May 30, 2018

Optimal portfolio in corporate pension plans: risk shifting and risk management

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Abstract

We derive the optimal corporate pension portfolio policy in a consolidated setting in the presence of PBGC insurance. The paper's result formalizes the forces of risk shifting and risk management that shape the form of the corporate pension portfolio. As in Rauh (2009), the risk-shifting and risk-management incentives increase when a sponsoring company runs into financial trouble. Unlike Rauh (2009), we show that risk management must not constitute a force countering risk shifting. On the contrary, for a company registering serious financial problems, the strategies driven by risk-shifting and risk-management motives are both extreme.

JEL classification: C61; D92; G11; G22; G23; G31; G32.

Keywords: optimal corporate pension portfolio policy; defined benefit pension plans; PBGC insurance; financial distress; risk shifting; risk management.

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†I am indebted to David Love and Joshua Rauh for insightful comments on an earlier version of this paper.

The effect that the Pension Benefit Guaranty Corporation (PBGC) insurance exerts on the finances of corporate defined benefit (DB) pension funds is a topic that today attracts a particular attention, as numerous distress terminations of DB plans occurred recently.

One of the fundamental issues addressed is whether or not the existence of the PBGC insurance leads to more risk-taking by corporate pension plans. The PBGC insurance effect advances the moral hazard issue, linked to the existence of the DB plan guarantee; firms in financial difficulty choose to underfund their pension plans and to invest in stocks (Sharpe, 1976; Treynor, 1977). The evidence is mixed as to the existence of the PBGC insurance effect in practice. This effect is documented by Bodie et al. (1985, 1987), Crossley and Jametti (2013), Guan and Lui (2016) and Bartram (2018), whereas Hsieh et al. (1994), Gallo and Lockwood (1995), Petersen (1996), Coronado and Liang (2005), Rauh (2009) and An et al. (2013) propose contradicting results.

The main argument that the literature puts forward to explain why in practice sponsoring firms in financial difficulty do not invest more in equity is that the risk-shifting incentive is being opposed by the risk-management incentive (Rauh, 2009). If the pension assets perform well, the firm can manage to avoid bankruptcy and will then face lower funding requirements, whereas in the case of bankruptcy, the PBGC will take responsibility for the generated pension deficit; however, if bankruptcy is avoided, but assets have performed poorly, large contributions may be necessary and can themselves financially threaten the company or at least prevent the firm from making profitable investments. The first mechanism is risk shifting, and tends to increase the pension risk level; the second mechanism is risk management, and tends to decrease the pension risk level. Love et al. (2011) explore the incentives of risk shifting and risk management in a two-period model that is solved via simulations.[1] They suggest that an analysis in an intertemporal setting could bring additional insights.[2] This paper's contribution is to formalize the forces

[1] In their model, the crucial assumption is that workers are unable to hedge firm-specific risk and thus demand a compensation for the financial risk in their pension benefit.

[2] For instance, one is then able to consider different measures of liabilities and analyze how the chosen measure relates to the form of the portfolio policy.

of risk shifting and risk management in a continuous-time setting. Analytical decision rules are obtained, and we judge whether or not the risk-management incentive counters the risk-shifting incentive for sponsoring firms in financial trouble.

A second issue in the corporate pension literature that deserves a formalization is how corporate policies evolve when moving from the non-consolidated to the consolidated environment. Considering the pension policy, moving from the non-consolidated to the consolidated perspective means moving from the traditional to the corporate pension policy view (Bodie et al., 1985).[3] In practice, sponsoring firms make decisions within a consolidated setting (Friedman, 1983; Bodie et al., 1985; Bergstresser et al., 2006; Rauh, 2006; Shivdasani and Stefanescu, 2010). When isolating the elements in the policies that change when moving from the non-consolidated to the consolidated perspective, one is able to determine and analyze the mechanisms that are inherent to the consolidated environment.

This paper's analysis proceeds in three steps. First, we derive the pension portfolio policy and the corporate investment and financing policies in non-consolidated environments. Second, we derive the three policies in a consolidated environment without the PBGC put. Third, we derive the three policies in a consolidated setting with the PBGC put. Moving from the first to the second step, we isolate the changes in the three policies induced by consolidation. Moving from the second to the third step, we isolate the changes induced by the incorporation of the PBGC put.

We prove that when moving from the non-consolidated to the consolidated setting, interactions between the pension portfolio policy and the corporate investment and financing policies emerge. The pension portfolio policy additionally hedges against the firm variables; the corporate investment and financing policies additionally hedge against the pension plan variables. We find that in the consolidated environment, the optimal policies of the firm and pension plan are driven

[3] The traditional view emphasizes that the DB pension plan is to be managed separately from the firm and in the best interests of the plan's participants. The corporate view considers the firm's extended balance sheet, which incorporates the pension assets and liabilities, and aims to manage the pension plan in the best interests of the firm's equityholders.

by speculative and hedging motives, and in each optimal policy, the hedging activity considers, apart from the state variables, both the variables related to the pension plan and those related to the firm. This result constitutes an extension with respect to the existing literature, which has fundamentally regarded the pension portfolio policy from the corporate perspective solely as a hedging tool, used to optimize the firm's investment or financing decisions (Merton, 2006).

We find that the effects of the PBGC insurance depend on whether or not the PBGC put is in the money (ITM). When the put is out of the money (OTM), the sponsoring company is healthy or the pension plan is funded. In this case, the portfolio policy is comparable to the no-put portfolio policy. When the put is ITM, the firm ran into financial trouble and the plan is underfunded. In this configuration, the portfolio policy is crucially distorted by the presence of the PBGC put. The fact that the speculative and hedging terms are divided by one plus the delta of the put importantly impacts on the portfolio behavior.

The corporate pension portfolio rule that we obtain formalizes the forces of risk shifting and risk management. When the put is ITM, we show that the risk-shifting and risk-management incentives increase, a result compatible with Rauh (2009). However, we find that the risk-management incentive must not have the stabilizing property on the portfolio policy surmised by Rauh (2009). Indeed, the put-induced distortions on both of the incentives are preoccupying. For a put that is deeply ITM, the whole portfolio policy appears as extreme.

The paper is organized as follows. In the first section, the model is built and solved. The results are discussed in the second section. The last section concludes.

1 The model

We consider a firm that has created a DB pension plan for its employees.

1.1 The balance sheets

One first studies the firm's and DB plan's simplified balance sheets (BS) separately. One obtains, respectively, the following:

$$BS\ (1)\ \begin{array}{|c|c|}\hline A^F & D^F \\ & X^F \\ \hline \end{array}$$

$$BS\ (2)\ \begin{array}{|c|c|}\hline A^P & L^P \\ & X^P \\ \hline \end{array}$$

where A^F and A^P, respectively represent the firm's and pension plan's assets, D^F is the firm's debt, L^P are the pension plan's liabilities, and $X^F \equiv A^F - D^F$ and $X^P \equiv A^P - L^P$ are the firm's equity and DB plan's net position, respectively.

The consolidated balance sheet incorporates both the firm's and DB plan's items. It is written as follows:

$$BS\ (3)\ \begin{array}{|c|c|}\hline A^F & D^F \\ A^P & L^P \\ & X^{F+P} \\ \hline \end{array}$$

where $X^{F+P} \equiv X^F + X^P$ is the consolidated balance sheet equity.[4]

Let us introduce the PBGC insurance. The sponsoring company has the ability to pass the burden of its DB plan deficit, if such a deficit occurs, to the PBGC. The sponsoring firm buys a put P^P, written on the DB pension plan's assets and with the pension liabilities as the strike. The put can only be exercised in the case of the sponsoring firm's bankruptcy. The option is American-type; the exercise time is stochastic and corresponds to the firm's bankruptcy.[5] The put value at maturity T is written as $P^P(T) = Max(L^P(T) - A^P(T); 0)1_{A^F(T)<D^F(T)}$. P^P is ITM when the pension plan is underfunded *and* the firm is bankrupt.

The balance sheet has the following form:

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[4] Merton (2006) proposes a similar balance sheet form.

[5] The characteristics of the PBGC put are described in Marcus (1985, 1987) and Pennacchi and Lewis (1994).

	A^F	D^F
BS (4)	A^P	L^P
	P^P	X

with $X \equiv X^F + X^P + P^P$ being the consolidated balance sheet equity with the PBGC put.

1.2 The optimization programs

Let us define the optimization programs. BS (4) constitutes the benchmark balance sheet.

The firm manager chooses policies that are optimal from the equityholders' point of view.[6] , [7] He has the following program at date t:

$$MaxE_t \left[U(Z(T)) \right] \tag{1}$$

where E_t stands for the expectation, conditional on the information available in t, U is the utility function, assumed to be increasing and concave in Z and respecting the Inada conditions: $\lim_{Z \to \infty} U_Z = 0$ and $\lim_{Z \to 0} U_Z = \infty$, with U_Z being the derivative of U with respect to Z, and

$$Z \equiv \alpha \left(A^F - D^F \right) + \beta \left(A^P - L^P \right) + \gamma P^P \tag{2}$$

with $\alpha, \beta, \gamma = \{0, 1\}$. T is the firm's horizon.[8]

The definition of Z allows the analysis of several relevant cases:

(i) $\alpha = 1$ and $\beta = \gamma = 0$: non-consolidated firm-only balance sheet - BS (1);

(ii) $\beta = 1$ and $\alpha = \gamma = 0$: non-consolidated DB plan-only balance sheet - BS (2);

(iii) $\alpha = \beta = 1$ and $\gamma = 0$: consolidated balance sheet without the PBGC put, as materialized by BS (3);

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[6] The compatibility, or not, of the interests of equityholders, participants and PBGC is discussed in the second section.

[7] Agency issues between the manager and equityholders are thus ignored.

[8] For BS (1), BS (3) and BS (4), one directly concludes that T is the firm's horizon. Regarding BS (2), T is the pension plan's horizon, where T normally coincides with the firm's horizon, as the pension plan's normal functioning is conditional on the firm being in activity. The pension plan's horizon could turn out to be shorter than the firm's horizon in the case of a voluntary termination of the pension plan.

(iv) $\alpha = \beta = \gamma = 1$: consolidated balance sheet with PBGC put, as represented by BS (4).

The optimization program is then $MaxE_t\left[U(X^F(T))\right]$, $MaxE_t\left[U(X^P(T))\right]$, $MaxE_t\left[U(X^{F+P}(T))\right]$ and $MaxE_t\left[U(X(T))\right]$ in cases (i), (ii), (iii) and (iv), respectively.

We assume that the sponsoring firm is risk averse. Let us begin by noting that Sundaresan and Zapatero (1997) make the same assumption. They work in a non-consolidated DB plan-only setting and use the program we are proposing for case (ii).

The firm's risk aversion stems from financial constraints that the firm is facing. These constraints are the result of informational asymmetries in capital markets which limit the firm's ability to raise external finance. Because of these capital market imperfections, the firm could be forced to forego profitable investment projects in some states of the world. As a consequence, the firm hedges to avoid situations of underinvestment. Hedging allows to ensure that the firm has anytime sufficient internal funds at its disposal to invest in attractive investment projects. The assumption of informational imperfections in capital markets is made by Greenwald and Stiglitz (1987), for instance.[9] These authors model the firm as maximizing the expected utility of future equity value, which is compatible with the program we are proposing for case (i).

We close by adding that the case we are dealing with in this paper is characterized by the presence of limited liability. Gollier et al. (1997) emphasize that this kind of problem should be analyzed under the assumption of risk aversion. The reason for that is that under risk neutrality, firms invest in the riskiest projects they have, which is incompatible with the behavior that is observed in practice. The principles that are driving the behavior of a risk averse decision maker are more realistic. The authors see the decision maker's risk aversion as "a proxy for taking into account the imperfection of capital markets, and more specifically the fact that the firm's shareholders cannot perfectly diversify their own portfolios".

Let us focus on the decisions to be made optimally, which will enable the definition of our

[9] Assumptions in the same spirit can be found in Froot et al. (1993), Greenwald and Stiglitz (1993) or Rochet and Villeneuve (2005).

problem control variables. The major decisions a firm makes are those related to investment and financing. The main task faced in managing the DB plan is the definition of a portfolio strategy.[10]

The variables representative of the firm's investment and financing decisions are the assets to equity ratio $x_{A^F} \equiv \frac{A^F}{X^F}$ and the debt to equity ratio $x_{D^F} \equiv \frac{D^F}{X^F}$, respectively, with $x_{A^F} - x_{D^F} = 1$. The investment decision is defined by the A^F level. One considers that the firm's total asset value is representative of the chosen level of investment activity. When expanding, the total asset value increases; when downsizing, the latter decreases. When new investment projects are undertaken, new assets are acquired; when eliminating existing projects, some assets are sold.[11] Expanding (downsizing) also increases (reduces) the expected discounted value of future cash flows.[12]

The DB plan's portfolio policy is defined by the proportions of the plan's assets A^P to invest in the stock market index S and in the riskless asset η. We have:

$$A^P = X_S S + X_\eta \eta \tag{3}$$

where X_S and X_η the number of assets S and η, respectively. The portfolio strategy is then characterized by the plan's asset proportions $x_S \equiv \frac{X_S S}{A^P}$ to attribute to S and $x_\eta \equiv \frac{X_\eta \eta}{A^P}$ to give to η, with $x_S + x_\eta = 1$.

The optimization program is thus the following:

$$Max_{x_{A^F}, x_S} E_t \left[U(Z(T)) \right] \tag{4}$$

In cases (i), (ii), (iii) and (iv), one obtains the optimization programs $Max_{x_{A^F}} E_t \left[U(X^F(T)) \right]$, $Max_{x_S} E_t \left[U(X^P(T)) \right]$, $Max_{x_{A^F}, x_S} E_t \left[U(X^{F+P}(T)) \right]$ and $Max_{x_{A^F}, x_S} E_t \left[U(X(T)) \right]$, respectively.

[10] The second pension plan's decision - the funding level - is constrained by law (Rauh, 2009).

[11] For Biais et al. (2010), downsizing indeed leads to the liquidation of a fraction of the firm's assets. For DeMarzo and Fishman (2007), investment is a decision to expand or contract the firm, the size of the latter being measured by its (physical) scale.

[12] Following DeMarzo and Fishman (2007), when rescaling the firm, all of the cash flows associated with the firm are also rescaled.

1.3 The variables dynamics

The relevant variables obey the following dynamics:

$$\frac{dA^F(t)}{A^F(t)} = \mu_{A^F}(t, Y(t))dt + \sigma_{A^F}(t, Y(t))dB^{A^F}(t) \tag{5}$$

$$\frac{dD^F(t)}{D^F(t)} = \mu_{D^F}(t, Y(t))dt + \sigma_{D^F}(t, Y(t))dB^{D^F}(t) \tag{6}$$

$$\frac{dS(t)}{S(t)} = \mu_S(t, Y(t))dt + \sigma_S(t, Y(t))dB^S(t) \tag{7}$$

$$\frac{d\eta(t)}{\eta(t)} = r(t, Y(t))dt \tag{8}$$

$$\frac{dL^P(t)}{L^P(t)} = \mu_{L^P}(t, Y(t))dt + \sigma_{L^P}(t, Y(t))dB^{L^P}(t) \tag{9}$$

$$\frac{dc(t)}{c(t)} = \mu_c(t, Y(t))dt + \sigma_c(t, Y(t))dB^c(t) \tag{10}$$

$$\frac{dw(t)}{w(t)} = \mu_w(t, Y(t))dt + \sigma_w(t, Y(t))dB^w(t) \tag{11}$$

where c represents the contribution flow from the firm to the DB plan and w is the withdrawal flow from the DB plan, taking the form of pension benefits paid to the retired employees. $\mu_i(t, Y(t))$, the bounded function of time t and the vector of K state variables Y, denotes the expectation of the instantaneous variation rate of i, $\sigma_i(t, Y(t))$, the bounded function of t and Y, is its standard deviation, $B^i(t)$ stands for a standard Brownian motion, instantaneously correlated with $B^j(t)$ with coefficient ρ_{ij}, where $dB^i(t)dB^j(t) = \rho_{ij}dt$, $-1 \leq \rho_{ij} \leq 1$ and $i, j = \left\{A^F, D^F, S, L^P, c, w\right\}$. $r(t, Y(t))$ is the instantaneously riskless interest rate, which is assumed to depend on t and Y.

K stochastic state variables are present in the economy. The k-th variable Y_k dynamics is written as follows:

$$\frac{dY_k(t)}{Y_k(t)} = \mu_{Y_k}(t, Y(t))dt + \sigma_{Y_k}(t, Y(t))dB^{Y_k}(t) \tag{12}$$

where $\mu_{Y_k}(t, Y(t))$ and $\sigma_{Y_k}(t, Y(t))$, bounded functions of t and Y, are the expectation and standard deviation, respectively, of the instantaneous variation rate of Y_k, and $B^{Y_k}(t)$ stands for a standard Brownian motion, instantaneously correlated with $B^{Y_l}(t)$ with coefficient $\rho_{Y_k Y_l}$, where $dB^{Y_k}(t)dB^{Y_l}(t) = \rho_{Y_k Y_l}dt$, $-1 \leq \rho_{Y_k Y_l} \leq 1$ and $k, l = \{1, 2, ..., K\}$.

State variables can be the main variables themselves, i.e. A^F, D^F, S, L^P, c and w. As a consequence, the given dynamics (5) to (11) incorporate interactions between these main variables. Other state variables can be the interest rate, the wage, inflation, exchange rates.

The pension plan's liabilities L^P are stochastic. The view of liabilities adopted in this paper is thus the projected benefit obligation (PBO) view, in line with Black (1989) and Lucas and Zeldes (2006). When valuing liabilities, one discounts with the expected return rate of securities of equivalent risk. However, one can move directly to the accumulated benefit obligation (ABO) view simply by assuming that $\sigma_{L^P} = 0$. The discount rate is then the riskless interest rate.

1.4 The optimal policies

Table 1 presents the optimal investment and portfolio policies in cases (i) to (iv).

Appendix A develops the proof.

Appendix B proposes a technical description of the structure of the optimal policies.

Table 1: Optimal policies in cases (i), (ii), (iii) and (iv)

	firm's optimal investment policy x_{A^F}	DB plan's optimal portfolio policy x_S
cases (i) and (ii)	$x^i_{A^F} = -\frac{J_{X^F}}{J_{X^F X^F} X^F} \frac{\mu_{A^F}-\mu_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{\sigma_{A^F D^F}-\sigma^2_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $+\frac{c}{X^F} \frac{\sigma_{A^F c}-\sigma_{D^F c}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\sum_k \frac{J_{X^F Y_k} Y_k}{J_{X^F X^F} X^F} \frac{\sigma_{A^F Y_k}-\sigma_{D^F Y_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$	$x^{ii}_S = -\frac{J_{X^P}}{J_{X^P X^P} X^P} \frac{X^P}{A^P} \frac{\mu_S - r}{\sigma^2_S}$ $+\frac{L^P}{A^P} \frac{\sigma_{S L^P}}{\sigma^2_S}$ $-\frac{c}{A^P} \frac{\sigma_{Sc}}{\sigma^2_S}$ $+\frac{w}{A^P} \frac{\sigma_{Sw}}{\sigma^2_S}$ $-\sum_k \frac{J_{X^P Y_k} Y_k}{J_{X^P X^P} X^P} \frac{X^P}{A^P} \frac{\sigma_{S Y_k}}{\sigma^2_S}$
case (iii)	$x^{iii}_{A^F} = -\frac{J_{X^{F+P}}}{J_{X^{F+P} X^{F+P}} X^{F+P}} \frac{X^{F+P}}{X^F} \frac{\mu_{A^F}-\mu_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{\sigma_{A^F D^F}-\sigma^2_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{A^P}{X^F} x_S \frac{\sigma_{A^F S}-\sigma_{D^F S}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $+\frac{L^P}{X^F} \frac{\sigma_{A^F L^P}-\sigma_{D^F L^P}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $+\frac{w}{X^F} \frac{\sigma_{A^F w}-\sigma_{D^F w}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\sum_k \frac{J_{X^{F+P} Y_k} Y_k}{J_{X^{F+P} X^{F+P}} X^{F+P}} \frac{X^{F+P}}{X^F} \frac{\sigma_{A^F Y_k}-\sigma_{D^F Y_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$	$x^{iii}_S = -\frac{J_{X^{F+P}}}{J_{X^{F+P} X^{F+P}} X^{F+P}} \frac{X^{F+P}}{A^P} \frac{\mu_S - r}{\sigma^2_S}$ $-\frac{X^F}{A^P} \frac{x_{A^F}\left(\sigma_{S A^F}-\sigma_{S D^F}\right)+\sigma_{S D^F}}{\sigma^2_S}$ $+\frac{L^P}{A^P} \frac{\sigma_{S L^P}}{\sigma^2_S}$ $+\frac{w}{A^P} \frac{\sigma_{Sw}}{\sigma^2_S}$ $-\sum_k \frac{J_{X^{F+P} Y_k} Y_k}{J_{X^{F+P} X^{F+P}} X^{F+P}} \frac{X^{F+P}}{A^P} \frac{\sigma_{S Y_k}}{\sigma^2_S}$
case (iv)	$x^{iv}_{A^F} = -\frac{J_X}{J_{XX} X} \frac{X}{X^F} \frac{\mu_{A^F}-\mu_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{\sigma_{A^F D^F}-\sigma^2_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{A^P\left(1+P^P_{A^P}\right)}{X^F} x_S \frac{\sigma_{A^F S}-\sigma_{D^F S}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{L^P\left(-1+P^P_{L^P}\right)}{X^F} \frac{\sigma_{A^F L^P}-\sigma_{D^F L^P}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $+\frac{w}{X^F} \frac{\sigma_{A^F w}-\sigma_{D^F w}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\frac{1}{X^F} \sum_{k=1}^K P^P_{Y_k} Y_k \frac{\sigma_{A^F Y_k}-\sigma_{D^F Y_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$ $-\sum_k \frac{J_{X Y_k} Y_k}{J_{XX} X} \frac{X}{X^F} \frac{\sigma_{A^F Y_k}-\sigma_{D^F Y_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^F D^F}}$	$x^{iv}_S = -\frac{J_X}{J_{XX} X} \frac{X}{A^P\left(1+P^P_{A^P}\right)} \frac{\mu_S - r}{\sigma^2_S}$ $-\frac{X^F}{A^P\left(1+P^P_{A^P}\right)} \frac{x_{A^F}\left(\sigma_{S A^F}-\sigma_{S D^F}\right)+\sigma_{S D^F}}{\sigma^2_S}$ $-\frac{L^P\left(-1+P^P_{L^P}\right)}{A^P\left(1+P^P_{A^P}\right)} \frac{\sigma_{S L^P}}{\sigma^2_S}$ $+\frac{w}{A^P\left(1+P^P_{A^P}\right)} \frac{\sigma_{Sw}}{\sigma^2_S}$ $-\frac{1}{A^P\left(1+P^P_{A^P}\right)} \sum_{k=1}^K P^P_{Y_k} Y_k \frac{\sigma_{S Y_k}}{\sigma^2_S}$ $-\sum_k \frac{J_{X Y_k} Y_k}{J_{XX} X} \frac{X}{A^P\left(1+P^P_{A^P}\right)} \frac{\sigma_{S Y_k}}{\sigma^2_S}$

The subscripts on J (the indirect utility function) and on P^P denote partial derivatives. σ_{op} stands for the covariance between any variables o and p.

In the three cases (i), (iii) and (iv), one obtains the firm's optimal financing policy x_{D^F} by recalling that $x_{D^F} = x_{A^F} - 1$. All of the portfolio terms in x_{D^F}, with the exception of the second term, are identical to the x_{A^F} terms. The second term, $-\frac{\sigma_{A^F D^F} - \sigma^2_{D^F}}{\sigma^2_{A^F} + \sigma^2_{D^F} - 2\sigma_{A^F D^F}}$ in x_{A^F}, becomes $-\frac{\sigma^2_{A^F} - \sigma_{A^F D^F}}{\sigma^2_{A^F} + \sigma^2_{D^F} - 2\sigma_{A^F D^F}}$ in x_{D^F}.

2 Results

2.1 Overview of the main results

a. When moving from the non-consolidated to the consolidated setting, interactions between the pension portfolio policy and the corporate investment and financing policies emerge. The pension portfolio policy additionally hedges against the firm variables; the corporate investment and financing policies additionally hedge against the pension plan variables.

b. The effects of the incorporation of the PBGC put are the following.

When the put is OTM, the sponsoring company is healthy or the pension plan is funded. In this case, the put effect on all the three policies is limited. The pension portfolio policy is comparable to the no-put portfolio policy. The corporate investment and financing policies are comparable to the no-put policies.

When the put is ITM, the firm ran into financial trouble and the plan is underfunded. In this configuration, the pension portfolio policy is crucially distorted by the presence of the PBGC put. The fact that the speculative and hedging terms are divided by one plus the delta of the put acquires a major importance. The corporate investment and financing policies do not register crucial put-induced distortions.

c. The pension portfolio policy in the presence of PBGC insurance provides a formalization of the forces of risk shifting and risk management.

d. The focus of this paper's analysis is on the ITM-put case. As Rauh (2009), we show that the risk-shifting and risk-management incentives increase in this configuration. Yet, contrary to Rauh (2009), we find that the risk-management incentive must not have a stabilizing property on the portfolio policy. Indeed, the put-induced distortions on the two incentives are preoccupying. For a put that is deeply ITM, the whole portfolio policy appears as extreme.

e. To conclude, one should consider other factors than risk management to explain the empirical fact that corporate sponsors in financial difficulty invest more in safe securities.

2.2 The mechanisms driving the consolidated optimal policies

When moving from the non-consolidated to the consolidated perspective, new hedge funds emerge in the optimal policies: the firm (DB plan) additionally hedges against the DB plan (firm) variables. These hedges are representative of the interactions between the pension portfolio policy and the corporate investment and financing policies. They constitute the mechanism that is inherent to the consolidated environment.

In the existing literature, the pension portfolio policy from the corporate perspective has been fundamentally regarded solely as a hedging tool, used to optimize the firm's investment or financing decisions. The general advice appears to be low equity investment in the pension fund, enabling more risk-taking in operating activities or in the capital structure (Merton, 2006). According to this approach, the pension portfolio rule, as defined by Eq. x_S^{iii} in Table 1, would reduce to the hedging term for the firm variables.[13] , [14] The view proposed in this paper appears to be more general than that adopted in the existing literature. The optimal pension portfolio policy incorporates the element described in this literature. However, it also includes other elements: first the speculative fund, representative of the risk-reward arbitrage, then hedging terms for the stochastic variables characteristic of the pension plan's (and not only firm's) activity, and eventually the state-variable hedge fund. All the terms encountered in Eq. x_S^{iii} together form the optimal pension portfolio policy under a fairly general understanding. In addition, we show

[13] One refers to the A^F- D^F hedging term, i.e., $-\frac{X^F}{A^P}\frac{x_{A^F}(\sigma_{SA^F}-\sigma_{SD^F})+\sigma_{SD^F}}{\sigma_S^2}$.
The latter term will constitute the sole element of the optimal portfolio policy if the optimization program takes the form $Min_{x_S}\sigma^2_{X^{F+P}}$, under the assumption of non-stochastic L^P and w variables.

[14] The equations referenced in section 2 are shown in Table 1.

a mechanism not yet derived in the literature: how the firm's conventional decisions evolve when moving from the separate to the consolidated perspective. Here, the main modification is that these rules now incorporate hedging terms for the pension plan variables.

To conclude, in our general setting, the firm and pension plan's optimal policies are driven by speculative and hedging motives, and in each optimal policy, the hedging activity considers, apart from the state variables, both the variables related to the pension plan and those related to the firm.

2.3 The PBGC insurance effect in a consolidated environment

2.3.1 The crucial PBGC put-induced distortion

Let us begin with a simple intuition on how the PBGC put impacts the agent's optimal behavior. When the PBGC put is introduced, the agent optimizes with respect to the former (no-put) payoff augmented by the put. Intuition then leads us to assume that her optimal behavior should be riskier: in the case of an unfavorable outcome, she has the possibility of passing the burden of part of the pension liabilities to the insuring company by exercising the put.

When observing the optimal portfolio policy x_S^{iv}, one notices that the speculative and hedging positions are divided by $1 + P_{A^P}^P$. This is the crucial put-induced distortion, and one would like to understand its source. The dynamics of payoff X (see Eq. (27)) reveal that because of the put introduction, the investment in the risky asset has a standard deviation of $\left(1 + P_{A^P}^P\right)\sigma_S$ and no longer σ_S. The investment in the risky asset thus becomes less risky, and the higher $P_{A^P}^P$ (in absolute value), the lower is the risk attached to this investment. The fact that the standard deviation of the investment in the risky asset is now $\left(1 + P_{A^P}^P\right)\sigma_S$ impacts crucially on the speculative position and on the hedging demand. Both are divided by the term multiplying σ_S, that is $1 + P_{A^P}^P$.

We shall consider two cases: the put is OTM or ITM. First, when the put is OTM, its value and its sensitivities are close to zero. In this case, x_S^{iv} is comparable, in its mechanisms, to x_S^{iii}. The portfolio policy in the presence of the put is comparable to the no-put portfolio policy. Second, when the put is ITM, the absolute value of sensitivities increases. In particular, for an option deeply ITM, $P_{A^P}^P$ could approach -1. The portfolio policy thus registers substantial modifications with respect to the OTM-option case. For an option that is deeply ITM, dividing a term by $1 + P_{A^P}^P$ can lead to a substantial increase in its absolute value.

2.3.2 Formalization of the forces of risk shifting and risk management

Results in the existing literature suggest that adding a put to the agent's payoff increases her optimal risk exposure. In the non-pension literature, Gollier et al. (1997) conclude that under limited liability, the agent is in fact endowed with a free put option, and she always increases her risk exposure with respect to the full-liability case. Also, Ross (2004) finds that adding a put to the payoff makes the agent more willing to take risk. In the DB pension literature, the original results about the moral hazard triggered by the PBGC guarantee by Sharpe (1976) and Treynor (1977) also point out an increase in the pension risk.

However, Rauh (2009) describes a more contrasted picture in the DB plan context. Rauh (2009) emphasizes that, for a sponsoring firm approaching distress, two types of mechanisms - risk shifting and risk management - determine together the chosen pension risk level. On the one hand, risk shifting implies that the firm is tempted to increase the pension risk degree, as if the assets perform well, the firm can manage to avoid bankruptcy and will then face lower funding requirements, whereas in the case of bankruptcy, the PBGC will take responsibility for the generated pension deficit. On the other hand, risk-management incentives work in the opposite direction: they tend to decrease the pension risk level. If bankruptcy is avoided, but assets have performed poorly, large contributions may be necessary and can themselves financially threaten the company or at least prevent the firm from making profitable investments. Rauh (2009) concludes that on average, the second effect dominates among U.S. firms.

Let us build a link between the mechanisms described above and our modeling. One first recalls that the terms of an optimal portfolio strategy, as defined by Eq. x_S^{iv} for instance, structurally incorporate two element types. The first term is the speculative fund, representative of the risk-taking behavior; the remaining terms are hedging elements, which represent risk-management mechanisms. One can consider that the speculative term is here, in a larger sense, a representation of the risk-shifting mechanism, as in the case of bankruptcy, the consequences of this risk-taking will be effectively borne by the PBGC. One thus obtains a solution defining the optimal portfolio policy, which includes both the risk-shifting term and the risk-management terms.

We have seen that in policy x_S^{iv}, the speculative and hedging terms are divided by one plus the delta of the put. Therefore, when the firm is in financial trouble and the plan is underfunded, both the risk-shifting and risk-management incentives are more pronounced, as predicted by Rauh (2009).

2.3.3 The nature of risk management

Many authors find that in practice, sponsoring firms adopt a less risky policy when their financial condition is weak. Rauh (2009) concludes that based on observed firm behavior, the risk-management incentive dominates among US firms.

This paper's understanding on the role of risk management within the pension portfolio policy is different from Rauh's (2009).

Risk management must not lead to a decrease in the riskiness of the pension portfolio policy.[15]

Looking at x_S^{iv}, hedging increases (decreases) the optimal risky asset demand if the sum of all of the terms except the first one is positive (negative). Whether a hedging position increases or decreases the optimal risky asset demand depends on its sign, the sign of the involved variables, the sign of the involved correlation.

Also, in practice, there is the problem that it is not easy to observe the firm's hedging behavior. Measuring the presence and the extent of hedging is not straightforward. Hedging can be done

[15] In its myopic part, risk management responds to the objective of minimizing the variance of the payoff (here X) return rate; yet, risk management does not necessarily imply a decrease in the risky asset demand of the pension portfolio.

via an investment in futures, forwards, or options; yet, it can be also done by other means, like altering real operating decisions (Smith and Stulz, 1985). Smith and Stulz (1985) adopt the following definition of hedging: hedging a state variable reduces the dependence of firm value on changes in the state variable.

But whatever the sign of the hedging demand, the fact that it is so importantly distorted by the put presence is rather preoccupying, as is the put-induced distortion in the speculative position. In the case of a deeply ITM-option, both the speculative and hedging positions can be seen as extreme. The portfolio policy as a whole is then aggressive.

To conclude, first, the effect of an ITM-put on the speculative position is preoccupying. Second, the fact that the risk-management position is affected in a comparable way by an ITM-put does not solve the problem of the put-induced distortion in the speculative position. As a consequence, we agree with Rauh (2009) that an ITM-put does lead to an increase in both the risk-shifting and the risk-management incentives. However, contrary to Rauh (2009), we see the put-induced distortions on the two incentives as preoccupying. For a deeply ITM-put, the whole portfolio policy is extreme.

2.3.4 The influence of the participants and of the PBGC

Considering the behavior of sponsoring firms in practice, if risk management does not explain why firms in financial difficulty do not invest more in equities, then what is the explanation? Other forces than risk management could be at work. One of the aspects to consider is the influence that the participants and the PBGC are exerting on the sponsoring firm.

Nowadays, the participants' and the PBGC's awareness of the risks that they are facing is high. When the situation is threatening to these agents, they are aware of that and react accordingly. As an illustration, let us imagine the following situation in today's world: a firm currently invests 60% of its pension portfolio in risky assets; however, as the firm's financial condition has deteriorated, it is now optimal for the firm to invest 90% in risky assets. If the firm decides to move to the 90%

portfolio, the participants and the PBGC will evidently protest against this increase.

It seems important to understand when the interests of the sponsoring firm cease to be compatible with the interests of the participants and of the PBGC.

The participants' main objective is to obtain the promised pension amount. In the distress termination case, the amount they receive could be lower than what was agreed upon with the firm, as the PBGC guarantee is subject to a ceiling. Participants thus aim to avoid the pension plan termination. The PBGC's goal is also to avoid the plan termination, in order not to have to pay for the pension deficit. The participants and the PBGC thus disagree with pension portfolio policies that are not sound financially and can be thus destabilizing for the firm and the plan. The policy x_S^{iv} in the case of an ITM-put is not financially sound. This policy is the optimal strategy from the point of view of shareholders, yet it is threatening to both the participants and the PBGC due to its extreme character.

When the firm is in financial trouble and the plan is underfunded, the participants and the PBGC become preoccupied and put pressure on the firm's management not to adopt the policy x_S^{iv}. Participants can influence the firm's manager directly. The plan management is assumed to be done in the participants' best interest. Therefore, the participants' opinion must be taken into account, in part at least, by the firm's manager.

The PBGC's influence on the pension portfolio policy is less direct. Yet, the PBGC is vigilant before and after distress. The PBGC monitors the financial condition of the pension plans that it covers, via the Early Warning Program for instance.[16] The recent episode with American Airlines shows that the PBGC is vigilant when sponsoring companies run into financial trouble. Also, regulation gives additional tools to the PBGC to prevent the occurrence of large losses. For

[16] The Early Warning Program constitutes a control tool for the PBGC over troubled companies (http://www.pbgc.gov/prac/risk-mitigation.html). Under this program, the PBGC adopts a preemptive attitude, trying to avoid the future occurrence of large losses. The underlying mechanism consists of a monitoring system for companies with weak credit ratings or underfunded pension plans. Representing the program's focus are corporate transactions that could jeopardize pensions and lead to a pronounced increase in the risk of long-run loss to the PBGC. Of particular interest are transactions that can importantly weaken the financial support for a pension plan, as the breakup of a controlled group or a leveraged buyout. If the transaction to occur is judged to pose a threat, the PBGC aims to negotiate additional protections for the pension plan.

instance, the PBGC can decide to terminate a plan before maturity.

2.3.5 Other possible explanations for the pension equity proportion decrease in distress

Signaling A company that is running into financial problems is closely observed, and the firm is aware of that. One of the company's objectives is to keep up appearances, and to act like as if the firm were to be financially sound again. A pension equity proportion increase would be certainly interpreted as a bad signal concerning the firm's future. On the contrary, an equity proportion decrease would lead to the desired signaling effect, i.e. that the firm will be financially sound again, and also, that the firm is doing what is possible to protect the pension plan's participants.

The pension portfolio policy interpreted as a hedging tool If the pension portfolio policy from the company's point of view is regarded only as a hedging tool, used to optimize the firm's investment or financing decisions, the advice for the firm is a low pension equity investment, to enable more risk-taking in operating activities or in the capital structure (Merton, 2006). From this perspective, the firm in distress decreases the pension equity allocation because it needs to take more risk elsewhere.

Consolidation may not hold in distress If consolidation does not hold in distress, the firm may conduct a portfolio policy different from x_S^{iv}.

Other aspects to consider Some other factors, firm-specific, play a role when defining the pension portfolio policy, i.e. firm size, ownership structure, managerial power, industry structure, labor/union power, politics/bailouts.

3 Conclusion

A large part of the empirical literature on corporate pension plans finds that firms in financial difficulty with underfunded plans do not exploit the moral hazard triggered by the PBGC insurance and invest in safer securities. The argument evoked is that the risk-shifting incentive is dominated by the risk-management incentive. This argument implies that it is in the shareholders' interest to invest in safer securities when the company's financial condition is weak. This argument also implies that the participants and the PBGC do not have to worry about the pension portfolio policy when the company runs into financial trouble. On the contrary, this paper's main result is that both risk-shifting and risk-management incentives can become extreme when the firm is in financial difficulty, and that the risk-management incentive must not have the stabilizing property put forward in the literature. We emphasize that the participants and the PBGC should remain cautious when the sponsoring firm runs into financial trouble. It is likely that one of the factors that shape the firm's behavior is the (more or less direct) influence that the participants and the PBGC are exerting on the firm's decisions.[17] The participants' and PBGC's impact on the pension fund finances is an issue that deserves further study, both theoretical and empirical.

[17] There is evidence that depending on the sponsoring firm, equityholders' interests are taken into account to a more or less large extent. Cocco and Volpin (2007) find that in the UK, pension plans of indebted companies with a higher proportion of insider trustees (who are also executive directors of the sponsoring company) than independent trustees invest a higher proportion of pension assets in equities.

APPENDICES

APPENDIX A: SOLUTION TO THE OPTIMIZATION PROGRAM

a. The Z dynamics

Let us determine the dynamics of Z. One differentiates Eq. (2) by taking account of the DB plan's asset composition, defined by Eq. (3), and of the flows of contributions c and withdrawals w, to obtain the following:

$$
\begin{aligned}
dZ &= \alpha\left(dA^F - dc - dD^F\right) \\
&\quad +\beta\left(X_S dS + X_\eta d\eta + dc - dw - dL^P\right) \\
&\quad +\gamma dP^P
\end{aligned}
\tag{13}
$$

The Z dynamics is written as follows:

$$
\begin{aligned}
\frac{dZ}{Z} &= \frac{\alpha X^F}{Z}\left(x_{A^F}\frac{dA^F}{A^F} - \frac{c}{X^F}\frac{dc}{c} - x_{D^F}\frac{dD^F}{D^F}\right) \\
&\quad +\frac{\beta A^P}{Z}\left(x_S\frac{dS}{S} + x_\eta\frac{d\eta}{\eta} + \frac{c}{A^P}\frac{dc}{c} - \frac{w}{A^P}\frac{dw}{w} - \frac{L^P}{A^P}\frac{dL^P}{L^P}\right) \\
&\quad +\frac{\gamma P^P}{Z}\frac{dP^P}{P^P}
\end{aligned}
\tag{14}
$$

Using $x_{A^F} - x_{D^F} = 1$ and $x_S + x_\eta = 1$, the Z dynamics becomes:

$$
\begin{aligned}
\frac{dZ}{Z} &= \frac{\alpha X^F}{Z}\left(x_{A^F}\left(\frac{dA^F}{A^F} - \frac{dD^F}{D^F}\right) + \frac{dD^F}{D^F} - \frac{c}{X^F}\frac{dc}{c}\right) \\
&\quad +\frac{\beta A^P}{Z}\left(x_S\left(\frac{dS}{S} - \frac{d\eta}{\eta}\right) + \frac{d\eta}{\eta} + \frac{c}{A^P}\frac{dc}{c} - \frac{w}{A^P}\frac{dw}{w} - \frac{L^P}{A^P}\frac{dL^P}{L^P}\right) \\
&\quad +\frac{\gamma P^P}{Z}\frac{dP^P}{P^P}
\end{aligned}
\tag{15}
$$

b. The P^P dynamics

One needs to derive the put P^P dynamics.

As $P^P(t, A^P, L^P, Y_1, Y_2, ..., Y_K)$, applying Ito's lemma to the function P^P yields:

$$dP^P = P_t^P dt + \sum_m P_m^P dm + \frac{1}{2} \sum_m \sum_n P_{mn}^P dmdn \tag{16}$$

with $m, n = \left\{A^P, L^P, Y_1, Y_2, ..., Y_K\right\}$ and P_m^P denoting the partial derivative of the put with respect to the variable m, P_{mn}^P its second derivative with respect to m and n.

The Black-Scholes (1973) non-arbitrage riskless portfolio π takes the form:

$$\pi = -P^P + \sum_m P_m^P m \tag{17}$$

The self-financing condition implies:

$$d\pi = -dP^P + \sum_m P_m^P dm \tag{18}$$

Combining Eqs. (16) and (18) yields:

$$d\pi = -P_t^P dt - \frac{1}{2} \sum_m \sum_n P_{mn}^P dmdn \tag{19}$$

The portfolio π, being riskless, earns the riskless rate r. The π dynamics follows:

$$d\pi = r\pi dt \tag{20}$$

The combination of Eqs. (17), (19) and (20) implies:

$$-P_t^P dt - \frac{1}{2} \sum_m \sum_n P_{mn}^P dmdn = r(-P^P + \sum_m P_m^P m)dt \tag{21}$$

Replacing Eq. (21) in Eq. (16), the put dynamics becomes:

$$\frac{dP^P}{P^P} = \left[r + \sum_m P_m^P \frac{m}{P^P}(\mu_m - r)\right] dt + \sum_m P_m^P \frac{m}{P^P}\sigma_m dB^m \tag{22}$$

where the variable m dynamics is considered under its general form: $\frac{dm}{m} = \mu_m dt + \sigma_m dB^m$.

One can rewrite Eq. (22) as follows:

$$\begin{aligned} \frac{dP^P}{P^P} &= \left[\begin{array}{c} r + P_{A^P}^P \frac{A^P}{P^P}(\mu_{A^P} - r) \\ +P_{L^P}^P \frac{L^P}{P^P}(\mu_{L^P} - r) + \sum_{k=1}^K P_{Y_k}^P \frac{Y_k}{P^P}(\mu_{Y_k} - r) \end{array}\right] dt \\ &\quad + P_{A^P}^P \frac{A^P}{P^P}\sigma_{A^P} dB^{A^P} \\ &\quad + P_{L^P}^P \frac{L^P}{P^P}\sigma_{L^P} dB^{L^P} + \sum_{k=1}^K P_{Y_k}^P \frac{Y_k}{P^P}\sigma_{Y_k} dB^{Y_k} \end{aligned} \tag{23}$$

Let us develop the parameters of the A^P dynamics.

The DB plan's assets A^P are invested in the assets S and η, in the proportions x_S and x_η, respectively, with $x_S + x_\eta = 1$, implying:

$$\frac{dA^P}{A^P} = x_S\left(\frac{dS}{S} - \frac{d\eta}{\eta}\right) + \frac{d\eta}{\eta} \tag{24}$$

Incorporating the S and η dynamics, as defined by Eqs. (7) and (8), respectively, one obtains:

$$\frac{dA^P}{A^P} = \left[x_S(\mu_S - r) + r\right] dt + x_S\sigma_S dB^S \tag{25}$$

The put P^P dynamics follows:

$$\begin{aligned} \frac{dP^P}{P^P} &= \left[\begin{array}{c} r + P_{A^P}^P \frac{A^P}{P^P}(x_S(\mu_S - r)) \\ +P_{L^P}^P \frac{L^P}{P^P}(\mu_{L^P} - r) + \sum_{k=1}^K P_{Y_k}^P \frac{Y_k}{P^P}(\mu_{Y_k} - r) \end{array}\right] dt \\ &\quad + P_{A^P}^P \frac{A^P}{P^P} x_S\sigma_S dB^S \\ &\quad + P_{L^P}^P \frac{L^P}{P^P}\sigma_{L^P} dB^{L^P} + \sum_{k=1}^K P_{Y_k}^P \frac{Y_k}{P^P}\sigma_{Y_k} dB^{Y_k} \end{aligned} \tag{26}$$

c. The Z dynamics (cont.)

One replaces the dynamics of A^F, D^F, c, S, η, w, L^P and P^P, as defined by Eqs. (5), (6), (10), (7), (8), (11), (9) and (26), respectively, in Eq. (15), and factorizes the resulting Z dynamics with respect to x_S and $\frac{L^P}{Z}$. One obtains the following:

$$\begin{aligned}\frac{dZ}{Z} = &\left[\begin{array}{c} \frac{\alpha X^F}{Z}\left(x_{A^F}\left(\mu_{A^F}-\mu_{D^F}\right)+\mu_{D^F}-\frac{c}{X^F}\mu_c\right) \\ +\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)x_S\left(\mu_S-r\right) \\ +\frac{L^P}{Z}\left(-\beta\mu_{L^P}+\gamma P^P_{L^P}\left(\mu_{L^P}-r\right)\right) \\ +\frac{\beta A^P}{Z}\left(r+\frac{c}{A^P}\mu_c-\frac{w}{A^P}\mu_w\right) \\ +\frac{\gamma P^P}{Z}\left(r+\sum_{k=1}^K P^P_{Y_k}\frac{Y_k}{P^P}\left(\mu_{Y_k}-r\right)\right)\end{array}\right]dt \\ &+\frac{\alpha X^F}{Z}\left(x_{A^F}\left(\sigma_{A^F}dB^{A^F}-\sigma_{D^F}dB^{D^F}\right)+\sigma_{D^F}dB^{D^F}-\frac{c}{X^F}\sigma_c dB^c\right) \\ &+\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)x_S\sigma_S dB^S \\ &+\frac{L^P}{Z}\left(-\beta+\gamma P^P_{L^P}\right)\sigma_{L^P}dB^{L^P} \\ &+\frac{\beta A^P}{Z}\left(\frac{c}{A^P}\sigma_c dB^c-\frac{w}{A^P}\sigma_w dB^w\right) \\ &+\frac{\gamma P^P}{Z}\left(\sum_{k=1}^K P^P_{Y_k}\frac{Y_k}{P^P}\sigma_{Y_k}dB^{Y_k}\right)\end{aligned} \tag{27}$$

d. The optimization program

The firm manager solves the optimization program (4) under the constraints of the Z and Y_k dynamics, as defined by Eqs. (27) and (12), respectively.

e. The first order conditions

Let the indirect utility function J be defined as:

$$J(Z(t),Y(t),t) \equiv \max_{x_{A^F},x_S} E_t\left[U(Z(T))\right] \tag{28}$$

with J increasing, strictly concave in Z, once differentiable with respect to t and twice differentiable with respect to Z and Y.

The Hamilton-Jacobi-Bellman optimality condition states:

$$0 = \max_{x_{A^F}, x_S} DJ(Z(t), Y(t), t) \tag{29}$$

where D the Dynkin operator, the Dynkin of J being defined by:

$$\begin{aligned} DJ &= J_t + J_Z Z \mu_Z + \frac{1}{2} J_{ZZ} Z^2 \sigma_Z^2 \\ &\quad + \sum_k J_{Y_k} Y_k \mu_{Y_k} + \frac{1}{2} \sum_k \sum_l J_{Y_k Y_l} Y_k Y_l \sigma_{Y_k Y_l} \\ &\quad + \sum_k J_{Z Y_k} Z Y_k \sigma_{Z Y_k} \end{aligned} \tag{30}$$

where the subscripts on J denote partial derivatives and σ_{op} stands for the covariance between any variables o and p, while the variable Z dynamics is considered under its general form $\frac{dZ}{Z} = \mu_Z dt + \sigma_Z dB^Z$.

Replacing the parameters of the Z dynamics with their formulations as defined in Eq. (27) and deriving DJ with respect to x_{A^F} and x_S yields:

$$\begin{aligned} 0 &= J_Z Z \frac{\alpha X^F}{Z} \left(\mu_{A^F} - \mu_{D^F} \right) \\ &\quad + J_{ZZ} Z^2 \left[\begin{array}{c} \left(\frac{\alpha X^F}{Z} \right)^2 \left[x_{A^F} \left(\sigma_{A^F}^2 + \sigma_{D^F}^2 - 2\sigma_{A^F D^F} \right) + \left(\sigma_{A^F D^F} - \sigma_{D^F}^2 \right) \right] \\ + \frac{\alpha X^F}{Z} \frac{c}{Z} (\beta - \alpha) \left(\sigma_{A^F c} - \sigma_{D^F c} \right) \\ + \frac{\alpha X^F}{Z} \frac{A^P}{Z} \left(\beta + \gamma P_{A^P}^P \right) x_S \left(\sigma_{A^F S} - \sigma_{D^F S} \right) \\ + \frac{\alpha X^F}{Z} \frac{L^P}{Z} \left(-\beta + \gamma P_{L^P}^P \right) \left(\sigma_{A^F L^P} - \sigma_{D^F L^P} \right) \\ - \frac{\alpha X^F}{Z} \frac{w}{Z} \beta \left(\sigma_{A^F w} - \sigma_{D^F w} \right) \\ + \frac{\alpha X^F}{Z} \frac{\gamma P^P}{Z} \sum_{k=1}^K P_{Y_k}^P \frac{Y_k}{P^P} \left(\sigma_{A^F Y_k} - \sigma_{D^F Y_k} \right) \end{array} \right] \\ &\quad + \sum_k J_{Z Y_k} Z Y_k \left[\frac{\alpha X^F}{Z} \left(\sigma_{A^F Y_k} - \sigma_{D^F Y_k} \right) \right] \end{aligned} \tag{31}$$

$$
\begin{aligned}
0 \;=\;& J_Z Z\left[\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\left(\mu_S-r\right)\right] &(32)\\
&+J_{ZZ}Z^2\left[\begin{array}{c}
\left(\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\right)^2 x_S\sigma_S^2\\
+\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\frac{\alpha X^F}{Z}\left(x_{A^F}\left(\sigma_{SA^F}-\sigma_{SD^F}\right)+\sigma_{SD^F}\right)\\
+\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\frac{c}{Z}\left(\beta-\alpha\right)\sigma_{Sc}\\
+\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\frac{L^P}{Z}\left(-\beta+\gamma P^P_{L^P}\right)\sigma_{SL^P}\\
-\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\frac{w}{Z}\beta\sigma_{Sw}\\
+\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\frac{\gamma P^P}{Z}\sum_{k=1}^K P^P_{Y_k}\frac{Y_k}{P^P}\sigma_{SY_k}
\end{array}\right]\\
&+\sum_k J_{ZY_k}ZY_k\frac{A^P}{Z}\left(\beta+\gamma P^P_{A^P}\right)\sigma_{SY_k}
\end{aligned}
$$

f. The optimal policies

Using Eqs. (31) and (32), one determines the following optimal policies, respectively:

$$
\begin{aligned}
x_{A^F} \;=\;& -\frac{J_Z}{J_{ZZ}Z}\frac{Z}{\alpha X^F}\frac{\mu_{A^F}-\mu_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}} &(33)\\
&-\frac{\sigma_{A^FD^F}-\sigma^2_{D^F}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&-\frac{c\left(\beta-\alpha\right)}{\alpha X^F}\frac{\sigma_{A^Fc}-\sigma_{D^Fc}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&-\frac{A^P\left(\beta+\gamma P^P_{A^P}\right)}{\alpha X^F}x_S\frac{\sigma_{A^FS}-\sigma_{D^FS}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&-\frac{L^P\left(-\beta+\gamma P^P_{L^P}\right)}{\alpha X^F}\frac{\sigma_{A^FL^P}-\sigma_{D^FL^P}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&+\frac{w\beta}{\alpha X^F}\frac{\sigma_{A^Fw}-\sigma_{D^Fw}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&-\frac{\gamma}{\alpha X^F}\sum_{k=1}^K P^P_{Y_k}Y_k\frac{\sigma_{A^FY_k}-\sigma_{D^FY_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}\\
&-\sum_k\frac{J_{ZY_k}Y_k}{J_{ZZ}Z}\frac{Z}{\alpha X^F}\frac{\sigma_{A^FY_k}-\sigma_{D^FY_k}}{\sigma^2_{A^F}+\sigma^2_{D^F}-2\sigma_{A^FD^F}}
\end{aligned}
$$

$$
\begin{aligned}
x_S \quad = \quad & -\frac{J_Z}{J_{ZZ}Z}\frac{Z}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{\mu_S-r}{\sigma_S^2} \\
& -\frac{\alpha X^F}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{x_{A^F}\left(\sigma_{SA^F}-\sigma_{SD^F}\right)+\sigma_{SD^F}}{\sigma_S^2} \\
& -\frac{c\left(\beta-\alpha\right)}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{\sigma_{Sc}}{\sigma_S^2} \\
& -\frac{L^P\left(-\beta+\gamma P^P_{L^P}\right)}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{\sigma_{SL^P}}{\sigma_S^2} \\
& +\frac{w\beta}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{\sigma_{Sw}}{\sigma_S^2} \\
& -\frac{\gamma}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\sum_{k=1}^{K}P^P_{Y_k}Y_k\frac{\sigma_{SY_k}}{\sigma_S^2} \\
& -\sum_k\frac{J_{ZY_k}Y_k}{J_{ZZ}Z}\frac{Z}{A^P\left(\beta+\gamma P^P_{A^P}\right)}\frac{\sigma_{SY_k}}{\sigma_S^2}
\end{aligned} \tag{34}
$$

One obtains the optimal policies x_{A_F} and x_S presented in Table 1 by recalling that

- in case (i), the firm-only configuration: $\alpha = 1$, $\beta = \gamma = 0$ and $Z = X^F$;

- in case (ii), the DB pension plan-only configuration: $\beta = 1$, $\alpha = \gamma = 0$ and $Z = X^P$;

- in case (iii), the consolidated balance sheet without the PBGC put: $\alpha = \beta = 1$, $\gamma = 0$ and $Z = X^{F+P}$;

- in case (iv), the consolidated balance sheet with PBGC put: $\alpha = \beta = \gamma = 1$ and $Z = X$.

APPENDIX B: DESCRIPTION OF THE OPTIMAL POLICIES STRUCTURE

The equations referenced in this Appendix are shown in Table 1.

a. Case (i): The firm's policies in a non-consolidated perspective

The optimal $x^i_{A^F}$ and $x^i_{D^F}$ are composed of four terms. The preference-dependent speculative fund follows the objective of building the financially most interesting risk-reward couple $\left(\mu_{A^F} - \mu_{D^F}; \sigma^2_{A^F} + \sigma^2_{D^F} - 2\sigma_{A^F D^F}\right)$, while taking into account the relative risk tolerance degree, as represented by the coefficient $-\frac{J_{X^F}}{J_{X^F X^F} X^F}$. One then encounters the preference-independent portfolio variance minimizing hedge against the firm´s debt D^F (the firm´s assets A^F) in $x^i_{A^F}$ ($x^i_{D^F}$). The preference-independent contribution hedge term follows, covering the stochastic evolutions of the contribution process. The last term defines a preference-dependent state-variable hedge fund, which constitutes a cover against the variations of the K state variables influencing the evolution of the economy.

b. Case (ii): The pension plan's policy in a non-consolidated perspective

Five terms are present in the optimal x^{ii}_S. The usual speculative fund, preference dependent via the coefficient $-\frac{J_{X^P}}{J_{X^P X^P} X^P}$ and optimizing the risk-reward couple $(\mu_S - r; \sigma^2_S)$, is followed by four hedge funds. The first three funds are preference independent and cover the variations in the plan's liabilities L^P, the contributions to the fund c and the withdrawals from the fund w, respectively. The last optimal x^{ii}_S term, preference dependent, is a hedge against the state variable variations.

c. Case (iii): Consolidated optimal policies without PBGC put

Six terms emerge in the optimal $x^{iii}_{A^F}$: the preference-dependent speculative fund, building on the risk-reward arbitrage between $\mu_{A^F} - \mu_{D^F}$ and $\sigma^2_{A^F} + \sigma^2_{D^F} - 2\sigma_{A^F D^F}$, four preference-independent hedge funds against the D^F, S, L^P and w variations and the preference-dependent state-variable hedge fund.

When comparing to the optimal investment policy in the firm-only (i) configuration, as materialized by Eq. $x^i_{A^F}$, several differences are observed. First, three new hedge funds emerge in the consolidated case, which cover the S, L^P and w variations. The investment policy is now impacted by the hedging demand for stochastic variables characteristic of the DB pension plan. Second, the contribution c hedge fund disappears in the consolidated case. The contributions are paid by the firm and received by the DB pension plan, leading the corresponding hedge fund to cancel out in the consolidated configuration. Third, in the speculative fund and in the state-variable hedge fund, risk tolerance is measured with respect to X^{F+P} and no longer X^F.

For the optimal $x^{iii}_{D^F}$, the modifications are similar to those registered for the optimal $x^{iii}_{A^F}$.

With regard to the optimal x^{iii}_S, one observes five terms: the preference-dependent speculative fund, representing the risk-reward arbitrage between $\mu_S - r$ and σ^2_S, three preference-independent hedge funds against the $A^F$and D^F, L^P and w variations and the preference-dependent state-variable hedge fund.

When comparing to the non-consolidated DB plan-only (ii) case, characterized by the optimal portfolio policy defined by Eq. x^{ii}_S, one notices the following differences. First, a new hedge fund emerges against the $A^F$and D^F variations: the DB pension plan's optimal portfolio policy is now impacted by a hedging demand for stochastic variables characteristic of the firm. Second, the contribution c hedge fund cancels out. Third, in the speculative fund and in the state-variable hedge fund, risk tolerance is now measured with respect to X^{F+P} and not X^P.

To summarize, when moving from the non-consolidated to the consolidated firm's optimal investment and financing policies and DB plan's optimal portfolio strategy, new hedge funds emerge: the firm (DB plan) additionally hedges against the DB plan (firm) variables. More precisely, in the optimal $x^{iii}_{A^F}$ and $x^{iii}_{D^F}$, one additionally hedges against the DB plan's S, L^P and w; in the optimal x^{iii}_S, one additionally hedges against the firm's $A^F$and D^F.

The second major difference is the elimination of the contribution c hedge fund in all three optimal policies. The non-emergence of the contribution hedge fund occurs because the consolidated approach leads to the consideration of the pension assets (liabilities) as the firm's assets (debt). Consequently, the effect of cash flows directed from the firm to the pension plan is seen as neutral.

d. Case (iv): Consolidated optimal policies with PBGC put

Eq. $x^{iv}_{A^F}$ shows that seven terms characterize the optimal investment policy: the preference-dependent speculative fund, representative of the risk-reward arbitrage between $\mu_{A^F} - \mu_{D^F}$ and $\sigma^2_{A^F} + \sigma^2_{D^F} - 2\sigma_{A^F D^F}$, five preference-independent hedge funds against the variations of D^F, S, L^P, w and the put P^P driven state variables and the preference-dependent state-variable hedge fund.

When compared to the no-PBGC-put case, the optimal investment policy thus being formulated as defined by Eq. $x^{iii}_{A^F}$, the differences are as follows. First, the S hedge fund is now multiplied by $1 + P^P_{A^P}$. Second, the L^P hedge fund becomes multiplied by $1 - P^P_{L^P}$. Third, the put P^P driven state-variable hedge fund emerges. Fourth, in the speculative fund and in the preference-dependent state-variable hedge fund, risk tolerance is measured with respect to X and no longer X^{F+P}.

For the optimal $x^{iv}_{D^F}$, the modifications are similar to those registered for the optimal $x^{iv}_{A^F}$.

With regard to x^{iv}_S, the optimal portfolio policy incorporates six terms: the preference-dependent speculative fund, building on the arbitrage between $\mu_S - r$ and σ^2_S, four preference-independent hedge funds against the variations of A^F and D^F, L^P, w and the put P^P driven state variables and the preference-dependent state-variable hedge fund.

When compared to the optimal portfolio policy characteristic of the no-PBGC-put case, as defined by Eq. x^{iii}_S, several differences are observed. First, all of the terms are divided by $1+P^P_{A^P}$. Second, the L^P hedge fund is multiplied by $1 - P^P_{L^P}$. Third, the put P^P driven state-variable hedge fund emerges. Fourth, in the speculative fund and in the preference-dependent state-variable hedge fund, risk tolerance is now measured with respect to X and not X^{F+P}.

To summarize, the PBGC put incorporation leads to major modifications. First, and most important, the terms of the DB plan's optimal portfolio policy become all divided by $1 + P^P_{A^P}$. Second, in the firm's optimal investment and financing policies, the DB plan's portfolio policy related term - the S hedge fund - becomes multiplied by $1 + P^P_{A^P}$. Third, there are two differences in all three optimal policies: the L^P hedge fund becomes multiplied by $1 - P^P_{L^P}$, and the put P^P driven state-variable hedge fund emerges.

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