The Role of the Leverage Effect in the Price Discovery Process of Credit Markets

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Abstract

Starting from the analysis of a levered firm's capital structure, I show that corporate default risk becomes measurable through the so-called leverage effect observed between equity prices and implied stock option volatilities. In this model, the firm's debt-to-asset ratio governs both the iso-elasticities of equity variance and default probability relative to equity prices. I use a large dataset of S&P 500 firms and an extended timeframe (2008-2018) to examine the model's empirical implications. I find that the impact of the corporate leverage in the transmission mechanism to the CDS market is uniform across firms and robust to market conditions. Although the stock market generally dominates the price discovery process, a cluster of highly-leveraged firms exhibits a CDS market share around or above 50%. Under the effect of corporate leverage, the credit market attracts informed tradind and arbitrage resources.

JEL classification: G13; G30; G32

Keywords: Capital structure; Corporate leverage; Leverage effect; Implied volatility; Price discovery; Cointegration.

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1. Introduction

The standard financial theory suggests that credit and equity markets should be integrated. Indeed, under the stylized assumptions of the Modigliani-Miller world, individual stock and corporate debt securities are reasonably close substitutes, thus removing the risk limits to the arbitrage[1] of the firm's capital structure by rational investors who play the equity against a portfolio of corporate debt and credit derivatives (e.g., Kapadia and Pu, 2012). In the process, neither market should dominate the price discovery of credit risk. However, influential studies have shown that insider trading may occur in the credit derivatives market and impound the price discovery process (Acharya and Johnson, 2007; Kryzanowski, Perrakis, and Zhong, 2017). Such breaches to market efficiency raise the question as to which market attracts informed trading and arbitrage resources.

In this paper, I put forward a structural model of the leverage effect to interpret the dominance of credit markets in the price discovery process. In this model, the corporate leverage governs the transmission of price information between the credit and the stock market. When the firm's financial leverage is low, the informational content of the credit market is low and produces low-intensity signals. As a result, credit traders are mostly noise or liquidity traders, and the bulk of the price discovery process primarily occurs in the stock market, in line with multiple empirical studies (e.g., Hilscher, Pollet, and Wilson, 2015). Conversely, when the firm's financial leverage gradually increases, rational and sophisticated credit investors acquire a gradual advantage in the gathering and the processing of information related to the firm's credit quality. As credit traders tend to monopolize the incorporation of private information into prices, the transmission to the stock market intensifies due to the effect of the corporate leverage, pushing stock traders to chase the trend and morph into noise traders without their knowing. Everything happens as if the corporate leverage made informed trading migrate to the credit market.

My structural model for price transmission via the corporate leverage draws on the following insight: the economic concept of elasticity provides a way to capture the joint correlation between the evolution of the firm's equity market price, on the one hand, and the firm's credit quality, on the

[1] See Shleifer and Summers (1990), Shleifer and Vishny (1997) for the literature on the limits to arbitrage.

other. The first contribution of the paper is to show that the credit-equity elasticity is the adequate medium to convey one of the critical indicators of the firm's financial health—the debt-to-equity ratio. This approach relies on previous works that have already introduced money capital in the production theory of the firm to solve for the optimal capital structure in a partial equilibrium setting (see, for example, Vickers (1970) and Turnovsky (1970)).

The concept of elasticity, however, also provides a straightforward modeling framework to account for the negative correlation between stock prices and their volatilities. At the root of the constant-elasticity-of-variance (CEV) paradigm of Cox (1996) lies the negative correlation observed between stock returns and changes in stock volatility.[2] A simplified explanation for these statistical relationships is the "leverage effect," a term coined by Black (1976) to explain the empirical relation between stock price returns and changes in volatility.[3]

The article's second theoretical contribution, therefore, is to introduce a structural formulation of the asymmetry of the equity implied volatility surface that fits in the CEV framework under the guise of the variance-equity elasticity. This approach explicitly involves the firm's financial leverage and is valid for a broad scope of companies or market conditions. It provides both a structural interpretation of the variance-equity iso-elasticity and a practical method for estimating this parameter, free of the statistical biases inherent to an econometric approach. In doing so, I provide some theoretic elements of a response to the old question summarized by Figlewski and Wang (2000): Is the so-called "leverage effect" a real effect of the firm's financial leverage?

The third theoretical contribution of the article is to show that the CEV approach to the leverage effect implies the constant elasticity of default probabilities relative to stock prices. In this spirit, a simple relationship must bind together the credit-equity iso-elasticity and the variance-equity iso-elasticity. The intuition for this result is that both elasticities are two separate facets of the same

[2]Researchers have extensively documented this statistical relationship in the case of realized volatility (e.g., Christie, 1982; French, Schwert, and Stambaugh, 1987; Duffee, 1995). In parallel, as early as 1987, the asymmetry of the Black-Scholes volatility surface implied from stock options market prices arose as a robust empirical pattern.

[3]The conventional explanation proceeds as follows. The decline of the stock price entails an increase in the firm's debt-equity ratio, which in turn increases the firm's riskiness. Since the creditors' claim on the firm reduces to the debt face value, the bulk of the variations in the firm's asset value should concentrate on the equity component. The equity volatility, either realized or implied from option prices, should thereby rise mechanically.

fundamental firm metrics, namely the debt-to-asset ratio.

My model offers far-reaching empirical implications. First, the credit-equity iso-elasticity hypothesis brings support to the presence of a long-run equilibrium relationship between a firm's credit spreads and equity prices. By capturing the nonlinear effect of the firm's leverage, this cointegrating vector is distinct from the linear combination of the credit spread and the stock price already investigated in the literature (e.g., Narayan, Sharma, and Thuraisamy, 2014).

Second, the model provides new testable hypotheses concerning the price discovery process at work in credit markets. If equity and credit prices are co-integrated, there must be an error-correction mechanism reflecting arbitrage across equity and credit markets. It becomes then possible to measure the contributions made by each market to the price discovery process.

Third, as already underscored in the literature (Kapadia and Pu, 2012), exogenous barriers to arbitrage such as funding constraints and liquidity risks interfere with the co-movements in the equity and credit markets. This paper hypothesizes that the nonlinear impact of the leverage effect may be one of the endogenous sources for the lack of integration between the credit and equity markets.

I use a large dataset of S&P 500 firms and an extended timeframe (2008-2018) to examine the transmission of pricing information from the stock market to the credit default swap (CDS) market. By identifying the genuine price innovations arising in the stock market, I offer an empirical methodology to identify the non-linear impact of the financial leverage on the information flow transiting to the credit market. This leveraged transmission mechanism to the CDS market appears (i) more intense than a linear direct transmission channel, (ii) uniform across firms, (iii) robust to market conditions.

Most firms in the sample reject the null hypothesis of no (leverage-)cointegration between their equity and CDS markets. For these firms, I rely on the vector-error correction (VEC) approach of Gonzalo and Granger (1995) to study the respective contributions of each market to the price discovery process. The CDS market share appears to be low and below 30% for the vast majority of firms, consistent with the CDS "sideshow" hypothesis (Hilscher, Pollet, and Wilson, 2015).

However, a small cluster of highly-leveraged firms exhibits a CDS market share around or above 50%. This new finding provides reliable evidence for the role of the leverage effect in the price discovery process.

This paper relates to the vast empirical literature that investigates the price discovery process in credit markets. The conventional view states that credit pricing information primarily flows from stock markets to credit markets due to lower transaction costs (e.g., Hilscher, Pollet, and Wilson, 2015). The alternative view underscores the role of private information in the flow of pricing information from credit markets to stock markets (e.g., Acharya and Johnson, 2007). The most recent literature suggests that both credit and equity markets should potentially lead and lag the other market (Marsh and Wagner, 2016; Lee, Naranjo, and Velioglu, 2018). By studying the endogenous, non-linear impact of the firm's capital structure, this paper departs from a work of literature mainly focused on exogenous and linear transmission effects.

The article proceeds as follows. Section 2 reviews the literature related to the leverage effect. Section 3 briefly expounds the microeconomic foundations for the credit-equity elasticity using a partial equilibrium approach. Section 4 contains the main theoretical contribution. After introducing a structural approach to the so-called "leverage effect," it links the variance-equity iso-elasticity to the firm's financial leverage, and finally derives the iso-elasticity of default probabilities relative to stock prices. Section 5 discusses the economic implications of the theory. Section 6 describes the data used in the empirical analysis. Section 7 presents empirical evidence on the role of the financial leverage in the transmission of price information from the equity market to the credit market. Section 8 concludes the article.

2. Related Literature on the Leveraged Effect

The approach developed in this article relies on the class of CEV stock price processes introduced by Cox (1996). The CEV paradigm has found some empirical support, especially in the pricing of warrants (Lauterbach and Schultz, 1990). A natural outlet of the CEV framework is the modeling of corporate hybrid products for which the issuing firm's financial leverage is a key concept. However, only econometric approaches to the estimation of CEV parameters appear in the CEV literature (Beckers, 1980; De Spiegeleer, Schoutens, and Van Hulle, 2014). Although researchers have long recognized the strong influence of the firm's debt-equity ratio (Schroder, 1989), to the best of my knowledge, there is no estimation procedure of the CEV elasticity parameter that explicitly takes into account the fundamental factors of the firm such as the financial leverage.

It appears that somewhat limited empirical support has been advanced to substantiate the specific role of the firm's financial leverage in the so-called leverage effect. Christie (1982) finds a significant influence for large firms. Duffee (1995) does not confirm this result for a broader sample of firms. Figlewski and Wang (2000) find evidence for a "down market effect" highlighting the part of negative stock returns on realized volatilities. It further underscores the significant asymmetry in the role of the firm's financial leverage. French, Schwert, and Stambaugh (1987) dismiss its causal role in the asymmetric return-volatility relationship, thereby raising the alternate hypothesis of a volatility feedback effect (see, e.g., Bekaert and Wu, 2000; Wu, 2001). More recently, the question has shifted on the data sampling frequencies used when estimating stock volatilities. Bollerslev, Litvinova, and Tauchen (2006) find empirical evidence of the leverage effect over intraday frequencies. Aït-Sahalia, Fan, and Lin (2013) underscore the asymptotic biases inherent in traditional volatility estimators used in the context of high-frequency data.

From the implied volatility perspective, Toft and Prucyk (1997) produce strong empirical evidence in support of the leverage effect hypothesis for implied volatilities. They employ a structural option pricing model derived from Leland's (1994) structural model of leveraged equity. More recently, Hibbert, Daigler, and Dupoyet (2008) explore behavioral alternatives to the leverage and the volatility feedback hypotheses. As noted by Figlewski and Wang (2000), however, practitioners routinely invoke the leverage effect to account for the empirically observed skew pattern in the equity implied volatility surface, even in the case of asset classes such as foreign exchange rates for which financial leverage is no longer a self-evident concept. In this regard, stochastic volatility option pricing models, such as the well-known Heston (1993) or SABR (Hagan et al., 2002) models, routinely take into account the correlation between the stock price process and its variance. Nevertheless, to the best of our knowledge, no exogenously-specified option pricing model explicitly takes into account the firm's financial leverage per se.

Finally, credit risk also provides a perspective on the leverage effect. From an empirical point of view, researchers such as Campbell and Taksler (2003) have extensively documented the positive correlation between corporate bond yields and equity volatility. Along with the rapid growth of credit derivatives, researchers have underscored similar links between credit default swap (CDS) spreads, and stock option implied volatilities. Cremers et al. (2008) highlight the explanatory power of both the levels and the steepness of the equity implied volatility in interpreting the variations of long-term CDS spreads. Carr and Wu (2010) show empirically that for long maturities and low strike prices, the credit risk contribution becomes as significant as the stock volatility contribution in the pricing of out-of-the-money options.

3. Microeconomic Background

This section lays the economic foundations for the *credit-equity elasticity*.[4] The main quantity of interest is the elasticity of a firm's cost of debt available in the market, r, with respect to the firm's equity market value, S:

$$e := \frac{\mathrm{d}r/r}{\mathrm{d}S/S}. \tag{1}$$

Following the seminal work by Vickers (1970, 1987), I put forward a neoclassical, static model of the firm which takes into account money capital. I consider a situation in which, given the exogenous market rate of interest on the debt, the firm's management is free to select not only an optimal level of production but also an optimal mix of debt and equity to finance this production, by a single set of decisions at the initial time. The comparative static properties of this optimal capital structure will then allow determining the impact of an exogenous deviation in the cost of debt on the firm's equity market value. Let us introduce the following notation.

[4] In the sequel of the article, I reserve the term *elasticity* to situations where a change of δ % in a dimensionless financial quantity x generates a change of $e\delta\%$ in the quantity x^e for a δ close to 0.

x : n-vector of input factors x_i of production used by the firm (physical capital).

w : n-vector of unit factor costs of inputs x_i.

$f(x)$: the firm's production function. As is conventional in production theory, f is assumed to have positive partial derivatives f_i and a negative definite matrix of second partial derivatives f_{ii} (convexity assumption).

p : unit selling price of the firm which is expected to prevail in all future periods. It is assumed to be independent of the production plans of the firm (price-taking assumption).

E : book value of firm's equity.

D : book value of firm's debt, assumed to be equal to its market value.

r : average market rate of interest on the total debt.

ρ : stockholder's required rate of return on equity capital. The equity owners being risk-averse, their capitalization rate ρ is assumed to depend on the debt-equity mix employed in the firm. Following Turnovsky (1970), we shall assume that ρ is a function of the financial leverage, $\rho\left(\sigma D/E\right)$, with $\rho(0) > 0$ and $\rho' \geq 0$. This functional form enables the equity owners' capitalization rate, given a level of the the firms' business risk, to increase with the debt-to-equity ratio.

σ : business risk, measured by the standard deviation of the firm's net operating income.

It is convenient to write the firm's profit function as:

$$\pi(x) := pf(x) - wx. \tag{2}$$

The expected profit $\overline{\pi}(x)$ is the average income stream generated by the firm on a perpetual time scale. As ρ is the capitalization rate applicable to the owners' equity, $\overline{\pi}(x)/\rho$ will be the market value of this income stream. Subtracting the financial debt service then yields the profit being earned for the benefit of the residual owners of the firm:

$$S := \frac{\overline{\pi}(x) - rD}{\rho}, \tag{3}$$

which is also the firm's equity market value.

In their quest for an optimal capital structure, the planning problem faced by the firm's management is to select x, D and E so as to maximize the net discounted present value of the firm:

$$\phi(x,D,K;r) := \frac{\overline{\pi}(x) - rD}{\rho} - E, \tag{4}$$

subject to the following money capital budget constraint:

$$D + E \geq g(x), \tag{5}$$

where the function $g(x)$ describes the requirements in money capital to operate the firm and sustain production.[5] Without loss of generality, the money capital budget constraint (5) can be restricted to an equality, meaning that there is no capital saturation (e.g., Lange, 1936).

Proposition 1 (Credit-equity elasticity). *In a partial equilibrium framework, the credit-equity elasticity of an optimal capital structure is given by the ratio of the expected return on equity over the cost of total debt:*

$$e = -\frac{\rho S}{rD}. \tag{6}$$

Proof. See Appendix A. □

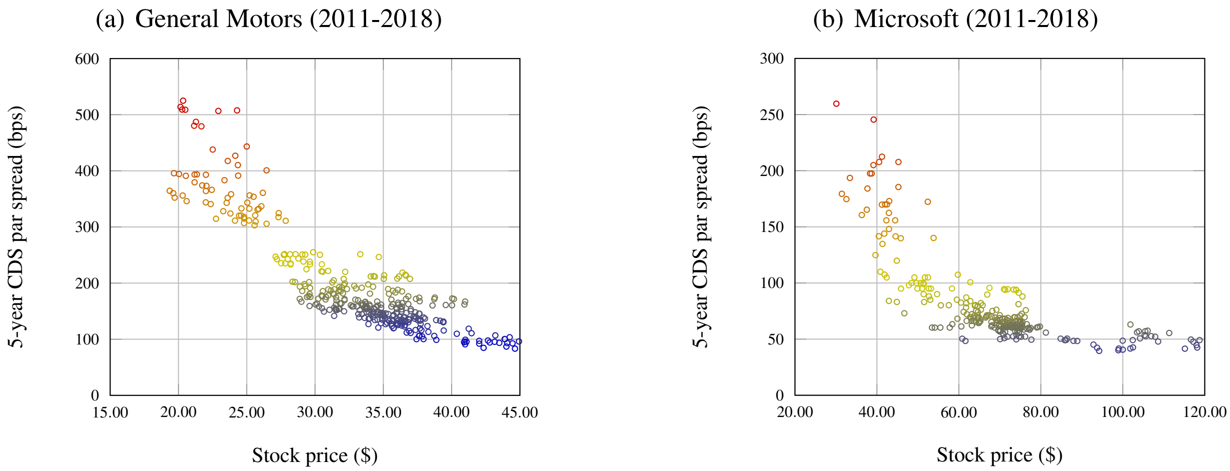
Equation (6) provides important economic insights into the credit-equity elasticity. It suggests that any credit-equity parametric relationship should: *(i)* be monotonic, *(ii)* capture the loose credit-equity de-correlation when stock prices increase, and *(iii)* capture the sharp credit-equity re-correlation when stock prices fall. This relationship should also be convex.

Figure 1 illustrates the credit-equity relationship with scatter plots of 5-year CDS par spreads against prices of the common stock for General Motors and Microsoft over the period 2011-2018.

[5] It will be convenient to think of the money capital requirement function g as the sum of: (i) a global investment of financial capital in the working capital assets (cash, account receivables, inventories, liquidity cushion, etc.), and (ii) specific investments of financial capital relative to each production factor x_i. In the sequel, there is no need to specify g more precisely. For further discussion of money capital requirements, see Vickers (1970, 1987), Turnovsky (1970), or Arzac (1975).

Figure 1. Credit-equity elasticity

This figure plots weekly CDS par spreads (5-year, senior unsecured contract) against weekly closing prices for the common equity. Time period: October, 2011 to September, 2018. Data source: Thomson Reuters.



The monotonicity and the convexity predicted by Equation (6) are easily recognizable. When the equity market value rises significantly, the firm's improved financial health is expected to enhance its creditworthiness, and default probabilities should tend smoothly towards zero (case of Microsoft). Conversely, a significant fall in the equity market value is expected to signal greater odds of financial distress and default probabilities should increase sharply (case of General Motors).

4. The Model

In this section, I build a new model of corporate credit risk which captures the credit-equity elasticity whose microeconomic foundations are laid out in Section 3.

4.1 Structural framework

I now introduce a simple structural framework to build a new approach to the leverage effect. The following set of assumptions is standard in simple versions of the structural framework of corporate default risk (e.g., Merton, 1974; Black and Cox, 1976; Leland, 1994; Leland and Toft, 1996).

ASSUMPTION 1 (Perfect capital markets). *There are no transaction costs and no problems with indivisibilities of assets or informational asymmetries. Securities short-selling is possible with full use of the proceeds; borrowing and lending occur at the same rate of interest. Investors act as price-takers.*

ASSUMPTION 2 (Risk-neutral probability measure). *Trading in assets takes place continuously in a dynamically complete financial market.*

As in similar studies, assuming a non-stochastic interest rate term structure is warranted by the negligible effect of stochastic interest rates on structural models (e.g., Leland, 1998).

ASSUMPTION 3 (Risk-free interest rate). *A riskless asset paying a known constant interest rate* r_f *exists.*

ASSUMPTION 4 (Firm value dynamics). *As in Merton (1974), Black and Cox (1976), Leland (1994), Leland and Toft (1996), the firm's value evolves according to a risk-neutral diffusion-type stochastic process with constant volatility of rate of return:*

$$\mathrm{d}V_t = \mu_V V_t \mathrm{d}t + \sigma_V V_t \mathrm{d}W_t, \tag{7}$$

where $\mu_V = r_f - \delta$ *is the expected return of the firm,* δ *is the net cash outflow rate from the firm paid to claim-holders by unit time (dividend pay-outs to stockholders as well as interest payments to debt-holders),* σ_V^2 *is the instantaneous variance of the return on the firm, and* W_t *is a Wiener process.*

ASSUMPTION 5 (Capital structure). *The firm has issued two types of claims: equity* E *and an amount* D *of debt interest and principal due to be repaid at time horizon* T *under the form of a default-risky bond, which is paying interests in the form of a discrete stream of coupons. The rule of absolute priority governs the distribution of assets to bondholders in case of bankruptcy, which happens as soon as the firm is unable to make contractual cash payments to its claim-holders.*

In Merton's zero-coupon bond framework (1974), default can only occur through net-worth insolvency, when the firm's assets are exhausted at debt horizon. Consequently, the residual claim on the firm is seen as a simple European-style call option on the firm's asset value struck at the debt face value. As Black and Cox (1976) recognized, however, the risk of cash-flow-based insolvency may force stockholders to sell assets to meet the firm's dividend payout policy or its debt servicing obligations.[6] To capture this liquidity risk component of corporate default risk, I make the crucial economic assumption of a continuous default boundary to let default events occur before the debt horizon.

ASSUMPTION 6 (Drift to default). *As in Black and Cox (1976), Leland (1994), Leland and Toft (1996), the default may happen before maturity as soon as the firm value falls below a certain reorganization boundary, under which bondholders get the right to bankrupt or force the reorganization of the firm. The default boundary V_B is assumed to be an exogenous constant smaller than the debt face value D_T.*

While this structural framework is not simple enough to be solved analytically without additional assumptions (such as perpetual debt or continuous interest payment as in Leland, 1994), closed-form solutions will not be needed in the sequel of the article. I now turn to the structural analysis of the leverage effect.

4.2 Structural approach to the "leverage effect"

As their claim does not protect equity holders against a fall of the firm value below the default barrier (before potentially rising anew after successful restructuring), the opportunity loss materializes for equity holders in a short, down-and-in option on the firm value. Consequently, the levered equity becomes a *down-and-out* European-style call option on the firm's asset value, and the firm's

[6] Kim, Ramaswamy, and Sundaresan (1993) argue that the sale of assets to pay debt interests "alters the firm's investment policy and hence its future cash-flows," thereby diminishing the total value of the firm. As an alternative to the assumption of assets' perfect liquidity, I will assume that liquidity risk (i.e., the risk to the firm of not meeting its short-term financial commitments) may force the firm to issue new securities.The issuance of new securities can be limited to the sale of additional common equity through rights issues to avoid the dilution of existing claim-holders. In particular, the subsequent discussion precludes reorganization strategies to resolve financial distress through strategic debt servicing (Mella-Barral and Perraudin, 1997).

volatility remains linked to the equity instantaneous volatility σ by the traditional relationship:

$$\sigma S = S_v \sigma_V V, \tag{8}$$

where $S_v := \partial S / \partial V$ denotes the equity market value differential with respect to the firm value. Equation (8) is not able to provide any direct insight on the leverage effect since both the equity value and its sensitivity with respect to the firm's value move apart when the firm is viable, and stockholders' call option is primarily in the money. To gain further insight into the convexity of the leverage effect, I need to differentiate Equation (8) and to make an assumption on the shape of the equity volatility. I will remain within a general and parsimonious setting, however, by assuming a time-homogeneous local volatility surface.

ASSUMPTION 7 (Local volatility). *The instantaneous equity volatility is a time-homogeneous deterministic function of the stochastic stock price process, that is,* $\sigma(S_t, t) \equiv \sigma(S_t)$.

Equipped with Assumption 7, it is now possible to differentiate Equation (8) which leads to the following structural formulation of the leverage effect.

Proposition 2. *(a) Under the structural Assumptions 1–7, the logarithmic slope of the equity local volatility surface is linked to the firm's financial leverage by the relationship:*

$$\frac{\partial \sigma}{\partial \ln(S)} = -\sigma \cdot (\ell - \varepsilon_\ell), \tag{9}$$

where $\ell := D/V$ *is the debt-to-asset ratio, and where the adjusting term to the financial leverage is given by:*

$$\varepsilon_\ell := \frac{S S_{vv}}{S_v^2}. \tag{10}$$

(b) Exogenous market conditions, as well as endogenous structural conditions, can ensure that the adjustment term ε_ℓ *to the firm's financial leverage is negligible in front of the debt-to-asset ratio* ($\varepsilon_\ell \ll \ell$)*. In these conditions, the slope of the equity instantaneous volatility surface is determined at first order by the firm's financial leverage* ℓ.

Proof. See Appendix B. □

The previous result provides the missing link between the logarithmic slope of the local volatility surface and an adjusted value of the firm's corporate leverage. Note that the debt-to-asset ratio, one of the traditional measures of the company's financial leverage for the financial analysts, is specified with market values instead of book values. This modeling choice reflects not only the firm's tangible assets and working capital but also its intangible assets and growth opportunities.

Limiting oneself to Equation (9) in order to substantiate the role of the firm's financial leverage in the so-called leverage effect presents serious shortcomings. The local volatility surface is not observable and not even model-free, as it depends on the model of the stock price dynamics. A model-free formulation of the volatility surface—such as the Black-Scholes implied volatility surface, which is directly observable in the market—would be preferable. I can therefore introduce the Black-Scholes implied volatility $\widehat{\sigma}(K,T)$ for a given strike price K and given time to maturity T, that is, the volatility number to be input in the Black-Scholes-Merton model (Black and Scholes, 1973) in order to match the observed European-style call price $C(K,T)$ in the options market. I define the slope of the implied volatility surface (the "skew") in the log-strike space as:

$$\widehat{\Sigma} := \frac{\partial \widehat{\sigma}(K,T)}{\partial k}, \tag{11}$$

where F_T is the stock's forward price at maturity T and $k := \ln(K/F_T)$ is the log-moneyness variable.

The local volatility slope appearing in Equation (9) is known to be a good predictor of the asymmetry of the implied volatility surface observed in options markets (see, for example, Gatheral, 2006). More precisely, the local volatility skew is twice as steep as the implied volatility skew for short times to expiration.[7] However, as the following technical result derived by Hagan and Woodward (1999) via singular perturbation theory shows, a stronger pattern holds thanks to Assumption 7 and the assumption of short times to expiration may safely be relaxed.

[7] Gatheral (2006) formally proves this result, as well as comparable results linking local and implied volatilities under various modeling assumptions.

Lemma 1 (Hagan and Woodward, 1999). *Under Assumption 7 of a time-homogeneous local volatility surface, the instantaneous volatility surface $\sigma(\cdot)$ may be inferred from the volatility surface $\widehat{\sigma}(\cdot)$ implied by the options market through the following affine transformation:*

$$\widehat{\sigma}(K) = \sigma\left(\frac{S_0 + K}{2}\right), \tag{12}$$

where K is the option strike price and S_0 is the initial equity market price.

Proof. See Appendix C. □

An immediate consequence of Lemma 1 is that the local volatility slope appearing in Equation (9) is twice as steep as the implied volatility skew. As a result, substituting the implied volatility skew into Equation (9) leads to a structural interpretation of the negative relationship between implied volatility and strike price.

Proposition 3 (Structural leverage effect). *Under structural Assumptions 1–7, the implied volatility skew is linked to the debt-to-asset ratio:*

$$\widehat{\Sigma} = -\frac{\sigma}{2} \cdot (\ell - \varepsilon_\ell), \tag{13}$$

where $\ell = D/V$ and the adjusting term to the financial leverage is given by Equation (10).

The reader will notice that, on top of the leverage effect, Equation (13) captures a form of volatility feedback effect, the alternate hypothesis traditionally advanced for the asymmetry of the implied volatility surface (e.g., Bekaert and Wu, 2000; Wu, 2001).[8] In our current structural framework, it is thus the role of the firm's financial leverage to dampen or magnify a possible volatility feedback effect.

[8]The economic mechanism goes as follows. As an increase in stock market volatility raises expected stock returns (Campbell and Hentschel, 1992), current stock prices then decline to adjust to these revised expectations. As a result, an increase in volatility is correlated with negative stock returns, thus raising the value of out-of-the-money stock options and the implied volatility skew.

4.3 The variance-equity iso-elasticity

To reflect the leverage effect observed empirically on both realized and implied volatility, it is standard practice to resort to the broad class of constant-elasticity of variance (CEV) stock price processes introduced by Cox (1996).

ASSUMPTION 8 (Constant elasticity variance). *The stock price process evolves as the solution to the following stochastic dynamics:*

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma_{CEV} S_t^{\frac{\beta}{2}} \mathrm{d}W_t, \qquad (0 \leq \beta \leq 2) \tag{14}$$

where β is a characteristic exponent, σ_{CEV} is a positive constant diffusive parameter, and W_t is a standard Brownian motion.

Notice that we may conveniently omit the drift term μ since it has no qualitative effect on the dynamics of the process. The CEV framework encompasses a vast spectrum of models ranging from the Bachelier normal model ($\beta = 0$) and the square-root diffusion model ($\beta = 1$) of Cox and Ross (1976) to the Black-Scholes paradigm, which corresponds to the particular case of $\beta = 2$.

The CEV framework is entirely consistent with the approach to credit risk developed so far. Indeed, contrary to the geometric Brownian motion, the CEV diffusion can hit zero with positive probability as soon as $\beta < 2$ (see, for example, Linetsky and Mendoza, 2010), at which time the firm enters a state of default.[9] As a result, the CEV framework is sufficiently rich to enable the following structural interpretation of the variance-equity iso-elasticity.

Proposition 4 (Variance-equity iso-elasticity). *Under Assumptions 1–8, the variance-equity iso-elasticity is given by $\beta - 2$ and amounts to twice the firm's adjusted financial leverage:*

$$|\beta - 2| = 2(\ell - \varepsilon_\ell), \tag{15}$$

[9]Owing to their diffusive characteristics, however, short-term default probabilities are small and fail to produce realistic credit spreads for short expirations (Carr and Linetsky, 2006). This feature renders the CEV dynamics well-suited to model hybrid products subject to default risk only for medium- to long-term expirations.

where ℓ is the debt-to-asset ratio, and ε_ℓ is given by Equation (10). At second order, the equity volatility undergoes intensified impacts from the leverage when equity is a concave claim ($\varepsilon_\ell < 0$), and reduced impacts from the leverage when equity is a convex claim ($\varepsilon_\ell > 0$). The case of an un-levered firm ($\ell \equiv 0$) leads to the Black-Scholes paradigm ($\beta = 2$), in which the absence of any impact from the leverage is synonymous with a constant local volatility function as well as a flat implied volatility surface.

Proof. See Appendix D. □

Equation (15) establishes the role of the variance elasticity as a driving force of the asymmetry of the implied volatility surface. Moreover, it suggests that the CEV model is the simplest model nesting the Black-Scholes paradigm to account for the leverage effect. In this spirit, a link must hold between the variance-equity iso-elasticity and the firm's financial leverage. It comes as no surprise that some authors call the CEV characteristic exponent β a *leverage coefficient* (Das and Sundaram, 2007), although the authors recognize that "there is no direct interpretation of this parameter within the Merton framework." The academic literature devoted to the CEV stock price process has rarely addressed the practical problem of parameter estimation.[10] To the best of my knowledge, Equation (15) is the first theoretical result to provide an unambiguous structural estimate for the variance-equity iso-elasticity.[11]

4.4 The credit-equity iso-elasticity

To exploit the approach to the variance-equity elasticity developed in Section 4.3, I now introduce a simple economic model to link credit spreads with equity volatilities for which I need additional modeling assumptions. First, I allow for a more accurate description of corporate default risk. In addition to cash-flow-based insolvency risk, which is best modeled by the diffusion

[10] Beckers (1980) initiates an econometric approach to the variance-equity iso-elasticity estimation. Schroder (1989) is the first to outline the influence of the firm's debt-equity ratio. More recently, De Spiegeleer, Schoutens, and Van Hulle (2014) further elaborate upon it in the context of the modeling of hybrid securities.

[11] This systematic link with the debt-to-asset ratio comes as no surprise considering the static-tradeoff theory of capital structure (Myers, 1984), in which firms maximize their value by targeting the debt-to-value ratio. Put differently, when the firm optimizes its capital structure by substituting equity to debt and vice-versa, the impact on the equity market value should be commensurate to the ℓ-target.

of the firm value through a reorganization boundary (see Assumption 6), I consider the possibility of a "blue-sky" default through a jump to zero of the stock price upon a credit event.[12]

Assumption 9 (Jump to default). *The stock price jumps to zero upon the default event, meaning that the instantaneous default probability is well defined.*

Second, I slightly reinforce Assumption 2 on the completeness of the financial market. The purpose is to exhibit optional equity structures liable to replicate the main features of a conventional default swap instrument,[13] and to match the higher moments of the implied volatility surface, such as the volatility "skew."

Assumption 10 (Option continuum). *Equity options are continuously tradeable within a significant range of exercise prices before the default event.*

If the positive correlation between default swap spreads and the levels and slopes of the implied volatility surface is well known from empiricists (Cremers et al., 2008), theoretical models that account for this close empirical relationship are still lacking. It is possible, however, to rely on sensitivity-matching analysis to get a better understanding of the links between default probabilities and the dynamics of the implied volatility surface. Grounded in the replication of a default swap instrument by an equity option structure, the following result provides a workable relationship between the default swap spread and the implied volatility skew.

Lemma 2 (Zimmermann, 2015). *Under Assumptions 9–10, the firm's risk premium on its debt at a given maturity, r, is linked at first order to the at-the-money implied volatility,* $\widehat{\sigma}_{ATM}$*, and the implied volatility skew,* $\widehat{\Sigma}$*, as follows:*

$$r = k \cdot \widehat{\sigma}_{ATM} \cdot |\widehat{\Sigma}|, \tag{16}$$

[12] Jumps are essential for capturing the wide spectrum of exogenous drivers of corporate default that do not qualify as "structural:" natural hazards or disasters, catastrophic events, specific news, systemic financial risks, massive fraud risk inside the firm, and so on. Moreover, the introduction of jumps is crucial for reflecting credit risk affecting specific categories of firms, those with few assets in place financed through debt, and whose equity market value derives primarily from growth opportunities (such as patents, trademarks, and so on). If growth stocks are less risky than value stocks owing to the costly reversibility of assets in place (Zhang, 2005), jumps in the equity market value are critical for capturing countercyclical prices of default risk affecting those firms.

[13] A conventional default swap instrument is a bilateral contract with zero entry cost that provides an insurance payoff upon a default event to its buyer in return for a quarterly running fee paid to the seller.

where the constant normalizing factor k is typically independent of the equity volatility and reflects the expected recovery rate on the debt.

Proof. See Appendix E. □

I now combine the insights from Lemma 2 with the variance-equity iso-elasticity of Proposition 4 to derive the main result of the paper. The next Proposition provides a structural estimate for the credit-equity iso-elasticity, e, introduced in Section 3.

Proposition 5 (Credit-equity iso-elasticity). *Under Assumptions 1–10, the credit-equity iso-elasticity e is equal to the variance-equity iso-elasticity and amounts to twice the firm's adjusted financial leverage:*

$$e = |\beta - 2| = 2(\ell - \varepsilon_\ell), \tag{17}$$

where ℓ is the debt-to-asset ratio and ε_ℓ is given by Equation (10). At second order, the effect of the leverage on credit spreads is more pronounced for capital structures whose equity is a concave claim ($\varepsilon_\ell < 0$) and reduced for capital structures whose equity is a convex claim ($\varepsilon_\ell > 0$). The case of an un-levered firm ($\ell \equiv 0$) is consistent with the Black-Scholes paradigm ($\beta = 2$) in which the perfect de-correlation between credit spreads and stock prices ($e = 0$) means that the stock price process cannot reach zero and that the default probability reduces to zero.

Proof. See Appendix F. □

5. Model Implications

In this section, I show that the model's two main results, Propositions 4 and 5, provide new refutable hypotheses for future empirical research.

5.1 The credit-equity power relationship

The main implication of Proposition 5 is that over a small period Δt, the firm's credit spread, r_t, should follow a power relationship relative to the stock price, S_t:

$$r_{t+\Delta t} = r_t \cdot \left(\frac{S_t}{S_{t+\Delta t}}\right)^{2(\ell-\varepsilon_\ell)}. \tag{18}$$

As a power exponent, the firm's (adjusted) financial leverage thus appears as the measure of the cost of debt responsiveness to the impulses of equity market value. In other words, the corporate leverage should intervene in a non-linear way when it comes to dampening or amplifying the transmission of price information from stock markets to credit markets and vice versa.

Notice how power relationships such as Equation (18) precisely fit the microeconomic requirements of Proposition 1. In contrast with alternative parameterizations of credit spreads by stock prices based on logarithmic or exponential functions, the power function ensures sound boundary conditions when stock prices fall close to zero or tend to infinity.[14] Another comparative advantage lies in the scalability of inputs, which can be multiplied by any factor without altering relevant empirical aspects.[15]

5.2 Information transmission between stock and credit markets

Under the form of Equation (18), the credit-equity iso-elasticity hypothesis provides new testable hypotheses concerning the transmission mechanisms between stock and credit markets. Taking the logarithm of both sides, it suggests the following transmission mechanism:

$$(\text{CDS return})_t = -2(\ell - \varepsilon_\ell)_t \times (\text{Stock return})_t + \varepsilon_t, \tag{19}$$

[14]The superior capability of power parametric functions for data fitting is not an isolated case in the financial domain. Also known as the family of constant relative risk aversion (CRRA) in the economic literature, the power family is widely used in economics and other social sciences (e.g., Wakker, 2008; Gabaix, 2009).

[15]For example, credit risk should be an invariant across the different quoting currencies of the firm's common stock.

where ε_t is a microstructure noise. In this mechanism, the interaction between the firm's leverage and the stock (resp. CDS) returns constitutes the signal which primarily matters in predicting CDS (resp. stock) returns.

Professional arbitrage of the capital structure aims at bringing stock and CDS prices toward fundamentals. Such arbitrageurs are mostly hedge funds or private equity firms. They are "highly specialized investors who combine their knowledge with resources of outside investors to take large positions" (Shleifer and Vishny, 1997). Primarily concerned by the interaction signal, they use their knowledge and information to forecast the firm's adjusted corporate leverage, $\ell - \varepsilon_\ell$. Having perfect elasticity demand for the CDS at the price of its substitute stock portfolio, they also need the resources of outside capital to take vast positions in the CDS market.

By contrast, the uninformed traders do not observe the signal and learn from public prices only. Lacking expertise about the interaction signal, they turn into noise traders as soon as the signal intensifies and contains more informational content.

5.3 Price discovery process

At first sight, Equation (19) suggests that both credit and equity markets should potentially alternate in price leadership, consistent with the most recent literature (e.g., Marsh and Wagner, 2016; Lee, Naranjo, and Velioglu, 2018). More fundamentally, taking the logarithms of both sides of Equation (18) suggests that there is a long-run equilibrium relationship between the log-price of credit and the leveraged log-price of equity. If equity and credit prices are co-integrated, there must be an error-correction mechanism reflecting arbitrage across the equity and credit markets. The question becomes which of the two co-integrated markets is the first to absorb the new information and dominates the price discovery process.

6. The Data

For this study, I consider daily closing CDS quotes for the most widely traded, North American reference entities. To build as much as possible a large and representative CDS universe, I impose three requirements. The first constraint is for bid-ask CDS quotes to be available in Thomson Reuters over an extended 10-year sample period running from September 20, 2008, to November 1, 2018. In particular, the firm must not have undergone any major credit event (corporate default, merger, or acquisition) leading to an early exit from the dataset over the sample period. The second constraint is for the corresponding common stocks to continuously trade on the S&P 500 stock index over the full sampling period. Finally, we ask for the historical leverage ratio to be available in Thomson Reuters over the full sampling period. For all the reference entities satisfying the previous three requirements, all the CDS quotes, stock market data, and leverage data are then consistently retrieved from the Thomson Reuters database.[16]

The final single name CDS list comprises a total of 204 corporate credits from the S&P 500 index. For consistency, I consider only CDS par spreads corresponding to U.S.-dollar denominated contracts on the most liquid tenor (5 years), the lowest seniority (senior unsecured debt), and the same restructuring clause (Modified Restructuring). Thomson Reuters provides end-of-day prices by collecting daily single-name CDS quotes from over 30 contributors around the world and applying a rigorous screening procedure to eliminate outliers or doubtful data. Final CDS quotes are thus composite mid spreads calculated by Thomson Reuters and expressed in basis points. The timing for the end-of-day composite calculation is in T+1 (5:00 am GMT). As this last update takes place after the end of trading for U.S. stocks, there is no bias in detecting information flows from stock markets to credit markets.

To measure the firm's financial leverage, I use the ratio of total debt book value to enterprise value:

$$\frac{\text{Short-term Debt} + \text{Long-term Debt}}{\text{Market Capitalization} + \text{Total Debt} + \text{Minority Interest} + \text{Preferred Stock} - \text{Cash}}. \tag{20}$$

The sample thus comprises estimates of the debt-to-asset ratio over the period 2008-2018. Notice that the fluctuations of the firm's market capitalization on top of the changes in total debt book

[16]Mayordomo, Peña, and Schwartz (2010) offer an in-depth comparative study of the Thomson Reuters database and five other public sources of corporate CDS prices.

Table 1. Descriptive statistics

The table reports summary statistics for firm characteristics (Panel A), overall equity and CDS returns (Panel B), and equity and CDS returns with opposite signs (Panel C). The sample consists of 204 U.S. firms over the period September 20, 2008, to November 1, 2018, including only trading days with available CDS spread observations and equity returns. Sample statistics are computed across all observations. Data source: Thomson Reuters.

	5^{th} perc.	25^{th} perc.	Median	Mean	75^{th} perc.	95^{th} perc.	SD	Observations
Panel A: firm-level statistics								467,330
Firm CDS level (mid-price, bps)	25	45	70	113	120	315	188	
Firm leverage (debt to assets)	0.07	0.15	0.23	0.29	0.37	0.69	0.20	
Firm size (mkt. cap., $bn)	4.59	11.29	22.29	44.55	48.51	178.32	58.49	
Firm debt (book value, $bn)	1.34	3.41	6.73	27.85	13.51	99.11	88.92	
Daily observations	974	2,320	2,499	2,291	2,515	2,520	453	
Panel B: equity and CDS returns								467,330
Equity daily return (%)	−2.82	−0.78	0.05	0.03	0.88	2.76	2.06	
CDS daily return (%)	−3.36	−0.10	0.00	−0.02	0.06	3.36	4.31	
Panel C: opposite-sign equity and CDS returns								206,506
Equity daily return (%)	−3.20	−0.91	0.03	−0.04	0.93	2.90	2.25	
CDS daily return (%)	−4.26	−0.75	−0.01	−0.02	0.21	4.53	4.49	

	Firms	Mean CDS level (bps)	Mean leverage	Mean debt ($bn)	Mean size ($bn)	Observations
Panel D: business sector-level statistics						
Basic Materials	15	122	0.25	4.65	15.63	34,630
Consumer Cyclicals	32	160	0.27	9.49	27.72	79,126
Consumer Non-Cyclicals	23	63	0.21	11.13	52.07	56,099
Energy	18	108	0.23	10.82	62.68	40,033
Financials	36	136	0.44	107.49	47.96	74,178
Healthcare	21	61	0.19	11.17	62.96	48,218
Industrials	30	87	0.25	19.01	41.83	69,609
Technology	11	157	0.23	12.97	71.13	26,213
Telecommunications	3	147	0.41	65.74	120.96	5,884
Utilities	15	120	0.48	14.54	17.27	33,340

value entail daily variations in the leverage data set.

Table 1 provides summary statistics for the CDS levels, the leverage data, and the characteristics of the firms in the sample.

7. Empirical Analysis

In this section, I first provide empirical evidence for the effect of the firm's financial leverage in the transmission of price information from the stock market to the credit market.

7.1 Identifying pure stock innovations

I first describe the methodology for identifying true innovations in the stock market due to information revelation. For each firm, I run a regression of stock percentage changes on a constant, four lags of CDS percentage changes to absorb any transmission of delayed information from the credit market, and four stock return lags to capture any autocorrelation in the stock market. To take the elasticity of CDS returns relative to stock returns into account as predicted by the model and Equation (19), the specification also includes interactions of the CDS returns (both contemporaneous and lagged) with the firm's leverage. This approach starts from the conventional view that credit pricing information primarily flows from stocks to CDS (e.g., Hilscher, Pollet, and Wilson, 2015).

Specifically, I estimate the following specification for each firm i:

$$\begin{aligned}(\text{Stock return})_{i,t} = \alpha_i &+ \sum_{k=0}^{4}\left[\beta_{i,k} + \frac{\beta^{\ell}_{i,k}}{(\text{Leverage})_{i,t}}\right](\text{CDS return})_{i,t-k} \\ &+ \sum_{k=1}^{4}\gamma_{i,k}(\text{Stock return})_{i,t-k} + \varepsilon_{i,t}.\end{aligned} \tag{21}$$

I view the residuals $\varepsilon_{i,t}$ from each of these regressions as independent innovations arriving in the stock market. These innovations are either not relevant or just not appreciated by the credit market at the time. By contrast, the coefficients $\beta_{i,0}$ and $\beta^{\ell}_{i,0}$ are akin to linear and "leveraged" measures of the feedback information flowing from the CDS market to the stock market. This approach is similar to the one by Acharya and Johnson (2007) who isolate CDS market innovations at time t by controlling for both stock and CDS returns between t and $t-k$.

The contemporaneous linear response $\beta_{i,0}$ is statistically significant at the 5% level for 22% of the firms. The contemporaneous leveraged response, $\beta^{\ell}_{i,0}$, is even more significant at 33%. For the sake of robustness, I then consider the following aggregated measures:

$$\beta_i := \sum_{k=0}^{4} \beta_{i,k}, \qquad \beta^{\ell}_i := \sum_{k=0}^{4} \beta^{\ell}_{i,k}, \tag{22}$$

These measures capture the overall feedback information flowing from the CDS market to the stock market at the firm level. The aggregated linear response β_i remains significant at the 5% level for only 18% of the firms. However, the level of statistical significance of the aggregated leveraged response β^{ℓ}_i now rises to at least 49% of the firms. This gap in the level of statistical significance stands a first hint as to the role of the leverage effect in the feedback price transmission from credit markets to stock markets.

Table 2 sorts the firms into quintiles based on their aggregated response and examines the average characteristics for firms within each quintile. Panel A of Table 2 shows that the linear aggregated response β_i can be positive, in stark contrast with structural models of default risk. Neither the CDS level nor the leverage appears to vary much across quintiles. When combining p-values within quintiles, only the lowest and the highest quintiles display statistical significance. By contrast, Panel B of Table 2 shows that the aggregated leveraged response β^{ℓ}_i is negative for most firms, in line with structural models of credit risk. Moreover, the high degree of combined statistical significance is almost uniform across quintiles. In this sense, β_i and β^{ℓ}_i appear as complementary measures of the feedback transmission channel existing from the CDS market to the stock market.

Table 2. Feedback information from CDS to stock markets

In the first stage, we run for each firm i the time-series regression:

$$(\text{Stock return})_{i,t} = \alpha_i + \sum_{k=0}^{4}\left[\beta_{i,k} + \frac{\beta^{\ell}_{i,k}}{(\text{Leverage})_{i,t}}\right](\text{CDS return})_{i,t-k} + \sum_{k=1}^{4}\gamma_{i,k}(\text{Stock return})_{i,t-k} + \varepsilon_{i,t} \quad (21)$$

p-values are calculated via standard errors corrected for heteroscedasticity and serial correlation (Newey-West, 1987). In the second stage, firms are ranked into quintiles based on the first-stage estimates of $\beta_i = \sum_{k=0}^{4}\beta_{i,k}$ (resp. $\beta^{\ell}_i = \sum_{k=0}^{4}\beta^{\ell}_{i,k}$), Q1 being the quintile with the smallest (most negative) estimates and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the medians (across firms) of the time-series means of the characteristics for each firm. Within each quintile, p-values across firms are combined via Fisher's sum of logarithms method. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

	Q1	Q2	Q3	Q4	Q5
Panel A: Properties of firms in different β-quintiles					
Median β_i	−0.335	−0.126	−0.022	0.136	0.463
Within-quintile p-value	0.000***	0.011**	1.000	0.851	0.000***
Median CDS level (bps)	102	77	66	69	81
Median firm leverage	0.29	0.19	0.20	0.23	0.36
Panel B: Properties of firms in different β^{ℓ}-quintiles					
Median β^{ℓ}_i	−1.687	−0.946	−0.523	−0.112	0.364
Within-quintile p-value	0.000***	0.000***	0.000***	0.999	0.014**
Median CDS level (bps)	68	82	96	76	80
Median firm leverage	0.23	0.27	0.26	0.25	0.21

7.2 Evidence of leveraged information

I can now exploit the stock price innovations identified in the previous section to study the information flow from stock markets to credit markets. To bring to light the leverage effect predicted by the model and Equation (19), the specification of expected CDS returns includes interactions of the stock returns (both contemporaneous and lagged) with the firm's financial leverage. The specification also contains four lags of CDS percentage changes to purge the credit market of any residual autocorrelation. Finally, I allow the information flow to vary conditionally to specific market conditions.

More specifically, I estimate the following panel specification by pooled regression:

$$(\text{CDS return})_t = a + \sum_{k=0}^{4} b^{\ell}_k (\text{Leverage})_t (\text{Stock innovation})_{t-k} + \sum_{k=1}^{4} c_k (\text{CDS return})_{t-k} + e_t, \quad (23)$$

where the first-stage residuals $\hat{\varepsilon}_{i,t}$ provide a proxy for the real stock innovations. The linear combination $\sum_{k=0}^{4} b_k^{\ell}$ offers a measure of the "leveraged" information flowing unconditionally from the stock market to the credit market. The stock market direction allows to condition specification (23) by using separate regression coefficients on the positive and negative part of each of the five lagged stock innovation terms. Similarly, conditioning upon stock innovation intensity enables to obtain more granular insights into the leveraged information flow.

Table 3 reports estimates for the specification (23). The main finding here is that the overall flow of leveraged information from stock to credit markets is highly significant at the 0.1% threshold. The measure has the awaited negative sign predicted by structural models of credit risk. Its value (-0.462) is significantly higher than the flow of direct, unleveraged information (-0.310). Moreover, this finding is robust when conditioning upon the stock market direction. Column (B) shows that both the responses to positive and negative lagged innovations keep negative signs and the same magnitudes. The finding is also robust to the intensity of the stock information flow. The distribution of stock innovations being symmetrical, the lowest and highest deciles correspond to stock volatility above 67% per annum. When conditioning upon these extreme innovations, column (C) reveals that aggregated responses still keep negative signs and the same magnitudes.

To investigate the firm conditions in which leveraged information typically flows from stock to credit markets, I also estimate specification (23) conditionally upon different credit conditions:

$$\begin{aligned}(\text{CDS return})_t = a + \sum_{k=0}^{4}\left[b_k^{\ell} + b_k^{\ell,D}(\text{Dummy})_t\right](\text{Leverage})_t \times (\text{Stock innovation})_{t-k} \\ + \sum_{k=1}^{4} c_k (\text{CDS return})_{t-k} + e_t,\end{aligned} \tag{24}$$

where the first-stage residuals $\hat{\varepsilon}_{i,t}$ proxy the real stock innovations. I interpret the linear combination $\sum_{k=0}^{4} b_k$ (resp. $\sum_{k=0}^{4} b_k^{D}$) as a measure of the unconditional (resp. conditional) leveraged information flow from the stock market to the credit market. To investigate the role of the firm's credit quality, I first condition by the credit spread level. I build three dummy variables to allocate the CDS level variation between the top quintile of the CDS distribution (above 142 basis points,

Table 3. Leveraged information from stock to CDS markets

This table reports OLS panel estimates and t-statistics for the coefficients of the following pooled regression:

$$(\text{CDS return})_t = a + \sum_{k=0}^{4}\left[b_k + b_k^{\ell}(\text{Leverage})_t + b_k^{\ell,D}(\text{Leverage})_t(\text{Dummy})_t\right](\text{Stock innovation})_{t-k} + \sum_{k=1}^{4} c_k(\text{CDS return})_{t-k} + e_t \tag{23}$$

The first column reports the baseline model (no leverage, no dummy). Column (A) reports unconditional estimates (no dummy). Column (B) reports estimates conditioned on positive stock innovations ($\hat{\varepsilon} > 0$) and negative stock innovations ($\hat{\varepsilon} < 0$). Column (C) reports estimates conditioned on positive stock innovations in the top decile and negative stock innovations in the lowest decile. Column (D) reports estimates conditioned on CDS levels in the lowest quintile Q_1 (< 40 bps), medium quintiles, and the top quintile Q_5 (> 142 bps), respectively. Column (E) reports estimates conditioned on the leverage in the lowest quintile Q_1 (< 0.14), medium quintiles, and the top quintile Q_5 (> 0.41), respectively. t-statistics in parentheses are calculated via firm-clustered standard errors corrected for heteroscedasticity and serial correlation. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

	Baseline model	(A)	(B)	(C)	(D)	(E)
a	-0.0001	-0.0001^{*}	-0.0003^{***}	-0.0004^{***}	-0.0001^{*}	-0.0001
	(-1.61)	(-2.11)	(-4.11)	(-3.38)	(-2.04)	(-1.69)
$\sum_{k=0}^{4} b_k$	-0.310^{***}					
	(-50.22)					
$\sum_{k=0}^{4} b_k^{\ell}$		-0.462^{***}				
		(-34.86)				
$\sum_{k=0}^{4} b_k^{\ell,\hat{\varepsilon}>0}$			-0.397^{***}	-0.437^{***}		
			(-24.73)	(-8.30)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\varepsilon}>0,\text{ top}}$				-0.358^{***}		
				(-17.23)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\varepsilon}<0}$			-0.511^{***}	-0.796^{***}		
			(-33.12)	(-15.40)		
$\sum_{k=0}^{4} b_k^{\ell,\hat{\varepsilon}<0,\text{ lowest}}$				-0.498^{***}		
				(-24.95)		
$\sum_{k=0}^{4} b_k^{\ell,Q_1}$					2.424^{***}	-2.816^{***}
					(19.19)	(-14.39)
$\sum_{k=0}^{4} b_k^{\ell,Q_{2,3,4}}$					-0.444^{***}	-0.341^{***}
					(-30.48)	(-23.97)
$\sum_{k=0}^{4} b_k^{\ell,Q_5}$					-0.747^{***}	-1.215^{***}
					(-22.90)	(-32.74)
$\sum_{k=0}^{4} c_k$	0.039^{***}	0.045^{***}	0.044^{***}	0.044^{***}	0.044^{***}	0.043^{***}
	(9.48)	(16.45)	(16.25)	(16.26	(16.15)	(15.67)
Obs.	455,690	455,690	455,690	455,690	455,690	455,690

corresponding approximately to a credit rating equal to or lower than A3/A-), the intermediary three quintiles (between 142 and 40 basis points), and the lowest quintile (below 40 basis points), respectively. Similarly, I probe the role of the firm's level of indebtedness by setting three dummy variables to allocate the leverage variation between the top quintile of the leverage distribution (above 0.41), the three intermediary quintiles (between 0.14 and 0.41), and the lowest quintile (below 0.14).

Table 3 reports estimates for the specification (24). Column (D) shows that the conditioned flow measure becomes positive when conditioning by low CDS levels. This unexplained positive response could signal either a low degree of informed trading in the CDS market or the absence of substantive information concerning credit risk. In other words, top CDS levels seem to impound a very substantial part of the leveraged price transmission. This finding could suggest a regime of informed revision of CDS quotes under conditions of financial stress.

Table 3 reveals a similar phenomenon when conditioning by top levels of indebtedness. The (unconditioned) response of column (A) increases from -0.462 to -1.215 in column (E), more than threefold an increase in intensity. Highly leveraged firms seem to produce even more informed revisions of CDS quotes instead of mechanical price transmission, once again suggesting the occurring of insider trading (Acharya and Johnson, 2007).

7.3 Leveraged information at the firm level

The pooled regression described above forces all firms to have the same dynamic properties, except as captured by market-conditioning dummy variables. I now estimate separate dynamics for each firm by allowing for firm fixed effects. To compare the intensity of the leveraged, non-linear information flow with the direct transmission of information, I also include five lags of unleveraged stock innovations. This alternative specification allows testing for differences among nested models at the firm level by running a likelihood ratio (LR) test. The LR test statistic then measures whether the inclusion of leveraged regressors significantly improves the goodness of fit of the regression model.

Specifically, I estimate the following specification for each firm i:

$$\begin{aligned}(\text{CDS return})_{i,t} = a_i + \sum_{k=0}^{4} \left[b_{i,k} + b^{\ell}_{i,k}(\text{Leverage})_{i,t}\right] \times (\text{Stock innovation})_{i,t-k} \\ + \sum_{k=1}^{4} c_{i,k}(\text{CDS return})_{i,t-k} + e_{i,t}\end{aligned} \tag{25}$$

where the first-stage residuals $\hat{\varepsilon}_{i,t}$ provide a proxy for the real stock innovations.

Table 4 reports estimates for the specification (25). Panel A shows the summary statistics for the estimated linear responses b_i. The mean is -0.017 and statistically insignificant, consistent with the findings of previous studies (Acharya and Johnson, 2007). By contrast, the mean of the leveraged response b^{ℓ}_i is -1.303 and significant, thereby validating the non-linear role of the leverage. As a robustness check, I also run for each firm an LR test of the null hypothesis $\mathrm{H}_0 : b^{\ell}_{i,k} = 0$ $(0 \leqslant k \leqslant 4)$. This procedure provides a collection of independent LR test statistics and p-values. I then use Fisher's combined probability test to fusion these p-values and to assess whether the inclusion of leveraged predictor variables improves the model's goodness of fit. Panel A reveals that this combined LR p-value is highly significant.

Panel B sorts the firms into quintiles based on their aggregated leveraged response and examine the median firm characteristics of each. The combined LR p-value turns out to be highly significant for four quintiles out of five. Moreover, there is a uniform distribution of firm characteristics across quintiles. These observations suggest that a specific category of firms does not impound the leveraged flow of information from stock to CDS markets.

7.4 Contributions to price discovery

I now study the contribution of each market to the price discovery process. Taking the logarithms of both sides of Equation (18) suggests that the CDS price and the leveraged stock price should be co-integrated. This feature suggests adopting the classical vector error-correction model (VECM) approach to price discovery formalized by Gonzalo and Granger (1995).

In a first stage, I test at the level of each firm i the co-integration of the two price series by

Table 4. Leveraged information: two-stage cross-sectional estimation

In the first stage, I run for each firm i the time-series regression:

$$(\text{CDS return})_{i,t} = a_i + \sum_{k=0}^{4}\left[b_{i,k} + b^{\ell}_{i,k}(\text{Leverage})_{i,t}\right](\text{Stock innovation})_{i,t-k} + \sum_{k=1}^{4} c_{i,k}(\text{CDS return})_{i,t-k} + e_{i,t}. \quad (25)$$

LR measures rejection of the null hypothesis $\mathrm{H}_0: b^{\ell}_{i,k} = 0$ $(0 \leqslant k \leqslant 4)$ by the likelihood ratio test. In the second stage, firms are ranked into quintiles based on the first-stage estimates of $b^{\ell}_i = \sum_{k=0}^{4} b^{\ell}_{i,k}$, Q1 being the quintile with the smallest (most negative) estimates and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the medians (across firms) of the time-series means of the characteristics for each firm. Within each quintile, p-values across firms are combined via Fisher's sum of logarithms method. ***, ** and * denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Data source: Thomson Reuters.

Panel A: Univariate properties of b and b^{ℓ}					
Average b_i	-0.017		Average b^{ℓ}_i	-1.303	
t-statistic	(-0.14)		t-statistic	(-1.74)	
			LR (p-value)	0.000^{***}	
Min	-3.277		Min	-142.903	
Max	18.000		Max	16.520	
	Q1	Q2	Q3	Q4	Q5
Panel B: Properties of firms in different b^{ℓ}-quintiles					
Median b^{ℓ}_i	-3.216	-0.930	-0.128	0.372	1.679
LR (p-value)	0.000^{***}	0.000^{***}	0.094	0.002^{***}	0.000^{***}
Median CDS level (bps)	59	77	129	97	74
Median firm leverage	0.22	0.22	0.32	0.29	0.20
Median firm size ($bn)	24.06	22.62	17.04	21.19	32.47
Median firm debt ($bn)	7.04	5.90	5.95	7.41	6.54

examining whether the residuals of the following regression are stationary:

$$(\ln \text{CDS})_{i,t} = \alpha_i + \beta^{\ell}_i \times (\text{Leverage})_{i,t} \times (\ln \text{Stock})_{i,t} + \eta_{i,t}. \quad (26)$$

The augmented Dickey-Fuller (ADF) test statistic enables to reject the null hypothesis of co-integration by invalidating the presence of a unit root in the residuals $\hat{\eta}_{i,t}$.

In a second stage, for those firms which are co-integrated, I measure the contribution of each market to the price discovery process by estimating the following VECM:

$$
\begin{aligned}
(\text{CDS return})_{i,t} &= \lambda_1 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{4} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{4} c_{1,k} (\text{CDS return})_{i,t-k} + u_{i,t}, \\
(\text{Stock return})_{i,t} &= \lambda_2 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{4} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{4} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + v_{i,t},
\end{aligned}
\tag{27}
$$

where the first-stage residuals $\widehat{\eta}_{i,t}$ provide error-correcting terms, the lagged stock and CDS returns capture market imperfections, and $u_{i,t}$ and $v_{i,t}$ are i.i.d. disturbances. If the stock market is contributing significantly to the price discovery process, then λ_1 should be negative and statistically significant as the CDS market adjusts to incorporate information discovered in the stock market. Conversely, if the CDS market dominates the price discovery process, λ_2 should be positive and statistically significant. Error-correcting adjustments must occur in either the stock market or the CDS market or in both to maintain the long-run equilibrium relationship between the two series. The market which reflects new information most rapidly is the one dominating the price discovery process.

Comparing the error-correction coefficients λ_1 and λ_2 allows estimating the information share of each market through their relative speed of adjustment to the long-run equilibrium relationship. Following the literature on credit price discovery (e.g., Narayan, Sharma, and Thuraisamy, 2014), I use the Gonzalo and Granger (GG, 2015) measure for the CDS market share:

$$
S_{\text{CDS}} := \frac{\hat{\lambda}_2}{\hat{\lambda}_2 - \hat{\lambda}_1}, \tag{28}
$$

provided that $\hat{\lambda}_1 \neq \hat{\lambda}_2$. Notice that $0 \leqslant S_{\text{CDS}} \leqslant 1$ as soon as $\hat{\lambda}_1$ and $\hat{\lambda}_2$ have the expected negative and positive sign, respectively. If $\hat{\lambda}_1 = 0$, there is no evidence of price discovery in the stock market ($S_{\text{CDS}} = 1$). If $\hat{\lambda}_2 = 0$ there is no evidence of price discovery in the CDS market ($S_{\text{CDS}} = 0$).

Table 5 reports the contributions to the price discovery process made by the equity and credit markets for the firms that reject the null hypothesis of no cointegration at the 5% threshold. Panel

Table 5. CDS and equity market shares in the price discovery process

In the first stage, for the firms whose ADF test statistic rejects at the 5% level the null of a unit root in the residuals of the regression:

$$(\ln \mathrm{CDS})_{i,t} = \alpha_i + \beta_i^{\ell} \times (\mathrm{Leverage})_{i,t} \times (\ln \mathrm{Stock})_{i,t} + \eta_{i,t}, \tag{26}$$

the following VECM is estimated:

$$\begin{aligned}(\text{CDS return})_{i,t} &= \lambda_1 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{4} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{4} c_{1,k} (\text{CDS return})_{i,t-k} + u_{i,t}, \\ (\text{Stock return})_{i,t} &= \lambda_2 \widehat{\eta}_{i,t-1} + \sum_{k=1}^{4} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{4} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + v_{i,t}.\end{aligned} \tag{27}$$

In the second stage, firms whose $\hat{\lambda}_1$ is significant at the 5% level are ranked into quintiles based on the first-stage estimate of the GG measure S_{Stock} of stock market contribution to price discovery (Panel B). Firms whose $\hat{\lambda}_2$ is significant at the 5% level are ranked into quintiles based on the first-stage estimate of the GG measure S_{CDS} of CDS market contribution to price discovery (Panel C). The summary statistics reported for each quintile are the averages (across firms) of the time-series means of the characteristics for each firm. Data source: Thomson Reuters.

	Co-integration threshold					
	1%	5%	10%			
Panel A: co-integrated firms						
Number of firms	99	159	188			
Percentage of firms	48.5	77.9	92.2			
	Q1	Q2	Q3	Q4	Q5	Total
Panel B: price discovery in the stock market						
Average stock market share, S_{Stock} (%)	58.1	74.4	87.2	98.7	100.0	83.5
Number of firms	24	24	24	24	23	119
Average CDS level (bps)	146	100	83	91	111	106
Average firm leverage	0.33	0.32	0.27	0.24	0.30	0.29
	Q1	Q2	Q3	Q4	Q5	Total
Panel C: price discovery in the CDS market						
Average CDS market share, S_{CDS} (%)	5.4	30.9	38.6	44.4	67.1	34.7
Number of firms	5	5	5	5	3	23
Average CDS level (bps)	119	70	100	95	211	111
Average firm leverage	0.33	0.30	0.44	0.18	0.56	0.34

A shows that nearly 80% of the firms in the sample are co-integrated and qualify for the VECM stage. Only 16 firms out of 204 do not hint at co-integration at all, while 29 firms barely miss the 5% rejection threshold.

Panel B reports the properties of co-integrated firms whose error-correcting coefficient $\hat{\lambda}_1$ is statistically significant at least at the 5% level. With a stock market share well above 50%, roughly two thirds of the firms in the sample exhibit a high degree of price discovery in the stock market. This result is consistent with the CDS "sideshow" hypothesis (Hilsher, Pollet, and Wilson, 2015) for which informed traders favor the stock market to the CDS market because of transaction costs.

Panel C reports the firms whose CDS market provides an alternative forum for price discovery. In this case, the error-correcting coefficient $\hat{\lambda}_2$ must be significant at the 5% level at least. Confined to the highest quintile, Panel C reveals strong evidence for the role of the leverage effect in the price discovery process. With a market share above 60%, the top quintile contains highly-leveraged firms with CDS levels above 200 basis points, a level corresponding to a credit rating lower than Baa2/BBB.

8. Conclusions

In this article, I show that a parsimonious structural framework is sufficient to build a theoretical model to connect the firm's financial leverage and the variance-equity elasticity. This elasticity amounts to twice the debt-to-assets ratio—a standard measure of the corporate leverage. This key feature enables to put the so-called "leverage effect" into a credit risk perspective, thus giving its full meaning to a four-decade-old term (Black, 1976). It provides a non-linear mechanism of information transmission between the equity and credit markets.

An empirical analysis over a large dataset of S&P 500 firms and an extended timeframe (2008-2018) highlights the nonlinear role of the corporate leverage in the transmission of price information between stock and CDS markets. The study shows that such activity is intense and robust to market conditions. It affects all firms uniformly, irrespective of their level of indebtedness or their CDS spread quoted in the market.

In line with previous studies, two-thirds of the firms in the sample see their price discovery process widely dominated by the equity market, with stocks impounding more than 70% of the process. However, I find a significant portion of highly-leveraged firms for which half of the discovery process or more is occurring in the CDS market. The leverage effect could explain some of the pricing discrepancies observed between stock and CDS markets. These mispricings were usually attributed to some CDS market inefficiencies such as illiquidity or opaqueness by the recent literature. By contrast, the leverage effect provides an economic rationale to the limits of capital structure arbitrage.

Appendix A. Proof of Proposition 1

The firm's structural planning problem can be rewritten as the maximization of the following Lagrangian expression:

$$\mathscr{L}(x,D,E,\lambda;r) := \frac{\overline{\pi}(x)-rD}{\rho} - E + \lambda\left[D+E-g(x)\right], \tag{A1}$$

where λ is the Lagrange multiplier. Differentiating with respect to x, D, E, and λ respectively results in the following set of first order conditions for the optimum:

$$\frac{\partial\mathscr{L}}{\partial x} = \frac{\overline{\pi}'(x)}{\rho} - \lambda g'(x) \equiv 0, \tag{A2}$$

$$\frac{\partial\mathscr{L}}{\partial D} = \frac{-r\rho - [\overline{\pi}(x)-rD]\,\rho'\sigma/E}{\rho^2} + \lambda \equiv 0, \tag{A3}$$

$$\frac{\partial\mathscr{L}}{\partial E} = -\frac{[\overline{\pi}(x)-rD]\,\rho'\sigma D/E^2}{\rho^2} - 1 + \lambda \equiv 0, \tag{A4}$$

$$\frac{\partial\mathscr{L}}{\partial\lambda} = D+E-g(x) \equiv 0. \tag{A5}$$

Assuming the existence of such an optimal financial structure, we can now examine the comparative statics of the solution. Let $x^*(r)$, $D^*(r)$ and $E^*(r)$ the decision variables that maximize the firm's net present value where r is the exogenous parameter of interest. The value function satisfies:

$$\phi^*(r) := \phi(x^*,D^*,E^*,\lambda^*;r) = \max_{x,D,E} \phi(x,D,E;r). \tag{A6}$$

The envelope theorem provides provides the response of changes in the firm's net present value to changes to the given cost of debt in a straightforward way:

$$\frac{\mathrm{d}\phi^*(r)}{\mathrm{d}r} = \frac{\partial\mathscr{L}}{\partial r}\Big|_{x^*,D^*,E^*} = -\frac{D^*(r)}{\rho}. \tag{A7}$$

Since the objective function ϕ is the difference between the equity market value S and the equity book value E, the response in the firm's equity market value to a deviation in the cost of debt is then given by:

$$\frac{\mathrm{d}S^*(r)}{\mathrm{d}r} = -\frac{D^*(r)}{\rho} + \frac{\mathrm{d}E^*(r)}{\mathrm{d}r}. \tag{A8}$$

In the specific case of a newly setup firm, the amount of book equity raised depends crucially on the cost of debt available in the market on firm's inception. Depending on on the firm regarding debt and equity as complementary of substitute forms of finance, the derivative of the optimally chosen book equity, E^*, may be either positive (substitutes) or negative (complements). Equation (A8) highlights the dependence of the equity market value sensitivity to r on this optimal debt-equity mix.

In the more general case of an already-setup, non-restructuring firm operating on a going concern basis, the amount of book equity becomes a fixed constraint. As a given parameter of the model, $\overline{E}$ loses any sensitivity to the market cost of debt, and the term dE^*/dr cancels from the right hand side of Equation (A8). Multiplying both sides by the ratio r/S, we obtain the expression for the credit-equity elasticity given by Equation (6).

Appendix B. Proof of Proposition 2

(a) The firm's business risk, σ_V, being a constant independent from the capital structure (Assumption 4), Equation (8) can be differentiated with respect to the equity market value to get:

$$\frac{\partial \sigma}{\partial S} = \frac{\partial}{\partial S}\left(S_v \sigma_V \frac{V}{S}\right) = \frac{\partial S_v}{\partial S} \times \sigma_V \frac{V}{S} + S_v \sigma_V \times \frac{S-V}{S^2}. \tag{B1}$$

Using the chain rule $\partial_S = \partial_V / S_v$ yields:

$$S\frac{\partial \sigma}{\partial S} = \frac{S_{vv}}{S_v}\sigma_V V - S_v \sigma_V \frac{D}{S}. \tag{B2}$$

The unknown asset volatility $\sigma_V = \sigma S/(S_v V)$ is given by Equation (8) and can now be eliminated to find:

$$S\frac{\partial \sigma}{\partial S} = \sigma \frac{SS_{vv}}{S_v^2} - \sigma \frac{D}{V}, \tag{B3}$$

which yields Equation (9) in a straightforward way.

(b) For a reorganization boundary close to the debt face value ($V_B \uparrow D_T$), the debt becomes a quasi-sure claim and loses any sensitivity to the firm value ($D_v \downarrow 0$). At the same time, the equity becomes a levered position on the firm whose sensitivity with respect to the firm's value increases toward unity ($S_v \uparrow 1$), and more importantly, the convexity of the equity's profile vanishes ($S_{vv} \downarrow 0^+$). In other words, if debt covenants (such as positive net-worth agreements) were restrictive enough to protect the debt face value, the corrective term to the leverage in Equation (9) would fully vanish ($\varepsilon_\ell \downarrow 0^+$).

It is unrealistic, however, to assume all debt to be safety-covenant protected to the degree that the continuous reorganization barrier approaches the debt face value. Nevertheless, an exhaustive sensitivity analysis of barrier options shows that comparable results hold in a wide range of circumstances, whether they be exogenous or endogenous. For example, when exogenous market conditions lead to a re-leveraging of the firm ($V \downarrow V_B$), the probability of hitting the reorganization boundary is going to absorb all the equity time value, thus leading to $S_v \uparrow 1$ and $S_{vv} \downarrow 0^+$. As a result, the corrective term ε_ℓ will vanish as well. Similarly, endogenous conditions such as a debt refinancing process with an extension of the debt horizon ($T \uparrow \infty$), or a significant increase in the firm's riskiness ($\sigma_V \uparrow \infty$), will lead to similar results for ε_ℓ.

More generally, Table 6 reports values for ε_ℓ computed with one of the simplest closed-form model nested in my structural framework. Leland's (1994) structural model presents the advantage of allowing for an endogenous setting of the bankruptcy boundary V_B and generating not only convex but concave profiles for equity, while providing an analytic solution for the equity value function at the same time. To allow for comparison, numerical assumptions are those from Toft and Prucyk (1997). The maximal bankruptcy boundary corresponds to the case of debt protected by strict net-worth covenants. In this case, $V_B^{\max}$ exceeds the debt principal and equity is therefore a concave function which leads to small, negative values for ε_ℓ. For a large range of bankruptcy boundaries around the critical boundary V_B^* (linear equity), equity remains a quasi-linear function of the firm's asset value, thus ensuring the lowest values for ε_ℓ compared to the corporate leverage. Finally, the minimal bankruptcy boundary $V_B^{\min}$ corresponds to the case of an endogenous boundary. In this optimal configuration for equity holders, it takes low asset values and deep states of financial distress for the equity convexity to produce values of ε_ℓ of the same magnitude as the corporate leverage.

Table 6. Numerical magnitude of the financial leverage adjustment ε_ℓ

This table reports the numerical magnitude of ε_ℓ in Leland's (1994) structural model as a function of the firm's asset value V for 5 different bankruptcy boundaries: (a) endogenous bankruptcy ($V_B^{\min} = 39.25$), (b) convex equity ($V_B = 55.00$), (c) linear equity ($V_B^* = 65.00$), (d) concave equity ($V_B = 85.00$), (e) strict net-worth covenant ($V_B^{\max} = 111.11$). The size of ε_ℓ compared to the corresponding corporate leverage ℓ is also reported. Pricing assumptions are those of Toft and Prucyk (1997): annual debt interest charge $C = 8$, corporate tax rate $\tau = 0.35$, bankruptcy costs $\alpha = 0.1$, payout rate $\delta = 0.082$, $r = 0.08$ per annum, $\sigma_V = 0.20$.

		Bankruptcy boundary									
		$V_B^{\min} = 39.25$		$V_B = 55.00$		$V_B^* = 65.00$		$V_B = 85.00$		$V_B^{\max} = 111.11$	
V	ℓ	ε_ℓ	ε_ℓ/ℓ	ε_ℓ	ε_ℓ/ℓ	ε_ℓ	ε_ℓ/ℓ	ε_ℓ	ε_ℓ/ℓ	ε_ℓ	ε_ℓ/ℓ
40	0.71	0.49	*0.69*	-	-	-	-	-	-	-	-
60	0.63	0.28	*0.44*	0.06	*0.09*	-	-	-	-	-	-
80	0.56	0.18	*0.32*	0.09	*0.16*	0	0	-	-	-	-
100	0.50	0.12	*0.24*	0.07	*0.14*	0	0	-0.08	*(0.15)*	-	-
120	0.45	0.08	*0.18*	0.05	*0.11*	0	0	-0.10	*(0.23)*	-0.07	*(0.15)*
140	0.42	0.06	*0.14*	0.04	*0.09*	0	0	-0.10	*(0.24)*	-0.15	*(0.36)*
160	0.38	0.05	*0.13*	0.03	*0.08*	0	0	-0.09	*(0.23)*	-0.17	*(0.45)*
180	0.36	0.04	*0.11*	0.02	*0.07*	0	0	-0.07	*(0.21)*	-0.17	*(0.47)*
200	0.33	0.03	*0.09*	0.02	*0.06*	0	0	-0.06	*(0.19)*	-0.16	*(0.48)*

Appendix C. Proof of Lemma 1

In the sequel, I simplify the argument of Hagan and Woodward (1999) to prove Equation (12) by singular perturbation theory. I make the assumption of a *separable* local volatility surface: $\sigma(S_t,t) \equiv \alpha(t)\sigma(S_t)$. For the sake of notational simplicity, I will assume zero interest rates. The stock pays no dividends, which implies a zero drift under the risk-neutral probability measure associated with the money market account. The stock price diffuses according to the dynamics:

$$dS_t = \alpha(t)\sigma(S_t)S_t dW_t. \tag{C1}$$

The undiscounted risk-neutral value $C(S,t) = \mathbb{E}\{(S_T - K)^+|S\}$ of a European-style call option with strike K and time to maturity T evolves according to the Black-Scholes-Merton partial differential equation (PDE):

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2(S)S^2\frac{\partial^2 C}{\partial S^2} = 0, \tag{C2}$$

subject to appropriate boundary and terminal conditions.

- *Re-scaling procedure.* Denoting $f : x \longmapsto x\sigma(x)$ and $\varepsilon := f(K) \ll 1$, I introduce the following change of variables:

  $$\tau := \int_t^T \alpha^2(u)du, \qquad x := \frac{S-K}{\varepsilon}, \tag{C3}$$

  in order to re-scale the call value as $\widetilde{C}(\tau,x) := C(t,S)/\varepsilon$. The new PDE in the variables (τ,x) verified by the re-scaled call value is as follows:

  $$-\frac{\partial \widetilde{C}}{\partial \tau} + \frac{1}{2}\frac{f^2(K+\varepsilon x)}{f^2(K)}\frac{\partial^2 \widetilde{C}}{\partial x^2} = 0. \tag{C4}$$

  Expanding in power series of ε, we note that:

  $$f^2(K+x\varepsilon) = f^2(K)\left(1 + 2\frac{f'(K)}{f(K)}x\varepsilon\right) + O(\varepsilon^2). \tag{C5}$$

  Substituting in Equation (C4), the PDE can now be written at first order in ε:

  $$\frac{\partial \widetilde{C}}{\partial \tau} - \frac{1}{2}\frac{\partial^2 \widetilde{C}}{\partial x^2} = \nu x\varepsilon\frac{\partial^2 \widetilde{C}}{\partial x^2} + O(\varepsilon^2), \tag{C6}$$

  where $\nu := f'(K)/f(K)$. Expanding the re-scaled price $\widetilde{C}$ in power series of ε as $\widetilde{C}^0 + \varepsilon\widetilde{C}^1 +$

$O(\varepsilon^2)$, we are led to solve the following system of PDEs at first order in ε:

$$\begin{cases} \dfrac{\partial \widetilde{C}^0}{\partial \tau} - \dfrac{1}{2}\dfrac{\partial^2 \widetilde{C}^0}{\partial x^2} = 0, & \widetilde{C}^0(0,x) = x^+, \\ \dfrac{\partial \widetilde{C}^1}{\partial \tau} - \dfrac{1}{2}\dfrac{\partial^2 \widetilde{C}^1}{\partial x^2} = \nu x \dfrac{\partial^2 \widetilde{C}^0}{\partial x^2}, & \widetilde{C}^1(0,x) = 0. \end{cases} \tag{C7}$$

- *Solving the re-scaled problem.* Standard techniques apply to solve the first heat-like PDE. The solution for $\widetilde{C}^0$ is given by:

$$\widetilde{C}^0(\tau,x) = xN\left(\frac{x}{\sqrt{\tau}}\right) + \sqrt{\frac{\tau}{2\pi}} e^{-x^2/2\tau}, \tag{C8}$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \widetilde{C}^0}{\partial x} = N\left(\frac{x}{\sqrt{\tau}}\right), \qquad \frac{\partial^2 \widetilde{C}^0}{\partial x^2} = \frac{e^{-x^2/2\tau}}{\sqrt{2\pi\tau}}, \qquad \frac{\partial \widetilde{C}^0}{\partial \tau} = \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}. \tag{C9}$$

In the same way, the solution for $\widetilde{C}^1$ is given by:

$$\widetilde{C}^1(\tau,x) = \nu x \tau \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}, \tag{C10}$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \widetilde{C}^1}{\partial \tau} = \frac{(\nu x\tau + \nu x^3)e^{-x^2/2\tau}}{4\tau\sqrt{2\pi\tau}}, \qquad \frac{\partial^2 \widetilde{C}^1}{\partial x^2} = \frac{(-3\nu x\tau + \nu x^3)e^{-x^2/2\tau}}{2\tau\sqrt{2\pi\tau}}. \tag{C11}$$

Moreover, we notice that:

$$\widetilde{C}^1(\tau,x) = \tau \nu x \frac{\partial \widetilde{C}^0}{\partial \tau}. \tag{C12}$$

Substituting Equation (C12) in the re-scaled price expansion of $\widetilde{C}$, we obtain the solution for the re-scaled price at first order in ε:

$$\widetilde{C}(\tau,x) = \widetilde{C}^0(\tau,x) + \varepsilon\tau\nu x \frac{\partial \widetilde{C}^0}{\partial \tau} + O(\varepsilon^2) = \widetilde{C}^0\left(\tau + \varepsilon\tau\nu x + O(\varepsilon^2), x\right). \tag{C13}$$

- *Solving for the option price in the physical space.* The unscaled call price may then be deduced as follows:

$$C(t,S) = \varepsilon\widetilde{C}(\tau,x) = \varepsilon\widetilde{C}^0(\tau(1+\varepsilon\nu x)+O(\varepsilon^2),x) = \widetilde{C}^0\left(\varepsilon^2\tau(1+\varepsilon\nu x)+O(\varepsilon^4),\varepsilon x\right). \quad \text{(C14)}$$

Noting that $\varepsilon x = S-K$, we obtain the option price with respect to physical variables:

$$C(t,S) \approx \widetilde{C}^0(\tau^*, S-K), \quad \text{(C15)}$$

where $\tau^* \simeq \varepsilon^2\tau\left(1+\nu(S-K)\right)$. We also note that $\varepsilon = f(K)$ may be developed around the midpoint $(K+S)/2$ for spot prices close to the call strike K:

$$f^2(K) = f^2\left(\frac{K+S}{2}\right)\left(1+\frac{f'\left(\frac{K+S}{2}\right)}{f\left(\frac{K+S}{2}\right)}(K-S)\right)+o(K-S), \quad \text{(C16)}$$

which gives at leading order:

$$\tau^* = \tau f^2\left(\frac{K+S}{2}\right)+O(K-S). \quad \text{(C17)}$$

- *The Black-Scholes-Merton case.* The preceding whole line of reasoning may be applied to the pure Black-Scholes model, which means performing the same calculations for the following stock price dynamics:

$$dS_t = \widehat{\sigma}_K S_t dW_t, \quad \text{(C18)}$$

where $\widehat{\sigma}_K$ is the constant Black-Scholes implied volatility at strike K and expiry T. In this specific case we note that f is the identity function while $\tau = \widehat{\sigma}_K^2(T-t)$, $\nu = 1/K$ and $\varepsilon = K$. Applying Equation (C15) with the previous parameters, the Black-Scholes price is then given by $\widetilde{C}^0(\tau^*_{BS}, S-K)$ where we have at leading order:

$$\tau^*_{BS} \simeq \widehat{\sigma}_K^2(T-t)\left(\frac{K+S}{2}\right)^2+O(K-S). \quad \text{(C19)}$$

- *Linking local volatility with implied volatility.* As the option price observed in the market is both given by the local volatility model (C15) and the Black-Scholes model, we can write:

$$\widetilde{C}^0(\tau^*, S-K) = \widetilde{C}^0(\tau^*_{BS}, S-K). \tag{C20}$$

As $\widetilde{C}^0$ is strictly increasing in the re-scaled time to maturity variable τ, we thus obtain $\tau^* = \tau^*_{BS}$. Substituting Equations (C17) and (C19) in this last relationship, we get at leading order the following relationship which is valid for stock prices in the vicinity of the strike price:

$$\widehat{\sigma}_K^2(T-t) \simeq \sigma^2\left(\frac{K+S}{2}\right)\int_t^T \alpha^2(u)du. \tag{C21}$$

This is Equation (12) in the case $\alpha \equiv 1$.

Appendix D. Proof of Proposition 4

In the CEV framework, the instantaneous variance of stocks returns is $v_t = \sigma^2_{CEV} S_t^{\beta-2}$ and its percentage change is as follows:

$$\frac{\mathrm{d}v_t}{v_t} = \frac{(\beta-2)\sigma^2_{CEV} S_t^{\beta-3}\mathrm{d}S_t}{\sigma^2_{CEV} S_t^{\beta-2}} = (\beta-2)\frac{\mathrm{d}S_t}{S_t}. \tag{D1}$$

The parameter $\beta-2$ appears to tie the percentage changes in the variance to the percentage changes in the share price. It is thus recognized as the variance-equity isoelasticity.

The CEV diffusion coefficient, σ_{CEV}, has the dimension of $S_t^{1-\beta/2}/\sqrt{t}$. We account for this fact by introducing an effective log-normal volatility σ_0 with the standard dimension of $1/\sqrt{t}$, and by positing:

$$\sigma_{CEV} = \sigma_0 S_0^{1-\beta/2}. \tag{D2}$$

The CEV local volatility is then given by:

$$\sqrt{v_t} = \sigma_0 \left(\frac{S_t}{S_0} \right)^{\frac{\beta}{2}-1}, \tag{D3}$$

which provides the logarithmic slope of the CEV local volatility surface:

$$\frac{\partial \sqrt{v_t}}{\partial \ln(S)} = \frac{\beta - 2}{2} \sqrt{v_t}. \tag{D4}$$

Since the CEV local volatility is time-homogeneous, the implied volatility skew, $\widehat{\Sigma}_{CEV}$, is easily obtained as half of the logarithmic slope of the CEV local volatility (see Lemma 1) given by Equation (D4). As a result:

$$\widehat{\Sigma}_{CEV} = \frac{\beta - 2}{4} \sqrt{v_t}. \tag{D5}$$

It is then possible to equate the structural implied volatility skew (13) with the CEV implied volatility skew (D5) to obtain Equation (15).[17]

Appendix E. Proof of Lemma 2

A natural option structure matching the moments of default swap instrument is the *credit risk reversal* (Ilinski, 2003), built from long out-of-the-money put options and short at-the-money call options. The long out-of-the-money put option is intended to replicate the default swap payoff on the occurrence of a credit event, that is, upon a jump to zero of the stock price. Simultaneously, the at-the-money call option is intended to provide the exposure to the third moment of the implied volatility surface. A specific choice for the geometry of the credit risk reversal structure offers no entry cost and as little convexity as possible between the two option exercise prices.[18] This last

[17]To get the final connection between the CEV framework and the firm's financial leverage, it could be tempting to equate Equations (9) and (D4) for local volatility, in order to obtain a straightforward link between the CEV characteristic exponent β and the debt-to-asset ratio ℓ. As already explained, however, the local volatility surface is not model-free and its logarithmic slope has no reason to be a market invariant across models. I therefore need to resort to a stronger market invariant observed for stock options, namely the typical asymmetry of the implied volatility surface when quoted in the Black-Scholes-Merton model (1973).

[18]It is still possible to use more complex structures, such as combinations of risk reversal or put spreads, to match the higher-order sensitivities of the default swap instrument more closely.

feature ensures an approximate static replication of the default swap instrument.[19]

To match the first two moments of a *binary* default swap, let us show that once the put strike, K_p, has been chosen arbitrarily, the call strike, K_c, and the put (resp. call) quantity n_p (resp. n_c) should be chosen as follows:

$$\begin{cases} K_c = F_T^2/K_p, \\ n_p = F_T/K_p, \\ n_c = -1. \end{cases} \tag{E1}$$

Hedging the structure with forward contracts, I can assume no dividends, no carrying costs as well as no implied volatility skew for the sake of simplicity. The Black-Scholes formulae can be used to calculate the upfront cost of the credit risk reversal as follows:

$$\begin{aligned} n_pP - C &= [n_pK_pN(d_1) - n_pF_TN(d_2) - F_TN(d_1) - K_cN(d_2)]e^{-rT} \\ &= [(n_pK_p - F_T)N(d_1) - (n_pF_T - K_c)N(d_2)]e^{-rT} \\ &= 0, \end{aligned} \tag{E2}$$

where $d_1 = \ln(F_T/K_c)/(\sigma\sqrt{T}) + \sigma\sqrt{T}/2$ and $d_2 = d_1 - \sigma\sqrt{T}$. Similarly, the convexity γ of the credit risk reversal is zero since:[20]

$$\frac{\gamma_c}{\gamma_p} = \frac{N'(d_1)}{N'(d_2)} = \exp\left(-\frac{\ln(F_T/K_c) + \ln(K_p/F_T)}{2}\right) = \exp\left(\frac{2\ln n_p}{2}\right) = n_p. \tag{E3}$$

The expected payoff of the delta-hedged credit risk reversal upon a default event appears to be tightly constrained by its geometry. In case of jump to zero of the stock price, the delta-hedged credit risk reversal pays off the put notional n_pK_p minus its initial delta δF_T in cash:

$$\text{JtD} = F_T - \delta F_T = (1 - n_p\delta_p + \delta_c)F_T, \tag{E4}$$

where δ_p (resp. δ_c) is the initial hedge ratio of the put (resp. call). Denoting δ_p^0 (resp. δ_c^0) the delta

[19] A binary default swap instrument is an instrument making a single payment of 1\$ in case of a default event.

[20] The Black-Scholes convexity is $\gamma_p = \gamma_c = N'(d_1)/(S_0\sigma\sqrt{T})$ where $N'(x) = \exp(-x^2/2)/\sqrt{2\pi}$.

of the put (resp. call) option struck at F_T, the call-put parity yields $\delta_c^0 - \delta_p^0 = 1$. With a strike K_p sufficiently close to F_T, we have $n_p\delta_c^0 - \delta_c^0 \simeq 1$. Substituting into Equation (E5) yields:

$$\text{JtD} \simeq [n_p(\delta_p^0 - \delta_p) - (\delta_c^0 - \delta_c)]F_T. \tag{E5}$$

Recall that the sensitivity of the delta is then given by $\partial\delta/\partial K = -\gamma F_T/K$ in the Black-Scholes model. Applying this general result for K_p and K_c sufficiently close to F_T, a first-order Taylor expansion yields:

$$\delta_p^0 - \delta_p \approx -F_T\overline{\gamma}_p \ln(F_T/K_p) \qquad \left(\text{resp.} \quad \delta_c^0 - \delta_c \approx -F_T\overline{\gamma}_c \ln(F_T/K_c)\right), \tag{E6}$$

where $\overline{\gamma}_p$ (resp. $\overline{\gamma}_c$) is the average gamma between F_T and K_p (resp. K_c). Substituting into Equation (E5), the expected loss amount upon default turns out to depend explicitly on the log-distance between the strikes:

$$\text{JtD} \simeq \overline{\gamma}F_T^2 \ln(K_c/K_p), \tag{E7}$$

where $\overline{\gamma} = n_p\overline{\gamma}_p = \overline{\gamma}_c$ is the average convexity between the strikes.

In the presence of an implied volatility skew $\sigma_p > \sigma_{ATM} > \sigma_c$, the upfront premium of the credit risk reversal has to be adjusted for the put (resp. call) implied volatility σ_p (resp. σ_c). At first order, the adjustment to the premium is:

$$\text{RR} \simeq (\sigma_p - \sigma_{ATM}) \times n_p\nu_p - (\sigma_{ATM} - \sigma_c) \times \nu_c, \tag{E8}$$

where ν_p (resp. ν_c) is the put (resp. call) sensitivity to volatility[21] calculated at σ_{ATM}. Using the fact that $n_p\nu_p = n_p\sigma_{ATM}\gamma_p F_T^2 T = \sigma_{ATM}\gamma_c F_T^2 T = \nu_c$, the annualized premium is:

$$\frac{\text{RR}}{T} \simeq (\sigma_p - \sigma_c)\sigma_{ATM}\overline{\gamma}F_T^2. \tag{E9}$$

[21]The Black-scholes sensitivity to the implied volatility σ is given by $\sigma\gamma F_T^2 T$.

Finally, the fair spread of a binary default swap instrument may be assimilated to the annualized premium to be paid for protection against the expected loss amount. Dividing Equations (E7) and (E9), the fair spread is given by:

$$\sigma_{ATM} \cdot \frac{\sigma_p - \sigma_c}{\ln(K_c/K_p)}, \tag{E10}$$

which is Equation (16).

Appendix F. Proof of Proposition 5

Recall from Equation (D3) that the CEV local variance of stock returns can be written as:

$$v_t = \sigma_0^2 \left(\frac{S_0}{S_t} \right)^{2-\beta}. \tag{F1}$$

Inserting the CEV formulation of the implied volatility skew given by Equation (D5) into the left-hand-side of Equation (F1) yields:

$$\frac{4}{|\beta - 2|} \times \sqrt{v_t} \times |\widehat{\Sigma}_{CEV}| = \sigma_0^2 \left(\frac{S_0}{S_t} \right)^{2-\beta}. \tag{F2}$$

Assuming that the best proxy for the instantaneous volatility $\sqrt{v_t}$ is the at-the-money implied volatility, $\widehat{\sigma}_{ATM}$, the left-hand side of Equation (F2) can now be connected to the company's risk premium r_t on its debt appearing in the economic model linking credit spreads with implied volatilities derived in Lemma 2. As a consequence, the volatility leverage effect is found to imply the constant elasticity of debt risk premia relative to stock prices:

$$r_t = \text{const} \cdot \left(\frac{S_0}{S_t} \right)^{2-\beta}, \tag{F3}$$

where the constant is homogeneous to the log-normal variance σ_0^2.

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