Investment and asset securitization with an option-for-guarantee swap ☆

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Abstract

We address investment and financing decisions of entrepreneurs entering into option-for-guarantee swaps (OGSs). OGSs increase investment option values. The entrepreneur first accelerates investment but then postpones investment as funding gaps rise. Guarantee costs increase with project risks when funding gaps are small or large enough and otherwise the opposite holds true. Investment is postponed as the project risk, effective tax rate or bankruptcy costs increase. Surprisingly, the higher the project risk, the more the entrepreneur borrows and optimal leverage is much higher than predicted by previous models. Entrepreneurs can succeed in securitizing their assets by OGSs.

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1. Introduction

China has experienced rapid economic growth since the 1980s. Due to the very strict requirements for the initial public offerings (IPOs) of equity, many Chinese companies are unable to access equity financing and trips to the debt market are a last resort. According to Liu et al. (2016), during 2013, the volume of bank loans represented 84.5%, bond issuances represented 10.5%, and equity offerings represented only 1.3% of the total capital raised through formal channels in China. Debt financing is therefore an major source of capital for Chinese companies. However, although SMEs play a vital role in promoting economic growth over the world and they represent 99% of all the Chinese companies, it is difficult for them to get loans from a bank. For this reason, a large number of SMEs have to give up investing in a project even it is very profitable. This situation gets even more serious due to the recent global financial crisis.

To overcome the financing constraints, some Chinese SMEs turn to insurers and enter into some equity default swaps (EDSs) agreement with an insurer and a lender (bank). According to Dybvig, Shan and Tang (2016), the amount of guaranteed loans was more than twice as much as that of equity financing during this period while, according to the World Bank Enterprise Surveys, more than 79% of the bank loan must be insured.

On the other hand, Peer to peer (P2P) lending is getting more and more popular, which plays a significant role for the inclusive finance. Since the well-known Mandatory Payment Arrangements are forbidden in China, to protect inexperienced investors, many P2P landing platforms ask a third party to provide a guarantee for the lending and guarantee service gets much more popular than before.

There are three popular types of loan guarantees in China: equity-, fee- and option-for-guarantee swaps, denoted by EGSs, FGSs and OGSs respectively. The OGSs appeared for the first time in Shenzhen of China, in August 2002, when Shenzhen Bak Battery Limited liability company (BB) signed an OGS swap agreement with Shenzhen High-Tech Investment Group CO., LTD (HTI) and obtained 30 million RMB yuan from a bank with the guarantee of the latter. In return for the guarantee, the BB granted the HTI an American call option to buy two percent of the BB's equity, of which the value was worth 20 million RMB yuan. Thanks to this swap, the BB grew rapidly. In the year of 2011, its products represent 5% of the global market and and ranked 7th in its industry over the world. In fact, the BB repurchased that option, the value of which increased to 118 million RMB yuan, by negotiating with the HTI in December 2003. At present, a half of the public listed companies in Shenzhen, including the well-known Huawei, BYD, Skyworth and so on, have greatly benefited from the loan guarantees provided by the HTI, according to our recent interview with the president, Dr. Suhua Liu, of the corporation.

An OGS is a three-party agreement between a bank/lender, an insurer and an SME/borrower, where a borrower obtains a loan from a lender at a given interest rate and if he defaults on the loan, the insurer must pay all the outstanding interest and principal to the lender. In return, the borrower must give the insurer a call option to buy a given fraction of equity at a given exercise price per share.

OGSs are similar with credit default swaps (CDSs), which are designed to transfer the credit exposure of fixed income products between parties. In a CDS, the purchaser of the swap makes payments (the CDS "fee" or "spread") up until the maturity date of a contract to the seller of the swap. In return, the seller agrees to pay off a third party debt if this party defaults on the loan, see Rutkowski and Armstrong (2009) among others.

OGSs seem more similar with equity default swaps (EDSs) discussed by Mendoza and Linetsky (2011), which are designed to deliver a protection payment to the EDS buyer at the time of the triggering event defined as the stock price decline below a pre-specified lower triggering barrier level. In exchange, the EDS buyer makes periodic premium payments at time intervals at the equity default swap rate up to the triggering event or the final maturity, whichever comes first. For this reason, we think of OGSs as a new type of EDSs since all of them are derivatives essentially underlying the value of equity of a firm.

Roughly speaking, in addition to overcoming financing constraints, OGSs actually force a borrower to reserve a part of his profits when his firm is well profitable to pay his debt once the firm falls in distress. Doing so, insurers play a key role but they do not obtain net earnings from the swap, since we assume the guarantee market is fully competitive.[1] As a result, such

[1] We assume this also because a lot of guarantee companies in China are supported by the government, the main goal of which is to stimulate investment. On the other hand, it

guarantee succeeds in smoothing the borrower's cash flow and can improve the social welfare level greatly, especially when idiosyncratic risk is taken into account, as argued by Yang and Zhang (2015). In view of this, OGSs are a considerable financial innovation, worth developing a formal OGSs' theory.

This paper continues a long line of research originating in McDonald and Siegel (1986) and Dixit and Pindyck (1994) using a real options model to study firm's investment and financing decisions. But differently, we address the decisions with a swap agreement to overcome financing constraints.

Our paper is closely related with Yang and Zhang (2013), Wang et al. (2015), Xiang and Yang (2015) and Yang and Zhang (2015). The first three papers investigate another similar but different swap, i.e. EGS. OGSs bring insurers more flexibility benefits but it leads to a much more challenging problem, since we have to endogenously fix a new exercising threshold and two different default thresholds instead of one for EGSs to solve the problem. The last one by Yang and Zhang (2015) explores OGSs for the first time. However, they are silent about dynamic investment under irreversibility and uncertainty and their main goal is to make clear which are the better of OGSs and EGSs in a static capital structure model. They show that in view of utility-based prices, OGSs are generally better than EGSs. The advantage increases quickly with the firm's cash flow level and is generally more pronounced when risk aversion, cash flow risk or the correlation be-

is true that the guarantee market would be not so competitive and furthermore, in many cases, it is impossible for an entrepreneur (borrower) to invest in a project if he does not get a guarantee. Accordingly, insurers should share the value of the option to invest in a project with entrepreneurs, as argued by Gan et al. (2016), who, however, focus on FGSs and EGSs instead of OGSs we consider here.

tween the cash flow and the market increases. It is therefore interesting to further clarify how OGSs affect dynamic investment decisions. To this end, we examine the interaction between investment and financing decisions of a firm entering into OGSs. Without a doubt, it is more challenging to solve our model than previous ones. For example, we must solve the Nash equilibrium of a game between an insurer and a borrower and find four optimal stoping times, i.e. the investment time of a project, the exercising time of an insurer's option and two default times. Furthermore, in contrast to Yang and Zhang (2015), our focus is to study how OGSs influence the value of the option to invest and how they affect investment threshold, coupon payments, leverage and default thresholds. We also consider how a funding gap impacts on investment time and the value of the option, and how a borrower's incentive behavior changes after he enters into an OGS. In particular, according to Chinese taxation regulations, our model introduces a new factor to avoid double taxation. In this way, we show that the benefit for an entrepreneur entering into OGSs is much more than that uncovered by previous studies and we therefore propose a new explanation for why loan guarantees are so popular in China.

In short, our model reveals that an OGS agreement can not only overcome financing constraints but also increase the value of the option to invest greatly. An entrepreneur (borrower) first accelerates and then postpones investment as the funding gap rises. The guarantee cost increases with project risk when the funding gap is small or large enough and otherwise the opposite holds true. Investment is postponed as project risk, effective tax rate or bankruptcy cost rise. In contrast to a classical case where debt is fully protected without a guarantee, the higher the project risk, the more an entrepreneur borrows. In particular, the value of the option to invest increases remarkably due to OGSs and the increased amount rises quickly with effective tax rate. Interestingly, our model shows that the optimal leverage is much higher than that predicted by Leland (1994). Thanks to the swap, entrepreneurs raise much more funds than required to invest and in a sense they succeed in 'securitizing' their assets.

The rest of the paper proceeds as follows. Section 2 presents the model. Section 3 discusses the pricing of corporate claims and guarantee costs, optimal financing and optimal investment. Section 4 considers model implications. Section 5 concludes. Technical developments are included in the Appendix.

2. Model setup

We consider an entrepreneur who possesses the option to invest in a single project at any time in the future, which requires a one-time investment cost I. We assume the entrepreneur finances the cost via his own funds (internal financing) and the remaining funding gap is raised by debt financing. It is common that such investment has a high default probability, lacks collateral and involves severe information asymmetry between a borrower and a lender.[2] Therefore we assume the entrepreneur cannot borrow money directly from a lender (bank) and he chooses to enter into an option-for-guarantee swap. Under this swap, the entrepreneur obtains a loan from a lender (bank)

[2] By contrast, we presume insurers are powerful in accurately identifying the default risk and specifically, there is no information asymmetry between borrowers and insurers.

guaranteed by an insurer after granting the insurer a perpetual American call option to purchase a fraction ϕ, called guarantee cost, of his equity at the strike price K per share.[3] Once the entrepreneur defaults, the insurer must make a compensatory payment instead of the borrower, paying all the outstanding interest and principal to the lender. For tractability reasons, we assume debt is interest-only (consol) with its coupon rate being constant c and it is issued only at the investment time remaining unchanged until a default occurs.

After the investment option is exercised, we assume the project generates earnings before interest and tax (EBIT) δ_t, which is given by the following geometric Brownian motion:

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dZ_t, \quad \delta_0 > 0 \text{ given}, \tag{1}$$

where μ is the mean appreciation rate, $\sigma > 0$ is the volatility rate, and Z is a standard Brownian motion defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q})$. The filtration $\{\mathcal{F}_t\}_{t\geq 0}$ describes the flow of information available to investors and $\mathbb{Q}$ is a given risk-neutral probability measure.

After debt is in place, at any time $t > 0$, the entrepreneur has the option to default on the outstanding debt, the exercising of which leads to the liquidation of his firm. The bankruptcy (liquidation) time might be either earlier or later than the time when the call option of the insurer is exercised and so it is referred to as pre-exercise or post-exercise default time respectively in the following text.

[3] Without loss of generality, we assume the number of shares of stock is just one.

Once the entrepreneur defaults, the outside lenders (creditors) take over and liquidate the firm. We assume the liquidation value of the firm is equal to a fraction $(1-\alpha)$ of its value and the remaining fraction α is lost due to bankruptcy costs. Moreover, the creditors will gain an extra compensatory payment from the guarantee company so that the lump-sum payment received by the creditors equals c/r before tax at default time, where r is the risk-free interest rate. For this reason, thanks to the option-for-guarantee swap, the loan is actually risk-free.

Last, similar to Goldstein et al. (2001), we assume a simple tax structure that includes corporate and personal taxes, where interest payments are taxed at a personal rate τ_i, effective dividends are taxed at τ_d, and corporate profits are taxed at τ_c, with full loss offset provisions.

3. The pricing of corporate securities, guarantee cost, and optimal investment

In this section, using a backward method, we first provide the pricing of contingent claims after investment, calculate the fair guarantee cost of the option-for-guarantee swap, and jointly determine the optimal bankruptcy policy by the entrepreneur and the optimal exercising policy by the insurer. Then, we consider the optimal financing and investment problems.

3.1. The risk-neutral prices of corporate securities

To price a claim, we must specify its cash flow in advance. We note that default is endogenously decided by the entrepreneur and naturally the default threshold must depend on whether the insurer's call option is exercised or not. For this reason, to define all the claims we discuss here, the key is to determine four thresholds (or equivalently four stopping times), i.e. the investment threshold, the exercise threshold of the insurer's option, the pre-exercise and post-exercise default threshold, which are all independent of time since our model is time-homogeneous, and we therefore denote them by δ_i, δ_k, δ_{bo}, δ_b respectively throughout the text. Noting that the four thresholds are endogenously determined by different investors, we first take them and guarantee cost ϕ as given in this subsection to price contingent claims and then in the next subsection derive their values, which actually constitute a Nash equilibrium of a game between the entrepreneur and insurer.

At any time $t \geq 0$ after investment but prior to both bankruptcy and the exercising of the insurer's call option, the entrepreneur gains an instantaneous cash flow $(1-\tau_f)(\delta_t - c)$ from the firm, where $\tau_f = 1-(1-\tau_c)(1-\tau_d) > 0$ represents the corporate effective tax rate. The guarantee company holds an American call option and the lender receives the cash flow c before tax. As the EBIT δ_t of the firm becomes sufficiently small and hits from above the pre-exercise threshold δ_{bo}, the firm is liquidated. On the other hand, if the EBIT δ_t of the firm rises sufficiently high and hits from below the exercise threshold δ_k, the insurer exercises the American call option and obtains a fraction ϕ of equity after paying ϕK to the entrepreneur.

Since our model is time-homogenous, we assume without loss of generality that the current time is zero and the current cash flow rate $\delta_0 = x$. To facilitate exposition, we denote by $E(x,c)$ the value of the levered firm's equity and by $E^j(x,c)$, $D^j(x,c)$ and $V^j(x,c)$ the value of the entrepreneur's equity, of debt and of the levered firm, respectively, where $j \in \{a,b\}$ and superscripts "a" and "b" indicate the value of that claim after and before the exercising of the insurer's option, respectively.

Using the well known risk-neutral pricing theory, we derive the prices of the corporate securities under the assumption that the guarantee cost, the exercise and default thresholds are given in advance. To do so, we utilize the backward induction method. That is, we begin with deriving the prices after the insurer's call option is already exercised and then turn to the pricing prior to the exercise time. We directly list the results below and the details of our derivations are relegated to the appendices.

First, we consider the case where the investment has taken place and the insurer's option is already exercised. The value of the levered firm's equity, of the entrepreneur's equity and of debt without a guarantee are given respectively by

$$E(x,c) = (1-\tau_f)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) - (1-\tau_f)\left(\frac{\delta_b}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{\delta_b}\right)^{\beta_-}, \quad (2)$$

$$E^a(x,c) = (1-\phi)E(x,c), \quad (3)$$

$$D^a(x,c) = (1-\tau_i)\left[\frac{c}{r} + \left(\frac{(1-\tau_f)(1-\alpha)}{(1-\tau_i)}\frac{\delta_b}{r-\mu} - \frac{c}{r}\right)\left(\frac{x}{\delta_b}\right)^{\beta_-}\right], \quad (4)$$

where $\beta_- = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0$, is the negative root of the characteristic equation $\frac{1}{2}\sigma^2 x(1-x) + \mu x - r = 0$. The other positive root of the characteristic equation is denoted by β_+, which is $\beta_+ = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} > 1$, and will be cited later.

Remark 1. *For some given investment threshold, exercising threshold, and two default thresholds, all the prices of contingent claims in our model are easy to derive and explain from economic intuition. For example, The value* *of equity in (2) is that of the total future cash flow received by shareholders if default does not happen, less a deduction equal to the value of the total cash flow received starting at default time. Or equivalently, the equity value equals the present value of the cash flow without possibility of default plus the value of the option to default.*

Next, we turn to the situation where the investment has already taken place but the insurer's option is not exercised yet. In the same way, we derive that the value of equity owned by the entrepreneur and of debt without a guarantee are given respectively by

$$\begin{aligned} E^b(x,c) =& (1-\tau_f)\left(\frac{x}{r-\mu}-\frac{c}{r}\right)+\Pi_1(x)\Big[E^a(\delta_k,c)+\phi K-(1-\tau_f) \\ & \times\left(\frac{\delta_k}{r-\mu}-\frac{c}{r}\right)\Big]-\Pi_2(x)(1-\tau_f)\left(\frac{\delta_{bo}}{r-\mu}-\frac{c}{r}\right), \end{aligned} \tag{5}$$

$$\begin{aligned} D^b(x,c) =& (1-\tau_i)\frac{c}{r}+\Pi_1(x)\left[D^a(\delta_k,c)-(1-\tau_i)\frac{c}{r}\right] \\ & +\Pi_2(x)\left(V(\delta_{bo})-(1-\tau_i)\frac{c}{r}\right), \end{aligned} \tag{6}$$

where $V(\delta_{bo})$ is the liquidation value, i.e. the value of the unlevered firm at default time, given by

$$V(\delta_{bo}) = (1-\alpha)(1-\tau_f)\frac{\delta_{bo}}{r-\mu},$$

and

$$\Pi_1(x) = \frac{\delta_{bo}^{\beta_+} x^{\beta_-} - \delta_{bo}^{\beta_-} x^{\beta_+}}{\delta_{bo}^{\beta_+} \delta_k^{\beta_-} - \delta_{bo}^{\beta_-} \delta_k^{\beta_+}}, \quad \Pi_2(x) = \frac{\delta_k^{\beta_-} x^{\beta_+} - \delta_k^{\beta_+} x^{\beta_-}}{\delta_{bo}^{\beta_+} \delta_k^{\beta_-} - \delta_{bo}^{\beta_-} \delta_k^{\beta_+}} \tag{7}$$

are respectively the value of the claim on nothing but one dollar received at the exercise time of the insurer's option prior to default, and that of the claim on nothing but one dollar received at the default time before the exercising of the option, conditional on the current cash flow level being x, which satisfies $\delta_{bo} < x < \delta_k$. The values defined by (7) are key to price contingent claims since they give the prices of the claims like the well-known Arrow securities and using them, we can easily explain the prices given by (5) and (6). In particular, the value, denoted by $V^G(x)$, of the American call option can be directly written as

$$V^G(x) = \Pi_1(x)[E(\delta_k, c) - K]^+ \text{ for } \delta_{bo} < x < \delta_k, \tag{8}$$

and naturally, the value of the insurer's call option equals $\phi V^G(x)$.

3.2. Optimal exercise and default thresholds with the fair guarantee cost

Based on previous computations, in this subsection we derive the fair guarantee cost, optimal exercise threshold decided by the insurer, and both pre-exercise and post-exercise default threshold decided by the entrepreneur.

First, taking the default threshold δ_b as the decision variable and maximizing the value of the entrepreneur's equity given by (3), we immediately derive the following optimal post-exercise default threshold:

$$\delta_b = -\frac{\beta_-}{1-\beta_-}\frac{(r-\mu)c}{r}. \tag{9}$$

Second, according to (3) and (7), taking δ_{bo} as the decision variable and maximizing the value of the entrepreneur's equity given by (5), we derive the optimal pre-exercise default threshold δ_{bo} satisfying the following algebra equation:

$$\begin{aligned}&\left(\frac{E^a(\delta_k,c)+\phi K}{1-\tau_f}-\left(\frac{\delta_k}{r-\mu}-\frac{c}{r}\right)\right)(\beta_- - \beta_+)\delta_{bo}^{(\beta_+ + \beta_- -1)}-\left(\frac{\delta_{bo}}{r-\mu}-\frac{c}{r}\right)\\&\times\left(\beta_+\delta_k^{\beta_-}\delta_{bo}^{(\beta_+ -1)}-\beta_-\delta_k^{\beta_+}\delta_{bo}^{(\beta_- -1)}\right)+\frac{\delta_k^{\beta_-}\delta_{bo}^{\beta_+}-\delta_k^{\beta_+}\delta_{bo}^{\beta_-}}{r-\mu}=0.\end{aligned} \tag{10}$$

Third, according to (2) and (7), taking δ_k as the decision variable and maximizing the value of the insurer's option given by (8), we derive the optimal exercise threshold δ_k satisfying

$$\begin{aligned}&\left((1-\beta_-)\tfrac{\delta_k}{r-\mu}+\tfrac{\beta_- c}{r}+\tfrac{\beta_- K}{1-\tau_f}\right)\delta_{bo}^{\beta_+}\delta_k^{\beta_-}+(\beta_- - \beta_+)\left(\tfrac{\delta_b}{r-\mu}-\tfrac{c}{r}\right)\left(\tfrac{\delta_k}{\delta_b}\right)^{\beta_-}\delta_{bo}^{\beta_-}\delta_k^{\beta_+}\\&-\left((1-\beta_+)\tfrac{\delta_k}{r-\mu}+\tfrac{\beta_+ c}{r}+\tfrac{\beta_+ K}{1-\tau_f}\right)\delta_{bo}^{\beta_-}\delta_k^{\beta_+}=0.\end{aligned} \tag{11}$$

Last, we turn to the guarantee cost. Naturally, we assume the swap agreement is signed just when the investment takes place. For this reason, we take the current cash flow level as the investment threshold, i.e. $x=\delta_i$, satisfying $\delta_{bo}<x=\delta_i<\delta_k$. Denote by $D_{guar}(\delta_i)$ the value of the insurer's compensatory payment to the bank/lender, which is taxed at a personal tax rate τ_i. Clearly, in order to fully protect the debt, the value $D_{guar}(\delta_i)$ at the investment time must satisfy the following equation:

$$D^b(\delta_i, c) + \frac{1-\tau_i}{1-\tau_f} D_{guar}(\delta_i) = \frac{c}{r}(1-\tau_i). \tag{12}$$

Substituting (6) into (12), we get

$$D_{guar}(\delta_i) = (1-\tau_f)\left[\Pi_1(\delta_i)\left(\frac{c}{r} - \frac{D^a(\delta_k, c)}{1-\tau_i}\right) + \Pi_2(\delta_i)\left(\frac{c}{r} - \frac{V(\delta_{bo})}{1-\tau_i}\right)\right]. \tag{13}$$

Clearly, the first term of the right-hand side in (13) represents the value of the insurer's compensatory payment at the exercise time of the insurer's call option, and the second term captures the value of the insurer's compensatory payment at the default time. This is quite in agreement with intuition.

We assume the market is competitive and therefore, at the investment time, an insurer's compensatory payment $D_{guar}(\delta_i)$ to a bank/lender must equal the value of the granted call option, i.e.

$$(1-\tau_f)\left[\Pi_1(\delta_i)\left(\frac{c}{r} - \frac{D^a(\delta_k, c)}{1-\tau_i}\right) + \Pi_2(\delta_i)\left(\frac{c}{r} - \frac{V(\delta_{bo})}{1-\tau_i}\right)\right] = \phi V^G(\delta_i). \tag{14}$$

From (8) and (14), the fair guarantee cost ϕ is given by

$$\phi = \frac{(1-\tau_f)\left[\Pi_1(\delta_i)\left(\frac{c}{r} - \frac{D^a(\delta_k)}{1-\tau_i}\right) + \Pi_2(\delta_i)\left(\frac{c}{r} - \frac{V(\delta_{bo})}{1-\tau_i}\right)\right]}{\Pi_1(\delta_i)[E(\delta_k, c) - K]^+}. \tag{15}$$

In short, for a given guarantee cost ϕ and coupon rate c of debt, we solve a set of nonlinear simultaneous equations given by (9)∼(11) to derive the exercise threshold δ_k and the default thresholds δ_b and δ_{bo}, all of which are independent of investment threshold δ_i but a function of the coupon rate c and guarantee cost ϕ. On the other hand, for a given investment threshold δ_i and coupon rate c of debt, we solve a set of nonlinear simultaneous equations given by (9)∼(11) and (15), and conclude that the guarantee cost, exercise threshold, pre-exercise and post exercise default threshold are a function of the investment threshold δ_i and coupon rate c, which are determined in the next subsection.

Remark 2. *Unlike previous studies, say Yang and Zhang (2015) and Xiang and Yang (2015), the second term of the left side of (12) is added a factor* $1-\tau_f$ *in the denominator. This tax assumption is realistic in China and more reasonable as well: First, debt in our model carries no risk and therefore, it should still bring tax shields even after borrowers have defaulted. Second and more importantly, according to the Chinese Administration of Taxation (Circular Cai Shui [2007] No. 27), the compensatory payment of an insurer can be deduced from the insurer's income that must be taxed. As a result, the insurer actually needs to pay less compensatory payment to the lender. This new tax structure is also quite in agreement with the Chinese government's guarantee promotion policies, which encourage insurers to offer guarantee services for SMEs. We emphasize that, due to our previous assumption that the guarantee market is fully competitive, all tax shields are harvested at last by entrepreneurs through paying less guarantee costs to insurers thanks to (14). Naturally, under this tax assumption, debt financing is particularly profitable and so the optimal leverage will get much higher than that without* *a guarantee, as shown in the following Section 4.*

3.3. Financing and investment polices

Generally speaking, the financing amount or almost equivalently the coupon rate c is determined by the funding gap to start a project. However, after entering into the option-for-guarantee swap, an entrepreneur can finance his investment without financing difficulties. For this reason, we take the coupon rate as a decision variable rather than the exogenously given one as assumed before, and consider the optimal capital structure in this subsection. After that, taking all the items discussed before into account, we address the pricing and timing of the option to invest in the project defined by (1).

First, thanks to the swap, debt is actually risk-free and consequently the entrepreneur can raise $D_0(c) \equiv (1 - \tau_i)c/r$ of money at investment time in return for the coupon payment c from a competitive market. That is, the amount of money he must invest in the project is just the balance $I - D_0(c)$. Hence, denoting by $FV(x)$ the value of the entrepreneur's option to invest, he should solve the following optimization problem for the financing and investment polices:

$$FV(x) = \sup_{\delta_i, c \geq 0} \mathbb{E}\left\{ e^{-rt_i} \left[E^b(\delta_i, c) - (I - D_0(c)) \right] \right\}, \tag{16}$$

where t_i is the first hitting time of the investment threshold δ_i, which is a stopping time on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{Q})$.

According to Dixit and Pindyck (1994), we have $\mathbb{E}[e^{-rt_i}] = (x/\delta_i)^{\beta_+}$.

Thus, the optimization problem of (16) is rewritten as

$$FV(x) = \sup_{\delta_i \geq 0, c \geq 0} \left(E^b(\delta_i, c) + D_0(c) - I \right) \left(\frac{x}{\delta_i} \right)^{\beta_+}. \tag{17}$$

This is a nonlinear programming problem. Due to the fact that the value $E^b(\delta_i, c)$ given by (5) depends on the exercise threshold δ_k, default threshold δ_{bo} and guarantee cost ϕ, which are the functions of the two decision variables δ_i and c given by (10), (11) and (15) respectively, we provide the following numerical algorithm for solving (17) instead of presenting its first-order condition: First, we fix an interval, denoted by Co, consisting of the candidates for the optimal coupon rate c and an interval, denoted by Inv, consisting of the candidates for the optimal investment threshold δ_i. Second, for a given accuracy level, we choose a grid

$$\left\{ (c^j, \delta_i^k), j \in \{1, \cdots, M\}, k \in \{1, \cdots, N\} \right\} \subseteq Co \times Inv.$$

Last, we take a two-dimensional search scheme on the grid to get the optimal investment threshold δ_i^* and optimal coupon rate c^*, which maximize the value $(E^b(\delta_i, c) + D_0(c) - I)(x/\delta_i)^{\beta_+}$ of the option to invest.

To sum up, our model states that investment and financing directly interact with each other under the swap agreement. For a given guarantee cost, the exercising of the insurer's option and defaulting polices by the entrepreneur are independent of investment policy but the latter has a direct effect on the guarantee cost, which has an important impact on the exercise threshold and the two default thresholds. All of the guarantee cost, investing, exercising and defaulting polices have an effect on optimal coupon rate and vice versa. In fact, to determine the guarantee cost, exercising, defaulting, investment and financing polices, we have solved a system of nonlinear equations simultaneously given by (9)$\sim$(11), (15) and the first-order conditions (two equations) of the optimization problem (17).

4. Model implications

After the analytical derivations in the previous text, we provide numerical analysis in this section. To do so, following Mauer and Sarkar (2005) among others in the real options literature, we take the following baseline parameter values unless otherwise stated: the risk-free interest rate r=5%, expected growth rate of the firm's earnings μ=2%, its volatility σ=30%, personal tax rate τ_i=25%, corporate profit tax rate τ_c=35%, dividend tax rate τ_d=20% (i.e. the effective tax rate $\tau_f = 0.48$), bankruptcy loss rate α=35%, initial earnings before interest and tax $\delta_0 = x = 1$, irreversible investment cost $I = 10$, the strike price of the American call option K=5.

The above-defined baseline parameter values are carefully selected to exclude some obviously uninteresting cases, say the case of that the current cash flow level satisfies $x > \min(\delta_i, \delta_k)$. That is, we do not consider the following two situations: First, the entrepreneur exercises the investment option immediately; Second, the insurer exercises his call option once the swap agreement is signed.

4.1. The effects of the funding gap and project risk on the entrepreneur's and insurer's polices and guarantee cost

We assume an entrepreneur first utilizes his own funds to start a project but there is a funding gap, which he must cover by external financing. Our

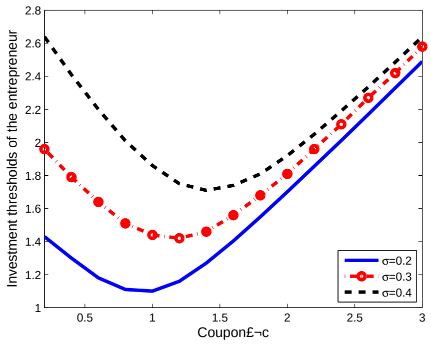

Figure 1: The investment threshold vs. coupon rate

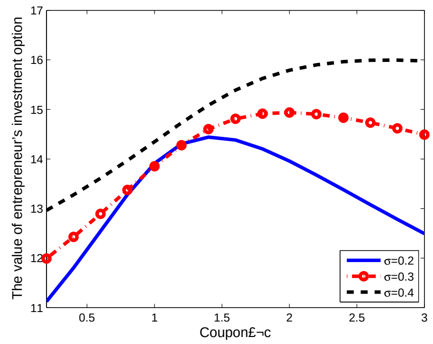

Figure 2: The value of the investment option vs. coupon rate.

model argues that he can cover the gap by entering into an option-for-gurantee swap. Naturally, we wonder how the funding gap or equivalently the corresponding coupon rate and project risk (volatility) impact on the pricing and timing of the investment option and insurer's call option, both the pre-exercise and post-exercise default threshold, and the guarantee cost. This subsection answers these problems.

Figure 1 depicts the impact of the coupon payment of debt on investment thresholds for three different volatility levels (σ=0.2, σ=0.3, σ=0.4). As we expected, no matter what capital structure is (i.e. how much the coupon rate c is), the bigger the business risk of the firm, the larger the value of the entrepreneur's option to invest and thus the later the investment threshold. It is seen that there is a U-shaped relation between investment thresholds and the coupon payments: Investment thresholds decrease first and then increase with the coupon payments of debt. The reason of the result is that a higher coupon payments of debt has two opposite effects: one is to accelerate investment since it increases tax shields and the other is to delay

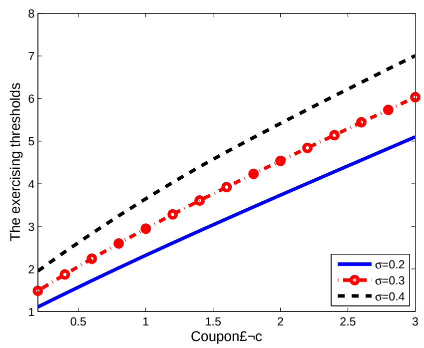

Figure 3: The exercise threshold of the insurer's call option vs. coupon rate.

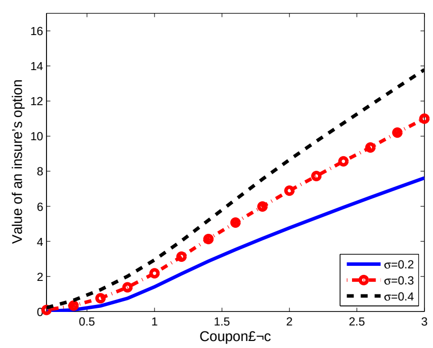

Figure 4: The value of the insurer's call option vs. coupon rate.

investment because it increases bankruptcy costs as well as the guarantee cost. These conclusions are in agreement with Xiang and Yang (2015) who consider another but similar swap, called the equity-for-guarantee swap.

Figure 2 illustrates the value of the option to invest as a function of the coupon payment for three different volatilities. It shows that the value first rises and then declines with a growth of the coupon rate and the three curves are globally concave, meaning there is an optimal coupon rate to maximize the value of the option to invest for any given volatility of the project. This actually explains why it is interesting for us to discuss the optimal capital structure problem in (16). It happens just because of the same reason with why the curves in Figure 1 are convex. In addition, Figure 2 states a well-known phenomenon. That is, for a given coupon payment c, an increase in the project risk (volatility) induces a larger value of the investment option.

Figure 3 depicts the exercise threshold of the insurer's option as a function of the coupon payment for three different volatilities. It says that a growth of the coupon payment of debt gets rise to an increase in the exercise threshold,

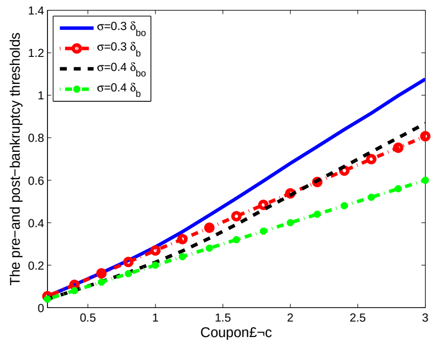

Figure 5: The pre- and post-exercise default threshold vs. coupon rate.

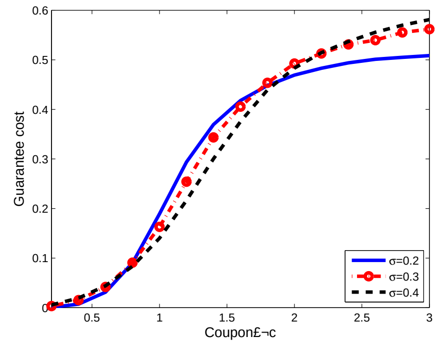

Figure 6: The guarantee cost vs. coupon rate.

meaning that the exercising time of the call option is postponed. This results from the fact that a higher coupon payment means a larger compensatory payment and thus a higher value of his call option is needed in return for his guarantee, as documented by Figure 4. What's more, the larger the coupon rate, the less the value of equity, i.e. the less the insurer obtains upon exercising his option and therefore his waiting is more valuable. The results displayed in Figure 4 are evident since according to (14), the value of an insurer's call option is just the insurer's compensatory payment, which naturally increases with the associated coupon rate.

In addition, for a given coupon payment of debt, Figures 3 and 4 indicate that the higher the project risk, the larger the value and the higher the exercise threshold of the insurer's option. This is consistent with the conclusions about the investment option displayed in Figures 1 and 2.

Figure 5 explains that as we expect, both the pre-exercise and post-exercise default threshold increase with the funding gap but decrease with the project risk. In particular, the pre-exercise default threshold is bigger than the post-exercise one as reported by Yang and Zhang (2015), and their spread is very small when the coupon rate is low but it increases with a growth of the rate.

Figure 6 says that the guarantee cost increases with the coupon rate. This is obvious. However, we find from this figure that there can be ambiguity as to whether the guarantee cost increases or decreases with project risk. In fact, such cost might increase since a higher project risk leads to a larger default probability and consequently the insurer demands a bigger fraction (guarantee cost) of equity in return for his more likely default loss. On the other hand, from the well-known option pricing theory, the higher the project risk, the bigger the value of the insurer's call option and so a less fraction of equity is enough to match his guarantee commitment.

4.2. Asset substitution and debt overhang

Markets are generally imperfect and thus it is important to select a suitable capital structure. To assess capital structure, we usually consider inefficiencies arising from asset substitution and debt overhang. Naturally we wonder whether and how OGS induces the inefficiencies arising from asset substitution discussed by Jensen and Meckling (1976) and debt overhang by Myers (1977).

To see if there is an inefficiency from asset substitution and make a comparison between an OGS and its peer EGS, Figure 7 depicts the sensitivity of the value of the entrepreneur's claim $E^b(x, c)$ to the volatility σ as a function of the cash flow level x. Following the literature on corporate finance, the sensitivity is measured by the derivative of the value of the entrepreneur's claim with regard to the volatility of the firm's cash flow. The figure shows

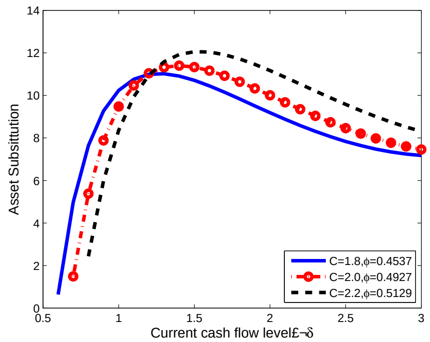

Figure 7: The sensitivity $\partial E^b(x,c)/\partial\sigma$ of the value $E^b(x,c)$ of the shareholders' claim to the volatility σ as a function of the cash flow level x

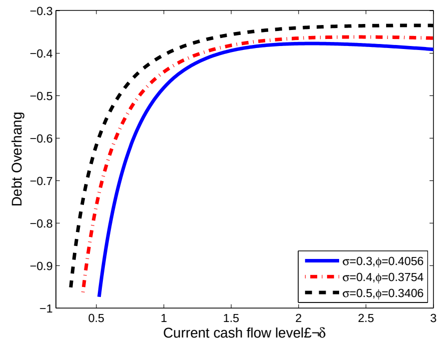

Figure 8: The sensitivity $\partial E^b(x,c)/\partial A - 1$ of the value $E^b(x,c)$ of the shareholders' claim to the firm's value A versus the cash flow level x.

that the derivatives are always positive, implying that entrepreneurs will invest in a higher risk project once they have such selection. The larger the guarantee cost, the stronger the incentive. This happens because under the swap agreements, debt is not protected in the entrepreneur's view though the existing creditor/lender is sufficiently protected since the swap transfers his risk exposure entirely to the insurer. Jensen and Meckling (1976) documents that when debt is not protected, there is a strong risk-shifting incentive for shareholders to invest in high risk project since they harvest all the profit if they succeed but transfer loss to the creditors if they fail. For this reason, our model suggests that to enter into the swap, insurers should in advance limit the project in which the borrower is permitted to invest the loan if he has several investment opportunities in the future.

To make clear if there is a debt overhang problem, Figure 8 illustrates the sensitivity of the value of the shareholders' claim $E^b(x,c)$ to the unlevered firm's value A as a function of the cash flow level x, where the value A is equal to $(1-\tau_f)x/(r-\mu)$ when the firm's EBIT is x. The sensitivity is measured by the derivative $\partial E^b(x,c)/\partial A$ minus one, meaning that the net gain harvested by shareholders after investing a unit of capital. The figure shows that for different volatility of the firm's earnings and the cash flow level, the net profit is always negative, and thus there is a debt overhang problem if OGSs or EGSs are signed. It implies that the entrepreneur is unwilling to inject his own funds to make his firm solvent even if he has money. This conclusion reminds the insurer that after entering into such swap contract, the entrepreneur will not invest his funds in the project any more and so it seems unrealistic. As a matter of fact, an SME might reinvest its retaining earnings and grow with the time. It turns out that this inconsistency results from the fact that an SME may be endowed with some growth options that we do not take into account in this paper, see e.g. Hackbarth and Mauer (2012).

In addition, Figures 7 and 8 show that the OGS induces less inefficiencies arising from both asset substitution and debt overhang than its peer EGS.

4.3. The effects of the tax policy, project risk and bankruptcy loss on investment and financing decisions

In this subsection, we address the effects of project risk σ, effective corporate tax rate τ_f and bankruptcy loss rate α on investment timing, financing structure, default thresholds, leverage level, guarantee cost, and exercise threshold of the insurer's option under optimal capital structure.

To make a sharp comparison, we introduce a benchmark model where debt is fully protected as assumed by Leland (1994). That is, debt issued is risk-free, and the firm must be liquidated once its EBIT hits a threshold

Table 1: Investment and financing decisions under optimal capital structure

	$FV(x)$	δ_i	$E^b(\delta_i, c)$	c^*	L^*	δ_{bo}	ϕ	δ_b	δ_k	IP
Panel A1: The benchmark model										
τ_f=0.35	15.78	1.71	35.55	0.11	0.04	NA	NA	0.12	NA	NA
τ_f=0.45	12.67	2.01	34.21	0.20	0.08	NA	NA	0.25	NA	NA
τ_f=0.50	11.20	2.20	33.69	0.23	0.10	NA	NA	0.32	NA	NA
Panel A2: Financing with an option-for-guarantee swap										
τ_f=0.35	16.67	1.65	16.11	1.41	0.57	0.42	0.23	0.38	3.39	5.68
τ_f=0.45	15.28	1.76	9.02	1.89	0.76	0.62	0.44	0.51	4.31	20.59
τ_f=0.50	14.72	1.82	7.23	2.02	0.81	0.69	0.51	0.54	4.61	31.44
Panel B1: The benchmark model										
σ=0.30	11.78	2.12	33.88	0.22	0.09	NA	NA	0.29	NA	NA
σ=0.40	12.73	2.87	48.08	0.13	0.04	NA	NA	0.17	NA	NA
σ=0.50	13.66	3.79	64.89	0.06	0.01	NA	NA	0.08	NA	NA
Panel B2: Financing with an option-for-guarantee swap										
σ=0.30	14.94	1.79	7.81	1.97	0.79	0.67	0.49	0.53	4.49	26.78
σ=0.40	16.00	2.42	9.89	2.71	0.80	0.77	0.56	0.54	6.56	25.64
σ=0.50	17.14	3.21	12.01	3.70	0.82	0.91	0.62	0.56	9.15	25.44
Panel C1: The benchmark model										
α=0.40	11.74	2.12	34.63	0.17	0.07	NA	NA	0.25	NA	NA
α=0.50	11.67	2.13	35.69	0.10	0.04	NA	NA	0.17	NA	NA
α=0.60	11.63	2.14	36.40	0.05	0.02	NA	NA	0.11	NA	NA
Panel C2: Financing with an option-for-guarantee swap										
α=0.40	14.83	1.80	8.32	1.94	0.78	0.65	0.47	0.52	4.45	26.30
α=0.50	14.62	1.82	9.15	1.89	0.76	0.62	0.44	0.51	4.38	25.24
α=0.60	14.44	1.84	9.97	1.83	0.73	0.59	0.42	0.49	4.29	24.11
Panel D1: The benchmark model										
K=NA	11.78	2.12	33.88	0.22	0.09	NA	NA	0.29	NA	NA
Panel D2: Financing with an option-for-guarantee swap										
K= 5	14.94	1.79	7.88	1.97	0.78	0.67	0.48	0.53	4.49	26.78
K=10	14.90	1.79	8.04	1.96	0.78	0.66	0.51	0.53	5.74	26.50
K=20	14.88	1.79	8.14	1.94	0.78	0.65	0.57	0.42	8.07	26.30

Note. This table reports the following quantities: The value $FV(x)$ of investment option when the current cash flow rate is $x = 1$; investment threshold δ_i; the value $E^b(\delta_i, c)$ of the entrepreneur's equity at investment time; optimal coupon c^*; optimal leverage L^*; pre-exercise default threshold δ_{bo}; guarantee cost ϕ; post-exercise default threshold δ_b; exercise threshold δ_k; the increased percentage IP of the value of the option to invest due to OGSs.

from above, at which the residual value of the firm collected by debt holders is equal to the after-tax value of the outstanding debt. Like the OGS, such entire protection is another candidate contract that makes it possible for a borrower like an SME to get a loan from a lender. Table 1 indicates several new findings shown below.

First, thanks to the OGS, the value of the option to invest increases extremely, as seen in Panels A1 and A2. The increased percentage soars up quickly with a growth of the effective tax rate τ_f. For example, if the tax rate is 35%, the value of the option to invest is 15.78 under the benchmark model and it is 16.67 if a borrower enters into an OGS, i.e. the increased percentage is approximately 5.68%. By contrast, the percentage soars up to 20.59% if the effective tax rate increases to 45%, which is roughly a typical rate in China. This is because thanks to an OGS, debt is risk-free and so it generates tax shields reaching the maximum value: In essence, it brings the issuing firm tax shields still even it defaults. Indirectly, this phenomenon results from our assumption that insurers' compensatory payments make them eligible to pay less income tax, according to the Chinese Administration of Taxation (Circular Cai Shui [2007] No. 27). For this reason, an OGS can not only overcome the financing constraints faced by an SME but also it can increase the firm value (i.e. the value of the option to invest) greatly.

Second, in contrast to the benchmark model, where the coupon rate decreases fast with project risk, it is surprising that the coupon rate in our model increases sharply with the project risk if the OGS is signed as displayed in Panels B1 and B2. In fact, in the borrower's view, debt is actually not protected under the OGS and thus he would default later if project risk is large for a given coupon rate of debt. As a result, the bankruptcy loss gets less for investing in a higher risk project and naturally he would issue more debt. In fact, all the bankruptcy losses and tax shields are harvested by a borrower since both a lender and insurer obtains eventually what he pays in our model. Moreover, Panels B1 and B2 report that in contrast to the benchmark model, the optimal default boundary increases with the cash flow risk under our OGS augmented model. This phenomenon results from the fact that under the OGS model, the higher the cash flow risk, the more the optimal coupon as shown in the table.

Third, from Panels C1 and C2 of the table, the effect of the bankruptcy loss rate under benchmark model is inconsistent with that under our OGS model, though the former is somewhat weaker than the latter. At first sight, this is in conflict with our expectation but actually it happens because the issued amount of debt under an OGS is much bigger than that under the benchmark model and therefore the firm value is more sensitive to the bankruptcy loss rate in our model.

Fourth, Panel D2 of the table explains that the strike price K has a weak impact on investment and financing: The higher the strike price, the higher the guarantee cost but the less the optimal coupon and the value of the investment option by a moderate amount. However, this does not mean that the optimal strike price should be the minimum value zero, which actually simplifies the OGS into an EGS. The reason is that, as displayed by Figures 7 and 8, the OGS induces less inefficiencies arising from both asset substitution and debt overhang than its peer EGS. In particular, if idiosyncratic risk is taken into account, OGSs are much better than EGSs as argued by Yang and Zhang (2015).

Last, the optimal leverage ratio (say 57%) under the OGS is much higher than that (say 4%) under the benchmark model. To begin with, the latter seems implausibly low since it is much lower than predicted by Leland (1994). It turns out that the interest payments are taxed at a personal rate $\tau_i = 0$ in Leland (1994) rather than $\tau_i = 25\%$ taken here. In addition, in our OGS augmented model, debt brings full tax benefits to the issuing firm no matter if default happens or not. This argument is implied by (12) and explained in Remark 2. For this reason, debt should be issued much more than usual and as a result, the optimal leverage ratio increases remarkably. We emphasize that the optimal amount of money borrowed is much higher than the funding gap, meaning that in a sense, entrepreneurs have in effect securitized their assets after entering into the swap.

5. Conclusions

Recently and particularly during the post-financial crisis period, it gets more difficult for an SME to obtain a loan from a bank. To solve such problem, a lot of credit guarantee schemes are popular over the world. For example, the Chinese government encourages insurers to provide SMEs with a credit guarantee by giving insurers many extra premiums. However, this government guarantee would lead to a moral hazard, which actually induces the recent global financial crisis. As its important supplement, a credit guarantee scheme in a market economy should be effectively developed. In particular, an option-for-guarantee swap (OGS) invented by China entrepreneurs is powerful to overcome financing constraints essentially originated from asymmetric information between borrowers and lenders, meaning that the borrowers learn more about their productivity than the lenders.

In this paper, we assume there is no information asymmetry between the borrowers and the insurers and we develop a model that analyzes the investment and financing strategies of an entrepreneur (borrower), who signs a three-party agreement, i.e. OGS, with a bank/lender and an insurer. We compare our OGS augmented model with the classical capital structure analyzed by Leland (1994), where debt is fully protected, and shed light on the entrepreneur's investment, financing, and bankruptcy strategies after the agreement is signed.

We find that an entrepreneur entering into an OGS first accelerates and then postpones investment as the coupon rate of debt rises. The investment threshold increases with the project risk, corporate effective tax rate, bankruptcy costs.

In particular, we reveal that OGSs can significantly increase the value of the investment option in addition to overcoming the financing difficulties experienced by a lot of small- and medium-sized enterprisers over the world. The latter is well known but the former is largely omitted. The increased value of the option to invest soars up with a growth of the effective tax rate. In contrast to the classical capital structure model, the optimal issued amount of debt increases with project risk and the optimal amount of money borrowed is much higher than required to start the project. In this sense, after entering into an OGS, entrepreneurs can actually succeed in securitizing their assets.

In a word, we succeed in providing a new and sufficiently strong argument for why loan guarantees are so popular in China. However, like most borrowing, OGSs would induce inefficiencies arising from asset substitution and debt overhang. Moreover, our conclusions depend on the assumption that insurers are powerful in accurately identifying default risk and there is no information asymmetry between borrowers and insurers. This suggests that to establish extremely powerful guarantee corporations is particularly important to stimulate investment and finally improve social welfare level. We leave the loan guarantee issue under asymmetric information for future research.

Appendix. The pricing of contingent claims

According to Section 3 of Tan and Yang (2017), we present the following approach to price the contingent claims discussed in the paper.

A general formula. A contingent claim here can be considered as a derivative instrument underlying EBIT δ. Specifically, we assume its current cash flow rate ζ is a linear function of the EBIT δ, i.e. $\zeta_s = a_1\delta_s + a_2, s \geq t$, with t representing the current time, and a_1 and a_2 being constants, up to stopping time $T_{\mathcal{D}} \equiv \inf\{s \geq t : \delta_s \notin \mathcal{D}\}$, which is the time of first departure of δ from a domain $\mathcal{D}$. Here we assume the current time is zero, i.e. $t = 0$, without loss of generality since our model is time-homogeneous, meaning all decisions and prices are independent of time. At the stopping time $T_{\mathcal{D}}$, the claim generates a lump-sum dividend, which is a function, denoted by $g(\cdot)$, of the cash flow rate $\delta_{T_{\mathcal{D}}} \in \partial\mathcal{D}$, where $\partial\mathcal{D}$ represents the boundary of the domain $\mathcal{D}$. After time $T_{\mathcal{D}}$, its claimant obtains nothing. According to dynamic asset pricing theory, see e.g. Duffie (2001), the price of the claim is a function of x independent of time, denoted by $F(x)$, and it is given by

$$F(x) = \mathbb{E}\Big[\int_0^{T_{\mathcal{D}}} e^{-rs}(a_1\delta_s + a_2)ds + e^{-rT_{\mathcal{D}}} g(\delta_{T_{\mathcal{D}}})|\delta_0 = x\Big],\ x \in \mathcal{D},$$

and

$$F(x) = g(x), \quad x \in \partial\mathcal{D}. \tag{18}$$

According to a standard method, we conclude that the function $F(\cdot)$ must satisfy the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 F_{xx}(x) + \mu F_x(x) - rF(x) + (a_1 x + a_2) = 0, \quad x \in \mathcal{D}.$$

Its general solution is

$$F(x) = R_1 x^{\beta_+} + R_2 x^{\beta_-} + \frac{a_1 x}{r - \mu} + \frac{a_2}{r},\ \ x \in \mathcal{D}, \tag{19}$$

where R_1 and R_2 are constants to be determined by the boundary conditions (18), and β_+ and β_- are the positive and negative root of the quadratic equation:

$$\frac{1}{2}x(x-1) + \mu x - r = 0.$$

The pricing of equity after the insurer's option is exercised. Under this special situation, we note that $\mathcal{D} = (\delta_b, \infty)$, $\zeta = (1-\tau_f)(x-c)$, i.e. $a_1 = 1 - \tau_f$

and $a_2 = -(1-\tau_f)c$ for $x \in \mathcal{D}$, and the boundary conditions

$$F(\delta_b) = 0 \quad \text{and} \quad \lim_{x\to\infty} F(x) = (1-\tau_f)\left[\frac{x}{r-\mu} - \frac{c}{r}\right]. \tag{20}$$

Substituting (19) into (20), we derive (2) directly noting that $F(x) = E(x,c)$ here. Naturally (3) holds also.

The value of debt after the insurer's option is exercised. To price the value of debt after exercising the call option, we note that $\mathcal{D} = (\delta_b, \infty)$, $\zeta = (1-\tau_i)c$, i.e. $a_1 = 0$ and $a_2 = (1-\tau_i)c$ for $x \in \mathcal{D}$, and the boundary conditions

$$F(\delta_b) = (1-\alpha)\frac{\delta_b}{r-\mu} \quad \text{and} \quad \lim_{x\to\infty} F(x) = (1-\tau_i)\frac{c}{r}. \tag{21}$$

Substituting (19) into (21), we derive (4) at once.

We now determine the optimal post-exercise default threshold δ_b. For this aim, we use the smooth-pasting condition, see e.g. Dixit and Pindyck (1994) or Song et al. (2014): $\frac{\partial E^a(x,c)}{\partial x}|_{x=\delta_b} = 0$. Thanks to (3), this condition leads to the optimal post-exercise default threshold given by (9).

The value of equity before the insurer's option is exercised. For this end, we note that $\mathcal{D} = (\delta_{bo}, \delta_k)$, $\zeta = (1-\tau_f)(x-c)$, i.e. $a_1 = 1-\tau_f$ and $a_2 = -(1-\tau_f)c$ for $x \in \mathcal{D}$, and the boundary conditions

$$F(\delta_{bo}) = 0 \quad \text{and} \quad F(\delta_k) = E^a(\delta_k, c) + \phi K. \tag{22}$$

Substituting (19) into (22), we derive (5).

The value of debt before the insurer's option is exercised. In the same way, we note here that $\mathcal{D} = (\delta_{bo}, \delta_k)$, $\zeta = (1 - \tau_i)c$, i.e. $a_1 = 0$ and $a_2 = (1 - \tau_i)c$ for $x \in \mathcal{D}$, and the boundary conditions

$$F(\delta_{bo}) = (1 - \alpha)(1 - \tau_f)x/(r - \mu) \quad \text{and} \quad F(\delta_k) = D^a(\delta_k, c). \tag{23}$$

Substituting (19) into (23), we derive (6).

The value of the claim on nothing but one dollar received at the exercise time of the insurer's option prior to default. This claim is similar with an Arrow security. To derive its value, we note here that $\mathcal{D} = (\delta_{bo}, \delta_k)$, $\zeta = 0$, i.e. $a_1 = 0$ and $a_2 = 0$ for $x \in \mathcal{D}$, and the boundary conditions

$$F(\delta_{bo}) = 0 \quad \text{and} \quad F(\delta_K) = 1. \tag{24}$$

Substituting (19) into (24), we derive the first equation of (7). The second one is similarly derived. After that, (8) is obvious based on our derivations here.

Optimal pre-exercise default threshold decided by the entrepreneur and optimal exercise threshold decided by the insurer. On account of that the two decisions depend on each other, we must fix a Nash equilibrium of the game between the entrepreneur and insurer. For this end, using the following smooth-pasting conditions

$$\left.\frac{\partial E^b(x,c)}{\partial x}\right|_{x=\delta_{bo}} = 0 \quad \text{and} \quad \left.\frac{\partial[\phi V^G(x)]}{\partial x}\right|_{x=\delta_k} = \left.\frac{\partial[\phi(E(x,c) - K)]}{\partial x}\right|_{x=\delta_k},$$

we derive (10) and (11). We emphasize that while computing the second condition mentioned above, guarantee cost ϕ, which is given by (15), must be considered as a constant since the exercising of the insurer's option happens after the swap agreement is signed.

A benchmark model. For comparison, we introduce a benchmark model, where an SME does not enter into an option-for-guarantee swap contract, but instead, debt is still fully protected since we assume otherwise no lender would like to provide an SME with a loan. Clearly, the value of debt is still $D_0(c)$ defined before and there must be a bankruptcy threshold, denoted by δ_b, such that once its EBIT reaches δ_b, the SME is liquidated. Thus due to the fully protected covenant, it follows that

$$(1-\alpha)(1-\tau_f)\frac{\delta_b}{(r-\mu)} = (1-\tau_i)\frac{c}{r},$$

That is,

$$\delta_b = \frac{(1-\tau_i)(r-\mu)}{(1-\tau_f)(1-\alpha)}\frac{c}{r}.$$

Therefore, the value of the entrepreneur's claim after investment equals

$$E(x,c) = (1-\tau_f)(\frac{x}{r-\mu} - \frac{c}{r}) - (1-\tau_f)(\frac{\delta_b}{r-\mu} - \frac{c}{r})\left(\frac{x}{\delta_b}\right)^{\beta_-}.$$

We now consider optimal investment and financing policies under the benchmark model. For a given investment threshold, δ_i, optimal coupon rate

c^* must satisfy the following first-order condition:

$$\left.\frac{\partial[E(\delta_i, c) + D_0(c)]}{\partial c}\right|_{c=c^*} = 0.$$

The optimal coupon payments rate c^* is therefore a solution of the following transcendental equation:

$$(1-\beta_-)(1-\tau_f)\left[\frac{(1-\tau_i)}{(1-\tau_f)(1-\alpha)}\frac{1}{r} - \frac{1}{r}\right]\left(\frac{\delta_i}{\delta_b}\right)^{\beta_-} + \frac{\tau_f - \tau_i}{r} = 0.$$

For a given coupon rate c and investment threshold δ_i, the value of the entrepreneur's investment option is given by

$$\overline{FV}(x) = \left[E(\delta_i, c) - \Big(I - D_0(c)\Big)\right]\left(\frac{x}{\delta_i}\right)^{\beta_+}.$$

Accordingly, for a given coupon rate c, the optimal investment threshold δ_i^* that maximizes the value $FV(x)$ of the entrepreneur's claim is a solution of the following equation with the variable being δ_i:

$$\begin{aligned}&\beta_+\left[\left(\frac{\delta_i}{r-\mu} - \frac{c}{r}\right) - \left(\frac{\delta_b}{r-\mu} - \frac{c}{r}\right)\left(\frac{\delta_i}{\delta_b}\right)^{\beta_-} + \frac{1-\tau_i}{1-\tau_f}\frac{c}{r} - \frac{I}{1-\tau_f}\right]\\ &- \frac{\delta_i}{r-\mu} + \beta_-\left(\frac{\delta_b}{r-\mu} - \frac{c}{r}\right)\left(\frac{\delta_i}{\delta_b}\right)^{\beta_-} = 0.\end{aligned}$$

Hence, the value of the option to invest is

$$\overline{FV}(x) = \left[E(\delta_i^*, c^*) + D_0(c^*) - I\right]\left(\frac{x}{\delta_i^*}\right)^{\beta_+}. \tag{25}$$

The increased percentage (IP) of the value of the option to invest due to an OGS. This percentage is defined from (17) and (25) by

$$IP \equiv \frac{FV(x) - \overline{FV}(x)}{\overline{FV}(x)}, \tag{26}$$

where x represents the current cash flow level as before.

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