Creditor's holdup and the setting of private appropriation in a control contract between shareholders

H. de La Bruslerie

S. Gueguen[1]

Abstract

Debt is analyzed in relation to the conflict between three parties, a controlling shareholder, outside investors and creditors. We follow Jensen and Meckling's (1976) and Myers' (1977) intuitions that a high leverage may result in excess value appropriation by creditors while at the same time acting to discipline private benefits appropriation. A contingent claim valuation model is used to show that debt is *also* a key governance variable because it can moderate or enhance private benefits and because incentivization triggers a transfer of value to creditors. We show that debt is a complex regulation tool in an agency contract approach, as it is simultaneously an expropriation device and a limitation tool. Debt is a disciplinary tool for shareholders, but to avoid a holdup by creditors, we also need to discipline the disciplinary tool.

JEL: G30/ G32 / G34

Keywords: creditors' holdup, private benefits, controlling shareholders, debt leverage, corporate governance

---

[1]Respectively, Professor of Finance, DRM Finance, University Paris Dauphine, Place du Maréchal de Lattre 75116 Paris, France; e-mail: hlb@dauphine.psl.eu.
Assistant Professor of Finance, University of Cergy-Pontoise, 33 Boulevard du Port, 95011 Cergy-Pontoise, France; e-mail: simon.gueguen@u-cergy.fr.
We would like to thank S. Attaoui, C. At, P. Poncet, P. Six, L. Xu, Y. Yasuda and S. Zhao for their helpful comments and suggestions. A former version under the title "Debt and private benefits appropriation by a controlling shareholder: Introducing a creditors' holdup effect" was presented at internal finance seminars at the University of Lorraine, at the University of Franche-Comté, at Neoma Business School, and in conferences at the 2017 CIG Conference in Lausanne, the 34th annual AFFI conference in Valence and the 2017 WFS Conference in Sardinia.

Creditor’s holdup and the setting of private appropriation in a control contract between shareholders

Abstract

Debt is analyzed in relation to the conflict between three parties, a controlling shareholder, outside investors and creditors. We follow Jensen and Meckling’s (1976) and Myers’ (1977) intuitions that a high leverage may result in excess value appropriation by creditors while at the same time acting to discipline private benefits appropriation. A contingent claim valuation model is used to show that debt is *also* a key governance variable because it can moderate or enhance private benefits and because incentivization triggers a transfer of value to creditors. We show that debt is a complex regulation tool in an agency contract approach, as it is simultaneously an expropriation device and a limitation tool. Debt is a disciplinary tool for shareholders, but to avoid a holdup by creditors, we also need to discipline the disciplinary tool.

1. Introduction

Jensen and Meckling’s (1976) seminal paper analyzes the relationship between entrepreneurs’ ownership stake, non-financial revenues (perquisites, etc.), and debt. Among others, debt is an incentive tool for the managing shareholder. She can choose the asset volatility and divert wealth from the bondholders to the shareholders. However, debtholders are not naïve, and efficient markets will integrate it. Creditors can invest in monitoring, such as implementing debt provisions and limiting the riskiness of the projects. From the viewpoint of the controlling shareholder or the managers, debt may be seen as an expropriation device similar to control enhancement mechanisms. It helps to control more economic resources. This is well known, and it led Jensen and Meckling to develop a theory whereby outside equity holders will monitor the manager–owner who rules the firm. Debt in Jensen’s (1986) framework is considered a disciplinary tool. It is a limiting device to control free cash-flow misuse by managers or controlling shareholders similar to other control enhancement mechanisms.

Jensen and Meckling's initial focus is to look at ownership structure, not financial structure. The agency relationship between creditors and the managing shareholder is not in their scope. This does not mean that it does not exist but that they chose not to develop it. In a footnote, they mention that they do not consider the case to be symmetric to the one-way wealth transfer from creditors to shareholders, in which "*bondholders (...) can force management to take actions which would transfer wealth from the equity holder to the bondholders (...). One can easily construct situations where such actions could make the bondholder better off, hurt the equity holders, and actually lower the total value of the firm". They do not consider such a possibility and recognize that this assumption "allows us to avoid the incentive effect associated with bondholders potentially exploiting stockholders*" (footnote 49, page 339). A large strand of the literature has focused on the capital structure decision integrating agency costs and problems. The basic framework is the agent-principal relationship between managers and shareholders should now refer to creditors as a category of investors who are specifically exposed to financial distress and bankruptcy risk. As stated by Dewatripont and Tirole (1994) "capital structure is a disciplinary device for managers as well as an incentive scheme for outsiders" (page 1049). The latter are both shareholders and debt holders. Myers (1977) has identified the "debt overhang" problem where, in the context of investment, shareholders will not finance a valuable investment if such investment would result in a large increase in the value of existing debt. Shareholders would bear the full cost of investing but, due to the transfer to debtholders, will get only part of the corresponding value creation. "Debt overhang" problem becomes more critical when the firm gets close to bankruptcy. The point has been revisited since then and extended to "effort" as opposed to "under-investment" problems (He 2011). The idea is similar and relies on the fact that an increase in the value of existing debt acts as a tax and thus reduces incentives for shareholders to incur the cost of increasing firm value.

This avenue of research has been developed in the context of private benefits, i.e., the agency conflict between outside investors and a controlling shareholder. Private benefits have been recognized in the managers / global shareholder context with private benefits seized by the agent without any investment in equity (Dewatripont and Tirole 1994; DeMarzo and Sannikov 2006; DeMarzo and Fishman 2007; Lambrecht and Myers 2008). The main result is that private benefits or rents are endogenously determined and interfere with the capital structure.

In this situation, debt may help to extract private benefits, but it may also be a tool to limit and pressure the wealth appropriation by managers or the controlling shareholder. The purpose of this paper is to explore this intuition in a controlling shareholder's private benefits scheme with outside shareholders (La Porta et al. 1998, 1999, 2000) as well as in a managers' perquisite conflict with dispersed shareholders. The difference is that the controlling shareholder both extracts private benefits and largely invests in equity capital. The managers/global shareholders conflict appears as a special case of controlling/outsider shareholder by assuming a null investment in the firm's equity. In these two contexts, debt plays a similar role in the incentivization of the dominant shareholder/manager ruling the firm as it places more resources at the disposal of the controlling shareholders and facilitates tunneling activities (Claessens et al. 2002; Paligorova and Xu 2012; Buchuk et al. 2014; Qian and Yeung 2014). Debt may be seen as an expropriation device similar to control enhancement mechanisms. Referring to leverage leads the bankruptcy risk to be taken into account. This eventuality is implied in the debt contract and is integrated by the shareholders as the probability of distress will constrain both the equity value and private benefit appropriation. Debt may divert additional wealth both to and from the creditors and may condition private benefits appropriation from outside shareholders. It makes the regulation story more complex, as the game is between three parties rather than two (He 2011). As a result, debt is a sophisticated regulation tool in an agency contract approach, as it is both an expropriation device and a limitation tool. Debt is a disciplinary tool for shareholders, but to avoid a holdup, we also need to discipline the disciplinary tool. Here holdup describes the appropriation by the incumbent creditors of the additional value created by incentivized shareholders in a context of private benefits.

Ownership concentration and control can potentially have an impact on a firm's financing decisions, particularly its choices regarding leverage. Private benefits are an incentive ensuing from an implicit control contract between the controlling shareholder and the outside shareholders. Our understanding of the relationship between dominant shareholder ownership and firm debt levels is addressed in recent corporate governance studies (Faccio et al. 2010; He 2011; Liu and Tian 2012) and is somewhat limited. Morellec et al. (2017) is an exception as it aims at jointly analyzing the two types of conflict of interest among shareholders and with creditors. Our motivation is similar; we explore the two-way effect of debt as a disciplinary tool for controlling shareholders and as a tool to transfer wealth to creditors. Indebtedness curbs the controlling shareholders' private appropriation and at the same time the private benefit incentive initiates a holdup to creditors. We develop a simple theoretical relation between controlling shareholders, private benefits, and corporate debt levels. This paper highlights an asymmetric and self-regulated relationship between debt levels and controlling shareholders' private benefits. First, it is known that the controlling shareholder is incentivized to increase debt in order to dominate more of the firm's resources and to transfer some risk to creditors. On the other hand, more leverage induces a risk of default and weighs on the controlling shareholders' wealth. We introduce a third effect due to the endogenous incentivization of the controlling shareholder through a control contract between both categories of shareholders. Enhancing firm profitability will result in a transfer of value to creditors, whose debt value improves because of the lower default risk. This holdup to the creditor is an opportunity cost to any shareholder but is asymmetrically shared between outside and controlling shareholders. The capital structure decisions and private benefits choices depend on the trade-off between these three effects. Although the first and the second are well identified in the literature, the third is relatively new in a corporate governance framework as we explicitly introduce a difference between controlling and outside shareholders.

Following John and Kedia (2006), this paper outlines the two main issues resulting from the concentration of power by a controlling group. The first is the existence of private benefits as an implicit compensation scheme of the controlling shareholder, and the second is the choice of a debt structure with senior standard debt and equity, as the latter belongs at the same time to the controlling shareholders and to outside investors. We show that the two questions are linked in a financial governance framework. We explicitly identify the creditors' holdup problem, which should be integrated into a three-party equilibrium based on incentivization. Moreover, we introduce two contractual frameworks of benefits incentivizing a controlling shareholder, and we compare private direct expropriation with an additional sharing of public profit awarded to the controller.

Our study contributes to the extant literature on the relationship between private benefits and capital structure in several ways. First, we develop a model using the option valuation framework. This justifies referring to a risk-neutral hypothesis, as in Liu and Miao (2006), DeMarzo and Fishman (2007), Morellec et al. (2012), or Morellec et al. (2018). Second, we emphasize the role of debt leverage in agency conflicts because the controlling shareholders often find it easier to modify the leverage ratio than to modify their share of capital. However, our analysis also applies to situations involving dominant managers who are incentivized to appropriate benefits through an implicit contract with shareholders. We refer to the existence of an implicit control contract between the controller and outside shareholders, whose argument is the amount of private benefits seized by one party to reduce managerial costs and/or to enhance the value growth process. Outside shareholders will implicitly accept a trade-off because of the incentive characteristics of private benefits. In the context of dominant control, we show that the existence of a control contract is possible and that the debt-level decision is of the utmost importance for its design. We add to the literature on claim design by introducing expropriation of private benefits and by integrating the possibility of a debt value holdup by creditors. We explicate an asymmetric and disciplinary relationship between debt and private benefits. Aside from the standard Merton case, which uses the simple framework of zero coupon debt with a fixed nominal value and maturity, we follow Leland (1994), He (2011), Barsotti et al. (2012), Morellec et al. (2012), Attaoui and Poncet (2013), and Morellec et al. (2018) who refer to a dynamic framework with a bankruptcy risk defined with regard to the continuous coupon payment of perpetual debt. Contrary to the previous literature, our framework allows to consider private appropriation by the controlling shareholders as endogenous and not exogenous. Moreover, we distinguish between two possible designs for rewarding the controlling shareholder, implicit private benefits and the explicit profit-sharing design.

As a result, we show that creditors will benefit from a holdup situation through the additional value drift resulting from incentivized controllers and managers. We determine a threshold debt leverage ratio *above* which debt is moderately disciplinary and relatively helpful for private appropriation. Contrary to He (2011), we show that *a rise in indebtedness* is a way to avoid the creditors' holdup problem. Low leverage below the threshold exposes shareholders to a transfer of value to creditors. Moreover, debt influences the solution of the controlling–outside shareholders' conflict and applies enhanced disciplinary pressure to the controlling shareholder. A self-regulation mechanism is identified that constrains the appropriation of private benefits. This self-regulation mechanism will develop differently according to the two contractual frameworks of incentivizing benefits, i.e., direct private expropriation and additional sharing of public profit. We derive practical and testable implications.[2]

The remainder of this paper is organized as follows. Section 2 reviews the literature related to the topic. Section 3 presents the model and identifies the private benefits contracts. analyzes the effect of debt on the shareholders' wealth of the controlling shareholder. Section 4 identifies the specific role of debt in a corporate governance framework. Section 5 shows the conditions for a control contract between the controller and outside shareholders. The conclusion follows.

2. Review of the literature

The links between corporate governance and debt were first identified by Jensen and Meckling (1976). Capital structure is not solely explained by shareholders' value maximization. Debt is seen as a disciplinary tool that limits the free cash flow that can be used discretionarily by managers (Jensen, 1986). This first approach analyzes debt in the traditional agency conflict between managers and shareholders, in which the managers are willing to entrench (Claessens et al. 2002). Debt increases when the pressure of the controlling or majority shareholders develops. The conclusions are twofold. On the one hand, Harris and Raviv (1988) and Stulz (1988) suggest that debt is positively related to the managers' equity ownership. On the other hand, some empirical studies confirm that the managers' equity ownership negatively affects the firm debt level (Jensen, Solberg and Zorn 1992). Indeed, when managers hold a large stake in the firm's capital, they become less diversified, which may cause them to reduce debt levels to limit the default risk. These diverging results have shifted the focus toward the

[2] This paper brings different and complementary contributions to La Bruslerie (2016). Technically the former is developed in a fixed maturity Merton-like debt valuation model which is more limited compared to the Leland approach used in the current paper. Moreover the La Bruslerie (2016) paper focuses on the feasibility of a private aapropiation contract between controlling shareholders and outside investors where private benefits are expropriated upstream before debt repayment at the maturity date. The former framework doesn't propose any analysis of the creditor's holdup problem and the strategic management of debt leverage, which is addressed here.

possibility of a nonlinear relationship between ownership structure and indebtedness. Mikkelson and Partch (1989) find a negative relationship between inside ownership and leverage. Holderness et al. (1999) find no relationship and show that managerial stock ownership does not increase with the leverage ratio. Brailsford et al. (2002) propose an empirical test that highlights a nonlinear relation between the percentage of capital held by managers and the debt levels. Symmetrical to debt is cash holding. Excess cash holding is linked to managerial power and discretionary decisions of the controlling shareholder. It contributes less to firm value in a control situation with poor corporate governance (Belkhir et al. 2014).

Controlling ownership enlarges the above analyses, which mainly focus on agency conflicts with managers. Recent empirical studies on corporate governance show the prevalence of firms with a dominant shareholder (La Porta et al. 1997, 1998, 2002; Faccio et al. 2002, 2003). This situation is quite common in Europe. Even in the US, many corporations are actually controlled by large shareholding groups (Holderness 2009; Albuquerque and Schroth 2010). Furthermore, the world’s most common form of controlling ownership is family ownership. Since the dominant shareholder may extract private benefits of control at the expense of outside shareholders (La Porta et al. 1999), the fundamental agency problem turns out to be between controlling shareholders and outside shareholders (Bebchuk and Neeman 2010). This situation is identified in the literature through the idea of tunneling (Young et al. 2008). Private benefits are at the same time the symptom, the goal, and the regulating variable of an implicit contract between controlling shareholders and outside investors. Private benefits introduce a long-term perspective, and an implicit agency relationship develops over time. It is set in an implicit contract framework in which ownership is determined by the controlling shareholder, who takes into account the expected profit and characteristics of the firm (Gibbons 2002; At et al. 2006). The first consequence is that private appropriation of benefits appears as the cost associated with a concentration of power and control by the dominant shareholders. Barclay and Holderness (1989) and Dyck and Zingales (2004) provide evidence of private benefits when trades of blocks are set at a premium compared with the market price. The characteristics of private benefit appropriation are empirically studied by Leuz et al. (2002). In an international comparison, Bhattacharya et al. (2002) are also led to the conclusion of the existence of private benefits for controlling shareholders. The empirical relation between private benefits and leverage is analyzed in relatively few papers (except Kang and Kim 2006; Faccio et al. 2010; Liu and Tian 2012; La Bruslerie 2016, Morellec et al. 2018).

The role of debt in corporate governance depends on the structure of corporate ownership and control. Indeed, debt can play two contrasting roles in relation to financial governance. On the one hand, in the traditional manager–shareholder conflict, debt is seen as a disciplinary device that limits managerial opportunism in widely held corporations (Jensen and Meckling 1976; Jensen 1986). On the other hand, in firms dominated by controlling shareholders, debt is used to enhance the voting power of the controlling shareholders and to expropriate the outside shareholders further (Claessens et al. 2002; Paligorova and Xu 2012; Qian and Yeung 2014; Buchuk et al. 2014). The role of debt in the conflict between controlling and outside shareholders also involves third parties, such as banks or other creditors. Debt imposes limits on the behavior of controlling shareholders, and outside investors publicly know its amount. This external limitation interferes with the process of appropriating private earnings. The literature on the role of debt in the agency relationship between managers and shareholders has been well established since Myers (1977) and Dewatripont and Tirole (1994). Debt appears to be the "safest security" for outside investors because of the asymmetry of information enjoyed by creditors (Myers and Majluf 1984; Modigliani and Perotti 2000). From a theoretical point of view, a payment default transfers the control from the borrower to the lender (Grossman and Hart 1982; Aghion and Bolton 1992). The relationship between debt levels and control is seen as a positive device to protect the controller's situation (Harris and Raviv 1988) or to allow a "risk-shifting effect" (Zhang 1998). Debt enhances the economic power of the controlling shareholder without modifying the structure of ownership. Risky debt introduces specific bankruptcy costs to the creditors and increases the probability of default. On the other side, the "debt overhang" problem (Myers, 1977) creates a specific cost to shareholders, and as such, it is internalized in the controlling shareholder's decisions (Blazy et al. 2013). In the context of a supposed situation of control linked to the presence of a family, Ellul et al. (2009) provide a comprehensive empirical study on leverage that shows the balancing forces between managing the control and the expropriation possibilities on the one side and the disciplinary effect introduced by the risk of bankruptcy on the other side. Considering US firms, Nielsen (2006) empirically documents the existence of a trade-off between a tightly levered financial structure and low shareholding. A similar result is also highlighted in European family firms by Croci et al. (2010). Debt leverage is also shown to increase in a transfer of control process, particularly after the announcement of an offer up to after the completion of the acquisition (Jandik and Lallemand 2017).

On theoretical grounds, Harris and Raviv (1990), Zhang (1998), and Almeida and Wolfenzon (2005) address the problem of debt level and controlling–outside shareholder conflict within the context of information asymmetry. The optimal claim design is analyzed in the agency contracting literature. For instance, Gale and Hellwig (1985) introduce implicit incentive contracts and outline the importance of debt contracts in solving problems concerning the asymmetry of information. Bolton and Scharfein (1990) analyze one-period risky cash flows that can partly be diverted by managers. Berglof and Von Thadden (1994) and Dewatripont and Tirole (1994) show the coexistence of multiple outside claims, which can be interpreted as debt and outside equity. Berglof and Von Thadden distinguish between two categories of debt holders with long-term lenders and short-term bank credit lines as part of an optimal contract. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) describe financial contracting in a setting of private benefits through cash flow appropriation. They also introduce two categories of debt, long-term debt and a line of credit. Similarly, in a continuous-time setting, Liu and Miao (2006) examine the controlling shareholder's optimal choice of capital structure. The interaction between debt and ownership structure is analyzed in a global governance framework by John and Khedia (2006) and Lambrecht and Myers (2008). In a recent paper, Burkart et al. (2014) analyze the private diversion of the future profit resulting from a takeover. They focus on the financing constraint as debt fills the gap between the takeover cost and the bidder's equity. As in At et al. (2006), an optimal compensation contract using private benefits is identified for the controlling shareholder. It is shown that the level of investor protection plays an important role in funding the acquisition, as private benefits are not pledgeable.

This problem is also analyzed theoretically through models derived from an option framework. Debt is first presented as a zero-coupon bond (Merton 1974) with a fixed known maturity. This framework is equivalent to a one-period model. In Leland (1994) (or Uhrig-Hombourg 2005), debt is a perpetual constant coupon bond and default is endogenous. The definition of failure is different from that in Merton's scheme, in which the default is triggered at maturity by simply comparing the asset value of the firm with the nominal bond due at maturity. In the continuous set-up of permanent debt, the cash flow generated by the assets should cover the coupon flow to be paid indefinitely to the debtholders. This analysis leads to the uncertain time of bankruptcy being determined endogenously. Morellec (2004), Lambrecht and Myers (2008), and He (2011) analyze leverage in a contingent claims framework when the managers are disciplined by outside equity holders. He (2011) refers to the Leland (1994) model. He shows that debt will introduce specific "debt overhang" costs to the shareholders that will endogenously affect the managers/shareholder efforts. Morellec et al. (2012) develop a dynamic model in which the cash flow is partly appropriated by the managers. This modifies the failure risk of the firm and interferes with the capital structure decision. A trade-off is identified between the tax subsidy advantage of debt and the liquidation costs. An extension to the payout policy is proposed by Barsotti et al. (2012) and for junior-type debt by Attaoui and Poncet (2013). However, the question of private benefits in an optimal contracting set-up is not addressed by the latter. In a recent paper, Morellec et al. (2018) use a similar dynamic setting and identify two types of agency costs with relation to creditors and private benefits appropriation. However, debt is only seen as a positive transfer tool to the shareholders as the later can renegotiate debt when the firm goes bankrupt. By so doing, they can extract value from creditors who are willing to avoid liquidation. The creditors enter in their three parties setting, however they are passive and do not benefit from any transfer of value, negating the debt overhang problem.

3. Modeling debt and private benefits contracts

The model stands from the controlling shareholder's point of view. The goal is to analyze how debt modifies the negotiation terms of her control contract with outside investors.

A. Valuation framework

We refer to the framework developed in a continuous setting by Leland (1994). Creditors, the controlling shareholder, and outside investors have claims on the firm's assets. At time $t = 0$ the drift ruling the asset's growth is inflated with value creation resulting from the incentive flow produced by the controller's action.[3] In a risk-neutral framework, the market is complete, and the firm's assets are tradable and contractible. The firm uses debt and equity to finance them. However, the incentive contract resulting from the controlling shareholder's activity is implicit and is not a tradable asset. It results in a payment added to the asset value because of extra profitability drawn from the economic environment or resulting from cost savings. It can also be thought that the managers do not need to be monitored strictly, so the monitoring cost previously expended by the firm is saved and adds continuously to the asset's drift as a percentage, η. The firm value A follows the process as shown by formula (1a):

$$\frac{dA}{A} = (\mu + \eta)dt + \sigma dW \tag{1a}$$

The previous formula differs from the literature, which usually refers to a negative cash outflow considered as a dividend payment to shareholders or a coupon payment to bondholders. The assumption of a possible positive cash inflow finds its source back in Merton (1974) or Black and Cox (1976).[4]

As a result, the process followed by the asset is inflated by a net η cash inflow. This framework is symmetrically opposed to the cash outflows paid by the firm to security holders as dividends or interest payments in Leland's scheme. We suppose that this global inflow resulting from the incentivization of the controlling shareholder is continuously proportional to the asset value, ηA. In a risk-neutral set-up, the asset value follows a geometric Brownian motion. Its drift is $r + \eta$ (with r as a risk-free rate):[5]

$$\frac{dA}{A} = (r + \eta)dt + \sigma dW \tag{1b}$$

[3] This state variable is standardly based on the firm's asset dynamic. Ebit or cash-flow dynamics have been suggested by Goldstein et al. (2001). They privilege an Ebit dynamic as it is invariant to capital structure and to the way the Ebit "pie" is shared (p. 488). In our framework, we cannot assume that an Ebit state variable is exogenous as we want to model the incentivization effect of the payments between the different claimants.

[4] For Merton (1974), a payout in the firm's drift equation can be either negative or positive (p.450). Black and Cox (1976) refer explicitly to "the net total payout made, or inflow received, by the firm", which is identified in the valuation equation with either a positive or a negative sign (see Equation 1, p.352).

[5] See Merton (1973), Equation 7, p.452, in a no arbitrage framework. As mentioned in Black and Cox (1976), the instantaneous return is the risk free rate, so the instantaneous mean of the price should be adapted for the cash in(out)flow stream (Equation 1, p. 352). Similar settings are made in Leland (1994), Barsotti et al. (2012), Equation 1, and Attaoui and Poncet (2013), Assumption 2.

At inception, before setting an implicit contract of control that introduces appropriation, the firm comprises equity, E, and debt, D, belonging respectively to shareholders and lenders. Just before the setting at time 0, $V_0 = A_0 = E + D$. Debt is contractually set and incumbent creditors will only accept to renegotiate debt at cost for the shareholders. Once the contract is agreed upon, a positive continuous cash flow proportional to A resulting from the controlling shareholder's action adds to the asset drift. However, the controlling shareholder will divert some of the additional flow. This shareholder owns a stake, α, in the equity. Globally considered from the lender's point of view, these benefits are appropriated within the shareholders' group between a controlling shareholder and the minor investors. We can refer equivalently to the scheme of managers ruling a dispersed-ownership firm and incentivized by seizing a share of the additional cash flow.[6] Apparently, the financial situation of the creditors seems unchanged as they own a perpetual debt and receive a previously stated continuous coupon payment, C.

The nominal amount of debt was set at inception, and we do not need to refer to it to define bankruptcy as no reimbursement is scheduled. [7] We suppose, as in Leland (1994), that solvency is only linked to the possibility of financing a coupon payment with the issue of equity. It will stop if the equity value is below zero. At default, the firm cannot raise capital to pay its creditors. Bankruptcy does not define itself with regard to the nominal value of debt, D_0. This approach is different, as it does not yield a closed-form solution. Attaoui and Poncet (2013) links prior-to-maturity default with an interest payment lower than the net cash outflows drawn from the firm. We do not address the issue of the global capital structure with debt priority (e.g., Attaoui and Poncet 2013) but that of a mix with private benefits ranking before net equity.

[6] In our scheme, we rule out the possibility of a pure predatory controlling shareholder who expropriates cash flow without any value creation. Then, the additional drift becomes purely negative. This situation is detrimental to both outside investors and creditors. The latter will react by monitoring the controller. They will limit asset substitution, cash-flow diversion, or dividend payment by introducing provisions to constrain the controlling shareholder's behavior. This is the standard Jensen and Meckling disciplinary role of debt.

[7] We only know that the nominal value of debt at inception D_0 is linked to the perpetual nominal coupon flow C in the real world by a risk adjusted valuation rate r_0^{debt} such that $D_0 = C/r_0^{debt}$.

In a risk-neutral framework, the value of a perpetual claim, F, continuously paying a coupon C, where the assets' drift is inflated by a proportional cash inflow $(1-\gamma)\eta$ according to (1), follows the differential equation (Leland 1994; p.1241):

$$\frac{1}{2}\sigma^2 A^2 F_{AA} + (r+\eta)AF_A - rF + C = 0 \qquad (2)$$

The general solution submitted to bounding conditions is (Leland 1994, Eq. 33 to 35):

$$F = X_0 + X_1 A^{-y} + X_2 A^{-z} \qquad (3)$$

with

$$z = \frac{\left\{(r+\eta-0.5\sigma^2) + [(r+\eta-0.5\sigma^2)^2 + 2\sigma^2 r]^{\frac{1}{2}}\right\}}{\sigma^2}$$

$$y = \frac{\left\{(r+\eta-0.5\sigma^2) - [(r+\eta-0.5\sigma^2)^2 + 2\sigma^2 r]^{\frac{1}{2}}\right\}}{\sigma^2}$$

Default occurs when the assets reach the minimum value A_b. At that time, the assets are liquidated, and the creditors suffer liquidation costs calculated as a percentage, *l*, of the remaining assets. The other binding condition for debt is that it converges to a perpetual cash flow valued at the risk-free rates when the assets converge to infinity. The debt *D* claim satisfies:

$$F = D \to (1-l)A_b \text{ for } A = A_b \qquad (4)$$

$$F = D = \frac{C}{r} \text{ for } A \to \infty \qquad (5)$$

with $\eta > 0$; we obtain $z \geq 0$ and $z < \frac{2(r+\eta)}{\sigma^2}$. Similarly, we find $y \leq 0$. As a result, we obtain $X_1 = 0$ in Equation (3) to satisfy Condition (4). Bounding Condition (5) gives the value $X_0 = \frac{C}{r}$. At the limit value,

$A = A_b$, the value of debt, *D*, satisfies Equation (4): $D(A_b) = (1-l)A_b = \frac{C}{r} + X_2 A^{-z}$. We obtain X_2, and the debt value is:

$$D(A) = \frac{C}{r} + \left[(1-l)A_b - \frac{C}{r}\right]\left(\frac{A}{A_b}\right)^{-z} \tag{6a}$$

Rearranging produces the well-known result that the debt value is a weighted average between a pure risk-free perpetuity and the current liquidation value of the firm when bankrupt. The weight is the present value of 1 dollar of liquidated assets in the event that a default has occurred. It is also termed as the risk-neutral probability that a default will occur.

$$D(A) = \frac{C}{r}\left(1 - \left(\frac{A}{A_b}\right)^{-z}\right) + [(1-l)A_b]\left(\frac{A}{A_b}\right)^{-z} \tag{6b}$$

When valuing a firm, we need to account for tax deductibility gains. As interest is tax deductible, the present value of the tax savings will add to the equity and debt value. We define τ as the tax rate. The present value of tax shield $TS(A)$ in a continuous and risk-free setting is $\frac{\tau C}{r}$. When *A* is high, the tax shield value tends towards that value. For low values of *A* nearing the default threshold from above, the tax shield value is null. The tax shield claim is valued using Equation (3), but we need to adapt the bounding conditions:

$TS = 0$ for $A = A_b$

$TS = \frac{\tau C}{r}$ for $A \to \infty$

$$TS(A) = \frac{\tau C}{r} - \frac{\tau C}{r}\left(\frac{A}{A_b}\right)^{-z} \tag{7}$$

Bankruptcy costs are claims due to third parties when default occurs. They are estimated as a percentage, *l*, of the assets at default, that is, when $A = A_b$. As a result, they amount to lA_b. When the assets' value is very high, the eventuality of bankruptcy is null and the *ex ante* bankruptcy costs are negligible. This gives the boundary condition for $BC(A)$.

$$BC = lA_b \text{ for } A = A_b$$

$$BC = 0 \text{ for } A \rightarrow \infty$$

$$BC(A) = lA_b \left(\frac{A}{A_b}\right)^{-z} \quad (8)$$

The total market value, *v(A)*, adds the tax shield and the bankruptcy cost to the asset value. We derive the equity market value, *E(A)*, by subtracting the debt value from the total market value.

$$v(A) = A + TS(A) - BC(A) = A + \frac{\tau C}{r} - \frac{\tau C}{r}\left(\frac{A}{A_b}\right)^{-z} - lA_b\left(\frac{A}{A_b}\right)^{-z} \quad (9)$$

$$E(A) = v(A) - D(A) = A - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - A_b\right]\left(\frac{A}{A_b}\right)^{-z} \quad (10)$$

The firm's equity market value does not depend on parameter *l* because the loss rate is the creditors' problem. From (10), we derive the threshold value, A_b, which triggers the default as the one when the equity value is null $E(A_b) = 0$ because when $v \rightarrow$D, it is no longer possible to issue equity to finance any interest payments. The value A_b should be set as a limit condition in which $\frac{dE}{dA} \rightarrow 0$ when $A \rightarrow A_b$. This "smooth-pasting" condition gives the A_b value:

$$A_b = \frac{z}{1+z}\frac{(1-\tau)C}{r} \quad (11)$$

As *z* is positive, we obtain $A_b < (1-\tau)\frac{C}{r}$. The right side of the inequality is the after-tax risk-free value of the debt. Looking at Equation (10), the derivative of equity *E(.)* with regard to *A* is positive but decreasing. The equity value increases with the asset value but is a convex function of the firm's assets (Leland 1994; Barsotti et al. 2012).

B. Design of private appropriation

We will analyze "private" benefits as extra benefits shared with the minor shareholder. They rank controlling shareholder equally with standard equity claimants.

Private benefits are designed as a specific share of the net worth after debt flow payment; as such, they will appear as a specific additional right given to a specific category of shareholders. They have a contingent claim feature. Here private benefits become extra profits. They are a cake-sharing rule and present contingent claim features that incentivizes the controlling shareholder. A further remark to add is that this appropriation scheme is no longer "private" but publicly legitimate, as the controlling shareholder stands equally with other shareholders to share the equity cake. However, this right does not have priority rank within the shareholder group. In that framework, private benefit reduces to a sharing rule within the net worth public cake. It does not change anything for external creditors and the incentive of the controlling shareholder is to get a more than proportional share of the cake. This modeling can be ex ante sustainable for both parties, and shareholders as a whole group are compensated by the net worth. Equity considered globally remains a call option whose valuation relies simply on the standard Merton model. It has the contingent claim feature incentivizing the controlling shareholder. As a consequence, private benefits are no longer private, but become a public sharing rule contract that entails lower private costs borne by the controlling shareholder. The latter is still exposed to monitoring costs in her controlling job, but expropriation, legal, or reputational risks fade away as this additional benefit contract becomes explicit and legitimate. This alternate design may be a competitor to the private benefit framework we refer to above. In this framework the controlling shareholder/managers reward themselves with specific additional rights on the net public equity of the firm. Still ranking as a last resort creditor after the lenders, they will be paid with a larger share of equity capital. The controlling shareholder is compensated by a share of capital, α', which is larger compared to their original investment, α. This compensation is similar to a stock option-like scheme (hereafter SO contract). For instance, the controlling shareholder is granted zero price stock options or free new shares, giving him a γ% specific right on the net equity.

The incentivization as an extra share of equity capital awarded to the controlling shareholder gives $\gamma = (\alpha' - \alpha)$. Wealth appropriation ensuing from so-called private benefits PB is:

$$PB(A,C) = \gamma E(A,C) = \gamma \left\{ A - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - A_b\right] \left(\frac{A}{A_b}\right)^{-z} \right\} \tag{12}$$

At inception before any new issue of debt, the controlling shareholders' wealth is $\alpha' E(A)$.

The terms of an implicit contract are agreed ex ante between minor investors and the controlling investor. The key characteristic of this implicit contract is that the drift in the creation of value is a positive function of the private benefits, PB_t, appropriated at time t by the controlling shareholder. We state: $\eta = \eta(PB_t) = \eta(\gamma)$. This incentive condition is controlled by the parameter γ.

4. Analysis and role of debt

When the bankruptcy threshold A_b is small, debt $D(A)$ has a value that increases with the continuous coupon, C. However, when cash-flow payment C is high, the value of A_b increases, the probability of default becomes higher, and the value of debt converges down to the liquidation value $(1-l)A_b$ (see Equation 6b). Two opposite forces explain the debt value: One is the coupon flow value, and the other is the present value of the net liquidation flow in the event of default. We need to identify the coupon level that gives the optimal debt value balancing these two forces. The coupon level that maximizes the debt value, C^{max}, is such that $\frac{dD}{dC} = 0$. Solving this first-order condition gives (see Annex 1.1, Equation A2):

$$C^{max} = \frac{rA(1+z)}{z(1-\tau)} \left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} \tag{13}$$

As seen in Equation (13), a maximum affordable coupon payment exists. It is a positive function of the asset value. It permits the identification of a maximum affordable debt value from the creditors' point of view (see Annex 1.1, Equation A3).

$$D^{max} = \frac{A}{(1-\tau)} \left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} \tag{14a}$$

The maximum debt capacity is a positive function of the asset value. It depends on the initial asset size *A*, and on the firm characteristics through *z*. Calling leverage $\lambda = D/A$, we define:

$$\lambda^{max} = \frac{1}{(1-\tau)} \left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} \tag{14b}$$

We observe that D^{max} and λ^{max} are directly influenced by the firm's choices ruling the *z* value. The *z* parameter is a function of the assets' volatility, σ. It also depends on the assets' drift, η. The specific case of no drift simplifies the formula to $z = \frac{2r}{\sigma^2}$ (Leland 1994). When *C* is small (i.e., below C^{max}), or equivalently when the leverage is low and below λ^{max}, the debt value increases with the size of the coupon flow. The sign turns negative when the risk of bankruptcy becomes overwhelming.

A. The setting of debt level

The setting of debt is in the managers'/controlling shareholder's hands. Equation (6a) shows a second term in the right side member that has a negative sign since it results from Equation (11) that $A_b < (1-\tau)\frac{C}{r}$. The $(1-\tau)\frac{C}{r}$ value is the theoretical debt value in a risk free context taking into account the tax shield profit. The first way to limit creditors' wealth is to increase the default limit, which in turn will increase the risk-neutral probability of default $\left(\frac{A_b}{A}\right)^z$. The question is actually more complex, as A_b is increasing in *z* and any increase in A_b will also result in a better liquidation value for creditors in the event of default. To assess the net effect, we need to examine further the strategic determinants of default channeled through the manageable variable z. Therefore, we analyze the derivative of debt with respect to z (Annex 1.2, Equation A5), as we know that z is positively linked to the value creation drift η. This derivative is always positive; in a first step the creditors are systematically winning from the value creation.

As a result, the value of debt increases when a value creation incentivization scheme is implemented through a private benefit contract with the controlling shareholders. This value is a transfer from shareholders to debtholders. In particular, outside shareholders suffer from this wealth transfer, which is not justified by any wealth creation from debtholders.

Proposition 1 ("Creditors' holdup"). As the derivative of the debt value with respect to z is strictly positive, the higher drift in value creation is partly or totally captured by the existing creditors.

This creditor's holdup effect is directly controlled by the private benefits appropriation rate. We assume positive incentivization with $\frac{d\eta}{d\gamma} > 0$. A transfer of value initiates with debt and private benefits incentives. The debt value increases with the higher growth rate of the firm's assets. Creditors benefit from value-creative incentivization in the firm. However, debt may stimulate appropriate decisions in a control situation to avoid or limit creditors' holdup. The controller has in hand many tools to play with:

The first tool is σ, that is, the choice of assets' volatility. It is well analyzed in the literature since the Leland (1994) case without value creation and private benefits. In such a situation, we obtain $z = 2r/\sigma^2$. The only usable determinant available in this case is asset volatility through an asset substitution policy (Bigus 2002; Garvey and Mawani 2005; Tarentino 2013). For instance, an increase in volatility, substituting less risky assets with more risky ones, will decrease z and consequently decrease the debt value.

- Another tool is debt leverage. As identified in Equation (15b), the debt value increases first with the size of the coupon flow. Above the maximum leverage, the debt value decreases as the bankruptcy fear overcomes the payment effect.

When introducing incentivization and three parties' agency conflict, asset substitution mechanism (Jensen and Meckling 1976) and re-leveraging can be re-interpreted not as a device to extract value from creditors but to limit transfers of value to creditors.

Proposition 2. In a case in which the debt value increases with the setting of a private benefits scheme of incentivization, a way to avoid or limit the holdup by creditors is to substitute assets or to increase the firm's leverage.

In a private benefit appropriation scheme, the contract between the controlling shareholder and outside investors is grounded in the common goal of avoiding a side value transfer to creditors. This justifies private appropriation to maintain outside investors' wealth and limit the creditor's holdup by raising debt leverage.

B. Illustration

Graph 1 shows the situation of the debt value when the value creation rate is between 0% and 11% and the perpetual coupon flow is between 0 and 0.11.

INSERT GRAPH 1

The debt value decreases for high debt levels, that is, high coupon flows. For A = 1, the debt increases first with the coupon level; for η = 11%, the debt value levels off at 0.79, corresponding to a coupon flow of 6%. Above, the debt value declines to the floor of the minimum value after bankruptcy costs, specifically 0.70. For a lower creation drift of η = 1%, the debt value still increases but reaches its maximum at a coupon of 8%, displaying a debt value of 0.77. The maximum value depends on A_b, which is a positive function of the coupon. The condition for a negative slope is more easily met with a high drift. An increasing drift gives increasing z values. For coupon flows ranging from 1% to 6%, debt value increases with the additional drift value. It corresponds to situations in which dD/dz is positive. For larger coupon rates, debt value decreases with drift, signaling that the coupon is above the C_{max} value defined by Equation (14). The graph illustrates that increasing the additional drift to a local maximum and increasing the coupon rate afterward decreases the value of the creditors' claim.

C. The choice of debt for outside and controlling shareholders

The agency problem is proprietarily supported by the outside equity investors; meanwhile, they implicitly agree on a control situation and an incentivization deal to increase the growth of assets. An increase in leverage, i.e. issuing new debt, will curb and potentially balance the loss in equity value A. We need to integrate the use of cash raised by the debt issue. The first immediate idea to restore outside shareholders' wealth is to increase the amount of debt to reduce the share of equity financing. It will result in an increase in the total coupon paid to creditors. The new issue of debt is $D_2 = k.D$. This amount replaces equity financing and is given back to the shareholders as an exceptional dividend or a share buyback. It cumulates with the previous historical debt D_1 and k is the relative share of new issue of debt compared with the total debt $D = D_1 + D_2$ and $k = D_2/D$. The shareholders will use debt and question the amount of coupon flow the firm will pay. After implementing a new debt D_2, the outside shareholders' wealth consists in their stake of equity and the share of the new raised debt financing a dividend. Globally this equity reimbursement results in a jump in leverage and counters the creditor's holdup. We need to maximize the outside shareholders' wealth with regard to *C*. It comprises the new share of equity after awarding a part of it to the controlling shareholder, and a part of the proceeds of the debt issue:

$$Max\ w_O = (1-\alpha')E(C) + k(1-\ \alpha)D(C) \tag{15a}$$

The controlling shareholder's wealth covers both market equity value, the cash flow resulting from the debt issuing and private benefits featured as an additional share of equity ($\alpha' > \alpha$).

$$Max\ w_C = \alpha' E(C) + k\alpha D(C) \tag{15b}$$

Considering the global shareholders' wealth, we sum the two:

$$Max\ w_{O+C} = E(C) + kD(C) \quad (15c)$$

In the general case we do not find a closed-form solution for the optimal coupon C^* for the global shareholders' wealth maximization, but we show (see Annex1.3) that a function $C^* = f(\tau)$ exits with $f'(\tau) > 0$ and $C^* > C_1$ for any $\tau > 0$. Shareholders wealth maximization commands a univocal increase in indebtedness, and the debt issuance is larger as the tax rate gets higher.

Assuming that the tax rate is null, we get an analytical solution for the new debt issue resulting from the incentivization and the first step transfer of value to the creditors (see Annex 1.3 Equation A10). In this case, the new optimal debt from the global shareholders' point of view is:

$$C^* = \frac{C_1.[1+lz]}{(1+z)l} \quad (16a)$$

Because of its tractability we consider hereafter the null tax rate case, knowing that optimal solutions exist in any other cases and that these optimal debt values are unique and increasing with τ.

Alternatively, we define a ratio specifying the relative increase in debt compared to the initial debt amount before the holdup situation D_1, which corresponds to the initial coupon flow C_1:

$$\frac{C^*}{C_1} = \frac{1+lz}{(1+z)l} \quad (16b)$$

Since l <1, we verify that $C^* > C_1$.The ratio C^*/C_1 is also lower with z. As the value creation process is high, the increase in debt is capped by the rise of the probability of default (see Annex 1.3 Equation A11). The variation in indebtedness is directly linked to incentivization and its result. This justify analyzing the two dimensions of incentivization and leverage decision jointly. However, this result is demonstrated in a situation where all shareholders are homogeneous and share the same information set, and where there is no difference between a controlling shareholder who is incentivized and outside shareholders allowing the former to receive an additional part of equity.

This result holds in the general case of a positive tax rate. Even if we do not yield a closed form solution for the optimal debt value for the global shareholders, we show in the Annex 1.3 that this optimal value exists, is increasing with the tax rate and has a negative second derivative evidencing a maximum for the shareholders' wealth (see Equation A12). This maximum is above the initial debt level and shows that globally the shareholders should react by increasing the debt level of the firm (Equation A17).

- Situation of outside shareholders

We now formulate the outside shareholders' wealth as a fraction of the total shareholders' wealth knowing that they abandon a share γ of total equity. Equivalently to (15a) we set:

$$w_O = (1-\alpha)(w_O + w_C) - \gamma\, E(C) \quad (17a)$$

Similarly, the controlling shareholders' wealth is:

$$w_C = \alpha(w_O + w_C) + \gamma\, E(C) \quad (17b)$$

The target debt ratio is higher for the outside shareholders compared with the controlling shareholders' optimum (see Annex 1.4, Equation (A20)):

$$C^{*O} > C^{*C} \quad (18)$$

This means that as the outside shareholders abandon private benefits to the controller, they need to develop relatively more deleveraging to offset the creditors' holdup. Our Proposition 3 outlines that the two categories of shareholder share a common target.

Proposition 3. An increase in debt leverage is a way to limit the wealth transfer from shareholders to creditors in a situation of incentivized additional value creation. The controlling shareholder is specifically incentivized to protect outside shareholders and to manage the conflict between outside shareholders and creditors by raising leverage up.

In re-leveraging the firm to limit the transfer of value to creditors, all shareholders will share common goal. The leverage target value is set for a given value of the parameter z; we recall that z is a function of γ as the drift in value creation η is itself a positive function γ.

The optimal leverage is larger than the initial one previous the setting of a value creation plan meaning that an increase in debt will benefit the outside shareholders' wealth up to $C^{*C,O}$ (see Equation 18). The optimal debt level for the controlling shareholder, C^{*C} is lower than the C^{*O} optimal value considered by the outside shareholders. The contract of incentivization improves the situation of outside shareholders as the debt leverage target for the controller is higher. Incentivization boosts releveraging compared with a no incentivization situation where all shareholders will be in the same situation with regards to the creditors. This is demonstrated analytically (see Annex 1.3 Equation A20) even if we do not get general analytical expressions for optimal debt leverage for respectively outside and controlling shareholders. Later we will use numerical simulations.

Proposition 4 The appropriation of private benefits leads to a higher demand of debt for outside shareholders compared to the controlling shareholder, resulting in a stronger limitation of the creditors' holdup phenomenon which will benefit relatively more to them and partially offset the private benefits they award to the controller.

The first way for outside investors to avoid a creditors' holdup is to increase the coupon flow from C_1 up to C^{*O} and seek a new value of debt $D^{*}=D(C^{*O})$ corresponding to the optimal value C^{*O}. This illustrates the need for a value repatriation mechanism to balance a creditor's holdup due to an initial debt overhang situation.

This means increasing debt up to the leverage ratio $\lambda^{*} = \frac{D^{*}}{A}$. Then the controlling shareholder will increase the bankruptcy risk by inflating the coupon flow to be paid. Ceteris paribus, this will induce an upside jump in the firm's debt leverage. The same story applies when the debt leverage jumps following a substantial share repurchase. The wealth transfer hypothesis from bondholders to stockholders has been empirically documented by Maxwell and Stephens (2003) with regard to share repurchases. The risk of firm's debt increases outstandingly after a share repurchase offer linked to the awarding of stock options to executives (Jun et al. 2009). Controlled firms will empirically experience greater bondholder' losses at the announcement of a share repurchase. Jandik and Lallemand (2017) show that debt leverage increases after the announcement of an acquisition up to its completion.

The increase in leverage is specifically due to the absence of protection of outside shareholders who do not profit from any private benefits. The motivation of the controlling shareholder is twofold. She has incentive to protect himself and outside shareholders as a whole, by limiting the creditors' holdup. She is also incentivized by the agreement of a private benefits scheme between himself and the outside shareholders. She seeks to identify a trade-off situation between the holdup by creditors and private benefits appropriation from the point of view of outside shareholders who are aware of the situation. The protection of outside shareholders is a side effect of the control contract, where the controlling shareholder will limit the creditor's holdup by raising the debt leverage.

The optimal debt level from the whole shareholders' point of view, C^*, shows a negative derivative with regard to z (see Annex 1.3 Equation A11). As z is increasing with the creation of value rate η, a balancing mechanism develops where the increase of debt to offset the creditor's holdup is moderated by the aim not to be exposed to a larger bankruptcy risk. An increase in z means a higher default threshold and a higher probability of default. The outside shareholders look at long-term benefit from the value creation process. The potential for a jump in leverage is capped by the liquidation costs. The higher the liquidation cost, the smaller the relative increase in debt after the holdup as the derivative of Equation (16a,b) with regard to l is negative. This conclusion is coherent to that found in He (2011), who identifies a negative relationship between leverage and incentive effort because a heavily indebted firm close to financial distress is particularly exposed to a strong debt overhang phenomenon. To develop an effort without being exposed to a strong creditor's holdup, the managers should lower leverage. However, in our controlling/outside shareholding context, this is not true as outside shareholders are more exposed to a creditor's holdup than is the controlling shareholder, who is directly incentivized and will make specific efforts.

Another way to balance the phenomenon is to increase the risk of the firm by developing asset substitution. We know from Leland's polar case that z and σ are inversely related. We obtain the standard result that the equity value increases with the assets' volatility. This example shows that asset substitution cannot be seen only as a simple way to extract value from creditors to shareholders. When a situation of control appears with private benefits appropriation, the first direct consequence is the transfer of value to the creditors; then, asset substitution is a way not only to expropriate value but to limit the transfer of value seized by creditors and detrimental to shareholders. The basic reason is the holdup by creditors without any causal contribution to the economic creation of value.

D. Tax rates and numerical simulations

The existence of an optimal leverage value C^* is demonstrated using the assumption of a null tax rate. The introduction of positive tax rates will not change the conclusions although we do not get simple analytical solution. Positive tax rates will only move the optimal leverage levels up. This results from the tax shield benefits ensuing from indebtedness. It increases the outside shareholders' wealth and leads to more indebted firms. Starting with an initial debt flow of 3%, the optimal debt after incentivization corresponds to a new coupon flow of 7.5% for a null tax rate. It stays unchanged is the tax rate is 10%. But the optimal indebtedness has a coupon flow of 8.5% when the tax rate τ is 30%. Introducing tax rate lets the analysis unchanged but magnifies indebtedness from the shareholders point of view. Graph 2 shows the situation of the outside shareholders' wealth when the perpetual coupon flow moves up from 0.03 to 0.09 for different tax rate laying between 0% and 50%. Numerical simulations show similar patterns for the controlling shareholder's wealth.

INSERT GRAPH 2

The C^* optimal coupon flows are decreasing with the additional value creation drift. This result is formally demonstrated in Annex 1.3 Equation (A11) for τ =0%. We know that z is a positive function of the drift η. The optimal coupon debt level for the controlling shareholder C^{*C} has the same derivative with regard to the drift as C^*. Numerical simulations for different tax rate show that the decreasing relation between the optimal debt level for the whole shareholders and the drift value is decreasing for positive tax rates. The new debt optimal leverage curve C* is higher with increasing tax rates as higher debt is preferable because of the tax shield effect. The negative relation shows a balancing effect when the drift resulting from incentivization increases strongly, a lower transfer of value from the creditors is needed.

In the following numerical simulations, we have made explicit the positive incentivization between the appropriation rate γ and the additional drift η in value creation. As a consequence of the incentivization agreement, the drift η is supposed to be increasing with γ at a decreasing pace. The increase is bounded as we can assume that the potential of creation of value is limited by increasing costs to improve the firm's efficiency. In the numerical simulations we set $\eta=a_0.(\gamma- 2\gamma^2)$ with a_0=0.2. The controller wealth benefits from two effects: first he receives an additional share of the equity cake and, second, he benefits from the value due to the increased value creation process inflated by a value η. However the threshold of bankruptcy increases with z (i.e. η) and the event of a bankruptcy will lead to the loss of both private benefits and the initial percentage of owned equity. This situation is more severe compared to outside shareholders' as the latter will only lose their (reduced) equity shares. The increase in leverage pulls value back from the creditors to the shareholders. This mechanical effect develops only if the controller is incentivized with a γ value minimum enough to trigger an effort. The Graph 3 shows the optimal equilibrium loci for the controlling outside shareholders. For the controller, it shows increases in debt leverage that is below the optimal increase from the global shareholders' viewpoint; for the outside shareholders is shows increases in debt that is higher. The optimum solution for the controller corresponds to increase in wealth coming from a mix between transfers of value from the creditors, and profits from private benefits, and increase in the owned equity value. The plain curve in graph 3 is steep for null tax rates. Positive tax rates will yield lower debts value (because of the tax subsidy), so the shareholders can afford higher debt leverage. The equity value is also higher and the incentive γ "given" by the outside shareholder can go up to 25% of the equity for a 30% tax rate. Globally Graph 3 shows that the debt optimal curves have similar shapes when introducing tax rate. These simulations are in line with the analytical analysis evidencing that the optimal debt level increases with the tax rate (see above and Annex 1.3 Equation A17).The curves are simply moved upward when a tax subsidy effect transfers value to the shareholders. The difference of shape is important for the controller and the outside investors. The former when she is awarded private benefits sees her optimal leverage to decrease and is relatively less willing to increase the firm's leverage. The disciplinary role of debt is binding first to the controlling shareholder. The reason is that bankruptcy risk affects specifically the flow of private benefits. Avoiding too risky leverage is important to protect the golden eggs' hen. The disciplinary role of debt is specific to the controlling shareholder seizing private benefits. The bankruptcy risk will threaten the specific private benefits appropriated by the controlling shareholder.

GRAPH 3

The optimal debt setting is for outside shareholders. Their contract loci are displayed in dashed line. The outside shareholders will "pay" the private benefits to incentivize the controller. They lose a share γ of equity. They need a further increase in leverage to pull back value from the creditors. The more they accept to abandon in appropriation rate, the more they look for an increase in debt. Numerical simulations in Graph 3 show that the contract loci for the outside shareholder are increasing with γ. In Graph 3, the controller is the first exposed to the disciplinary effect of leverage. The divergence of goals appears as the gap between the optimal contract curves widens between the controller and the outside shareholders; and it widens with increasing drift.

5. The design of a control contract between the controller and outside shareholders

A. The setting of a contract

The setting of an incentive contract with private benefits is illustrated in Graph 4 which considers only null tax rates[8] The origin of the y-axis starts with an initial debt of 3%. (i.e. a coupon flow of 0.03 scaled by a total asset value of 1) The total wealth of shareholders is 0.4926 (with a total asset value scaled to 1). It is equally split between outside shareholders and the controller as their respective share of equity is 50%. Both know that a potential for creation of value exists and may be captured only if the controller is incentivized with some private benefits. Point A maximizes their

[8] Numerical simulations in this Section start with an initial debt with a coupon flow of 0.03 scaled by an asset value of 1. The risk free rate is 5% resulting to a risk free debt value of 0.6. Morellec et al. (2012) risk free rate parameter is 0.0421. The initial risk free debt to total asset ratio is 0.6. The parameter σ is set to 0.32. Morellec et al. (2012) use 0.2886. Liquidation costs percentage is set to 0.30. Morellec et al. (2012) refer to 0.4852, Branch (2002) takes a parameter value of 0.44, and Leland (1994) takes 0.50.

respective wealth only by leveraging and implementing a transfer of value from the creditors to them. The gain resulting by moving to the pure leveraging situation A is a common hypothetical situation where a re-leveraging decision is made without any creation of additional value, no incentivization and no effort are implemented. At this point, the optimal increase in debt results in a total coupon of 6.5%. Since no incentivization scheme has been implemented ($\gamma = 0$), outside and controlling shareholders agree on the optimal re-leveraging. The equity global value is then 0.5297. The mechanical transfer of value from debtholders to shareholders is +0.0371 (i.e. 3.7% of the asset value which is set to 1). The optimal increase in leverage corresponds to a coupon flow more than doubled. When the controlling share is 50%, the gain is halved between the two categories of shareholders (+0.0185). This situation is a pure predatory holdup by shareholders as no effort is necessary and no value is created. It cannot be motivated by any prior increase of value in the initial debt featuring a creditor's holdup. This potential of transfer of value introduces an iso-profit curve corresponding to the same increase in wealth for the controller resulting from mix of decisions of re-leveraging and incentivization (plain black line in Graph 4). This iso-profit curve defines a region where no effort is implemented as the controller is better off in purely re-leveraging.

GRAPH 4

Point A is not acceptable for the controller's viewpoint as the profit is not a feasible equilibrium. He will receive no private benefits. As it is common knowledge that a potential of creation of value exists, she will improve her wealth by being rewarded and develop an effort. The iso-profit curve for the controlling shareholder is below his optimal debt level curve. Point Z in Graph 4 ends the iso-profit curve for the controller and corresponds to a no change in debt leverage and an incentive award γ set to 4.5%. It is strictly dominated by the optimal point B ending the controller's optimal curve AB, where his wealth is 0.2756. Then region above the curve AB defines an effort zone below which no gains resulting from the incentivized creation of value will occur. A choice of private benefits defined by point B, i.e. γ=7.5%, will give the controller an additional creation of value of +0.0293 which is above the pure gain from a simple re-leveraging decision in point A (+0.0185). This point B is a maximum incentivizing award because any increase in private benefit above γ=7.5% will not result in any additional debt issue. This point B defines a vertical line BX. No bargaining agreement can locate on the right side on this line. A bargaining process can only initiate on the left side of the line BX. Point B is important as it defines a region for possible bargain solutions.

However, the point B is not a solution for the outside shareholders as it stands very far away from their optimal curve of contract (see graph 4, dotted line). They will lose -0.0426 compared to their initial situation before any decision. A global equilibrium cannot be settled as the net profit is negative. They will propose a 7.5% incentive only if debt leverage is moved up towards their optimal 0.0725 coupon flow (Point X in Graph 4). If the leverage is moved up to the optimal situation for outside shareholders, the respective gain will be +0.0139 for the controller and +0.0050 for the outside shareholders. They ask for a leverage to balance the cost of the incentivization plan *and* to repatriate the creditor's holdup. In such a situation the debt level strongly departs from the controller's optimal locus C^{*C}. Any incentivization γ, i.e. creation of value, will widen the conflict between the controlling and the outside shareholders. The gap between the two curves widens with γ.

A bargaining process respectively starting from B upward and X downward will initiate as the controller have in hand the decision to develop efforts. A feasible solution will be outside shareholders to accept standing on the global shareholders contact curve C^{*O+C} at point Z. It defines an incentivization scheme shared by a bargain among the shareholders group. Point Z on the curve will correspond to a 0.0625 optimal coupon debt as calculated for the global shareholder's point of view. The profit is lower for the outside shareholders departing from their optimal locus C^{*O}. However, combined with an incentivization of 7.5% they are left with a net profit of +0.0017 compared to their initial situation without any agreement. The controller who accepts to raise the debt leverage up to a 0.0625 coupon flow will gain +0.0201. He will gain more than a pure predatory re-leveraging without any effort. The total profit considered at the global shareholders' level is +0.0218. It is less that the pure predatory transfer of value from the creditors' (+0.0371). However, it is based on the existence of an incentive contract that creates additional value to the creditors *and* initiates a repatriation mechanism necessary to pull it back to the shareholders. In our analysis this mechanism is re-leveraging. More precisely, pegged with the incentivization contract, the shareholders' wealth is positively linked to the issue of new debt that allows to fund a transfer back to the shareholders as an exceptional dividend.

A process of bargain initiates starting from the point B to go to better solutions characterized by more debt and private benefits appropriation. A common possible second-best target is the leverage ratio corresponding from the global shareholders point of view at the controller's optimal private benefits. This is a simple trade-off for the controlling shareholder who is balancing additional debt possibly hurting her wealth by raising the default risk, and more private appropriation. This is not a simple trade-off for the outside shareholders who will improve their wealth both in using more debt and/or more private benefits. More private benefits have a double effect to them: negative with more appropriation by the controller, positive with more incentivization which leads to additional value creation.

We show the opportunity of a negotiation process and its feasibility through the shared trade-off rationale between private appropriation by the controller and debt increase. The condition for a convergence is that, given the incentivization plan maximizing the creation of value for the controller with no change in leverage (here γ=7.5%), the total gain for the whole shareholder (+0.0218) is above the no effort profit for the controller (+0.0185). That gives enough room to find a profit sharing with the outside shareholders. This total gain for the whole shareholders is determined using the optimal re-leveraging decision for the shareholders globally considered, here moving up the coupon flow from 0.03 to 0.0625, combined with a private benefits contract optimal to the controller. The limiting values for the bargaining process (for a γ value fixed at 7.5%) are given by the points Z' and B'. The first one defines a minimum increase in wealth of +0.0185 for the controller. He will not accept to develop an effort if his increase in wealth is lower compared to a situation of pure leveraging without effort. This gives him a maximum affordable coupon level of 0.0625. Outside shareholders are dependent on the controller's will to develop effort. Their agreement is conditioned by a positive increase in wealth. This gives point X' who defines a minimum increase of the coupon flow of 0.06. The coupon flow range between 0.06 and 0.065 defines a set of feasible agreement for the given value of γ. Of course, this later variable is itself part of the negotiation. This defines a conic region of admissible bargain solutions centered around the global shareholders optimal incentivization and debt choices (curve C^{*O+C}). In the

Graph 5, the region for admissible agreements is AZ'B'. The incentive contract is defined by the setting of an admissible combination of the two key variables: private benefits award γ and the re-leveraging decision C^*.

GRAPH 5

The condition for a private benefits contract incentivizing creation of value is that the set of joint decisions corresponding to (i) the optimal private benefits for the controller without any re-leveraging decision, and (ii) the optimal re-leveraging decision from the the global shareholders' level given the best incentivization plan, creates more value compared to the profit to the controller resulting from a pure predatory leveraging decision. Then a re-leveraging decision takes place as a balancing mechanism in a context of potential creditor's holdup.

If the condition is met a bargain process may develop to share the extra profit resulting from incentivization. If not, no effort is implemented. Rational outside shareholders know that giving private benefits is necessary to find equilibrium and repatriate the creditors' holdup. Moreover, they know that a minimum private benefit amount is necessary to capture the potential of value creation. The two decisions of incentivization and re-leveraging are joint decisions; the first one resulting in the second.

Numerical simulations evidence the level of a maximum private benefit rate to trigger effort by the controller corresponding to the no increase in debt situation and featured by point B in Graphs 4 and 5. This defines a no bargain zone on the right-hand side of the point B. For a zero-tax rate, the maximum γ is 7,5%. If the tax rate is 30% the maximum γ is 25%. The maximum private benefit award increases with the tax rate. This explains because a tax rate will introduce a tax subsidy for the whole shareholders. Everything being equal, it is a stronger incentive to increase leverage to capture the tax shield instead of triggering an effort. The private benefits should compensate in the controller's wealth the tax shield advantage. As a result, a paradoxical consequence is that tax rate discourages the controller to trigger an effort to capture additional value as it widens the region under the AB curve.

The productivity of the controller's incentivization is controlled by the parameter a_0.[9] In our simulation we have considered values ranging between 0.1 to 0.5 meaning that an incentive stake of 10% of the equity will result in an increase in the value creation drift between 0.9% and 4.5%. The controlling shareholders optimal debt curve AB in the Graph 5 is decreasing with the private benefits award. The point A is given for any value of the a_0 parameter as it corresponds to γ=0%, i.e. a pure jump in leverage. For the given set of parameters, it does not depend on the initial value of the debt coupon C_1. We consider different initial coupon debt values between 0.02 to 0.05.[10] The position of the point B depends of the steepness of the controller's optimal contract curve. The value of γ_{max} corresponding to the point B is important as it delimits the conic region of feasible agreements. The negative steepness of the AB curve clearly depends on the productivity of the controller's best efforts.

Looking at the outside investors, their optimal curve AX is upward oriented an its steepness increases with the effort's productivity. This means the region between the two curves defines the large disagreement about the optimal debt level to negotiate. The productive incentivization will result in larger disagreement and thus makes the negotiation process more necessary. Oppositely non-productive efforts (i.e. a_0=0) would lead to a pure transfer of value from creditors (no value creation and no private benefits).

a_0	C_1=0.02	C_1=0.03	C_1=0.04	C_1=0.05
0.1	8.5%	7.5%	6.0%	4.5%
0.2	8.0%	7.5%	6.0%	4.5%
0.3	7.5%	7.0%	6.0%	4.0%
0.4	7.0%	7.0%	6.0%	4.0%
0.5	7.0%	6.5%	5.5%	4.0%

Table 1 Maximum incentive stake of equity awarded to the controlling shareholder

[9] The additional drift in creation of value is set: $\eta=a_0.\gamma *(1-\gamma)$.

[10] Using a risk-free rate of 5% it gives a risk free value of debt ranging between 0.4 and 1.0. Scaled by total assets of 1, it results in debt to total assets ratios which characterize moderately to heavily indebted firm.

(maximum incentive stake is γ_{max} corresponding to the point B location on the x-axis in Graph 5; maximum wealth for different initial coupon flow C1 varying from 0.02 to 0.05; drift is positive in appropriation rate: $\eta=a_0.\gamma *(1-\gamma)$, with a0= varying from 0.1 to 0.5; asset A=1; risk free rate r =5%; volatility of the firm's assets σ =32%; corporate tax rate τ equal to 0%; bankruptcy cost l=32%; ownership α=50%)

Table 1 shows values of the maximum incentivizing award γ_{max}. They are calculated for different levels of initial debt C_1 and for different values of the productivity of the incentivization, a_0. The value of γ_{max} is decreasing with the productivity of the effort developed by the controller. Interestingly, γ_{max} is also decreasing in the initial debt level. A highly indebted firm will never implement a very high incentivization scheme (γ=4% for C_1=0.05, compared to γ=8% for C_1=0.02), because in this case the controlling shareholder will prefer not to increase leverage. The incentivization stakes resulting from the of parameters are relatively limited. Awarding of stakes of capital between 4 to 8% do not imply massive dilution of the outside shareholders.

B. Practical and empirical implications

Our propositions are testable, mainly propositions 3 and 4. They identify a link between leverage jumps and the setting of incentivization through private benefits. However, private benefits are implicit contracts and are not observable. If successful, they will result in higher drifts in value creation flowing into the firm, at least for the part captured by the firm. We may also expect an increase in asset substitution (proposition 3 and 3bis). The previous results have been drawn with no reference to the equity stake held by the controller. We only assume that a major shareholder has control or that the managers have a dominant position. Changes in the debt level without a change in ownership are potential signals of private benefits appropriation (or modification) for investors. A jump in debt and the agreement of an incentive contract will demonstrate a reaction to the holdup problem. The basic empirical implication of the above model is to identify a jump in a firm's debt leverage after a change in the incentive compensation scheme. This will occur in the situation of a controlled firm.

The identification of incentive contracts is quite difficult. We can hypothesize that incentive contracts could be proxied in the following ways:

- For managers, the awarding of a massive SO plan. The difficulty is the continuous award of SOs in a public firm, as SOs are cumulative and roll over. The identification of a massive break-up is sometimes hazardous.
- Jumps in leverage may also be tracked by repurchase decisions. If these offers are contemporaneous with new incentivization schemes, we can hypothesize a causal relationship. The aim to limit transfer of value to creditors and to repatriate value to the shareholders may explain share repurchases (Jun et al. 2009). The development of share repurchases is also identified after an acquisition (Jandik and Lallemand 2017).
- The debt changes level off as bankruptcy threatens. Controllers and minor investors are then exposed to the same event of default. In a situation of high financial distress risk, debt has no specific role in private benefits appropriation compared with a SO contract. Thus, assuming that the private costs, m_C, are higher than m_{SO}, the SO contract should dominate, and private expropriation should be low. We should expect for highly levered firms that stock option-like compensation contracts will be dominant in controlled firms. This feature supposes looking at paired firms, associating controlled and non-controlled firms. Highly indebted firms should show a higher probability of setting up SO incentive contracts to appropriate an additional part of the benefits. Conversely, controlling shareholders will use private expropriation in less indebted firms. The latter will as a consequence show lower public profitability after private appropriation.
- A new control situation is an event that will introduce a break. A transfer of control resulting from an acquisition may call into question the current incentive scheme of the manager or the new controlling shareholder, and open new possibilities for value creation. As such, we expect it to trigger a debt leverage jump. If the controller is able to initiate a positive drift in value creation, she will try to avoid a transfer of value to the creditors. This has been documented in Jandik and Lallemand (2017) within and after the acquisition process. The target firm's leverage ratio increases by 5.5% after the completion compared with before. Interestingly, the abnormal returns of buyers of targets who raise their debt leverage during the acquisition is shown to be higher compared to other targets. This is in line with the creditor's holdup

mitigation and value repatriation mechanisms. Another expected consequence can be asset substitution toward more risk (Bigus 2002; Garvey and Mawani 2005; Tarentino 2013). A rise in the economic risk of the firm after an M&A transaction without an increase in the debt conditions and without a rise in investors' profitability suggests appropriation by a controlling shareholder.

C. Alternate scheme of private upstream appropriation

The above analysis has been developed using an appropriation scheme featured as an award of an equity stake at zero cost to the controlling shareholder. To check the robustness of our model we will now turn toward another private appropriation scheme. Here we assume that private benefits are levied by the controlling shareholder before displaying the firm's cash-flow. Private benefits rank first before the debtholder and outside shareholders' payments. The nature of private benefits is to rely on the raw profit generated by the firm before public profit is disclosed to the general investors and the market. We refer to a design of private benefits that gives them priority before net equity compensation. For instance, industrial organization gives the possibility to extract private benefits. Within industrial groups or pyramidal structure, it is easy for controlling shareholders to extract benefits using transfer pricing or cash–flow "tunneling" (Johnson et al., 2000; Claesens et al. 2002; Young et al. 2008). Transfers of firm's cash-flow outside the firm to third parties or to blockholders are also ways of private appropriation. The economic explanation for seniority of private benefits lies in the existence of costs privately borne by the controlling shareholder. This seniority is part of the contract between shareholders and is not legally warranted. These costs have two components: (i) specific costs linked to the diversion of private benefits in an implicit framework, and (ii) monitoring costs exposed in the specific work of controlling shareholders to control the managers and improve the firm's decision process. The first component covers both the legal, judicial, and criminal risk and the hiding cost of "tunneling" and diverting some part of the raw profit (Johnson et al. 2000). We assume that these costs are proportional to the assets under management. In designing the implicit contract, the controlling shareholder wants to first cover their private costs. This implicit contract is ex ante trustable and optimally designed by both the controlling and outside shareholders. This contract is featured as from a controlling shareholder's point of view or equivalently from a manager point of view. Both can be incentivized by private benefits and will hold a share of equity capital. Of course, the controlling shareholder's stake of equity capital is designed to be high enough to give him control over the firm. This is a difference of scale, not of rationale.

As in La Bruslerie (2016) and Morellec et al. (2018), we now assume that the controlling shareholder seizes part of the extra generated inflow at the source. We define γ as the percentage of the η inflow rate appropriated by the controlling shareholder. The controlling shareholder (or, equivalently, the managers) diverts part of the additional free cash flow that is generated. However, the variable γ is not exogenous, as supposed in Morellec et al.'s (2012) setting. It is part of the problem as the implicit regulation variable in the contract of control between the controlling shareholder and the outside shareholders. In a risk-neutral set-up, the asset value follows a geometric Brownian motion. Its drift is $r + \eta - \eta\gamma$ (with r as a risk-free rate):

$$\frac{dA}{A} = (r + \eta - \eta\gamma)dt + \sigma dW \qquad \text{(1b')}$$

with

$$z = \frac{\left\{(r + \eta - \eta\gamma - 0.5\sigma^2) + [(r + \eta - \eta\gamma - 0.5\sigma^2)^2 + 2\sigma^2 r]^{\frac{1}{2}}\right\}}{\sigma^2}$$

This gives values of debt and value of equity similar to Equations (6a), (9) and (10). However the threshold value for default is lower as it refers to a z value which is lower. Then bankruptcy is less probable and debt is less risky.

In this framework, private benefits PB are a claim on the firm's assets that produces a continuous cash flow, $\gamma\eta A$, diverted from the global incentivization cash flow. They are valued as perpetuity in a continuous risk-free context. Of course, their value collapses to zero when the firm defaults.

In the new drift framework defined by Equation (1b'), the global value of the firms covers the market value, v(A), and a non-tradable claim on private benefits, which is a partial counterparty of the economic value creation due to incentivization. We obtain $w(A) = v(A) + PB(A)$. The controlling shareholder's wealth consolidates a fraction of the market-valued equity, α, which gives him control and private benefits, PB described in Equation (A21) (see Annex 1.5).

$$w_c(A) = \alpha E(A) + PB(A)$$

After taking the leverage decision, the controlling shareholder's wealth covers both market equity value, the cash flow resulting from the debt issuing and private benefits.

$$Max\ w_C = \alpha E(C) + k\alpha D(C) + PB \quad (19)$$

In the Equation (1b') framework, the amount of private benefits *PB* is positively linked to the appropriation rate, γ. The optimal debt level for the controlling shareholder exits and is defined by (see Annex 1.5 Equation A23):

$$C^{**} = \frac{C_1[1+lz]}{(1+z)\left\{l+\frac{\gamma\eta}{r\alpha}\right\}} \quad (20)$$

Equation (20) is demonstrated in Annex 1.5 assuming a null tax rate. Comparing Equation (20) with the optimal value for the whole shareholders in the other private benefits scheme (see Equation 16a), we get $C^{**}<C^{*}$. Numerical simulations show that it is still valid with positive tax rates. The optimal indebtedness moves up with tax rate τ because of the tax shield benefits to any shareholders. We get wealth curves very similar to Graph 2. The optimal coupon flow starts from 3% to 7% for τ ranging from 0% to 40%. With increasing tax rates, the controlling shareholder's wealth increases so the reward of indebtedness is pull up with τ.

The outside shareholders have a value (1-α) of the global equity value after increase in debt (see Equation (15c) and Equation (A8) in Annex 1.3). The optimal solution for their debt level is C^* (see Annex 1.3 Equation (A9) and Equation (A10) in the null tax case). We have $C^{**}<C^{*}$.[11] In a first range up to C^{**} the controller's wealth increase with C above its initial value C_1 at inception of the incentivization plan. The rationale for the controlling shareholder is shared with outside shareholders up to a level of debt such that $D^{**}=D(C^{**})$. The private benefits part of the controller's wealth is left unchanged with C (see Equation A22 in Annex 1.5). The two optimal debt levels for controlling

[11] Remember that in this framework z is defined by Eq (1b') and is lower tthan the z value in the SO incentive contract.

shareholder and outside investors are not identical. Both look for an increase in debt, however *t*heir interests are not strictly aligned with regard to the creditor's holdup. The controlling shareholder holds private benefits that will increase the bankruptcy risk and whose existence is exposed to a failure risk.[12] The discrepancy between C* and C** is increasing with l and γ and decreasing with σ. The derivative of C** with regard to z is negative (see Equation A24 Annex 1.5) and is larger than the comparable derivative for the C* leverage target in the initial appropriation scheme Equation A11 Annex 1.3). Incentivization will raise the z value and will cap more rapidly the increase in debt.

Proposition 5 Within an upstream private appropriation scheme the controlling shareholder is less incentivized to increase debt leverage compared to the target ratio aimed at by the outside shareholders. Compared with the private benefits compensation using equity award to the controlling shareholder; re-leveraging is rapidly binding as bankruptcy risk appears sooner.

Overall the analysis of private benefits, transfer of value through debt overhang, and the setting of an incentivization contract between outside and the controlling shareholders stays unchanged when referring to an alternate scheme of compensation. The optimal releveraging choices are different and open a second-best trade-off situation to offset the creditors' holdup.

6. Conclusion

This paper addresses the question of the relationship between private benefits and the choice of a debt structure with senior standard debt and equity, as the latter belongs at the same time to the controlling shareholder and to outside investors. Each of these two questions corresponds to an agency conflict that is addressed in the literature. Establishing a link between the two issues is uncommon. The relation becomes more complex with three parties. We show that the two issues are linked in a financial governance framework. We can no longer separate the issues when a holdup problem is identified and

[12] The value of z is lower in this Section 5.C compared to z used in Section 3 because of the negative $\eta\gamma$ term. The probability of failure is positively linked to z.

when a controlling shareholder is incentivized through private benefits. Incentivization will not profit outside investors as creditors are the first to be enriched by safer claims.

As a consequence, we predict a jump in leverage to protect outside shareholders. Our paper derives some testable implications and proposes an empirical test. The traditional disciplinary role of debt is analyzed as a limitation on the free cash flow of the firm. Indebtedness limits the discretionary misuse of resources. A slightly more sophisticated view is that debt increases the probability of bankruptcy and as such limits the time scope of entrenchment behavior and cash flow appropriation or diversion. We show that debt is also *per se* an appropriation device for creditors. This will trigger a set of reactions as the shareholders will try to limit the transfer of value to the creditors. Several possibilities are explored, among which a non-intuitive solution is demonstrated: Shareholders will increase the debt leverage to increase the risk of default and lower the debt value. The controlling shareholder may also target higher equity ownership or a lower appropriation rate. She can also maximize the value creation drift, which is beneficial (a) to the controller himself and (b) to the creditors. The shareholders are the losers in the holdup area. However, we show that, with regard to the creditors' holdup problem, the two categories of shareholders may bargain on an incentive contract designing private benefits and resulting in releveraging. Debt is an appropriation device for creditors in the context of value creation. The disciplinary tool is no longer disciplinary, and creditors also need to be disciplined.

References

Aghion P., Bolton P. "An Incomplete Contract Approach to Financial Contracting". *Review of Financial Studies* 59 (1992): 473-494

Albuquerque, R. and Schroth, E. "Quantifying Private Benefits of Control from a Structural Model of Block Trades". *Journal of Financial Economics* 96(1) (2010): 33-55

Almeida H., Wolfenzon D. "A Theory of Pyramidal Ownership and Family Business Groups". *Journal of Finance* 61(6) (2006): 2637-2680

At C., Chappe N., Morand P.H., Thomas L. « Rémunération optimale des dirigeants : Faut-il interdire les bénéfices privés ? », *Revue d'Economie Politique*, 116(6) (2006) : 831-846.

Attaoui S., Poncet P. "Capital Structure and Debt Priority", *Financial Management* 42(4) (2013): 737-775

Barclay M., Holderness C. "Private Benefits from Control of Public Corporations". *Journal of Financial Economics* 25(2) (1989): 371-395

Barsotti F., Mancino M.E., Pontier M. « The Role of a Firm's net Cash Payouts in Leland (1994) Model" *Economic notes by Monte dei Paschi di Sienna* 41(3) (2012):115-144

Bebchuk L., Neeman Z. "Investor Protection and Interest Group Politics" *Review of Financial Studies* 23 (2010): 1089-1119

Belkhir M., Boubaker S., Derouiche I. "Control-ownership Wedge, Board of Directors, and the Value of Excess Cash" *Economic Modelling* 39 (2014): 110-122

Berglöf, E., Von Thadden E.L. "Short-term versus Long-term Interests: Capital Structure with Multiple Investors," *Quarterly Journal of Economics* 109 (1994): 1055-1084.

Bhattacharya U., Daouk H., Welker, M. "The World Price of Earning Opacity". Accounting Review 78 (2003): 641-678

Bigus J. "Bankruptcy Law, Asset Substitution Problem, and Creditor Conflicts". *International Review of Law and Economics* 22(2) (2002): 109-132

Black F., Cox J. "Corporate Securities: Some Effects of Bond Indenture Provisions". *Journal of Finance* 31 (1976): 351-367

Blazy R., Deffains B., Umbhauer G., Weil L. "Severe or Gentle Bankruptcy Law: Which Impact on Investing and Financing Decisions?", *Economic Modelling* 34 (2013): 129-144

Bolton P., Scharfstein D. "A theory of Predation Based on Agency Problem in Financial Contracting", *American Economic Review* 80 (1990):94-106

Brailsford T. J., Barry O.L., Pua S .L.H. “On the Relation between Ownership Structure and Capital Structure. *Accounting and Finance* 42 (2002): 1-26

Branch B. “The cost of bankruptcy: A review” *International Review of Financial Analysis* vol11(2002):39-57

Buchuk D., Larrain B., Munoz F., Urzua I. “The internal Capital Market of Business Groups: Evidence from Intra-group Loans” *Journal of Financial Economics* 112 (2014); 190-212

Burkart M., Gromb D., Mueller H., Panunzi F. “Legal Investor Protection and Takeovers”. *Journal of Finance* 69(3) (2014):1129-1165

Claessens S., Djankov S., Fan J., Lang L. “Disentangling the Incentive and Entrenchment Effects of Large Shareholdings”. *Journal of Finance* 57(6) (2002): 2741–2771

Croci E., Doukas J., Gonenc H. “Family Control and Financing Decision”. *European Financial Management* 17 (2010):860-897

DeMarzo P., Fishman M. “Optimal long-term financial contracting”. (2007) *Review of Financial Studies* 20(5) (2007): 2079-2128

DeMarzo P., Sannikov Y. “Optimal Security Design and Dynamic Capital Structure in a Continuous-time Agency Model,” *Journal of Finance* 61 (2006): 2681-2724.

Dewatripont M., Tirole J. “A Theory of Debt and Equity: Diversity of Securities and Manager–Shareholder Congruence” *Quarterly Journal of Economics*, 109 (1994): 1027-1054

Dyck A., Zingales L. "Private Benefits of Control: An International Comparison". *Journal of Finance,* 59 (2004): 537-600

Ellul, A., Guntay L., Lel U.. “Blockholders, Debt Agency Costs and Legal Protection”. FRB International Finance Discussion Paper No. 908. Available at SSRN: http://ssrn.com/abstract=68737 (2009)

Faccio M., Lang L., “The Ultimate Ownership of Western European Corporations” *Journal of Financial. Economics* 65 (2002): 365–395

Faccio, M., Lang, H.P.L., Young, L.S. “Debt and Expropriation” EFMA 2001 Lugano Meetings, http://ssrn.com/abstract=23972 (2003)

Faccio M., Lang L., Young, L.S. “Pyramiding vs. Leverage in Corporate Groups: International Evidence” *Journal of International Business Studies* 41 (2010): 88-104

Gale D., Hellwig M. “Incentive-compatible Debt Contracts: The One-period Problem” *Review of Economic Studies* 52 (1985): 647-663

Garvey, G. T., Mawani, A. “Risk-taking Incentives of Executive Stock Options and the Asset Substitution Problem”. *Accounting & Finance*, 45 (2005): 3-23

Gibbons R. “Relational Contracts and the Theory of the Firm”. *Quarterly Journal of Economics*, 117(2002):39-83

Goldstein R., Ju N., Leland H. “An Ebit-based model of Dynamic Capital Structure”. *Journal of Business*, 74(4) (2001): 483-512

Grossman S., Hart O. “Corporate Financial Structure and Managerial Incentives”. In *The Economics of Information and Uncertainty*, edited by J.J. McCall, 123-155. Chicago: University of Chicago press, 1982

Harris M., Raviv A. “Corporate Control Contests and Capital Structure. *Journal of Financial Economics* 20 (1988): 55-86

Harris M., Raviv A. “Capital Structure and the Informational Role of Debt”. *Journal of Finance 45 (1990): 321-350*

He Z. “A Model of Dynamic Compensation and Capital Structure,” *Journal of Financial Economics* 100 (2011): 351-366.

Holderness C.G., Krosner R., Sheehan D. “Were the Good Old Days that Good? Changes in Managerial Stock Ownership Since the Great Depression”. *Journal of Finance* 54 (1999): 435-469

Holderness C.G. “The Myth of Diffuse Ownership in the United Sates”. *Review of Financial Studies* 22 (2009): 1377-1408.

Jandik T., Lallemand J. “Do Capital Structure Adjustments by Takeover Targets Influence Acquisition Gains?”. *The Financial Review* 52(2) (2017):171-198

Jensen M. “Agency Costs of Free Cash Flow, Corporate Finance and Takeovers”. *American Economic Review* 76 (1986): 323-329

Jensen M., Meckling W. “Theory of the Firm: Managerial Behaviour, Agency Costs, and Ownership Structure”. *Journal of Financial Economics* 3 (1976): 305-60

Jensen G., Solberg D. and Zorn T. “Simultaneous Determination of Insider Ownership, Debt, and Dividend Policies”. *Journal of Financial and Quantitative Analysis* 27 (1992): 247-263

John K., Kedia S. “Design of Corporate Governance: Role of Ownership Structure, Takeovers and Bank Debt”, working paper, NYU, www.ssrn.com, (2006)

Johnson S., La Porta R., Lopez de Silanes F., Shleifer A. “Tunnelling”. *American Economic Review*, papers and proceedings, 90 (2000): 22-27

Jun S., Jung M., Walking R. “Share repurchase, executive options and wealth changes to stockholders and bondholders”, *Journal of Corporate Finance* 15 (2009): 212-229

Kang J., Kim J. “Private Benefits of Control and Firm Leverage: An Analysis of Korean Firms”. *Review of Quantitative Finance and Accounting* 27 (2006): 439-463

La Bruslerie, H. de “Does Debt Curb Controlling Shareholders’ Private Benefits? Modelling in a Contingent Claim Framework”. *Economic Modelling* 58 (2016): 263-282

La Porta R., Lopez-de-Silanes F., Shleifer A., and Vishny R. “Legal Determinants of External Finance”. *Journal of Finance* 52 (1997): 1131-1150

La Porta R., Lopez-de-Silanes F., Shleifer A., and Vishny R. “Law and Finance”. *Journal of Political Economy* 106 (1998): 1113-1150

La Porta R., Lopez-de-Silanes F., Shleifer A, and Vishny R. “Investor Protection and Corporate Valuation”. *Journal of Finance* 57 (2002): 1147-1170

Lambrecht B., Myers S. "Debt and Managerial Rents in a Real-options Model of the Firm," *Journal of Financial Economics* 89 (2008): 209-231

Leland H. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure". *Journal of Finance* 49(4) (1994): 1213-1252

Leuz C., Nanda D., Wysocki P. "Earnings Management and Investor Protection: An International Comparison" *Journal of Financial Economics* 69(3) (2003): 505-527

Liu H., Miao, J. "Managerial Preferences, Corporate Governance and Financial Structure", working paper, www.ssrn.com (2006)

Liu Q., Tian G. "Controlling Shareholder, Expropriation and Firm's Leverage Decision: Evidence from Chinese Non-tradable Share Reform" *Journal of Corporate Finance* 18 (2012): 782-803

Maxwell W., Stephens C., "The Wealth Effects of Repurchases on Bondholders", *Journal of Finance* 58 (2003): 895-919

Merton R. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates". *Journal of Finance* 29(2) (1974): 449-470

Mikkelson W., Patch M. "Manager's Voting Rights and Corporate Control". *Journal of Financial Economics* 25 (1989): 263-290

Modigliani F., Perotti, E. "Security Versus Bank Finance: Legal enforcement and investors' protection", *International Review of Finance* 1(2) (2000):81-96

Moraux F., Navatte P. "Strategic Management of Private Benefits in a Contingent Claim Framework", *Bankers, Markets and Investors*, n°122 (2013): 48-56,

Morellec E. "Can Managerial Discretion Explain Observed Leverage Ratios?" *Review of Financial Studies* 17 (2004): 257-294.

Morellec E., Nikolov B., Schurhoff N. "Corporate Governance and Capital Structure Dynamics" *Journal of Finance* 67 (2012), 803-848

Morellec E., Nikolov B., Schurhoff N. "Agency conflicts around the world", *Review of Financial Studies* 31(11) (2018): 4232-4287

Myers S.C. "The Determinants of Corporate Borrowing" *Journal of Financial Economics*, 5 (1977): 147-175.

Myers S.C., Majluf N.S. "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have". *Journal of Financial Economics* 20 (1984): 187-221.

Nielsen A. "Corporate Governance, Leverage and Dividend Policy", EFA 2006 Zurich Meetings. Available at SSRN: http://ssrn.com/abstract=906339 (2006)

Paligorova T., Xu Z. "Complex Ownership and Capital Structure" *Journal of Corporate Finance* 18 (2012): 701-716

Qian M., Yeung Y. "Bank Financing and Corporate Governance" *Journal of Corporate Finance* 32 (2015): 258-270

Stulz R. "Managerial Control of Voting Rights: Financing Policies and the Market for Corporate Control". *Journal of Financial Economics* 20 (1988): 25-54

Tarantino E. "Bankruptcy Law and Corporate Investment Decisions". *Journal of Banking and Finance* 37(7) (2013): 2490-2500

Uhrig-Hombourg M. "Cash Flow Shortage as an Endogenous Bankruptcy Reason". *Journal of Banking and Finance*, 29 (2005): 1509–1534

Young M.N., Peng N.W., Ahlstrom D., Bruton G.D., Jiang Y. "Corporate Governance in Emerging Economies: A review of the Principal-Principal Perspectives". *Journal of Management Studies* 45 (2008); 197-220

Zhang G. "Ownership Concentration, Risk Aversion and the Effect of Financial Structure on Investment Decisions". *European Economic Review* 42 (1998): 1751-1778

Appendix

Annex 1.1 Derivative of debt with regard to the coupon flow $\frac{dD}{dC}$

We rewrite *D(A)* (see Equation (6)) using Equation (11) which identifies A_b:

$$D(A) = \frac{C}{r} + \left[(1-l)\left(\frac{z}{1+z}\frac{(1-\tau)C}{r}\right) - \frac{C}{r}\right]\left(\frac{z}{1+z}\frac{(1-\tau)C}{rA}\right)^{z}$$

Rearranging

$$D(A) = \frac{C}{r} - \frac{C}{r}\left(\frac{C}{r}\right)^{z}\left[\frac{1+z(\tau+l(1-\tau))}{1+z}\right]\left(\frac{z}{1+z}\frac{(1-\tau)}{A}\right)^{z} \quad \text{(A1)}$$

The first order condition is:

$$\frac{dD}{dC} = sgn\left\{1 - \left(\frac{C}{r}\right)^{z}\left[1 + z(\tau + l(1-\tau))\right]\left(\frac{z}{1+z}\frac{(1-\tau)}{A}\right)^{z}\right\} = 0$$

Manipulation gives:

$$C^{max} = \frac{rA(1+z)}{z(1-\tau)}\left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} \quad \text{(A2)}$$

A maximum affordable coupon exists maximizing the debt value is identified. Replacing (A2) in Equation (11) in the text gives the highest possible default threshold A_b^{max}. We can plug into Equation (6) the values C^{max} and A_b^{max} to get the maximum value of debt $D^{max}(C^{max}, A_b^{max})$:

$$D^{max} = \frac{C^{max}}{r}\left[1 - \left[\frac{1+z(\tau+l(1-\tau))}{1+z}\right]\left(\frac{A_b^{max}}{A}\right)^{z}\right]$$

with $A_b^{max} = \frac{z}{1+z}\frac{(1-\tau)C^{max}}{r}$.

Using (A2) gives:

$$D^{max} = \frac{A(1+z)}{z(1-\tau)}\left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} - \frac{A(1+z)}{z(1-\tau)}\left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z}\left(\left(\frac{A(1+z)}{z(1-\tau)}\right)^{z}\left[\frac{1}{1+z(\tau+l(1-\tau))}\right]\right)\left[\frac{1+z(\tau+l(1-\tau))}{1+z}\right]\left(\frac{z}{1+z}\frac{(1-\tau)}{A}\right)^{z}$$

It simplifies to:

$$D^{max} = \frac{A}{(1-\tau)}\left[\frac{1}{1+z(\tau+l(1-\tau))}\right]^{1/z} \quad \text{(A3)}$$

Annex 1.2. Derivative $\frac{dD}{dz}$

We first recall the sign of the partial derivatives of $z = z(\eta, \sigma^2)$: $\frac{dz}{d\eta} > 0$.

The derivative $\frac{dA_b}{dz} = sgn\frac{1}{(1+z)^2} > 0$. The risk neutral probability of default $\left(\frac{A_b}{A}\right)^z$ increases with *z*.

Starting from (A1):

$$\frac{dD(z)}{dz} = sgn\frac{d\left[-\frac{C}{r}f(z)\right]}{dz} \text{ with } f(z) = \left[\frac{1+z(\tau+l(1-\tau))}{1+z}\right]\left(\frac{z}{1+z}\frac{(1-\tau)}{A}\frac{C}{r}\right)^z \quad \text{(A4)}$$

We consider *h'(z)* the log-derivative of:

$$h(z) = \log(f(z)) = \log\left(1 + z(\tau + l(1-\tau))\right) - \log(1+z) + z\log\left(\frac{A_b}{A}\right)$$

The derivative *h'(z)* has the sign of:

$$h'(z) = \frac{(\tau + l(1-\tau)) - 1}{\left(1 + z(\tau + l(1-\tau))\right)(1+z)} + \log\left(\frac{A_b}{A}\right)$$

Recalling the negative sign of $(-\frac{C}{r})$ in Equation (A4), the derivative of debt versus *z* is negative when

$$A_b exp\left(\frac{(\tau+l(1-\tau))-1}{\left(1+z(\tau+l(1-\tau))\right)(1+z)}\right) \geq A \quad \text{(A5)}$$

This is always verified because *l* and *τ* are between 0 and 1 and $(\tau + l(1-\tau))$ is lower than 1. Then the turning point stands below the default threshold. As the value A_b is a binding limit to *A*, the condition (A5) is never satisfied and the derivative of debt value with regard to *z* and to value creation *η* is always positive.

Annex 1.3 Global shareholders' point of view

The value of total debt is related to the perpetual coupon flow *C* by Equations (6b) and (11) in the text.

$$D(C) = \frac{C}{r}\left(1 - \left(\frac{A}{A_b}\right)^{-z}\right) + [(1-l)A_b]\left(\frac{A}{A_b}\right)^{-z}$$

$$= \frac{C}{r}\left(1 - \left(\frac{A}{A_b}\right)^{-z}\right) + \left[(1-l)\frac{z}{1+z}\frac{(1-\tau)C}{r}\right]\left(\frac{A}{A_b}\right)^{-z}$$

$$= \frac{C}{r}\left\{\left(1 - \left(\frac{A}{A_b}\right)^{-z}\right) + \left[(1-l)\frac{z}{1+z}(1-\tau)\right]\left(\frac{A}{A_b}\right)^{-z}\right\} = \frac{C}{r}\{\dots\}$$

The above equation shows that the total debt value is strictly proportional to the coupon flow *C*. We recall that the expression $\left(\frac{A}{A_b}\right)^{-z}$ is homogenous to a probability of bankruptcy.

Using the last equation, we set:

$$k = \frac{D_2}{D} = \frac{(C-C_1)\frac{1}{r}\{\dots\}}{C\frac{1}{r}\{\dots\}} = 1 - \frac{C_1}{C} \quad \text{(A6)}$$

From Equation (15c) in the text we set the total shareholders' wealth w_{0+C} replacing *E* and *D* by respectively Equations (10) and (6a) by their equations.

$$w_{O+C} = E(C) + kD(C)$$

$$= A - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - A_b\right]\left(\frac{A}{A_b}\right)^{-z} + k\left[\frac{C}{r} + \left[(1-l)A_b - \frac{C}{r}\right]\left(\frac{A}{A_b}\right)^{-z}\right]$$

We replace *k* by (A7) and use the variable $AA = \left(\frac{(1-\tau)}{r}\frac{z}{1+z}\frac{1}{A}\right)^{z}$:

$$AA = C^{-z}.\left(\frac{A}{A_b}\right)^{-z} \quad \text{(A7)}$$

$$w_{O+C} = A + \frac{(C\tau)}{r} - \frac{C_1}{r} + AA.C^z.\frac{(1-\tau)C}{r} - AA.C^z.\frac{z}{1+z}\frac{(1-\tau)C}{r}$$

$$+ AA.C^z\left[(1-l)\frac{z}{1+z}\frac{(1-\tau)C}{r} - \frac{C}{r}\right] - \frac{C_1}{C}AA.C^z\left[(1-l)\frac{z}{1+z}\frac{(1-\tau)C}{r} - \frac{C}{r}\right]$$

We now derive this expression with respect to *C* and express it as a function of *C* and the tax-rate *τ*:

$$g(C,\tau) = \frac{1}{r}\left\{\begin{matrix} \tau - AA(\tau).C^z.\left[lz + \tau\big(1 + z(1-l)\big)\right] \\ +AA(\tau).C^{z-1}.C_1.\frac{z}{1+z}.\left[1 + lz + \tau z(1-l)\right] \end{matrix}\right\} \quad \text{(A8)}$$

The first-order condition for a maximum C^* is obtained by solving the equation:

$$g(C^*,\tau) = 0 \quad \text{(A9)}$$

$g(C,\tau)$ is a regular continuously derivable function. There exists an implicit function $C^* = f(\tau)$ solving (A8). We cannot find a closed-form solution for this function. However, stating that the tax rate *τ* is null allows us to cancel out the term proportional to *C* and to obtain a closed-form solution. We have: $g(C,0) = \frac{1}{r}\left[-AA(\tau).C^z.lz + AA(\tau).C^{z-1}.C_1.(1+lz).\frac{z}{1+z}\right]$. Solving $g(C,0) = 0$ for $\tau = 0$ and $C > 0$ gives the unique solution:

$$C^*_{\tau=0} = \frac{C_1.[1+lz]}{(1+z)l} \tag{A10}$$

Since l <1, C^*/C_1 is larger than one which means that there will be a positive debt issuance. We compute the derivative of C^*/C_1 with regard to z:

$$\frac{d(\frac{C^*}{C_1})}{dz} = l(1+z)l - (1+lz)l = l^2 - l = (l-1)l < 0 \tag{A11}$$

The derivative is negative as l is below one. The variable C^*/C_1 decreases with z (but always remains higher than one).

We now check the second order condition (in the general case where $\tau \geq 0$). We verify that:

$$\frac{dg}{dC}(f(\tau),\tau) < 0$$

From (A7) we find:

$$\frac{dg}{dC}(C,\tau) = \frac{1}{r}\left\{\begin{array}{c} z.AA(\tau).C^{z-1}.[lz+\tau[1+z(1-l)]] \\ +(z-1)AA(\tau).C^{z-2}.C_1.\frac{z}{1+z}.[1+lz+\tau z(1-l)] \end{array}\right\}$$

$$\frac{dg}{dC}(C,\tau) = \frac{1}{c}\ \left\{z.g(C,\tau) - \frac{1}{r}[AA(\tau).C^{z-1}.C_1.\frac{z}{1+z}.[1+lz+\tau z(1-l)] + \tau z]\right\}$$

With (A8) we obtain the second-order condition (all the remaining terms are negative):

$$\frac{dg}{dC}(f(\tau),\tau) < 0 \tag{A12}$$

The regular form of g also allows us to study the derivative of the implicit function $f(\tau)$:

$$f'(\tau) = \frac{d(C^*)}{d\tau} = -\frac{\frac{dg}{d\tau}(C^*,\tau)}{\frac{dg}{dC}(C^*,\tau)}$$

Using (A11) this means that the sign of $f'(\tau)$ is the same as the sign of $\frac{dg}{d\tau}(C^*,\tau)$. Rearranging (A8):

$$g(C,\tau).r = \tau + AA(\tau).C^{z-1}\left\{\begin{array}{c} \tau.[-C.[1+z(1-l)] + C_1.\frac{z^2}{1+z}(1-l)] \\ -C.lz + C_1.\frac{z}{1+z}.(1+lz) \end{array}\right\} \tag{A13}$$

Calling $h(C,\tau)$ the right hand side of this equation:

$$h(C,\tau) = \tau.[-C.[1+z(1-l)] + C_1.\frac{z^2}{1+z}(1-l)] - C.lz + C_1.\frac{z}{1+z}.(1+lz)$$

or

$$h(C,\tau) = -C\{\tau.[1+z(1-l)] - lz\} + C_1.\left\{\tau.\frac{z^2}{1+z}(1-l) + \frac{z}{1+z}.(1+lz)\right\} \quad \text{(A14)}$$

$$\frac{dh}{d\tau}(C,\tau) = -C.[1+z(1-l)] + C_1.\frac{z^2}{1+z}(1-l)$$

or

$$\frac{dh}{d\tau}(C,\tau) = \frac{1}{\tau}(h(C,\tau) + C.lz) - C_1(\frac{z}{1+z}.\frac{1+lz}{\tau}) \quad \text{(A15)}$$

With $AA(\tau) = \left(\frac{(1-\tau)}{r}\frac{z}{1+z}\frac{1}{A}\right)^z$, we get: $AA'(\tau) = -\frac{z}{(1-\tau)}AA$ (A16)

Deriving (A13):

$$\frac{dg}{d\tau}(C,\tau).r = 1 + C^{z-1}\left(AA'(\tau).h(C,\tau) + AA(\tau).\frac{dh}{d\tau}(C,\tau)\right)$$

Using (A15) and (A16) gives:

$$\frac{dg}{d\tau}(C,\tau).r = 1 - \frac{z}{(1-\tau)}AA.C^{z-1}.h(C,\tau) + C^{z-1}.AA.\left[\frac{1}{\tau}(h(C,\tau) + C.lz) - C_1.\frac{z}{1+z}.\frac{1+lz}{\tau}\right]$$

We now use (A9) in order to study the sign of the derivative at *C**. Using (A9) and (A13):

$$AA.f(\tau)^{z-1}.h(f(\tau),\tau) = -\tau$$

$$\frac{dg}{d\tau}(f(\tau),\tau).r = \frac{z\tau}{(1-\tau)} + AA(\tau).f(\tau)^{z-1}.\frac{1}{\tau}\left(f(\tau).lz - C_1.\frac{z}{1+z}.(1+lz)\right)$$

For any $f(\tau) \geq \frac{C_1.[1+lz]}{(1+z)l}$, which is true for $\tau = 0$, we obtain:

$$\frac{dg}{d\tau}(f(\tau),\tau) > 0$$

$$f'(\tau) > 0 \quad \text{(A17)}$$

This proves that *C** is a growing function of the tax rate, and that there will always be a positive net issuance ($C^* > C_1$).

Annex 1.4 Situation of the outside shareholders

Using the variable AA we know that

$$E(C) = A - \frac{(1-\tau).C}{r} + AA.C^{z+1}.\frac{(1-\tau)}{r}.\frac{1}{1+z}$$

$$E'(C) = \frac{(1-\tau)}{r}.(AA.C^z - 1)$$

We start from Equation (17a) in the text. The derivative of w_0 with regard to C is (with $g(.)$ defined in Eq. A8):

$$w'_0 = (1-\alpha).g(.) - \gamma E' \quad \text{(A18)}$$

At $C^* = f(\tau)$, we know that $g(C^*,\tau) = 0$. With (A7) we also have $AA.C^z < 1$ so that $E'(C) < 0$. It means that the derivative of the outside shareholders wealth around the (collective) optimal coupon C^* is positive:

$$w'_0(C^*) > 0$$

Outside shareholders want more debt that shareholders considered as a whole. This is not surprising since outside shareholders receive higher proportion of the debt issuance compared to their effective share of equity (net of private benefits).

Symmetrically, considering the controlling shareholder and deriving (17b) we have:

$$w'_C = \alpha.g(.) + \gamma E' \quad \text{(A19)}$$

$$w'_c(C^*) < 0$$

The controlling shareholder wants less debt that shareholders considered as a whole. The optimal coupon is higher for outside shareholders than for the controlling shareholder:

$$C^{*O} > C^{*C} \quad \text{(A20)}$$

Annex 1.5 Controlling shareholder - Alternate scheme of upstream private benefits

The private benefits defined as upstream expropriation ranking before equity $PB(A)$ follows the boundary limit conditions:

$$PB = 0 \text{ for } A = A_b$$

$$PB = \frac{\gamma\eta A}{r} \to \infty \text{ for } A \to \infty$$

$$PB(A) = \frac{\gamma\eta A}{r} - \frac{\gamma\eta A_b}{r}\left(\frac{A}{A_b}\right)^{-z} \quad \text{(A21)}$$

Equation (19) in the text identifies the controller's wealth

$w_C(C) = \alpha\left\{A - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - A_b\right]\left(\frac{A}{A_b}\right)^{-z} + k\left[\frac{C}{r} + \left[(1-l)A_b - \frac{C}{r}\right]\left(\frac{A}{A_b}\right)^{-z}\right]\right\} + \frac{\gamma\eta A}{r} - \frac{\gamma\eta A_b}{r}\left(\frac{A}{A_b}\right)^{-z}$

We replace k by (A7), use the variable $A_b = \frac{z}{1+z}\frac{(1-\tau)C}{r}$ and refer to $AA = \left(\frac{(1-\tau)}{r}\frac{z}{1+z}\frac{1}{A}\right)^z$. It yields:

$$w_C(C) = \alpha\left\{A - \frac{(1-\tau)C}{r} + \left[\frac{(1-\tau)C}{r} - \frac{z}{1+z}\frac{(1-\tau)C}{r}\right]AA.C^z + \frac{C}{r} + \left[(1-l)\frac{z}{1+z}\frac{(1-\tau)C}{r} - \frac{C}{r}\right]AA.C^z - \frac{C_1}{r} - \frac{C_1}{C}AA.C^z\left[(1-l)\frac{z}{1+z}\frac{(1-\tau)C}{r} - \frac{C}{r}\right]\right\} + \frac{\gamma\eta A}{r} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{(1-\tau)C}{r}AA.C^z \quad \text{(A22)}$$

We state that the tax rate τ is zero. The terms of first order in C cancel and we use $BB = \left[\frac{(1-l)}{r}\frac{z}{1+z} - \frac{1}{r}\right]$.

$$w_C(C) = f(C) = \alpha\left\{A - \frac{C_1}{r} + \frac{C}{r}AA.C^z - \frac{C}{r}\frac{z}{1+z}AA.C^z + C[BB]AA.C^z - C_1AA.C^z[BB]\right\} + \frac{\gamma\eta A}{r} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{C}{r}AA.C^z$$

$$f(C) = \alpha\left(A - \frac{C_1}{r}\right) + \frac{\gamma\eta A}{r} + \alpha C^z(-C_1AA[BB]) + \alpha C^{z+1}\left\{\frac{1}{r}AA - \frac{1}{r}\frac{z}{1+z}AA + [BB].AA\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA.C^{z+1}$$

$$f(C) = \alpha\left(A - \frac{C_1}{r}\right) + \frac{\gamma\eta A}{r} + C^z\left\{\alpha(-C_1AA[BB]) + \alpha C\left\{\frac{1}{r}AA - \frac{1}{r}\frac{z}{1+z}AA + [BB].AA\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA.C\right\}$$

We derive with regard to C:

$$f'(C) = zC^{z-1}\left\{\alpha(-C_1AA[BB]) + \alpha C\left\{\frac{1}{r}AA - \frac{1}{r}\frac{z}{1+z}AA + [BB].AA\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA.C\right\} + C^z\left\{\alpha\left\{\frac{1}{r}AA - \frac{1}{r}\frac{z}{1+z}AA + [BB].AA\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA\right\}$$

Setting the derivative to zero and simplifying by C^{z-1}

$$f'(C) = z\left\{-\alpha(C_1AA[BB]) + \alpha C.AA\left\{\frac{1}{r} - \frac{1}{r}\frac{z}{1+z} + [BB]\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA.C\right\} + C\left\{\alpha AA\left\{\frac{1}{r} - \frac{1}{r}\frac{z}{1+z} + [BB]\right\} - \frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA\right\} = 0$$

$$C(1+z)\left\{\alpha AA\left\{\frac{1}{r}-\frac{1}{r}\frac{z}{1+z}+[BB]\right\}-\frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA\right\}-\alpha(C_1 AA[BB])=0$$

We get the optimal coupon flow maximizing the controller's wealth:

$$C^{**C}=\frac{\alpha(C_1 AA[BB])}{(1+z)\left\{\alpha AA\left\{\frac{1}{r}-\frac{1}{r}\frac{z}{1+z}+[BB]\right\}-\frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}AA\right\}}$$

$$=\frac{zC_1[BB]}{(1+z)\left\{\left\{\frac{1}{r}-\frac{1}{r}\frac{z}{1+z}+[BB]\right\}-\frac{\gamma\eta}{r}\frac{z}{1+z}\frac{1}{r}\frac{1}{\alpha}\right\}}$$

$$C^{**C}=\frac{zC_1[-1-lz]}{(1+z)\left\{\{-lz\}-\frac{\gamma\eta}{r}z\frac{1}{\alpha}\right\}}=\frac{C_1[1+lz]}{(1+z)\left\{l+\frac{\gamma\eta}{r}\frac{1}{\alpha}\right\}} \quad \text{(A23)}$$

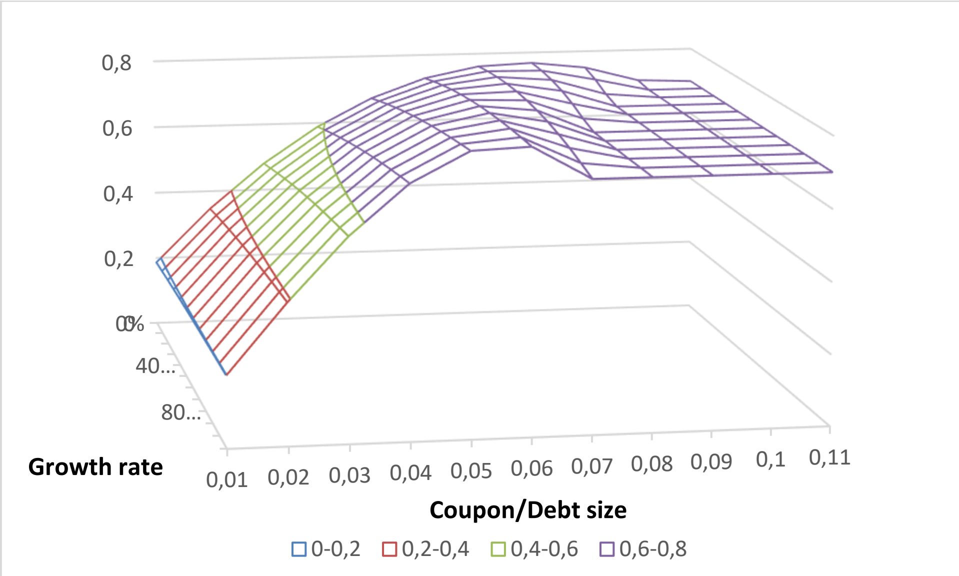
The second term between brackets in the denominator is positive.

We compute the derivative of C^{**}/C_1 with regard to z (where $DD=(l+\frac{\gamma\eta}{r\alpha})$):

$$\frac{d(\frac{C^{**}}{C_1})}{dz}=l(1+z).DD-(1+lz)(DD)$$

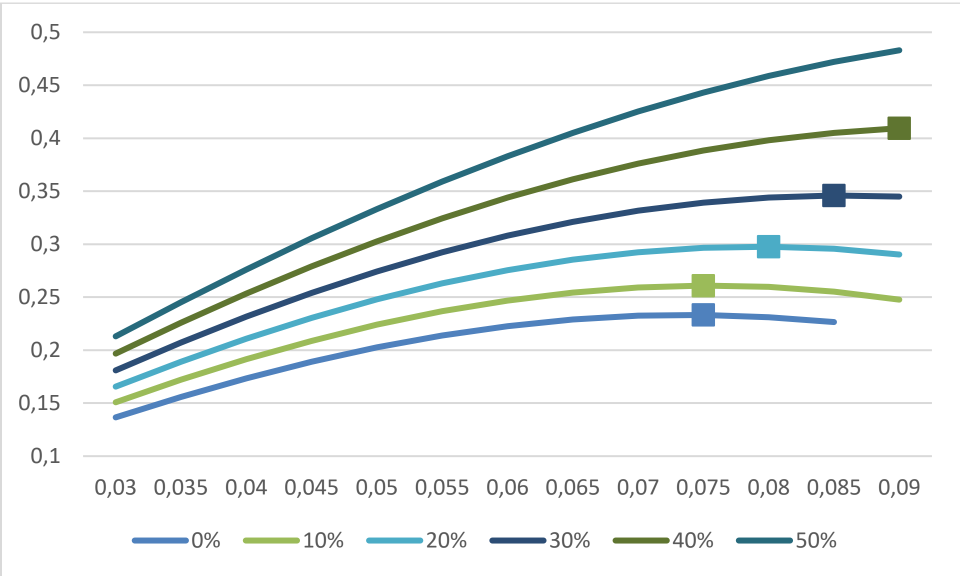
$$sgn\frac{d(\frac{C^{**}}{C_1})}{dz}=l-1<0 \quad \text{(A24)}$$

The derivative is negative. The variable C^{**}/C_1 decreases with z. It starts very high meaning a strong issue of new debt D_2 for low values of z and decreases to a lower bound where the new debt issue is null. This lower bound is set when C^{*}/C_1 is equal to 1.



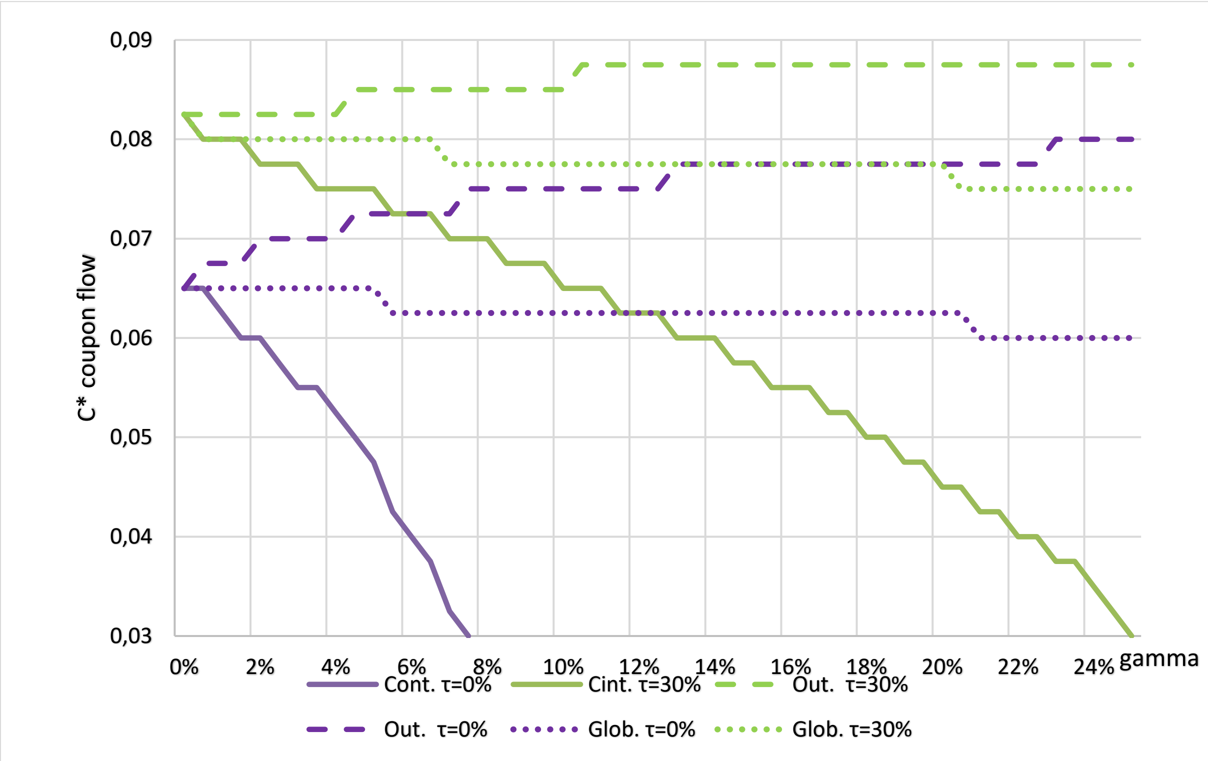
Graph 1 Debt Value

Coupon flow varying from 0.01 to 0.11, drift growth rate is additional drift η varying from 0% to 11%, asset A=1, risk free rate r =5%, volatility of the firm's assets σ =32%, corporate tax rate τ=0%, bankruptcy cost l=30%



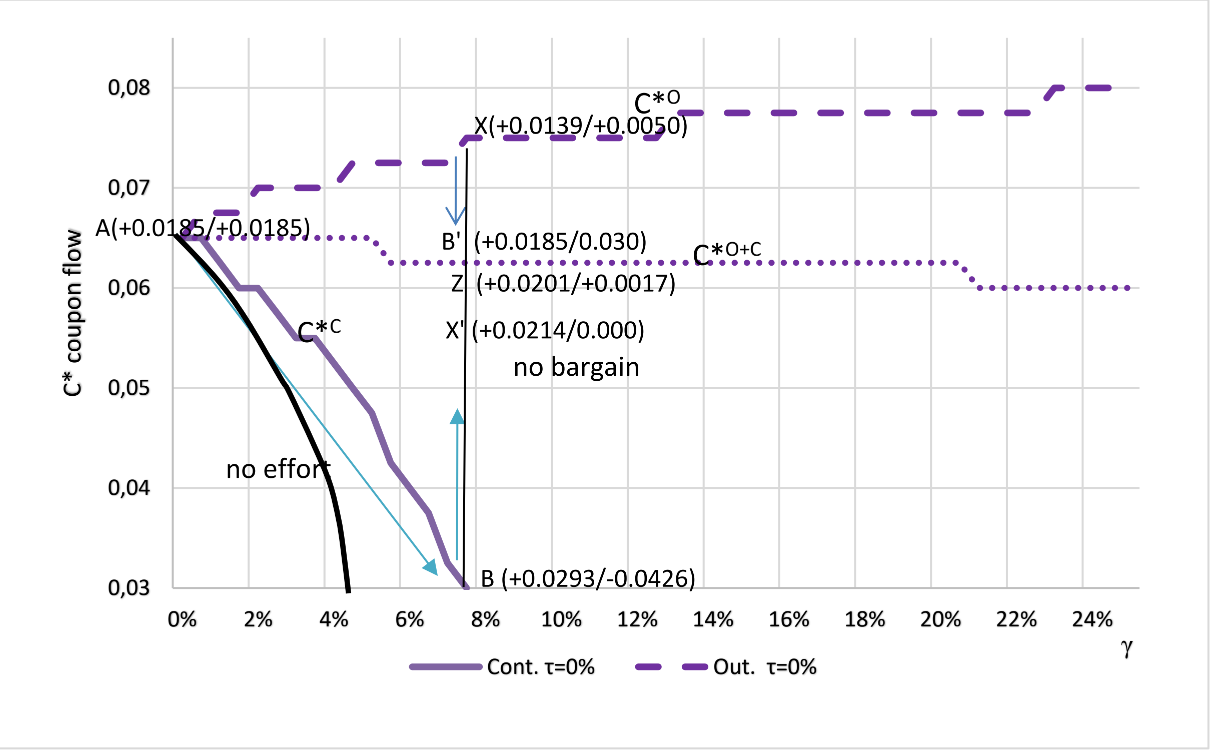
Graph 2 Tax rate and Outside shareholder's wealth

Outside shareholders' wealth with coupon flow varying from 0.03 to 0.09; initial coupon flow C_1=0.03; growth rate is additional drift η= 5%; asset A=1; risk free rate r =5%; volatility of the firm's assets σ =32%; corporate tax rate τ=0%; bankruptcy cost l=30%; appropriation rate γ=0.2; ownership α=50%; dots are maximum values for a given tax rate.



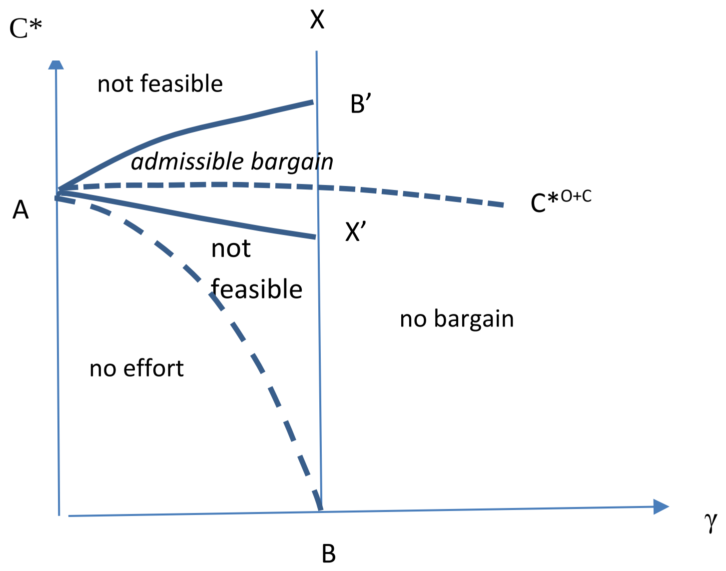
Graph 3 Optimal debt for outside shareholders and controlling shareholder

Outside shareholders' wealth: dashed line; controller' wealth: plain line; whole shareholders: dotted line; maximum wealth for different coupon flow flow varying from 0.03 to 0.085; initial coupon flow C_1=0.03; appropriation rate γ varying from 0% to 30%; drift is positive in appropriation rate: $\eta = a_0 \cdot \gamma * (1-\gamma)$, with a_0=0.2; asset A=1; risk free rate r =5%; volatility of the firm's assets σ =32%; corporate tax rate τ equal to 0% and 30%; bankruptcy cost l=32%; ownership α=50%



Graph 4 Optimal incentivization and re-leveraging settings

Outside shareholders' wealth: dashed line; controller' wealth: plain line; whole shareholders: dotted line; maximum wealth for different coupon flow varying from 0.03 to 0.085; initial coupon flow C_1=0.03; appropriation rate γ varying from 0% to 30%; drift is positive in appropriation rate: $\eta = a_0 . \gamma * (1-\gamma)$, with a_0=0.2; asset A=1; risk free rate r =5%; volatility of the firm's assets σ =32%; corporate tax rate τ equal to 0%; bankruptcy cost l=32%; ownership α=50%; data between parentheses after the points A, B, B',X, X' and Z are the increases in wealth from the origin of the axes situation (i.e. coupon low of 0.03 and γ=0%) for respectively the controlling and the outside shareholders.



Graph 5 - Definition of a feasible region of bargaining solution