Strategic Debt Restructuring and Asset Substitution

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Abstract

This paper examines whether debt renegotiation mitigates the agency costs associated with asset substitution. Inspired by the studies of Mella-Barral and Perraudin (1997) and Leland (1998) we have developed an analytic continuous time model of a firm that has the option to switch to a higher risk activity and renegotiate the terms of the debt. Our model creates a tradeoff between increasing firm

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volatility and decreasing growth rate which characterizes the potential for asset substitution. This study finds that debt renegotiation substantially reduces agency costs for moderate to high substitution potential, but it increases the costs for low substitution opportunities. Further evidence suggests that the effectiveness of debt renegotiation in mitigating the costs of asset substitution is highly influenced by the equityholders' bargaining power. Our results show that agency costs are eliminated by granting equityholders the entire bargaining power.

Keywords: asset substitution, capital structure, debt renegotiation
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1. Introduction

The possibility of debt restructuring upon financial distress is a central element in the corporate finance literature. In particular, a number of studies examine the effect of debt renegotiability on capital structure and wealth transfer between firm's claimants. Asset substitution problem, first described by Jensen and Meckling (1976), is another well-studied issue in the corporate financial theory. The purpose of this paper is to combine these two strands of literature into one model and, moreover, to examine whether debt renegotiation can mitigate the asset substitution problem.

The framework of this study is a unified dynamic model of capital structure and risk selection. Our model relates to the dynamic contingent claims models of Leland (1998) and Ericsson (2000). Their models examine asset substitution by considering that equityholders have the flexibility to change firm risk after debt is contracted. Equityholders' risk-shifting incentives reduce the ex post optimal firm value and, hence, generate agency costs. Agency costs are expressed as the difference between the ex ante (before debt is contracted) and the ex post (after debt is contracted) optimal firm values. However, both papers do not examine debt renegotiation and ignore opportunity costs of risk-shifting. Staying in the tradition of this approach, several relevant studies (e.g., Henessy and Tserlukevich, 2004; Ju and Ou-Yang 2005; Mauer and Sarkar, 2005) consider that the asset substitution problem is equivalent to a pure risk-shifting problem. Intuitively, a setting that only relates risk-shifting to volatility changes, can serve as a preliminary step in studying equityholders' risk-taking incentives. According to Décamps and Djembissi (2007) the inclusion of risk-shifting costs in the asset substitution problem is fundamental: "two problems jointly define asset substitution (i) a pure risk-shifting problem acting on the volatility of the growth rate of the cash flows, and (ii) a first order stochastic dominance problem acting on the risk adjusted expected growth rate of the cash flows".

In the present paper, we adopt the assumption that equityholders have the ability to switch to a high-risk activity, but this decision entails an opportunity cost. This cost is reflected by decreasing the growth rate of the high-risk profile. Notably, our study creates a tradeoff between increasing firm's volatility and decreasing growth rate which characterizes asset substitution opportunities. Risk-shifting costs are indeed a crucial determinant of the equityholders' incentives for asset substitution and, hence, the agency problem.

Although the asset substitution problem has been widely studied in the literature, the role of debt renegotiation in mitigating the costs of asset substitution has received relatively little attention. The study of Flor (2011) offers a first insight into the relationship between debt renegotiation and risk-shifting incentives. However, this framework of analysis is restrictive in the sense that asset substitution is only used as a threat to debt renegotiation. Our study builds upon Leland (1998), who focuses on the joint determination of risk policy and capital structure when the terms of the debt are non-renegotiable, and both Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000), who examine strategic debt renegotiation.

The influential studies of Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) highlight the significance of incorporating debt renegotiability in determining the firm's optimal capital structure. All these studies incorporate debt renegotiation under the assumption that equityholders service the debt strategically. Debt renegotiation takes the form of take-it or leave-it offers (from the equityholders to debtholders or from debtholders to the equityholders) that lead to temporal coupon reductions. In this paper, we establish a unified model of renegotiable debt that accounts for taxes and renegotiation costs. Mella-Barral and Perraudin (1997) asset pricing model is nested in our model framework. By setting the renegotiation costs and taxes to zero along with take-it or leave-it offers, and assuming no risk flexibility, our model reduces to Mella-Barral and Perraudin (1997). Francois and Morellec (2002), Roberts and Sufi (2009), Pawlina (2010) and Favara et al. (2012, 2017) are also important studies on debt renegotiation to consider.

Our analysis illustrates that the possibility of debt renegotiation mitigates in most cases the asset substitution problem. In particular, the impact of debt renegotiation on risk-shifting is highly influenced by the distribution of bargaining power between equityholders and debtholders. Altogether, our model shows that debt renegotiability, in combination with high equityholders' bargaining power, can serve as a mechanism to eliminate the asset substitution problem.

The rest of the paper is structured as follows. Section 2 extends the proposed model in order to include risk-shifting considerations. Section 3 determines optimal risk policies and provides a measure of agency costs. Section 4 presents the numerical results of our study. Finally, Section 5 concludes. The generalization of Mella-Barral and Perraudin (1997) asset pricing model to incorporate tax advantages to debt and renegotiation costs is presented in Appendix A. Remaining proofs and derivations are given in Appendices B and C.

2. Model Setup

The current section develops the proposed dynamic model of debt renegotiation and asset substitution that incorporates tax advantages, renegotiation costs and bargaining power of firm's claimants. Subsection 2.1 presents a review of the Mella-Barral and Perraudin (1997) asset pricing model of strategic default and debt renegotiation. Subsection 2.2 generalizes the basic model to include the tax benefits to leverage and renegotiation costs. Subsection 2.3 modifies the generalized model to incorporate the bargaining power of firm's claimants. Finally, Subsection 2.4 extends the developed model to study the asset substitution problem.

2.1 THE BASIC MODEL

Mella-Barral and Perraudin (1997) consider a one-product firm selling a unit of its product for a price, p_t, where the price of the product is assumed to follow geometric Brownian motion, while incurring constant production costs, w, per unit output. In their model, bankruptcy reduces the firm's efficiency, by generating diminished net earnings per unit output. The degree to which net earnings decline, is a measure of the magnitude of direct costs of bankruptcy. After bankruptcy, the new owners of the firm are the debtholders who have the option to liquidate the firm or maintain the firm's operations with diminished net earnings.

Mella-Barral and Perraudin (1997) determine the optimal capital structure and debt service under three different scenarios. In the first scenario, the contracted coupon payments cannot be renegotiated and bankruptcy is triggered endogenously by equityholders in order to maximize the value of equity. In the second scenario, equityholders can make take-it or leave-it offers to creditors regarding contracted payments. The debt service is a piecewise function of the output price, p_t, applying to intervals which are uniquely determined by the optimal trigger values of p_t. As a consequence, debtholders accept a reduced service flow within the renegotiation region. In the third scenario, the alternative polar case of debt renegotiation is examined in which debtholders have maximum bargaining power and can make take-it or leave-it offers to equityholders regarding debt service. Here, debtholders have an incentive to maximize the value of their claim and minimize the value of equity. Throughout the analysis, it is assumed that the levered firm has issued perpetual debt with face value, b/r, and coupon, b.

Because we are interested in risky debt, we further assume that debt principal is no less than the liquidation value, γ, and hence, $\gamma < b/r$.

2.2 GENERALIZED ASSET PRICING MODEL

In order to examine the effect of potential risk-shifting, Mella-Barral and Perraudin (1997) model is generalized to include the tax advantages of debt and renegotiation costs. Similar to a large strand of the literature we assume that the cash flow to equityholders is taxed at a rate, τ, while the coupon payments are tax deductible. Fan and Sundaresan (2000) assume that the tax advantage is lost inside the renegotiation region and is restored when the full coupon payment is made. The tax structure adopted here is based on this reasonable assumption.

In addition, the analysis herein assumes a costly debt renegotiation process. The proposed model formulates the Mella-Barral and Perraudin (1997) model so as to include renegotiation cost by assuming that renegotiation of debt service incurs a cost, δ, per unit of time when the full contracted coupon, b, is not paid. The incidence of this cost will be entirely on equityholders in the case of equityholder offers and on debtholders in the case of debtholder offers. We define the renegotiation cost function $C(p)$ over the interval $[p_c, p_s)$, where p_c is the output price that triggers bankruptcy and p_s is the lowest price at which equityholders pay the contracted payment, b. Subsequently, we determine $C(p)$ as the value of a claim that provides a payment δ when the output price p_t lies in the interval $[p_c, p_s)$ and zero when $p_t \geq p_s$. It follows that the renegotiation cost function $C(p)$ satisfies ordinary differential equations

$$rC(p) = \begin{cases} \delta + \mu p C'(p) + \frac{\sigma^2}{2} p^2 C''(p) & \text{for } p \in [p_c, p_s), \\ \mu p C'(p) + \frac{\sigma^2}{2} p^2 C''(p) & \text{for } p \in [p_s, \infty), \end{cases} \tag{1}$$

where μ is the rate of return and σ is the risk of the asset return.

Analytic solutions for equity and debt values as well as the optimal service flow function for each renegotiation scenario (no-renegotiation, with equityholder offers, with debtholder offers) are obtained in Appendix A. Furthermore, a detailed mathematical treatment of the generalization of the basic model to incorporate corporate taxes and renegotiation costs is also presented.

2.3 NASH BARGAINING GAME

Having determined the model of optimal capital structure for the non-renegotiable debt and the two limit cases of debt renegotiation, we pursue a Nash equilibrium analysis in order to take into consideration the bargaining power of firm's claimants. Mella-Barral and Perraudin (1997) presume that there are only two extreme debt renegotiation scenarios, the cases that either equityholders or debtholders hold all the bargaining power. Our analysis extends this approach by considering that the bargaining power is distributed among the firm's claimants. By adopting the notation made by Fan and Sundaresan (2000), we define η to be the equityholders' bargaining power that takes values in the interval $[0,1]$. Since η is the equityholders' bargaining power coefficient, $1-\eta$ corresponds to the debtholders' bargaining power. Here, the firm's claimants bargain over the net firm value $W(p)-C(p)$ with equityholders receiving a fraction $\theta^*\Big(W(p)-C(p)\Big)$ and debtholders receiving the remaining $(1-\theta^*)\Big(W(p)-C(p)\Big)$. The sharing rule θ^* is optimal if it is the solution of the Nash bargaining game. Given that the payoffs of equityholders and debtholders for quitting renegotiation are zero and X(p), respectively, the Nash bargaining solution can be expressed as

$$\begin{aligned}\theta^* &= \underset{\theta}{\operatorname{argmax}}\left[\left(\theta\Big(W(p)-C(p)\Big)\right)^{\eta}\left((1-\theta)\Big(W(p)-C(p)\Big)-X(p)\right)^{1-\eta}\right]\\ &= \eta-\eta\frac{X(p)}{W(p)-C(p)},\end{aligned} \tag{2}$$

where $X(p)$, $W(p)$ and $C(p)$ are given by Equations (A-4), (A-18) and (A-20) respectively, with $p_{c1}=p_c$ and $p_{s1}=p_s$. Given the optimal sharing rule θ^*, we may now derive the values of equity $V(p)$ and debt $D(p)$ as well as the optimal renegotiation trigger, p_s. The equity value is given by

$$V(p)=\begin{cases}\eta\Big(W(p)-X(p)-C(p)\Big) & \text{for } p\in[p_c,p_s),\\ (1-\tau)\left(\frac{p}{r-\mu}-\frac{w+b}{r}\right)+\Big[\eta\Big(W(p_s)-X(p_s) & \\ -C(p_s)\Big)-(1-\tau)\left(\frac{p_s}{r-\mu}-\frac{w+b}{r}\right)\Big]\left(\frac{p}{p_s}\right)^{\lambda_1} & \text{for } p\in[p_s,\infty).\end{cases} \tag{3}$$

The value of debt can be expressed as

$$D(p) = \begin{cases} \eta X(p) + (1-\eta)\left(W(p) - C(p)\right) & \text{for } p \in [p_c, p_s), \\ \frac{b}{r} + \Big[\eta X(p_s) + (1-\eta)\Big(W(p_s) & \\ -C(p_s)\Big) - \frac{b}{r}\Big]\left(\frac{p}{p_s}\right)^{\lambda_1} & \text{for } p \in [p_s, \infty), \end{cases} \tag{4}$$

where p_s is determined from the smoothness condition $\frac{\partial D}{\partial p}|_{p\uparrow p_s} = \frac{\partial D}{\partial p}|_{p\downarrow p_s}$ and equals

$$p_s = -\frac{\lambda_1}{1-\lambda_1}\frac{(1-\tau)(1-\eta(1-\xi_0))\,w + (1-\tau+\eta\tau)\,b}{(1-\eta(1-\xi_1))\,r\,(1-\tau)}(r-\mu). \tag{5}$$

Applying the optimality condition $W'(p_c) = 0$ allows for determining the default trigger, p_c. The latter is given by the implicit solution of the following equation

$$(1-\tau)\frac{r(\lambda_1 - 1)\,p_c}{r-\mu} - \lambda_1(\gamma r + \delta + (1-\tau)\,w) + \lambda_1(\tau b + \delta)\left(\frac{p_c}{p_s}\right)^{\lambda_2} = 0. \tag{6}$$

From Equation (6) it can be seen that if taxes and renegotiation costs are zero ($\tau = \delta = 0$), the bankruptcy triggers for the cases of all-equity financing and debt financing with renegotiable debt are the same, $p_c^* = p_c$. The optimal debt service is found by substituting $D(p)$ into the ordinary differential equation

$$rD(p) = s(p) + \mu p D'(p) + \frac{\sigma^2}{2}p^2 D''(p), \quad p \in [p_c, p_s]. \tag{7}$$

Given that $s(p) = b$ if $p > p_s$, we can directly obtain the expression for optimal debt service $s(p)$

$$s(p) = \begin{cases} \eta r\gamma + (1-\tau)(1-\eta)\left(p - w - \frac{\delta}{1-\tau}\right) & \text{for } p \in [p_c, p_x), \\ (1-\tau)\left[(1-\eta)\left(p - w - \frac{\delta}{1-\tau}\right) + \eta(\xi_1 p - \xi_0 w)\right] & \text{for } p \in [p_x, p_s), \\ b & \text{for } p \in [p_s, \infty), \end{cases} \tag{8}$$

where p_x is the optimal default trigger of the debtholders running the firm and is given by Equation (A-5).

2.4 RISK STRATEGY SELECTION

The proposed asset pricing model is further extended by considering that firm volatility σ and growth rate μ are not constant and the firm is flexible to change its risk strategy. So far, a large strand of the literature studying the asset substitution problem (Leland, 1998; Ericsson, 2000) has assumed that altering the firm risk is costless. A necessary modification is to suppose that risk-shifting imposes a cost on equityholders. Risk-shifting cost is reflected in lowering the growth rate μ associated with the high volatility strategy. To study this extension of the model, we assume that equityholders have the option to choose between two firm activities related to different volatilities and growth rates. We denote by H and L the high- and low-risk activities, respectively, imposing the assumption that $\sigma_H > \sigma_L$ and $\mu_H < \mu_L$. The inclusion of risk-shifting alters the discussed model in a simple fashion. More specifically, the solutions for the firm, debt and equity values are determined by substituting μ, λ_1 and λ_2 with μ_i, λ_1^i and λ_2^i to the corresponding expressions of the previous section, where, $i = H, L$.

In order to investigate the influence of a given tradeoff between decreasing growth rate and increasing firm volatility on the risk strategy choice, we concentrate on non-renegotiable debt. We start by examining two extreme cases, the case that equityholders always choose the high-risk activity and the polar case that the safe activity is always chosen. The following lemma shows that when $\mu_H = \mu_L$ and $\sigma_H > \sigma_L$, the value of equity $\hat{V}(p)$ becomes equal to $\hat{V}_H(p)$.

Lemma 1. *If the terms of the debt cannot be renegotiated and the increase in risk is costless ($\mu_H = \mu_L$, $\sigma_H > \sigma_L$) then, equityholders always choose the high-risk profile. This condition implies that the firm goes into liquidation at the trigger price p_b^H.*

Here, equity value is an increasing and convex function of the output price p. In turn, this implies that there is a positive relationship between equity value and the volatility of the output price, p. Figure 1 shows that $\forall p \in (0, \infty)$, $\hat{V}_H(p) > \hat{V}_L(p)$ thus, equityholders always choose the high-risk profile and provoke liquidation at the trigger price p_b^H. Equityholders' optimal decision is driven by the fact that the bankruptcy trigger is decreasing with the volatility, so a necessary condition for never being optimal by equityholders to switch to the high-risk profile is $p_b^H > p_b^L$. This condition occurs when the cost of increasing the firm risk is extremely high, hence, for a given increase in firm volatility $\Delta\sigma = \sigma_H - \sigma_L$ the decline in growth rate $\Delta\mu = \mu_L - \mu_H$ is enormous. The following lemma shows that when

$\mu_H << \mu_L$ and $\sigma_H > \sigma_L$, the value of equity $\hat{V}(p)$ becomes equal to $\hat{V}_L(p)$.

Lemma 2. *If the terms of the debt cannot be renegotiated and $p_b^L < p_b^H$ then, equityholders always choose the low-risk profile. This condition implies that the firm goes into liquidation at the trigger price p_b^L.*

Here, the cost of increasing the firm risk is so high that it is optimal for equityholders to never adopt the high-risk profile. More precisely, the condition $p_b^L < p_b^H$ ensures that $\hat{V}_L(p) > \hat{V}_H(p)\ \forall p \in (0, \infty)$ (see Figure 2). Mathematical proofs of these two lemmas are presented in the Appendix B. The findings of the above analysis are consistent with the study of Décamps and Djembissi (2007).

When the firm is financed with renegotiable debt then an analytical proof for the equityholders' incentives cannot be obtained. The analysis therefore relies on numerical methods. As expected, the findings show that debt renegotiation does not affect the optimal decision of equityholders for never switching to the safe (high) activity when $\mu_H = \mu_L$ ($\mu_H << \mu_L$).

However, in practice, equityholders immediately choose the high-risk activity in order to exploit potential tax advantages, whereas for larger values of the output price it may be optimal to switch to the low-risk activity (see Figure 3). Consequently, our analysis focuses on more realistic cases, where neither always adopting the safe or the high-risk profile is optimal. This approach is achieved by formulating the presented model so as to create a reasonable tradeoff between firm volatility σ and growth rate μ and is described in the following section.

3. Determination of Optimal Risk Choices and Agency Costs

The firm chooses its optimal risk switching point p_{HL} depending on whether the switching decision can be committed ex ante or will be selected ex post. The ex ante determination of optimal risk strategy is characterized by the claimholders' incentives to maximize the initial firm value. On the other hand, the ex post optimal risk profile is characterized by the equityholders' incentives to maximize the value of equity. The loss in firm value that arises from equityholder's incentives is used as a proxy for the magnitude of the agency problem. This measure of agency costs, first established by Leland (1998), has also been adopted by several studies (Ericsson, 2000; D*é*camps and Djembissi, 2007; Djembissi, 2011) on asset substitution.

The switching trigger p_{HL} characterizes the firm's optimal risk strategy, thus implying that the firm adopts the low volatility strategy when $p \geq p_{HL}$ and the high volatility strategy when $p < p_{HL}$. Note that higher volatility is associated with a lower growth rate, hence, if $\sigma_H > \sigma_L$ then also $\mu_H < \mu_L$. The following subsections determine the ex ante and ex post optimal risk policies and capital structure for both cases of debt renegotiation.

3.1 NON-RENEGOTIABLE DEBT

3.1.a Valuation of Claims for Non-Renegotiable Debt

First, we determine the values of the firm's debt and equity by considering that $p_b < p_{HL}$. The specified ordering implies $\sigma = \sigma_H$, $\mu = \mu_H$ when

$p_b \leq p < p_{HL}$ and $\sigma = \sigma_L$, $\mu = \mu_L$ when $p \geq p_{HL}$. Debt value is obtained by solving Equation (A-6) with no-bubbles condition, default condition at p_b, value matching and smooth pasting conditions at the risk switching point p_{HL}. The general solution to Equation (A-6) in the high and low volatility regions is given by

$$\begin{aligned}\hat{D}(p) = \hat{DH}(p) &= \frac{b}{r} + \hat{a}_{1H} p^{\lambda_1^H} + \hat{a}_{2H} p^{\lambda_2^H} \qquad p \in [p_b, p_{HL}), \\ = \hat{DL}(p) &= \frac{b}{r} + \hat{a}_{1L} p^{\lambda_1^L} + \hat{a}_{2L} p^{\lambda_2^L} \qquad p \in [p_{HL}, \infty),\end{aligned} \tag{9}$$

where the analytical determination of constants $\hat{a}_{1H}, \hat{a}_{2H}, \hat{a}_{1L}$ and $\hat{a}_{2L}$ is presented in Appendix C.

Similarly, equity value is the general solution to Equation (A-7) in the high and low volatility regions

$$\begin{aligned}&\hat{V}(p) = \\ &\hat{VH}(p) = (1-\tau)\left(\frac{p}{r-\mu_H} - \frac{w+b}{r}\right) + \hat{b}_{1H} p^{\lambda_1^H} + \hat{b}_{2H} p^{\lambda_2^H} \quad p \in [p_b, p_{HL}), \\ &\hat{VL}(p) = (1-\tau)\left(\frac{p}{r-\mu_L} - \frac{w+b}{r}\right) + \hat{b}_{1L} p^{\lambda_1^L} + \hat{b}_{2L} p^{\lambda_2^L} \quad p \in [p_{HL}, \infty),\end{aligned} \tag{10}$$

where $\hat{b}_{1H}, \hat{b}_{2H}, \hat{b}_{1L}$ and $\hat{b}_{2L}$ are determined from the boundary conditions. More precisely, the unknowns equity coefficients are defined by the absence of bubbles condition, the bankruptcy condition at p_b, the continuity conditions both in levels and in first derivatives at p_{HL} (cf. Appendix C).

Firm value in both volatility regions is the sum of debt value and equity value

$$\hat{W}(p) =$$

$$\hat{WH}(p) = \hat{DH}(p) + \hat{VH}(p) \qquad p \in [p_b, p_{HL}), \tag{11}$$

$$\hat{WL}(p) = \hat{DL}(p) + \hat{VL}(p) \qquad p \in [p_{HL}, \infty).$$

The default trigger p_b is endogenously determined by equityholders to maximize the value of their claim. The latter is implicitly determined by the smoothness condition at $p = p_b$

$$\hat{h}_1\left(p_b, p_{HL}, b, \hat{\epsilon_1}\right) = \frac{\partial \hat{VH}\left(p, p_b, p_{HL}, b, \hat{\epsilon_1}\right)}{\partial p}|_{p=p_b} = 0, \tag{12}$$

where $\hat{\epsilon_1} = (\sigma_H, \sigma_L, w, \mu_H, \mu_L, r, \tau)$ is the exogenous parameter vector.

3.1.b Optimal Risk Strategy for Non-Renegotiable Debt

As in Leland (1998), our study measures agency costs by the difference between the ex ante and the ex post optimal firm values. The ex ante optimal risk switching trigger p_{HL}, default trigger p_b and coupon b are determined by maximizing the initial value of the firm

$$\max_{p_b, p_{HL}, b} \hat{W}\left(p, p_b, p_{HL}, b, \hat{\epsilon_2}\right)|_{p=p_0} \tag{13}$$

subject to the smooth pasting condition (12), where the exogenous parameter vector is $\hat{\epsilon_2} = (\sigma_H, \sigma_L, w, \mu_H, \mu_L, r, \tau, \gamma, \xi_1, \xi_0)$. Depending on the value of p_0, $\hat{W}$ equals $\hat{WH}$, if $p_b \leq p_0 < p_{HL}$ and $\hat{WL}$, if $p_0 \geq p_{HL}$.

The ex post optimal risk strategy is determined by obtaining the maximum of (13) subject to (12) and the super contact optimality condition, first documented by Dumas (1991). The latter is given by

$$\begin{aligned}\hat{h}_2\left(p_b, p_{HL}, b, \hat{\epsilon_1}\right) &= \frac{\partial^2 \hat{VL}}{\partial p^2}|_{p\uparrow p_{HL}} - \frac{\partial^2 \hat{VL}}{\partial p^2}|_{p\downarrow p_{HL}} \\ &= 0.\end{aligned} \tag{14}$$

The boundary condition (14) ensures that the risk switching point p_{HL} is selected optimally so as to maximize equity value. Here, two optimal solutions are determined, one with $p_b \leq p_0 < p_{HL}$, and one with $p_0 \geq p_{HL}$. The ex ante and ex post optimal risk strategies corresponding to the larger initial firm value are selected.

3.2 RENEGOTIABLE DEBT

3.2.a Valuation of Claims for Renegotiable Debt

With renegotiation, our analysis adopts the assumption that during the renegotiation process the firm's risk remains unchanged, hence, $p_c \leq p_x \leq p_s \leq p_{HL}$. Consequently, it is assumed that the high-risk level is chosen if $p < p_{HL}$ and the low-risk level if $p \geq p_{HL}$. First, the value of the firm in the high- and low-risk regions can be written on the form

$$\begin{aligned}&W(p) = \\ &WH(p) = \begin{cases} (1-\tau)\left(\frac{p}{r-\mu_H} - \frac{w}{r}\right) + a_{1s}p^{\lambda_{1H}} + a_{2s}p^{\lambda_{2H}} & p \in [p_c, p_s), \\ (1-\tau)\left(\frac{p}{r-\mu_H} - \frac{w}{r}\right) + \frac{\tau b}{r} + a_{1H}p^{\lambda_{1H}} + a_{2H}p^{\lambda_{2H}} & p \in [p_s, p_{HL}), \end{cases} \\ &WL(p) = (1-\tau)\left(\frac{p}{r-\mu_L} - \frac{w}{r}\right) + \frac{\tau b}{r} + a_{1L}p^{\lambda_{1L}} + a_{2L}p^{\lambda_{2L}} \qquad p \in [p_{HL}, \infty).\end{aligned} \tag{15}$$

Constants $a_{1s}, a_{2s}, a_{1H}, a_{2H}, a_{1L}$ and a_{2L} are determined from asymptotic condition, default condition, value matching and smooth pasting requirements at p_s and p_{HL} (cf. Appendix C).

The renegotiation cost function satisfies the set of ordinary differential Equations (1). It follows that

$$
\begin{aligned}
&C(p) = \\
&CH(p) = \begin{cases} \frac{\delta}{r} + b_{1s}p^{\lambda_{1H}} + b_{2s}p^{\lambda_{2H}} & p \in [p_c, p_s), \\ b_{1H}p^{\lambda_{1H}} + b_{2H}p^{\lambda_{2H}} & p \in [p_s, p_{HL}), \end{cases} \\
&CL(p) = b_{1L}p^{\lambda_{1L}} + b_{2L}p^{\lambda_{2L}} \qquad p \in [p_{HL}, \infty).
\end{aligned}
\tag{16}
$$

Again, boundary conditions are the zero renegotiation costs at default, the no-bubbles condition and the value matching and smoothness conditions at p_s and p_{HL}. Unknown constants $b_{1s}, b_{2s}, b_{1H}, b_{2H}, b_{1L}$ and b_{2L} are defined by Equation (C-5) in the Appendix.

In this setting the debt value function is the solution to Equation (A-14) with $\hat{s}(p) = s(p)$ and is given by

$$
\begin{aligned}
D(p) = DH(p) &= \begin{cases} \eta XH(p) + (1-\eta)\left(WH(p) - CH(p)\right) & p \in [p_c, p_s), \\ \frac{b}{r} + c_{1H}p^{\lambda_{1H}} + c_{2H}p^{\lambda_{2H}} & p \in [p_s, p_{HL}), \end{cases} \\
= DL(p) &= \frac{b}{r} + c_{1L}p^{\lambda_{1L}} + c_{2L}p^{\lambda_{2L}} \qquad p \in [p_{HL}, \infty),
\end{aligned}
\tag{17}
$$

where the unknown constants c_{1H}, c_{2H}, c_{1L} and c_{2L} are determined by the appropriate boundary conditions (see Appendix C for complete derivation). The after bankruptcy firm value $XH(p)$ is given by Equation (A-4), where

μ and λ_1 are replaced by μ_H and λ_1^H of the high-risk strategy. Moreover, the trigger p_x is the one that maximizes the value of $XH(p)$ and is given by Equation (A-5) after substituting λ_1 and μ with λ_1^H and μ_H, respectively.

The value of the firm's equity in both volatility regions will then equal

$$\begin{aligned} V(p) = & \\ VH(p) = WH(p) - DH(p) - CH(p) & \quad p \in [p_c, p_{HL}), \\ VL(p) = WL(p) - DL(p) - CL(p) & \quad p \in [p_{HL}, \infty). \end{aligned} \tag{18}$$

When equityholders make take-it or leave-it offers to debtholders regarding debt service, the bankruptcy trigger p_c, is chosen by the equityholders in order to maximize the value of equity, $VH'(p_c) = 0$. In the opposite case that bondholders' bargaining power η is maximum, p_c is the root of the smooth pasting condition $DH'(p_c) = 0$. Since, the after bankruptcy firm value $XH(p)$ is independent of the default trigger p_c, it follows that p_c is the root of the smoothness condition

$$h_1(p_c, p_s, p_{HL}, b, \epsilon_1) = \frac{\partial (WH - CH)}{\partial p}|_{p=p_c} = 0, \tag{19}$$

where $\epsilon_1 = (\sigma_H, \sigma_L, w, \mu_H, \mu_L, r, \tau, \gamma, \delta)$. A closed form expression for p_c is not possible to be determined but a solution can be obtained numerically given p_s, p_{HL}, b and ϵ_1.

The renegotiation trigger p_s is found by applying the smooth pasting condition to the value of the firm's debt

$$h_2\left(p_c, p_x, p_s, p_{HL}, b, \epsilon_2\right) = \frac{\partial DH}{\partial p}|_{p\uparrow p_s} - \frac{\partial DH}{\partial p}|_{p\downarrow p_s} = 0, \tag{20}$$

where $\epsilon_2 = (\sigma_H, \sigma_L, w, \mu_H, \mu_L, r, \tau, \gamma, \delta, \xi_1, \xi_0, \eta)$. Again, an analytical expression for p_s cannot be determined but a solution can be derived from numerical algorithms given p_c, p_x, p_{HL}, b and ϵ_2.

3.2.b Optimal Risk Strategy for Renegotiable Debt

Similarly to the no-renegotiation case, our analysis studies the agency problem by measuring the difference between the ex ante and ex post maximal firm values. In each case, the optimal debt structure is associated with the coupon payment b that maximizes the initial firm value.

When the risk switching strategy can be committed before debt is in place, the firm will optimally choose its coupon b, risk switching trigger p_{HL}, default p_c and renegotiation p_s triggers to maximize the initial firm value

$$\max_{p_c, p_s, p_{HL}, b} W\left(p, p_c, p_x, p_s, p_{HL}, b, \epsilon_3\right)|_{p=p_0} \tag{21}$$

subject to the smoothness conditions (19) and (20). The exogenous parameter vector is $\epsilon_3 = (\sigma_H, \sigma_L, w, \mu_H, \mu_L, r, \tau, \gamma)$.

When the risk switching strategy is determined ex post, after debt is contracted, the optimal risk switching trigger p_{HL} will be chosen so as to maximize the value of equity. Our analysis, imposes continuity of the second derivatives of equity value at p_{HL} to ensure that p_{HL} is ex post optimal.

This additional condition is the so-called super contact condition that can be written as

$$
\begin{aligned}
h_3\left(p_c, p_x, p_s, p_{HL}, b, \epsilon_2\right) &= \frac{\partial^2 VL}{\partial p^2}|_{p\uparrow p_{HL}} - \frac{\partial^2 VL}{\partial p^2}|_{p\downarrow p_{HL}} \\
&= 0.
\end{aligned}
\tag{22}
$$

Consequently, the firm will choose the optimal ex post risk policy to maximize the initial firm value (21) subject to the conditions (19), (20) and (22).

Here, our model distinguishes two cases for the valuation of expression (21), one with $ps \leq p_0 < p_{HL}$, and one with $p_0 \geq p_{HL}$. Thus, $W(p_0) = WH(p_0)$ in the former case and $W(p_0) = WL(p_0)$ in the latter. Our analysis keeps the solution and its associated optimal risk profile that corresponds to the larger initial firm value.

4. Numerical Implementation

4.1 NUMERICAL RESULTS

In this section, a numerical implementation of the developed model is performed. More precisely, a series of numerical examples investigate the presented modeling framework and analyze the characteristics of the risk-shifting problem. Table I shows the base-case parameter values that are adopted in our analysis. The parameter values considered here are standard in the relevant literature.

$\boxed{\text{Insert Table I Here}}$

Tables II, III, IV and V list for several cases of debt renegotiation the optimal ex ante and optimal ex post risk profiles and optimal capital structure. Table II reports the numerical results for the non-renegotiable debt, while Tables III, IV and V focus on renegotiable debt with bargaining power η being equal to $1, 0.5$ and 0, respectively.

Insert Tables II, III, IV and V Here

Examining these tables reveals the following observations:

1. In our model, when the firm's risk profile can be contracted ex ante to maximize firm value, the firm will choose the high-risk activity for low values of the output price to take advantage of debt tax shields. When the firm's risk profile is determined ex post to maximize equity value, the firm will choose the high-risk activity for greater values of the output price. Table II illustrates this point with optimal risk switching trigger p_{HL} increasing from 0.67 in the ex ante case to 1.12 in the ex post case. In Tables III, IV and V the optimal risk switching trigger p_{HL} increases from $0.88, 0.90$ and 0.91 in the ex ante case to $1.47, 2.38$ and 3.28 in the ex post case, respectively. This finding reflects for all the cases of debt renegotiation the severity of the asset substitution problem.

2. The results of the model indicate that agency costs of renegotiable debt are lower comparing to non-renegotiable debt. In particular, the increase in equityholders' bargaining power leads to a bigger decline in agency costs corresponding to renegotiable debt. For $\eta = 0$, agency

costs associated with renegotiable debt are the highest (9.63%), but are still lower than agency costs of non-renegotiable debt (10.02%). If equityholders and debtholders have equal bargaining power $\eta = 0.5$, the agency costs of debt are reduced (6.81%). Finally, if equityholders hold all the bargaining power $\eta = 1$, the agency costs are the lowest (3.92%).

3. Optimal leverage ratios decrease relative to the ex ante case, confirming the predictions of Leland (1998) and Décamps and Djembissi (2007). In particular, when the debt is non-renegotiable the asset substitution problem tends to be more severe and this decrease in leverage ratios becomes maximum.

4. As expected, our model predicts high optimal leverage ratios when the terms of the debt cannot be renegotiated. Notably, when the debt is renegotiable, the degree of leverage depends crucially on the bargaining power of firm's claimants. Precisely, Tables III, IV and V show that the larger the bargaining power of equityholders η, the lower the optimal leverage ratios. This result can be explained considering that creditors anticipate debt renegotiation and impose a limit on debt capacity. Thus, the value of debt issued by the firm will be positively related to the creditors' bargaining power, $1 - \eta$.

5. Not surprisingly, yield spreads increase significantly with respect to the ex ante case for both renegotiable and non-renegotiable debt. As in Mella-Barral and Perraudin (1997) our results indicate that servicing the debt strategically will substantially increase yield spreads, reflect-

ing the high riskiness of renegotiable debt. Remark however that an increase in the bargaining power of equityholders η leads to a reduction in the yield spreads of debt. This finding underscores the existence of a positive relationship between yields of corporate debt and agency costs.

6. The renegotiation trigger p_s decreases relative to the ex ante case for all cases of renegotiable debt. The interpretation of this observation is the following. When the firm's risk profile is determined ex post, the average firm risk is greater. As risk increases, the postbankruptcy firm value increases as well. Consequently, the bondholders' bargaining position strengthens and, thus, the ex post optimal renegotiation trigger p_s falls.

4.2 SENSITIVITY ANALYSIS

To enhance the interpretation of our findings, we conduct a sensitivity analysis of the ex post maximal firm value W_0 (or $\hat{W}_0$), the optimal risk switching p_{HL} and renegotiation p_s triggers, the leverage ratio L, the yield spread YS and the agency costs AC with respect to the main model parameters, for both renegotiable and non-renegotiable debt. The baseline values of the model parameters are listed in Table I.

Figures 4 and 5 illustrate the optimal capital structure and risk profile as a function of σ_H for non-renegotiable and renegotiable debt, respectively. As volatility increases, the risk switching trigger p_{HL}, the renegotiation trigger p_s, the agency costs and the yield spreads increase as well. This can be explained by considering that the potential of risk-shifting becomes greater with σ_H. As expected, leverage and maximal firm value fall with σ_H when the terms of the debt cannot be renegotiated. Less expected is the slight increase in leverage ratio of renegotiable debt. The latter finding can be attributed to the fact that leverage rises substantially with σ_H when the firms' optimal risk policy is determined ex ante. Note that as the asset substitution problem becomes more severe, the agency costs of renegotiable debt are significantly lower than those of non-renegotiable debt.

Figures 6 and 7 examine the effect of different growth rates μ_H. Here, the increase in μ_H reduces the opportunity cost of employing the high-risk activity and, thus, implies a higher potential for asset substitution. Consequently, the optimal risk switching trigger p_{HL} and yield spreads for both non-renegotiable and renegotiable debt increase with μ_H. Raising μ_H has two opposite effects on optimal capital structure. When the optimal risk switching trigger p_{HL} is low ($p_{HL} \leq p_0$), a moderate increase in μ_H leads to a decrease in the ex post firm value and a corresponding increase in agency costs of debt (that is agency costs are positively related to the equityholders' risk-shifting incentives). When the optimal risk switching trigger p_{HL} is high ($p_{HL} > p_0$), an increase in μ_H raises the initial firm value and as a consequence lowers the agency costs of asset substitution (that is growth rates are reversely related to the probability of default). For the current parametrization, the first effect dominates if the debt is non-renegotiable and the second one if the debt is renegotiable. In particular, our results indicate that the optimal risk switching trigger p_{HL} associated with renegotiable debt is greater than with non-renegotiable debt. This finding stems from the model's underlying assumption that tax benefits are suspended inside the renegotiation region and, thus, when the debt is renegotiable, the firm will optimally switch to the high-risk activity at a higher trigger price p_{HL}.

Figure 6 shows that agency costs of non-renegotiable debt increase with μ_H but experience a slight decrease as the opportunity cost of employing the high-risk strategy approaches zero. Not surprisingly, optimal leverage ratios decrease compared to their ex ante values. When the firm's claimants renegotiate debt service, agency costs fall with μ_H (see Figure 7). In this case the ex post optimal switching trigger p_{HL} is significantly greater than p_0 and the firm initially adopts the high-risk profile. Consequently, firm value increases with μ_H and, hence, the optimal leverage ratios and renegotiation trigger p_s increase as well.

Figure 8 considers changes in the renegotiation cost δ. Higher costs produce lower equity and debt values, because the incidence of the renegotiation cost is assumed to be equally distributed between creditors and debtors ($\eta = 0.5$). Accordingly, optimal leverage ratios decrease with δ and, hence, yield spreads are higher. Agency costs, optimal risk switching p_{HL} and renegotiation p_s triggers are relatively flat. As expected, the renegotiation cost parameter δ plays no role when the debt in non-renegotiable, since it does not affect the optimal switching policy.

Figure 9 charts the effect of equityholders' bargaining power η. Not surprisingly, the maximum firm value is obtained when bondholders hold all the bargaining power, $\eta = 0$. Increasing equityholders' bargaining power η reduces significantly the debt capacity and, hence, the optimal leverage ratios. It can be seen that equityholders' bargaining power coefficient η is negatively related to the risk switching trigger p_{HL}. Consequently, yield spreads and agency costs of renegotiable debt fall with η, reflecting the reduction in average firm risk. The explanation for this finding is fairly straightforward. As η increases, equityholders are more willing not to loose their increasing option value to renegotiate coupon payments, thus their risk-shifting incentives are mitigated. Less expected is that the renegotiation trigger p_s decreases slightly, despite the increase in equityholders' bargaining power η. This happens because an increase in η has two opposite effects on the value of p_s. First, it increases the probability of strategic default and accordingly the trigger price p_s at which debt renegotiation commences for obvious reasons. Second, it reduces the debt value and its associated coupon payment b, which tends to reduce equityholders' strategic considerations and, hence, the renegotiation trigger, p_s. For the current set of parameters, the second effect dominates. Note that η has no impact on the firm's optimal capital structure and risk profile when the terms of the debt cannot be renegotiated.

Figure 10 examines the effect of the growth rate of high-risk profile μ_H on the agency costs of renegotiable and non-renegotiable debt. The opportunity cost of adopting the high-risk activity declines in volatility, and, hence, agency costs of asset substitution rise for both types of debt. The effect of debt renegotiation in mitigating the equityholders' risk-shifting incentives is positive if the asset substitution problem is severe (the right region in the subfigures). When the potential for asset substitution is low (the decrease in the growth rate μ_H is large relative to the increase in the volatility σ_H) - the left region in the subfigures - and debtholders hold most of the bargaining power (η close to 0), then debt renegotiation worsens the asset substitution problem. Our numerical results suggest that agency costs of renegotiable debt highly depend on the equityholder's bargaining power, η. For moderate to high values of η ($\eta > 0.5$ for the baseline parameters), the agency costs of renegotiable debt are considerably lower than those of non-renegotiable debt.

Figure 11 presents agency costs at varying levels of equityholders' bargaining power coefficient η and growth rate of high-risk profile μ_H, for both renegotiable and non-renegotiable debt. For most values of the couples (μ_H, η), a debt contract that allows the firm to renegotiate coupon payments, substantially reduces the agency costs of asset substitution. However, when the risk-shifting potential is low and the shareholders' bargaining power η is close to zero, it is better choosing the non-renegotiable debt.

5. Conclusion

This paper studies the relationship between asset substitution and the firm's option to renegotiate debt contracts. Our debt renegotiation model is inspired by the asset pricing model of Mella-Barral and Perraudin (1997), that focuses on strategic debt service, and it integrates key elements of the models of Leland (1998), Fan and Sundaresan (2000) and Décamps and Djembissi (2007). The proposed model includes debt renegotiability by modifying the basic model to take into account the tax advantage to leverage, the renegotiation costs and the equityholders' bargaining power. For comparison purposes, the case of non-renegotiable debt is also examined.

Our model also incorporates asset substitution in a setting where equityholders' risk-shifting incentives alter not only the volatility, but also the growth rate of the firm's assets. Specifically, our study assumes that risk-shifting is not costless and, hence, the opportunity cost of adopting the high-risk profile is reflected in the reduction of its growth rate. The analysis framework creates a tradeoff between the increase in volatility and the decrease in growth rate which characterizes asset substitution opportunities. On the one hand, when the increase in risk is large with respect to the decrease in growth rate, the potential for asset substitution is high. On the other hand, when the increase in firm's risk is accompanied by a relatively high opportunity cost, asset substitution opportunities are low.

Numerical implementation of the discussed model determines the firm's optimal capital structure and risk policy for both types of debt (renegotiable and non-renegotiable). Agency costs, leverage ratios and yield spreads of debt are calculated for a range of model parameters. Our results indicate that for modest to high risk-shifting opportunities, debt renegotiation mitigates agency costs of asset substitution. In contrast, when equityholders' risk-shifting incentives are weak, it may then be optimal for the firm to design debt contracts that are non-renegotiable. A central element in determining the efficiency of debt renegotiation is the bargaining power of firm's claimants. Precisely, our analysis shows that agency costs of asset substitution can be significantly reduced by strengthening the bargaining position of equityholders.

Appendix A

DERIVATION OF GENERALIZED ASSET PRICING MODEL

Mella-Barral and Perraudin (1997) asset pricing model is generalized to include tax advantages of debt and renegotiation costs. Consider a firm that sells one unit of its product for p_t while experiencing constant production costs w. It is assumed that p_t follows the geometric Brownian motion

$$dp_t = \mu dt + \sigma p_t dB_t,$$

where μ is the expected growth rate and σ is the volatility of p_t. Since the firm is less efficient after bankruptcy, the firm income $p_t - w$ before bankruptcy, diminishes to $\xi_1 p_t - \xi_0 w$ after bankruptcy, where $\xi_1 \leq 1$ and $\xi_0 \geq 1$. We make the assumption that the firm earnings are taxed at a corporate tax rate τ but that coupon payments are tax deductible. Hence, the firm net earnings flows are $(1-\tau)(p_t - w)$ and $(1-\tau)(\xi_1 p_t - \xi_0 w)$ before and after bankruptcy, respectively.

With the tax structure in place, the value of the firm under pure equity financing is the solution to the ordinary differential equation

$$r\breve{W}(p) = (1-\tau)(p-w) + \mu p \breve{W}'(p) + \frac{\sigma^2}{2} p^2 \breve{W}''(p) \qquad \text{(A-1)}$$

with general solution,

$$\breve{W}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + a_1 p^{\lambda_1} + a_2 p^{\lambda_2},$$

where

$$\lambda_1 = \frac{\sigma^2 - 2\mu - \sqrt{\left(\sigma^2 - 2\mu\right)^2 + 8\sigma^2 r}}{2\sigma^2},$$

$$\lambda_2 = \frac{\sigma^2 - 2\mu + \sqrt{\left(\sigma^2 - 2\mu\right)^2 + 8\sigma^2 r}}{2\sigma^2}$$

and the unknown constants a_1, a_2 are determined from the appropriate boundary conditions. The value matching condition $\breve{W}(p_c^*) = \gamma$, ensures that the firm value at closure equals the liquidation value of the firm γ. In addition, assuming that asset prices are free of bubbles the following condition holds $\lim_{p\to\infty} \breve{W}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right)$. The optimal liquidation trigger p_c^* is the one that maximizes the firm value, and is determined from the smooth pasting condition, $\breve{W}'(p_c^*) = 0$.

Solving for $\breve{W}(p)$ and p_c^* we derive the following expressions

$$\breve{W}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + \left[\gamma - (1-\tau)\left(\frac{p_c^*}{r-\mu} - \frac{w}{r}\right)\right] \left(\frac{p}{p_c^*}\right)^{\lambda_1} \quad \text{for } p \geq p_c^*, \tag{A-2}$$

$$p_c^* = -\frac{\lambda_1}{1-\lambda_1}\frac{(1-\tau)\,w + r\gamma}{(1-\tau)\,r}\,(r-\mu)\,. \tag{A-3}$$

For $p < p_c^*$, $\breve{W}(p) = \gamma$.

Alternatively, the model examines an initially levered firm that after declaring bankruptcy, ends up as an all-equity firm operated by its former debtholders. The total value of the pure equity firm in the hands of the new owners after bankruptcy $X(p)$, satisfies Equation (A-1) after the firm net earnings flow $(1-\tau)(p-w)$ has been replaced by $(1-\tau)(\xi_1 p - \xi_0 w)$. Associated with this equation are the boundary conditions $X(p_x) = \gamma$ (value matching) and $\lim_{p\to\infty} X(p) = (1-\tau)\left(\frac{\xi_1 p}{r-\mu} - \frac{\xi_0 w}{r}\right)$ (no-bubbles). Consequently, $X(p)$ is given by

$$X(p) = (1-\tau)\left(\frac{\xi_1 p}{r-\mu} - \frac{\xi_0 w}{r}\right) + \left[\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu} - \frac{\xi_0 w}{r}\right)\right] \left(\frac{p}{p_x}\right)^{\lambda_1} \quad \text{for } p \geq p_x, \tag{A-4}$$

where p_x is the optimal liquidation trigger of the creditors which is defined by the smooth pasting condition $X'(p_x) = 0$ and is equal to

$$p_x = -\frac{\lambda_1}{1-\lambda_1} \frac{(1-\tau)\,\xi_0 w + r\gamma}{(1-\tau)\,\xi_1 r} (r-\mu). \tag{A-5}$$

Similarly, for $p < p_x$, $X(p) = \gamma$.

A.1 Without renegotiation

If the terms of the debt are non-negotiable, the firm has an incentive to maximize its value and declare bankruptcy when coupon payment failure occurs. The firm issues debt in order to benefit from the favorable tax system and promises to pay debtholders the contracted coupon, b until liquidation takes place at p_b. Under these assumptions, the values of debt and equity are the solutions to the ordinary differential equations

$$r\hat{D}(p) = b + \mu p\hat{D}'(p) + \frac{\sigma^2}{2}p^2\hat{D}''(p), \tag{A-6}$$

$$r\hat{V}(p) = (1-\tau)(p-w-b) + \mu p\hat{V}'(p) + \frac{\sigma^2}{2}p^2\hat{V}''(p). \tag{A-7}$$

The general solutions to Equations (A-6) and (A-7) are

$$\hat{D}(p) = \frac{b}{r} + b_1 p^{\lambda_1} + b_2 p^{\lambda_2},$$

$$\hat{V}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w+b}{r}\right) + b_3 p^{\lambda_1} + b_4 p^{\lambda_2},$$

where the unknown quantities b_1, b_2, b_3 and b_4 are determined by the default conditions at the trigger p_b and the no-bubbles conditions. Therefore, under the no-arbitrage assumption, the value of debt at default must equal that of debtholders' outside option, $\hat{D}(p_b) = X(p_b)$ and the value of equity at default must equal zero, $\hat{V}(p_b) = 0$. Furthermore, asset prices are free of bubbles, hence, $\lim_{p\to\infty} \hat{D}(p) = \frac{b}{r}$ and $\lim_{p\to\infty} \hat{V}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w+b}{r}\right)$. Because the quantity p^{λ_2} tends to infinity as p goes to infinity, the constants b_2 and b_4 are zero. Consequently, $b_1 = -p_b^{-\lambda_1}\left[X(p_b) - \frac{b}{r}\right]$ and the value of debt is given by

$$\hat{D}(p) = \frac{b}{r} + \left[X(p_b) - \frac{b}{r}\right]\left(\frac{p}{p_b}\right)^{\lambda_1} \quad \text{for } p \geq p_b. \tag{A-8}$$

Similarly, $b_3 = -p_b^{-\lambda_1}\left[(1-\tau)\left(\frac{p_b}{r-\mu} - \frac{w+b}{r}\right)\right]$ and the value of equity is given by

$$\hat{V}(p) = (1-\tau)\left[\frac{p}{r-\mu} - \frac{w+b}{r} - \left(\frac{p_b}{r-\mu} - \frac{w+b}{r}\right)\left(\frac{p}{p_b}\right)^{\lambda_1}\right] \quad \text{for } p \geq p_b. \tag{A-9}$$

The default trigger point, p_b, is the one that maximizes equity value and is computed from the smoothness condition $\hat{V}'(p_b) = 0$. The bankruptcy trigger p_b is equal to

$$p_b = -\frac{\lambda_1}{1-\lambda_1}\frac{w+b}{r}(r-\mu). \tag{A-10}$$

Firm value $\hat{W}(p)$ equals the sum of the levered firm's equity and debt values, $\hat{V}(p) + \hat{D}(p)$ and is given by

$$\begin{aligned}\hat{W}(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + \frac{\tau b}{r} + \Bigg[X(p_b) - \frac{\tau b}{r} - \\ (1-\tau)\left(\frac{p_b}{r-\mu} - \frac{w}{r}\right)\Bigg]\left(\frac{p}{p_b}\right)^{\lambda_1} \quad \text{for } p \geq p_b.\end{aligned} \tag{A-11}$$

It should be noted that if $p_b \geq p_x$ then the value of $X(p_b)$ is given by Equation (A-4). If $p_b < p_x$, then $X(p_b) = \gamma$, and hence, debtholders will prefer to lead the firm into bankruptcy at p_b rather than operate the firm with diminished earnings. This condition occurs if

$$r\gamma < b < \frac{\xi_0 w + \frac{r\gamma}{1-\tau}}{\xi_1} - w.$$

A.2 With renegotiation

If equityholders make take-it or leave-it offers on coupon payments to creditors, the optimal function of coupon payments $\hat{s}(p)$ is a uniquely determined function of the output price p. If the output price p reaches the default trigger p_{c1}, then immediate liquidation takes place. As the output price increases, the trigger level p_{s1} is reached and equityholders have an incentive to pay the contracted payment to bondholders. Debt payment is less than the initially contracted coupon when the output price p lies in the interval $[p_{c1}, p_{s1})$. Specifically, for production cost $w > -\lambda_1 b(r-\mu)/[(1-\tau)\,\xi_0(r-\lambda_1\mu)]$, the debt service becomes negative. In this case, the company experiences operating losses and creditors inject cash to cover them. The optimal debt service $\hat{s}(p)$ is determined by the following equation

$$rX(p) = \hat{s}(p) + \mu p X'(p) + \frac{\sigma^2}{2} p^2 X''(p) \tag{A-12}$$

and is given by

$$\hat{s}(p) = \begin{cases} r\gamma & \text{for } p \in [p_{c1}, p_x), \\ (1-\tau)\,(\xi_1 p - \xi_0 w) & \text{for } p \in [p_x, p_{s1}), \\ b & \text{for } p \in [p_{s1}, \infty). \end{cases} \tag{A-13}$$

The presence of renegotiation costs does not affect debt values. Hence, debt value $D_1(p)$ satisfies the equation

$$rD_1(p) = \hat{s}(p) + \mu p D_1'(p) + \frac{\sigma^2}{2} p^2 D_1''(p). \tag{A-14}$$

For $p < p_{s1}$, the value of debt equals the creditor's outside option, $D_1(p) = X(p)$. For $p \geq p_{s1}$, the general solution to Equation (A-14) is

$$D_1(p) = \frac{b}{r} + c_1 p^{\lambda_1} + c_2 p^{\lambda_2}.$$

No-bubbles condition implies that $\lim_{p \to \infty} D_1(p) = \frac{b}{r}$ and the constant c_2 equals zero. The unknown constant c_1 and the renegotiation trigger p_{s1} are determined from the value matching $D_1(p_{s1}) = X(p_{s1})$ and smooth pasting conditions $D_1'(p_{s1}) = X'(p_{s1})$ at p_{s1}. Thus, the value of debt equals

$$D_1(p) = \begin{cases} X(p) & \text{for } p \in [p_{c1}, p_{s1}), \\ \frac{b}{r} + \left[X(p_{s1}) - \frac{b}{r}\right]\left(\frac{p}{p_{s1}}\right)^{\lambda_1} & \text{for } p \in [p_{s1}, \infty), \end{cases} \tag{A-15}$$

where p_{s1} is given by

$$p_{s1} = -\frac{\lambda_1}{1-\lambda_1} \frac{(1-\tau)\,\xi_0 w + b}{\xi_1 r (1-\tau)} (r-\mu). \tag{A-16}$$

The general solution to firm value $W(p)$ is determined by

$$W(p) = \begin{cases} (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + d_1 p^{\lambda_1} + d_2 p^{\lambda_2} & \text{for } p \in [p_{c1}, p_{s1}), \\ (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + \frac{\tau b}{r} + d_3 p^{\lambda_1} + d_4 p^{\lambda_2} & \text{for } p \in [p_{s1}, \infty), \end{cases} \tag{A-17}$$

where the constants d_1, d_2, d_3 and d_4 are computed from the no-bubble condition $\lim_{p \to \infty} W(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + \frac{\tau b}{r}$, the default condition $W(p_{c1}) = \gamma$ and the value matching and smooth pasting conditions at p_{s1}. Applying the above boundary conditions to Equation (A-17) the firm value, $W(p)$ equals

$$W(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) - \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b}{r}\left(\frac{p}{p_{s1}}\right)^{\lambda_2} + \left[\gamma + \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b}{r}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} - (1-\tau)\left(\frac{p_{c1}}{r-\mu} - \frac{w}{r}\right)\right]\left(\frac{p}{p_{c1}}\right)^{\lambda_1} \tag{A-18}$$

for $p \in [p_{c1}, p_{s1})$,

$$W(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) + \frac{\tau b}{r}\left[1 - \frac{\lambda_2}{\lambda_2 - \lambda_1}\left(\frac{p}{p_{s1}}\right)^{\lambda_1}\right] + \left[\gamma + \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b}{r}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} - (1-\tau)\left(\frac{p_{c1}}{r-\mu} - \frac{w}{r}\right)\right]\left(\frac{p}{p_{c1}}\right)^{\lambda_1} \tag{A-19}$$

for $p \in [p_{s1}, \infty)$.

We generalize the basic model so as to incorporate renegotiation costs by assuming that debt renegotiation incurs a cost δ per unit of time when equityholders do not pay the full contracted payment, b. Therefore, the renegotiation cost function $C(p)$ satisfies Equation (1) with general solution

$$C(p) = \begin{cases} \frac{\delta}{r} + e_1 p^{\lambda_1} + e_2 p^{\lambda_2} & \text{for } p \in [p_{c1}, p_{s1}), \\ e_3 p^{\lambda_1} + e_4 p^{\lambda_2} & \text{for } p \in [p_{s1}, \infty). \end{cases}$$

No-bubbles conditions include $\lim_{p\to\infty} C(p) = \frac{\delta}{r}$ while the absence of arbitrage implies that $C(p_{c1}) = 0$. Associated with this pair of equations are also the value matching and smoothness conditions at p_{s1}:

$$\frac{\delta}{r} + e_1 p_{s1}^{\lambda_1} + e_2 p_{s1}^{\lambda_2} = e_3 p_{s1}^{\lambda_1} + e_4 p_{s1}^{\lambda_2},$$

$$\lambda_1 e_1 p_{s1}^{\lambda_1 - 1} + \lambda_2 e_2 p_{s1}^{\lambda_2 - 1} = \lambda_1 e_3 p_{s1}^{\lambda_1 - 1} + \lambda_2 e_4 p_{s1}^{\lambda_2 - 1}.$$

Solving Equation (1) subject to the boundary conditions, the following expressions for the renegotiation cost function $C(p)$ are derived

$$C(p) = \frac{\delta}{r}\left[1 + \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(\frac{p}{p_{s1}}\right)^{\lambda_2} - \left(\frac{p}{p_{c1}}\right)^{\lambda_1}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2}\right)\right] \quad \text{for } p \in [p_{c1}, p_{s1}), \tag{A-20}$$

$$C(p) = \frac{\delta}{r}\left[\frac{\lambda_2}{\lambda_2 - \lambda_1}\left(\frac{p}{p_{s1}}\right)^{\lambda_1} - \left(\frac{p}{p_{c1}}\right)^{\lambda_1}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2}\right)\right] \quad \text{for } p \in [p_{s1}, \infty). \tag{A-21}$$

Finally, the value of equity will be $W(p) - D_1(p) - C(p)$, and is given by

$$V_1(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w}{r}\right) - \frac{\gamma r + \delta}{r} - \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b + \delta}{r}\left(\frac{p}{p_{s1}}\right)^{\lambda_2} + \left[\frac{\gamma r + \delta}{r} + \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b + \delta}{r}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} - (1-\tau)\left(\frac{p_{c1}}{r-\mu} - \frac{w}{r}\right)\right]\left(\frac{p}{p_{c1}}\right)^{\lambda_1} \quad \text{for } p \in [p_{c1}, p_x), \tag{A-22}$$

$$V_1(p) = (1-\tau)\left[\frac{(1-\xi_1)\,p}{r-\mu} - \frac{(1-\xi_0)\,w}{r}\right] - \frac{\delta}{r} - \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b + \delta}{r}\left(\frac{p}{p_{s1}}\right)^{\lambda_2} + \left[\frac{\gamma r + \delta}{r} + \frac{\lambda_1}{\lambda_2 - \lambda_1}\frac{\tau b + \delta}{r}\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} - (1-\tau)\left(\frac{p_{c1}}{r-\mu} - \frac{w}{r}\right)\right]\left(\frac{p}{p_{c1}}\right)^{\lambda_1} - \left[\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu} - \frac{\xi_0 w}{r}\right)\right]\left(\frac{p}{p_x}\right)^{\lambda_1} \quad \text{for } p \in [p_x, p_{s1}), \tag{A-23}$$

$$V_1(p) = (1-\tau)\left(\frac{p}{r-\mu} - \frac{w+b}{r}\right) + \left[\frac{\gamma r+\delta}{r} + \frac{\lambda_1}{\lambda_2-\lambda_1}\frac{\tau b+\delta}{r}\right.$$
$$\left.\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} - (1-\tau)\left(\frac{p_{c1}}{r-\mu} - \frac{w}{r}\right)\right]\left(\frac{p}{p_{c1}}\right)^{\lambda_1} - \left(X\left(p_{s1}\right) - \frac{b}{r} + \right.$$
$$\left.\frac{\lambda_2}{\lambda_2-\lambda_1}\frac{\tau b+\delta}{r}\right)\left(\frac{p}{p_{s1}}\right)^{\lambda_1} \tag{A-24}$$
$$\text{for } p \in [p_{s1}, \infty).$$

The default trigger, p_{c1} is the solution of the optimality condition $V'(p_{c1}) = 0$ that can be written as

$$(1-\tau)\frac{r\left(\lambda_1-1\right)p_{c1}}{r-\mu} - \lambda_1\left(\gamma r+\delta+(1-\tau)\,w\right) + \lambda_1\left(\tau b+\delta\right)\left(\frac{p_{c1}}{p_{s1}}\right)^{\lambda_2} = 0. \tag{A-25}$$

While Equation (A-25) can be expressed in closed form, a closed form solution for p_{c1} is not available and can be found from root finding algorithms. It is notable that if taxes and renegotiation costs are zero, $p_{c1} = p_c^*$.

The optimal capital structure in the polar case of debt renegotiation is determined under the assumption that debtholders make take-it or leave-it offers regarding debt service $q(p)$ to equityholders. The trigger values for default p_{c2} and renegotiation p_{b2} define the boundaries of the interval over which coupon payment is less than the initially contracted payment. Particularly when production cost $w > -\lambda_1 b(r-\mu)/(r-\lambda_1\mu)$, the debt service is negative, meaning that creditors are willing to temporarily inject cash into company in order to meet its operating losses. In this setting, the value of equity inside the interval $[p_{c2}, p_{b2})$ is determined by the outside option of equityholders, hence, equity value is given by the no-renegotiation case, $\hat{V}(p)$. Consequently, the default trigger of non-renegotiable debt and renegotiation trigger coincide and $p_b = p_{b2}$. Therefore, the optimal debt service $q(p)$ equals

$$q(p) = \begin{cases} (1-\tau)\,(p-w) & \text{for } p \in [p_{c2}, p_b), \\ b & \text{for } p \in [p_b, \infty). \end{cases} \tag{A-26}$$

Because debtholders have maximum bargaining power, the incidence of a costly renegotiation is entirely on them, and hence, the debt value for costly renegotiation is $W(p) - \hat{V}(p) - C(p)$. Since, the default trigger p_{c2} is optimally chosen by debtholders to maximize the value of their claim, it is the solution of the optimality condition

$$(1-\tau)\,\frac{r\,(\lambda_1 - 1)\,p_{c2}}{r-\mu} - \lambda_1\,(\gamma r + \delta + (1-\tau)\,w) + \lambda_1\,(\tau b + \delta)\left(\frac{p_{c2}}{p_b}\right)^{\lambda_2} = 0. \tag{A-27}$$

Note that if taxes and renegotiation costs are absent, the default triggers for the unlevered firm and the levered firm with renegotiable debt coincide, $p_c^* = p_{c1} = p_{c2}$.

Appendix B

PROOFS OF LEMMAS

Proof of Lemma 1. We prove that $\hat{V}_H(p) > \hat{V}_L(p)$ for all $p > p_b^H$. If $\sigma_H > \sigma_L$ and $\mu_H = \mu_L$ then $p_b^H < p_b^L$ and $\hat{V}_H(p) > \hat{V}_L(p) = 0 \ \forall p \in [p_b^H, p_b^L)$. Furthermore, for all $p \geq p_b^L$:

$$\hat{V}_H(p) > \hat{V}_L(p) \Rightarrow$$

$$\left(\frac{w+b}{r} - \frac{p_b^H}{r-\mu_H}\right)\left(\frac{p}{p_b^H}\right)^{\lambda_1^H} > \left(\frac{w+b}{r} - \frac{p_b^L}{r-\mu_L}\right)\left(\frac{p}{p_b^L}\right)^{\lambda_1^L} \Rightarrow$$

$$\left(\frac{p}{p_b^H}\right)^{\lambda_1^H}\left(\frac{p_b^L}{p}\right)^{\lambda_1^L} > \frac{\frac{w+b}{r} - \frac{p_b^L}{r-\mu_H}}{\frac{w+b}{r} - \frac{p_b^H}{r-\mu_L}} \Rightarrow$$

$$\left(\frac{p}{p_b^H}\right)^{\lambda_1^H}\left(\frac{p_b^L}{p}\right)^{\lambda_1^L} > \frac{\frac{w+b}{r} - \frac{-\frac{\lambda_1^L}{1-\lambda_1^L}\frac{w+b}{r}(r-\mu_L)}{r-\mu_L}}{\frac{w+b}{r} - \frac{-\frac{\lambda_1^H}{1-\lambda_1^H}\frac{w+b}{r}(r-\mu_H)}{r-\mu_H}} \Rightarrow$$

$$\left(\frac{p}{p_b^H}\right)^{\lambda_1^H}\left(\frac{p_b^L}{p}\right)^{\lambda_1^L} > \frac{\frac{1}{1-\lambda_1^L}\left(\frac{w+b}{r}\right)}{\frac{1}{1-\lambda_1^H}\left(\frac{w+b}{r}\right)}$$

$$\left(\frac{p}{p_b^H}\right)^{\lambda_1^H}\left(\frac{p_b^L}{p}\right)^{\lambda_1^L} > \frac{1-\lambda_1^H}{1-\lambda_1^L} \Rightarrow$$

$$p > \left[\left(\frac{1-\lambda_1^H}{1-\lambda_1^L}\right)\frac{\left(p_b^H\right)^{\lambda_1^H}}{\left(p_b^L\right)^{\lambda_1^L}}\right]^{\frac{1}{\lambda_1^H-\lambda_1^L}}. \tag{B-1}$$

If inequality (B-1) is satisfied for $p = p_b^L$ then it is also satisfied $\forall p \geq p_b^L$. Consequently,

$$p_b^L > \left[\left(\frac{1-\lambda_1^H}{1-\lambda_1^L}\right)\frac{\left(p_b^H\right)^{\lambda_1^H}}{\left(p_b^L\right)^{\lambda_1^L}}\right]^{\frac{1}{\lambda_1^H-\lambda_1^L}} \Rightarrow$$

$$\left(\frac{p_b^L}{p_b^H}\right)^{\lambda_1^H} > \frac{1-\lambda_1^H}{1-\lambda_1^L} \Rightarrow$$

$$\left(\frac{-\frac{\lambda_1^L}{1-\lambda_1^L}}{-\frac{\lambda_1^H}{1-\lambda_1^H}}\right)^{\lambda_1^H} > \frac{1-\lambda_1^H}{1-\lambda_1^L} \Rightarrow$$

$$\frac{\lambda_1^L}{\lambda_1^H}\left(\frac{\lambda_1^L\left(1-\lambda_1^H\right)}{\lambda_1^H\left(1-\lambda_1^L\right)}\right)^{\lambda_1^H-1} > 1 \Rightarrow$$

$$\left(\frac{\lambda_1^L}{\lambda_1^H}\right)^{\lambda_1^H}\left(\frac{1-\lambda_1^L}{1-\lambda_1^H}\right)^{1-\lambda_1^H} > 1 \Rightarrow$$

$$\left(\frac{1-\lambda_1^L}{1-\lambda_1^H}\right)^{1-\lambda_1^H} > \left(\frac{\lambda_1^L}{\lambda_1^H}\right)^{-\lambda_1^H} \Rightarrow$$

$$\left(\frac{1-\lambda_1^L}{1-\lambda_1^H}\right)^{\frac{1-\lambda_1^H}{\lambda_1^H-\lambda_1^L}} > \left(\frac{\lambda_1^L}{\lambda_1^H}\right)^{\frac{-\lambda_1^H}{\lambda_1^H-\lambda_1^L}} \Rightarrow$$

$$\left(1+\frac{1}{\frac{1-\lambda_1^H}{\lambda_1^H-\lambda_1^L}}\right)^{\frac{1-\lambda_1^H}{\lambda_1^H-\lambda_1^L}} > \left(1+\frac{1}{\frac{-\lambda_1^H}{\lambda_1^H-\lambda_1^L}}\right)^{\frac{-\lambda_1^H}{\lambda_1^H-\lambda_1^L}} \Rightarrow$$

$$f\left(\frac{1-\lambda_1^H}{\lambda_1^H-\lambda_1^L}\right) > f\left(\frac{-\lambda_1^H}{\lambda_1^H-\lambda_1^L}\right)$$

$$\frac{1-\lambda_1^H}{\lambda_1^H-\lambda_1^L} > \frac{-\lambda_1^H}{\lambda_1^H-\lambda_1^L} \Rightarrow$$

$$1-\lambda_1^H > -\lambda_1^H \Rightarrow$$

$$1 > 0,$$

where $f(x) = \left(1 + \frac{1}{x}\right)^x$ is monotonically increasing $\forall x > 0$ and $\lambda_1^L < \lambda_1^H < 0$. $\square$

Proof of Lemma 2. If $p_b^L < p_b^H$ then $\hat{V}_L(p) > \hat{V}_H(p) = 0 \ \forall p \in [p_b^L, p_b^H)$. Our result holds if the following condition is met: $\hat{V}_L'(p) > \hat{V}_H'(p)$ for all $p \geq p_b^H$. We have for all $p \geq p_b^H$:

$$
\begin{aligned}
\frac{p}{1-\tau}\left(\hat{V}_L'(p) - \hat{V}_H'(p)\right) &= \frac{p}{r-\mu_L} - \frac{p}{r-\mu_H} + \lambda_1^L\left(\frac{w+b}{r} - \frac{p_b^L}{r-\mu_L}\right)\left(\frac{p}{p_b^L}\right)^{\lambda_1^L} \\
&- \lambda_1^H\left(\frac{w+b}{r} - \frac{p_b^H}{r-\mu_H}\right)\left(\frac{p}{p_b^H}\right)^{\lambda_1^H} \\
&> \frac{p_b^H}{r-\mu_L} - \frac{p_b^H}{r-\mu_H} + \lambda_1^L\left(\frac{w+b}{r} - \frac{p_b^L}{r-\mu_L}\right)\left(\frac{p}{p_b^H}\right)^{\lambda_1^L} \\
&- \lambda_1^H\left(\frac{w+b}{r} - \frac{p_b^H}{r-\mu_H}\right)\left(\frac{p}{p_b^H}\right)^{\lambda_1^H} \\
&> \left[p_b^H\left(\frac{1}{r-\mu_L} - \frac{1}{r-\mu_H}\right) + \lambda_1^L\left(\frac{w+b}{r} - \frac{p_b^L}{r-\mu_L}\right)\right. \\
&\left. - \lambda_1^H\left(\frac{w+b}{r} - \frac{p_b^H}{r-\mu_H}\right)\right]\left(\frac{p}{p_b^H}\right)^{\lambda_1^L} \\
&> \left[\left(\frac{p_b^L}{r-\mu_L} - \frac{w+b}{r}\right)\left(1-\lambda_1^L\right)\right. \\
&\left. - \left(\frac{p_b^H}{r-\mu_H} - \frac{w+b}{r}\right)\left(1-\lambda_1^H\right)\right]\left(\frac{p}{p_b^H}\right)^{\lambda_1^L} = 0.
\end{aligned}
$$

$\square$

Appendix C

NON-RENEGOTIABLE DEBT

C.1 Debt and Equity Coefficients

First, we determine the debt coefficients. The asymptotic condition $\lim_{p\to\infty} \hat{DL}(p) = \frac{b}{r}$ implies that $\hat{a}_{2L}$ equals zero. The default condition at $p = p_b$ is given by

$$\hat{DH}(p_b) = \begin{cases} XH(p_b) & \text{if } p_x \leq p_b, \\ \gamma & \text{if } p_x > p_b, \end{cases}$$

where $XH(p_b)$ is defined as in Equation (A-4) with $\mu = \mu_H$ and $\lambda_1 = \lambda_1^H$. More precisely, if $p_x \leq p_b$ the default condition becomes

$$\hat{a}_{1H} p_b^{\lambda_1^H} + \hat{a}_{2H} p_b^{\lambda_2^H} + \frac{b}{r} - (1-\tau)\left(\frac{\xi_1 p_b}{r-\mu_H} - \frac{\xi_0 w}{r}\right) - \left[\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu_H} - \frac{\xi_0 w}{r}\right)\right]\left(\frac{p_b}{p_x}\right)^{\lambda_1^H} = 0,$$

whereas if $p_x > p_b$ the default condition is

$$\hat{a}_{1H} p_b^{\lambda_1^H} + \hat{a}_{2H} p_b^{\lambda_2^H} + \frac{b}{r} - \gamma = 0.$$

The remaining unknown coefficients are obtained from the value matching $\hat{DH}(p_{HL}) = \hat{DL}(p_{HL})$ and smooth pasting $\hat{DH}'(p_{HL}) = \hat{DL}'(p_{HL})$ conditions at the optimal risk-shifting point

$$\hat{a}_{1H} p_{HL}^{\lambda_1^H} + \hat{a}_{2H} p_{HL}^{\lambda_2^H} - \hat{a}_{1L} p_{HL}^{\lambda_1^L} = 0,$$

$$\lambda_1^H \hat{a}_{1H} p_{HL}^{\lambda_1^H - 1} + \lambda_2^H \hat{a}_{2H} p_{HL}^{\lambda_2^H - 1} - \lambda_1^L \hat{a}_{1L} p_{HL}^{\lambda_1^L - 1} = 0.$$

Solving for the debt coefficients $\hat{a}_{1H}, \hat{a}_{2H}$ and $\hat{a}_{1L}$ if $p_x \leq p_b$, yields the expression

$$\begin{bmatrix} \hat{a}_{1H} \\ \hat{a}_{2H} \\ \hat{a}_{1L} \end{bmatrix} = \begin{bmatrix} p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ \lambda_1^H p_{HL}^{\lambda_1^H - 1} & \lambda_2^H p_{HL}^{\lambda_2^H - 1} & -\lambda_1^L p_{HL}^{\lambda_1^L - 1} \\ p_b^{\lambda_1^H} & p_b^{\lambda_2^H} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ (1-\tau)\left(\frac{\xi_1 p_b}{r-\mu} - \frac{\xi_0 w}{r}\right) + \left[\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu} - \frac{\xi_0 w}{r}\right)\right]\left(\frac{p_b}{p_x}\right)^{\lambda_1^H} - \frac{b}{r} \end{bmatrix}, \tag{C-1}$$

whereas if $p_x > p_b$, gives

$$\begin{bmatrix} \hat{a}_{1H} \\ \hat{a}_{2H} \\ \hat{a}_{1L} \end{bmatrix} = \begin{bmatrix} p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ \lambda_1^H p_{HL}^{\lambda_1^H - 1} & \lambda_2^H p_{HL}^{\lambda_2^H - 1} & -\lambda_1^L p_{HL}^{\lambda_1^L - 1} \\ p_b^{\lambda_1^H} & p_b^{\lambda_2^H} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \gamma - \frac{b}{r} \end{bmatrix}. \tag{C-2}$$

Similarly, we obtain analytical expressions for the value of equity coefficients by solving Equation (A-7) in the high and low volatility regions, subject to the following boundary conditions. In the absence of arbitrage, the no-bubbles condition $\lim_{p \to \infty} \hat{VL}(p) = (1-\tau)\left(\frac{p}{r-\mu_L} - \frac{w+b}{r}\right)$ holds and $\hat{b}_{2L}$ is zero. The default condition $\hat{VH}(p_b) = 0$ at $p = p_b$, can be written as

$$(1-\tau)\left(\frac{p_b}{r-\mu_H} - \frac{w+b}{r}\right) + \hat{b}_{1H} p_b^{\lambda_1^H} + \hat{b}_{2H} p_b^{\lambda_2^H} = 0,$$

while the value matching $\hat{VH}(p_{HL}) = \hat{VL}(p_{HL})$ and smooth pasting $\hat{VH}^{'}(p_{HL}) = \hat{VL}^{'}(p_{HL})$ conditions at $p = p_{HL}$ are equal to

$$\hat{b}_{1H} p_{HL}^{\lambda_1^H} + \hat{b}_{2H} p_{HL}^{\lambda_2^H} - \hat{b}_{1L} p_{HL}^{\lambda_1^L} = 0,$$

$$\lambda_1^H \hat{b}_{1H} p_{HL}^{\lambda_1^H - 1} + \lambda_2^H \hat{b}_{2H} p_{HL}^{\lambda_2^H - 1} - \lambda_1^L \hat{b}_{1L} p_{HL}^{\lambda_1^L - 1} = 0.$$

Consequently, the equity coefficients $\hat{b}_{1H}, \hat{b}_{2H}$ and $\hat{b}_{1L}$ are given by

$$\begin{bmatrix} \hat{b}_{1H} \\ \hat{b}_{2H} \\ \hat{b}_{1L} \end{bmatrix} = \begin{bmatrix} p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ \lambda_1^H p_{HL}^{\lambda_1^H - 1} & \lambda_2^H p_{HL}^{\lambda_2^H - 1} & -\lambda_1^L p_{HL}^{\lambda_1^L - 1} \\ p_b^{\lambda_1^H} & p_b^{\lambda_2^H} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ (1-\tau)\left(\frac{w+b}{r} - \frac{p_b}{r-\mu_H}\right) \end{bmatrix}. \tag{C-3}$$

RENEGOTIABLE DEBT

C.2 Firm, Renegotiation Cost and Debt Coefficients

As a starting point, the firm coefficients are obtained. Boundary conditions include the bankruptcy condition at $p = p_c$

$$(1-\tau)\left(\frac{p_c}{r-\mu_H} - \frac{w}{r}\right) + a_{1s} p_c^{\lambda_1^H} + a_{2s} p_c^{\lambda_2^H} = \gamma.$$

The asymptotic condition $\lim_{p\to\infty} WL(p) = (1-\tau)\left(\frac{p}{r-\mu_L} - \frac{w}{r}\right) + \frac{\tau b}{r}$ implies that $a_{2L} = 0$. Moreover, the value matching and smooth pasting requirements at

$p = p_s$ and $p = p_{HL}$ can be written as

$$a_{1s} p_s^{\lambda_1^H} + a_{2s} p_s^{\lambda_2^H} - a_{1H} p_s^{\lambda_1^H} - a_{2H} p_s^{\lambda_2^H} - \frac{\tau b}{r} = 0,$$

$$\lambda_1^H a_{1s} p_s^{\lambda_1^H - 1} + \lambda_2^H a_{2s} p_s^{\lambda_2^H - 1} - \lambda_1^H a_{1H} p_s^{\lambda_1^H - 1} - \lambda_2^H a_{2H} p_s^{\lambda_2^H - 1} = 0,$$

$$a_{1H} p_{HL}^{\lambda_1^H} + a_{2H} p_{HL}^{\lambda_2^H} - a_{1L} p_{HL}^{\lambda_1^L} = 0,$$

$$\lambda_1^H a_{1H} p_{HL}^{\lambda_1^H - 1} + \lambda_2^H a_{2H} p_{HL}^{\lambda_2^H - 1} - \lambda_1^L a_{1L} p_{HL}^{\lambda_1^L - 1} = 0.$$

Solving for the unknown coefficients $a_{1s}, a_{2s}, a_{1H}, a_{2H}$ and a_{1L} gives

$$\begin{bmatrix} a_{1s} \\ a_{2s} \\ a_{1H} \\ a_{2H} \\ a_{1L} \end{bmatrix} = \begin{bmatrix} p_c^{\lambda_1^H} & p_c^{\lambda_2^H} & 0 & 0 & 0 \\ p_s^{\lambda_1^H} & p_s^{\lambda_2^H} & -p_s^{\lambda_1^H} & -p_s^{\lambda_2^H} & 0 \\ \lambda_1^H p_s^{\lambda_1^H - 1} & \lambda_2^H p_s^{\lambda_2^H - 1} & -\lambda_1^H p_s^{\lambda_1^H - 1} & -\lambda_2^H p_s^{\lambda_2^H - 1} & 0 \\ 0 & 0 & p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ 0 & 0 & \lambda_1^H p_{HL}^{\lambda_1^H - 1} & \lambda_2^H p_{HL}^{\lambda_2^H - 1} & -\lambda_1^L p_{HL}^{\lambda_1^L - 1} \end{bmatrix}^{-1} \begin{bmatrix} \gamma - (1 - \tau)\left(\frac{p_c}{r - \mu_H} - \frac{w}{r}\right) \\ \tau \frac{b}{r} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{C-4}$$

Determining the renegotiation cost coefficients, the asymptotic condition $\lim_{p \to \infty} CL(p) = 0$ implies that b_{2L} is zero. The remaining boundary conditions include the default condition at $p = p_c$

$$\frac{\delta}{r} + b_{1s} p_c^{\lambda_1^H} + b_{2s} p_c^{\lambda_2^H} = 0$$

and the value matching and smoothness conditions at $p = p_{HL}$ and $p = p_s$

$$\frac{\delta}{r} + b_{1s} p_s^{\lambda_1^H} + b_{2s} p_s^{\lambda_2^H} - b_{1H} p_s^{\lambda_1^H} - b_{2H} p_s^{\lambda_2^H} = 0,$$

$$\lambda_1^H b_{1s} p_s^{\lambda_1^H - 1} + \lambda_2^H b_{2s} p_s^{\lambda_2^H - 1} - \lambda_1^H b_{1H} p_s^{\lambda_1^H - 1} - \lambda_2^H b_{2H} p_s^{\lambda_2^H - 1} = 0,$$

$$b_{1H} p_{HL}^{\lambda_1^H} + b_{2H} p_{HL}^{\lambda_2^H} - b_{1L} p_{HL}^{\lambda_1^L} = 0,$$

$$\lambda_1^H b_{1H} p_{HL}^{\lambda_1^H - 1} + \lambda_2^H b_{2H} p_{HL}^{\lambda_2^H - 1} - \lambda_1^L b_{1L} p_{HL}^{\lambda_1^L - 1} = 0.$$

Thus, the renegotiation cost coefficients $b_{1s}, b_{2s}, b_{1H}, b_{2H}, b_{1L}$ are given by

$$\begin{bmatrix} b_{1s} \\ b_{2s} \\ b_{1H} \\ b_{2H} \\ b_{1L} \end{bmatrix} = \begin{bmatrix} p_c^{\lambda_1^H} & p_c^{\lambda_2^H} & 0 & 0 & 0 \\ p_s^{\lambda_1^H} & p_s^{\lambda_2^H} & -p_s^{\lambda_1^H} & -p_s^{\lambda_2^H} & 0 \\ \lambda_1^H p_s^{\lambda_1^H - 1} & \lambda_2^H p_s^{\lambda_2^H - 1} & -\lambda_1^H p_s^{\lambda_1^H - 1} & -\lambda_2^H p_s^{\lambda_2^H - 1} & 0 \\ 0 & 0 & p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ 0 & 0 & \lambda_1^H p_{HL}^{\lambda_1^H - 1} & \lambda_2^H p_{HL}^{\lambda_2^H - 1} & -\lambda_1^L p_{HL}^{\lambda_1^L - 1} \end{bmatrix}^{-1} \begin{bmatrix} -\frac{\delta}{r} \\ -\frac{\delta}{r} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{C-5}$$

Finally, the debt coefficients c_{1H}, c_{2H}, c_{1L} and c_{2L} are determined. Again, the no-bubbles requirement $\lim_{p\to\infty} DL(p) = \frac{b}{r}$, ensures that c_{2L} equals zero. The value matching conditions at $p = p_s$ and $p = p_{HL}$ and the smooth pasting condition at $p = p_{HL}$ yield the following expressions

$$c_{1H}p_s^{\lambda_1^H} + c_{2H}p_s^{\lambda_2^H} + \frac{b}{r} - \eta\Bigg[(1-\tau)\left(\frac{\xi_1 p_s}{r-\mu_H} - \frac{\xi_0 w}{r}\right) + \Bigg(\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu_H} - \frac{\xi_0 w}{r}\right)\Bigg)\left(\frac{p_s}{p_x}\right)^{\lambda_1^H}\Bigg] - (1-\eta)\Bigg[(1-\tau)\left(\frac{p_s}{r-\mu_H} - \frac{w}{r}\right) - \frac{\delta}{r} + a_{1s}p_s^{\lambda_1^H} + a_{2s}p_s^{\lambda_2^H} - b_{1s}p_s^{\lambda_1^H} - b_{2s}p_s^{\lambda_2^H}\Bigg] = 0,$$

$$c_{1H}p_{HL}^{\lambda_1^H} + c_{2H}p_{HL}^{\lambda_2^H} - c_{1L}p_{HL}^{\lambda_1^L} = 0,$$

$$\lambda_1^H c_{1H}p_{HL}^{\lambda_1^H-1} + \lambda_2^H c_{2H}p_{HL}^{\lambda_2^H-1} - \lambda_1^L c_{1L}p_{HL}^{\lambda_1^L-1} = 0.$$

Solving for the debt coefficients c_{1H}, c_{2H} and c_{1L} one gets

$$\begin{bmatrix} c_{1H} \\ c_{2H} \\ c_{1L} \end{bmatrix} = \begin{bmatrix} p_{HL}^{\lambda_1^H} & p_{HL}^{\lambda_2^H} & -p_{HL}^{\lambda_1^L} \\ \lambda_1^H p_{HL}^{\lambda_1^H-1} & \lambda_2^H p_{HL}^{\lambda_2^H-1} & -\lambda_1^L p_{HL}^{\lambda_1^L-1} \\ p_s^{\lambda_1^H} & p_s^{\lambda_2^H} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -\frac{b}{r} + \eta\Big[(1-\tau)\left(\frac{\xi_1 p_s}{r-\mu_H} - \frac{\xi_0 w}{r}\right) + \Big(\gamma - (1-\tau)\left(\frac{\xi_1 p_x}{r-\mu_H} - \frac{\xi_0 w}{r}\right)\Big)\left(\frac{p_s}{p_x}\right)^{\lambda_1^H}\Big] + \\ (1-\eta)\Big[(1-\tau)\left(\frac{p_s}{r-\mu_H} - \frac{w}{r}\right) - \frac{\delta}{r} + (a_{1s}-b_{1s})\,p_s^{\lambda_1^H} + (a_{2s}-b_{2s})\,p_s^{\lambda_2^H}\Big] \end{bmatrix}. \tag{C-6}$$

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Figures.

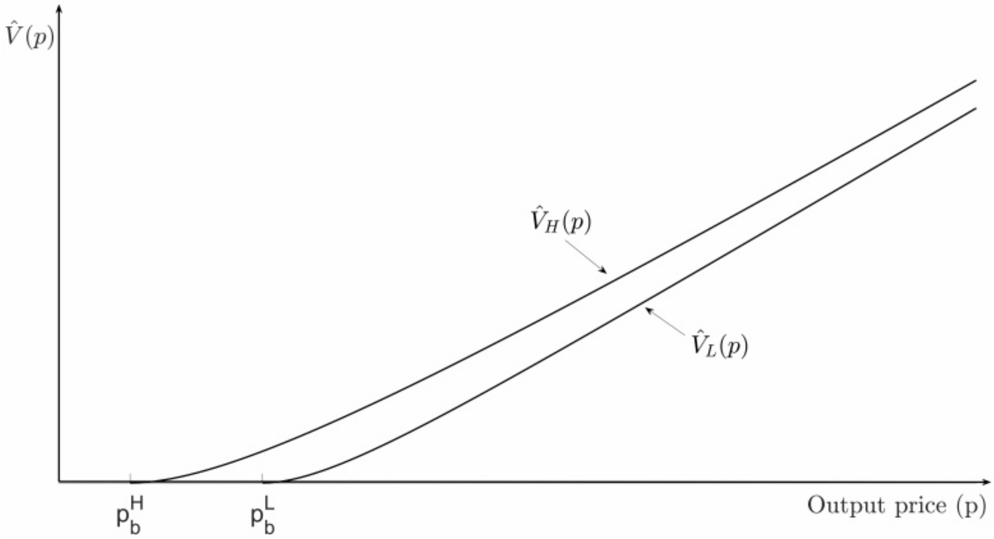

Figure 1. No-renegotiation case, $\mu_H = \mu_L$ and $\sigma_H > \sigma_L$.

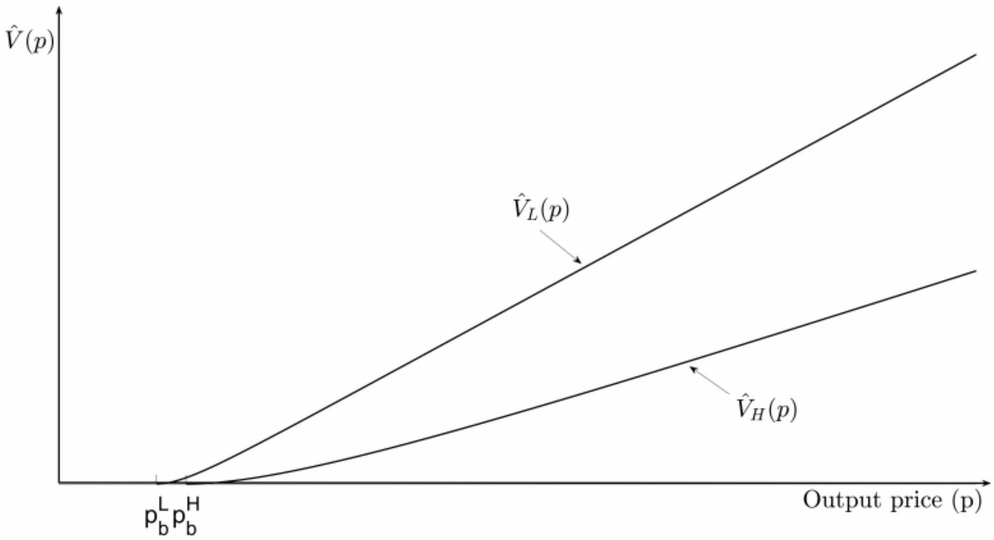

Figure 2. No-renegotiation case, $p_b^H > p_b^L$, $\mu_H < \mu_L$ and $\sigma_H > \sigma_L$.

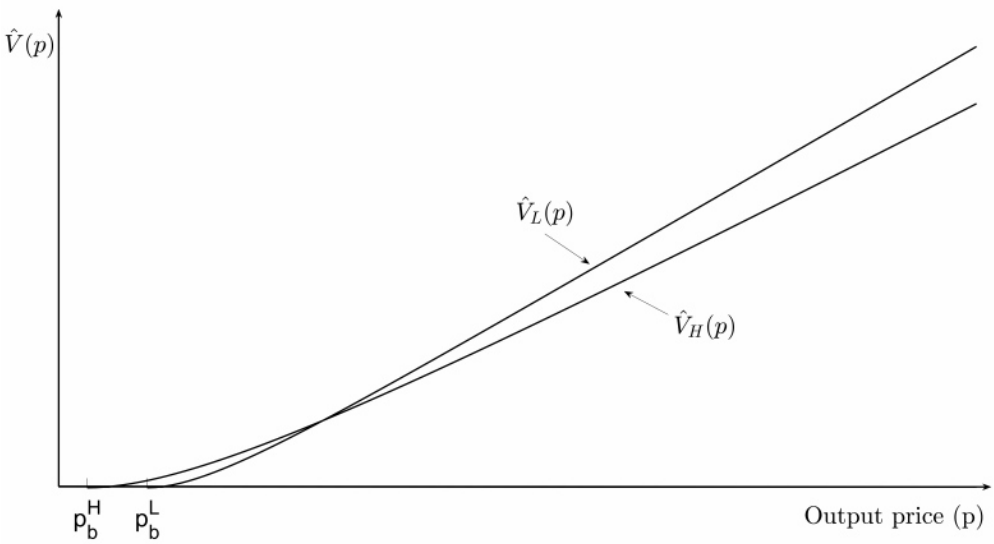

Figure 3. No-renegotiation case, $p_b^H < p_b^L$, $\mu_H < \mu_L$ and $\sigma_H > \sigma_L$.

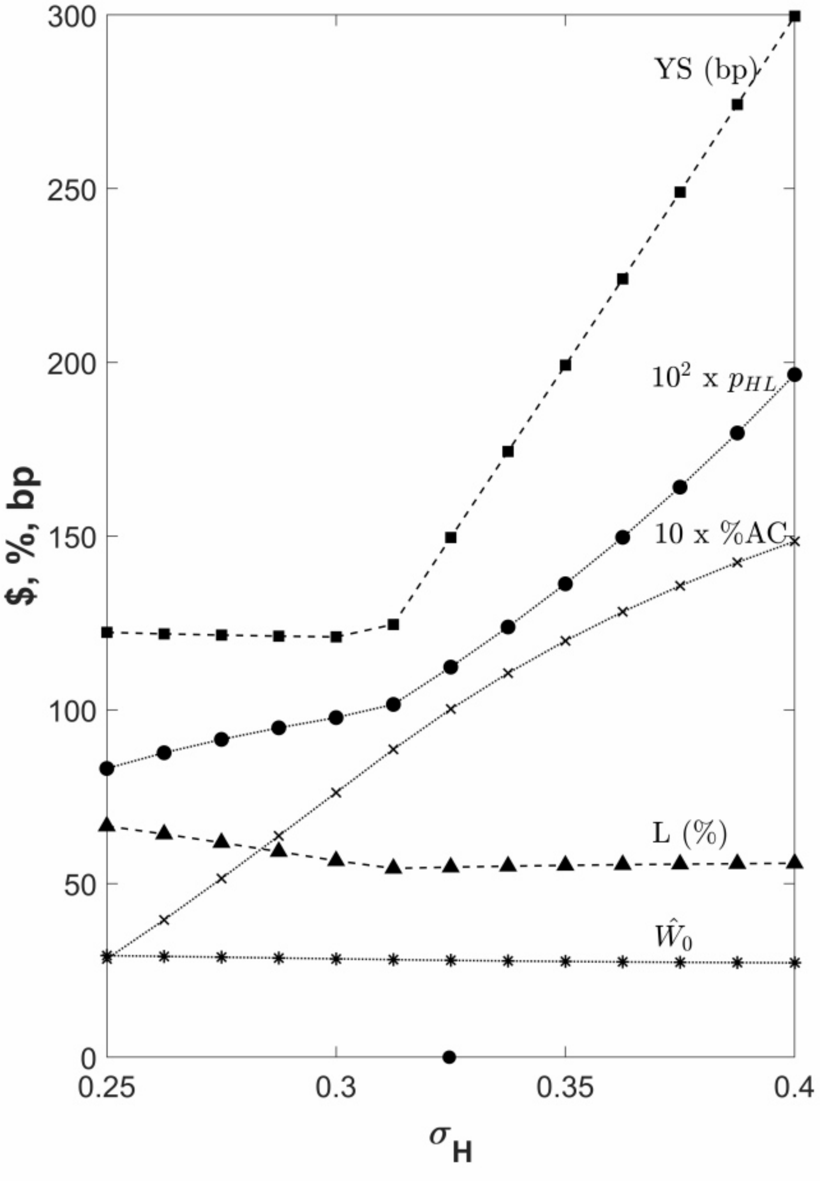

Figure 4. Optimal capital structure and risk profile as a function of σ_H for non-renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \mu_L = 0.035, \mu_H = 0.0275, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of σ_H.

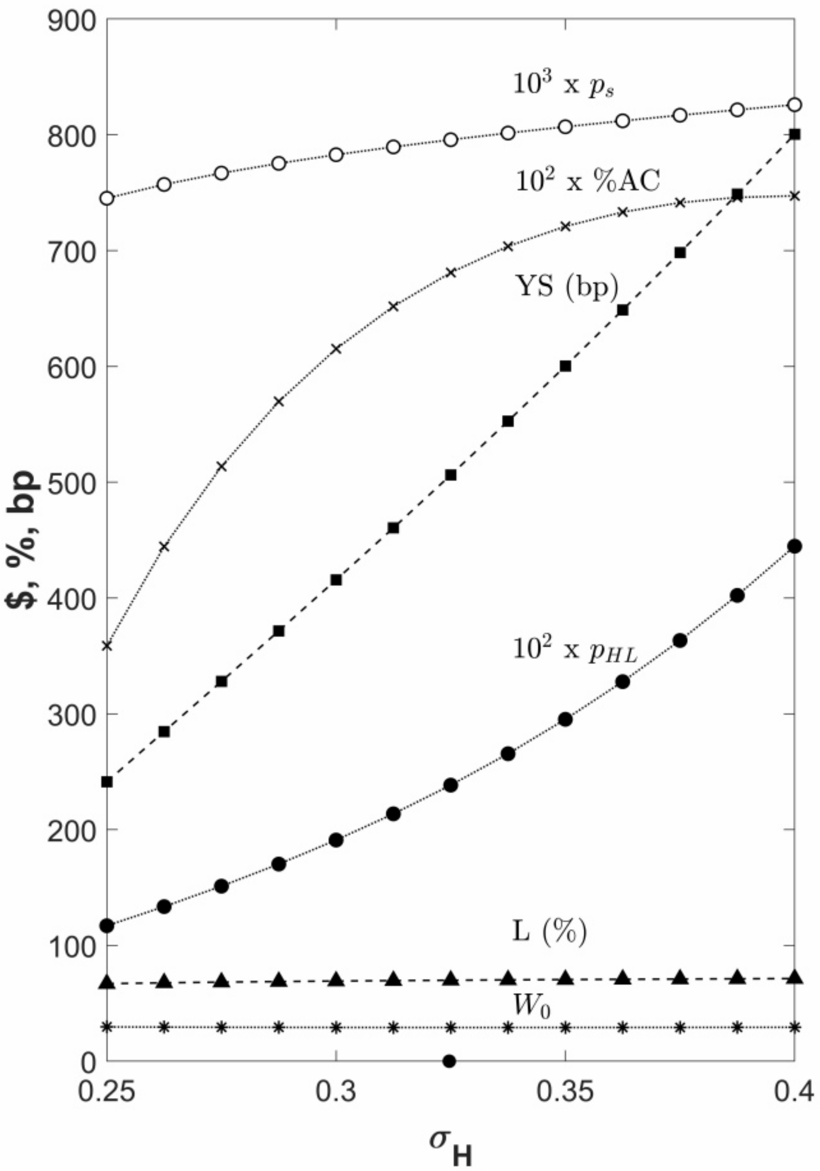

Figure 5. Optimal capital structure and risk profile as a function of σ_H for renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \mu_L = 0.035, \mu_H = 0.0275, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \delta = 0.15, \eta = 0.5, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of σ_H.

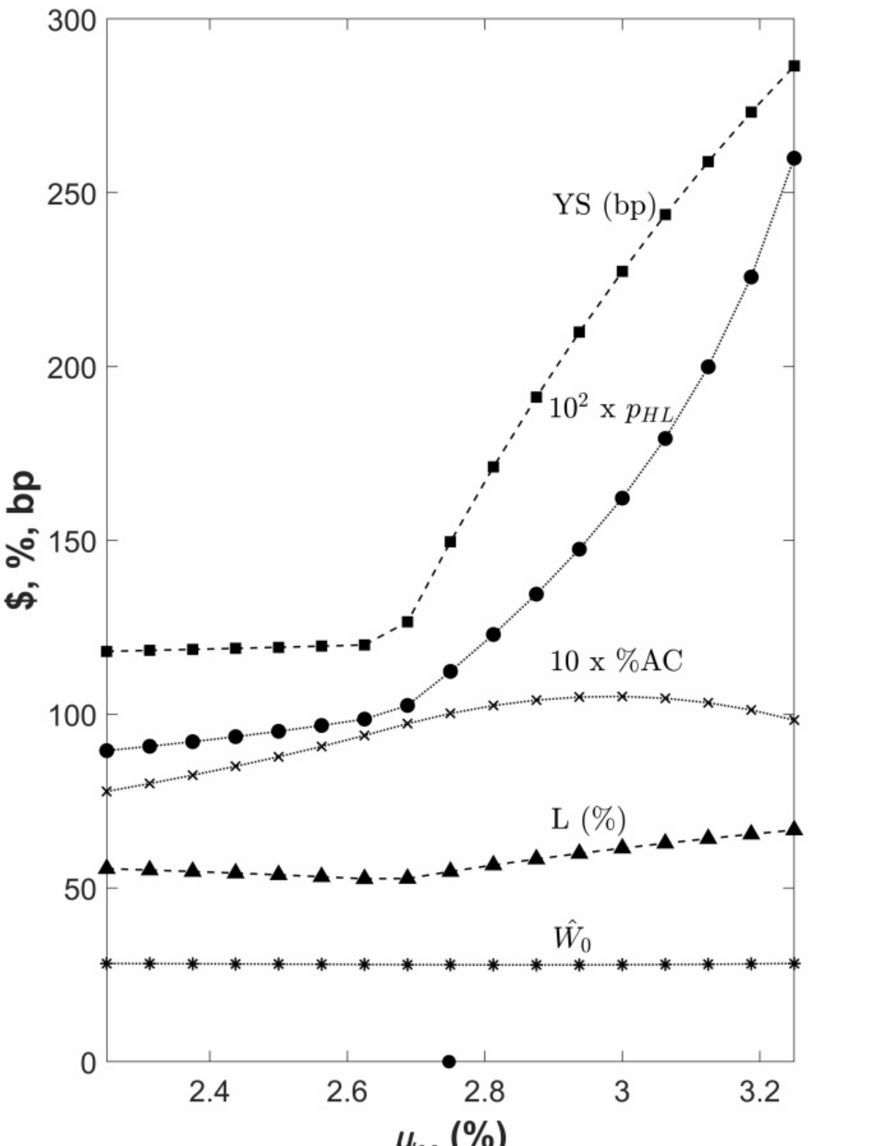

Figure 6. Optimal capital structure and risk profile as a function of μ_H for non-renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \sigma_H = 0.325, \mu_L = 0.035, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of μ_H.

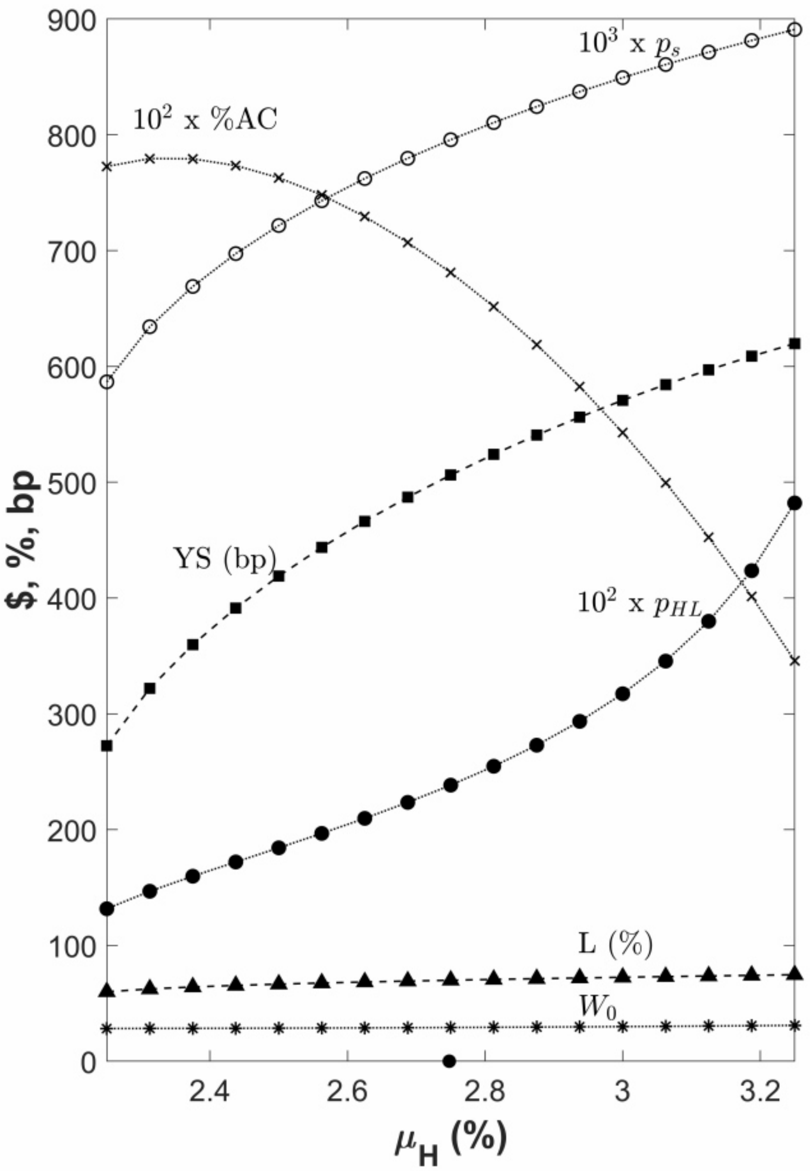

Figure 7. Optimal capital structure and risk profile as a function of μ_H for renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \sigma_H = 0.325, \mu_L = 0.035, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \delta = 0.15, \eta = 0.5, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of μ_H.

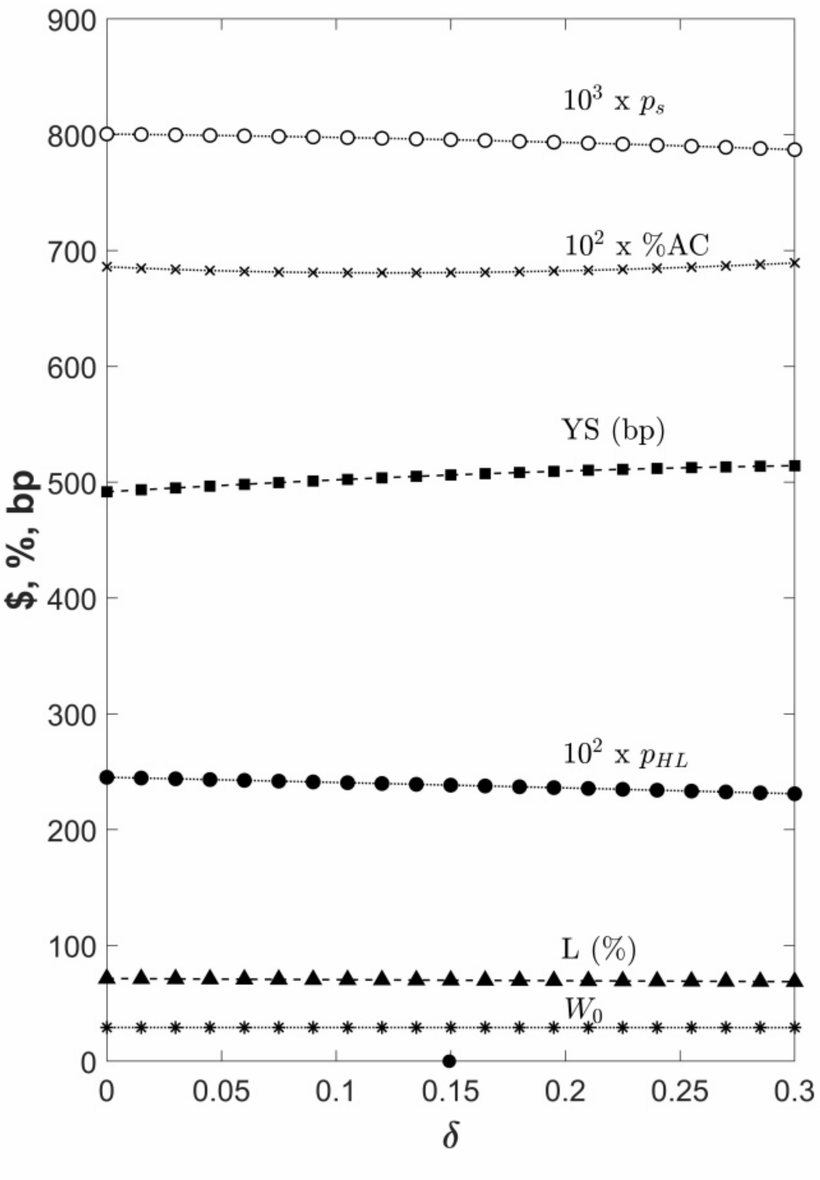

Figure 8. Optimal capital structure and risk profile as a function of δ for renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \sigma_H = 0.325, \mu_L = 0.035, \mu_H = 0.0275, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \eta = 0.5, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of δ.

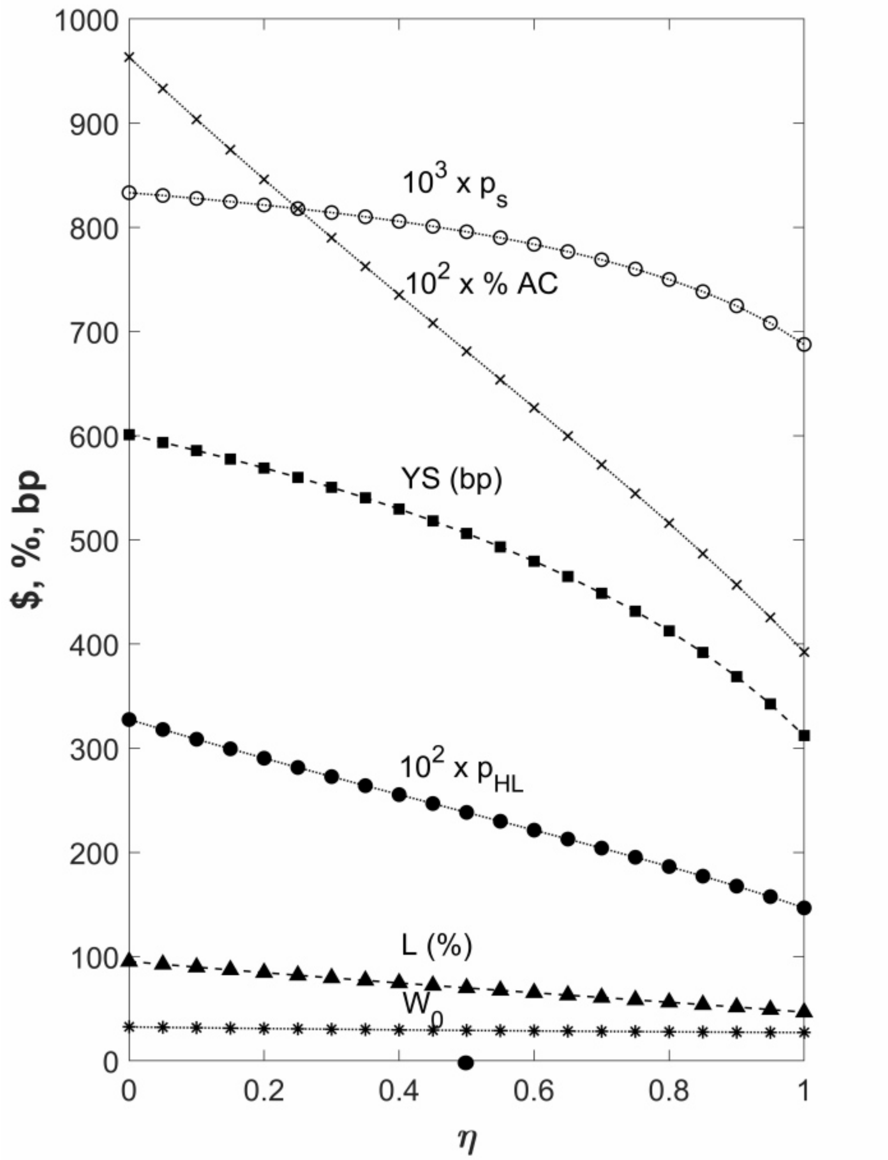

Figure 9. Optimal capital structure and risk profile as a function of η for renegotiable debt. Base-case parameters values: $\sigma_L = 0.2, \sigma_H = 0.325, \mu_L = 0.035, \mu_H = 0.0275, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \delta = 0.15, \tau = 0.35$ and $p_0 = 1$. The dot marker on the horizontal axis corresponds to the base-case value of η.

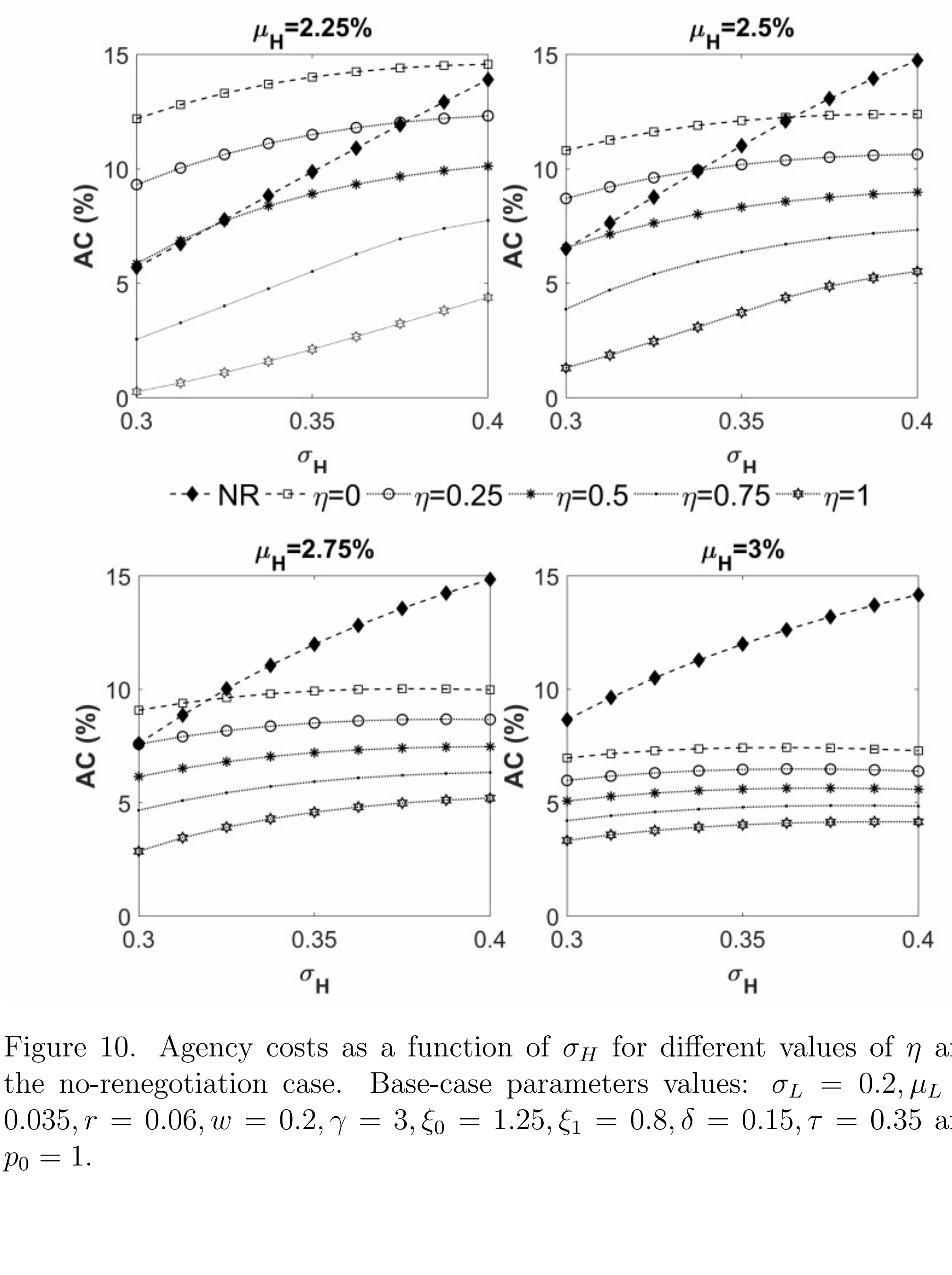

Figure 10. Agency costs as a function of σ_H for different values of η and the no-renegotiation case. Base-case parameters values: $\sigma_L = 0.2, \mu_L = 0.035, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \delta = 0.15, \tau = 0.35$ and $p_0 = 1$.

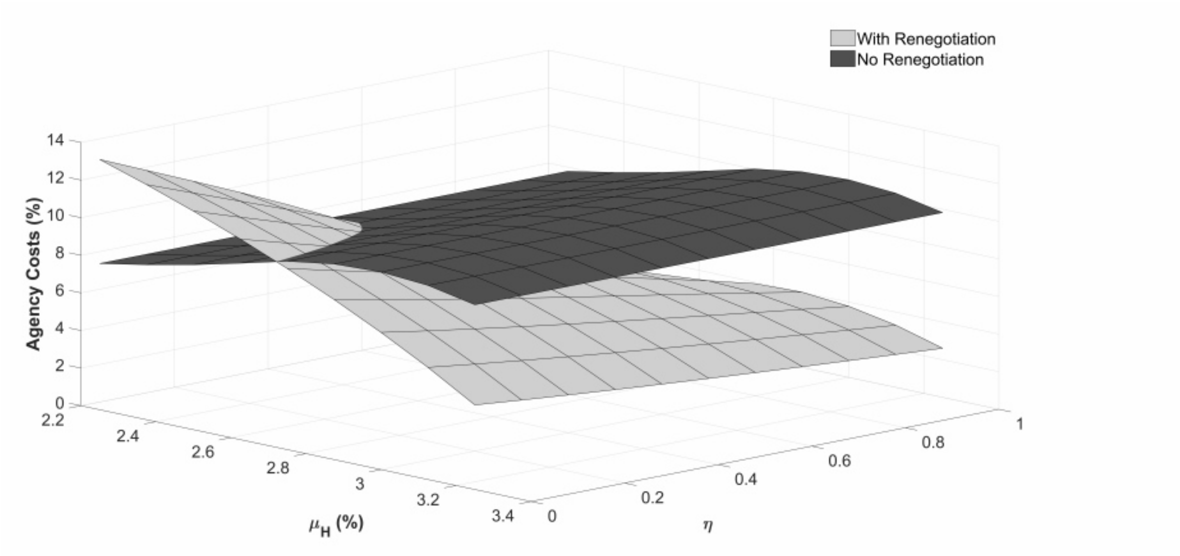

Figure 11. Agency costs as a function of μ_H and η for both cases of debt renegotiation. Base-case parameters values: $\sigma_L = 0.2, \sigma_H = 0.325, \mu_L = 0.035, r = 0.06, w = 0.2, \gamma = 3, \xi_0 = 1.25, \xi_1 = 0.8, \delta = 0.15, \tau = 0.35$ and $p_0 = 1$.

Tables.

Table I. Base-case parameters: σ_L and σ_H are the volatilities of the low- and high-risk activities, μ_L and μ_H are the growth rates of the low- and high-risk activities, r is the risk-free interest rate, w is the production costs value, γ is the liquidation value of the firm, ξ_0 and ξ_1 are the direct bankruptcy costs, δ is the renegotiation costs value, η is the bargaining power of equityholders, τ is the tax rate and p_0 is the initial output price.

σ_L	σ_H	μ_L	μ_H	r	w	γ
0.2	0.325	0.035	0.0275	0.06	0.2	3

ξ_0	ξ_1	δ	η	τ	p_0
1.25	0.8	0.15	0.5	0.35	1

Table II. Optimal capital structure and risk selection for non-renegotiable debt. Here, $\hat{W}_0$ is the optimal firm value at $p = p_0$, b is the optimal coupon payment, p_x is the optimal liquidation trigger of the firm operated by debtholders after bankruptcy, p_b is the levered firm's optimal liquidation trigger, p_{HL} is the optimal risk switching trigger, L (expressed in percentage form) is the optimal leverage ratio ($\hat{D}/\hat{W}$), YS (measured in basis points) is the yield spread on debt ($b/\hat{D} - r$) and AC (determined by the percentage difference between the ex ante and ex post maximal firm values) is the measure of agency costs.

Without Renegotiation	$\hat{W}_0$	$L(\%)$	$YS(bp)$	$AC(\%)$
Ex ante	30.67	77.51	138.46	-
Ex post	27.88	54.71	149.56	10.02

Without Renegotiation	b	p_x	p_b	p_{HL}
Ex ante	1.76	0.16	0.47	0.67
Ex post	1.14	0.16	0.29	1.12

Table III. Optimal capital structure and risk selection with equityholder offers, $\eta = 1$. Here, W_0 is the optimal firm value at $p = p_0$, b is the optimal coupon payment, p_c is the optimal default trigger, p_x is the optimal liquidation trigger of the firm operated by debtholders after bankruptcy, p_s is the optimal renegotiation trigger, p_{HL} is the optimal risk switching trigger, L (expressed in percentage form) is the optimal leverage ratio (D/W), YS (measured in basis points) is the yield spread on debt $(b/D - r)$ and AC (determined by the percentage difference between the ex ante and ex post maximal firm values) is the measure of agency costs.

Equityholder Offers	W_0	$L(\%)$	$YS(bp)$	$AC(\%)$
Ex ante	28.19	48.33	211.11	-
Ex post	27.12	46.64	312.12	3.92

Equityholder Offers	b	p_c	p_x	p_s	p_{HL}
Ex ante	1.10	0.13	0.16	0.88	0.88
Ex post	1.15	0.13	0.16	0.69	1.47

Table IV. Optimal capital structure and risk selection for renegotiable debt with equal bargaining power between debtors and creditors, $\eta = 0.5$. Here, W_0 is the optimal firm value at $p = p_0$, b is the optimal coupon payment, p_c is the optimal default trigger, p_x is the optimal liquidation trigger of the firm operated by debtholders after bankruptcy, p_s is the optimal renegotiation trigger, p_{HL} is the optimal risk switching trigger, L (expressed in percentage form) is the optimal leverage ratio (D/W), YS (measured in basis points) is the yield spread on debt $(b/D - r)$ and AC (determined by the percentage difference between the ex ante and ex post maximal firm values) is the measure of agency costs.

With renegotiation, $\eta = 0.5$	W_0	$L(\%)$	$YS(bp)$	$AC(\%)$
Ex ante	31.11	70.51	243.37	-
Ex post	29.12	69.94	506.17	6.81

With renegotiation, $\eta = 0.5$	b	p_c	p_x	p_s	p_{HL}
Ex ante	1.85	0.12	0.16	0.90	0.90
Ex post	2.25	0.13	0.16	0.80	2.38

Table V. Optimal capital structure and risk selection with debtholder offers, $\eta = 0$. Here, W_0 is the optimal firm value at $p = p_0$, b is the optimal coupon payment, p_c is the optimal default trigger, p_x is the optimal liquidation trigger of the firm operated by debtholders after bankruptcy, p_s is the optimal renegotiation trigger, p_{HL} is the optimal risk switching trigger, L (expressed in percentage form) is the optimal leverage ratio (D/W), YS (measured in basis points) is the yield spread on debt $(b/D - r)$ and AC (determined by the percentage difference between the ex ante and ex post maximal firm values) is the measure of agency costs.

Debtholder Offers	W_0	$L(\%)$	$YS(bp)$	$AC(\%)$
Ex ante	35.61	97.48	263.75	-
Ex post	32.48	95.31	600.87	9.63

Debtholder Offers	b	p_c	p_x	p_s	p_{HL}
Ex ante	3.00	0.11	0.16	0.91	0.91
Ex post	3.72	0.12	0.16	0.83	3.28