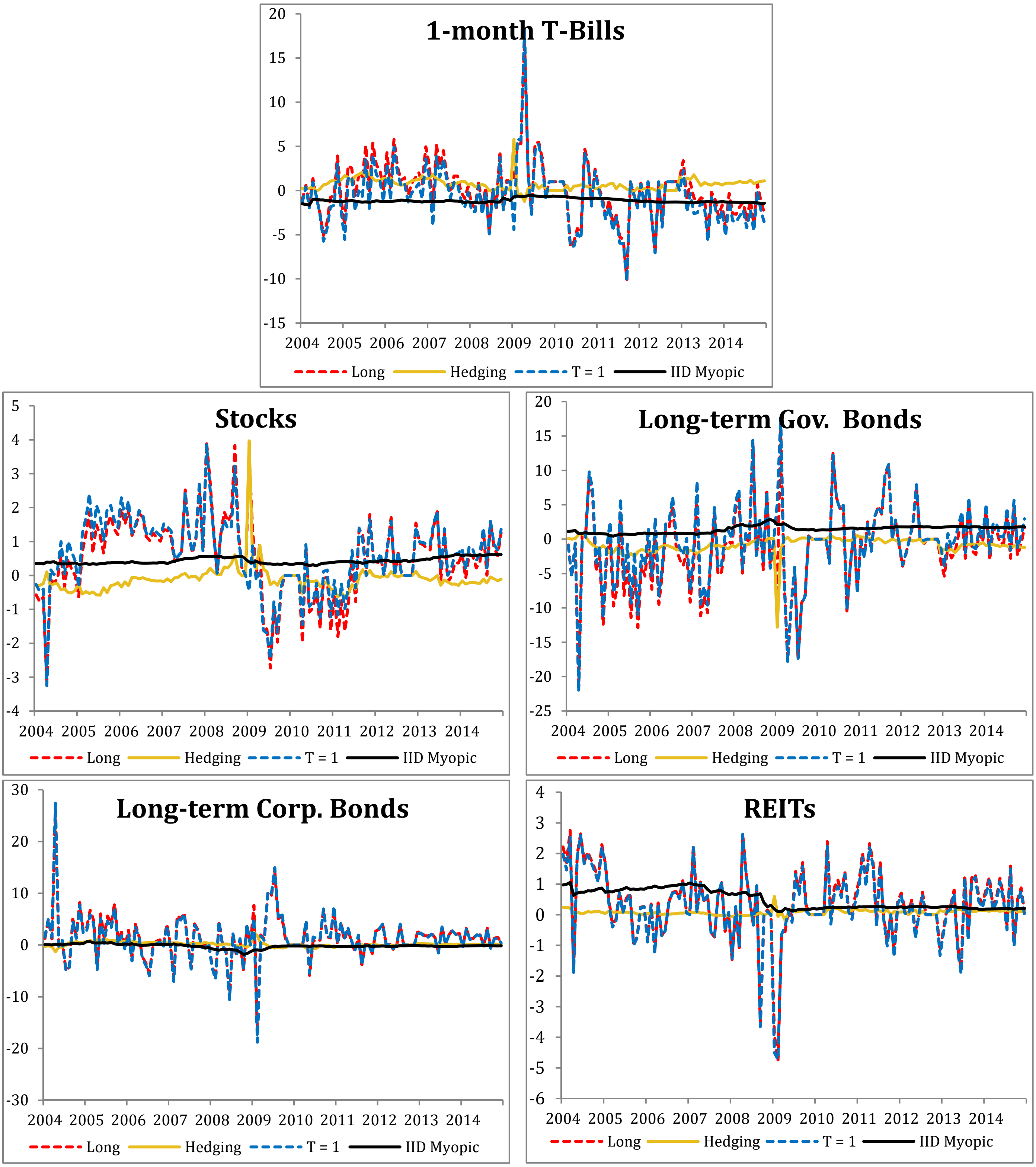

Figure 2

Recursive Optimal Portfolio Weights: HFRI Fund Weighted Composite Index (FWC) ($\gamma = 5$)

The plots refer to an expanding autoregressive model with only lagged asset returns. The Gaussian IID myopic demand is also plotted as a benchmark.

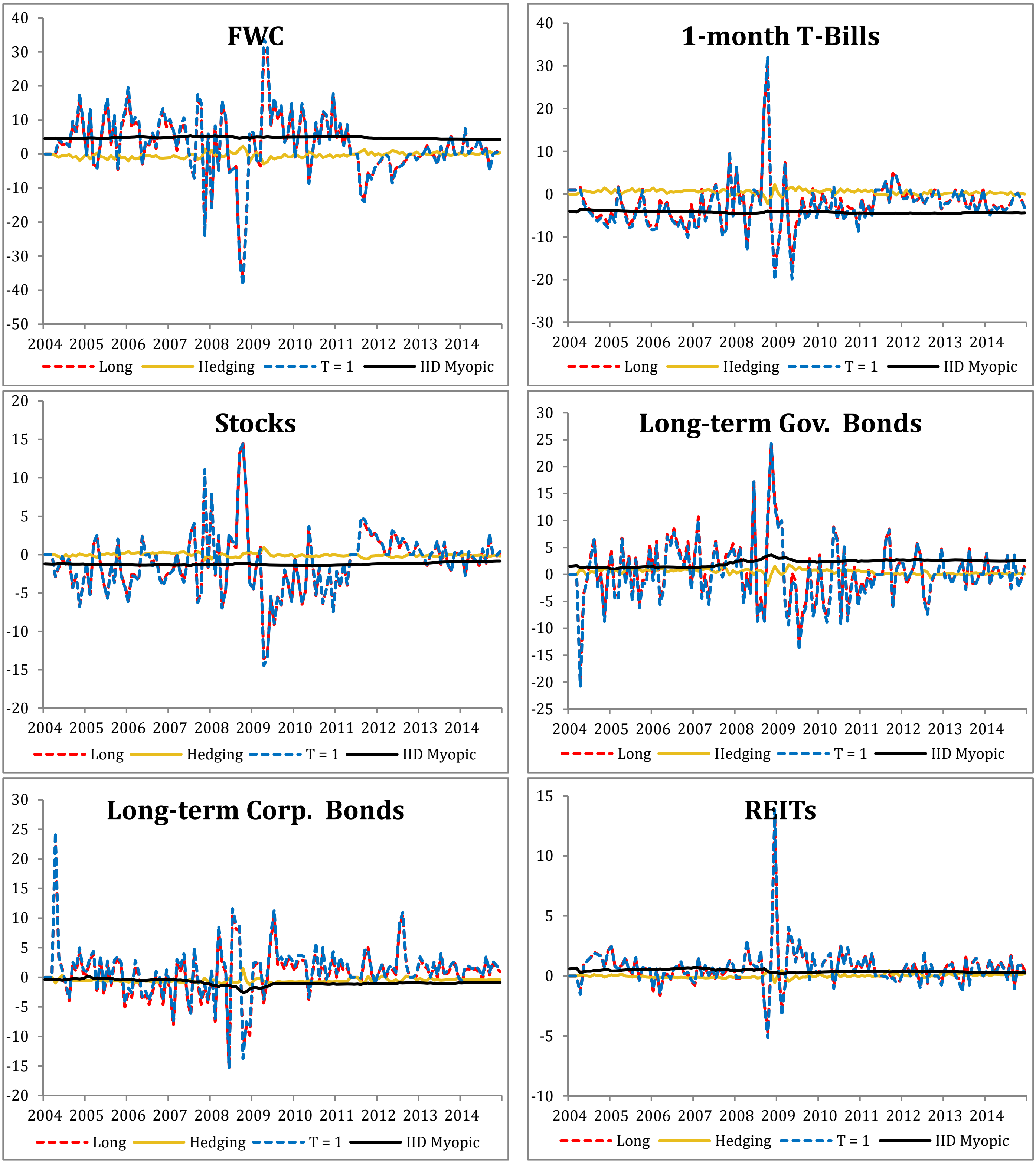

Figure 3

Realized, Recursive Optimal Portfolio Weights: HFRI Fund of Funds Composite (FFP) (γ = 5)

The plots refers to an expanding autoregressive model with only lagged asset returns. The Gaussian IID myopic demand is also plotted as a benchmark.

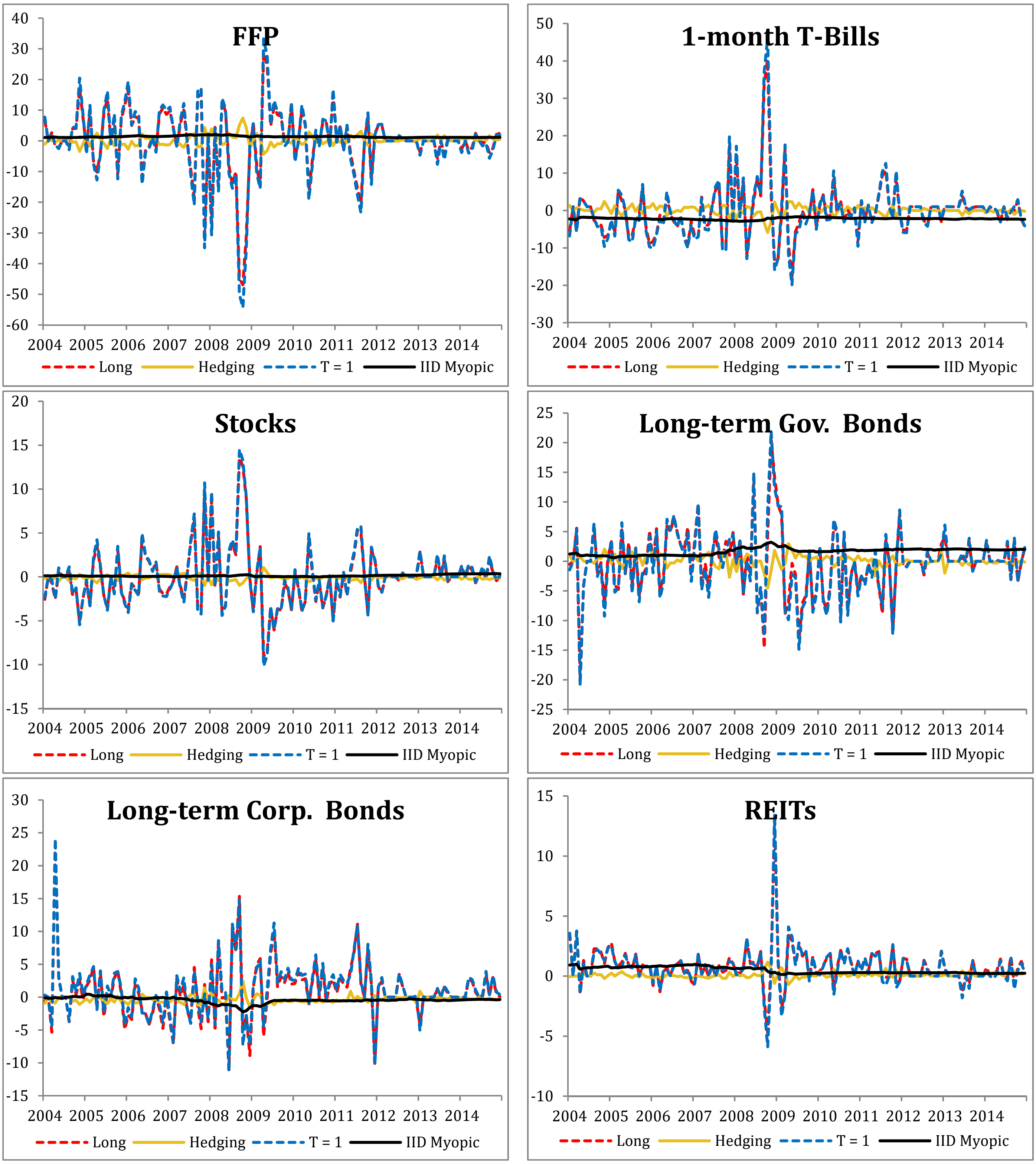

Figure 4

Comparisons of Realized Performance Indicators for the Top Models

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS performance measures obtained with references to a recursive portfolio exercises for the sample 2004:01 – 2014:12. Each measure refers to either a benchmark asset menu that excludes HF strategies or to extended menus that include either a composite value-weighted index of all HF strategies or to one strategy at the time. In the case of skewness and kurtosis, we report 90% confidence intervals based on a delta-method approximation of their standard error.

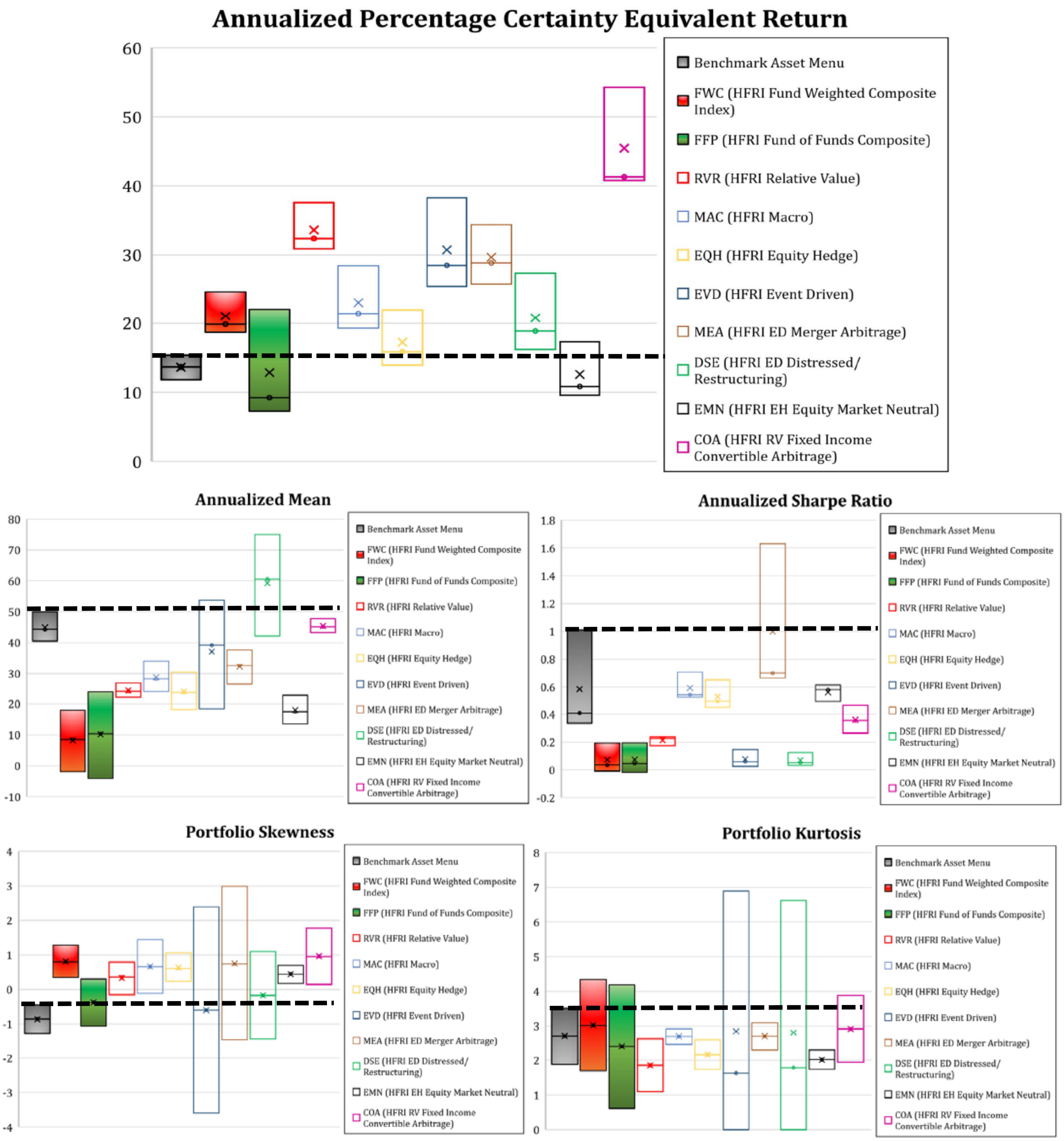

Figure 5

Comparisons of Realized Performance Indicators for the Median Expanding VAR Models

The plots represent the mean (as a solid horizontal line), median (as a cross), and realized 90% range (as a bin) of OOS performance measures obtained with references to a recursive, portfolio exercises for the sample 2004:01 – 2014:12. Each measure refers to either a benchmark asset menu that excludes HF strategies or to extended menus that include either a composite value-weighted index of all HF strategies or to one strategy at the time. In the case of skewness and kurtosis, we report 90% confidence intervals based on a delta-method approximation of their standard error.