Optimal Corporate Strategy under Uncertainty

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Abstract

The purpose of this paper is to develop a discrete-time model for optimal corporate business strategy that incorporates both endogenous and exogenous factors and is consistent with the value-based criterion for maximizing the shareholders wealth. The model is feasible for practical implementation.

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1. INTRODUCTION

Optimal corporate strategy in a dynamic setting involves mainly portfolio revisions through which managers of a multi-business or multi-division corporation restructure their portfolio of strategic business units (SBU) periodically to maximize the market value of the firm. In the real options literature, theories have been developed and applied to model and explain managerial flexibility in investment projects and in the operational decisions. With a few exceptions — for example, Grenadier (1995, 1996, and 1999) and Smit and Ankum (1993), Smit, Smith and Trigerogis (2004) — the real-options approach has not been applied to corporate strategic planning. On the other hand, the corporate strategic planning has focused on the strategic environments and characteristics of corporate business units, and less on the managerial flexibility that real options make available to corporate managers. In several studies, for example the work of Grenadier (1995, 1996, and 1999), under the assumption of risk-neutrality and within the continuous-time framework, researchers have developed models that have combined theories of real options and business strategies. By doing so, these researchers obtained some interesting new results in corporate financial management. However, a key assumption in these studies is that the future homogeneous cash flows of a firm are generated from a single investment project or production process. Thus, the interrelations among the cash flows of different business units or divisions within a corporation or across corporations have not explicitly taken into account corporate strategic decisions making. Therefore, some interesting numerical results from these studies are not directly applicable to the strategic decisions of a multi-business or multi--division corporation.

The changing business conditions both endogenous and exogenous make it necessary for managers of a multi-business corporation to conduct periodic portfolio revisions in strategic planning. The central problem in the optimal corporate business strategy involves the following three components: (1) the portfolios of strategic business units (SBUs) of a corporation should be frequently revised by adding new units, deleting existing units, or reallocating corporate assets among existing units; (2) the evaluation and selection of an optimal corporate business strategy should follow the value-based criterion to maximize the shareholders wealth, using properly specified valuation models as decision criteria, and; (3) the relevant endogenous and exogenous dynamic factors should be incorporated into the valuation models and the decision processes.

The purpose of this paper is to develop a discrete-time model for optimal corporate business strategy that both incorporates the above three components and is feasible for practical implementation. In Section 2, intertemporal capital asset pricing model (ICAPM) formulated by Merton (1973) and its special feature that is most suitable for corporate strategic planning decisions are discussed. In Section 3, the application of the ICAPM to evaluate a firm's SBUs and the decision rules for acquiring or divesting a business consistent with a value-based approach are discussed. Both the building and the restructuring of a corporate portfolio of SBUs are explored in Section 4. In Section 5, we formulate and solve the problem of optimal corporate investment in a new risky venture. In the final section, we address the parameter estimation question and summarize the findings and implications of the paper.

2. THE MERTON CAPITAL ASSET PRICING MODEL

Allowing investors to consider the possible shifts in their future investment opportunity set in their optimal investment-consumption decisions, Merton (1973) extended the static Markowitz portfolio selection model to a dynamic optimal portfolio decision model and developed an ICAPM. The two main results in Merton (1973) are (1) the traditional two-fund separation theorem formulated by Tobin (1958) no longer holds in the investors' optimal portfolio decisions; and (2) in the equilibrium risk-return relation for each asset, the *relevant*, or *systematic*, risk of an asset includes not only its *market-volatility risk* (measured by the covariance between the asset return and the return on the market portfolio), but also its *state-variable risk* (measured by the covariance between the asset return and the possible shifts in an investor's future investment opportunities). The exposure to possible shifts in future investment opportunities faced by investors is generally caused by changes in the state variables such as inflation, market interest rate, production technology, consumer preferences, foreign exchange rates, and so on. Therefore, in contrast to a single risk factor (referred to as beta risk) in the traditional Sharpe-Lintner-Mossin capital asset pricing model, there are multiple sources of risk in the equilibrium risk-return relationship for the ICAPM.

As will become clearer later in this paper, the ICAPM that allows capturing the characteristics of a corporation's SBUs through its state-variable risks is a better valuation model than the traditional CAPM for corporate strategic planning decision making. In the ICAPM, the relevant risks of any asset in equilibrium consist of both the market-volatility risk and the state-variable risk. Formally, the ICAPM can be recast in terms of the certainty-equivalent (CEQ) form of cash flows, as follows:

$$V_{jt} = \frac{1}{\gamma}\left\{\bar{D}_{jt+1} - \left[\lambda_1 COV\left(\tilde{D}_{jt+1}, \tilde{D}_{Mt+1}\right) - \lambda_2 COV\left(\tilde{D}_{jt+1}, \tilde{\theta}_{t+1}\right)\right]\right\}, \tag{1}$$

where

V_{jt} = the equilibrium value of firm or asset *j* at the beginning of period *t;*

γ = one plus the risk-free rate of interest;

$\tilde{D}_{jt+1}$ = the uncertain future cash profit of firm or asset *j*, a random variable with expected value $\bar{D}_{jt+1}$ and variance σ_j^2 (assumed constant over time for simplicity);

$\tilde{D}_{Mt+1}$ = the aggregate uncertain future cash profit of all firms or assets at the beginning of period *t*+1;

$\tilde{\theta}_{t+1}$ = the uncertain changes in the state variable (or state variables if it denotes a vector);

λ_1 = the equilibrium market price of volatility risk;

λ_2 = the equilibrium market price of state-variable risk; and

COV = the covariance operator.

Equation (1) states that the equilibrium value of firm or asset *j* is equal to the present value of the CEQ of its future cash profits discounted at the risk-free interest rate. The CEQ of an uncertain cash profit is equal to its expected value minus the appropriate risk premium of the cash profit, which consists of market-volatility risk $\left(COV\left(\tilde{D}_j, \tilde{D}_M\right)\right)$ and state-variable risk $\left(COV\left(\tilde{D}_j, \tilde{\theta}\right)\right)$, weighted by their respective factors, λ_1 and λ_2. The traditional CAPM is static in nature and is developed without an explicit consideration of possible shifts in the investment opportunity set. Therefore, the traditional CAPM does not include the uncertain changes in the state variable, $\tilde{\theta}$, and its equilibrium risk-return relation does not include the relevant state-variable risk, $COV\left(\tilde{D}_j, \tilde{\theta}\right)$. In other words, the traditional CAPM is a special case of the ICAPM. By omitting the $COV\left(\tilde{D}_j, \tilde{\theta}\right)$ term in equation (1), the equilibrium value of a firm or asset *j* in the CEQ form of cash flows in the traditional CAPM is as follows:

$$V_{jt} = \frac{1}{\gamma}\left[\bar{D}_{jt+1} - \lambda\, COV\left(\tilde{D}_{jt+1}, \tilde{D}_{Mt+1}\right)\right] \quad , \tag{2}$$

where V_{jt} = the equilibrium value of firm or asset *j* at the beginning of period *t* in the traditional CAPM;

λ = the equilibrium market price of volatility risk in the traditional CAPM.

Equation (2) also states that the equilibrium value of firm *j* is equal to the CEQ of its future cash profits discounted at the risk-free rate of interest. However, in the determination of the CEQ of a cash profit, only the market-volatility risk $\left(COV\left(\tilde{D}_{jt+1}, \tilde{D}_{Mt+1}\right)\right)$ is included in the appropriate risk premium in the traditional CAPM; in contrast, both the market-volatility risk $\left(COV\left(\tilde{D}_j, \tilde{D}_M\right)\right)$ and the state-variable risk $\left(COV\left(\tilde{D}_j, \tilde{\theta}\right)\right)$ are included in the appropriate risk premium in the ICAPM. Therefore, the traditional CAPM *understates* the firm's or asset's systematic risk if its cash profits are positively correlated with the uncertain changes in the state variable, and it *overstates* the firm's or asset's systematic risk if its cash profits are negatively correlated with the uncertain changes in the state variable.

3. VALUE-BASED EVALUATION OF STRATEGIC BUSINESS UNITS

A successful and complete corporate strategic plan requires an appropriate value-based criterion for decision making. The ICAPM described in the previous section provides a proper value-based criterion for evaluating strategic business units of a corporation.

Suppose that firm *j* owns a portfolio of *n* SBUs with future cash flows denoted by, $\tilde{D}_{jk}$ where $k = 1, 2, \ldots, n$. The total cash profit of firm *j* is a sum of the cash profits of all the SBUs owned by the firm. Thus, $\tilde{D}_j = \sum_{k=1}^{n} \tilde{D}_{jk}$. Utilizing the ICAPM as given by equation (1), the equilibrium value of the *k-th* SBU of firm *j* can be expressed as follows:

$$V_{jk} = \frac{1}{\gamma}\left\{\tilde{D}_{jk} \quad - \left[\lambda_1 COV\left(\tilde{D}_{jk}, \tilde{D}_M\right) \quad - \lambda_2 COV\left(\tilde{D}_{jk}\ , \tilde{\theta}\right)\right]\right\} \tag{3}$$

Similar to the equilibrium value of a firm, the equilibrium value of a SBU is equal to the present value of the CEQ of its cash profits discounted at the risk-free rate of interest. In the determination of CEQ of the cash profits of a SBU, the relevant risk premium consists of both the SBU'S market-volatility risk and the state-variable risk. To gain further insight into the value-based criterion for evaluating the SBUs within a firm, we next analyze these two components of systematic risk separately.

3.1 A SBU's Market-Volatility Risk

As shown in equation (3), the market-volatility risk of the *k-th* SBU in firm *j* is as follows:

$$COV\left(\tilde{D}_{jk}, \tilde{D}_M\right)$$

$$= COV\left(\tilde{D}_{jk}, \tilde{D}_j + \tilde{D}_W\right) = COV\left(\tilde{D}_{jk}, \tilde{D}_{jk} + \sum_{\substack{i=1 \\ i \neq k}}^{n} \tilde{D}_{ji} + \tilde{D}_W\right)$$

$$= Var(\tilde{D}_{jk}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} COV(\tilde{D}_{jk}, \tilde{D}_{ji}) + COV(\tilde{D}_{jk}, \tilde{D}_W) \tag{4}$$

where $\tilde{D}_W$ = the aggregate uncertain future cash profits of all *other* firms

(i.e., all firms, except firm *j*, in the market).

Therefore, in the value-based valuation model for each SBU, the market-volatility risk consists of the following three key components:

(i) The variance of the cash profits of the *k-th* SBU in firm *j*, that is Var ($\tilde{D}_{jk}$), to be called the *k-th* SBU's "own risk";

(ii) The covariance between the *k-th* SBU's cash profits and that of all other SBUs in firm *j*, i.e., $\sum_{\substack{i=1 \\ i \neq k}}^{n} COV(\tilde{D}_{jk}, \tilde{D}_{ji})$, to be called the *k-th* SBU's "internal risk";

(iii) The covariance between the *k-th* SBU's cash profits and the aggregate cash profits of all other firms $\left(i.e., COV\left(\tilde{D}_{jk}, \tilde{D}_W\right)\right)$, which we will call as the *k-th* SBU's "external risk".

The numerous factors influencing a SBU's market-volatility risk clearly indicate that evaluation of any SBU cannot and should not be carried out in isolation, without examining its relations to other SBUs in the firm and to other firms in the market. In other words, the value-based criterion for evaluating a SBU calls for an integrated approach. Without such a complete evaluation approach, the decisions in corporate strategic planning will be biased and non-optimal.

3.2 A SBU's State-Variable Risk

As indicated in equation (3), the state-variable risk of the *k-th* SBU in firm *j* is measured by the covariance of its cash profits with the unfavorable uncertain changes in the state variable, $\left(COV\left(\tilde{D}_{jk},\tilde{\theta}\right)\right)$. A positive covariance between the cash profits of the *k-th* SBU and the unfavorable uncertain changes in the state variable indicates that the cash profits of the SBU tend to be higher in the event that an unfavorable state occurs. For example, if the uncertain rate of inflation in the economy is the unfavorable state variable of the major concern in corporate strategic planning decisions, a SBU with $COV\left(\tilde{D}_{jk},\tilde{\theta}\right)>0$ tends to have a larger cash profit during the period of higher rate of inflation, and hence, it will be valued higher by the investors and corporate financial managers. Such a business unit will be termed an *inflation-hedging* SBU and it will command a greater value in the marketplace. Likewise, a SBU with $COV\left(\tilde{D}_{jk},\tilde{\theta}\right)<0$ tends to have a smaller cash profit during the period of higher rate of inflation, and hence, it will be valued lower because it is an *inflation-adverse* SBU.

The sign and magnitude of $COV\left(\tilde{D}_{jk},\tilde{\theta}\right)$ indicate the state-variable risk of the *k-th* SBU. More relevant in the current context of strategic planning, this state-variable risk can be used to measure the SBU's strategic advantages if it is operating in a monopolistic environment or disadvantages if it has no protection from the entry of competitors. Since the state-variable risks of all SBUs within the firm are measured by $\left(COV\left(\tilde{D}_{j},\tilde{\theta}\right)=\sum_{k=1}^{n}COV\left(\tilde{D}_{jk},\tilde{\theta}\right)\right)$, by changing the relative sizes of SBUs in the firm and by changing $COV\left(\tilde{D}_{jk},\tilde{\theta}\right)$ of all SBUs under its control through advertisement and marketing expenditures as well as research and developments expenditures, a firm's central management can adjust its state-variable risk or strategic risk in its strategic planning.

3.3 The Value-Based Criterion in Acquiring or Divesting of a SBU

The ICAPM for evaluating SBUs provides a useful tool in corporate strategic planning—it is a value-based approach and it explicitly incorporates the characteristics of strategic businesses. As we have already pointed out, the equilibrium value of a SBU is determined by both its market-volatility risk and its state-variable risk. Furthermore, the stochastic relationship of the SBU's cash profits and that of other SBUs within the firm as well as that of other firms in the market should be considered in the process of evaluating a SBU. In other words, the value-based approach to the evaluation of a SBU requires a comprehensive and integrated view of the unit. A SBU cannot be evaluated independently.

It can be easily seen that the *risk-adjusted net present value* (RANPV) *rule* can be employed in the acquisition or the divestiture of a SBU in corporate strategic planning decisions. The managerial flexibility in strategic planning in buying or selling a business unit is the real options available to central management managers of a multi-business corporation. The rule calls for acquiring a new SBU if its RANPV is greater than zero.

The RANPV of acquiring a new SBU is defined as follows:

$$RANPV_k = V_k - B_k$$

$$= \frac{1}{\gamma}\left\{\bar{D}_k - \left[\lambda_1 COV\left(\tilde{D}_k, \tilde{D}_M\right) - \lambda_2 COV\left(\tilde{D}_k, \tilde{\theta}\right)\right]\right\} - B_k, \qquad (4)$$

where B_k = the *market asking price* for the *k-th* SBU.

Note that the RANPV rule identifies when the central manager should exercise the *real call option* to acquire a SBU, where the call option's exercise value and strike price are stochastically determined by the state variables. If the RANPV_k, as shown in equation (4) is greater than zero, then the *k-th* SBU is worth being acquired because the acquisition of the SBU will increase the value of the firm. On the other hand, acquiring a SBU with a negative RANPV will reduce the value of the firm, which will not be in the best interest of the firm's shareholders.

Similarly, we can easily devise a simple rule for divesting a SBU under the control of the firm. That is, the central management of a multi-business corporation has a *real put option* to sell a SBU with an uncertain strike price. If the RANPV_k as shown by equation (9) below is positive, the central management's *real put option* should be exercised and the *k-th* SBU currently under control should be sold:

$$RANPV_k = S_k - V_k$$

$$= S_k - \frac{1}{\gamma}\left\{\bar{D}_k - \left[\lambda_1 \, COV\left(\tilde{D}_k, \tilde{D}_M\right) - \lambda_2 \, COV\left(\tilde{D}_k, \tilde{\theta}\right)\right]\right\}, \qquad (5)$$

where S_k = the *bid price* for the *k-th* SBU.

Following the positive RANPV decision rules in equations (4) or (5), we know the optimal exercise decisions for central management of a multi-business corporation and that the acquisition or the divestiture of a SBU will increase the value of the firm. Therefore, the managers following the value-maximization rule should acquire a SBU if the expression in equation (4) is greater than zero, and should sell a SBU if the expression in equation (5) is greater than zero. Note that these decision rules are perfectly consistent with the value-based criterion in corporate strategic planning.

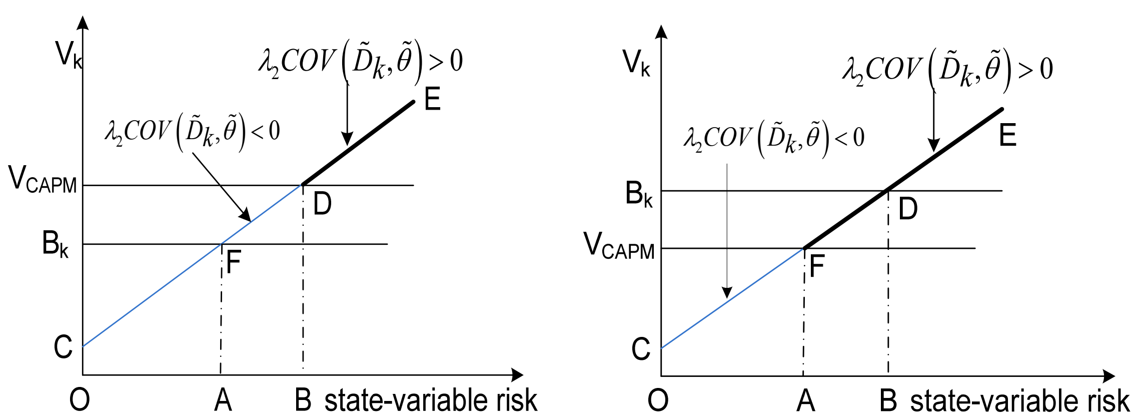

Figure 1: Acquiring or selling a new SBU

To get an intuition for when the firm should purchase or sell the *k-th* SBU, Figure 1provides a simple example. Ignoring the state-variable risk, the market equilibrium price V_k (=V_{CAPM}) and the market asking price B_k are determined. The firm should buy the SBU when $V_{\text{CAPM}} > B_k$ (left panel) and sell the SBU when $V_{\text{CAPM}} < B_k$ (right panel). When considering the state-variable risk, however, this conclusion does not hold any more. In the left panel, the firm will buy the *k-th* SBU only when the state-variable risk is larger than A (i.e., the market equilibrium price with ICAPM lies on line

CE and only when V_k lies on FDE, the NPV is positive). In contrast, in the right panel, the firm will buy *k-th* SBU when the state-variable risk is larger than B (i.e, the market equilibrium price with ICAPM is larger than the asking price in DE).

4. BUILDING AND RESTRUCTURING A PORTFOLIO OF SBUs

The portfolio selection problem deals with an optimal allocation of the initial *cash* among the feasible set of securities; while the problem of *portfolio revision* deals with an optimal restructure of the *portfolio of securities*. The two problems are different in that the revision problem calls for liquidating some of the currently held securities that will incur some selling costs. Similarly, building a portfolio of SBUs and restructuring (a term we will employ rather than revising) an existing portfolio of SBUs are different in that the restructuring of an existing portfolio of SBUs requires the selling of some of the SBUs currently held by the firm. In this section we first investigate the problem of building a portfolio of SBUs with initial cash, and then turn to the problem of restructuring a portfolio of existing SBUs.

4.1 Optimal Selection of an SBU Portfolio

Using the ICAPM we can unambiguously determine the equilibrium values of a firm as well as a SBU. Therefore, the ICAPM along with mathematical programming can be combined to solve the problem of building an optimal portfolio of SBUs with budget constraints. To see this, let us utilize the relation $\tilde{D}_j = \sum_{k=1}^{n} \tilde{D}_{jk}$, where $\tilde{D}_j$ is the random cash profits of firm *j* and $\tilde{D}_{jk}$ is the random cash profits of the *k-th* SBU in firm *j*, and express the equilibrium value of firm *j* as follows:

$$
\begin{aligned}
V_j &= \frac{1}{\gamma}\left\{\bar{D}_j - \left[\lambda_1\, COV\left(\tilde{D}_j, \tilde{D}_M\right) - \lambda_2\; COV\left(\tilde{D}_j, \tilde{\theta}\right)\right]\right\} \\
&= \frac{1}{\gamma}\left\{\sum_{k}^{n} \bar{D}_{jk} - \lambda_1\left[\sum_{k}\sum_{i} COV\left(\tilde{D}_{jk}, \tilde{D}_{ji}\right) + \sum_{k} COV\left(\tilde{D}_{jk}, \tilde{D}_W\right)\right] + \lambda_2 \sum_{k} COV\left(\tilde{D}_{jk}, \tilde{\theta}\right)\right\} \\
&= \sum_{k} a_{jk} - \sum_{k}\sum_{i} COV\left(\tilde{b}_{jk}, \tilde{b}_{ji}\right), \qquad (6)
\end{aligned}
$$

where $a_{jk} = \frac{1}{\gamma}\left[\bar{D}_{jk} - \lambda_1 COV\left(\tilde{D}_{jk}, \tilde{D}_W\right) + \lambda_2 COV\left(\tilde{D}_{jk}, \tilde{\theta}\right) \right]$

$$\tilde{b}_{jk} = \sqrt{\frac{\lambda_1}{\gamma}}\tilde{D}_{jk}.$$

Therefore, the equilibrium value of firm *j* has been expressed in terms of two major components: (1) the sum of SBUs' expected cash profits adjusted for their respective *external-external risks* as well as *state-variable risks*; and (2) the sum of the SBUs' *internal-internal risks* and *internal-external risks*.

The problem of building an optimal portfolio of SBUs is to allocate the available initial amount of cash among the SBUs so that the total value of the firm is maximized. Using the expression of equilibrium value of a firm given by equation (6), the problem of building an optimal portfolio of SBUs can be formulated as the following constrained maximization problem:

$$\underset{< X_k >}{Max}\ V = \sum_k X_k a_k - \sum_k \sum_i X_k X_i COV\left(\tilde{b}_k, \tilde{b}_i\right) \tag{7}$$

Subject to $\underset{k}{\Sigma} X_k B_k = C$ and $X_k \leq 1$,

where X_k = the fraction of the *k-th* SBU purchased;

B_k = the purchase price of the *k-th* SBU;

C = the total initial amount of cash available for business investment;

a_k *and* $\tilde{b}_k$ are as defined in equation (6).

The maximization problem in equation (7) is a standard quadratic programming problem which can be solved by any quadratic programming routine. The above formulation can also be used to handle the strategic planning problem with non-financial constraints. For mutually exclusive SBUs, say the *k-th* and the *i-th* SBUs, we simply add one additional constraint that $X_k X_i = 0$ to the above maximization model. For the contingent SBUs, say SBU_I is a prerequisite for SBU_k, we simply add the constraint that $X_k \leq X_1$ to the maximization problem in (7).

4.2 Optimal Restructuring of Business Portfolio

As we have noted earlier, the problem of portfolio revision is different from the problem of portfolio selection. Therefore, the optimal building of the SBU portfolio model described in

Section 4.2 cannot be used to restructure a business portfolio without modification. The reason is that a firm with an existing portfolio of SBUs is "locked in" because of the restructuring costs (e.g., investment banking fees, legal fees, consulting fees for integration, and any adverse tax consequences) associated with restructuring a SBU portfolio.

In the following, we shall formulate a model for optimal restructuring of a SBU portfolio, and explicitly consider restructuring costs that are incurred in changing a SBU portfolio. The following additional notation will be used:

χ_k = the fraction of the *k-th* SBU held in the portfolio before the restructuring;

H_k = the current market value of the *k-th* SBU;

δ_k = the increase in the fraction of the *k-th* SBU held in the restructured portfolio, $\delta_k \geq 0$;

ω_k = the decrease in the fraction of the *k-th* SBU held in the restructured portfolio, $\omega_k \geq 0$;

υ_k = the fraction of the *kth* SBU held in the restructured portfolio, a decision variable in the portfolio restructuring decisions;

β_k = the proportional buying disposal cost of the *kth* SBU

φ_k = the proportional selling disposal cost of the *kth*SBU.

For simplicity, both restructuring costs are assumed to be proportional to the amount of the SBU sold.. By definition, a change in the holdings of the *kth*SBU $(\delta_k \text{ or } \omega_k)$ equals the difference between the fraction of the unit held in the old portfolio (v_k) and the fraction held in the restructured portfolio (υ_k):

$$\delta_k - \omega_k = \upsilon_k - \chi_k . \tag{8}$$

Note that a firm can either buy or sell or not change the amount of a SBU in the portfolio. Hence, either δ_k or ω_k , or both will equal zero, and thus we have

$$\delta_k \omega_k = 0 \tag{9}$$

In addition, the market value of the restructured portfolio must equal the market value of the old portfolio less the restructuring costs incurred to assemble the restructured portfolio, therefore, the following identity must hold:

$$\sum_k^m \upsilon_k H_k = \sum_k^m \chi_k H_k = \sum_k^m \left(\delta_k \beta_k H_k + \omega_k \varphi_k H_k\right). \tag{10}$$

Consequently, the problem of optimal restructuring the SBU portfolio can be formulated as the following maximization model:

$$\underset{<\upsilon_k>}{Max}\, V = \sum_k \upsilon_k \ \beta_k - \sum_k \sum_i \upsilon_k \upsilon_i COV\left(\tilde{b}_k \ , \tilde{b}_1\right) \qquad (11)$$

Subject to constraints expressed in equations (8), (9), and (10),

Similar to that in the problem of portfolio selection, the standard computational procedures applicable to quadratic programming problems can be used to solve the problem of business portfolio restructuring.

4.3 The Optimal Level of New Business Investment

In corporate strategic planning, central management often is faced with the problem of deciding the amount of investment in a risky new business. In the following, we shall apply the ICAPM to derive a model for the optimal level of corporate investment in a new SBU. Let us suppose that a *new* SBU which promises a random return per dollar invested, denoted by $\tilde{\rho}$, with

$$\begin{aligned}
&E(\tilde{\rho}) = \bar{\rho}\,, \\
&Var\left(\tilde{P}\right) = \sigma_P{}^2\,, \\
&COV\left(\tilde{\rho}\,, \tilde{D}_j\right) = \sigma_{\rho j}\,, \\
&COV\left(\tilde{\rho}\,, \tilde{D}_m\right) = \sigma_{\rho m}\,, \\
&COV\left(\tilde{\rho}\,, \tilde{\theta}\right) = \sigma_{\rho\theta}\,.
\end{aligned} \qquad (12)$$

To simplify our analysis, the new business is assumed to have a constant stochastic return to scale. Let I_j be the amount to be invested in the new SBU by firm *j* and, for simplicity, that amount will be raised through the sale of additional common stock. If we denote the post-investment total cash profits of firm *j* by $\tilde{D}'_j = \tilde{D}_j + I_j\tilde{\rho}$ and the *post-investment* total cash profits of all firms by $\tilde{D}'_m = \tilde{D}_m + I_j\tilde{\rho}$, then using the ICAPM, the new equilibrium value of the firm is given by

$$V'_j = \frac{1}{\gamma}\left\{\bar{D}'_j - \left[\lambda_1 COV\left(\tilde{D}_j\,, \tilde{D}_m\right) - \lambda_2 COV\left(\tilde{D}'_j\,, \tilde{\theta}\right)\right]\right\}$$

$$= \frac{1}{\gamma}\left\{\left(\bar{D}_j + I_j\bar{\rho}\right) - \left[\lambda_1\left(\sigma_{jm} + I_j\sigma_{\rho m} + I_j\sigma_{\rho j} + I_j{}^2\sigma_\rho{}^2\right) - \lambda_2\left(\sigma_{j\theta} + I_j\sigma_{\rho\theta}\right)\right]\right\} \quad (13)$$

Therefore, the net increase in the value of the firm (i.e., the change in the value of the firm brought about by the new investment after subtracting the cost of the investment in the new SBU) is given by

$$\Delta V_j = \frac{I_j}{\gamma}\left\{ (\bar{\rho}-\gamma) - \left[\lambda_1 \left(\sigma_{\rho m} + \sigma_{\rho j} + I_j \sigma_{\rho}^2 \right) + \lambda_2 \sigma_{\rho\theta} \right] \right\} \tag{14}$$

Therefore, the optimal level of investment in the new SBU for a value-maximizing firm can be derived by differentiating equation (14) and setting the result equal to zero. The solution is

$$I_j^* = Max\left\{ 0, \frac{1}{2\lambda_1 \sigma_{\rho}^2}\left[(\bar{\rho}-\gamma) - \lambda_1 \left(\sigma_{\rho m} + \sigma_{\rho j} \right) + \lambda_2 \sigma_{\rho\theta} \right] \right\}. \tag{15}$$

Equation (15) indicates that the optimal level of new investment in the risky venture is determined by the characteristics of the investment as well as the firm making the investment decision. Specifically, the investment's *internal-internal risk* (σ_{ρ}^2), *internal-external risk* $(\sigma_{\rho j})$, *external-external risk* $(\sigma_{\rho m})$, and *state variable risk* $(\sigma_{\rho\theta})$ are among the determining factors that set the optimal amount of new investment in the risky venture. Any new investment that has low or no "*internal-external risk*" $(i.e., \sigma_{\rho j} = 0)$ would provide benefits of diversification to the firm and will be considered a preferable investment. Some major U.S. corporations, such as Northwest Industries Inc., have pursued the diversification strategy to reduce the "*internal-external risk.*"

From (15), another alternative for reducing the internal-external risk is to invest a project negatively correlated to the state variable risk $(\sigma_{\rho\theta} < 0)$. This is an intuitive investment wisdom. The firm does not like uncertainty about future cashflows. If it invests in a SBU whose payoff covaries positively with the state $\tilde{\theta}$, one that pays off well when the economy is wealthy and pays off badly when the economy is poor, this will make the future payoff of the firm more volatile. In this setting, the firm should reduce investment in the project. Instead, investing a project covarying negatively with the state variable will make the future payoffs more smooth. As a simple example, Figure 2 exhibits that state-variable risk changes the optimal investment in a new SBU significantly. Without this risk, the firm will invest amount of $I_{j,CAPM}$ in the new SBU. Incorporate the state-variable risk makes the investment convex. That is, the firm invests in the new SBU only when $\lambda_2 \sigma_{\rho\theta}$>A, and after that, the investment is linearly increasing according to $\lambda_2 \sigma_{\rho\theta}$. In this example, the state variable risk is less than zero over interval OB and becomes positive after point B. From the perspective of the firm, it overinvests over OB and underinvests after B.

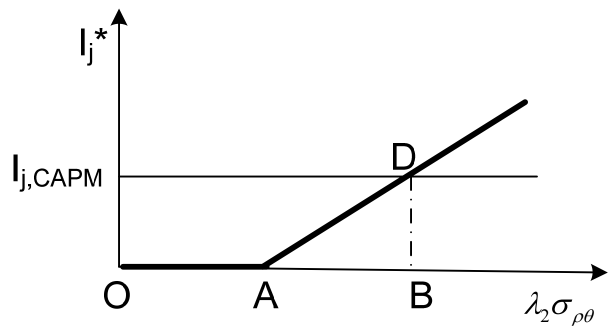

Figure 2: Optimal investment in a new SBU

5. **CONCLUSION**

The option games played by corporate managers in their business strategies are effectively combinations of theories of real options and game strategies. Based on the framework of risk-neutrality and continuous-time models, option games have been studied and published in the literature by several scholars. And we have seen some interesting and useful insights from these studies. The focus of this paper is to provide a conceptual framework in a discrete-time model that can be used for the option games that can be implemented more directly in the real world. For future work in this area, it should be of interest and importance to incorporate into the real options framework the cash-flow valuation models under risk that explicitly consider the state variable or strategic factors.

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