Arbitrage Strategies between French Treasury Inflation Linked and Nominal Bonds: Solving the Puzzle

Béatrice de Séverac*

José S. da Fonseca**

Abstract

This paper examines mispricing between French Treasury inflation-linked bonds and nominal bonds. It investigates whether mispricing exists on the French bond market by constructing arbitrage strategies between these two asset types. We consider self-financing portfolios hedged against various types of interest rate risk, represented by different order durations. Implementation of arbitrage strategies is supported by an innovative zero-coupon pricing methodology based on piecewise constant forward rates. We find that arbitrage profits remain even after controlling for level, slope, and hump risks. Econometric analysis shows that changes in the difference between forward nominal and real rates explain the detected profits.

Keywords: arbitrage; duration; inflation-linked bonds; breakeven inflation rate; forward breakeven inflation; real interest rates; inflation risk.

JEL Classification: E43; G01; G12

*bseverac@parisnanterre.fr; Université de Paris Nanterre, France; 200 Avenue de la République, Nanterre cedex, Nanterre 92001, France.

**jfonseca@fe.uc.pt; Center for Business and Economics Research (CeBER), Faculty of Economics, University of Coimbra, Portugal, Av. Dias da Silva 165, Coimbra, ZIP Code 3004-512, Portugal.

Corresponding author : Béatrice de Séverac

1. Introduction

The Treasury bond and Treasury inflation-linked (IL) bond markets are two of the largest and most actively traded fixed-income markets in the world. Despite this, there is persistent mispricing across these two markets (Fleckenstein et al. (2014)). A large amount of research suggests that IL bonds are less liquid than conventional Treasury bonds (Auckenthaler et al. (2015), Fleming and Krishnan (2012), Gürkaynak et al. (2010), Campbell et al. (2009)). This finding has several implications. First, the illiquidity drives a wedge between real yields and IL bond yields (Grishchenko and Huang (2013)). Second, it complicates estimation of the breakeven inflation rate, which reflects inflation expectations and represents compensation to investors for bearing inflation risk. This compensation includes both expected inflation and an inflation risk premium because of inflation uncertainty. The inflation risk premium may include a component related to the illiquidity of the IL bond market. However, the level of bias incurred by these frictions on IL bond prices remains a topic of debate, as no consensus has emerged on estimates of the magnitude of the liquidity premium embedded in these securities. Since it can be accepted that market narrowness can deviate market prices from their fair prices, this mispricing can lead to arbitrage opportunities. Illiquidity can drive market prices down and IL bond yields up (Grishchenko and Huang (2013)). To explore whether there is mispricing, Fleckenstein et al. (2014) implement arbitrage strategies that highlight the possible divergence between the price of nominal Treasury bonds and inflation swapped TIPS that exactly replicate conventional cash flows. These authors conclude that nominal bonds are consistently overpriced relative to TIPS. Simon (2015) attempts to explain this TIPS–Treasury puzzle. This author finds that the difference in liquidity and credit risk premium in IL and nominal bonds explains the persistent nature of the puzzle. Going further, Driessen, Nijman and Simon (2017) show that the level of liquidity affects TIPS, whereas swap yields include a liquidity premium. They also recall that the price of an asset can differ from its replicated cash flow counterpart due to specific liquidity effects.

The present work differs from the aforementioned articles since it sets up a self-financing strategy based directly on the underlying assets constituted by nominal and IL Treasury bonds to compare the price of these two assets. One of the main purposes here is to identify and explain arbitrage opportunities between the French nominal and IL Treasury bond markets. To our knowledge, this is the first article to study directly the no-arbitrage relation between these two bond market segments.

This study builds on prior and concurrent research from three strands of the literature on IL bonds. The first relies on the vast literature on inflation expectations, with the inflation risk premium obtained based on IL bond market prices or not. In this strand, Grishchenko and Huang (2013) estimate the magnitude of the inflation risk premium. Pflueger and Viceira (2001b) as well as Gürkaynak et al. (2010) and

Zeng (2013) use breakeven inflation rates as relevant time series to determine the liquidity premium in IL bonds.

The second strand addresses the performance of IL bonds as an asset class and their role in portfolios. Recent studies that favor IL bonds include Mkaouar, Prigent and Abid (2017), Brière and Signori (2012), and Cartea, Saúl, Toro (2002), which illustrate the benefits of introducing TIPS in portfolios.

The third strand focuses on the pricing of these assets. The first authors to set an arbitrage model to price TIPS are Jarrow and Yildirim (2003) and Chen, Liu, Cheng (2010). More recently, affine term structure models are developed by D'Amico, Kim and Wei (2018), Andreasen, Christensen and Cook (2017). Abrahams, Adrian, Crump, Moench and Yu (2016) present a Gaussian affine model for pricing IL bonds that adjusts for the relative illiquidity of this type of bond and generates estimates of the inflation premium. A special focus on the relative pricing of nominal and indexed Treasuries is implemented by Fleckenstein et al. (2014), Simon (2015), and Driessen et al. (2017). The approach of the present work is closely related to these papers.

To set up arbitrage strategies, self-financing bond portfolios are constructed. To hedge these portfolios against interest rate risk, three hedging methods are implemented that rely on the number of duration orders measures used and the corresponding hedging ratios. The first uses a duration measure of order one (D^1) measuring interest rate level–risk. The second uses D^1 and D^2 (second-order duration), where D^2 measures yield curve slope risk. The third aims to hedge the portfolio against level, slope, and hump risks using respectively D^1, D^2, and D^3. Additionally, each hedging method includes a duration measuring IL bond sensitivity to the inflation indexation factor. However, the duration measures of interest rate risk depend on the considered pricing model. Jarrow and Yildirim (2003) develop a model of IL bonds consistent with the Vasicek duration. Following Jarrow (2010), arbitrage strategies are built that replicate a French nominal Treasury bond with several IL Treasury bonds, based on Vasicek duration. To determine the impact of the duration measures on the results of the arbitrage strategies, we implement a Fisher-Weil duration measure alternately to Vasicek duration. Thus, two kinds of duration measure are considered and each allows us to calculate in a specific way D^1, D^2, and D^3.

Duration measures depend on (nominal and real) zero-coupon yields. Real zero-coupon yields are not available on the French bond market and must be estimated. They are deduced thanks to an innovative method that modifies and widens the Jarrow and Yildirim (2003) method used to extract forward nominal and real zero-coupon yields from market prices. Our estimations rely on market prices of nominal and IL bonds issued by the French Treasury on a daily basis from January 1, 2013 to December 31, 2015.

Our main findings are as follows. First, we show that arbitrage profits exist even when the proportion of IL bonds is determined to avoid the risk of shift in level, slope, and hump of the nominal and real yield curve. This result must be compared with those of Fleckenstein et al. (2014), who show that American nominal Treasury bonds are overpriced relative to TIPS.

Second, we demonstrate that those profits are explained by forward breakeven inflation (FBEI), calculated as the difference between forward nominal and forward real zero-coupon yields, and which represent inflation expectations at future dates plus an inflation risk premium. Forward breakeven inflation computed for this purpose refers to, respectively, 1 year, 3 years, 5 years, and 7 years in the future, and each FBEI variable has a 1-year term. The choice of these FBEI variables aims to explain the arbitrage between nominal and IL bonds with the difference between forward rates extracted from the nominal and real term structures at non-overlapping maturity segments covering the short-, medium-, and long-term.

This paper makes several contributions. First, it contributes to the literature on the pricing of IL bonds by comparing the price of these assets with their nominal counterparts. Second, it shows that the type of duration measure, i.e., Vasicek duration versus Fisher-Weil duration, has no significant impact on the level of the noted arbitrage profits. Nevertheless, the introduction of higher-order durations improves significantly the arbitrage strategies. Third, the methodology employed to extract nominal and real zero-coupon yields fits perfectly the market prices of the two securities used, e.g., nominal and IL coupon-bearing bonds. This methodology imposes no assumption on the term structure shape, and it assumes only piecewise constant forward rates within the maturity segments between coupon bonds with neighboring maturities.

This article proceeds as follows. Section 2 begins with a formal analysis of the problem of hedging nominal bonds with IL bonds. Section 3 presents the data and estimations of the nominal and real zero-coupon yield curves. Section 4 describes the results of the arbitrage strategies and their analyses. Finally, Section 5 describes our conclusions.

2. Arbitrage strategy

We start with a formal analysis of the problem of setting an arbitrage strategy. The continuous time model of Jarrow and Yildirim (2003), henceforth JY2003, provides a convenient framework. This will lead to the definition of the long/short replicating strategy, which we test empirically.

2.1 Nominal and IL bond price

Our notations used have the following meanings:

r_n: nominal spot interest rate

r_r: real spot interest rate

$P_n(t,\tau)$: price at time t of a nominal zero-coupon bond (i.e., nominal discount function of a coupon bond payoff) maturing at time $t+\tau$

$P_r(t,\tau)$: price at time t of a real zero-coupon bond (i.e., real discount function) maturing at time $t+\tau$

IF_t: indexation factor of an indexed bond, which is the ratio between the value of the harmonized CPI (Consumer Price Index) excluding tobacco at time t (I_t) and the value of the same index at bond issuance date t_0 (I_{t0})

$B_{nom}(t,m)$: price on date t of a nominal bond that periodically pays C euros on each date $t+\tau$ between t and $t+m$, and C+VF euros on maturity date $t+m$; bond current price is equal to the sum of the present value of its payoffs, given by

$$B_{nom}(t,m) = \sum_{\tau=1}^{m} CP_n(t,\tau) + VFP_n(t,m) \qquad (1)$$

$B_{ilb}(t,m)$ is the price at date t of a Treasury inflation-indexed bond. The bond periodically pays C units of the CPI on each date $t+\tau$ between t and $t+m$, and C+VF units of the CPI on maturity date $t+m$. Taking into account the current value of the bond indexation factor IF_t, the bond price is equal to the sum of the present value of its nominal payoffs, given by:

$$B_{ilb}(t,m) = IF_t\left[\sum_{\tau=1}^{m} C\,P_r(t,\tau) + VF\,P_r(t,m)\right] \qquad (2)$$

In the context of IL bonds, breakeven inflation plays a key role and is defined by the difference between nominal and real interest rates with the same maturity:

$$bei(t,\tau) = r_n(t,\tau) - r_r(t,\tau) \qquad (3)$$

Breakeven inflation comprises expectations of inflation for maturity τ plus an inflation risk premium.

JY2003 set forth a pricing model for TIPS that relies on a foreign currency analogy, like Amin and Jarrow (1991), who price contingent claims on foreign currencies in an HJM context. JY2003 consider a hypothetical cross-currency economy under a no-arbitrage assumption, where the nominal currency corresponds to the domestic currency, real currency to the foreign currency, and the inflation index to the spot exchange rate. Hence, the model developed by JY2003 is a three-factor model.

Under historical probability P, the three-factor model of JY2003 is defined by the dynamics of its factors; the dynamic of the real forward rate is:

$$df_r(t,T) = \alpha_r(t,T)dt + \sigma_r(t,T)dw_r(t) \qquad (4)$$

and the dynamic of the nominal forward rate is:

$$df_n(t,T) = \alpha_n(t,T)dt + \sigma_n(t,T)dw_n(t) \qquad (5)$$

Finally, the evolution of the inflation index, which allows the logarithm of the inflation index process to be normally distributed is:

$$dI(t)/I(t) = \mu_I(t)dt + \mu_I(t)dw_I(t) \qquad (6)$$

where $dw_r(t)$, $dw_n(t)$ and $dw_I(t)$ are Brownian motions with the following correlations: $dw_n(t)dw_r(t) = \rho_{nr}dt$, $dw_r(t)dw_I(t) = \rho_{rI}dt$.

The difference between the nominal forward rate *($f_n(t,T)$)*, and the real forward rate *($f_r(t,T)$* is the FBEI *noted $f_{bei}(t,T)$*, which comprises inflation expectations plus an inflation risk premium.

To make the JY2003 model tractable, it is necessary only to specify $\sigma_r(t,T)$, the volatility function of the real forward rates and $\sigma_n(t,T)$, the volatility function of the nominal rates. We impose, like JY2003, an exponentially declining volatility for both functions:

$$\sigma_i(t,\tau) = \sigma_i e^{-\alpha_i(\tau)} \text{ for } i = r \text{ (real), } n \text{ (nominal)} \qquad (7)$$

which is an extension of the Vasicek (1977) model for the term structure, where σ_r and α_r (respectively σ_n and α_n) are constants. In this particular case, the zero-coupon bond return $dP_i(t,T)/P_i(t,T)$ follows a normal distribution.

2.2. Interest rate risk measures: duration measures

Two different types of duration measure are used. One is derived from the JY(2003) model. The other relies on the Fisher-Weil duration.

The Vasicek duration

The volatility function chosen by JY2003 is that of the extended Vasicek model. This exponential volatility implies that the duration of the real and nominal zero-coupon bond is:

$$D_i(t,\tau) = \left|\frac{dP_i(t,\tau)}{dr_i}\frac{1}{P_i(t,\tau)}\right| = \left(\frac{1-e^{-\alpha_i\tau}}{\alpha_i}\right) \text{ for } i = r \text{ (real), } n \text{ (nominal)} \qquad (8)$$

where α_r (α_n) are the elasticities of reversion of the real (nominal) short-term interest rate to its long-term value.

Fisher-Weil duration

The Fisher-Weil duration is based on the assumption that stochastic changes in interest rates always consist of parallel shifts of the term structure. The Fisher-Weil duration of a zero-coupon bond is the absolute value of the sensitivity of its price relative to the corresponding spot rate, that is:

$$D_i(t,\tau) = \left|\frac{dP_i(t,\tau)}{dr_i}\frac{1}{P_i(t,\tau)}\right| = \tau \qquad \text{for } i = r \text{ (real)}, n \text{ (nominal)} \qquad (9)$$

This equality results from the general definition of the price of $P(t,\tau) = e^{-\tau r(t,\tau)}$, where $r(t,\tau)$ is the τ period maturity spot rate.

The duration of the nominal and IL coupon-bearing bonds are weighted averages of the durations of their payoffs, which can be derived using equations (1) and (2).

Higher-order durations

Duration is the key variable in bond portfolio immunization strategies, whether used as the single interest rate risk measure, as in Fong and Vasicek (1984), or with higher-order interest rate risk measures, as in Nawalkha (1995) and Nawalkha, De Soto and Zhang (2003). Higher-order durations are based on higher-order price derivatives. Thus, a k^{th} order duration is based on the k^{th} order price derivative, i.e., for a zero-coupon bond:

$$D_i^k(t,\tau) = \left|\frac{d^k P_i(t,\tau)}{(dr_i)^k}\frac{1}{P_i(t,\tau)}\right| \qquad \text{for } i = r \text{ (real)}, n \qquad (10)$$

The higher-order durations of coupon-paying nominal and IL bonds are calculated with the same procedure used to calculate the first-order duration of these bonds. According to this procedure, the k order duration of a coupon bond, whether nominal or IL, is the weighted average of the k order duration of its payoffs.

Fong and Vasicek (1984) show that immunization strategies need to supplement the first-order duration, $D^{(1)}$, with minimization of the second-order duration, $D^{(2)}$ (convexity), to minimize the portion of interest rate risk not captured by $D^{(1)}$. The direct relation between immunization risk and convexity is confirmed empirically by Lacey and Nawalkha (1993), Nawalkha (1995) and Nawalkha *et al.* (2003), who propose the use of duration vectors composed of durations of different order to take into account different types of shifts on the term structure, e.g., level, slope, and hump, whose risks are measured, respectively, by D^1, D^2, and D^3.

This general definition of the duration of order k, D^k, can be specified in the case of JY2003 and Fisher-Weil.

For the JY2003 model, the order k duration is defined as follows:

$$D_{r_i}^k = \left(\frac{1-e^{-\alpha_i \tau}}{\alpha_i}\right)^k \qquad \text{for } i = r \text{ (real)},\ n \text{ (nominal)} \qquad (11)$$

In case of a parallel shift shock on the term structure, the order k duration is defined as follows:

$$D_{r_i}^k = \tau^k \qquad \text{for } i = r \text{ (real)},\ n \text{ (nominal)} \qquad (12)$$

2.3. Hedging strategies

Several strategies are employed to construct a zero-value, self-financing portfolio with one nominal bond and several IL bonds whose proportions ensure that the portfolio has a zero derivative relative to one or more risk factors. Hence, three sets of hedged portfolios are set up, one using only D^1 durations, the second using D^1 and D^2 durations, and the third using D^1, D^2, and D^3 durations. Introduction of D^1, D^2, and D^3 is relevant since the prices of nominal and IL bonds are affected by changes in term structure of interest rate due to the discount factor effect (Xu (2011)).

The proportions (w_i^{ilb}) of IL bonds required to replicate a nominal bond in portfolios based on duration of order k must respect the following system of k+2 equations:

$$(1)\ \ 1 + \sum_{i=1}^{i=k+2} w_i^{ilb} = 0$$

$$(2)\ \ D_{nom,r_r}^j + \textstyle\sum_{i=1}^{i=k+2} w_i^{ilb} D_{ilb,r_r}^j = 0 \quad \text{for j = 1 to k} \qquad (13)$$

$$(3)\ \ D_{nom,IF} + \textstyle\sum_{i=1}^{i=k+2} w_i^{ilb} D_{ilb,IF} = 0$$

where:

$D_{nom,rr}$ is the duration of the nominal bond relative to the real spot interest rate r_r, which is zero by definition;

$D_{ilb,rr}$ is the duration of the IL bond relative to the real rate r;

$D_{nom,IF}$ is the duration of the nominal bond relative to the inflation factor IF, which is zero by definition; and

$D_{ilb,IF}$ is the duration the IL bond relative to the inflation factor IF, whose representation is:

$$D_{ilb,IF} = \frac{1}{IF_t} \qquad (14)$$

Frequently, nominal interest rates are correlated with real interest rates and inflation. Hence, both types of bond depend directly or indirectly on three factors: nominal interest rates, real interest rates, and inflation.
According to JY2003, the IL zero-coupon bond does not depend directly on the nominal spot rate. Hence, the IL term structure depends only on two factors: the real rate and inflation. Symmetrically, the nominal zero-coupon bond depends on the nominal spot rate but does not depend on the real spot interest rate or the inflation factor. This implies that IL bonds have zero derivatives relative to nominal interest rates. Conversely, nominal bonds have zero derivatives relative to real interest rates and to inflation. Thus, $D_{nom,rr} = 0$ and $D_{nom,IF} = 0$.

2.4 Estimation of nominal and real zero-coupon yields

No data on real zero-coupon yields are public or available on the French bond market. Therefore, it is necessary to estimate the real zero-coupon yield curve from the market prices of French IL bonds. To avoid differences in estimating method, we apply the same method to estimate the nominal zero-coupon yield curve.

JY2003 propose a method to estimate zero-coupon bond prices that relies on a discrete time approach to modeling forward interest rates and, additionally, it accepts the assumption that forward rates are constant within piecewise segments of the maturity spectrum. Under this method, the theoretical price function of a coupon bond, both nominal and indexed, has the following representation:

$$B(t,m)=\sum_{\tau=1}^{m}C_{\tau}\exp\left(-\left(\sum_{i=1}^{K}f_{i}\phi(\tau,i)\right)\right) \qquad (15)$$

where C_τ is the bond payoff at date $t+\tau$, K is the number of piecewise maturity segments of constant forward rates, and f_i is the constant forward rate to be observed within the i^{th} maturity segment and $\phi(\tau,i)$ is the part of i maturity segment covered by the maturity of C_τ payoff. The lower limit of the shortest maturity segment is zero and its upper limit is $m(1)$. Similarly, the upper limits of the other maturity segments are $m(i)$, for i = 2, …K. In many cases, the C_τ payoff maturity covers more than one maturity segment. Hence, the part of the i^{th} maturity segment, $m(i)$, covered by the C_τ payoff maturity is defined as follows:

$$\begin{aligned}
&\phi(\tau,i)=m(i)-m(i-1)\text{ if } \tau\geq m(i),\\
&\phi(\tau,i)=\tau-m(i-1)\text{ if } m(i)>\tau\geq m(i-1)\text{, and}\\
&\phi(\tau,i)=0\text{ if } \tau<m(i-1)
\end{aligned} \qquad (16)$$

This paper proposes an alternative to JY2003 that consists of setting the limits between two piecewise segments at the maturity dates of the coupon bonds. Hence, the upper limit of the first maturity segment is the maturity of the coupon bond with shortest maturity, represented by $m(b(1))$. Similarly, the upper limits of the other maturity segments $m(b(i))$, for i = 2, …K, K being the number of bonds in the sample, are adjusted to the maturity of the K coupon bonds. Under this approach, the portion of the i^{th} maturity segment covered by the maturity of C_τ, $\phi(\tau,i)$ is defined as follows:

$$\phi(\tau,i) = m(b(i)) - m(b(i-1)) \text{if } \tau \geq m(b(i)),$$
$$\phi(\tau,i) = \tau - m(b(i-1)) \text{ if } m(b(i)) > \tau \geq m(b(i-1)), \text{ and} \tag{17}$$
$$\phi(\tau,i) = 0 \text{ if } \tau < m(b(i-1))$$

While the piecewise maturity limits in the JY 2003 model, *m(i)*, are chosen arbitrarily, in the innovative procedure proposed in this article, the corresponding *m(b(i))* limits are adjusted to the maturities of the coupon bonds in the sample. This procedure has the advantage of giving estimated prices that match perfectly the market prices.

3. Data description and estimation of zero-coupon yields

3.1. Data presentation and preliminary statistics

A database was constructed that comprises daily prices covering the period January 1, 2013 through December 31, 2015, or 783 daily market prices for nominal and IL bonds issued by the French Treasury. Market prices and inflation factors are extracted from the Datastream data base (Thomson Reuters). The inflation factor is the ratio between the value of harmonized CPI excluding tobacco on date t and its value at the issuance date t_0 of the IL bond. This inflation factor, or IF, is applied to IL bonds according to equation 2 and allows us to protect investor cash flows against inflation. The same database is used both for extracting zero-coupon yields and setting up hedging strategies.

The period begins in 2013 because too few French IL bonds were traded before that year. Two types of IL bonds are available on the French market. Some are indexed to the domestic Consumer Price Index (CPI) and named OATi. Others have been issued more recently (since July 2001) and are indexed to the euro area Harmonized Index of Consumer Prices and named OAT€i. As in Pericoli (2014), these two types of IL bonds are included in our sample. Before including these IL bonds, we took the precaution to calculate the correlation coefficients between the variations of the indexation factors (IF). As shown in Table 1, these coefficients are very high, not only within each group (OATis/OAT€is), but also between the two groups.

Table 1: Correlation coefficients among the indexation factor changes

	OAT€i-18	OAT€i-20	OAT€i-22	OAT€i-24	OATi-17	OATi-19	OATi-21	OATi-23
OAT€i-18	1							
OAT€i-20	0.999848	1						
OAT€i-22	0.999845	0.999859	1					
OAT€i-24	0.999805	0.99985	0.999821	1				
OATi-17	0.820918	0.82084	0.820749	0.820575	1			
OATi-19	0.820368	0.820282	0.820183	0.820008	0.999704	1		
OATi-21	0.820933	0.820845	0.82076	0.820574	0.999683	0.999666	1	
OATi-23	0.820436	0.820319	0.820251	0.820069	0.999664	0.999636	0.999628	1

*In this table, the different IL bonds are indicated by their inflation mode, I or EI, and their maturity dates.

For comparison purposes, we use a similar number of French Treasury nominal and IL bonds, even though there are many more available nominal bonds (OATs). Furthermore, the chosen nominal bonds have maturities close to those of the IL bonds. A total of 13 nominal bonds and 11 IL bonds with maturity ranges from 2016 to 2029 are taken into account in our sample.

3.2. Real and nominal zero-coupon yield estimations

As mentioned in Section 2, two alternative pricing methods are used to extract nominal and real zero-coupon bond prices: JY2003 piecewise constant forward rates and modified JY2003 piecewise constant forward rates. The estimations result from minimization of the sum of squared differences between market prices and estimated prices based on extracted zero-coupon yields, which in the case of IL bonds corresponds to:

$$\min \sum_{q=1}^{N} \left\{ B_{ilb}\left(t, m_q\right) - IF_t \left[\sum_{\tau=1}^{m} CP_r\left(t, \tau_q\right) + VFP_r\left(t, m_q\right) \right] \right\}^2 \tag{18}$$

and in the case of nominal bonds, corresponds to:

$$\min \sum_{q=1}^{N} \left\{ B_{nom}\left(t, m_q\right) - \left[\sum_{\tau=1}^{m_q} CP_n\left(t, \tau_q\right) + VFP_n\left(t, m_q\right) \right] \right\}^2 \tag{19}$$

where N is the number of bonds used in the estimation. To compare the ability of the implemented pricing procedures to reproduce market prices, the sum of the squared errors of each estimation is divided by the number of bonds used to compute the squared error per bond, *SQE*, which, for inflation-indexed bonds, is

$$SQE_{ilb} = \frac{\left\{ B_{ilb}\left(t, m\right) - IF_t \left[\sum_{\tau=1}^{m} CP_r\left(t, \tau\right) + VFP_r\left(t, m\right) \right] \right\}^2}{N} \tag{20}$$

and for NOMs is

$$SQE_{nom} = \frac{\left\{ B_{nom}\left(t, m\right) - \left[\sum_{\tau=1}^{m} CP_n\left(t, \tau\right) + VFP_n\left(t, m\right) \right] \right\}^2}{N} \tag{21}$$

The statistics (mean, standard deviation, maximum, and minimum) of SQE_{ilb} and SQE_{nom} related to the two models estimated are presented, respectively, in Table 2.

Table 2: Statistics for *SQE*

	SQE_{ilb}		SQE_{nom}	
	J2003Y	Modified JY2003	JY2003	Modified JY2003
Mean	28.26182	2.63588 E 10^{-5}	0.315437	1.29826E 10^{-5}
St. dev.	1.213068	0.000233141	0.781258	3.1909E 10^{-5}
Max.	36.3141	0.006450909	7.767054	0.000754167
Min.	26.07986	4.78232 E10^{-7}	0.030788	3.60719E 10^{-7}

Comparison of the SQE statistics presented in Table 2 confirms that the modified JY2003 method fits market prices almost perfectly. These results also show that the methods perform much better for nominal bonds than for indexed bonds.

Figures 1 through 4 graph the term structure generated from the two estimation methods for maturities ranging from 1 to 15 years.

Figure 1. JY2003
Nominal Term Structure

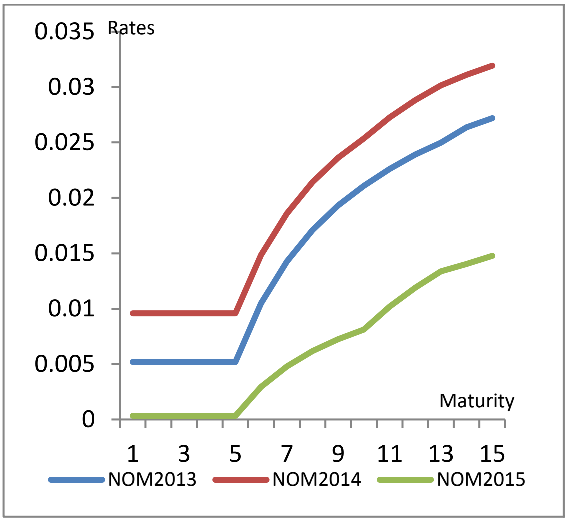
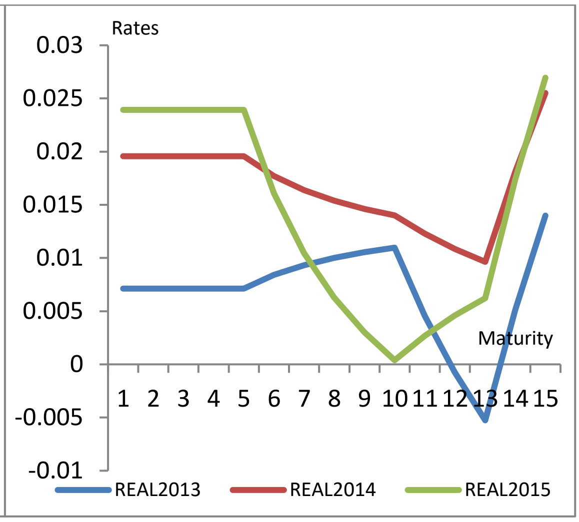


Figure 2. JY2003
Real Term Structure



NOM2013=Nominal term structure on the 1st day of 2013 NOM2014=Nominal term structure on the 1st day of 2014 NOM2015=Nominal term structure on the 1st day of 2015	REAL2013=Real term structure on the 1st day of 2013 REAL2014=Real term structure on the 1st day of 2014 REAL2015=Real term structure on the 1st day of 2015

Figure 3. Modified JY2003 Nominal
Term Structure

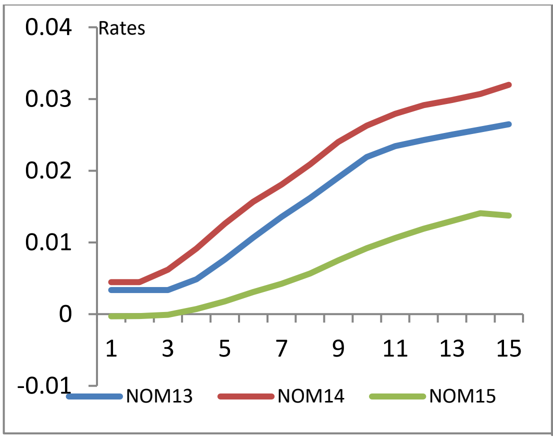
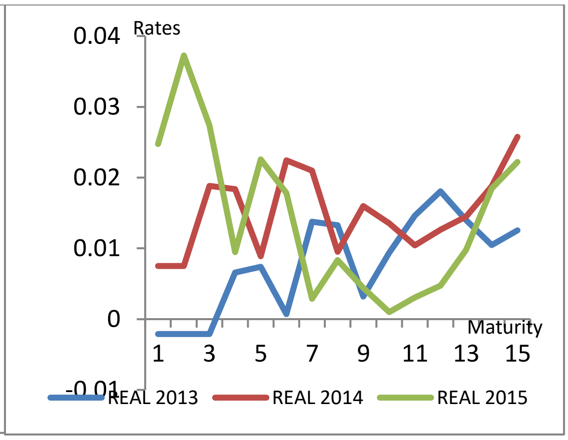


Figure 4. Modified JY2003 Real Term
Structure



NOM2013= Nominal term structure on the 1st day of 2013 NOM2014= Nominal term structure on the 1st day of 2014 NOM2015= Nominal term structure on the 1st day of 2015	REAL2013=Real term structure on the 1st day of 2013 REAL2014= Real term structure on the 1st day of 2014 REAL2015= Real term structure on the 1st day of 2015

Comparing these two sets of graphics, the nominal term structure is much smoother with the modified JY2003 model. On the other hand, yield curves present more landings, but inside the same shape.

Although the figures presented above evidence the changes in interest rate curves at one-year intervals, they cannot replace the information provided by the statistics on the zero-coupon interest rates shown in Tables 3 and 4.

Table 3. Statistics on zero-coupon spot rates given by the JY2003 method

	Real Interest Rates				*Nominal Interest Rates*			
Maturity	*5 Years*	*7 Years*	*10 Years*	*15 Years*	*5 Years*	*7 Years*	*10 Years*	*15 Years*
Mean	0.017576	0.01016	0.004599	0.025551	0.003915	0.010903	0.016144	0.022553
St. dev	0.004442	0.004226	0.007745	0.012662	0.003834	0.005486	0.00686	0.007549
Kurtosis	0.410469	-0.14496	-1.29447	124.2216	-0.64055	-1.46532	-1.41558	-1.17615
Skewness	-0.92331	-0.49948	-0.29193	-6.06007	0.397301	-0.00514	-0.10033	-0.3115

Table 4. Statistics on zero-coupon spot rates given by the modified JY2003 method

	Real Interest Rates				*Nominal Interest Rates*			
Maturity	*5 Years*	*7 Years*	*10 Years*	*15 Years*	*5 Years*	*7 Years*	*10 Years*	*15 Years*
Mean	0.013968	0.01020	0.005579	0.02263	0.006093	0.010265	0.016984	0.021749
St. dev	0.000348	0.000738	0.000286	0.001548	0.000154	0.00019	0.000253	0.000291
Kurtosis	-0.41486	415.514	74.38948	744.1544	-1.07922	-1.45824	-1.4161	-1.44479
Skewness	0.838013	-17.3448	26.90998	-6.06007	0.262618	-0.00211	-0.12404	-0.23985

The statistics shown in Tables 3 and 4 confirm, over the entire sample period, the monotonic increase in the nominal term structure, while a V shape dominates the real term structure, as illustrated by Figures 2 through 4. Kurtosis is also much higher for IL bonds than for nominal bonds. Similar results are observed in Pericoli (2014) on the French market between 2004 and 2014, which the author explains as resulting from the large segmentation and low liquidity of euro area inflation-linked bond markets.

4. Results and analysis of arbitrage strategies

The results from each strategy are presented, followed by analysis.

4.1 Implementation and results of the arbitrage strategies

Two different types (A and B) of self-financing portfolio are constructed. Types A and B aim to hedge the same nominal bond, but they comprise different IL bonds. Type A portfolios include the IL bonds with the highest maturities available in our sample, while type B portfolios include IL bonds whose maturities are lower than in type A portfolios.

Table 5: Portfolio constituents

Bond	Type A	Type B
Nominal bonds	OAT 2010-2 1/2%-2020	OAT 2010-2 1/2%-2020
IL Bonds	OAT€i-2012-1/4%-2018	BTAN I 2011-0.45%-2016
	OAT€i-2004-2 1/4%-2020	OAT-EI 2012-1/4%-2018
	OATi-2012-0.1%-2021	OAT€i-2004-2 1/4%-2020
	OATi-2008- 2.1%-2023	OAT-I 2012-0.1%-2021
	OATi-2011- 1.85%-2027	OAT-I 2008-2.1%-2023

According to JY2003, the entire term structure is governed by the short-term interest rate whose stochastic process is driven by the variable's elasticity of return to its long-term normal value, parameter α.

The first duration measure, derived from JY2003, is implemented with two different values of α ($\alpha = 0.005$, and $\alpha = 0.1$). Different values for α enable us to determine whether the speed of interest rate convergence to its normal value significantly affects the results (profit or loss) of the hedged portfolio. The second duration measure relies on the Fisher-Weil duration, which supposes that the most frequent shocks to the term structure consist of parallel movements of the entire term structure.

The hedged portfolios are constructed on a daily basis and their results (profit or loss) are based on market prices at the end of the holding period plus accrued interest during the portfolio holding period. Arbitrage results ($APG_{t,t+\Delta}$) are the difference between the portfolio value on the liquidation date and the portfolio value on the date the arbitrage is implemented (initial date). By construction, the initial portfolio value is zero.

$$APG_{t,t+\Delta} = V_{t+\Delta} - V_t \tag{22}$$

Where $APG_{t,t+\Delta}$ is the arbitrage result and Δ is the holding period.

Two holding periods are taken into account: 1-day and 10-day. This procedure yields 782 arbitrage results for the 1-day and 778 for the 10-day holding periods.

The number of IL bonds included in portfolios A and B depends on the order of the duration implemented. Three IL bonds are necessary with D^1, four with D^1 and D^2, and five for strategies with D^1, D^2, and D^3. While average maturity does not vary for nominal bonds since only one nominal bond is present (OAT 2010- 2 ½%-25-10-2020), average maturity differs and depends both on the order of duration implemented and on the nature of the IL bonds included, as shown in Table 6 below.

4.2. Arbitrage results

Table 6 shows basic statistics on the arbitrage strategy results $APG_{t,t+\Delta}$ (expressed in % of bond nominal value). As described above, the arbitrage strategies are developed for 1-day and 10-day holding periods. For each holding period (1 day or 10 days), three portfolios are built, one for each hedging method (D^1; D^1 and D^2; D^1 D^2 and D^3).

Table 6: Statistics on arbitrage results

Type A portfolios

Hedging Method	D^1			D^1 & D^2			D^1 & D^2 & D^3		
	H=1 day								
Duration measure	Mean	Std. dev.	*p*-value	Mean	Std. dev.	*p*-value	Mean	Std. dev.	*p*-value
Vasicek α= 0.005	0.0008	0.0269	0.3847	0.0006	0.0155	0.2793	0.0006	0.0259	0.4864
Vasicek α= 0.1	0.0008	0.027	0.3954	0.0007	0.0226	0.3895	0.0006	0.0682	0.7958
Fisher-Weil	0.0008	0.0269	0.3834	0.0006	0.0153	0.2761	0.0006	0.0249	0.4701
	H=10 days								
Vasicek α= 0.005	0.0089	0.0580	0.0000	0.0073	0.0415	0.0000	0.0181	0.1361	0.0000
Vasicek α= 0.1	0.0085	0.0584	0.0001	0.0088	0.0521	0.0000	0.0287	0.2470	0.0000
Fisher-Weil	0.0089	0.0580	0.0000	0.0072	0.0412	0.0000	0.0178	0.1324	0.0000

Type B portfolios

Hedging Method	D^1			D^1 & D^2			D^1 & D^2 & D^3		
	H=1 day								
Duration measure	Mean	Std. dev.	*p*-value	Mean	Std. dev.	*p*-value	Mean	Std. dev.	*p*-value
Vasicek α= 0.005	-0.0074	1.8122	0.9091	0.0005	0.0136	0.2869	0.0006	0.0134	0.2507
Vasicek α= 0.1	-0.0022	0.1831	0.7416	0.0005	0.014	0.2988	0.0005	0.0179	0.4273
Fisher-Weil	0.0435	1.1644	0.2961	0.0005	0.0137	0.2865	0.0006	0.0133	0,2487
	H=10 days								
Vasicek α= 0.005	0.4003	6.9282	0.1086	0.0049	0.0327	0.0000	0.0055	0.0330	0.0000
Vasicek α= 0.1	0.0017	0.4715	0.9193	0.0049	0.0331	0.0000	0.0048	0.0380	0.0000
Fisher-Weil	0.1403	3.4196	0.2544	0.0049	0.0327	0.0000	0.0055	0.0330	0.0000

Portfolio composition

Portfolios A composition

Hedging Method	D^1	D^1 & D^2	D^1 & D^2 & D^3
Nominal bond	OAT -2010-2 1/2%-2020	OAT -2010-2 1/2%-2020	OAT -2010-2 1/2%-2020
IL Bonds	OAT€i-2012- 1/4%-2018	OAT€i-2012- 1/4%-2018	OAT€i-2012- 1/4%-2018
	OAT€i-2012- 1/4%-2018	OAT€i-2012- 1/4%-2018	OAT€i-2004-2 1/4%-2020
	OATi -2012- 0,1%-2021	OAT€i-2004-2 1/4%-2020	OATi -2012- 0,1%-2021
	OATi -2008- 2.1%-2023	OATi -2012- 0,1%-2021	OATi -2008- 2.1%-2023
		OATi- 2008- 2.1%-2023	OATi -2011- 1.85%-2027

Portfolios B composition

Hedging Method	D^1	D^1 & D^2	D^1 & D^2 & D^3
Nominal bond	OAT -2010-2 1/2%-2020	OAT -2010-2 1/2%-2020	OAT -2010-2 1/2%-2020
IL Bonds	BTANi-2011-0.45%-2016	BTANi-2011-0.45%-2016	BTANi-2011-0.45%-2016
	OAT€i-2012- 1/4%-2018	OAT€i-2012-1/4%-2018	OAT€i-2012- 1/4%-2018
	OATi- 2012- 0.1%-2021	OAT€i-2004-2 1/4%-2020	OAT€i-2004-2 1/4%-2020
		OATi-2008-2.1%-2023	OATi -2012- 0.1%-2021
			OATi -2008- 2.1%-2023

None of the mean values of the results of the 1-day strategies significantly differs from zero. The opposite situation is observed in the 10-day strategies, where a much larger number of mean arbitrage profits are significantly different from zero. The profits from bond arbitrage portfolios stem from prices and accrued interest. The difference between the results in 1-day and 10-day holding periods suggests that bond prices adjust slowly to mispricing and the most important part of the arbitrage portfolio gains stems from the difference between accrued interest during the holding period of the nominal bonds (held long) and the IL bonds (held short). The 1-day holding period is not sufficient to evidence profits.

Type A portfolios held for a period of 10 days offer mean results that are positive and significantly different from zero, independent of the hedging method used against interest rate risks. On the contrary, Type B portfolios held for 10 days require a hedging method based at least on D^1 and D^2 to offer mean profits significantly different from zero. As a reminder, D^1 is relative to level risk while D^2 is related to slope risk, and D^3 is related to hump risk of interest rates.

The average maturity of IL bonds in type A portfolios is longer than in type B portfolios. Since the real yield curve is more flat in long maturity segments than in short maturity ones, the need to hedge against slope and hump risk is more important in portfolios with lower maturity than in portfolios with longer maturity. Finally, the duration measures used (Vasicek or Fisher-Weil duration) do not impact the arbitrage results.

4.3. Determinants of arbitrage results

Arbitrage profits are caused by irregularities between the IL and nominal bond markets, which end up in breakeven inflation. The advantage of FBEI over breakeven inflation is that it is not impacted by the correlation between the breakeven inflation of different maturities, which overlap. Hence, FBEI for different maturities seems well adapted to explain the arbitrage strategy results. To test this, we carry out regressions to explain the profits of the arbitrage strategies (whose mean is significantly different from zero) via changes in four FBEI variables with short-, medium-, and long-term time horizons.

It is important to determine in which maturity segments breakeven inflation has the more meaningful effect on the detected arbitrage profits. The forward rates used to calculate FBEI are the rates starting at the date corresponding to their maturity and lasting for 1 year. Hence, The forward breakeven inflation rate whose maturity is i, noted FBEI_iY, is the difference between nominal and real forward rates, such rates beginning in i year and ending at the end of the $i+1$ year[1].

As discussed above (paragraph 4.1), the arbitrage strategies are developed alternatively for 1-day and 10-day holding periods. Hence, the arbitrage portfolios constructed every day and held over 10 days are overlapped with 9 equivalent portfolios. To correct the autocorrelation effect caused by the portfolios overlapping, we include lagged values of the dependent variable in the regressions of the arbitrage portfolio profits, $APG_{t,t+\Delta}$, over the FBEI variables. Hence, the estimated equations take the following form:

$$APG_{t,t+\Delta} = \alpha + \beta_1 dFBEI1Y_{t,t+\Delta} + \beta_2 dFBEI3Y_{t,t+\Delta} + \beta_3 dFBEI5Y_{t,t+\Delta} + \beta_4 dFBEI7Y_{t,t+\Delta} + \sum_{i=1}^{K} \delta_i APG_{t-i,t-i+\Delta} + \varepsilon_{t,t+\Delta} \quad (23)$$

where $APG_{t,t+\Delta}$ is the gain of the arbitrage portfolio held between t and $t+\Delta$, $dFBEI1Y_{t,t+\Delta}$, $dFBEI3Y_{t,t+\Delta}$, $dFBEI5Y_{t,t+\Delta}$ and $dFBEI7Y_{t,t+\Delta}$ are the changes, between t

[1] FBEI_1Y is the difference between $F_n(1,2)$ and $F_r(1,2)$, where $F_i(1,2)$ is the rate running for the period beginning at the end of year 1 and ending at the end of year 2. The same applies for other maturities.

and $t+\Delta$, of the FBEI variables included in the regressions, $APG_{t-i,t-i+\Delta}$ is the i th lagged value of the dependent variable (i = 1,..., K), and $\varepsilon_{t,t+\Delta}$ is the error term. Lags of the dependent variable are included in the regression for the purpose of eliminating residuals autocorrelation. The search for the minimum number of lags required to remove residuals autocorrelation is guided by the Breusch-Godfrey test on autocorrelation.

The results of the regressions are shown in Table 8. Only strategies where profits exist are analyzed.

Table 7: Impact of forward breakeven inflation on arbitrage results

H=10 days

				Type A Portfolios					
Hedging method		D^1			D^1 & D^2			D^1 & D^2 & D^3	
	Vasicek	Vasicek	Fisher–Weil	Vasicek	Vasicek	Fisher–Weil	Vasicek	Vasicek	Fisher–Weil
Duration measure	α = 0.005	α = 0.1		α = 0.005	α = 0.1		α = 0.005	α = 0.1	
ΔFBEI_1Y	-0.6468**	-0.6907***	-0.6441**	-0.5999***	-0.7463**	-0.5955***	-0.8267	-1.1308	-0.8132
	0.0139	0.0091	0.0143	0.0042	0.0108	0.0040	0.1945	0.3721	0.1887
ΔFBEI_3Y	-0.3241**	-0.3619**	-0.3216**	-0.3903***	-0.4974***	-0.3865***	-0.9453**	-1.3472*	-0.9242**
	0.0354	0.0198	0.0368	0.0015	0.0039	0.0014	0.0134	0.0774	0.0127
ΔFBEI_5Y	-0.0643	-0.0855	-0.0630	-0.1362	-0.2030*	-0.1352	-0.4415*	-0.6495	-0.4329*
	0.5353	0.4137	0.5438	0.1021	0.0810	0.1001	0.0825	0.1985	0.0795
ΔFBEI_7Y	-0.0125**	-0.0118*	-0.0126**	-0.0114**	0.0065	-0.0104**	-0.0251*	-0.0480	-0.0236
	0.0469	0.0633	0.0461	0.026	0.3629	0.0381	0.0974	0.1096	0.1084
R^2	0.732	0.733	0.732	0.673	0.591	0.677	0.71246	0.648	0.714
B&G (P-value)	0.8430	0.8970	0.8380	0.1084	0.3508	0.1150	0.5429	0.8068	0.5587
Lag number		15			20			20	
				Type B Portfolios					
Hedging method		D^1			D^1 & D^2			D^1 & D^2 & D^3	
	Vasicek	Vasicek	Fisher–Weil	Vasicek	Vasicek	Fisher–Weil	Vasicek	Vasicek	Fisher–Weil
Duration measure	α = 0.005	α = 0.1		a = 0.005	a = 0.1		α = 0.005	α = 0.1	
ΔFBEI_1Y				-0.4169***	-0.4218***	-0.4170***	-0.4425***	-0.5105***	-0.4413***
				0.0030	0.0032	0.0030	0.0006	0.0011	0.0006
ΔFBEI_3Y				-0.1634**	-0.1575*	-0.1636**	-0.1480**	-0.1114	-0.1494**
				0.0466	0.0598	0.0465	0.0475	0.2204	0.0449
ΔFBEI_5Y				-0.1290**	-0.1315**	-0.1291**	-0.0992*	-0.1343**	-0.0986*
				0.0216	0.02156	0.0216	0.0515	0.0296	0.0525
ΔFBEI_7Y				0.0003	0.0002	0.0003	0.0014	0.0055	0.0013
				0.9246	0.9564	0.9242	0.6660	0.1440	0.6920
R^2				0.767	0.764	0.767	0.811	0.780	0.811
B&G stat (P-value)				0.2248	0.663	0.224	0.121	0.783	0.115
Lag number					25			25	

*** Significant at the 1% level; ** Significant at the 5% level; * Significant at the 10% level.

Table 7 presents the coefficient estimates, adjusted R^2, and Breusch-Godfrey (B&G) *p*-value which confirms the absence of residual autocorrelation in all regressions. The number of lagged values of the dependent variable included in the regressions is also given in the table.

The results of the regression differ between type A and type B portfolios. The difference between these two portfolio types is their composition, which affects the mean maturity of the IL bonds included in the portfolio. In type A portfolios, the mean maturity is closer to the nominal bond than for type B portfolios, where IL bonds have a shorter maturity.

Within each portfolio type, the result differs according to the hedging method used, which relies on the type of interest risk hedged (level, slope, or hump). On the other hand, the results are independent of the chosen duration measure implemented (Vasicek $\alpha = 0.05$, Vasicek $\alpha = 0.10$, and Fisher-Weil).

For type A portfolios, there is evidence that changes in one-year forward breakeven inflation (FBEI_1Y) explain the profits when the portfolio is hedged against level and slope, i.e., when D^1 and D^2 are implemented. The 3-year breakeven inflation changes explain significantly (at the level of 1% and 5%) the arbitrage results whatever the hedging method implemented. However, the 5-year breakeven inflation changes explain very poorly the arbitrage results. Changes in 7-year FBEI are significant at 1% level only in one case only, the most part is significant at 5%.

For type B portfolios, FBEI_1Y changes explain at a very high level (1%) the arbitrage results whatever the hedging measures. The changes in FBEI_3Y and FBEI_5Y explain significantly (at 1% and 5% levels, respectively) the arbitrage results. Conversely, changes in FBEI_7Y never explain arbitrage profits. This result is not surprising considering the average maturity of the IL bonds included in these portfolios, which is lower than 7 years.

Thus, it is possible to conclude that short maturities of breakeven inflation explain the arbitrage results. Portfolios of type A are more sensitive to a maturity of 3 years while portfolios of type B are more sensitive to a 1-year maturity. The maturity of the breakeven inflation to which a portfolio is sensitive depends on the average maturity of the IL bonds present in the portfolio.

5. Conclusion

This paper focuses on French Treasury inflation-indexed bonds, their evaluation, and their relative prices relative to nominal Treasury bonds. To compare the prices of these two assets, we build portfolios composed of a Treasury nominal bond and several Treasury IL bonds whose proportions are determined such that the portfolio is self-financing and hedged against interest rate risks. Three types of risk are involved: level, slope, and hump risk. The results show arbitrage opportunities between these two types of assets.

The arbitrage results show that choice of hedging method is important, since hedging against second- (slope risk) and third-order (hump risk) durations, and not only against first-order duration (level), is particularly important in the portfolios whose payoff maturities fall predominantly in yield curve segments with high slope and hump, usually the short and medium term. On the contrary, the type of duration measure used, which relies on the considered pricing model, does not impact the arbitrage results. If arbitrage profits are detected, it is important to discover which factor explains these profits. Based on ordinary least squares regressions, this paper shows that the changes in FBEI (the difference between nominal and real forward interest rates) with reference to short- and medium-term maturities play an important role in explaining arbitrage portfolio gains.

References

Abrahams M., Adrian, T., Crump R., Moench E., Yu, R. (2016). "Decomposing real and nominal yield curves", *Journal of Monetary Economics*, 84, 182-200.

Amin, K., Jarrow, R. (1991). "Pricing foreign currency options under stochastic interest rates", *Journal of International Money and Finance*, 10 (3), 310-329.

Andreasen M., Christensen J., Cook K. (2017). "The TIPS liquidity premium", *Federal Reserve Bank of San Francisco*, WP

Auckenthaler J., Kupfer A., Sendlhofer R. (2015). "The impact of liquidity on inflation-linked bonds: A hypothetical indexed bonds approach", *The North American Journal of Economics and Finance*, 32, 139-154.

Brière M., Signori O. (2012). "Inflation hedging portfolios: Economic regimes matter", *Journal of Portfolio Management*, 38, 43-58.

Campbell J. Y., Shiller R. J., Viceira L. M. (2009). "Understanding inflation-indexed bond markets", *National Bureau of Economic Research*, No. w15014.

Cartea A, Saúl J., Toro, J. (2012). "Optimal portfolio choice in real terms: Measuring the benefits of TIPS", *Journal of Empirical Finance*, 19(5), 721-740.

Chen, R., Liu, B., Cheng, X. (2010). "Pricing the term structure of inflation risk premia: Theory and evidence from TIPS", *Journal of Empirical Finance*, 17 (4), 702-721.

D'Amico, S., Kim, D.H., Wei, M. (2018). "Tips from TIPS: The informational content of Treasury inflation-protected security prices", *Journal of Financial and Quantitative Analysis*, 53 (1), 395-436.

Driessen J., Nijman T., Simon, Z. (2017). "The missing piece of the puzzle: Liquidity premiums in inflation-indexed markets", working paper.

Fisher, L., Weil, R. (1971). "Coping with the risk of interest fluctuations: Returns to bondholders from naive and optimal strategies", *The Journal of Business*, 44 (4), 408-431.

Fleckenstein M., Longstaff F., Lustig, H. (2014). "The TIPS-Treasury Bond Puzzle", *the Journal of Finance*, 69(5), 2151-2197.

Fleming M., Krishnan N. (2012). "The microstructure of the TIPS market", *Federal Reserve Bank of New York Economic Policy Review*, 18(1), 27-45.

Fong, G., Vasicek, O. (1984). "A risk minimizing strategy for portfolio immunization", *The Journal of Finance,* 39 (5), 1541-1546.

Grishchenko, O.V., Huang J. (2013). “The inflation risk premium: evidence from the TIPS Market”, *Journal of Fixed Income*, 22 (4), 5-30.

Gürkaynak R. S., Sack, B., Wright, J. H. (2010). “The TIPS yield curve and inflation compensation”, *American Economic Journal: Macroeconomics*, 2(1), 70-92.

Heath, D., Jarrow, R., Morton, A. (1992). “Bond pricing and the term structure of interest rates: A new methodology for contingent claim valuation”, *Econometrica,* 60 (1), 77-105.

Jarrow, R. (2010). “Hedging in a HJM model”, *Finance Research Letters*, 7, 8-13.

Jarrow, R., Yildirim, Y. (2003). “Pricing treasury inflation protected securities and related derivatives using an HJM model”, *Journal of Financial and Quantitative Analysis*, 38 (2), 337-358.

Lacey, N., and Nawalkha, S. (1993). “Convexity, risk and returns”, *The Journal of Fixed Income,* 3 (3), 72-79.

McCulloch, J.H. (1975). “The tax adjusted yield curve”, *The Journal of Finance*, 30 (3), 811-830.

McCulloch, J., Kochin, L. (2000). “The inflation premium implicit in the US real and nominal term structures of interest rates”, working paper, Ohio State University.

Mkaouar F., Prigent JL, Abid I. (2017). “Long-term investment with stochastic interest and inflation rates: The need for inflation indexed bonds”, *Economic Modelling*, 67, 228-247.

Nawalkha, S. (1995). “The duration vector: A continuous-time extension to default-free contingent claims”, *Journal of Banking and Finance,* 19 (8), 1359–1366.

Nawalkha, S., De Soto, G., Zhang, J. (2003). “Generalized M-vector models for hedging interest rate risk”, *Journal of Banking and Finance*, 1581–1604.

Nelson, C., Siegel, A. (1987). “Parsimonious modeling of yield curves”, *The Journal of Business*, 60 (4), 473-489.

Pericoli, M (2014). “Real term structure and inflation compensation in the euro area”, *International Journal of Central Banking*, 10 (1), 1-42.

Pflueger C., Viceira, L. (2011b). “Return predictability in the Treasury market: real rates, inflation, and liquidity”, *National Bureau of Economic Research,* WP N°16892.

Simon Z. (2015). “Not risk free: The relative pricing of euro area inflation-indexed and nominal bonds”, working paper, papers.ssrn.com.

Vasicek, O. (1977). “An equilibrium characterization of the term structure”, *Journal of Financial Economics*, 5 (2), 177-188.

Xu E. (2011). “On the performance drivers of US Treasury Inflation Protected Securities”, *International Review of Accounting, Banking and Finance*, 3(4), 49-70.

Zeng Z. (2013). “New tips from TIPS: Identifying inflation expectations and the risk premia of break-even inflation”, *The quarterly Review of Economics and Finance*, 53, 125-139.