Systemic Risk from Interbank Credit Markets?

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Abstract

This theoretical model analyzes the impact of interbank credit markets on systemic risk in the economy. Starting with a single bank's balance sheet, portfolio equilibria in financial markets are derived. As interbank activities are important for individual bank's portfolio management, we focus on a flow mechanism in interbank credit provision and identify a new source of systemic risk, apart from bank runs and direct contagion mechanisms. In a stochastic model of interbank market dynamics we can identify a potential dynamic instability. Defining the probability of interbank market stability as market resilience, the volatility of reserve flows may threaten the resilience of interbank markets, and by this the stability of the financial system. Central Bank policies can only stabilize the interbank market or substitute the interbank credit flows, while the risk comes from stochastic volatility shocks in other markets. Consequently, the importance of system-wide, macroprudential policies is stressed to support and ensure the resilience of the financial system.

EFM classification codes: 370, 560, 570

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1 Introduction

In September 2008, the U.S. investment bank Lehman Brothers had to apply for insolvency due to write-offs in the course of the financial crisis. This default of one single bank has first caused write-offs in interconnected banks' balance sheets and then spread not only within the financial system of the U.S. but even around the globe. Since then, research gained increasingly interest in the role of interbank markets and the consequences of interbank activities. The observed enormous domino effects of bank defaults has led to growing interest in financial interconnectedness in interbank networks and contagion channels (see, amongst others, Heider et al., 2015; Mistrulli, 2011; Vollmer and Wiese, 2014; Rünstler, 2016), which can ultimately result in a systemic threat (cf. Acemoglu et al., 2015). In 2018, the 10th anniversary of Lehman Brothers insolvency has taken place. Although the consequences of the crises led to a vast literature on how to identifiy, measure and alleviate systemic risk as well as first real world applications and implementations in the context of macroprudential policies, the banking system is still not resilient. What could trigger systemic risk in a single bank's balance sheet? This study provides an explanation based on the bank's portfolio management and the dynamics in the interbank credit market.

The definition of systemic risk is not clear-cut. In general, it deals with a harmed financial system, which could endanger overall economic activity (Di Cesare and Rogantini Picco, 2018) and as such it is multifarious and difficult to capture (Hansen, 2014). Early surveys by De-Bandt and Hartmann (2000) up to a recent contribution of Di Cesare and Rogantini Picco (2018) cover a vast range e.g. of systemic risk indicators, empirical studies and macroprudential policies but leave a blank space where the impact of interbank credit markets on systemic risk should be covered theoretically. While research on systemic risk is clearly dominated by empirical studies (Silva et al., 2017), we contribute to the theoretical background analysis of systemic risk factors in the financial system using a portfolio management approach with a focus on interbank credit markets.

In this work, we refer to the systemic risk definition of Danielsson (2002), who defines it as an endogenous risk, which is "inside" the system and arises due to the interaction of market participants. According to Paltalidis et al. (2015) the interbank loan market is one source of systemic risk, while Iori et al. (2006: 526) name interbank lending as "*one form of safety net for individual banks*". This trade-off between mutual insurance and systemic risk will also be the focus of this work. Systemic risk can arise when information or news shocks reduce the interbank credit supply, which can ultimately lead to a complete dry out of the interbank credit market. This interbank illiquidity was breath taking for the private market and provoked the central bank to take its role as lender of last resort in the post-crisis years. Furthermore, the ECB adopted unconventional monetary policies to avoid the transmission of its policies to be stuck in the interbank markets, and acted as interbank intermediary (cf. Giannone et al., 2012).

Related theoretical models on interbank lending, the central bank and systemic risk are rarely found. For instance, Freixas and Jorge (2008) investigate the effects of interbank market imperfections on the transmission of monetary policy. Their theoretical model takes a real shock into consideration, which induces a need for liquidity by individual banks. The liquidity shortage can be settled either by the central bank or by interbank market borrowing, which is characterized by asymmetric information. Freixas and Jorge (2008) show that these frictions in the interbank market cause an equilibrium with rationing in the credit market. Heider and Hoerova (2009) investigate the functioning of unsecured and secured interbank markets in the presence of credit risk in a portfolio choice model. More recent research of Hauck and Neyer (2014) or Bucher et al. (2017) refutes interbank market frictions to impede the monetary policy transmission. Hauck and Neyer (2014) analyze the impact of frictions in the European interbank market on bank loan supply and suppose to reverse the intermediary function assumed by the ECB by reduced interbank market participation costs or reduced liquidity costs, which arise from transactions in the interbank market. Bucher et al. (2017) model a bank run and the respective liquidity management of a single bank, which maximizes profits and minimizes liquidity costs under financial imperfections in the interbank market. Biondi and Zhou (2017) develop an agents-based model of the interbank market in a bilateral transaction analysis.

However, the dynamic effects of an individual bank's interbank lending activities within its portfolio management, which leads the interbank market in a stable or instable equilibrium, are not yet explored. In our theoretical analysis, the interbank market constitutes a well-functioning element in the resilient financial system, when everything goes as expected, but an exogenous shock can lead in an extreme case to a complete shut down of the interbank credit market. We present a theoretical framework, which identifies the importance of interbank lending dynamics for the resilience of the financial system. Our results suggest the importance of safeguarding the smooth provision of interbank credits over time to ensure the functioning of the financial system and to avoid the emergeny operations of the Central Bank.

To avoid a repetition of disastrous events, a wide range of macroprudential policies was born. Amongst others, they should safeguard the efficient liquidity transfer in the interbank market (see e.g. Furfine, 2001, Acharya and Yorulmazer, 2008). While interbank markets in general allow for risk-sharing across banks (Bhattacharya and Gale, 1987), interbank exposures can also result in systemic risk. This was also observed in the financial crisis, when interbank market rates increased significantly and transaction volumes tended to zero. Macroprudential - indicators of systemic risk distinguish between the cross-sectional dimension (contribution of individual banks) and the time-dimension (procyclicality of systemic risk) (Borio, 2003) Could macro-prudential policies improve the resilience of the financial system? To give an answer we acknowledge the importance of the interbank credit market and add to the discussion on the efficacy of macroprudential policies by combining the cross-sectional with the time dimension in an illustration of the dynamics in the interbank credit market as potential source of systemic risk.

2 Private banks

In this model, the (domestic) banking sector consists out of many competing banks. Banks operate in competitive markets for loans, deposits, and reserves (cf. Bianchi and Bigio, 2017). The banking sector can be sub-divided into two groups. Each group is represented by one bank, i for one group or j for the other group. Moreover, in this model banks behave symmetric. Thus, we condense our descriptions to the activities of the representative bank i of one group and start with the portfolio management of bank i.

2.1 Bank i's assets

For each bank we want to model the optimal portfolio strategy. Therefore we look at the **asset side** of the balance sheet, which records the use of funds. In the balance sheet of bank i (BS_i) we distinguish between three groups of assets, central bank reserves, credits, and securities. As first step, we introduce these assets with their respective properties. Each asset has a distinct risk-return combination, where risk refers to the effects on portfolio risk. For instance, central bank reserves in general reduce portfolio risk, but generate cost; credits and securities in general increase portfolio risks, but also earn returns. The following sub-sections will describe the function of each asset in the portfolio of the representative bank i.

Liquidity management: Liquid asset of central bank reserves First, the individual bank i decides on the amount of central bank (CB) reserves to hold (M_{CBi}) to cover its differentiated need of riskless liquidity. The central bank provides credits via refinancing operations (simplified as "reserves" or central bank money M_{CB}), which reduce the portfolio risk within bank i's balance sheet. Bank i's need of CB reserves consists out of the fulfillment of its minimum reserve requirement, which is imposed by the CB, and the so-called "autonomous factors", such as cash demanded by bank customers. Depending on its current stock of CB reserves, bank i can demand CB reserves, which are provided at the official interest rate i_R. On the other hand, bank i can deposit excess liquidity overnight at the CB. The market for central bank reserves can be regarded as the primary money market for the representative bank (e.g. Affinito, 2013). We assume that the central bank's reserves (M_{CB}) are always available and thus, portfolio risk decreases with increasing central bank reserves in the individual bank's balance sheet.

If we look at the ECB several instruments are used to manage the Euro Area banking sector's liquidity (see ECB, 2011, for a detailed description of its instruments), which can be reduced to three key interest rates. First, liquidity is mainly provided via the main refinancing operation (MRO), which traditionally satisfies approximately 74% of the liquidity needs of the Euro Area's banking sector (ECB, 2002). These credit transactions are subject to several rigidities, e.g.: (i) they do not take place daily but weekly, which reduces the availability of liquidity, (ii) their duration is constrained to two weeks, (iii) they are collateralized, which inherits a loss of interest profit, and could increase the marginal lending costs of individual banks in the Euro Area (see Hauck and Neyer, 2014). Another segment of CB liquidity demand is served by marginal lending facilities, which is an overnight market for liquidity from the CB. However, its costs are higher as compared to the MRO. The interest rate policy is complemented by the deposit facility, which represents an overnight deposit of our representative bank i, held at the CB.

In this model, we focus on the MRO due to its main application. Moreover, we abstract from the rigidities of the MRO to keep the analysis traceable and refer to the bank's deposits at the central bank as a central bank credit repayment.

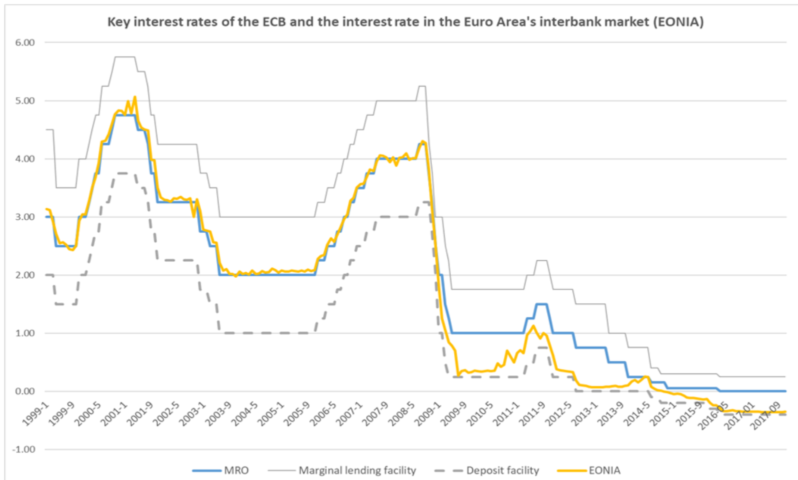
To provide an illustrative example, figure 1 shows the interest rate in the Euro Area's interbank market, EONIA, which is an index of the average unsecured interbank lending rate of selected banks. The EONIA generally lies above the CB deposit facility but below the marginal lending facility and is usually close to the MRO. Since May 2009, however, the EONIA is very low, shortly above the deposit facility. The spread between the CB's MRO rate and the interbank credit rate increased after the insolvency of Lehman Brothers in 2008, with lower returns from lending in the interbank market.

This is reasonable due to massive expansionary monetary policy of the CB, which led to a liquidity surplus within the aggregate interbank market. Consequently, banks decided to invest in the deposit facility of the CB or alternatively to provide credit in the interbank market (cf. Affinito, 2013).

Investment opportunity 1: Credits

In case of a liquidity surplus, the bank can apply its funds into different uses. Each bank carries out an investment decision on credit provision. Credits can be given to the bank's customers (the private sector) or to other private banks. Each of these investment opportunities can be substituted for one another. However, they have different characteristics with respect to their return, i.e. interest rate and risk. Generally, both investments earn a positive return and bear some risk.

First, bank i can provide its excess liquidity to other banks via an **interbank credit**. The amount of interbank credits given from bank i to bank j is denoted by CR_{IB_i}, where the first subindex refers to the credit providing bank (credit from bank i), and the second to the loan receiving bank (credit to bank j). As all j banks are identical all credits given by bank i to the j banks are identical such that we can simplify the notation to $CR_{IB_{ij}} = CR_{IB_i}$. Bank j's demand of an interbank credit, i.e. borrowing in the IB market is denoted as CR_{IB}^D. Real interbank markets are characterized by over-the-counter trade, in which contracts and terms of conditions are negotiated individually among lending and borrowing banks (e.g. Vollmer and Wiese, 2014; Bianchi and Bigio, 2017). The terms of loan provision foresee an individual interest rate, which in this model is denoted by i_{IB}. Interbank loans are not insured and often uncollateralized



Sources: ECB and https://www.emmi-benchmarks.eu/euribor-eonia-org/eonia-rates.html (EONIA)

Figure 1: CB interest rate and IB interest rate

(Furfine, 2001), which increases the lender's risk exposure. Furthermore, the lending bank does not have to hold minimum reserves on the interbank liquidity provided, which increases the associated risk. We will investigate the flow mechanism of interbank credits in detail in the following section *3.6.*

The second kind of **credits** a bank can provide are loans to the non-bank **private market** (CR_{Pi}) at the interest rate i_P. We will simplify with a general interest rate on loans to the private sector, as usual e.g. in overdraft facilities. Moreover, we will abstract from required minimum reserves, to which the bank would be obliged in reality due to the fact that it would constitute only a minor position in the balance sheet. The credit provision by bank i to the private market simultaneously creates a non-bank customer deposit (D_{Pi}), which constitutes a liability of bank i.

Investment opportunity 2: Securities

In addition to providing credits, bank i can invest in a number of different securities, such as domestic government bonds (B_i), foreign bonds (F_i) or shares (K_i). Each of the three representative groups has a distinct risk-return combination.

Domestic bonds: Bank i decides to hold a number of government bonds B_i. Bonds are defined in the standard way. They have an infinite life time, provide a given fixed interest payment of one unit, and can be traded at current market price P_B. Thus, we obtain the standard relation between bond prices and returns i_B, $P_B = \frac{1}{i_B}$. Consequently, their value in bank i's balance sheet equals $P_B B_i$. Due to the fact that we refer to government bonds, they are regarded as low risk investment with a presumably relatively low return.

An additional risk-return refers to an investment in **foreign bond**s (F_i), which are denominated in foreign exchange and subject to exchange rate volatility. This adds an additional risk compared to domestic bonds. Bank i decides on the value of foreign bonds, which are traded at the foreign currency price of $P_F = \frac{1}{i_F}$, where i_F is the foreign return. Consequently, the value of foreign bonds in domestic currency in bank i's balance sheet is $eP_F F_i$, where e denotes the exchange rate.

Third, the bank can invest in shares at the **stock market**. Bank i holds a number of shares K_i traded at the stock-market at price P_K. Therefore, value of stock shares in bank i's balance sheet is $P_K K_i$.The return of a share i_K is assumed to consist of a fixed dividend payment ρ plus the change in the share's price $\dot{P}_K$, $i_K = \frac{\rho}{P_K} + \frac{\dot{P}_K}{P_K}$. Share prices are highly volatile, and thus, they incur high risks as well as high returns. In static equilibrium however, no changes of prices are expected and the return of the share equals the reciprocal value of the equilibrium share price $i_K = \frac{\rho}{P_K} = \frac{1}{P_K}$. In summary, we assume that all securities do not fundamentally differ. They function in the same way but they address different segments of risk-return relations.

	share		foreign bond		domestic bond
returns	i_K	>	i_F	>	i_B
effect on portfolio risk	σ_K	>	σ_F	>	σ_B

Finally, a sub-segment of the stock market are shares, which are issued by bank i , K_{Bi}. Denoting P_{Kii} as price of bank i's share equity capital of bank i consists of stocks K_{Bii} of value $P_{Kii} K_{Bii}$. In line with existing literature, such as Adrian and Shin (2009), we assume that an increase in the general value of (purchased) securities increases equity in parallel, $P_{Kii} = P_K$. The value of equity capital moves in accordance with general share values.

2.2 Bank i's balance sheet

After having considered single entities of the asset side of the balance sheet of an individual bank i, the complete balance sheet is structured as follows.

Bank-i

assets	liabilities
(i) balance with central bank M_{CB_i}	
(ii) (interbank) lending to banks CR_{IB_i}	(vii) (interbank) liabilities to other banks L_{IB_i}
(iii) lending to non-banks CR_{Pi}	(viii) liabilities to non-banks D_P
(iv) government bonds $P_B B_i$	
(v) foreign bonds $eP_F F_i$	(ix) capital EK
(vi) shares $P_K K_i$	
balance sheet total BS_i	balance sheet total BS_i

The simplified asset side shows the use of funds and contains the following single entities (i) - (vi): Balance with the central bank, lending of bank i in the interbank market, lending of bank i to its customers in the private market, as well as the value of bank i's investment in government bonds, foreign bonds, and shares. We abstract e.g. from shares in affiliated enterprises, fiduciary assets or tangible assets, to keep the focus on interbank market activities instead of diluting the analysis with a high scope of other assets of minor importance. On the contrary, the simplified liability side depicts the sources of funds, which are used on the asset side. For instance, the credit provision to non-banks (iii) on the asset side is matched by the deposits of non-banks (viii) on the liability side of the balance sheet. This applies also to interbank activities, where bank i's balance sheet records the loans given, which means bank i's lending in the interbank market (ii) and/or loans taken, which means bank i's borrowing in the interbank market (vii). Therefore, bank i's liquid assets stem from the central bank, the interbank market, where L_{IBi} is the liquidity available to bank i, or its customers (D_p). While the bank is obliged to hold official minimum reserves on the liabilities to non-banks, liabilities to other banks do not require a liquidity back-up.

In our portfolio choice model we need to analyze the asset side of bank i's balance sheet. Hence, equation 1 describes the sum of the asset side of bank i' s balance sheet, which equals the balance sheet total.

$$M_{CB_i} + CR_{IB_i} + CR_{P_i} + p_B B_i + eP_F F_i + P_K K_i = BS_i \tag{1}$$

2.3 Bank i's optimal portfolio choice

The commercial bank tries to choose an optimal portfolio structure that maximizes expected utility of profits earned from this portfolio.

Bank i's portfolio profits and risks

The bank's portfolio profits π_i are simply the sum of elements that earn minus the cost-elements. In particular, a bank i earns from lending to other banks ($i_{IB}CR_{IB_i}$), providing credits to the private non-banking sector ($i_P CR_P$), holding government bonds ($P_B B_i^D$), holding foreign bonds ($P_F F_i^D e$), and holding stocks ($P_K K_i^D$). The portfolio generates costs for holding CB reserves ($-i_R M_{CB_i}^D$). By contrast, private deposits (D_p) are assumed to be non-interest-bearing and therefore do not enter the profit function. We do not assume any costs of portfolio management or bank production of services.

$$\pi_i = -i_R M_{CB_i} + i_{IB} CR_{IB_i} + i_P CR_P + i_B B_i^D + i_F F_i^D e + i_K K_i^D \tag{2}$$

However, banks not only earn profits with their portfolio, they also take risks. Each asset stands for a different risk-return characteristic. However, in principal, all earning assets ($CR_{IB_i}^S, CR_p, B_i^D, F_i^D, K_i^D$) are defined by positive contributions to profits, but also to the portfolio risk σ; each in it's particular way.

$$\sigma_i \left(M_{CB_i}, CR_{IB_i}, CR_p, B_i^D, F_i^D, K_i^D \right), \quad \frac{d\sigma_i}{dM_{CB_i}} < 0 \tag{3}$$

$$\frac{d\sigma_i}{dCR_{IB_i}} > 0, \quad \frac{d\sigma_i}{dCR_p} > 0, \frac{d\sigma_i}{dB_i^D} > 0, \quad \frac{d\sigma_i}{dF_i^D} > 0, \quad \frac{d\sigma_i}{dK_i^D} > 0$$

Bank i's expected utility of asset portfolio

Banks are risk averse and maximize the expected utility of the profits of their portfolio.

$$V_i = V_i \left(\pi_i, \sigma_i \right) \tag{4}$$

Bank i's optimization problem

The portfolio optimization problem can now be described as

$$\begin{aligned} \max \quad &: \quad V_i = V_i \left(\pi_i, \sigma_i \right) \\ s.t. \quad &: \quad M_{CB_i} + CR_{IB_i} + CR_{P_i} + P_B B_i^D + eP_F F_i^D + P_K K_i^D = BS_i \end{aligned} \tag{5}$$

Bank i's asset demand functions:

From the bank's optimization problem we can derive the bank's asset demand function as described in following proposition.

Proposition 1 *Bank i's optimal asset demand functions: Problem (5) and the respective FOC implicitly define bank i's asset demand functions for the 3 groups of assets*[1]

$$\begin{array}{rll} \textit{liqui. ass. (i)} & \textit{CB money} & M_{CB_i}^D = m_{CB_i}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}}) BS_i \\ \textit{credits (ii)} & \textit{IB credits} & CR_{IB_i} = cr_{IB_i}(\overset{(+)}{i_R}, \overset{(+)}{i_{IB}}, \overset{(-)}{i_P}) BS_i, \\ \textit{(iii)} & \textit{pr. credits} & CR_{P_i} = cr_{Pi}(\overset{(-)}{i_{IB}}, \overset{(+)}{i_P}) BS_i \\ \textit{securities (iv)} & \textit{gov. bonds} & B_i^D = b_i^D(\overset{(-)}{i_P}, \overset{(+)}{i_B}, \overset{(-)}{i_K}) BS_i \\ \textit{(v)} & \textit{for. bonds} & F_i^D = f_i^D(\overset{(-)}{i_B}, \overset{(+)}{i_F}) BS_i \\ \textit{(vi)} & \textit{shares} & K_i^D = \kappa_i^D(\overset{(-)}{i_B}, \overset{(+)}{i_K}) BS_i \end{array} \tag{6}$$

For a proof see appendix 6.2.

3 Financial markets

For simplicity we assume that asset markets are dominated by banks' financial activities. Thus, we assume that activities of the private sector are marginal and do not need to be explicitly modeled. We also assume that there are two representative banks, bank i and bank j, needed to describe all other banks and

[1] Small letters indicate shares of the balance sheet, e.g. $m_{CB_i} = M_{CB_i}/BS_i$.

show symmetries. All other banks behave like these two explicitly described banks.

3.1 Central bank money market

In this static model the CB money market is characterized by the CB's liquidity supply, which is demanded by the banking sector, i.e. our two representative banks i and j.

For simplicity, we reduce the CB's policy to its main instrument: the short-term refinancing rate (MRO). We assume, that this rate is determined according to an inflation target; it uses the Taylor rule.

Thus, the CB's policy is to determine the CB refinancing rate, shortly the reserve rate $i_R = const$. Thus, the market for CB money is $M_{CB_i} + M_{CB_j} = M_{CB}$. The full description the CB money market is summarized in the asset market equation system (7-i).

3.2 Interbank credit market

We assume a perfect interbank credit market without rigidities, asymmetries or frictions.

The banking sector can be sub-divided into two groups according to their position in the interbank credit market. Each bank can be either a lender or a borrower in the interbank credit market. Each group is represented by one bank, where i stands for a banking group, which will supply its liquidity surplus on the interbank credit market and acts as a lender (CR_{IB_i}), whereas j stands for the group with a liquidity deficit, which demands for credit on the interbank market, and as such represents the borrowers in the interbank market ($CR^D_{IB_j}$).

Bank j's demand in the interbank credit market is CR^D_{IBj}.This demand is matched by the credit supply of bank i. Thus, the interbank credit market is $CR_{IB_i} - CR^D_{IBj} = 0$. The full description of this market is summarized in the asset market equation system (7-ii).

3.3 Private credit markets

All banks give credits to the private sector. These credits have to match the private sector's demand for credits and hence simultaneously created deposits. Thus, the market for private credits and deposits $CR_{P_i} + CR_{P_j} = D_P$. The full description of this market is summarized in the asset market equation system (7-iii).

3.4 Security markets

The security markets can be described rather symmetrically.

Bonds (B, F), and stock markets (K):

Domestic bonds are demanded by all banks. The supply of the number of bonds is exogenous and the price is determined by market forces, $B_i^D + B_j^D = P_B B$. For foreign bonds we assume that the portfolio demand is described by values F_i^D in domestic currency. The value of foreign bonds in the domestic market segment is the given value of foreign bonds in foreign currency $(P_F F)$ revalued in domestic currency, $F_i^D + F_j^D = eP_F F$. The stock market is modelled symmetrically to the bond market as demand for stocks and a given number of shares and the stock price is the result of the respective market process, $K_i^D + K_i^D = P_K K$. The full description of this market is summarized in the asset market equation system (7-iv,-v and vi).

3.5 Complete financial market system and equilibrium

The complete system of financial markets can be summarized as

$$
\begin{array}{rl}
\text{markets for} & \\
\text{CB money (i)} & m_{CB_i}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_i + m_{CB_j}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_j = M_{CB} \\
\text{IB credits } i \text{ (ii)} & cr^S_{IB_i}(\overset{(+)}{i_{IB}}, \overset{(-)}{i_P})BS_i = cr^D_{IB_j}(\overset{(+)}{i_R}, \overset{(-)}{i_{IB}})BS_j \\
\text{priv. credits (iii)} & cr_{P_i}(\overset{(-)}{i_{IB}}, \overset{(+)}{i_P}, \overset{(-)}{i_B})BS_i + cr_{P_j}(\overset{(-)}{i_{IB}}, \overset{(+)}{i_P}, \overset{(-)}{i_B})BS_j = D_P(\overset{(-)}{i_P}) \\
\text{bonds (iv)} & b_i^D(\overset{(-)}{i_P}, \overset{(+)}{i_B}, \overset{(-)}{i_K})BS_i + b_j^D(\overset{(-)}{i_P}, \overset{(+)}{i_B}, \overset{(-)}{i_K})BS_j = P_B B \\
\text{foreign bonds (v)} & f_i^D(\overset{(-)}{i_B}, \overset{(+)}{i_F})BS_i + f_j^D(\overset{(-)}{i_B}, \overset{(+)}{i_F})BS_j = eP_F F \\
\text{shares (vi)} & \kappa_i^D(\overset{(-)}{i_B}, \overset{(+)}{i_K})BS_i + \kappa_j^D(\overset{(-)}{i_B}, \overset{(+)}{i_K})BS_j = P_K K
\end{array}
\tag{7}
$$

As we have six asset markets we can determine six endogenous variables, namely $i_{IB}, i_P, i_B, P_K, e, M_{CB}$. Each of these variables depends on the vector of exogenous variables, namely (i_R, i_F, B, F, K).

Proposition 2 *Financial market system (7) implicitly defines a vector of equilibrium rates of return and asset prices, namely $\tilde{\imath}_{IB}, \tilde{\imath}_P, \tilde{\imath}_B, \tilde{P}_K, \tilde{e}, \tilde{M}_{CB}$. Each of these variables depends on the vector of exogenous variables, namely (i_R, i_F, B, F, K); such as $\tilde{\imath}_{IB_i} = \tilde{\imath}_{IB_i}(i_R, i_F, B, F, K)$, $i_P = i_p(...)$, ..., and $\tilde{M}_{CB} = M_{CB}(i_R, i_F, B, F, K)$.*

For a proof see appendix 6.2

This portfolio equilibrium was realized in the individual bank's balance sheet perspective. Bank i takes individual portfolio decisions to realize this stock equilibrium in the end of the period. In the next section, we investigate a dynamic flow mechanism, which will lead the bank in the portfolio equilibrium.

3.6 Flow mechanism in the interbank credit market

Several studies name a credit boom as potential root cause of crises (see, e.g. Claessens, 2014, Dell'Arriccia et al., 2008). We acknowledge the importance of the interbank credit market and focus on the interbank credit market in aggregate (displayed in the financial market system in equation (ii)). On this market interbank credit demand and supply are matched. The interbank credit market is a very short term, overnight market, which induces a roll-over risk w.r.t. the portfolio management of the individual bank i. Departing from the notion, which is generally used in the literature, in this context "roll-over risk" is not associated with a sole interest rate risk to the borrowing bank but with a risk of the lender with the need to adjust its portfolio management. In this model, the banking sector can be sub-divided into two groups according to their position in the interbank credit market, where banks of group i supply their liquidity surplus and act as lenders, whereas banks of group j hold a liquidity deficit, demand credit on the interbank market, and as such represent the borrowers in the interbank market. Therefore, the interbank credit supply of bank i is modelled in a periodic flow concept, which can guide the financial system in a stable or instable equilibrium. This dynamic adjustment process is investigated below.

In the previous section we have already determined the stationary equilibrium of the asset stock. Thus, at the end of all flow adjustments there must be a stationary stock equilibrium as described in section 3.5. Further, the flow process has to be consistent with the asset stock equilibrium, such that the flow process will eventually end in the stock equilibrium. Thus, we need to model the flow adjustment accordingly. We model an adjustment period and assume that stock adjustments are not instantaneously but take some days for the flow process to terminate in the new stock equilibrium.

In general, the change in the stock of interbank credits recorded in bank i's balance sheet's asset side (i.e. interbank credit supply) ($\dot{C}R_{IB_i}$) is determined by newly created or revolved credit contracts as well as a dissolving of interbank credit provision . More precisely, $\dot{C}R^g_{IB}$ are newly created or revolved interbank credits that generate a gross increase in the stock of interbank credits, while dissolving existing interbank credits ($\dot{C}R^{\delta}_{IB_i}$) lead to a gross reduction of the stock of interbank credits of bank i. Consequently, the change in the stock of interbank credit provision recorded in bank i's balance sheet is decomposed into the creation of new credits and reduction of existing interbank credits.

$$\dot{C}R_{IB_i}(t) = \dot{C}R^g_{IB_i}(t) - \dot{C}R^{\delta}_{IB_i}(t) \tag{8}$$

First, the creation of interbank credits, $\dot{C}R^g_{IB_i}$: With the notion of being on the way to the new equilibrium the bank has an idea of the equilibrium credit demand $CR^D_{IB_i}(\overset{(+)}{i_R}, \overset{(-)}{i_{IB_i}})$. Simultaneously the bank knows about the current level of its credit supply CR_{IB_i}. As long as the credit demand exceeds the credit supply at equilibrium, the bank provides more credits to the market. In order to speed up this adjustment process the difference between the stock of credit demand $CR^D_{IB_i}$ and the already created supply CR_{IB_i} translates into a newly generated credit flow.

$$\dot{C}R^{g1}_{IB_i}(t) = b\left(CR^D_{IB_i}(i_R, i_{IB_i}) - CR_{IB_i}(t)\right),$$

where b is a parameter that translates the excess demand in stocks into a credit creating flow activity. Further, some of the existing stock of credit relations with other banks can be easily used for a revolving mechanism. The decision of bank i to roll-over credit to similar favorable conditions as in the last overnight credit does not take place automatically but with respect to the targeted portfolio equilibrium and the respective interest rate in the interbank credit market (i_{IB_i})

$$\dot{C}R^{g2}_{IB_i}(t) = \rho(i_{IB_i})\left(CR_{IB_i}(t)\right)^{1-\alpha},$$

with $\alpha < 1$ and ρ as a parameter that refers to the traditional relationships to borrowing banks (see e.g. Cocco et al., 2009; De la Motte et al., 2010; Afonso et al. 2013 on relationship lending). Roll-over credits increase underproportionately

Second, dissolving existing interbank credits, $\dot{C}R^{\delta}_{IB_i}$: While bank i is on its way to the new equilibrium stock of interbank credits, interbank lending is very short term and consists of mostly overnight credits, that are renewed or not. Thus, at every point in time t (every day) a large number of these credits is repaid. Again, for simplicity we assume that all credits are overnight credits and paid back every day. As we assume no defaults in normal interbank relations the full currently existing stock CR_{IB_i} at time t is dissolved

$$\dot{C}R^{\delta}_{IB_i}(t) = -CR_{IB_i}(t).$$

Net credit dynamics Total credit dynamics can now be described by bringing the two components together

$$\dot{C}R_{IB_i}(t) = b\left(CR^D_{IB_i}(i_R, i_{IB_i}) - CR_{IB_i}(t)\right) + \rho\left(CR_{IB_i}(t)\right)^{1-\alpha} - CR_{IB_i}(t). \tag{9}$$

Equation (9) is a non-linear, non-homogeneous differential equation in CR_{IB_i}.The equation

$$\dot{C}R_{IB_i}(t) = bCR^D_{IB_i} + \rho CR^{1-\alpha}_{IB_i} - (1+b)\,CR_{IB_i}$$

is graphically described in figure 2.and the properties of this differential equation are given by

$$\frac{d\dot{C}R_{IB_i}}{dCR_{IB_i}} = \rho\left(1-\alpha\right)CR_{IB_i}^{-\alpha} - \left(1+b\right)\begin{cases} > 0 \text{ for } CR_{IB_i}^{\alpha} > \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ = 0 \text{ for } CR_{IB_i}^{\alpha} = \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \\ < 0 \text{ for } CR_{IB_i}^{\alpha} < \rho\left(1-\alpha\right)\left(1+b\right)^{-1} \end{cases}$$

$$\frac{d\dot{C}R_{IB_i}^2}{d\left(CR_{IB_i}\right)^2} = \rho\left(-\alpha+\alpha^2\right)CR_{IB_i}^{-\alpha-1} < 0$$

A qualitative analysis of the dynamics of the process indicates that under the described standard conditions we have a stable dynamic process leading to the final stationary portfolio equilibrium (3.5). For this dynamic process we can also derive the stationary equilibrium at the end of the process when $CR_{IB_i}^D(\overset{(+)}{i_R},\overset{(-)}{i_{IB_i}}) - CR_{IB_i}\left(t\right) = 0$, and explicitly determine the steady state equilibrium credit supply[2]

$$CR_{IB_i} = \rho(\overset{(-)}{i_R},\overset{(+)}{i_{IB_i}})^{\frac{1}{\alpha}} \tag{10}$$

With (10) we have also shown that the portfolio equilibrium is consistently described.

Further, in order to give credits central bank reserves are needed to be available to the respective bank i. In other words, the adjustment towards a new portfolio equilibrium with a higher level of credit supply for bank i also means an adjustment of the portfolio equilibrium value of central bank money M_{CB}. However, in this context we are more interested in the interbank credit flow mechanism and thus want to focus on this mechanism. The flow management of reserves requires that the bank needs a reserve flow of $R\left(t\right) = \dot{C}R_{IB_i}\left(t\right)$ to give all credits described in the credit dynamics. In other words, as reserves are necessary for credit provision, the reserve flows $R\left(t\right)$ can be a direct credit constraint for credit expansion. Therefore, interbank credit managers determine the path of credit creation and plan the respective reserve flows for each time. However, as reserve in- and outflows are stochastic the planned reserves have to count for these stochastic element. If x is a random in- or outflow of reserves, with $E\left[x\right] = 0$ and $R^p\left(t\right)$ are the planned reserves, then credit managers would according to the expected availability $E\left[x\right]$ and reserve requirements $R\left(t\right)$ plan

$$R^p\left(t\right) = \dot{C}R_{IB_i}\left(t\right) - E\left[x\right] \tag{11}$$

[2]

$$\begin{aligned}
\dot{C}R_{IB_i}\left(t\right) &= 0 = bCR_{IB_i}^D + \rho(\overset{(-)}{i_R},\overset{(+)}{i_{IB_i}})CR_{IB_i}^{1-\alpha} - \left(1+b\right)CR_{IB_i} \\
0 &= \rho(\overset{(-)}{i_R},\overset{(+)}{i_{IB_i}})CR_{IB_i}^{1-\alpha} - CR_{IB_i} \\
1 &= \rho(\overset{(-)}{i_R},\overset{(+)}{i_{IB_i}})CR_{IB_i}^{-\alpha} \\
CR_{IB_i} &= \rho(\overset{(-)}{i_R},\overset{(+)}{i_{IB_i}})^{1/\alpha}
\end{aligned}$$

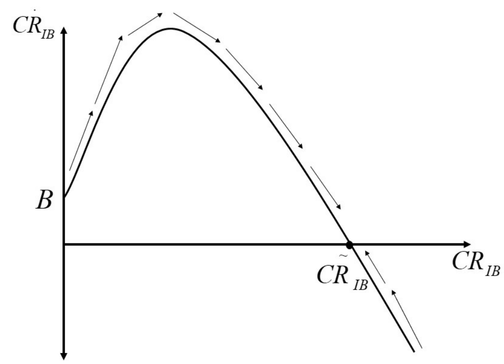


Figure 2: Dynamics and stability in the interbank credit market under normal conditions

Managers have a consistent plan, and therefore, in expected values ($E[x] = 0$) the plan works, and figure 2 represents the dynamics as long as expectations are fulfilled,

$$R^p(t) + E[x] = \dot{C}R^p_{IB_i}(t) = b\left(CR^D_{IB_i} - CR_{IB_i}(t)\right) + \rho\left(CR_{IB_i}(t)\right)^{1-\alpha} - (1+b)\,CR_{IB_i}(t)\,.$$

However, real conditions are sometimes different than the expected values. Therefore, we now describe what happens with the adjustment process if at random the bank realizes reserve values other than the expected. From the above discussion we know that existing stochastic reserves can restrict the credit creation $R(t) \geqq \dot{C}R_{IB_i}(t)$.Therefore, the manager planned reserves $R^p(t)$ and expected that the stochastic element of resource flows is zero ($E[x] = 0$). However, $x = R^R$ is stochastic and therefore in reality x may randomly realize the value $R^R < 0$ within the period of adjustment.Then, $R(t) = R^p(t) + R^R$and real credit creation is restricted to $\dot{C}R_{IB_i}(t) = R^p(t) + R^R$. As the planned credit creation at that point in time is $\dot{C}R^p_{IB_i}(t)$, the dynamics of the adjustment process change below this new reality. Adjusting to this real world observation $R(t) = R^p(t) + R^R < R^p(t)$ the bank will have to switch to a new adjustment path because of the realized reserve constraint. The new adjustment path is now[3]

[3]

$$\begin{aligned} \dot{C}R_{IB_i}(t) &= R(t) = R^p(t) + R^R \\ \dot{C}R_{IB_i}(t) &= R^R + b\left(CR^D_{IB_i} - CR_{IB_i}(t)\right) + \rho\left(CR_{IB_i}(t)\right)^{1-\alpha} - (1+b)CR_{IB_i}(t) \end{aligned}$$

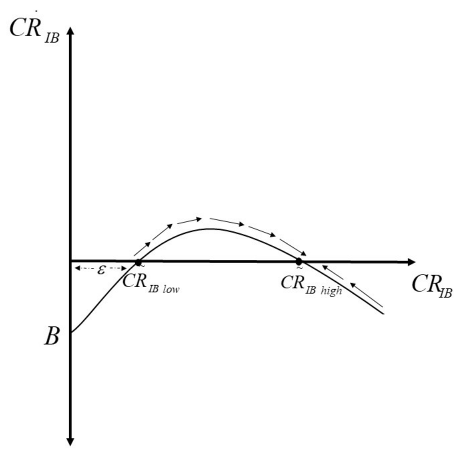


Figure 3: Dynamics and instability in the interbank market

$$\dot{C}R_{IB_i}(t) = R^R + bCR^D_{IB_i} + \rho' CR^{1-\alpha}_{IB_i} - (1+b)\,CR_{IB_i}$$

As long as the random shock $R^R < 0$ is sufficiently small in absolute terms $|R^R|$ the intersection with the vertical axis in figure (2) remains positive, and we have no general change in the adjustment dynamics. However, if in absolute terms the random shock $R^R < 0$ is sufficiently large $B = R^R + bCR^D_{IB_i}$ may turn negative, and the properties of the dynamic adjustment process may change. Figure 3 shows this new path for a negative B. Before we can discuss the implications for the dynamics we need to formally identify the new, low steady state and the reactions of this steady state with respect to changes in variables. We determine the low steady state for $CR^D_{IB_i} - CR_{IB_i} > 0$

$$0 = G = R^R + bCR^D_{IB_i} + \rho' CR^{1-\alpha}_{IB_i} - (1+b)\,CR_{IB_i} \tag{12}$$

As this equation cannot be solved explicitly we need to apply the implicit function theorem. Looking at figure 3 and using the implicit function theorem at a local point we can state that equation (12) implicitly defines a function CR_{IB_i} at potentially two equilibrium points, a low equilibrium point $CR^{low}_{IB_i}$, and a high equilibrium point $CR^{high}_{IB_i}$[4]

$$\begin{aligned} CR^{low}_{IB_i} &= CR^{low}_{IB_i}(R^R, ...), \qquad \text{with } \frac{CR^{low}_{IB_i}}{dR^R} > 0 \\ CR^{high}_{IB_i} &= CR^{high}_{IB_i}(R^R, ...), \end{aligned} \tag{13}$$

[4]Implicit function theorem: $0 = G = R^R + bCR^D_{IB_i} + \rho' CR^{1-\alpha}_{IB_i} - (1+b)\,CR_{IB_i}$ implicitly defines a function $CR^{low}_{IB_i} = CR^{low}_{IB_i}(R^R, ...)$ if $\frac{dG}{dCR_{IB_i}} \neq 0$: Thus, taking the derivative of G

In figure 3 we use this new path for a qualitative dynamic analysis. While figure 2 described a global overall stable process with only one equilibrium, figure 3 shows two equilibria. In this figure the high equilibrium is comparable to the one equilibrium in figure 2. We see a locally stable process as the $\dot{C}R_{IB_i}(t)$-*curve* has a negative slope around the high equilibrium. The high equilibrium is a locally stable point. This is different for the low equilibrium. Here, the slope is positive, which implies that the low equilibrium $CR_{IB_i}^{low}$ is locally unstable. As a result we have two dynamic regimes. At points larger than the low equilibrium level $CR_{IB_i}^{low}$ the credit creation process will stable move to the high equilibrium $CR_{IB_i}^{high}$, which is also the final portfolio equilibrium the banks would like to reach. However, if we look at points below the low equilibrium level $CR_{IB_i}^{low}$ the process is unstable and bank i would keep on decreasing credit creation. In this case credit creation of bank i is constrained by a too low reserve inflow $R(t)$ and it may reduce to zero. This brief discussion already allows to see that adjustment processes are no more only stable. If the stochastic shock described by a randomly much lower reserve inflow in the adjustment process is sufficiently large or CR_{IB_i} is still rather low, such that $CR_{IB_i}(t) < CR_{IB_i}^{low}$, the process becomes instable. This critical mechanism is studied in more detail in the next paragraph.

3.7 Resilience of interbank credit creation adjustments

Knowing of unstable credit creation processes in a bank that provides interbank credits, we will now discuss the aggregate process in the interbank market, and some elements that potentially affect the resilience of this market. The term *resilience* in this context stands for the chance of the market to be in a stable regime and automatically return to the stable portfolio equilibrium.

As the realized random reserve flow $x = R^R$ constrains the actual credit creation activity ($R(t) \geqq \dot{C}R_{IB_i}(t)$), and by that determines in (14) the shape of the actual adjustment process

$$d\dot{C}R_{IB_i}(t) = R^R + bCR_{IB_i}^D + \rho' CR_{IB_i}^{1-\alpha} - (1+b)\, CR_{IB_i}, \tag{14}$$

with respect to CR_{IB_i} gives

$$\begin{aligned} \frac{\partial G}{\partial CR_{IB_i}} &= (1-\alpha)\,\rho' CR_{IB_i}^{-\alpha} - (1+b) > 0 \qquad \text{for} \quad CR_{IB_i} < \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}} \\ &= (1-\alpha)\,\rho' CR_{IB_i}^{-\alpha} - (1+b) < 0 \qquad \text{for} \quad CR_{IB_i} > \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}. \end{aligned}$$

Thus, $CR_{IB_i}^{low} = CR_{IB_i} < \left(\frac{\rho'(1-\alpha)}{1+b}\right)^{\frac{1}{\alpha}}$ is the low credit level stationary equilibrium. The derivative of this implicit function is $\frac{\partial G}{\partial R^R} = 1$

$$\frac{dCR_{IB_i}}{dR^R} = -\frac{\frac{\partial G}{\partial R^R}}{\frac{\partial G}{\partial CR_{IB_i}}} = -\frac{1}{\frac{\partial G}{\partial CR_{IB_i}}} < 0$$

.

these random reserve flows must be studied in more detail.

While x is a particular realized value during the period of adjustment, we know more about the random distribution of this shock, and can use this knowledge to describe the probability that the process remains stable for any moment during the adjustment period.

First, as described before x is a random in- or outflow of reserves, with expectation $E[x] = 0$ and $Var = \sigma^2$. We specify the random distribution by choosing a normal distribution

$$X \sim \mathcal{N}(0, \sigma^2) \quad \text{with} \quad f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}. \tag{15}$$

Thus, we may be able to determine probabilities for each value x of the random in- or outflow of reserves during the adjustment period. However, we need to find out more about the probability of instability.

Second, according to figure 3 a process becomes instable if the low equilibrium $CR_{IB_i}^{low}$ is to the right side of the current credit stock $CR_{IB_i}(t)$ such that $CR_{IB_i}(t) \leq CR_{IB_i}^{low}$. As analyzed in the above section, $CR_{IB_i}^{low}$ is determined by the realized reserve flow x. From (13) we know that for a particular value x the derivative of the low equilibrium with respect to x is $\frac{CR_{IB_i}^{low}}{dx} > 0$. Using a local and linear approximation at $CR_{IB_i}^{low}$ we can rewrite $CR_{IB_i}^{low}$ as the linear function

$$CR_{IB_i}^{low}(x) = g(x) = mx - c \tag{16}$$

With this linear approximation and the random distribution (15) we can now arrive at proposition[5].

Proposition 3 *(probability of instability) Using the approximation in figure 3 for the low equilibrium $CR_{IB_i}^{low}(x)$ and random distribution (17) for the flow of random reserves, we can derive the probability that $\varepsilon \leq CR_{IB_i}^{low}$, and thus that the adjustment process becomes instable*

$$\mathbb{P}(f(X) \geq \varepsilon) = \int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx. \tag{17}$$

Therefore, if the current position of credit creation in figure 3 is $CR_{IB_i}(t) = \varepsilon$, we can now give a probability that the low equilibrium is to the right of ε,

---

[5]

$$\begin{aligned}
\mathbb{P}(f(X) \geq \varepsilon) &= \int_{x \leq \varepsilon} f_X(g^{-1}(x)) \frac{1}{|g'(g^{-1}(x))|} dx \\
&= \int_{x \geq \varepsilon} f_X\left(\frac{1}{m}(x-c)\right) \frac{1}{|m|} dx \\
&= \int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx
\end{aligned}$$

and thus we can give a probability of randomly falling in the instable region of the adjustment process. In other words, we determine the probability of an instable adjustment process.

If bank i is a representative credit provider in our system, all mechanisms described hold for the complete interbank credit market. Therefore, it is interesting to identify elements that would increase or decrease the probability of market instability, and identify the elements that affect *market resilience* (defined as the probability of market stability).

First, if ε increases the probability of instability decreases

$$\frac{d}{d\varepsilon}\mathbb{P}(f(X) \geq \varepsilon) = \frac{d}{d\varepsilon}\int_{\varepsilon}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(x-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} dx = -\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\left(\frac{1}{m}(\varepsilon-c)\right)^2}{2\sigma^2}} \frac{1}{|m|} < 0. \quad (18)$$

That is, close to the high equilibrium the probability of instability is low. The market is rather resilient.

Second, it is interesting to note that not necessarily the level of the reserve flows determines the probability of falling into an instable region of the adjustment process. If we take the derivative of (17) with respect to the variance σ^2 the probability of instability increases

$$\frac{d}{d\sigma^2}\mathbb{P}(f(X) \geq \varepsilon) = \frac{1}{|m|}\frac{1}{\sqrt{2\pi\sigma^2}}\frac{\varepsilon}{2\sigma^2} e^{-\frac{\varepsilon^2}{2m^2\sigma^2}} > 0. \quad (19)$$

Thus, high volatility of reserve flows, which may be generated even in other financial markets,[6] affects stability and resilience of the interbank credit market. When market volatility increases, shocks to individual banks in the group i are assumed to be correlated and thus, non-diversifiable, which implies that lenders in the interbank credit market are affected simultaneously. The interbank credit market may fall into an instable adjustment mechanism with more volatile reserve flows. That is, the interbank market is more likely to become instable if shock or developments somewhere in the financial system cause higher volatility of reserve flows.

4 Model implications

The theoretical considerations of the model presented above add to the general understanding of mechanisms and dynamics in the interbank market. The results show that dynamics in the interbank credit market can guide the financial system in an instable equilibrium and lead to systemic risk.

[6] Reasons for higher volatility in other financial markets are shown, for instance, by of Daniel et al. (1998). Their model shows that overconfidence of investors can cause stock market bubbles and increased volatility in asset markets.

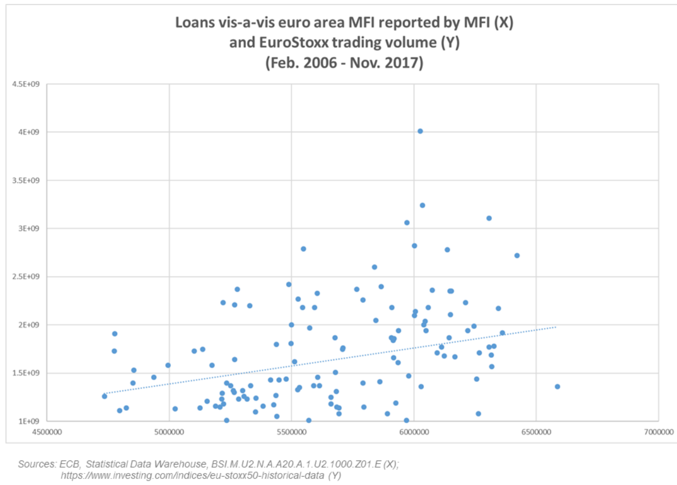


Figure 4: Relation between stock of IB credits and the trade volume of the EuroStoxx

Volatility in asset markets We assumed that the volatility in asset markets trigger these dynamics and as such, are well-connected to the liquidity management of the individual bank. The theoretical argument has been proven e.g. by Daniel et al. (1998) who show that overconfidence of investors can cause increased volatility in returns and asset markets, which subsequently would be transferred in the bank's balance sheet and portfolio management. In the same vein, Chuang and Lee (2006) investigate the overconfidence hypothesis in an empirical framework and find some evidence on it. In order to be more precise with respect to our model, we make a first check on the relation between other market's volatility and the interbank credit market with the help of empirical data. Data available as indicator of interbank loans are drawn from the ECB's dataset "Balance sheet items", which refers to the aggregated balance sheet of Euro Area MFIs. We take "Loans vis-a-vis euro area MFI reported by MFI excluding ESCB in the euro area (stock)" as proxy for our purposes. In figure 4 we show the relation of interbank credits and the trade volume of EuroStoxx stock market shares. Looking at this diagram we can suggest that the trade volume in the stock market seems to be connected with the interbank credit market ($corr$(IB credits, trading volume) = 0.290) . That is, active trading in the stock market goes along with an increase in interbank credit activities. The interbank credit market is related to balancing stock market trading activities. The relation of trade volume and asset price volatility is described in figure 5 ($corr$(trading volume, volatility index) = 0.511) Those two positive correlations suggest a positive correlation between the interbank credit provision and volatility, which has, however, to be further investigated and proved.

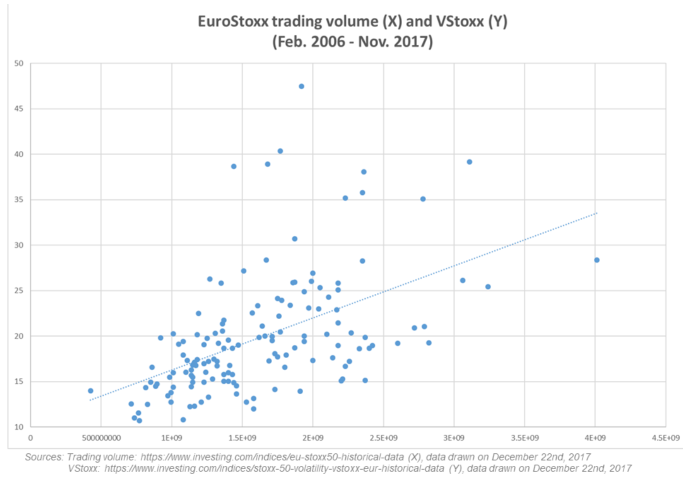


Figure 5: Relation of trade volume of the EuroStoxx and volatility index of the EuroStoxx

Central Bank policies In the extreme case, when banks are stuck in the low equilibrium loan provision in the interbank credit market could be completely driven down, the interbank credit market could dry out entirely. When this threat is present the Central Bank faces a trade-off between a bank's rescue, which simultaneously inherits a moral hazard issue, and the insolvency of a bank, which could cause contagious effects in the interbank market. In 2008, the ECB decided to rescue single banks and thereby the financial system on the whole. The Central Bank prevented the realization of systemic risk, at the cost of a moral hazard incentive in the banking sector (see e.g. Farhi and Tirole, 2012 on moral hazard and systemic bailouts). Monetary policy can try to revitalize the interbank credit market with the help of additional emergency liquidity supply at the cheapest refinancing rates. However, these emergency operations of the Central Bank can distort the traditional channels of monetary policy transmission (see, e.g. ECB (2011) for an overview on transmission channels in the Euro Area or Bernanke and Gertler (1995), Mishkin (1995), Modigliani (1971) on single transmission channels). However, if the Central Bank has to supply massive emergency liquidity, this could also boost or reduce all traditional channels with unforeseeable effects.

To safeguard the liquidity within the interbank credit market, either the Central Bank has to serve excess demand for credits in the interbank market or the shock on reserves could be reduced by the avoidance of volatility and bubbles. Therefore, macroprudential policies, risk indicators of triggers and amplifiers of volatility and early warning systems are important to be able to take counter measures.

Macroprudential policies Macroprudential policies were born to mitigate systemic risk. Several tools (e.g. a countercyclical capital requirement or a Liquidity Coverage Ratio (LCR)) have been implemented to ensure the liquidity of a bank at all times. In the light of our model, they should also safeguard the ability of a bank to provide interbank credits at all times, where a limited cash inflow constrained its credit provision. However, the discussion on the effectiveness of these tools is still underway

Further tools measure each bank's contribution to systemic risk within the concept of "Systemic Expected Shortfall" (see Archaya et al., 2010) or assess the interconnectedness across banks and systemic risk at the bank level, e.g. with respect to bank size, loan growth, leverage or loan maturity (the concept of $\Delta CoVar$). However, while these measures monitor an individual bank's contribution to systemic risk, they do not focus on the risk inherent in financial market, namely the interbank credit market but also other financial markets, which can transfer risk via volatility.

In our view, the risk of asset volatility with respect to its effects in the bank's portfolio and liqudity management, is still underrepresented in macroprudential policies and could gain further attention.

5 Conclusion

The lack of theoretical investigations of systemic risk has induced us to develop a theoretical model of the pivotal role of interbank credit markets in the financial system. Furthermore, we paid special attention to the interbank credit market and its dynamic adjustment processes without loosing track of the individual bank's portfolio management.

Starting with a single bank's balance sheet, we derived the portfolio equilibria in the respective financial markets. As interbank activities are important for individual bank's portfolio management, we focused on a flow mechanism in interbank credit provision and identified a new source of systemic risk, apart from bank runs and direct contagion mechanisms. This source is an instable equilibrium, which can be realized due to a stochastic process, which defines reserves available for interbank credit provision. Defining the probability of interbank market stability as market resilience, the volatility of reserve flows caused somewhere in the financial system may threaten the resilience of interbank markets, and by this of the financial system.

The resilience of the financial system could improved when liquidity in the interbank market is monitored, e.g. with the help of indicators, such as the liquidity coverage ratio of macroprudential policies. However, the risk is rooted in the volatility of reserve flows, which are caused by stochastic volatility shocks in other markets. The Central Bankcan try to stabilize the interbank credit market or substitute interbank credit flows with the help of emergency operations. While this provides quick relief, it could simultanously distort the effectivenss of traditional channels of monetary policy transmission.

Consequently, we stress the importance of system-wide, macroprudential policies, which should pay special attention to the risk of asset volatility and its effects in the bank's portfolio and liqudity management to support and ensure the resilience of the financial system.

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6 Appendix

6.1 A1 Proof of proposition 1: Asset demand function derived from portfolio choice model.

Problem

$$\pi_i = -i_R M_{CB_i} + i_{IB} CR_{IB_i} + i_P CR_P + i_B B_i^D + i_F F_i^D e + i_K K_i^D$$

$$\begin{aligned}
\max &: \quad V_i = V_i\left(\pi_i, \sigma_i\left(M_{CB_i}, CR_{IB_i}, B_i^D, F_i^D, K_i^D\right)\right) \\
s.t. &: \quad M_{CB_i}^D + CR_{IB_i} + CR_{P_i} + P_B B_i^D + eP_F F_i^D + P_K K_i^D = BS_i
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} &= V_i\left(\pi_i, \sigma\left(M_{CB_i}, CR_{IB_i}, B_i^D, F_i^D, K_i^D\right)\right) \\
&\quad -\lambda\left(M_{CB_i} + CR_{IB_i} + CR_{P_i} + P_B B_i^D + eP_F F_i^D + P_K K_i^D - BS_i\right)
\end{aligned}$$

Note: To focus on the importance of the transmission channels in general and to keep the analysis tracable, we assume the exchange rate e to be equal to 1.

Consequently, this yields the following first order conditions of our optimization problem.

F.O.C.

$$(i)\quad A_1 = \frac{d\mathcal{L}_i}{dM_{CB_i}} = -i_R + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial M_{CB_i}} - \lambda = 0,$$

$$\frac{d_i^2\mathcal{L}}{d\left(M_{CB_i}\right)^2} = A_{11} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(-)}{\partial^2 \sigma_i}}{\partial (M_{CB_i})^2} < 0, \qquad \frac{\partial^2 \sigma_i}{\partial (M_{CB_i})^2} > 0$$

$$(ii)\quad A_2 = \frac{d\mathcal{L}_i}{dCR_{IB_i}} = i_{IB} + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial CR_{IB_i}} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(CR_{IB_i}\right)^2} = A_{22} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(+)}{\partial^2 \sigma_i}}{\partial\left(CR_{IB_i}\right)^2} < 0$$

$$(iii)\quad A_3 = \frac{d\mathcal{L}_i}{dCR_{P_i}} = i_P + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial CR_{P_i}} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(CR_{P_i}\right)^2} = A_{33} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(+)}{\partial^2 \sigma_i}}{\partial\left(CR_{P_i}\right)^2} < 0$$

$$(iv)\quad A_4 = \frac{d\mathcal{L}_i}{dB_i^D} = i_B + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial B_i^D} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(B_i^D\right)^2} = A_{44} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(+)}{\partial^2 \sigma_i}}{\partial\left(B_i^D\right)^2} < 0$$

$$(v)\quad A_5 = \frac{d\mathcal{L}_i}{dF_i^D} = i_F + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial F_i^D} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(F_i^D\right)^2} = A_{55} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(+)}{\partial^2 \sigma_i}}{\partial\left(F_i^D\right)^2} < 0$$

$$(vi)\quad A_6 = \frac{d\mathcal{L}_i}{dK_i^D} = i_K + \frac{\partial V_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial K_i^D} - \lambda = 0, \quad \frac{d^2\mathcal{L}_i}{d\left(K_i^D\right)^2} = A_{66} = \frac{\partial V_i}{\partial \sigma_i}\frac{\overset{(+)}{\partial^2 \sigma_i}}{\partial\left(K_i^D\right)^2} < 0$$

$$(vii)\quad A_7 = \frac{d\mathcal{L}_i}{d\lambda} = M_{CB_i} + CR_{IB_i} + CR_{P_i} + P_B B_i + eP_F F_i + P_K K_i - BS_i = 0$$

As we would like to apply the implicit function theorem, which uses the Cramer's rule, we rewrite the system as

$$\begin{pmatrix} \overset{(-)}{A_{11}} & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & \overset{(-)}{A_{22}} & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & \overset{(-)}{A_{44}} & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & \overset{(-)}{A_{55}} & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & \overset{(-)}{A_{66}} & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} dM_{CB_i} \\ dCR_{IB_i} \\ dCR_{P_i} \\ dB_i^D \\ dF_i^D \\ dK_i^D \\ d\lambda \end{pmatrix} = \begin{pmatrix} di_R \\ -di_{IB} \\ -di_P \\ -di_B \\ -di_F \\ -di_K \\ dBS_i \end{pmatrix}$$

Implicit function theorem:

To apply the implicit function theorem we have to show that $|J| \neq 0:$ In order to simplify the calculations we rewrite this equation system and take equation

A_4 (line (iv)) as representing either bonds, foreign bonds or stock shares. Thus, we obtain a reduced system:

$$|J| = \begin{vmatrix} \overset{(-)}{A_{11}} & 0 & 0 & 0 & -1 \\ 0 & \overset{(-)}{A_{22}} & 0 & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & 0 & -1 \\ 0 & 0 & 0 & \overset{(-)}{A_{44}} & -1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

$$= A_{11} \begin{vmatrix} A_{22} & 0 & 0 & -1 \\ 0 & A_{33} & 0 & -1 \\ 0 & 0 & A_{44} & -1 \\ 1 & 1 & 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 0 & A_{22} & 0 & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

$$\begin{aligned} &= A_{11}A_{22} \begin{vmatrix} A_{33} & 0 & -1 \\ 0 & A_{44} & -1 \\ 1 & 1 & 0 \end{vmatrix} + (-1)\, A_{22}(-1) \begin{vmatrix} 0 & A_{33} & 0 \\ 0 & 0 & A_{44} \\ 1 & 1 & 1 \end{vmatrix} \\ &= A_{11}A_{22}A_{44} - A_{11}A_{22}A_{33} - A_{22}A_{33}A_{44} \end{aligned}$$

$$|J| = A_{11}A_{22}A_{44} - A_{11}A_{22}A_{33} - A_{22}A_{33}A_{44} < 0$$

Portfolio adjustments Portfolio adjustments due to changes in exogenous variables can be derived in the usual way. However, we do not go through this procedure for every single variable. We generally assume that direct (own) effects dominate cross effects and thus, the scheme of reactions should be in line with standard reactions and solve according to Cramer's rule:

Example 1 $\frac{dM_{CB_i}}{di_R}$:

$$|J_1| = \begin{vmatrix} \Delta & 0 & 0 & 0 & -1 \\ 0 & \overset{(-)}{A_{22}} & 0 & 0 & -1 \\ 0 & 0 & \overset{(-)}{A_{33}} & 0 & -1 \\ 0 & 0 & 0 & \overset{(-)}{A_{44}} & -1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

$$
\begin{aligned}
&= \Delta \begin{vmatrix} A_{22} & 0 & 0 & -1 \\ 0 & A_{33} & 0 & -1 \\ 0 & 0 & A_{44} & -1 \\ 1 & 1 & 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 0 & A_{22} & 0 & 0 \\ 0 & 0 & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \\ 1 & 1 & 1 & 1 \end{vmatrix} \\
&= \Delta A_{22} \begin{vmatrix} A_{33} & 0 & -1 \\ 0 & A_{44} & -1 \\ 1 & 1 & 0 \end{vmatrix} + (-1)\, A_{22}(-1) \begin{vmatrix} 0 & A_{33} & 0 \\ 0 & 0 & A_{44} \\ 1 & 1 & 1 \end{vmatrix} \\
&= \Delta A_{22} A_{44} - \Delta A_{22} A_{33} - A_{22} A_{33} A_{44}
\end{aligned}
$$

$$
\begin{aligned}
|J_1| &= \left[\Delta \overset{(-)}{A_{22}} \overset{(-)}{A_{44}} - \Delta \overset{(-)}{A_{22}} \overset{(-)}{A_{33}} - \overset{(-)}{A_{22}} \overset{(-)}{A_{33}} \overset{(-)}{A_{44}} \right], \qquad \Delta = 1 \\
|J_1| &< 0
\end{aligned}
$$

$$
\frac{dM_{CB_i}}{di_R} = \frac{|J_1|}{|J|} < 0
$$

In the same way we can calculate all other reactions. However, to save space we do not show all these calculations in this appendix. They may be obtained on request.

6.2 A2 Financial Markets and Equilibrium

Proof of proposition 2: Equilibrium price vector, derived using the implicit function theorem:

From the market system (7) we obtain a system of 6 functions $F_0....F_5$ depending on the 6 endogenous variables i_{IB}, i_P, i_B, e, P_K and M_{CB}. If these equations are linear independent we can apply the implicit function theorem.

$$
\begin{aligned}
\text{CB money (i)} \quad & F_0 = m_{CB_i}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_i + m_{CB_j}(\overset{(-)}{i_R}, \overset{(-)}{i_{IB}})BS_j - M_{CB} = 0 \\
\text{IB credits } i \text{ (ii)} \quad & F_1 = cr^S_{IB_i}(\overset{(+)}{i_{IB}}, \overset{(-)}{i_P})BS_i - cr^D_{IB_j}(\overset{(+)}{i_R}, \overset{(-)}{i_{IB}})BS_j = 0 \\
\text{private credits (iii)} \quad & F_2 = cr_{P_i}(\overset{(-)}{i_{IB}}, \overset{(+)}{i_P}, \overset{(-)}{i_B})BS_i + cr_{P_j}(\overset{(-)}{i_{IB}}, \overset{(+)}{i_P}, \overset{(-)}{i_B})BS_j - D_P(\overset{(-)}{i_P}) = 0 \\
\text{bonds (iv)} \quad & F_3 = b^D_i(\overset{(-)}{i_P}, \overset{(+)}{i_B}, \overset{(-)}{i_K})BS_i + b^D_j(\overset{(-)}{i_P}, \overset{(+)}{i_B}, \overset{(-)}{i_K})BS_j - P_B B = 0 \\
\text{foreign bonds (v)} \quad & F_4 = f^D_i(\overset{(-)}{i_B}, \overset{(+)}{i_F})BS_i + f^D_j(\overset{(-)}{i_B}, \overset{(+)}{i_F})BS_j - eP_F F = 0 \\
\text{shares (vi)} \quad & F_5 = \kappa^D_i(\overset{(-)}{i_B}, \overset{(+)}{i_K})BS_i + \kappa^D_i(\overset{(-)}{i_B}, \overset{(+)}{i_K})BS_i - P_K K = 0
\end{aligned} \tag{20}
$$

To keep the analysis manageable we use only one of the assets which are generally behaving similar. As private credits, domestic bonds, foreign bonds and shares are all assets with positive returns and increasing effects on risk, we describe the system by using only one of these assets as representative for all others. This simplifying assumption reduces the system substantially. Further, a detailed look at the CB money market shows, that this market is recursively related to the system and only required for determining the equilibrating supply of CB reserves. Thus, after rewriting this system we obtain a linear reduced system

$$\begin{pmatrix} \frac{\overset{(+)}{\partial CR_{IBi}}}{\partial i_{IB}} - \frac{\overset{(-)}{\partial CR^D_{IBj}}}{\partial i_{IB}} & 0 \\ 0 & \frac{\overset{(+)}{\partial K_i^D}}{\partial i_K} + \frac{\overset{(+)}{\partial K_j^D}}{\partial i_K} \end{pmatrix} \begin{pmatrix} di_{IB} \\ di_P \end{pmatrix} = \begin{pmatrix} -\frac{\overset{(-)}{\partial CR^D_{IBj}}}{\partial i_R} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \overset{(-)}{F_{11}} & 0 \\ 0 & \overset{(+)}{F_{22}} \end{pmatrix} \begin{pmatrix} di_{IB} \\ di_P \end{pmatrix} = \begin{pmatrix} -\overset{(-)}{F_{1\Delta}} di_R \\ 0 \end{pmatrix}$$

Here the numbering is defined as the first subindex gives the number of the market within equation system 20 and the second subindex gives the number of the endogenous variable according to following definition $1 : i_{IB}, 2 : i_P, 3 : i_P, 4 : e, 5 : i_K$. $F_{1\Delta}, F_{2\Delta}$ is the derivative of F_1 respectively of F_2 with respect to the exogenous variable di_R.

To apply the implicit function theorem $|J| \neq 0$:

Proof.

$$|J| = \begin{vmatrix} \overset{(-)}{F_{11}} & 0 \\ 0 & \overset{(+)}{F_{22}} \end{vmatrix} = \overset{(-)}{F_{11}} \overset{(+)}{F_{22}}$$

The sign turns positive, if we assume that direct effects are in general large in absolute values:

$$|J| = \overset{(-)}{F_{11}} \overset{(+)}{F_{22}} > 0.$$

■