Is Corporate Control Effective When Managers Face Investment Timing Decisions in Incomplete Markets?

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This paper presents a model of investment timing by risk averse managers facing incomplete markets and corporate control. Managers are exposed to idiosyncratic risks due to the dependence of their compensation on investment payoffs which are not spanned by other assets. We show that risk averse managers invest earlier than well-diversified shareholders would prefer, leading to significant agency costs. This effect can be mitigated if the manager is subject to corporate control. Our main finding is that the interaction of incompleteness and control results in two regimes. When the market is sufficiently close to being complete, control has a strong disciplinary effect and agency costs can be virtually eliminated. However, when idiosyncratic risk is too large, shareholders suffer agency costs and control is ineffective. An implication is that we would expect to see different investment behavior across industries or specific investments as the degree of incompleteness varied. It would also suggest that both the standard complete-markets real options model and the npv framework can play a role in describing investment timing.

Keywords: Real Options, Investment Timing, Incomplete Markets, Corporate Control

The standard real options approach to investment timing equates the opportunity to invest in a project with an American call option on the project, with the investment timing decision being analogous to the exercise of the option. The canonical real options models assume that investment payoffs can be perfectly spanned by existing assets or equivalently that decisions are made in complete markets (Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit and Pindyck (1994)). It is also implicit in the standard models that the owner of the option makes the exercise or investment decision. However, in reality, risk averse managers make investment decisions on behalf of shareholders, and investment payoffs cannot be perfectly hedged so managers face idiosyncratic risks and incomplete markets. Managers must also make decisions within the confines of corporate governance and controls. In this paper we consider the resulting agency conflicts arising between risk averse managers and well-diversified shareholders, and the impact of corporate control on the manager's investment timing. We ask whether corporate control is effective in reducing agency costs to shareholders which arise from the manager's timing of investment.

Idiosyncratic risk and incompleteness impact on the risk averse managers' investment timing choices. Managers are typically compensated with stock and stock options, the value of which depend upon the value of the firm's activities including their options to invest. However, investment payoffs cannot be perfectly spanned by existing assets and so idiosyncratic risks remain. Shareholders, on the other hand, are well-diversified and do not require compensating for idiosyncratic risk. Agency conflicts arise because the manager maximizes the value of the investment timing option based on his own preferences, whilst shareholders want to maximize firm value.[1] In our model, the investment pays out a lump-sum reward. The manager can offset some of the risk of this random payoff by trading in the market. The remaining unhedgeable risk is idiosyncratic and the manager's risk aversion causes him to prefer to invest earlier than the shareholder's prefer. He does this in order to reduce exposure to idiosyncratic risk, since the act of exercising or investing gives a one-off payment. This feature of our model is in common with Henderson (2006), see also Miao and Wang (2006).

When the investment timing choice of the manager reduces firm or shareholder value, the manager faces the threat of a control challenge. Corporate control is the right to determine the management of corporate resources; to hire, fire and set compensation (Jensen and Ruback (1983), Fama and Jensen (1983a,b)). In our framework, control challenges result in the possibility that the manager is terminated if he deviates too much from the shareholder's firm value maximization policy. This termination could arise from either internal or external mechanisms. Internal mechanisms involve control by the board of directors (Huson et al (2001)), and external control is exerted by the takeover market. Takeovers are thought to occur for disciplinary reasons to correct for bad management practice (Marris (1964), Manne (1965), Jensen (1986), Scharfstein (1988), Morck et al (1989)). Implicit in our model is the assumption that takeovers result in the manager's dismissal, consistent with evidence of Martin and McConnell (1991).

Although the model of corporate control described above is prevalent in the US, in most countries large firms are not widely held, but have controlling shareholders, who are often active in management (La Porta et al

[1] We abstract from capital structure issues and consider a firm with no debt.

(1999), Shleifer and Vishny (1997), see also Claessens et al (2000) and Faccio and Lang (2002)).[2] La Porta et al (1999) find "family control of firms appears to be common, significant, and typically unchallenged by other equity holders." In firms with a large shareholder (or family member) who is also a manager, monitoring and disciplining the manager becomes difficult.[3] We can additionally proxy for this situation in our model by taking the strength of control challenges to be (very small or) zero.

When there is no corporate control, perhaps because the manager is a large shareholder, we find risk averse managers invest much earlier than shareholders' prefer, resulting in large agency costs. The manager's investment threshold decreases away from the shareholder's threshold as risk aversion increases, and in the limit, approaches the zero npv threshold. Correspondingly, agency costs rise with managerial risk aversion. The value of the investment option to the manager is significantly less than the value to shareholders. At the other extreme where the manager does not require compensation for idiosyncratic risk, he invests exactly at the time leading to firm value maximization, leading to a redundancy of corporate control, at least in dealing with incomplete markets.[4]

[2]La Porta et al (1999) examine data on ownership structure of large companies in 27 countries and find that almost half of these firms are controlled by large shareholders. In total about 30% of companies are controlled by families and in 69% of family controlled firms, the family also participates in management. A US example is Microsoft, where Bill Gates owned 23.7% of the company in the mid 1990's. See also the model of succession in family firms of Burkart et al (2003).

[3]Note there is also a literature (see Shleifer and Vishny (1986)) where the large shareholder and manager are distinct, and the large shareholder monitors the manager. This case does not require separate consideration in our model.

[4]In this case, options are providing sufficient discipline for managers to time investment to maximize firm value.

When the manager facing idiosyncratic risks is also subject to corporate control, the risk of a control challenge always reduces agency costs to shareholders, in that the manager always chooses to invest at a threshold closer to the shareholder's preferred threshold. In fact, we show the manager subject to control invests at a threshold somewhere between the threshold of the shareholders and the threshold of an equivalent manager who is not subject to control.

Finally, we investigate the interaction between incompleteness and corporate control and ask "is corporate control effective when risk averse managers make investment timing decisions in incomplete markets?". Our main contribution is the finding that the effectiveness of control depends on the degree of incompleteness of the market. We find there are two regimes based on the correlation between the investment and the market or hedging opportunities, provided risk aversion is not too low. When the correlation is sufficiently high, there is little idiosyncratic risk and so the risk of termination dominates. Shareholders counteract the impact of managerial risk aversion in order to capture much of the value of the option, as the manager invests very close to the shareholders' threshold. In this case, control is effective as agency costs to shareholders can be virtually eliminated. The other regime is for sufficiently low correlation. In this case, idiosyncratic risk is significant and the risk of a control challenge never dominates. Shareholders lose a larger part of the option value, suffering agency costs, and control is ineffective.

Our model also allows us to answer the question of whether incomplete markets should be an important issue to shareholders. This depends greatly on whether there is a well-functioning market for corporate control. If there is not (for instance in many countries where firms have controlling shareholders who are also managers), the manager will make investment timing decisions which markedly differ from firm value maximizing ones, and shareholders suffer large agency costs. This is consistent with the broad evidence cited in Shleifer and Vishny (1997) on agency costs resulting from poor corporate control. In this case, market incompleteness is a further source of agency costs to shareholders in countries with weak corporate control. Conversely, if there is a well-functioning market for control, then control can have a significant impact on agency costs when idiosyncratic risk is not too large. However, when idiosyncratic risks are large, control has less of an impact, and incomplete markets do have a detrimental effect on shareholder wealth.

Whilst we study agency conflicts arising from managerial risk aversion and incompleteness, there are many forms of agency conflicts between managers and shareholders, including empire building, short-termism and overconfidence, see Stein (2003) for a review. Such studies usually concentrate on the impact of these agency issues on capital budgeting (under or overinvestment) as distinct from investment timing. An exception is the recent paper of Grenadier and Wang (2005) who extended the real options framework to account for issues of informational asymmetries and agency issues (unobserved effort and empire building). However their manager faces a complete market. Other papers incorporating shareholder-manager conflicts in contingent-claim models include Zweibel (1996), Morellec (2004) and Morellec and Smith (2006). However these papers are concerned with the impact of such conflicts on the firm's debt levels.[5] An important distinction be-

[5] Others have studied the impact of shareholder-debtholder conflicts on investment decisions using a real options approach, including Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), and Morellec (2001, 2004).

tween agency costs arising from incompleteness and others mentioned above is that in our model the manager has a concave utility function and maximizes expected utility. In contrast, the sources of agency costs described above either occur because the manager derives utility or benefit from private objectives or because he takes decisions based on a biased assessment of the probabilities of the outcomes (overconfidence).

Our findings can also be compared with the predictions of the literature of the impact of takeovers on managerial incentives to invest, which are in terms of under and over-investment rather than investment timing. The disciplinary theory suggests takeovers discipline managers who use free cash flow to make value-reducing investments. If managers are the empire-building type, the threat of takeover will lead to a decrease in their over-investment, whereas if managers were enjoying the quiet life, disciplinary theory would predict an increase in investment. In our model, the manager's risk aversion causes him to invest early and the threat of discipline mitigates this effect. Additionally, the magnitude of the effect of discipline depends greatly on how much idiosyncratic risk he faces and how risk averse he is. There is a stronger disciplinary effect if there is less idiosyncratic risk and higher risk aversion.

The paper most closely related to the present work is Hugonnier and Morellec (2006). Whilst they also consider a manager making an investment decision whilst facing incomplete markets and corporate control, their conclusions differ from ours. We give a more detailed discussion of the modeling differences in Appendix C, rather here we will highlight that they have important consequences for the economic predictions of the model. Hugonnier and Morellec (2006) only consider the joint impact of incompleteness and control and find that for reasonable levels of risk aversion, the manager invests close to the zero npv threshold, eroding almost all of the value of waiting. In this sense, they find control is ineffective, since shareholder's are not successful at encouraging the manager to invest close to their preferred threshold. In fact, if we consider a special case of their model where the manager is not subject to control, then the conclusion would be that the manager invests immediately (providing the investment has positive npv). This implies all option value is eliminated by incompleteness.

We extend the paper of Hugonnier and Morellec (2006) in a number of ways. First, we consider the impact of incompleteness and control separately on the manager's investment timing. This allows us to make comparisons between the potential investment behavior of managers in different corporate governance regimes, namely those with a well functioning market for corporate control versus those without. Second, we reward the manager based on the investment payoff as well as the value of his trading in the market and riskless bonds. In contrast, Hugonnier and Morellec (2006) model the reward as purely based on the value of the manager's financial holdings. Our formulation should be interpreted as one where the manager receives compensation based on the firm value, which in turn depends on the option to invest. This is desirable since stock options are an extremely popular form of executive compensation (see for example, Hall and Murphy (2002)).

There are significant advantages in our modeling approach. First, an advantage is that we obtain semi-closed form investment thresholds which enable us to provide some comparative statics and give ordering results on the thresholds analytically. Second, and more importantly, we obtain a richer set of conclusions. We find that without corporate control, the manager invests somewhere between the zero npv threshold and the shareholder's preferred threshold, depending on risk aversion and the level of incompleteness. When our manager also faces the threat of corporate control, we find that the degree of incompleteness influences the effectiveness of control. In our model, control is effective when the market is sufficiently close to being complete, but less effective when the market is too incomplete.

The model set-up and solution is presented in Section 1. Section 2 analyzes the implications of the model for investment timing and agency costs. We offer our conclusions in Section 3. Technical developments are contained in three Appendices - Appendix A contains proofs of the results in the paper, Appendix B gives some analytical comparison results concerning the thresholds, and Appendix C contains a more detailed comparison of our model with that of Hugonnier and Morellec (2006).

1 The Model

In our model, shareholders are well-diversified (and are unconcerned with idiosyncratic risks) but delegate investment decisions to a manager who is risk averse. We assume the manager faces a single irreversible investment decision which can be made over an infinite horizon.[6] The project pays a one-off amount V_τ at time τ for a cost K[7]. That is, exercise or investing at time τ (of the manager's choosing) yields the difference $(V_\tau - K)^+$.

Let the investment payoff V follow a geometric Brownian motion

$$dV = \nu V dt + \eta V dW \tag{1}$$

where W is standard Brownian motion. Denote by $\xi = \frac{\nu}{\eta}$ the Sharpe ratio of

[6]For simplicity we assume there is a single investment decision so the value of the firm is the value of the investment option. Recall, there is no debt in our model.

[7]All amounts are expressed in discounted units, or equivalently, we take the risk-free bond as numeraire.

the investment payoff. Although the investment payoff V is not traded, we assume there are other tradeable assets available to the manager, summarized by the market asset, denoted P. In contrast to standard real options models (Brennan and Schwartz (1985), McDonald and Siegel (1986), see also the textbook treatment of Dixit and Pindyck (1994)), here the payoff V is not perfectly spanned by traded assets, so the manager makes his investment timing decision in an incomplete market.

The market P also follows a geometric Brownian motion, with

$$dP = \mu P dt + \sigma P dB$$

where B is a standard Brownian motion correlated to W with $\rho \in (-1, 1)$ and $\rho^{\perp} = \sqrt{(1-\rho^2)}$. We can express the risk of V in terms of the risk of P plus some additional idiosyncratic risk. Write $dW = \rho dB + \rho^{\perp} dZ$ for a Brownian motion Z independent of B. Since P can be traded, B represents the hedgeable risk. The Brownian motion Z represents the idiosyncratic risk which cannot be hedged via the market asset. Denote by $\lambda = \frac{\mu}{\sigma}$ the Sharpe ratio of the market. We now consider the shareholder's and manager's investment timing problems in turn.

Shareholder's problem.

If well-diversified shareholders were to make the investment timing decision they would maximize firm value. Let $\mathcal{S}(v)$ denote the shareholder's value of the investment option, where v denotes the current (discounted) value of the project. Using standard arguments (Dixit and Pindyck (1994)), $\mathcal{S}(v)$ solves

$$0 = \frac{1}{2}\eta^2 v^2 \frac{\partial^2 \mathcal{S}}{\partial v^2} + \eta(\xi - \lambda\rho) v \frac{\partial \mathcal{S}}{\partial v}$$

subject to boundary, value-matching and smooth pasting conditions:

$$\mathcal{S}(0) = 0;\ \ \mathcal{S}(\bar{V}^s) = \bar{V}^s - K;\ \ \left.\frac{\partial \mathcal{S}}{\partial v}\right|_{\bar{V}^s} = I_{\{\bar{V}^s > K\}}$$

This gives the usual first passage time criteria where the manager invests the first time the (discounted) investment payoff V_t is greater than or equal to a constant threshold level $\bar{V}^s$. We solve for this threshold and associated value of the option to invest in the standard way to give the following result.

Proposition 1 *Denote by* $\beta = 1 - \frac{2(\xi - \lambda\rho)}{\eta}$ *the non-zero root of the quadratic*

$$\phi(\phi - 1)\eta^2/2 + \eta\phi(\xi - \lambda\rho) = 0.$$

Suppose $\beta > 1$. *Investment/exercise takes place at the first passage time* $\tau = \inf\{t : V_t \geq \bar{v}\}$ *for some constant* $\bar{v}$, *and the shareholder's value of the investment option (under any* $\bar{v}$*) is given by*

$$\mathcal{S}_{\bar{v}}(v) = \begin{cases} (\bar{v} - K)(\frac{v}{\bar{v}})^{\beta}; & v < \bar{v} \\ v - K; & v \geq \bar{v} \end{cases}$$

Maximizing the option value over possible thresholds $\bar{v}$ *gives*

$$\bar{v}^* = \bar{V}^s = \frac{\beta}{\beta - 1}K. \tag{2}$$

The shareholder's value of the investment option under their optimal threshold $\bar{V}^s$ *(or equivalently firm value maximization) is*

$$\mathcal{S}(v) \equiv \mathcal{S}_{\bar{V}^s}(v) = \begin{cases} (\bar{V}^s - K)(\frac{v}{\bar{V}^s})^{\beta}; & v < \bar{V}^s \\ v - K; & v \geq \bar{V}^s \end{cases} \tag{3}$$

The expression for the threshold in (2) gives the well known conclusion of real options that waiting to invest has value, since $\bar{V}^s > K$. Likewise, the formula in (3) takes the usual form - the payoff upon investment $\bar{V}^s - K$, multiplied by a stochastic discount factor which reflects the probability that investment will occur.

The shareholder's preferred investment timing and firm value will represent a benchmark against which the manager's behavior can be compared. It will also allow us to compute agency costs arising from the manager choosing a different trigger level at which to invest.

The Manager's Timing Problem Under Corporate Control

We now consider the manager's investment timing decision. The risk averse manager chooses when to pay the investment cost to receive the project value V. Since V is not tradeable, and its value is uncertain, the manager faces risk whilst waiting. However, the manager can also invest in the market asset P. This enables him to partially hedge the risk from the investment timing option.

Let X denote the manager's wealth from his holdings in the market asset P and the risk-free bond, discounted by the bond. The dynamics of X are

$$dX = \theta_t dP/P \tag{4}$$

where θ_t is the cash amount invested in the market asset at time t.

The risk averse manager chooses an investment time τ and a position θ in the market asset to maximize his expected utility. At the investment time, his position consists of two components – the payoff $(V_\tau - K)^+$ in addition to his accrued wealth X_τ from his holdings in the market. Implicit in this set-up is the assumption that the manager's private wealth is contingent on the value of the investment option or equivalently the value of the firm. This assumption is usually satisfied because most managers will be compensated in the form of stock and stock options and thus a significant proportion of their own private wealth will be dependent on the success of the company.

Without any sanctions by shareholders, the manager would choose an investment time to maximize his own objectives given he is risk averse. We will consider this special case later on. Here we consider the situation of a manager subject to corporate control (see also Zweibel (1996), Morellec (2004)). The manager will be penalized with a greater chance of facing a control challenge if his investment timing choice deviates further from the shareholders' firm value maximizing choice, given in Proposition 1.

Let the probability the manager faces a control challenge at time τ be given by

$$p(V_\tau) = 1 - e^{-\Phi(V_\tau)} \tag{5}$$

where

$$\Phi(v) = c[\mathcal{S}(v) - (v - K)]/K \tag{6}$$

The form of (6) reflects the shareholder value lost when the wrong threshold is chosen. The parameter c represents the strength or level of threat of the challenge. We can vary c to proxy for either the Anglo-Saxon model of corporate control (whereby internal control mechanisms as well as external mechanisms such as takeovers, play a role in disciplining management), or the large shareholder-manager model (where it is difficult to discipline the manager). The former would be consistent with a larger strength of threat, whilst a small or zero value for c would be appropriate in the latter model.

Note that Φ depends on the optimal threshold of the shareholders $\bar{V}^s$, and that $\Phi(v) = 0$ for $v \geq \bar{V}^s$. This implies there is no chance of a control challenge if the manager invests at (or above) the shareholder's preferred threshold, $\bar{V}^s$. In fact, as we will see later, the manager never chooses a threshold higher than $\bar{V}^s$ so this condition is for convenience and does not affect the results.

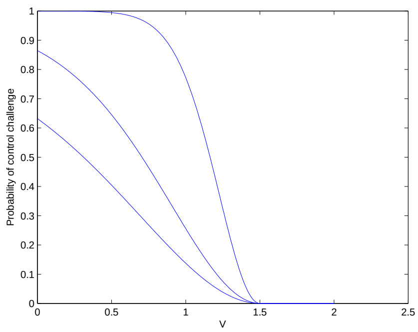

Figure 1: *The probability of a control challenge $p(V_\tau)$ as a function of the investment value at the exercise/investment time. The three lines from lowest to highest take $c = 1, 2, 10$. Parameters are $K = 1$, $\beta = 3$ giving the shareholder's threshold $\bar{V}^s = 1.5$.*

The probability of control challenge $p(V_\tau)$ is displayed in Figure 1 as a function of the manager's threshold. Under the chosen parameter values, the shareholder's preferred threshold is $\bar{V}^s = 1.5$. The different lines correspond to varying the parameter c. From lowest to highest, the lines take $c = 1, 2, 10$. As the strength of the challenge increases, at a fixed choice of exercise threshold, the probability of a challenge is higher. We see that as the manager's threshold deviates from $\bar{V}^s$, the probability of a challenge increases. Note that although (6) is well defined for $v \geq 0$, the manager will never choose a threshold below K.

We now return to state the problem solved by the risk averse manager facing control challenges and incomplete markets. If the manager is subject to a control challenge, he is terminated and no longer receives any compensation based on the outcome of the real investment. This is modeled via the indicator function I_τ. The manager solves the following problem:

$$\sup_{\tau \geq t} \sup_{(\theta_u)_{t \leq u \leq \tau}} \mathbb{E}[e^{-\zeta\tau} U(X_\tau + (V_\tau - K)^+ I_\tau) | X_t = x, V_t = v] \tag{7}$$

where

$$I_\tau = \begin{cases} 1; & \text{if the manager is not terminated} \\ 0; & \text{if the manager is terminated} \end{cases}$$

and ζ is a discount factor. We assume the risk averse manager has exponential utility, so $U(x) = -\frac{1}{\gamma} e^{-\gamma x}$.

We now remark on the formulation in (7). There are two decisions to be made by the manager - when to exercise the option to invest, and the portfolio choice problem of how much cash to hold in the market asset. It is appropriate to formulate the model such that there are no biases arising from the underlying portfolio choice problem influencing the manager's choice of exercise/investment time, τ. For a moment, consider only the portfolio choice component of the problem in (7), which can be written as

$$\sup_{(\theta_u)_{t \leq u \leq \tau}} \mathbb{E}[e^{-\zeta\tau} U(X_\tau) | X_t = x] \tag{8}$$

where now, τ is understood to be the horizon of the portfolio choice problem. We do not want the solution of this underlying problem to depend on the horizon time, τ. That is, we do not want the manager to have a preference for particular times τ in absence of the option to invest. He should be indifferent over horizons τ if he just faces the underlying portfolio choice problem in (8). We show in Appendix A, for the solution of (8) to be independent of the horizon τ, we need to have the particular specification of discount factor, $\zeta = -\frac{1}{2}\lambda^2$. This choice means that when the manager also has the real investment option and is choosing both the exercise time and the portfolio holdings according to (7), his underlying portfolio optimization problem is not biasing his choice of exercise time. A different choice of ζ would create artificial incentives to exercise early, or may even lead to a degenerate situation where the investment option should never be exercised.[8] In fact, in Appendix C we show that it is these discounting biases that are responsible for the conclusions of Hugonnier and Morellec (2006). Define

$$H(x,v) = \sup_{\tau \geq t} \sup_{\theta_u, t \leq u \leq \tau} \mathbb{E}\left[-\frac{1}{\gamma} e^{\frac{1}{2}\lambda^2(\tau - t)} e^{-\gamma(X_\tau + (V_\tau - K)^+ I_\tau)} | X_t = x, V_t = v\right]$$

Finding $H(x,v)$ at $t = 0$ is equivalent to solving (7) (with $\zeta = -\frac{1}{2}\lambda^2$). By time-homogeneity, we deduce the manager invests at the first passage time of V to a constant threshold $\bar{V}^c$,

$$\tau = \inf\{t : V_t \geq \bar{V}^c\}.$$

The following propositions characterize H and the threshold $\bar{V}^c$.

Proposition 2 *In the continuation region, H solves the following non-linear HJB equation*

$$0 = \frac{1}{2}\lambda^2 H + \xi\eta v H_v + \frac{1}{2}\eta^2 v^2 H_{vv} - \frac{1}{2}\frac{(\lambda H_x + \rho\eta v H_{xv})^2}{H_{xx}}$$

subject to boundary, value-matching and smooth-pasting conditions:

$$\begin{aligned} H(x,0) &= -\frac{1}{\gamma}e^{-\gamma x} && (9) \\ H(x,\bar{V}^c) &= -\frac{1}{\gamma}e^{-\gamma x}[1 + e^{-\Phi(\bar{V}^c)}(e^{-\gamma(\bar{V}^c - K)^+} - 1)] && (10) \\ H_v(x,\bar{V}^c) &= \frac{1}{\gamma}e^{-\gamma x}e^{-\Phi((\bar{V}^c)}\left\{\gamma e^{-\gamma(\bar{V}^c - K)^+}\right. \\ &+ \left.\Phi'(\bar{V}^c)(e^{-\gamma(\bar{V}^c - K)} - 1)\right\} && (11) \end{aligned}$$

[8] Note that the specification $\zeta = -\frac{1}{2}\lambda^2$ is not essential to solve the model. We could solve the model in (7) for a general discount factor ζ, although the resulting investment times would be biased towards early or late exercise.

We see from the above proposition that the investment timing problem of a manager facing incomplete markets and control can be expressed as the solution to a HJB equation with appropriate conditions, just as in the standard real options framework, see Proposition 1. In fact, there is a semi-closed-form solution to the above problem which we express in the following proposition.

Proposition 3 *Suppose $\beta > 1$. Define $\kappa(v) = 1-e^{-\gamma(v-K)}$ and $D(v) = 1-e^{-\Phi(v)}\kappa(v)$. The manager's constant investment threshold $\bar{V}^c$ is the solution to*

$$\frac{\beta}{\bar{V}^c}\left[D(\bar{V}^c)^{\rho^2} - D(\bar{V}^c)\right] = (1-\rho^2)e^{-\Phi(\bar{V}^c)}\left[\gamma e^{-\gamma(\bar{V}^c-K)} - \kappa(\bar{V}^c)\Phi'(\bar{V}^c)\right] \tag{12}$$

and the value function $H(x,v)$ is given by

$$H(x,v) = -\frac{1}{\gamma}e^{-\gamma x}\left[1+\left\{D(\bar{V}^c)^{1-\rho^2}-1\right\}\left(\frac{v}{\bar{V}^c}\right)^{\beta}\right]^{1/(1-\rho^2)}$$

The Manager's Timing Problem Without Corporate Control

We can easily recover the investment timing behavior of the manager who is not subject to control challenges, but does face incomplete markets. This proxies for a large shareholder-manager who is not disciplined by minority shareholders (La Porta et al (1999)). By setting $c = 0$ (or equivalently the function $\Phi(v) = 0$), there is no chance of the manager being subject to a control challenge. We obtain the following result.

Proposition 4 *The manager facing incomplete markets (but no control challenges) invests at the first passage time of V to the constant threshold $\bar{V}^0$,*

$$\tau = \inf\{t : V_t \geq \bar{V}^0\}$$

where $\bar{V}^0$ is the unique solution to

$$\bar{V}^0 - K = \frac{1}{\gamma(1-\rho^2)} \ln \left[1 + \frac{\gamma(1-\rho^2)\bar{V}^0}{\beta} \right]. \tag{13}$$

2 Model Implications

We now explore the economic implications of the model. We compare the various investment thresholds to analyze the effectiveness of control in moderating managerial investment behavior. Later in the section we consider the impact of incomplete markets on agency costs to shareholders and how such costs change when control challenges can occur. Finally we compare the value of the option to invest to each party.

2.1 Investment Timing

We focus first on the investment timing by the manager who is not subject to control, perhaps because he is a large shareholder. We have the following straightforward result.

Proposition 5 *(i)* $\lim_{\gamma\to 0} \bar{V}^0 = \bar{V}^s$ *and* $\lim_{\gamma\to\infty} \bar{V}^0 = K$
(ii) $\bar{V}^0$ *is decreasing in* γ*, hence* $K < \bar{V}^0 < \bar{V}^s$

Proposition 5 says a risk averse manager always chooses an investment threshold which is below the threshold the shareholders prefer. That is, the risk averse manager facing incomplete markets invests earlier than the shareholders want. This occurs because waiting to invest in our model involves being exposed to fluctuations in the payout received upon investment and hence waiting involves exposure to idiosyncratic risk. Investing locks in a value for the project and reduces risk. Since the manager is risk averse, he prefers to act earlier to lock in a certain value for the investment. He therefore acts too early compared with what the shareholders prefer. Higher levels of risk aversion lead to earlier investment times, and if the manager is extremely risk averse, he carries this behavior to its extreme and invests at the zero npv threshold. In this case, all option value of waiting is eliminated. On the other hand, if his risk aversion is close to zero, he selects the shareholder's threshold as he is unconcerned with idiosyncratic risk.

Now consider the investment threshold for the manager subject to corporate control. Figure 2 plots the three thresholds against the manager's risk aversion level. In each panel, the bold horizontal dotted line represents $\bar{V}^s = 1.5$, the shareholder's investment threshold level. As expected, this does not vary with risk aversion. The zero npv threshold in all panels is $K = 1$. The dashed line represents the threshold $\bar{V}^0$ at which a manager facing incomplete markets (but not corporate control) would invest. This threshold, as predicted by Proposition 5, is below the shareholder's threshold, is decreasing with risk aversion, and approaches the zero npv threshold as risk aversion gets large.

The solid lines on each panel represent the threshold $\bar{V}^c$ for the manager facing both incomplete markets and corporate control, for varying strengths of control challenges, c. From lowest to highest, the lines take values $c = 1, 3, 7, 10$. Notice first that (in each panel) $\bar{V}^c$ lies between the other two threshold levels – the corporate control causes the manager to moderate his investment timing to be closer to the time the shareholders prefer. This is as we would expect, given the chance of termination was higher the larger the deviation from firm value maximization. In Appendix B, we give analytical results in support of this ordering of the thresholds. Observe also that as the strength of the challenge rises, the threshold $\bar{V}^c$ moves closer to the shareholder's threshold. The risk of control by the shareholders causes

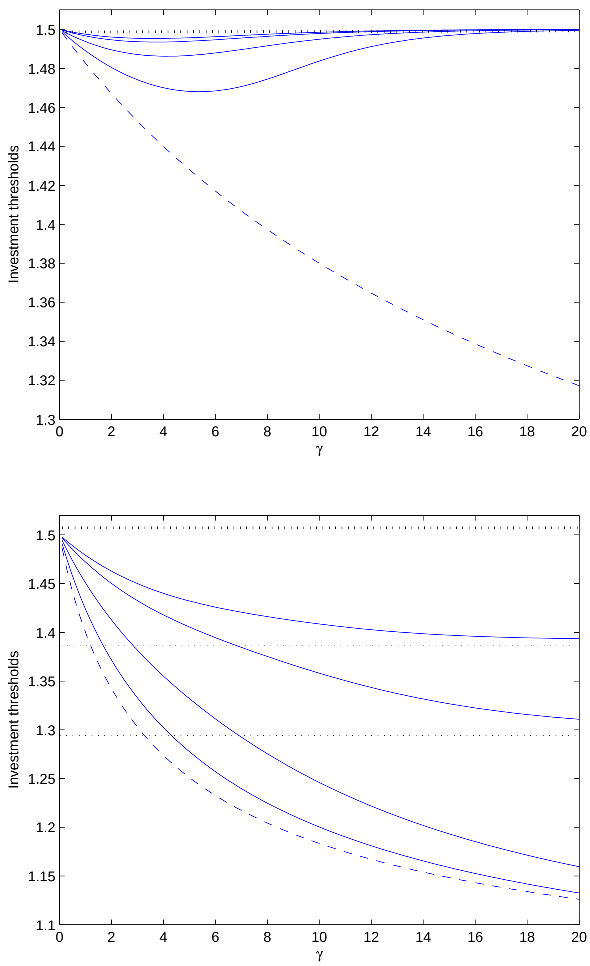

Figure 2: *Investment thresholds for shareholders and manager, as a function of managerial risk aversion. In each panel - the bold dotted horizontal line is* $\bar{V}^s = 1.5$*, the dashed curve is* $\bar{V}^0$ *and the solid lines are* $\bar{V}^c$ *for values of* $c = 1, 3, 7, 10$ *(from lowest to highest). The top panel takes* $\rho = 0.95$ *whilst the lower panel takes* $\rho = 0.5$*. Also shown in the lower panel are the limiting values for* $c = 7, 10$*. Other parameters are* $K = 1$*,* $\beta = 3$*.*

the manager to accept more exposure to idiosyncratic risk than he would otherwise want.

The behavior of the manager's threshold $\bar{V}^c$ is due to two influences – the impact of idiosyncratic risk and the risk of control by shareholders. The relative importance of these effects depends on the level of correlation, the manager's risk aversion, and the strength of the control threat. We now investigate how these effects interact. Compare the behavior of the threshold $\bar{V}^c$ in the upper panel (where $\rho = 0.95$) to that of the lower panel (where $\rho = 0.5$). We see in the top panel, $\bar{V}^c$ is non-monotonic and tends to the shareholder's threshold as risk aversion becomes large. In contrast, the lower panel shows the threshold (for various values of c) is monotonically decreasing with risk aversion. We can explain these differences as follows.

When risk aversion is low, the impact of each risk on the threshold is relatively small, and neither risk is dominant. For low risk aversion, the threshold $\bar{V}^c$ lies somewhere between the two limiting cases $\bar{V}^0$ and $\bar{V}^s$. When risk aversion is high, we have two possibilities, depending on the level of correlation. If correlation is high, there is little idiosyncratic risk and control risk is the dominant effect, causing $\bar{V}^c$ to be very close to $\bar{V}^s$. For high risk aversion levels, the manager's fear of termination leads him to select a threshold close to that which maximizes firm value. This results in the threshold being non-monotonic in risk aversion, as in the upper panel of Figure 2. If correlation is low, then idiosyncratic risk is large, and it outweighs the impact of control risk if the strength of the control threat is low. In this case, the threshold approaches $\bar{V}^0$ (which approaches K, the zero npv threshold) as risk aversion increases. We can see this in the lower panel for the lower values of c. However, if the strength of the control threat is large, then this can balance with the large idiosyncratic risk, and result in a threshold which approaches some level between $\bar{V}^0$ and $\bar{V}^s$ as risk aversion increases. In this case, neither risk has dominated the other. In the lower panel, we see this for the higher values of c where we have marked the limiting values on the graph. We observe that when correlation is low, the threshold is monotonically decreasing in risk aversion, regardless of the strength of the control (see the lower panel). Under the extremal case where correlation is zero (and all risk from fluctuations in the investment value is idiosyncratic) we can derive the limiting results and the value of c at which the threshold behavior switches between npv limit of K and higher limiting values. We have

Proposition 6 *Suppose* $\rho = 0$. *Let* $c^* = \frac{\beta}{1-(\frac{\beta-1}{\beta})^{\beta-1}}$.
If $c \leq c^*$ *then as* $\gamma \to \infty$, $\bar{V}^c \to K$.
If $c > c^*$ *then as* $\gamma \to \infty$, $\bar{V}^c \to \tilde{v}(c) \in (K, \bar{V}^s)$ *where* $\tilde{v}(c)$ *solves*

$$\frac{\beta}{\tilde{v}(c)} - \frac{c}{K}\left[1 - \frac{\tilde{v}(c)}{\bar{V}^s}\right] = 0$$

Figure 3 plots the thresholds in the situation where all risk is idiosyncratic for various values of c. For our choice of $\beta = 3$, we calculate $c^* = 5.4$. The figure confirms the result of the proposition as there is a visible difference in the limiting behavior between $c = 5$ and $c = 6$.

We have seen in the lower panel of Figure 2 that behavior consistent with Proposition 6 still occurs when $\rho = 0.5$. We also saw that when ρ was much higher ($\rho = 0.95$), markedly different threshold behavior occurred, since the threshold was then non-monotonic in γ. Using similar arguments to those used in Proposition 6, we find the critical value of correlation where the behavior of the thresholds switches is $\rho^2 = 0.5$.[9] We call values of

[9]The precise value of this critical value of correlation depends on the behavior of the function Φ near the shareholder's threshold. A different specification for Φ would give

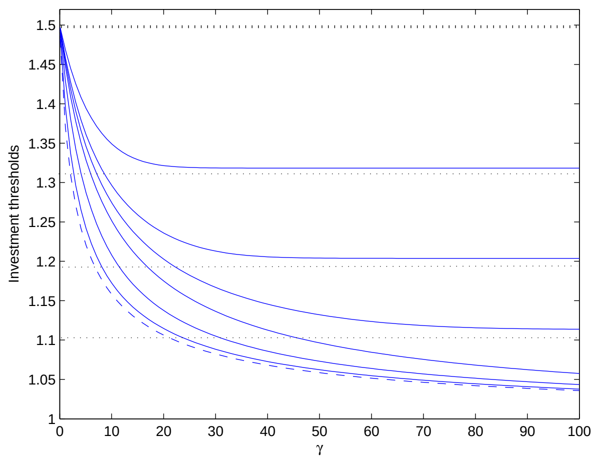

Figure 3: *Investment thresholds for the manager when $\rho = 0$, as a function of managerial risk aversion. The dashed curve is $\bar{V}^0$ and the solid lines are $\bar{V}^c$ for $c = 1, 3, 5, 6, 7, 10$ (from lowest to highest). The bold horizontal dotted line is $\bar{V}^s = 1.5$. Other parameters are $K = 1$, $\beta = 3$. The critical value of c for these parameters is $c^* = 5.4$. Also shown (by horizontal dotted lines) are the limiting values for $c = 6, 7, 10$.*

correlation such that $\rho^2 < 0.5$ the "low" correlation regime, and values such that $\rho^2 > 0.5$ the "high" correlation regime.

We now comment on the implications of our findings for the effectiveness of corporate control. As we saw in Figure 2 (and in the analytical results of Appendix B), control risk causes the risk averse manager to moderate his investment timing to be closer to what the shareholders prefer. We say corporate control is more effective if this occurs to a greater degree. Clearly one factor influencing the effectiveness of control is the strength of the challenge. More importantly, we have seen that the effectiveness of control also depends greatly on the level of correlation between the investment and the market. We investigate this further in Figure 4 where we plot thresholds against the value of correlation. We plot the thresholds for the manager who is not subject to control (dashed lines) and the manager subject to control with strength parameter $c = 4$ (solid lines). We compare two values of risk aversion – the non-bold lines take $\gamma = 2$ whilst the bold lines take $\gamma = 10$. We first observe that the thresholds for the manager without control are increasing in correlation. This is because as the correlation increases, there is less exposure to idiosyncratic risk whilst waiting to invest, and the manager can wait longer to invest at a higher threshold. When risk aversion is small, control causes the manager to choose a threshold between his preferred threshold $\bar{V}^0$ and that of the shareholders. Consistent with Figure 2, for small levels of risk aversion, the threshold $\bar{V}^0$ is not that far away from the shareholder's threshold. However, for higher levels of risk aversion, we can see the two regimes based on the level of correlation. The switch between the regimes can be seen in the figure when the threshold

a different critical value. However, the important observation is that there is a critical value, rather than its precise value.

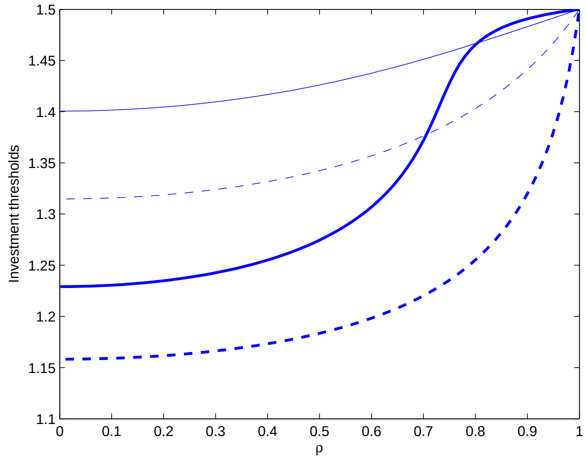

Figure 4: *Investment thresholds as a function of correlation. The dashed lines represent the threshold for the manager who is not subject to control,* $\bar{V}^0$*. The solid lines are thresholds for the manager subject to control,* $\bar{V}^c$*, with the strength of control being* $c = 4$*. The pair of bold lines are for risk aversion level* $\gamma = 10$*. The non-bold lines take* $\gamma = 2$*. Other parameters are* $K = 1$*,* $\beta = 3$ *giving* $\bar{V}^s = 1.5$.

changes from being convex to concave in correlation. When correlation is in the low regime, control increases the manager's choice of threshold, but it is still far from the shareholder's threshold. For the low correlation regime, the corporate control is relatively ineffective. In the high correlation regime, control causes the manager's threshold to move much closer to the shareholder's threshold. Control in the high correlation regime is very effective since it results in the manager investing close to the firm value maximizing threshold.

We can compare our findings with those of Hugonnier and Morellec (2006). They give a single threshold at which the manager subject to both incomplete markets and control invests (comparable to our threshold $\bar{V}^c$). As risk aversion increases in their model, the manager's threshold approaches the npv threshold, K. Thus in their model it appears that control is ineffective in moderating the investment behavior of the manager. Our conclusion is in stark contrast to their finding. We find that effectiveness of corporate control depends on the degree of incompleteness, and that for sufficiently high values of correlation and risk aversion, control significantly alters the manager's investment timing to be much closer to the shareholder's optimum.

2.2 Agency Costs

We now consider the agency costs that are experienced by shareholders as a result of the risk averse manager choosing the timing of investment. This enables us to evaluate whether the variations in investment thresholds translate into significant dollar-values and are therefore of economic importance. We can evaluate agency costs as being the difference in the shareholder's value of the option, and the shareholder value given the manager makes the investment timing decision:

$$\mathcal{A}_c(v) = \mathcal{S}(v) - \mathcal{S}_{\bar{V}^c}(v).$$

Similarly, we can also calculate the agency costs in the case where there is no corporate control:

$$\mathcal{A}_0(v) = \mathcal{S}(v) - \mathcal{S}_{\bar{V}^0}(v).$$

A comparison of these two costs will allow us to draw conclusions on the impact of control on agency costs to shareholders.

Figure 5 plots agency costs as a function of $\gamma(1-\rho^2)$. The dashed line corresponds to agency costs without control, $\mathcal{A}_0(v)$. In this case, agency costs only depend on risk aversion and correlation through the product $\gamma(1-\rho^2)$ (see (13)). For a fixed correlation, these costs increase monotonically with risk aversion, since the manager's threshold $\bar{V}^0$ is decreasing with risk aversion (see Proposition 5). If there is no well functioning market for corporate control (perhaps because the manager is also a large shareholder), then agency costs due to incomplete markets can be significant. For parameter values of $\gamma = 10$ and $\rho = 0.5$, the agency costs without control are around 25% of the shareholder value.

The solid lines on Figure 5 correspond to agency costs under control, $\mathcal{A}_c(v)$, for strength parameter $c = 1$. The threat of control always results in a reduction of agency costs to shareholders. However the magnitude of this reduction varies with parameters. As c increases, agency costs fall, since the manager's threshold is closer to the shareholder's threshold. Recall we said control was more effective if it caused a larger change in the manager's threshold, taking it closer to the shareholders' threshold. We can equivalently say control is more effective if it causes a larger reduction in agency costs to shareholders. We compare the impact of control on agency costs in two scenarios where $\gamma(1-\rho^2) = 1$ and hence the agency costs without control are identical. The higher solid line takes $\rho = 0.5$ and thus $\gamma(1-\rho^2) = 1$ corresponds to $\gamma = 4/3$. The lower solid line takes $\rho = 0.95$ and so $\gamma(1-\rho^2) = 1$ corresponds to $\gamma = 10$. We see when correlation is high, agency costs are virtually eliminated by control. However, when correlation is low, control only reduces agency costs by around 40%. As expected, the impact of control on agency costs depends on the level of correlation, or equivalently, the degree of incompleteness. This is exactly as we would expect, given our earlier analysis of the thresholds. In fact, we recall the two correlation regimes in which markedly differing threshold behavior emerged. The same two regimes apply to the analysis of agency costs. If the correlation is sufficiently high, there is little idiosyncratic risk, and corporate control is very effective in reducing agency costs. In the high correlation regime, agency costs are non-monotonic in risk aversion. However, when correlation is in the low regime, agency costs increase monotonically in risk aversion.

To summarize, if there is a market for corporate control, and if the investment is such that the situation is relatively incomplete, then control is quite inefficient in reducing agency costs to shareholders. However if the investment is in a "close to complete" market, then control is more effective, and agency costs can be negligible for moderate levels of risk aversion. We finally remark that our conclusions on agency costs differ from those made in Hugonnier and Morellec (2006). In their model, agency costs are monotonically increasing in risk aversion and control is relatively ineffective.

2.3 Value of the Option to Invest

In addition to comparing the investment thresholds, and agency costs to shareholders, we can compare the value the manager and shareholder's place on the option to invest. The shareholder's value of the option was given in Proposition 1. The manager's value of the investment option is obtained via certainty equivalence, and we obtain

Proposition 7 *(i) The manager's value of the option to invest when he faces incomplete markets and corporate control is*

$$\mathcal{M}_{\bar{V}^c}(v) = -\frac{1}{\gamma(1-\rho^2)} \ln\left[1 - \{1 - D(\bar{V}^c)^{1-\rho^2}\}(v/\bar{V}^c)^{\beta}\right]$$

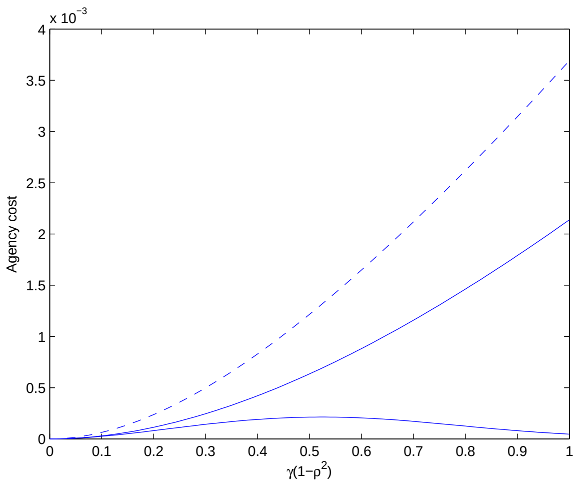

Figure 5: *Agency costs as a function of* $\gamma(1-\rho^2)$ *when* $v = 1 = K$. *The dashed line is agency costs for a manager who is not subject to control* $\mathcal{A}_0(v)$. *The solid lines correspond to agency costs for a manager subject to control* $\mathcal{A}_c(v)$. *Both take strength parameter* $c = 1$. *The higher solid line takes* $\rho = 0.5$ *and the lower solid line takes* $\rho = 0.95$. *We also take* $\beta = 3$.

(ii) The manager's value of the option to invest when he faces incomplete markets (but no control challenges) is

$$\mathcal{M}_{\bar{V}^0}(v) = -\frac{1}{\gamma(1-\rho^2)} \ln[1 - (1 - e^{-\gamma(1-\rho^2)(\bar{V}^0 - K)})(v/\bar{V}^0)^\beta]$$

In Figure 6 we plot the three option values against managerial risk aversion. In each panel, the highest is the shareholder's value, which does not vary with risk aversion. The upper panel takes correlation to be 0.95 whilst the lower panel takes $\rho = 0.5$. In each panel, the dashed line represents $\mathcal{M}_{\bar{V}^0}$, the manager's value given no corporate control. This is the value to the manager, given he faces incomplete markets. The value he places on the option to invest is lower than the shareholder's value, due to his aversion to idiosyncratic risk. For risk aversion $\gamma = 10$ and correlation $\rho = 0.95$, we see in the upper panel that the value is reduced to about 87% of the shareholder's value, whereas in the lower panel the effect is much larger and the value is only 53% of shareholder value because of the lower correlation and corresponding higher idiosyncratic risk. This shows that incompleteness alone can reduce the option value dramatically. When the manager also faces corporate control, the value $\mathcal{M}_{\bar{V}^c}$ is given by the solid lines in each panel. In the upper panel, we plot the value taking strength parameter $c = 1$. In this case, varying c only altered the value of the option marginally, so we do not plot other values. The risk of a control challenge reduces the value of the option to the manager further relative to the manager who is not subject to such risk. Again for $\gamma = 10$, in the upper panel, the value is about 84% of the shareholder's value. In the lower panel, we plot the option value for various values of the parameter c. For $c = 1$, the value is about 46% of the shareholder value, this falls to only 34% when $c = 10$. Both these values are when $\gamma = 10$. We can conclude that the risk of control can reduce

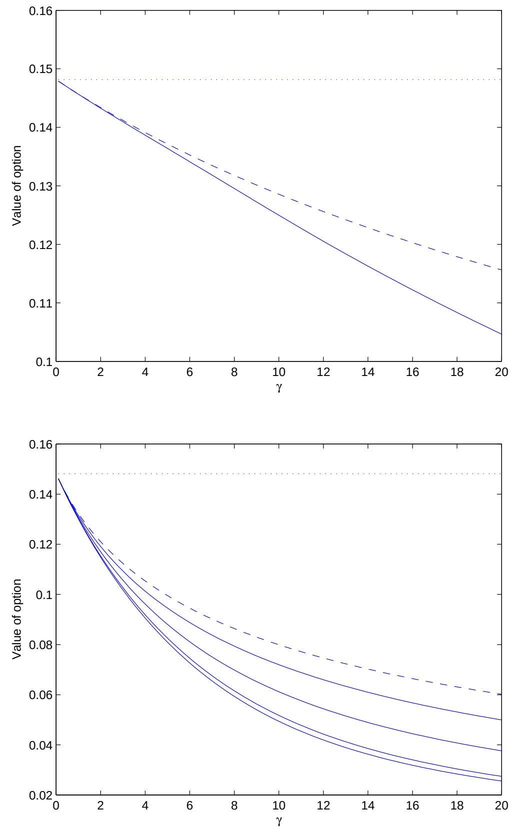

Figure 6: *Value of the Option to Invest, as a function of managerial risk aversion. The horizontal (dotted) line is the shareholder's value* $\mathcal{S}(v)$*. The (dashed) line is the manager's option value, given there is no corporate control,* $\mathcal{M}_{\bar{V}^0}$*. The solid lines are the manager's option value under control,* $\mathcal{M}_{\bar{V}^c}$ *for various values of* c*. The upper panel takes* $\rho = 0.95$ *whilst the lower panel takes* $\rho = 0.5$*. The solid line in the upper panel takes strength parameter* $c = 1$*. Other values of* c *gave a negligible difference. The solid lines in the lower panel correspond to (from highest to lowest) values* $c = 1, 3, 7, 10$*. Parameter values are* $K = 1, \beta = 3$*.*

the option value further relative to the situation without control, and that if the investment is such that there is a large idiosyncratic risk, the reduction in value is much greater and can be dramatic.

3 Conclusions

We present a model of investment timing by risk averse managers facing incomplete markets and corporate control. Idiosyncratic risks and control risks have opposing influences on the manager's investment behavior, and the manager trades-off between them. In fact, we show that the manager's trade-off results in two regimes based on the degree of incompleteness his investment poses. The risk of termination dominates over the effect of idiosyncratic risk when the latter is small, causing the manager to invest close to the shareholder's threshold. If idiosyncratic risk is not small, then if the control challenge is fairly weak, idiosyncratic risk dominates and the manager selects a threshold close to the zero npv threshold. However, for stronger control threats, neither effect dominates. Our findings are in contrast to those of Hugonnier and Morellec (2006), as idiosyncratic risk always dominated in their setting. Consequently, our model predictions are also different.

Our model predicts that when there is a well functioning market for corporate control, and when risk averse managers face a fairly "complete" market, they invest close to the time resulting in firm value maximization. Any evidence in favor of the standard real options model of investment timing is also evidence consistent with our prediction, provided it is gathered in a situation which is fairly complete, and where there was control risk. In contrast, such evidence is not consistent with the conclusions of Hugonnier and Morellec (2006). Although there are a number of papers supporting real options models, we mention Paddock, Siegel and Smith (1988) and Moel and Tufano (2002). The former use offshore oil reserve data, and the latter gold mines, both of which are likely to be relatively complete situations, given trading can be done in commodity futures. In addition, takeovers were commonplace in the oil and mining industries in the 1980's (see Jensen (1986)).

On the other hand, if a risk averse manager faces a relatively incomplete investment situation, then our model predicts the manager invests closer to the zero npv threshold since control is relatively ineffective. Increasing the strength of the control challenges improves effectiveness by encouraging the manager to choose a higher threshold. However, in reality, it is costly for shareholders to increase the control threat. There are costs of internal monitoring, as well as costs associated with terminating the manager (golden parachutes and other severance payouts) and hiring a new manager (both in terms of the search itself and the initial reduction in productivity whilst the manager settles in). It seems plausible that within the likely constraints on the strength of control, that control remains at levels which are ineffective when the investment's correlation with traded assets is low. The implication is that in countries such as the US (where the market for corporate control disciplines managers), we would expect to see differences in investment behavior across industries and indeed over specific investments, as the level of incompleteness varied. It appears that both the standard real options model and the npv framework can play a role in describing investment behavior of managers.

If there is no well-functioning market for corporate control (perhaps because of a large shareholder-manager being poorly disciplined), then we would expect to see the manager investing much earlier than shareholders prefer, resulting in large agency costs. The implication is that in many countries where large shareholder-managers are the norm, for investments for which there are no highly correlated traded assets in which to hedge, and for managers who are risk averse, the npv framework may become a reasonable approximating description of investment behavior.

In our framework, the manager's reward depends on the investment option itself, which proxies for stock option compensation. We noted that if the reward does not depend on the option itself, then the manager should not have any preference for the timing of investment, and certainly does not time investment such that firm value is maximized. In this sense, option compensation is partially aligning the interests of the manager with the shareholders, since the option causes the manager to have a preference for investment timing, albeit different from that of the shareholders. This is in agreement with the idea that options act as a corporate governance mechanism (see the survey of Becht et al (2005)). Once markets are incomplete however, options are not sufficient to discipline managers, and corporate control also acts as a disciplinary device in our model.

We mention here some issues concerning the robustness of our results. In our model, we assume the manager has exponential utility, which enables us to obtain semi-closed form results for investment thresholds and thus provide some analytical comparison results between thresholds. Whilst Hugonnier and Morellec (2006) use CRRA utility, this is not the reason for the differences in our conclusions and there is no reason to expect our main results would change under CRRA preferences. However, employing CRRA utility in our set-up would greatly complicate the analysis since we would require a two-dimensional problem which would be solved numerically. It did not complicate Hugonnier and Morellec's analysis because their manager's reward did not depend on the investment payoff. It is in fact the dependence of the reward on the investment payoff which distinguishes our conclusions from theirs. We also mention that in our model, investment payoffs are one-off rewards. Although many investments fit this description, others may better be described by a reward paying streams of cash-flows. Miao and Wang (2006) show that in this setting, incomplete markets alone cause the manager to delay, rather than speed-up investment. We would expect in this case that corporate control (this time influencing the manager to invest earlier) would still outweigh the effect of idiosyncratic risk when such risk was small, and therefore control would again be effective in this situation. Such results would be obtained however with a considerable loss of tractability and with little gain in terms of insight compared with the current model.

Of course the investment timing choice of managers will reflect other features not modeled here including strategic competition (Grenadier (2002)), unobserved effort and empire building (Grenadier and Wang (2005)) and private benefits (Morellec and Smith (2006)). It could be argued that incompleteness of markets is a more fundamental effect than many other types of agency conflicts since it does not require any special assumptions on managers' preferences, other than that they are risk averse. More importantly, we have demonstrated in this paper that the interaction of incomplete markets and corporate control can result in a variety of investment behavior by risk averse managers.

Appendix

A Remarks and Proofs

Remarks on the Manager's Optimization Problem in (7)

In this section we provide justification for the choice of discount factor $\zeta = -\frac{1}{2}\lambda^2$ in the manager's optimization problem in (7). As we discussed already, if we separate out the portfolio choice problem, then we do not want the manager to have a preference over the horizon τ. The choice $\zeta = -\frac{1}{2}\lambda^2$ ensures the manager facing the portfolio choice problem in (8):

$$\sup_{(\theta_u)_{t \le u \le \tau}} \mathbb{E}[e^{\frac{1}{2}\lambda^2 \tau} U(X_\tau) | X_t = x] \tag{14}$$

is indifferent over the choice of the horizon τ. The intuition is that under this formulation, the manager with the investment option to exercise does not already have an in-built preference for early or late exercise arising from the underlying portfolio choice problem. If he were to have such a preference, this would bias the exercise time of the option to invest.

Let $J(t,x) = e^{\frac{1}{2}\lambda^2 t} U(x)$ so (14) can be written as

$$\sup_{(\theta_u)_{t \le u \le \tau}} \mathbb{E}[J(\tau, X_\tau) | X_t = x].$$

If we can show that $J(t, X_t)$ is a super-martingale in general, and a martingale for the optimal θ ($J \le 0$), then

$$J(t,x) = \sup_{(\theta_u)_{t \le u \le \tau}} \mathbb{E}[J(\tau, X_\tau) | X_t = x]$$

and we can write

$$J(t,x) = \sup_{\tau} \sup_{(\theta_u)_{t \le u \le \tau}} \mathbb{E}[J(\tau, X_\tau) | X_t = x]$$

since $J(t,x)$ does not depend on the horizon τ.

We now show these properties. Applying Itô's formula to $J(t,X_t)$ and integrating gives

$$J(\tau,X_\tau)=J(t,X_t)+\int_t^\tau \frac{J(s,X_s)}{2}\left[\lambda-\gamma\theta_s\sigma\right]^2 ds-\int_t^\tau \gamma\theta_s\sigma J(s,X_s)dB_s$$

It follows that $\mathbb{E}J(\tau,X_\tau)\leq J(t,X_t)$ for any θ, and using the optimal strategy solving the problem (14), $\theta_s^*=\frac{\lambda}{\gamma\sigma}$, we have

$$\sup_{(\theta_u)_{t\leq u\leq\tau}}\mathbb{E}[J(\tau,X_\tau)]=J(t,X_t).$$

Hence $J(t,X_t)$ is a super-martingale in general and a martingale for the optimal θ.

Proof of Proposition 2

In the continuation region, $e^{\frac{1}{2}\lambda^2 t}H(x,v)$ is a martingale under the optimal strategy and a supermartingale otherwise. The HJB equation is derived using Ito's formula, giving

$$0=\frac{1}{2}\lambda^2 H+\xi\eta v H_v+\frac{1}{2}\eta^2 v^2 H_{vv}+\sup_\theta\left\{\theta\lambda\sigma H_x+\frac{1}{2}\theta\sigma^2 H_{xx}+\theta\sigma\rho\eta v H_{xv}\right\}$$

Optimizing over θ gives

$$0=\frac{1}{2}\lambda^2 H+\xi\eta v H_v+\frac{1}{2}\eta^2 v^2 H_{vv}-\frac{1}{2}\frac{(\lambda H_x+\rho\eta v H_{xv})^2}{H_{xx}}$$

We solve subject to the associated boundary condition, as well as at the constant exercise threshold $\bar{V}^c$, we must have value-matching and smooth-pasting conditions:

$$
\begin{aligned}
H(x,0) &= -\frac{1}{\gamma}e^{-\gamma x} && (15)\\
H(x,\bar{V}^c) &= \mathbb{E}\left[-\frac{1}{\gamma}e^{-\gamma(x+(\bar{V}^c-K)^+I_\tau};\tau<\infty\right] \\
&= -\frac{1}{\gamma}e^{-\gamma x}[1+e^{-\Phi(\bar{V}^c)}(e^{-\gamma(\bar{V}^c-K)^+}-1)] && (16)\\
H_v(x,\bar{V}^c) &= \frac{1}{\gamma}e^{-\gamma x}e^{-\Phi(\bar{V}^c)}\Big\{\gamma e^{-\gamma(\bar{V}^c-K)^+} \\
&+ \Phi'(\bar{V}^c)(e^{-\gamma(\bar{V}^c-K)}-1)\Big\} && (17)
\end{aligned}
$$

Proof of Proposition 3

We want to solve the non-linear pde subject to the associated boundary, value matching and smooth pasting conditions. Proposing a solution of the form $H(x,v) = -\frac{1}{\gamma}e^{-\gamma x}\Gamma(v)^g$ gives

$$0 = \left[v\Gamma_v\eta\,(\xi-\lambda\rho) + \frac{1}{2}\eta^2v^2\Gamma_{vv} + \frac{1}{2}\frac{\Gamma_v^2}{\Gamma}\eta^2v^2(g(1-\rho^2)-1)\right]. \qquad (18)$$

Choosing $g = \frac{1}{1-\rho^2}$ eliminates the non-linear term completely, leaving

$$0 = \left[v\Gamma_v\eta\,(\xi-\lambda\rho) + \frac{1}{2}\eta^2v^2\Gamma_{vv}\right] \qquad (19)$$

with corresponding conditions on $\Gamma(v)$ (translated from (15)-(17))

$$
\begin{aligned}
\Gamma(0) &= 1 && (20)\\
\Gamma(\bar{V}^c) &= [1+e^{-\Phi(\bar{V}^c)}(e^{-\gamma(\bar{V}^c-K)^+}-1)]^{1-\rho^2} && (21)\\
\frac{\Gamma_v(\bar{V}^c)}{\Gamma(\bar{V}^c)}\Gamma(\bar{V}^c)^{1/(1-\rho^2)} &= -(1-\rho^2)e^{-\Phi(\bar{V}^c)}\Big\{\gamma e^{-\gamma(\bar{V}^c-K)^+} && (22)\\
&+ \Phi'(\bar{V}^c)(e^{-\gamma(\bar{V}^c-K)}-1)\Big\}
\end{aligned}
$$

We propose a solution of the form $\Gamma(v) = Lv^\psi$, for some constant L which results in the fundamental quadratic in ψ,

$$\psi(\psi-1)\frac{\eta^2}{2} + \psi\eta(\xi-\lambda\rho) = 0.$$

The two roots of the quadratic are

$$\psi = \beta = 1 - \frac{2(\xi - \lambda\rho)}{\eta}, \qquad \psi = 0. \tag{23}$$

That is, there are one non-zero and one zero root. It can be seen that the general form of the solution must be $\Gamma(v) = Lv^{\beta} + B$, and (20) gives $B = 1$. We now have to decide when we can build a solution that value-matches and smooth-pastes. There are two possibilities. If $\beta \leq 0$ then $L = 0$, smooth pasting fails and there is no solution. In this case, the manager postpones indefinitely. If $\beta > 0$ the manager will exercise at time τ. We assume $\beta > 1$ such that the corresponding shareholder's problem is well defined and $\bar{V}^s < \infty$. In the case $\beta > 0$, (21) gives the constant $L < 0$, and thus

$$\Gamma(v) = 1 + \left\{[1 + e^{-\Phi(\bar{V}^c)}(e^{-\gamma(\bar{V}^c - K)^+} - 1)]^{1-\rho^2} - 1\right\}\left(\frac{v}{\bar{V}^c}\right)^{\beta} \tag{24}$$

Recalling $H(x,v) = -\frac{1}{\gamma}e^{-\gamma x}\Gamma(v)^g$, we find the solution is of the form given. We now solve for the optimal investment threshold $\bar{V}^c$ via (24) and the smooth-pasting condition (22). Manipulations result in the desired result.

Proof of Proposition 5 (i) We first show $\lim_{\gamma\to 0}\bar{V}^0 = \bar{V}^s$. We take the limit as $\gamma \to 0$ in (13). For $\beta > 1$, as $\gamma \to 0$,

$$(\bar{V}^0 - K) = \frac{1}{\gamma(1-\rho^2)}\ln\left(1 + \frac{\gamma(1-\rho^2)}{\beta}\bar{V}^0\right) \approx \bar{V}^0/\beta$$

giving $\lim_{\gamma\downarrow 0}\bar{V}^0 = \bar{V}^s$, where $\approx$ denotes equality to leading order. To show $\lim_{\gamma\to\infty}\bar{V}^0 = K$, take limits as $\gamma \to \infty$ in (13) immediately gives the right-hand-side tends to zero.

(ii) A tedious differentiation shows that $\bar{V}^0$ is decreasing in γ.

Proof of Proposition 6 When $\rho = 0$ we have from (12) that $\bar{V}^c$ solves

$$\frac{\beta}{w}\kappa(w) = \gamma e^{-\gamma(w-K)} + \kappa(w)|\Phi'(w)| \tag{25}$$

We look for a solution near K and try $w = K + F(\gamma)$. Then $\kappa(w) = 1 - e^{-\gamma F(\gamma)}$. Suppose $F(\gamma) \to 0$ but $\gamma F(\gamma) \to \infty$ so that $\kappa(w) \to 1$. Then we have

$$\frac{\beta}{K} = \gamma e^{-\gamma F(\gamma)} + |\Phi'(K)|$$

giving

$$F(\gamma) = \frac{1}{\gamma} \ln \left[\frac{\gamma}{\beta/K - |\Phi'(K)|} \right] = \frac{1}{\gamma} \ln(A\gamma)$$

where $A = 1/(\beta/K - |\Phi'(K)|)$.

Provided $\beta/K > |\Phi'(K)|$ or equivalently $A > 0$, then $\gamma F(\gamma) \to \infty$ but $F(\gamma) \to 0$ as required. Otherwise $F(\gamma)$ does not exist. Since

$$|\Phi'(K)| = \frac{c}{K} \left[1 - \left(\frac{\beta - 1}{\beta} \right)^{\beta - 1} \right]$$

the requirement that $A > 0$ is equivalent to $c < c^*$ where

$$c^* = \frac{\beta}{1 - (\frac{\beta-1}{\beta})^{\beta-1}}.$$

Now suppose $c > c^*$. Let y^* solve $M(y) = 0$ where

$$M(y) = \frac{\beta}{y} - \frac{c}{K} \left[1 - \left(\frac{y}{\bar{V}^s} \right)^{\beta-1} \right] \tag{26}$$

Note for $c > c^*$

$$M(K) = \frac{1}{K} \left[\beta - c \left(1 - \left(\frac{K}{\bar{V}^s} \right)^{\beta-1} \right) \right] = \frac{\beta}{K} \left[1 - \frac{c}{c^*} \right] < 0$$

and $M(\bar{V}^s) = \beta/\bar{V}^s > 0$ so there exists a solution to $M(y) = 0$. Also

$$M'(y) = -\frac{\beta}{y^2} + \frac{c}{K} \frac{\beta - 1}{y} \left(\frac{y}{\bar{V}^s} \right)^{\beta-1}$$

and using (26),

$$M'(y^*) = -\frac{\beta}{(y^*)^2} + \frac{c}{K} \frac{\beta - 1}{y^*} \left(\frac{y^*}{\bar{V}^s} \right)^{\beta-1} = \frac{1}{K(y^*)^2} \left[c(\beta - 1)y^* - \beta^2 K \right]$$

We see $M'(y^*) > 0$ for large enough y, and $M'(y^*) < 0$ for small y. Thus there is a unique root in $[K, \bar{V}^s]$.

Now try $w = y^* + L(\gamma)$. Then since w solves (25) we have

$$\left[\frac{\beta}{w} - |\Phi'(w)|\right] = \kappa(w)^{-1}\gamma e^{-\gamma(w-K)}$$

where $\kappa(w) = 1 - e^{-\gamma(y^*-K)-\gamma L(\gamma)}$. Letting $\gamma \to 0$ gives $\kappa(w)^{-1}\gamma e^{-\gamma(w-K)} \to 0$ and hence the left-hand-side must tend to zero. Hence we conclude that $w \to y^*$.

Proof of Proposition 7 (i) The manager's value of the option to invest is given by the solution to

$$H(x, v) = H(x + \mathcal{M}_{\bar{V}^c}, 0)$$

which calculates the certainty equivalent value of the option by comparing the value achieved with the option to that without the option but additional initial wealth. This additional initial wealth $\mathcal{M}_{\bar{V}^c}$ is the certainty equivalent option value.

(ii) Taking $c = 0$ or equivalently $\Phi(\bar{V}^c) = 0$ in (i) gives the result.

B Threshold Comparison Results

In this Appendix, we state and prove some results concerning the relationship between the thresholds of the shareholders, manager subject to control, and manager not subject to control. We saw in Figure 2 that the thresholds maintained the relationship $\bar{V}^0 < \bar{V}^c < \bar{V}^s$, so that the manager facing incomplete markets moderates his investment timing to be closer to what the shareholders want, when shareholders exert control. We can prove the following.

Proposition 8 *When the correlation $\rho = 0$, the thresholds are ordered as*

$$\bar{V}^0 \le \bar{V}^c \le \bar{V}^s$$

This result is for the special case of the model where there is effectively no market asset in which to hedge risk and so all risk from the fluctuating investment value is idiosyncratic. In this case, the expression for the threshold $\bar{V}^c$ simplifies dramatically and it is straightforward to prove the result.

For any value of ρ, we have result that the thresholds are ordered for small levels of risk aversion. Due to the complicated nature of the formulae, it was not straightforward to prove the ordering for general γ values, although our numerical investigations support the conclusion that the thresholds maintain their ordering.

Proposition 9 *For small γ, the thresholds are ordered as*

$$\bar{V}^0 \le \bar{V}^c \le \bar{V}^s$$

We now prove each of the above results.

Proof of Proposition 8

We know from Proposition 5 that $\bar{V}^0 \le \bar{V}^s$ for all values of ρ. When $\rho = 0$, the expression for $\bar{V}^c$ in (12) simplifies to

$$\bar{V}^c - K = \frac{1}{\gamma} \ln \left(1 + \frac{\gamma \bar{V}^c}{\beta + \bar{V}^c \Phi'(\bar{V}^c)} \right) \tag{27}$$

Similarly, the expression for $\bar{V}^0$ in (13) simplifies to

$$\bar{V}^0 - K = \frac{1}{\gamma} \ln \left(1 + \frac{\gamma \bar{V}^0}{\beta} \right) \tag{28}$$

Now suppose $\bar{V}^c > \bar{V}^s$. Then $\Phi'(\bar{V}^c) = 0$. This implies $\bar{V}^c = \bar{V}^0$, and combined with $\bar{V}^0 \le \bar{V}^s$ gives $\bar{V}^c = \bar{V}^0 \le \bar{V}^s$, which is a contradiction. So we must have $\bar{V}^c \le \bar{V}^s$. In this case, $\Phi'(\bar{V}^c) \le 0$. Observe the function

$$F(y,\alpha) = \ln\left(1 + \frac{\gamma y}{\beta + \alpha y}\right)$$

where $\gamma, \beta > 0$, is decreasing in α. Observe we can write (27) in terms of F as $\bar{V}^c - K = \frac{1}{\gamma}F(\bar{V}^c, \Phi'(\bar{V}^c))$ and (28) as $\bar{V}^0 - K = \frac{1}{\gamma}F(\bar{V}^0, 0)$. Since $\alpha = \Phi'(\bar{V}^c) \le 0$, so $F(y,\alpha) > F(y,0)$. This gives $\bar{V}^c \ge \bar{V}^0$. Thus $\bar{V}^0 \le \bar{V}^c \le \bar{V}^s$ as desired.

Proof of Proposition 9

First we expand the threshold $\bar{V}^0$ in powers of γ. Rewrite (13) as

$$e^{\gamma(1-\rho^2)(\bar{V}^0-K)} - 1 = \frac{\gamma(1-\rho^2)}{\beta}\bar{V}^0$$

Expanding in γ and putting $y = \bar{V}^0 - K$ gives

$$\left(y + \frac{1}{\gamma(1-\rho^2)}\frac{\beta-1}{\beta}\right)^2 = \frac{1}{\gamma^2(1-\rho^2)^2}\left(\frac{\beta-1}{\beta}\right)^2\left[1 + \frac{2K\gamma\beta(1-\rho^2)}{(\beta-1)^2}\right]$$

so

$$y = \frac{1}{\gamma(1-\rho^2)}\left(\frac{\beta-1}{\beta}\right)\left\{\frac{K\beta\gamma(1-\rho^2)}{(\beta-1)^2} - \frac{1}{2}K^2\beta^2\gamma^2\frac{(1-\rho^2)^2}{(\beta-1)^4} + ...\right\}$$

giving

$$\bar{V}^0 = \bar{V}^s - \frac{\gamma(1-\rho^2)\beta K^2}{2(\beta-1)^3} + O(\gamma^2) = \bar{V}^s - \gamma d_1 + O(\gamma^2)$$

where $d_1 = \frac{(1-\rho^2)\beta K^2}{2(\beta-1)^3} = \frac{(1-\rho^2)K^2}{2(\beta-1)^2}\left(\frac{\beta}{\beta-1}\right)$. Since $\beta > 1$, $d_1 > 0$, so $\bar{V}^0 < \bar{V}^s$. This is consistent with the result in Proposition 5 earlier.

Now we focus on the threshold $\bar{V}^c$ and again expand in powers of γ. First rewriting (12) gives

$$\begin{aligned} & e^{\Phi(\bar{V}^c)}e^{\gamma(\bar{V}^c-K)}(1 - e^{-\Phi(\bar{V}^c)}(1 - e^{-\gamma(\bar{V}^c-K)}))^{\rho^2} - e^{\Phi(\bar{V}^c)}e^{\gamma(\bar{V}^c-K)} \\ + \quad & e^{\gamma(\bar{V}^c-K)} - 1 = \frac{(1-\rho^2)}{\beta}\bar{V}^c\left\{\gamma - (e^{\gamma(\bar{V}^c-K)} - 1)\Phi'(\bar{V}^c)\right\} \end{aligned}$$

Expanding in γ gives

$$
\begin{aligned}
& e^{\Phi(\bar{V}^c)}\left(1+\gamma(\bar{V}^c-K)+\frac{1}{2}\gamma^2(\bar{V}^c-K)^2\right)\left\{\left(1-e^{-\Phi}\gamma(\bar{V}^c-K)\right.\right.\\
+ \quad & \left.\left. e^{-\Phi}\frac{1}{2}\gamma^2(\bar{V}^c-K)^2\right)^{\rho^2}-1\right\}+\gamma(\bar{V}^c-K)+\frac{1}{2}\gamma^2(\bar{V}^c-K)^2\\
= \quad & \frac{(1-\rho^2)}{\beta}\bar{V}^c\left\{\gamma-\Phi'\gamma(\bar{V}^c-K)-\Phi'\frac{1}{2}\gamma^2(\bar{V}^c-K)^2\right\}
\end{aligned}
$$

where we abbreviate $\Phi=\Phi(\bar{V}^c)$ and $\Phi'=\Phi'(\bar{V}^c)$. Expanding and collecting terms lower than $O(\gamma^2)$ gives

$$
(\bar{V}^c-K)+\frac{(1-\rho^2e^{-\Phi})\gamma(\bar{V}^c-K)^2}{2}=\frac{\bar{V}^c}{\beta}\left\{1-(\bar{V}^c-K)\Phi'-\frac{\gamma}{2}(\bar{V}^c-K)^2\Phi'\right\} \tag{29}
$$

Now propose that $\bar{V}^c=\bar{V}^s-\gamma d_2+O(\gamma^2)$ for some d_2. We aim to calculate d_2. Substitution into (29) gives

$$
\begin{aligned}
& \left(\frac{K}{\beta-1}-\gamma d_2\right)+\frac{1-\rho^2}{2}\gamma\left(\frac{K}{\beta-1}-\gamma d_2\right)^2\\
= \quad & \left(\frac{K}{\beta-1}-\frac{\gamma d_2}{\beta}\right)\left\{1-\left(\frac{K}{\beta-1}-\gamma d_2\right)\Phi'-\frac{\gamma}{2}\left(\frac{K}{\beta-1}-\gamma d_2\right)^2\Phi'\right\}
\end{aligned}
$$

Matching terms on each side of this equation shows terms cancel to $O(1)$. To $O(\gamma)$, we need an expression for Φ'. Differentiating (6) gives

$$
\Phi'(v)=\frac{c}{K}\left[\frac{\bar{V}^s-K}{\bar{V}^s}\beta\left(\frac{v}{\bar{V}^s}\right)^{\beta-1}-1\right]=\frac{c}{K}\left[\left(\frac{v(\beta-1)}{K\beta}\right)^{\beta-1}-1\right];\quad v<\bar{V}^s
$$

Substitution and expanding gives

$$
\Phi'(\bar{V}^c)\approx\frac{c}{K}\left[\left(\frac{(\beta-1)}{K\beta}\right)^{\beta-1}\left(\frac{\beta}{\beta-1}K-\gamma d_2\right)^{\beta-1}-1\right]\approx-\frac{(\beta-1)^2\gamma d_2}{\beta K}\frac{c}{K}
$$

Comparing terms of $O(\gamma)$ gives

$$
-d_2+\frac{1}{2}\frac{K^2}{(\beta-1)^2}(1-\rho^2)=-\frac{d_2}{\beta}+\frac{K}{\beta}\frac{c}{K}d_2
$$

and thus

$$d_2 = \frac{(1-\rho^2)K^2}{2(\beta-1)^2}\frac{\beta}{(\beta-(1-c))}$$

We now obtain comparisons between the thresholds. Since $\beta > 1$, $d_2 > 0$ and so $\bar{V}^c < \bar{V}^s$. Also $\frac{\beta}{(\beta-(1-c))} < \frac{\beta}{(\beta-1)}$ so $d_2 < d_1$ and hence $\bar{V}^0 < \bar{V}^c$. We thus have $\bar{V}^0 < \bar{V}^c < \bar{V}^s$ as desired.

C Comparison with Hugonnier and Morellec (2006)

In this appendix, we explore in more detail the differences in our framework and that of Hugonnier and Morellec (2006). These differences in formulation distinguish the conclusions of this paper from theirs, and contribute new economically important insights into the effects of incompleteness and corporate control upon investment timing and agency costs.

Hugonnier and Morellec (2006) consider simultaneously the impact of incomplete markets and corporate control on investment timing. They find that the manager invests close to the zero npv threshold, eroding almost all of the value of waiting. In this sense, they find control is ineffective, since shareholder's are not successful at encouraging the manager to invest closer to their preferred threshold.

In this Appendix, we highlight that two features of their model drive their results - first, their choice of discount factor and second, their manager's reward does not depend upon the investment payoff[10], rather it depends on wealth arising from holding the market and riskless bond. This wealth is scaled up or down (via a factor θ_τ) depending on whether he is terminated.

[10]Provided we make the reasonable assumption that the manager only invests in positive npv situations.

By highlighting the role of these factors in their conclusions, we can explain why our model gives more realistic outcomes.

We first draw an analogy between the treatment of discounting in Hugonnier and Morellec (2006) and this paper. In Hugonnier and Morellec's model, there is a subjective discount factor (which is analogous to our parameter ζ) and a parameter q which is a function of the subjective discount factor and other parameters of the model. We want to obtain a comparison to q in our setup. Since Hugonnier and Morellec use CRRA preferences, we take the limit as $R \to \infty$ (where R is the coefficient of constant relative risk aversion) of their parameter q to obtain the equivalent parameter under exponential utility, which we denote q^e.[11] We obtain (converting other model parameters to our notation)

$$q^e = \lim_{R \to \infty} q = (1 - \rho^2)[\zeta + \frac{1}{2}\lambda^2]$$

We now note that there is a link between the sign of q^e and the sign of $\zeta + \frac{1}{2}\lambda^2$. If $q^e > 0$, then this corresponds to $\zeta > -\frac{1}{2}\lambda^2$ in our model. Recall (see Appendix A and the discussion in the main text) we chose $\zeta = -\frac{1}{2}\lambda^2$ in our model, which corresponds to the choice $q^e = 0$. Hugonnier and Morellec (2006) assume $q > 0$, or in exponential utility terms, $q^e > 0$. We will elaborate now on the impact of this choice on their conclusions.

Although not considered in their paper, we can take the special case of the Hugonnier and Morellec (2006) model where there is no risk of a control challenge. In this case, the scale factor θ should be set to $\theta_\tau = 1$. In this case, the manager simply solves a portfolio choice problem to choose holdings in a market asset and a bond. In contrast to our model, their manager's

[11] Alternatively, we could derive q^e directly by re-solving their model with exponential utility.

reward does not depend on the investment payoff itself, and therefore, their manager should be indifferent over the investment timing (since it does not affect in any way his reward when there is no chance of termination). After some calculations, their manager's problem becomes (see their Theorem 1 with $\theta_\tau = 1$)

$$U(x) \sup_\tau \mathbb{E} e^{-(q/\Phi)\tau}$$

where $\Phi > 0$ and $U(x)$ are constants, x represents initial wealth. Under Hugonnier and Morellec's assumption that $q > 0$, the supremum above is trivially obtained at $\tau = 0$. Their manager is not indifferent to investment timing (as he certainly should be in this case) as this would require $q = 0$. Under the special case of their model without corporate control, their manager invests according to the npv rule, and all option value is eliminated. This has arisen because the assumption $q > 0$ means that the discounting is too great, inducing earlier (in this case, immediate) investment.

The reason we draw attention to this special case is that the assumption that $q > 0$ leads to a bias towards earlier investment times in the model with corporate control, which explains why the conclusions of Hugonnier and Morellec (2006) were that the manager facing both incompleteness and control invests close to the zero npv threshold.

Now let's consider what would happen in their model if we took $q = 0$. In the special case without control risks, this would imply the manager is indifferent over choice of investment times. However, in the model where there is control risk, the choice $q = 0$ would lead to the manager always choosing the shareholder's investment threshold. That is, the conclusion would be that there are no agency costs since the manager would always do what the shareholders wanted (see Hugonnier and Morellec (2006) [Theorem

2] with $q = 0$.)

We need to explain why our formulation does not give this degenerate conclusion, despite the fact that we choose the equivalent discounting to $q = 0$ (the choice $\zeta = -\frac{1}{2}\lambda^2$) in order to ensure there is no bias from the underlying portfolio choice problem. The reason is that our manager's reward depends on the investment payoff, see (7). In contrast, the manager in Hugonnier and Morellec (2006) has a reward which does not depend on the investment performance (except indirectly via termination). Our formulation has realistic conclusions both in the case where there is no corporate control and where the manager is subject to control challenges.

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