

Tail Risk and Expected Stock Returns^{*}

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ABSTRACT

This paper introduces new measures of tail risk and investigates their performance in predicting the cross-sectional variation in monthly returns on NYSE stocks over the sample period of July 1963-December 2006. The results indicate a positive and significant relation between tail covariance risk (TCR) and expected stock returns. A trading strategy that longs stocks in the highest-TCR quintile and shorts stocks in the lowest-TCR quintile produces average raw and risk-adjusted returns of 3.5% per annum. After controlling for size, book-to-market, momentum, short-term reversal, liquidity, volatility, and co-skewness, the positive relation between TCR and future returns remains economically and statistically significant.

Key words: tail risk, covariance risk, downside risk, systematic risk, idiosyncratic risk, total risk, expected stock returns.

JEL classification: G10, G11, C13.

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Tail Risk and Expected Stock Returns

In the mean-variance framework of Markowitz (1959), return and risk are to be measured by the expected value and variance of the probability distribution of portfolio returns. Although financial economists have generally accepted Markowitz's measure of return, they have not been completely satisfied with his suggested measure of risk. The focus on variance (or standard deviation) as the appropriate measure of risk implies that investors weigh the probability of negative returns equally against positive returns. If the empirical distribution of asset returns was symmetric and normal-tailed, then variance would be a proper measure of risk. However, it is a stylized fact that the distribution of many financial return series is generally skewed, fat-tailed and peaked around the mode. Furthermore, there is substantial evidence that investors often treat losses and gains asymmetrically. There is also a wealth of experimental evidence for loss aversion (see, e.g., Kahneman et al. (1990)). The choice therefore of mean-variance efficient portfolios is likely to give rise to an inefficient strategy for maximizing expected returns for financial assets while minimizing risk. Hence, it would be more desirable to rely on a measure of risk that is able to incorporate any non-normality in the return distributions and to account for investors' aversion to extreme losses.

Indeed, Markowitz himself had reservations about choosing variance as a measure of risk. Besides variance, Markowitz (1959) proposed an alternative measure of portfolio risk called semi-variance or lower partial moment (LPM) that depends on only those portfolio returns that fall below some target level of returns. LPM is defined as:

$$LPM = \int_{-\infty}^h (R - h)^2 f_p(R) dR, \quad (1)$$

where h is the target level of returns and $f_p(R)$ represents the probability density function of returns for portfolio p .¹ The main heuristic motivation for the use of semi-variance in place of variance is that minimization of semi-variance concentrates on the reduction of losses, whereas variance identifies extreme gains as well as extreme losses as undesirable. Too much expected return may be sacrificed in eliminating both extremes.

As shown in equation (1), semi-variance is the expected value of the squared negative deviations of the possible outcomes from an arbitrarily chosen point of reference (h): $E[\min(0, R - h)^2]$. In contrast, variance is the expected value of the squared deviations (whether positive or negative) of the possible outcomes from the mean of the random variable (μ): $E[(R - \mu)^2]$. This means that semi-variance evaluates the risks associated with different distributions by reference to a fixed point which is designated by the investor. The variance measure introduces no such refinement, but uses the means of the distributions, which

¹ For expected utility maximizing investors, Bawa (1975) provided a theoretical rationale for using semi-variance or lower partial moment as the measure of portfolio risk.

may vary widely, to make the judgments. Also, in computing semi-variance, positive and negative deviations contribute differently to risk, whereas in computing variance, a positive and a negative deviation of the same magnitude contribute equally to risk. In essence, then, since capital has an opportunity cost, the risk of an investment decision should be measured primarily by the prospect of failure to earn the return foregone. Semi-variance is more consistent with this concept of investment risk than ordinary variance.

Bawa and Lindenberg (1977) developed a mean-lower partial moment capital asset pricing model (EL-CAPM). The equilibrium pricing relationship of this model is formulated as:

$$E(R_i) - r_f = \beta_{LPM} \cdot [E(R_m) - r_f], \quad (2)$$

where $E(R_i)$ is the expected return on asset i , $E(R_m)$ is the expected return on the market portfolio, r_f is the risk-free interest rate, and β_{LPM} is a measure of downside systematic risk defined as:

$$\beta_{LPM} = \frac{CLPM(R_i, R_m)}{LPM(R_m)} = \frac{\int_{-\infty}^{+\infty} \int_{-\infty}^h (R_m - h)(R_i - h) f(R_m, R_i) dR_m dR_i}{\int_{-\infty}^h (R_m - h)^2 f(R_m) dR_m}, \quad (3)$$

where h is the target level of returns, $CLPM(R_i, R_m)$ is the co-lower partial moment below h of returns on the market portfolio with returns on security i , $LPM(R_m)$ is the lower partial moment of returns below h on the market portfolio, and $f(R_m, R_i)$ is the joint probability density function of returns on asset i and returns on the market portfolio.²

Earlier studies on the EL-CAPM model use alternative measures of downside market risk, β_{LPM} , based on different target level of returns.³ Specifically, h is measured by the risk-free interest rate (β_{rf}^-), zero rate of return on the market portfolio (β_0^-), or average excess market return ($\beta_{\mu_m}^-$):

$$\beta_{rf}^- = \frac{\text{cov}(R_i, R_m | R_m < r_f)}{\text{var}(R_m | R_m < r_f)}, \quad \beta_0^- = \frac{\text{cov}(R_i, R_m | R_m < 0)}{\text{var}(R_m | R_m < 0)}, \quad \beta_{\mu_m}^- = \frac{\text{cov}(R_i, R_m | R_m < \mu_m)}{\text{var}(R_m | R_m < \mu_m)}, \quad (4)$$

where μ_m is the average excess market return.⁴

In this paper, we examine the cross-sectional predictive power of β_{rf}^- , β_0^- , and $\beta_{\mu_m}^-$ for future stock returns. The results provide no evidence for a significant link between downside risk measures and the cross-section of expected returns. We think that the downside market risk in equation (4) may not contain useful information about *tail* events because the downside betas are estimated based on the 50th percentile of the

² Equation (3) can be written as $\beta_{LPM} = E[\min(0, R_m - h) \cdot (R_i - h)] / E[\min(0, R_m - h)^2]$.

³ See, for example, Jahankhani (1976), Price, Price, and Nantell (1982), and Harlow and Rao (1989).

⁴ Ang, Chen, and Xing (2006) show that β_{rf}^- , β_0^- , and $\beta_{\mu_m}^-$ are all highly correlated with each other with correlations greater than 0.96. Given these strong correlations they indicate that using any of the three measures of downside beta yields almost identical results.

market return distribution instead of using the extreme tails of the return distribution. Hence, we proposed an alternative measure of downside beta based on the observations in the 10% and 5% lower tails of the return distribution. However, these alternative measures focusing only on tail observations are also not successful in predicting the cross-sectional variation in stock returns.

The underlying assumption with the definition of downside risk measures given in equation (4) is that the prices of individual stocks decline when the market falls. However, this paper provides evidence that contradicts with this assumption. For each month t , one year of daily returns on the market portfolio from month t to $t-12$ (approximately 250 daily observations) are used to determine the tail observations for the market and also for individual stocks. First, we take the 10% lower tail of the market return distribution as our downside risk threshold and focus on the 25 days on which the market fell over the past one year. Then, we determine the 10% lower tail of the return distribution for individual stocks, i.e., the 25 days on which the individual stocks fell over the past one year. Finally, we count the number of common days for the market and individual stocks out of the 25 days on which the market fell.

We compute the percentiles for the number of common days. As presented below, the median number of common days is about 6.64, indicating that the price of a typical stock declines 6.64 days (on average) while the market lose value for 25 days over the past one year. Even the 99th percentile for the number of common days is only 10.26 (out of 25 days).

Percentiles for the number of common days (past 1 year)

1%	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	95%	99%
4.78	5.03	5.29	5.74	6.08	6.38	6.64	7.06	7.56	8.16	8.67	8.88	10.26

Similar results are obtained when we use the past two years of daily returns on the market portfolio (approximately 500 daily observations) to determine the tail observations for the market and individual stocks. As shown below, the median number of common days is about 14.04, indicating that the price of a typical stock declines 14 days while the market lose value for 50 days over the past two years. The 99th percentile for the number of common days is only 19.05 (out of 50 days).

Percentiles for the number of common days (past 2 years)

1%	5%	10%	20%	30%	40%	50%	60%	70%	80%	90%	95%	99%
10.49	11.12	11.54	11.98	12.51	13.32	14.04	14.59	14.99	15.91	17.49	18.42	19.05

These results indicate that individual stocks do not usually follow the market during the large falls of the market. More specifically, the 10% lower tails of the return distributions for individual stocks and the market portfolio do not match in terms of the days on which the extreme price movements are observed. Since the large falls of the market and the large falls of the stock prices occur on different days, the market-

based measures of downside risk do not reflect the actual firm-specific covariance risk. Therefore, instead of estimating downside risk for an individual stock based on the tails of the market return distribution, we propose a new measure of downside systematic risk based on the tails of the individual stock return distribution. For each month t , one year of daily returns on individual stocks from month t to $t-12$ are used to determine the tail observations for the individual stocks and also for the market. First, we take the 10% lower tail of the stock return distribution as our downside risk threshold and focus on the 25 days on which the stock price fell over the past one year. Then, we estimate downside beta using daily market returns corresponding to the days of extreme returns on individual stocks.

In addition to downside market beta, we investigate for future stock returns the cross-sectional predictive power of standard risk measures (market beta, total volatility, idiosyncratic volatility) as well as the widely-used measures of downside risk (value-at-risk and expected shortfall).⁵ The long-short portfolio analyses provide no evidence for a significant link between the conventional risk measures and the cross-section of expected returns.

In this paper, we introduce new measures of *tail risk* and investigate their performance in predicting the cross-sectional variation in stock returns. Our first measure called *tail total risk* (TTR) depends on only those stock returns that fall below some *disaster* level of returns. TTR is defined as the standard deviation of extreme daily returns that are in the 10% (or 5%) lower tail of the return distribution.⁶ Our second measure called *tail covariance risk* (TCR) is defined as:

$$TCR_i = \frac{\text{cov}(R_i, R_m | R_i < VaR_i^\alpha)}{\text{var}(R_m | R_i < VaR_i^\alpha)}, \quad (5)$$

where $VaR_i^{\alpha=10\%}$ ($VaR_i^{\alpha=5\%}$) denotes the cutoff point for the 10% (5%) lower tail of the return distribution of stock i , $\text{cov}(R_i, R_m | R_i < VaR_i^\alpha)$ is the covariance between extreme daily returns on stock i that are below VaR_i^α and the corresponding daily returns on the market portfolio, and $\text{var}(R_m | R_i < VaR_i^\alpha)$ is the variance of daily returns on the market portfolio that corresponds to the days of extreme returns on stock i that are below VaR_i^α . TCR is the slope coefficient from the regression of extreme daily returns on stock i that are below VaR_i^α on the corresponding days' returns of the market portfolio. Our last measure called *tail idiosyncratic risk* (TIR) is defined as the standard deviation of daily residuals from the regression of extreme daily returns on stock i against the corresponding daily returns on the market portfolio.

⁵ Value-at-risk (VaR) is defined as the expected maximum loss over a given time interval at a given confidence level. For example, if the given period of time is one day and the given probability is 1%, the VaR measure would be an estimate of the decline in the value of a stock that could occur with a 1% probability over the next trading day. In other words, if the VaR measure is accurate, losses greater than the VaR measure should occur less than 1% of the time. Expected shortfall (ES) is defined as the average value of those losses beyond the VaR threshold.

⁶ The cutoff point for the 10% (5%) lower tail of the return distribution of stock i is measured by the 10% (5%) value-at-risk of daily returns over the past one year denoted by $VaR_i^{\alpha=10\%}$ ($VaR_i^{\alpha=5\%}$).

We investigate the empirical performance of the newly proposed measures of tail risk in the cross-sectional pricing of NYSE stocks over the sample period of July 1963-December 2006. Univariate portfolio level analyses indicate that a trading strategy that longs stocks in the highest quintile of tail covariance risk (TCR) and shorts stocks in the lowest quintile of TCR yields average raw and risk-adjusted returns of 3.5% per annum. As a robustness check, we test whether the positive relation between tail covariance risk and the cross-section of expected returns holds once we exclude small, low-priced, illiquid, highly volatile, winner, loser, and reversal stocks. We find that the effect of tail covariance risk on stock returns does not disappear after a screen for size, price, liquidity, volatility, and past return characteristics.

We also control for the well-known cross-sectional effects including size and book-to-market (Fama and French (1992, 1993)), momentum (Jegadeesh and Titman (1993)), short-term reversal (Jegadeesh (1990)), liquidity (Amihud (2002)), volatility (Ang, Hodrick, Xing, and Zhang (2006)), and co-skewness (Harvey and Siddique (1999, 2000)). After controlling for these effects in bivariate sorts of portfolios, we estimate the cross-sectional premium of tail covariance risk to be in the range of 25 to 30 basis points per month and highly significant. Similar results are obtained from the firm-level Fama-MacBeth (1973) cross-sectional regressions.

We further examine the significance of a positive link between tail covariance risk and expected returns by presenting average returns on portfolios of tail covariance risk in excess of the size and book-to-market matched benchmark portfolios similar to Daniel and Titman (1997). After controlling for size and book-to-market simultaneously, the average return difference between the Low TCR and High TCR quintiles turns out to be positive and highly significant. For the size/book-to-market adjusted portfolios, we include additional controls for momentum, reversal, liquidity, volatility, and co-skewness. The results clearly indicate that the spreads in size and book-to-market adjusted returns between the TCR quintiles 5 and 1 remain positive and significant after controlling for momentum, short-term reversal, liquidity, volatility, and co-skewness. Thus, our predictive pattern of returns for tail covariance risk is not due to size, book-to-market, past return, liquidity, volatility, and co-skewness effects.

The paper is organized as follows. Section 1 contains the data and variable definitions. Section 2 discusses the average returns on portfolios of standard risk measures, downside risk measures, and the newly proposed measures of tail risk (tail covariance, tail total, tail idiosyncratic risk). Section 3 examines the significance of a cross-sectional relation between tail covariance risk and expected stock returns. Section 4 provides bivariate portfolio level analyses after controlling for size, book-to-market, momentum, short-term reversal, liquidity, volatility, and co-skewness. Section 5 investigates the cross-sectional predictive power of tail covariance risk after screening for small, low-priced, illiquid, highly volatile, winner, loser, and reversal stocks. Section 6 examines the predictive power of tail covariance risk using the characteristic matched benchmark portfolios. Section 7 presents results from the firm-level cross-sectional regressions. Section 8 provides additional robustness checks. Section 9 concludes the paper.

1. Data and Variable Definitions

The first data set includes all New York Stock Exchange (NYSE) financial and non-financial firms from the Center for Research in Security Prices (CRSP) for the period from July 1962 through December 2006. We use daily stock returns to estimate alternative measures of risk. The second data set is the COMPUSTAT, which is primarily used to obtain the book values for individual stocks. For each month from July 1963 to December 2006, the following variables are computed for each firm in the sample:

1.1 Standard Measures of Risk

Systematic Risk (SR): To estimate market beta (or systematic risk) of an individual stock, we assume a single factor return generating process:

$$R_{i,d} = \alpha_i + \beta_i R_{m,d} + \varepsilon_{i,d}, \quad (6)$$

where $R_{i,d}$ is the excess return on stock i on day d , $R_{m,d}$ is the excess return on the market portfolio on day d , and $\varepsilon_{i,d}$ is the idiosyncratic return on day d .⁷ The estimated slope $\hat{\beta}_{i,t} = \text{cov}(R_{i,d}, R_{m,d}) / \text{var}(R_{m,d})$ in equation (6) is the market beta (or systematic risk) of stock i . For each month t , one year of daily data from month t to $t-12$ are used to estimate systematic risk (SR) for month $t+1$:

$$SR_{i,t+1} = \frac{\text{cov}\left(\{R_{i,d}\}_{t-12}^t; \{R_{m,d}\}_{t-12}^t\right)}{\text{var}\left(\{R_{m,d}\}_{t-12}^t\right)}, \quad (7)$$

where $\{R_{i,d}\}_{t-12}^t$ is the excess daily return on stock i over the past one year and $\{R_{m,d}\}_{t-12}^t$ is the excess daily return on the market portfolio over the past one year.

While many cross-sectional studies in empirical asset pricing use within month daily data to estimate standard measures of risk, we follow Kothari, Shanken, and Sloan (1995) and Ang, Chen, and Xing (2006) and compute risk measures using daily returns over the past one year, from month t to month $t-12$. There are two reasons for choosing an annual horizon. First, we need a sufficiently large number of daily observations to compute downside and tail risk measures. One month of daily data provides too short a window for obtaining reliable estimates of tail risk based on extreme returns beyond some VaR threshold. As a robustness check, we also use six months of daily data to estimate tail risk measures. Second, Fama and French (1997), Lewellen and Nagel (2006), and Ang and Chen (2007) show that conditional betas with the market are time-varying. Hence, using intervals longer than one year may lead the estimates to be inaccurate. Fama and French (2006) also advocate estimating systematic risk based on daily returns within a year.

⁷ In our empirical analysis, $R_{m,d}$ is measured by the CRSP daily value-weighted index, i.e., the daily value-weighted average returns of all stocks trading at the NYSE, AMEX, and NASDAQ.

Total Risk (TR): Total risk (TR) of an individual stock is calculated as the standard deviation of daily excess returns in a year. For each month t , one year of daily stock returns from month t to $t-12$ are used to estimate total risk for month $t+1$:

$$TR_{i,t+1} = \sqrt{\text{var}\left(\{R_{i,d}\}_{t-12}^t\right)}, \quad (8)$$

where $\{R_{i,d}\}_{t-12}^t$ is the excess daily return on stock i over the past one year.

Idiosyncratic Risk (IR): Idiosyncratic risk (IR) of an individual stock is calculated as the standard deviation of daily residuals, $\varepsilon_{i,d}$, obtained from the single-factor return generating process in equation (6). For each month t , one year of daily residuals from month t to $t-12$ are used to estimate idiosyncratic risk for month $t+1$:

$$IR_{i,t+1} = \sqrt{\text{var}\left(\{\varepsilon_{i,d}\}_{t-12}^t\right)}, \quad (9)$$

where $\{\varepsilon_{i,d}\}_{t-12}^t$ is the daily idiosyncratic return on stock i over the past one year.

1.2 Downside Risk Measures

VaR: Value-at-Risk (VaR) of an individual stock is defined as the maximum loss expected on a stock over a certain holding period at a given confidence level (probability). There are three main decision variables that are required to estimate VaR – the confidence level, a target horizon, and an estimation model. In this paper, three confidence levels (99%, 95%, 90%) are used to estimate alternative measures of VaR. The time horizon is one year. Estimation model is based on the left tail of the empirical return distribution. We define VaR for month $t+1$ with coverage probability α , based on the information set in month t , as:

$$\Pr\left(\{L_{i,d}\}_{t-12}^t > VaR_{t+1}^\alpha \mid \Omega_t\right) = \alpha, \quad (10)$$

where VaR_{t+1}^α is the value-at-risk for month $t+1$ with coverage probability α which equals 1%, 5%, and 10% in our empirical analysis. $\{L_{i,d}\}_{t-12}^t$ denotes daily losses that are greater than VaR_{t+1}^α based on daily returns over the past one year and Ω_t denotes the information set known in month t .

The simplest way to estimate VaR is to use the sample quintile estimates based on historical return data – a nonparametric or completely model-free approach. For each month t , one year of daily data from month t to month $t-12$ are used to estimate VaR for month $t+1$:⁸

$$VaR_{t+1}^\alpha = Q_{1-\alpha}\left(\{L_{i,d}\}_{t-12}^t\right), \quad (11)$$

⁸ Assuming that there are 250 daily observations in a year, 1% VaR is calculated as the average of the second-lowest and third-lowest observations of 250 daily returns. 5% VaR is computed as the average of the 12th-lowest and 13th-lowest observations of 250 daily returns. 10% VaR is measured by the 25th lowest observation of 250 daily returns.

where $Q_{1-\alpha}(\cdot)$ is the empirical $(1-\alpha)$ quintile of the distribution of daily losses $\{L_{i,d}\}_{t-12}^t$ from month t to $t-12$.

ES: Expected shortfall (ES) originally proposed by Artzner et al. (1999) is defined as the conditional expectation of a loss given that the loss is beyond the VaR level. We define ES for month $t+1$ with coverage probability α , based on information in month t , as:

$$ES_{t+1}^{\alpha} = E\left(\{L_{i,d}\}_{t-12}^t \mid \{L_{i,d}\}_{t-12}^t > VaR_{t+1}^{\alpha}, \Omega_t\right). \quad (12)$$

Equation (12) can be viewed as a mathematical transcription of the concept ‘‘average loss in the worst 100α % cases’’.

We use a nonparametric approach and estimate ES using the sample quintile estimates based on historical return data. For each month t , one year of daily data from month t to $t-12$ are used to estimate ES for month $t+1$:⁹

$$ES_{t+1}^{\alpha} = \left(\frac{1}{\#\{L_{i,d}\}_{t-12}^t > VaR_{t+1}^{\alpha}} \right) \cdot \sum_{\{L_{i,d}\}_{t-12}^t > VaR_{t+1}^{\alpha}} L_t, \quad (13)$$

where $\#\{L_{i,d}\}_{t-12}^t > VaR_{t+1}^{\alpha}$ is the number of daily losses $\{L_{i,d}\}_{t-12}^t$ exceeding the VaR calculated in eq. (11).

It should be noted that the original VaR and ES values are negative because they are obtained from the left tail of the return distribution, but the original VaR and ES measures are multiplied by -1 before forming the long-short portfolios of VaR and ES and before running cross-sectional regressions. Therefore, we expect the cross-sectional relation between expected stock returns and downside risk measures (VaR and ES) to be positive, i.e, the more a stock can potentially fall in value the higher should be the expected return.

1.3 Tail Risk Measures

Tail Covariance Risk (TCR): To estimate tail covariance risk (TCR) of an individual stock, we assume a single factor return generating process based on the left tail of the return distribution:

$$L_{i,d} = a_i + b_i L_{m,d} + e_{i,d}, \quad (14)$$

where $L_{i,d}$ is the *extreme* daily excess return on stock i that are below the 10% VaR threshold and $L_{m,d}$ is the excess daily return on the market portfolio corresponding to the days of $L_{i,d}$ over the past one year, and $e_{i,d}$ is the *extreme* idiosyncratic return or regression residual in eq. (14). The estimated slope coefficient

⁹ Assuming that there are 250 daily observations in a year, 10% ES is computed as the average of 25 extreme daily return observations that are below the 10% VaR threshold.

$\hat{b}_{i,t} = \text{cov}(L_{i,d}, L_{m,d}) / \text{var}(L_{m,d})$ in eq. (14) is the tail covariance risk of stock i . For each month t , one year of daily extreme returns from month t to $t-12$ are used to estimate tail covariance risk (TCR) for month $t+1$:¹⁰

$$TCR_{i,t+1} = \frac{\text{cov}\left(\{L_{i,d}\}_{t-12}^t; \{L_{m,d}\}_{t-12}^t\right)}{\text{var}\left(\{L_{m,d}\}_{t-12}^t\right)}, \quad (15)$$

where $\{L_{i,d}\}_{t-12}^t$ is the extreme excess daily returns on stock i that are below the 10% VaR of past one year of daily data and $\{L_{m,d}\}_{t-12}^t$ is the excess daily returns on the market portfolio corresponding to the days of $\{L_{i,d}\}_{t-12}^t$ over the past one year.

Tail Total Risk (TTR): Tail total risk (TTR) of an individual stock is calculated as the standard deviation of daily extreme excess returns that are below the 10% VaR threshold. For each month t , one year of daily extreme stock returns from month t to $t-12$ are used to estimate total tail risk for month $t+1$:

$$TTR_{i,t+1} = \sqrt{\text{var}\left(\{L_{i,d}\}_{t-12}^t\right)}, \quad (16)$$

where $\{L_{i,d}\}_{t-12}^t$ is the extreme excess daily returns on stock i that are below the 10% VaR of past one year of daily data.

Tail Idiosyncratic Risk (TIR): Tail idiosyncratic risk (TIR) of an individual stock is calculated as the standard deviation of daily residuals, $e_{i,d}$, obtained from the single-factor return generating process in equation (14). For each month t , one year of daily extreme residuals from month t to $t-12$ are used to estimate tail idiosyncratic risk for month $t+1$:

$$TIR_{i,t+1} = \sqrt{\text{var}\left(\{e_{i,d}\}_{t-12}^t\right)}, \quad (17)$$

where $\{e_{i,d}\}_{t-12}^t$ is the *extreme* idiosyncratic return obtained from the 10% lower tail of the daily return distribution of stock i .

1.4 Control Variables

Size: Following the existing literature, firm size is measured by the natural logarithm of the market value of equity (a stock's price times shares outstanding in millions of dollars) for each stock.

¹⁰ Assuming that there are 250 daily observations in a year, tail covariance risk (TCR) is computed based on the regression given in equation (14) using 25 extreme daily return observations that are less than the 10% VaR threshold.

Book-to-market: Following Fama and French (1992), we compute a firm's book-to-market ratio using its market equity at the end of December of year $t-1$ and the book value of common equity plus balance-sheet deferred taxes for the firm's latest fiscal year ending in calendar year $t-1$.¹¹

Momentum: Following Jegadeesh and Titman (1993), momentum is defined as the cumulative return over the past 12 months by skipping a month between the portfolio formation period and the holding period, i.e., average monthly return from month $t-12$ to month $t-2$.

Short-term reversal: Jegadeesh (1990) and Lehmann (1990) provide evidence for short-term return reversals. These papers show that contrarian strategies that select stocks based on their returns in the previous week or month generate significant abnormal returns. In this paper, we investigate the robustness of a cross-sectional relation between tail covariance risk and expected returns after controlling for the past 1-month return.

Liquidity: Liquidity generally implies the ability to trade large quantities quickly, at low cost, and without inducing a large change in the price level. Following Amihud (2002), we measure stock illiquidity as the ratio of absolute stock return to its dollar trading volume:

$$ILLIQ_{i,t} = |R_{i,t}| / VOLD_{i,t}, \quad (18)$$

where $R_{i,t}$ is the return on stock i in month t , and $VOLD_{i,t}$ is the respective monthly volume in dollars. This ratio gives the absolute percentage price change per dollar of monthly trading volume. As discussed in Amihud (2002), $ILLIQ_{i,t}$ follows the Kyle's (1985) concept of illiquidity, i.e., the response of price to the associated order flow or trading volume. The measure of stock illiquidity given in equation (18) can be interpreted as the price response associated with one dollar of trading volume, thus serving as a rough measure of price impact.

Co-skewness: Following Harvey and Siddique (2000), we estimate co-skewness based on the following regression for each stock in our sample:

$$R_{i,d} = \alpha_i + \beta_i R_{m,d} + \gamma_i R_{m,d}^2 + \varepsilon_{i,d}, \quad (19)$$

where $R_{i,d}$ is the excess return on stock i on day d , and $R_{m,d}$ is the excess market return on day d . The co-skewness of stock i in month $t+1$ is the slope coefficient $\hat{\gamma}_{i,t+1}$ in equation (19) estimated with daily returns from month t to $t-12$.

¹¹ To avoid giving extreme observations heavy weight in our analysis, following Fama and French (1992), the smallest and largest 0.5% of the observations on book-to-market ratio are set equal to the next largest and smallest values of the ratio (the 0.005 and 0.995 fractiles).

2. Average Returns on Portfolios of Alternative Risk Measures

2.1 Average Returns on Portfolios of Downside Risk

Table 1 presents the average return, VaR, expected shortfall, market beta, total risk, market share, size, book-to-market, price, and illiquidity of individual stocks in the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on the absolute value of 1%, 5%, and 10% VaR of daily returns over the past one year. The results are reported for the sample period of July 1963 to December 2006.

In Panel A of Table 1, Portfolio 1 (Low VaR) contains the NYSE stocks with the lowest 1% VaR of daily returns in the previous year and Portfolio 5 (High VaR) includes the NYSE stocks with the highest 1% VaR of daily returns in the previous year. As shown in the second column of Panel A, the average 1% VaR of individual stocks is about 3.05% per day for Portfolio 1, 4.02% per day for Portfolio 2, 4.90% per day for Portfolio 3, 6.04% per day for Portfolio 4, and 8.31% per day for Portfolio 5. As shown in the first column of Panel A, the average return of individual stocks is about 0.64% per month for the Low VaR portfolio (Portfolio 1), 0.81% per month for Portfolio 2, 0.88% per month for Portfolio 3, 0.85% per month for Portfolio 4, and 0.60% per month for the High VaR portfolio (Portfolio 5). The average return difference between quintile 5 (High VaR) and quintile 1 (Low VaR) is about -0.05% per month with the Newey-West (1987) t-statistic of -0.20 .

As shown in Panels B and C of Table 1, similar results are obtained from the 5% VaR and 10% VaR measures. When portfolios are formed based on the 5% VaR of daily returns in the previous year, the average return difference between quintile 5 and quintile 1 is about -0.04% per month with the Newey-West t-statistic of -0.17 . When portfolios are formed based on the 10% VaR, the average return difference between the High VaR and the Low VaR portfolios is about -0.06% per month with t-stat. = -0.23 .

These results indicate economically and statistically insignificant relation between value-at-risk and the cross-section of expected returns over the sample period of July 1963 to December 2006.

A common observation in Table 1 is that there is a strong positive correlation among alternative measures of risk. For example, in Panel A, moving from Portfolio 1 to Portfolio 5, as the average 1% VaR increases from 3.05% to 8.31% per day, the average 1% expected shortfall increases from 3.60% to 10.28% per day, the average market beta increases from 0.51 to 1.23, and the average total risk increases from 1.31% to 3.37% per day. Another notable point is that stocks with high VaR (Portfolio 5) are, on average, have smaller market capitalization, higher book-to-market ratio (value stocks), lower price, and they are less liquid, whereas stocks with low VaR (Portfolio 1) are, on average, have larger market capitalization, lower book-to-market ratio (growth stocks), higher price, and they are more liquid. We observe exactly the same pattern in Panels B and C of Table 1, i.e., stocks with high downside risk are generally small, value, low-priced, and illiquid stocks.

Table 2 displays the average return, expected shortfall (ES), VaR, market beta, total risk, market share, size, book-to-market, price, and illiquidity of individual stocks in the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on the absolute value of 1%, 5%, and 10% ES of daily returns over the past 12 months. The results are reported for the same period of July 1963-December 2006.

In Panel A of Table 2, Portfolio 1 (Low ES) contains the NYSE stocks with the lowest 1% expected shortfall of daily returns in the previous year and Portfolio 5 (High ES) includes the NYSE stocks with the highest 1% expected shortfall of daily returns in the previous year. The second column of Panel A shows that the average 1% ES of individual stocks increases from 3.60% per day for Portfolio 1 to 10.60% per day for Portfolio 5. The first column of Panel A reports that moving from Portfolio 1 to Portfolio 5, the average return decreases from 0.65% to 0.58% per month. The average return difference between quintile 5 (High ES) and quintile 1 (Low ES) is about -0.07% per month with the Newey-West t-statistic of -0.29 . Similar results are obtained from the 5% and 10% expected shortfall measures. When portfolios are formed based on the 5% ES of daily returns in the previous year, the average return difference between quintile 5 and quintile 1 is only 6 basis points per month with the t-statistic of -0.24 . When portfolios are formed based on the 10% ES, the average return difference between the High ES and the Low ES portfolios is only 4 basis points per month with t-stat. = -0.17 . The findings in Table 2 provide evidence that expected shortfall has no predictive power for the cross-sectional variation in expected stock returns.¹²

2.2 Average Returns on Portfolios of Standard Risk Measures

Panel A of Table 3 presents the average return, systematic risk, total risk, idiosyncratic risk, market share, size, book-to-market, price, and illiquidity of individual stocks in the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their systematic risk (SR) estimated with daily returns over the past one year. Portfolio 1 (Low SR) contains the NYSE stocks with the lowest systematic risk and Portfolio 5 (High SR) includes the NYSE stocks with the highest systematic risk. Panel A shows that moving from quintile 1 to quintile 5, while the average market beta increases from 0.34 to 1.58, the average return increases only from 0.68% to 0.73% per month. The average return difference between quintile 5 and quintile 1 is only 0.05% per month with t-stat. = 0.28. This result indicates that the cross-sectional relation between market beta and expected stock returns is flat.

Panel B of Table 3 presents results from the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their total risk (TR) measured with the standard deviation of daily returns over the past one year. Portfolio 1 (Low TR) contains the NYSE stocks with the lowest total risk and

¹² Similar to our findings in Table 1, when portfolios are formed based on the expected shortfall, we find that stocks with high (low) expected shortfall are generally small (big), value (growth), low-priced (high-priced), and illiquid (liquid).

Portfolio 5 (High TR) includes the NYSE stocks with the highest total risk. As shown in Panel B, moving from Low TR to High TR portfolios, the average total risk increases significantly from 1.29% to 3.43% per day, whereas the average return decreases almost 1 basis point from 0.63% to 0.62% per month. The average return difference between quintile 5 and quintile 1 is only -0.008% per month with $t\text{-stat.} = -0.03$. This result indicates that there is no link between total risk and the cross-section of expected returns.

Panel C of Table 3 shows risk and return statistics from the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their idiosyncratic risk (IR) estimated with daily returns over the past one year. Portfolio 1 (Low IR) contains the NYSE stocks with the lowest idiosyncratic risk and Portfolio 5 (High IR) includes the NYSE stocks with the highest idiosyncratic risk. As shown in Panel C, moving from Low IR to High IR portfolio, the average idiosyncratic risk increases significantly from 1.17% to 3.22% per day, whereas the average return stays almost the same at 0.63% per month. The average return difference between quintiles 5 and 1 is only 5 basis points per month with $t\text{-stat.} = 0.02$, implying that idiosyncratic risk has no predictive power for the cross-section of stock returns.

2.3 Average Returns on Portfolios of Tail Risk

We have so far shown that neither downside risk (VaR, ES) nor standard risk measures (SR, TR, IR) can explain the cross-sectional variation in stock returns. In this section, we investigate whether there is a positive and significant relation between tail risk and the cross-section of expected returns.

Panel A of Table 4 presents the average return, tail covariance risk (TCR), systematic risk, total risk, market share, size, book-to-market, price, and illiquidity of individual stocks in the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their tail covariance risk. As discussed in Section 1.3, TCR is estimated using the extreme daily returns in the left tail of the empirical distribution. Assuming that there are 250 daily observations in a year, tail covariance risk (TCR) is computed based on the regression given in equation (14) using 25 extreme daily return observations that are less than the 10% VaR threshold.

In Panel A, Portfolio 1 (Low TCR) contains the NYSE stocks with the lowest tail covariance risk and Portfolio 5 (High TCR) includes the NYSE stocks with the highest tail covariance risk. Panel A shows that moving from quintile 1 to quintile 5, average return on TCR portfolios increases monotonically from 0.58% to 0.85% per month. The average return difference between quintiles 5 and 1 is about 0.27% per month with the Newey-West t -statistic of 2.77. A trading strategy that longs stocks in the highest TCR quintile and shorts stocks in the lowest TCR quintile produces average return of 27 basis points per month (or 3.24% per annum) from July 1963 to December 2006. This result indicates a positive and significant relation between tail covariance risk and the cross-section of expected returns.

A notable point in Panel A of Table 4 is that except for market beta there is no systematic pattern in the average total risk, market share, size, book-to-market, price and illiquidity of TCR portfolios. Moving

from the Low TCR to High TCR portfolios, average market beta increases from 0.72 to 1.18 although not monotonically. However, there is no significant difference between the average total risks of individual stocks in the Low TCR to High TCR portfolios, i.e., average total risk is 2.21% per day for Low TCR portfolio vs. 2.40% per day for High TCR portfolio. In addition, the last five columns of Panel A show that the average market share, average market capitalization, book-to-market ratio, price, and illiquidity of stocks in the Low TCR and High TCR portfolios are approximately the same. These results indicate that tail covariance risk is an independent risk factor that has almost no association with the well-known cross-sectional effects such as size, book-to-market, price, volatility and liquidity.

To better understand the interaction of TCR with other variables, we compute the cross-sectional correlation of tail covariance risk (TCR) with market beta, total volatility, size, book-to-market (BM), and illiquidity (ILLIQ) for each month from July 1963 to December 2006. Panel A of the Appendix reports the time-series averages of the cross-sectional correlations. A notable point in Panel A is that tail covariance risk has very low correlation with size, book-to-market, illiquidity, and total volatility. Consistent with the results presented in Table 4, TCR is only correlated with market beta although the correlation coefficient of 0.31 does not imply an economically large association of TCR with BETA.

Later in the paper, we provide different ways of dealing with the potential interaction of tail covariance risk with firm size, book-to-market, liquidity, past returns, total volatility, and co-skewness. Specifically, we test whether the positive relation between TCR and the cross-section of expected returns still holds once we control for size, book-to-market, momentum, short-term reversal, liquidity, total volatility, and co-skewness using the bivariate sorts of portfolios and the characteristic matched benchmark portfolios.

Panel B of Table 4 presents results for the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their tail total risk (TTR). As discussed in Section 1.3, TTR is estimated based on the standard deviation of extreme daily returns in the left tail of the empirical distribution. Assuming that there are 250 daily observations in a year, tail total risk (TTR) is calculated as the standard deviation of 25 extreme daily return observations that are less than the 10% VaR threshold. In Panel B, Portfolio 1 (Low TTR) contains the NYSE stocks with the lowest tail total risk and Portfolio 5 (High TTR) includes the NYSE stocks with the highest tail total risk. Panel B shows that moving from quintile 1 to quintile 5, there is no monotonically increasing or decreasing pattern in the average return of TTR portfolios. The average return difference between quintiles 5 and 1 is only -0.06% per month with $t\text{-stat.} = -0.32$, indicating that there is no link between that tail total risk and expected stock returns.

As presented in Panel C of Table 4, similar results are obtained from the tail idiosyncratic risk (TIR) defined as the standard deviation of residuals from regression equation (14) estimated with the extreme daily return observations that are less than the 10% VaR threshold. In Panel C, Portfolio 1 (Low TIR) contains the NYSE stocks with the lowest tail idiosyncratic risk and Portfolio 5 (High TIR) includes the NYSE stocks with the highest tail idiosyncratic risk. The second column of Panel C shows that the average TIR of

individual stocks increases from 0.61 for Portfolio 1 to 2.35 for Portfolio 5. The first column of Panel C reports that moving from Portfolio 1 to Portfolio 5, the average return decreases from 0.67% to 0.60% per month. The average return difference between quintiles 5 and 1 is about -0.07% per month with $t\text{-stat.} = -0.36$, implying that tail idiosyncratic risk has no predictive power for the cross-section of expected returns.

3. Average Returns on Portfolios of Tail Covariance Risk

To better understand the interaction of alternative risk measures, we compute the cross-sectional correlation of tail covariance risk (TCR) with tail total risk (TTR), tail idiosyncratic risk (TIR), systematic risk (SR), total risk (TR), idiosyncratic risk (IR), value-at-risk (VaR), and expected shortfall (ES) measures for each month from July 1963 to December 2006. Panel B of the Appendix reports the time-series averages of the cross-sectional correlations. A notable point in Panel B is that tail covariance risk has very low correlation with TTR, TIR, TR, IR, VaR, and ES. The correlation of TCR with these risk measures is in the range of 0.003 to 0.047. TCR seems to be correlated only with systematic risk (SR). The correlation between TCR and SR is about 0.31 which does not suggest an economically large association of TCR with SR. As expected, total risk, idiosyncratic risk, tail total risk, tail idiosyncratic risk, value-at-risk, and expected shortfall measures are found to be highly correlated.

Table 5 presents the average raw and risk-adjusted returns of individual stocks in the equal-weighted quintile portfolios that are formed by sorting the NYSE stocks based on their tail covariance risk. As shown in Panel A, for the full sample of NYSE stocks, moving from quintile 1 to quintile 5, average raw returns on TCR portfolios increase monotonically from 0.58% to 0.85% per month. The average raw return difference between quintiles 5 and 1 is about 0.27% per month with the $t\text{-statistic}$ of 2.77. The 5-1 difference in the FF-3 alphas is about 0.26% per month with $t\text{-stat.} = 2.99$. A trading strategy that longs stocks in the highest-TCR quintile and shorts stocks in the lowest-TCR quintile produces average raw and risk-adjusted returns of 26 to 27 basis points per month from July 1963 to December 2006.

Ang, Chen, and Xing (2006) emphasize the interaction of stock return volatility and downside beta and examine the cross-sectional relation between downside beta and expected returns after excluding stocks with the highest 20% volatility. They provide two reasons why high volatility stocks need to be eliminated before forming the long-short portfolios of downside beta. First, they find the measurement error in downside beta to be higher for stocks with high volatility. Second, Ang, Hodrick, Xing, and Zhang (2006) find that stocks with very high volatility have extremely low returns. Ang, Chen, and Xing (2006) point out that this volatility effect confounds the relationship between high downside beta and high return because high-volatility stocks tend to be high-beta stocks, but high-volatility stocks also tend to have lower average return.

To control for the potential interaction between stock return volatility and TCR, we exclude stocks with the highest 1%, 5%, 10%, and 20% volatility and our original findings remain intact. As presented in Table 5, the average raw return difference between the High TCR and Low TCR portfolios is in the range of

0.25% to 0.28% per month with the t-statistics ranging from 2.78 to 3.36. Similarly, the average risk-adjusted return difference between the High TCR and Low TCR portfolios is in the range of 0.24% to 0.26% per month with the t-statistics ranging from 2.86 to 3.48. Based on the average raw and risk-adjusted return differences, we find a positive and significant link between tail covariance risk and expected returns.

4. Portfolios of Tail Covariance Risk: After Controlling for Size, Book-to-Market, Momentum, Reversal, Liquidity, Volatility, and Co-skewness

This section tests whether there is a positive relation between tail covariance risk and future stock returns after controlling for size, book-to-market, momentum, short-term reversal, liquidity, total volatility, and co-skewness. We control for size by first forming quintile portfolios ranked based on market capitalization. Then, within each size quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel A of Table 6 shows that in each size quintile, the lowest (highest) TCR quintile has lower (higher) equal-weighted average returns. The column labeled “Average Returns” averages across the five size quintiles to produce quintile portfolios with dispersion in TCR, but which contain all sizes of firms. This procedure creates a set of TCR portfolios with near-identical levels of firm size and thus these TCR portfolios control for differences in size. After controlling for size, the equal-weighted average return difference between the Low TCR and High TCR portfolios is about 0.25% per month with the Newey-West t-statistic of 2.90. The 5-1 difference in the FF-3 alphas is 0.21% per month with the t-statistic of 2.80. Thus, market capitalization does not explain the high (low) returns to high (low) TCR stocks.

We also control for book-to-market (BM) by first forming quintile portfolios ranked based on the ratio of book value of equity to market value of equity. Then, within each BM quintile, we sort stocks into quintile portfolios ranked based on TCR so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel B of Table 6 shows that in each BM quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for BM. The average raw return difference between the Low TCR and High TCR quintiles is 0.26% per month with the t-statistic of 2.90. The 5-1 difference in the FF-3 alphas is also positive, 0.22% per month, and highly significant. Thus, book-to-market ratio does not explain the positive cross-sectional risk premium on the High TCR minus Low TCR portfolios.

We control for momentum (MOM) by first forming quintile portfolios ranked based on the cumulative monthly returns over the past 12 months by skipping a month between the portfolio formation

period and the holding period.¹³ At the beginning of each month t , stocks are ranked in ascending order on the basis of their cumulative returns from month $t-12$ to month $t-2$. Based on these rankings, 5 quintile portfolios are formed that equally weight the stocks contained in the top quintile, the second quintile, and so on. The top quintile portfolio is called the “losers” quintile (MOM 1) and the bottom quintile is called the “winners” quintile (MOM 5). Within each MOM quintile, we sort stocks into quintile portfolios ranked based on TCR so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel C of Table 6 shows that in each momentum quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for momentum. The average raw return difference between the Low TCR and High TCR quintiles is 0.24% per month with $t\text{-stat.} = 2.83$. The 5-1 differences in the FF-3 alphas is also positive, 0.23% per month, and highly significant. Thus, momentum does not explain the positive relation between TCR and the cross-section of future stock returns.

We control for short-term reversal (REV) by first forming quintile portfolios ranked based on the past 1-month return. Then, within each REV quintile, we sort stocks into quintile portfolios ranked based on TCR so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel D of Table 6 shows that in each reversal quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for short-term reversal. The average raw return difference between the Low TCR and High TCR quintiles is 0.29% per month with the t-statistic of 3.28. The 5-1 differences in the FF-3 alphas is also positive, 0.27% per month, with the t-statistic of 3.45. Thus, short-term reversal does not explain the high (low) returns to high (low) TCR stocks.

We control for liquidity by first forming quintile portfolios ranked based on the illiquidity measure of Amihud (2002). In Panel E of Table 6, ILLIQ 1 denotes a portfolio of stocks with the lowest illiquidity, whereas ILLIQ 5 denotes a portfolio of stocks with the highest illiquidity. Within each illiquidity quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel E of Table 6 shows that in each illiquidity quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for illiquidity. The average raw return difference between the Low TCR and High TCR quintiles is 0.24% per month with $t\text{-stat.} = 2.72$. The 5-1 differences in the FF-3 alphas is also positive, 0.20% per month, and highly significant. Thus, liquidity does not explain the positive relation between TCR and expected stock returns.

We control for total risk (TR) by first forming quintile portfolios ranked based on the total volatility measure. In Panel F of Table 6, TR 1 denotes a portfolio of stocks with the lowest total risk whereas TR 5

¹³ As discussed in Jegadeesh and Titman (1993), skipping a month avoids some of the bid-ask spread, price pressure, and lagged reaction effects.

denotes a portfolio of stocks with the highest total risk. Within each TR quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel F of Table 6 shows that in each TR quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for total risk. The average raw return difference between the Low TCR and High TCR quintiles is 0.21% per month with the t-statistic of 3.00. The 5-1 differences in the FF-3 alphas is also positive, 0.25% per month, and highly significant. Thus, total risk does not explain the high (low) returns to high (low) TCR stocks.

We control for co-skewness (COSKEW) by first forming quintile portfolios ranked based on the co-skewness measure given in equation (19). In Panel G of Table 6, COSKEW 1 denotes a portfolio of stocks with the lowest co-skewness whereas COSKEW 5 denotes a portfolio of stocks with the highest co-skewness. Within each COSKEW quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk so that quintile 1 (quintile 5) contains stocks with the lowest (highest) TCR. Panel G of Table 6 shows that in each COSKEW quintile, the lowest (highest) TCR quintile has lower (higher) average returns. The last two columns report the differences in average returns and the FF-3 alphas along with the Newey-West t-statistics after controlling for co-skewness. The average raw return difference between the Low TCR and High TCR quintiles is 0.24% per month with t-stat. = 2.65. The 5-1 differences in the FF-3 alphas is also positive, 0.27% per month, and highly significant. Thus, co-skewness does not explain the positive relation between tail covariance risk and the cross-section of expected returns.

5. Portfolios of Tail Covariance Risk: After Screening for Size, Price, Liquidity, Volatility, Winners, Losers, and Short-term Reversal

We have so far used bivariate portfolios to deal with the potential interaction of tail covariance risk with firm size, book-to-market, momentum, short-term reversal, volatility, liquidity, and co-skewness. We think that an alternative way of handling this problem is to exclude small, low-priced, illiquid, highly volatile, extreme winner, loser, and reversal stocks in the formation of TCR portfolios. Our screening process can be explained as follows:

(1) *Size*: To screen for size, all NYSE stocks are sorted for each month by firm size to determine the NYSE decile breakpoints for market capitalization. Then, we exclude all NYSE stocks with market capitalizations that would place them in the smallest NYSE size decile.

(2) *Price*: We exclude stocks whose price is less than \$5. Returns on low-priced stocks are greatly affected by the minimum tick of \$1/8, which may add noise to the construction of tail covariance risk.¹⁴

¹⁴ See Harris (1994) and Jegadeesh and Titman (2001).

(3) *Liquidity*: To screen for liquidity, all NYSE stocks are sorted for each month by the ratio of absolute stock return to its dollar volume to determine the NYSE decile breakpoints for the illiquidity measure. Then, we exclude all NYSE stocks that belong to the smallest NYSE liquidity decile (or the largest NYSE illiquidity decile).

(4) *Volatility*: To screen for total volatility, all NYSE stocks are sorted for each month by their total volatility to determine the NYSE decile breakpoints for the volatility measure. Then, we exclude all NYSE stocks that belong to the highest NYSE volatility decile.¹⁵

(5) *Winners*: To screen for past 12-month winners, all NYSE stocks are sorted for each month by their cumulative returns from month $t-12$ to month $t-2$ to determine the NYSE decile breakpoints for momentum. Then, we exclude the NYSE winner stocks that belong to the highest momentum decile. To screen for past 1-month winners, all NYSE stocks are sorted for each month by their past 1-month return to determine the NYSE decile breakpoints for short-term reversal. Then, we exclude the past 1-month winner stocks that belong to the highest reversal decile.

(6) *Losers*: To screen for past 12-month losers, all NYSE stocks are sorted for each month by their cumulative returns from month $t-12$ to month $t-2$ to determine the NYSE decile breakpoints for momentum. Then, we exclude the NYSE loser stocks that belong to the lowest momentum decile. To screen for past 1-month losers, all NYSE stocks are sorted for each month by their past 1-month return to determine the NYSE decile breakpoints for short-term reversal. Then, we exclude the past 1-month loser stocks that belong to the lowest reversal decile.

(7) *Short-term reversal*: As indicated by Jegadeesh (1990) and Lehman (1990), daily returns in month $t-1$ are the source of short-term reversal. Hence, to screen for short-term reversal, we estimate tail covariance risk using daily returns from month $t-12$ to month $t-2$, skipping daily returns in month $t-1$.

After screening for size, price, liquidity, volatility, winners, losers, and short-term reversal, equal-weighted quintile portfolios are formed every month from January 1963 to December 2006 by sorting the NYSE stocks based on tail covariance risk (TCR) calculated using the 10% lower tail of the daily return distribution over the past one year. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) tail covariance risk.

Table 7 reports the average returns in monthly percentage terms on TCR portfolios. The average raw return difference between quintiles 5 and 1 is in the range of 0.22% to 0.28% per month with the t-statistics ranging from 2.38 to 3.19. The 5-1 difference in the FF-3 alphas is in the range of 0.17% to 0.26% per month with t-statistics ranging from 2.13 to 3.24. Based on the average raw and risk-adjusted return differences, we

¹⁵ In Table 5, we report results after excluding stocks with the highest 1%, 5%, 10%, and 20% volatility. To be consistent with downside risk measures, total volatility in Table 5 is calculated as the standard deviation of daily returns over the past one year. In this section, following Ang, Hodrick, Xing, and Zhang (2006), we compute total volatility as the standard deviation of daily returns over the past one month.

find a positive and significant relation between tail covariance risk and expected returns after screening for small, low-priced, illiquid, highly volatile, extreme winner, loser, and reversal stocks.

6. Portfolios of Tail Covariance Risk: Alternative Controls for Size/BM, Momentum, Reversal, Liquidity, Volatility, and Co-skewness

In Table 8, we further examine whether the significantly positive relation between tail covariance risk and expected returns is not due to size, book-to-market, momentum, reversal, volatility, or liquidity effects. In column labeled “Size/BM adjusted,” we report the average returns in excess of the size and book-to-market matched benchmark portfolios similar to Daniel and Titman (1997). In the next five columns, we include additional controls for momentum (as measured by past 12-month returns), short-term reversal (as measured by past 1-month returns), illiquidity measure of Amihud (2002), total volatility, and co-skewness. For each additional control, we first perform a quintile sort based on the characteristic and then on tail covariance risk (TCR). Finally, we average the TCR quintiles across the characteristic quintiles and report size and book-to-market matched returns within each TCR quintile. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) tail covariance risk.

As presented in the first column of Table 8, after controlling for size and book-to-market simultaneously, the average return difference between the Low TCR and High TCR quintiles is about 0.21% per month with the t-statistic of 2.96. The last five columns of Table 8 clearly show that the spreads in size and book-to-market adjusted returns between the TCR quintiles 5 and 1 remain positive and significant after controlling for momentum, short-term reversal, liquidity, total risk, and co-skewness. In each case, the average return difference between the Low TCR and High TCR quintiles is in the range of 0.19% to 0.23% per month with the t-statistics ranging from 2.71 to 3.21. Thus, our predictive pattern of returns for tail covariance risk is not due to size, book-to-market, past return, liquidity, volatility, and co-skewness effects.

7. Firm-Level Cross-Sectional Regressions

We have so far investigated the significance of TCR as a determinant of the cross-section of future returns at the portfolio level. We now examine the cross-sectional relation between TCR and expected returns at the firm level using Fama and MacBeth (1973) regressions. We present the time-series averages of the slope coefficients from the regressions of one-month ahead stock returns on tail covariance risk (TCR), downside risk measures [Value at Risk (VaR), expected shortfall (ES)], and the control variables: log market capitalization (SIZE), log book-to-market ratio (BM), momentum (MOM), short-term reversal (REV), illiquidity (ILLIQ), co-skewness (COSKEW), total volatility (TVOL), and idiosyncratic volatility (IVOL). The average slopes provide standard Fama-MacBeth tests for determining which explanatory variables on

average have non-zero premiums. Monthly cross-sectional regressions are first run for the following univariate econometric specifications:

$$\begin{aligned}
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}TCR_{i,t} + \varepsilon_{i,t+1}, \\
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}VaR_{i,t} + \varepsilon_{i,t+1}, \\
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}ES_{i,t} + \varepsilon_{i,t+1},
\end{aligned} \tag{20}$$

where $R_{i,t+1}$ is the realized return on stock i in month $t+1$. The predictive cross-sectional regressions are run on the one-month lagged values of TCR, VaR, and ES.

Table 9 reports the time series averages of the slope coefficients over the 522 months from July 1963 to December 2006 for the NYSE stocks. The Newey-West adjusted t-statistics are given in parentheses. The univariate regression results show a positive and statistically significant relation between tail covariance risk and the cross-section of future stock returns. The average slope, $\lambda_{1,t}$, from the monthly regressions of realized returns on TCR alone is 0.15 with the Newey-West t-statistic of 2.07. The economic magnitude of the associated effect is similar to that documented in our earlier tables for the univariate and bivariate portfolios of TCR. Similar to our findings from the univariate portfolios, there is no significant relation between downside risk and the cross-section of expected returns because the average slopes on VaR and ES turn out to be statistically insignificant for all coverage probability levels (1%, 5%, and 10%).

Panel B of Table 9 reports the average slope coefficients from the bivariate regressions of returns on TCR after controlling for the downside risk measures:

$$\begin{aligned}
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}TCR_{i,t} + \lambda_{2,t}VaR_{i,t} + \varepsilon_{i,t+1}, \\
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}TCR_{i,t} + \lambda_{2,t}ES_{i,t} + \varepsilon_{i,t+1},
\end{aligned} \tag{21}$$

From these horserace regressions, the average slopes on TCR turn out to be in the range of 0.13 to 0.15 and statistically significant without any exception, whereas the average slopes on VaR and ES are found to be negative but statistically insignificant. These results indicate superior performance of tail covariance risk over the standard measures of downside risk in predicting the cross-sectional variation in stock returns.

Finally, monthly cross-sectional regressions are run for the following multivariate specification and its nested versions:

$$\begin{aligned}
R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t}TCR_{i,t} + \lambda_{2,t}SIZE_{i,t} + \lambda_{3,t}BM_{i,t} + \lambda_{4,t}MOM_{i,t} + \lambda_{5,t}REV_{i,t} + \lambda_{6,t}ILLIQ_{i,t} \\
&\quad + \lambda_{7,t}COSKEW_{i,t} + \lambda_{8,t}TVOL_{i,t} + \lambda_{9,t}IVOL_{i,t} + \varepsilon_{i,t+1}
\end{aligned} \tag{22}$$

As shown in Panel C of Table 9, the average slope coefficients on the individual control variables are generally consistent with earlier studies: the size effect is negative and significant, the value effect is positive and significant, stocks exhibit intermediate-term momentum and short-term reversals, and illiquidity is priced. Consistent with the findings of Harvey and Siddique (2000) and Ang, Hodrick, Xing, and Zhang (2006), the average slopes on co-skewness, total volatility, and idiosyncratic volatility turn out to be

negative, but statistically insignificant. Of primary interest in Panel C of Table 9, the average slope coefficients on TCR are in the range of 0.13 to 0.17 and statistically significant with all combinations of control variables.

Panel D of Table 9 provides evidence for the cross-sectional relation between TCR and expected returns after controlling for all variables considered in the paper. The average slope on TCR is estimated to be between 0.13 and 0.15 with the t-statistics ranging from 2.22 to 2.49. The clear conclusion is that firm-level cross-sectional regressions provide strong corroborating evidence for an economically and statistically significant positive relation between tail covariance risk and future returns, consistent with our earlier findings from the portfolio-level analyses.

8. Robustness Check

8.1 Alternative Measures of Tail Covariance Risk

As presented in equation (15), we have so far used one year of daily extreme returns from month t to $t-12$ to estimate tail covariance risk (TCR) for month $t+1$. Extreme daily returns are obtained from the lower 10% of the daily return distribution over the past 12 months. As a robustness check, in this section, we generate alternative measures of tail covariance risk and test their predictive power for the cross-section of expected returns.

The first alternative measure of TCR is computed based on the extreme daily returns from the lower 5% of the daily return distribution over the past 12 months. Instead of extremes beyond the 10% VaR of daily returns over the past one year, we use the daily extremes beyond the 5% VaR threshold. Panel A of Table 10 shows that, with the first alternative measure of TCR, the average raw return increases monotonically from 0.62% per month for the Low TCR portfolio to 0.87% per month for the High TCR portfolio. The average raw return difference between quintiles 5 and 1 is about 0.25% per month with the Newey-West t-statistic of 3.56. The average risk-adjusted return difference between the Low TCR and High TCR quintiles is also about 0.25% per month with t-stat. = 3.79.

The second alternative measure of TCR is computed based on the extreme daily returns from the lower 10% of the daily return distribution over the past 6 months. Instead of using the daily extremes in the previous 12 months, we utilize the daily extreme returns over the past 6 months when estimating tail covariance risk. Panel B of Table 10 reports similar results from the second alternative measure of TCR. Moving from quintile 1 to quintile 5, the average raw return increases monotonically from 0.57% to 0.86% per month. The average raw return difference between quintiles 5 and 1 is about 0.29% per month with the t-statistic of 4.22. The 5-1 difference in the FF-3 alphas is about 0.26% per month with t-stat. = 3.69.

The results based on the average raw and risk-adjusted return differences between the Low TCR and High TCR portfolios indicate a positive and significant relation between expected returns and alternative measures of tail covariance risk.

A notable point in Table 10 is that, similar to our earlier findings from the original measure of TCR, except for market beta there is no systematic pattern in the average total risk, market share, size, book-to-market, price and illiquidity of TCR portfolios. This result indicates that alternative measures of tail covariance risk are independent risk factors that have almost no interaction with the well-known cross-sectional effects such as size, book-to-market, price, volatility and liquidity.

8.2 Tail Covariance Risk Measured with Covariance and Correlation of Extremes

We have so far used a “beta” type measure of tail covariance risk (TCR^{beta}) in the cross-sectional pricing of individual stocks:

$$TCR_{i,t+1}^{\text{beta}} = \frac{\text{cov}\left(\left\{L_{i,d}\right\}_{t-12}^t; \left\{L_{m,d}\right\}_{t-12}^t\right)}{\text{var}\left(\left\{L_{m,d}\right\}_{t-12}^t\right)}, \quad (15')$$

where $\left\{L_{i,d}\right\}_{t-12}^t$ is the extreme excess daily returns on stock i that are below the 10% VaR of past 12 months of daily data and $\left\{L_{m,d}\right\}_{t-12}^t$ is the excess daily returns on the market portfolio corresponding to the days of $\left\{L_{i,d}\right\}_{t-12}^t$ over the past 12 months.

We now introduce and test the significance of “covariance” and “correlation” type measures of tail covariance risk in predicting the cross-sectional variation in stock returns. For each month t , one year of daily extreme returns from month t to $t-12$ are used to estimate the “covariance” and “correlation” type measure of tail covariance risk ($TCR^{\text{cov}}, TCR^{\text{corr}}$) for month $t+1$:

$$TCR_{i,t+1}^{\text{cov}} = \text{cov}\left(\left\{L_{i,d}\right\}_{t-12}^t; \left\{L_{m,d}\right\}_{t-12}^t\right), \quad TCR_{i,t+1}^{\text{corr}} = \frac{\text{cov}\left(\left\{L_{i,d}\right\}_{t-12}^t; \left\{L_{m,d}\right\}_{t-12}^t\right)}{\sqrt{\text{var}\left(\left\{L_{i,d}\right\}_{t-12}^t\right)} \cdot \sqrt{\text{var}\left(\left\{L_{m,d}\right\}_{t-12}^t\right)}}. \quad (23)$$

In Panel A of Table 11, TCR is computed as the “covariance” between extreme daily returns on individual stock and the market portfolio from the 10% lower tail of the daily return distribution of individual stocks over the past 12 months. The average raw return on TCR^{cov} portfolios increases monotonically from 0.59% per month for the Low TCR^{cov} portfolio to 0.88% per month for the High TCR^{cov} portfolio. The average raw return difference between quintiles 5 and 1 is about 0.29% per month with the Newey-West t-statistic of 3.15. The average risk-adjusted return difference between the Low TCR^{cov} and High TCR^{cov} quintiles is about 0.26% per month with t-stat. = 2.98.

In Panel B, TCR is computed as the “correlation” between extreme daily returns on individual stock and the market portfolio from the 10% lower tail of the daily return distribution of individual stocks over the past 12 months. Moving from low TCR^{corr} to high TCR^{corr} portfolios, the average raw return increases monotonically from 0.61% to 0.86% per month. The average raw return difference between quintiles 5 and 1

is about 0.25% per month with $t\text{-stat.} = 2.83$. The average risk-adjusted return difference between the Low TCR^{corr} to high TCR^{corr} quintiles is about 0.33% per month with $t\text{-stat.} = 4.13$.

Similar to our findings from the beta type measure of TCR, we obtain positive and significant relation between expected returns and the covariance and correlation type measures of TCR, i.e., stocks in the highest TCR^{beta} (TCR^{cov}) [TCR^{corr}] quintile have significantly higher average return than stocks in the lowest TCR^{beta} (TCR^{cov}) [TCR^{corr}] quintile. The average risk-adjusted returns on TCR portfolios indicate that after adjusting for market, size, and book-to-market factors, the positive cross-sectional premium on the High TCR minus Low TCR portfolios is in the range of 3.12% to 3.96% per annum, and highly significant.

9. Conclusion

This paper investigates the significance of a cross-sectional relation between alternative measures of risk and expected returns on NYSE stocks for the sample period of July 1963 to December 2006. The results provide no evidence for a significant link between one-month ahead stock returns and standard risk measures (market beta, total volatility, idiosyncratic volatility) that are estimated based on the entire distribution of daily returns over the past one year. The paper also examines whether the commonly used measures of downside risk (value-at-risk and expected shortfall) have any predictive power for the cross-sectional variation in stock returns. The long-short portfolio analyses indicate economically and statistically insignificant relation between one-month ahead expected returns and the 1%, 5%, and 10% VaR and expected shortfall measures that are estimated based on the left tail of the daily return distribution over the past one year.

A major contribution of this paper is to introduce new measures of *tail risk* and investigate their performance in predicting the cross-sectional variation in stock returns. The results provide strong evidence for a positive link between tail covariance risk and expected returns, whereas tail total risk and tail idiosyncratic risk have no influence on stock returns.

The univariate portfolio level analyses indicate that a trading strategy that longs stocks in the highest TCR quintile and shorts stocks in the lowest TCR quintile yields average raw and risk-adjusted returns of 3.5% per annum. As a robustness check, we exclude small, low-priced, illiquid, highly volatile, winner, loser, and reversal stocks and find that the effect of tail covariance risk on stock returns does not disappear after a screen for size, price, liquidity, volatility, and past return characteristics. In addition, we use two different methodologies to control for size, book-to-market, momentum, short-term reversal, liquidity, volatility, and co-skewness based on (i) the bivariate sorts of portfolios and (ii) the characteristic matched benchmark portfolios. The results from different portfolio sorting and firm-level Fama-MacBeth regressions indicate the cross-sectional premium of tail covariance risk to be positive and highly significant after controlling for these well-known risk factors and firm characteristics.

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Appendix. Average Cross Correlations

Panel A. Cross Correlations of Tail Covariance Risk with Control Variables

Panel A presents the time-series averages of the cross-sectional correlations of tail covariance risk (TCR), market beta (BETA), total volatility (TVOL), market capitalization (SIZE), book-to-market ratio (BM), and illiquidity (ILLIQ) for the sample period of July 1963-December 2006.

	TCR	BETA	TVOL	SIZE	BM	ILLIQ
TCR	1	0.305	0.056	0.052	-0.026	-0.018
BETA		1	0.484	0.141	-0.128	-0.106
TVOL			1	-0.498	0.163	0.264
SIZE				1	-0.369	-0.370
BM					1	0.163
ILLIQ						1

Panel B. Cross Correlations of Alternative Risk Measures

Panel B presents the time-series averages of the cross-sectional correlations of tail covariance risk (TCR), tail total risk (TTR), tail idiosyncratic risk (TIR), systematic risk (SR), total risk (TR), idiosyncratic risk (IR), value-at-risk (VaR), and expected shortfall (ES) for the sample period of July 1963-December 2006.

	TCR	TTR	TIR	SR	TR	IR	VaR	ES
TCR	1	0.015	0.012	0.305	0.047	0.003	0.038	0.039
TTR		1	0.904	0.284	0.692	0.690	0.567	0.734
TIR			1	0.168	0.575	0.583	0.449	0.604
SR				1	0.449	0.338	0.460	0.461
TR					1	0.989	0.942	0.964
IR						1	0.926	0.950
VaR							1	0.957
ES								1

TCR: Tail covariance risk is the slope coefficient in eq. (14) estimated with extreme daily returns over the past one year.

TTR: Tail total risk is the standard deviation of extreme daily returns over the past one year.

TIR: Tail idiosyncratic risk is the standard deviation of residuals in eq. (14).

SR: Systematic risk is the slope coefficient in eq. (6) estimated with daily returns over the past one year.

TR: Total risk is the standard deviation of daily returns over the past one year.

IR: Idiosyncratic risk is the standard deviation of residuals in eq. (6) estimated with daily returns over the past one year.

VaR: Value at risk is the 1% cutoff point of the left tail of the distribution of daily returns over the past one year.

ES: Expected shortfall is the average of losses beyond the 1% VaR of daily returns over the past one year.

Table 5. Portfolios of Tail Covariance Risk

This table presents average returns on the equal-weighted quintile portfolios that are formed by sorting NYSE stocks based on tail covariance risk (TCR). Portfolio 1 (Low TCR) contains NYSE stocks with the lowest tail covariance risk in the previous year and Portfolio 5 (High TCR) includes NYSE stocks with the highest tail covariance risk in the previous year. The row “High TCR – Low TCR” reports the difference in average monthly returns between the High TCR and Low TCR quintiles. The row “FF-3 Alpha Difference” reports Jensen’s alpha with respect to the 3-factor Fama-French (1993) model. Newey-West adjusted t-statistics are reported in parentheses. The results are presented for the full NYSE sample as well as for the sample excluding NYSE stocks with the highest 1%, 5%, 10%, and 20% volatility. The sample period is from July 1963 to December 2006.

	Full Sample	excluding stocks with highest 1% volatility	Excluding stocks with highest 5% volatility	excluding stocks with highest 10% volatility	excluding stocks with highest 20% volatility
Low TCR	0.5764	0.5721	0.6119	0.6233	0.6560
2	0.7033	0.7035	0.7058	0.7101	0.7002
3	0.8097	0.8119	0.8202	0.8145	0.8170
4	0.8350	0.8408	0.8672	0.8550	0.8663
High TCR	0.8469	0.8474	0.8640	0.8931	0.9026
High TCR – Low TCR	0.2705	0.2753	0.2521	0.2699	0.2466
t-statistic	(2.77)	(2.84)	(2.78)	(3.16)	(3.36)
FF-3 Alpha Difference	0.2599	0.2527	0.2374	0.2644	0.2373
t-statistic	(2.99)	(2.90)	(2.86)	(3.41)	(3.48)

Table 6 (continued)

Panel C. Controlling for Momentum

In Panel C, we first form quintile portfolios of NYSE stocks ranked based on their cumulative returns from month $t-12$ to month $t-2$ (MOM). Then, within each MOM quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (5) contains stocks with the lowest (highest) TCR. The column labeled “Average” averages across the five MOM quintiles to produce quintile portfolios with dispersion in TCR and with near-identical levels of momentum and thus these quintile TCR portfolios control for differences in momentum.

	MOM 1	MOM 2	MOM 3	MOM 4	MOM 5	Average	t-statistic
Low TCR	0.7146	1.0905	1.1414	1.1236	1.2404	1.0621	4.82
2	0.9637	1.2578	1.2590	1.1569	1.3046	1.1884	5.84
3	1.1196	1.2765	1.3176	1.2709	1.3776	1.2725	6.23
4	1.0379	1.3974	1.3257	1.2969	1.4312	1.2978	5.97
High TCR	0.9798	1.4155	1.3244	1.2850	1.5154	1.3040	5.52
				Return Difference		0.2419	2.83
				Alpha Difference		0.2251	2.95

Panel D. Controlling for Short-term Reversal

In Panel D, we first form quintile portfolios of NYSE stocks ranked based on their past 1-month return (REV). Then, within each REV quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (5) contains stocks with the lowest (highest) TCR. The column labeled “Average” averages across the five REV quintiles to produce quintile portfolios with dispersion in TCR and with near-identical levels of past 1-month return and thus these quintile TCR portfolios control for differences in short-term reversal.

	REV 1	REV 2	REV 3	REV 4	REV 5	Average	t-statistic
Low TCR	1.5996	1.1619	1.0627	0.9068	0.4246	1.0311	4.55
2	1.6502	1.3433	1.1661	1.0080	0.6936	1.1722	5.81
3	1.8625	1.4556	1.2959	1.0320	0.7506	1.2793	6.31
4	1.8831	1.4685	1.3807	1.1087	0.7543	1.3190	6.22
High TCR	1.9341	1.4983	1.2939	1.0844	0.7897	1.3201	5.47
				Return Difference		0.2890	3.28
				Alpha Difference		0.2672	3.45

Table 6 (continued)**Panel E. Controlling for Illiquidity**

In Panel E, we first form quintile portfolios of NYSE stocks ranked based on their illiquidity (ILLIQ). Then, within each ILLIQ quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (5) contains stocks with the lowest (highest) TCR. The column labeled “Average” averages across the five ILLIQ quintiles to produce quintile portfolios with dispersion in TCR and with near-identical levels of illiquidity and thus these quintile TCR portfolios control for differences in illiquidity.

	ILLIQ 1	ILLIQ 2	ILLIQ 3	ILLIQ 4	ILLIQ 5	Average	t-statistic
Low TCR	0.9431	1.0667	1.1212	1.0551	1.1336	1.0640	4.80
2	1.1044	1.2032	1.1158	1.2841	1.1833	1.1781	5.89
3	1.2004	1.2200	1.2787	1.4074	1.2967	1.2807	6.31
4	1.1568	1.3111	1.3367	1.3654	1.3011	1.2942	6.04
High TCR	1.1307	1.3114	1.3072	1.4286	1.3607	1.3077	5.36
				Return Difference		0.2438	2.72
				Alpha Difference		0.2021	2.53

Panel F. Controlling for Total Volatility

In Panel F, we first form quintile portfolios of NYSE stocks ranked based on their total volatility (TVOL). Then, within each TVOL quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (5) contains stocks with the lowest (highest) TCR. The column labeled “Average” averages across the five TVOL quintiles to produce quintile portfolios with dispersion in TCR and with near-identical levels of total volatility and thus these quintile TCR portfolios control for differences in total volatility.

	TVOL 1	TVOL 2	TVOL 3	TVOL 4	TVOL 5	Average	t-statistic
Low TCR	1.0612	1.1272	1.1582	1.1535	0.8772	1.0754	4.91
2	1.0925	1.2474	1.3763	1.1599	1.0009	1.1754	5.53
3	1.0533	1.3456	1.3952	1.4926	1.1960	1.2965	6.15
4	1.1175	1.3266	1.3923	1.4410	1.1512	1.2857	5.90
High TCR	1.1606	1.2985	1.4256	1.3572	1.2053	1.2894	5.94
				Return Difference		0.2140	3.00
				Alpha Difference		0.2493	4.11

Table 6 (continued)**Panel G. Controlling for Co-Skewness**

In Panel G, we first form quintile portfolios of NYSE stocks ranked based on their co-skewness measure (COSKEW). Then, within each COSKEW quintile, we sort stocks into quintile portfolios ranked based on tail covariance risk (TCR) so that quintile 1 (5) contains stocks with the lowest (highest) TCR. The column labeled “Average” averages across the five COSKEW quintiles to produce quintile portfolios with dispersion in TCR and with near-identical levels of co-skewness and thus these quintile TCR portfolios control for differences in co-skewness.

	COSKEW 1	COSKEW 2	COSKEW 3	COSKEW 4	COSKEW 5	Average	t-statistic
Low TCR	0.9529	1.0570	1.1908	1.1213	0.9984	1.0641	4.53
2	1.1334	1.0804	1.1106	1.2592	1.2431	1.1654	5.73
3	1.2390	1.2363	1.3708	1.3286	1.3471	1.3044	6.48
4	1.1864	1.2301	1.2712	1.3697	1.3895	1.2894	6.13
High TCR	1.2308	1.2848	1.3908	1.4192	1.1792	1.3010	5.54
				Return Difference		0.2369	2.65
				Alpha Difference		0.2653	3.15

Table 7. Portfolios of NYSE Stocks Sorted by Tail Covariance Risk after Screening for Size, Price, Liquidity, Volatility, Winners, Losers, and Short-term Reversal

After screening for size, price, liquidity, volatility, momentum, and reversal, the equal-weighted quintile portfolios are formed every month from January 1963 to December 2006 by sorting the NYSE stocks based on tail covariance risk (TCR) calculated using the lower 10% tail of the daily return distribution over the past one year. To screen for size, all NYSE stocks on CRSP are sorted for each month by firm size to determine the NYSE decile breakpoints for market capitalization. Then, we exclude all NYSE stocks with market capitalizations that would place them in the smallest NYSE size decile. To screen for price, we exclude stocks whose price is less than \$5. To screen for liquidity, all NYSE stocks are sorted for each month by the ratio of absolute stock return to its dollar volume to determine the NYSE decile breakpoints for the illiquidity measure. Then, we exclude all NYSE stocks that belong to the smallest NYSE liquidity decile (or the largest NYSE illiquidity decile). To screen for total volatility, all NYSE stocks are sorted for each month by their total volatility to determine the NYSE decile breakpoints for the volatility measure. Then, we exclude all NYSE stocks that belong to the highest NYSE volatility decile. To screen for past 12-month winners and losers, all NYSE stocks are sorted for each month by their cumulative returns from month t-12 to month t-2 to determine the NYSE decile breakpoints for momentum (or winners and losers). Then, we exclude the NYSE winner (loser) stocks that belong to the highest (lowest) momentum decile. To screen for past 1-month winners and losers, all NYSE stocks are sorted for each month by their past 1-month return to determine the NYSE decile breakpoints for short-term reversal. Then, we exclude the past 1-month winner (loser) stocks that belong to the highest (lowest) reversal decile. To screen for short-term reversal, we estimate tail covariance risk using daily returns from month t-12 to month t-2, skipping month t-1. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) tail covariance risk. This table reports the average returns in monthly percentage terms. The row “High TCR – Low TCR” refers to the difference in monthly returns between portfolios 5 and 1. The row “FF-3 Alpha Difference” reports Jensen’s alpha with respect to the 3-factor Fama-French model. Newey-West adjusted t-statistics are reported in parentheses.

Average Raw and Risk-Adjusted Returns on Portfolios of Tail Covariance Risk after Screening for

	Size	Price	Illiquidity	Volatility*	Past 12-month Winners	Past 12-month Losers	Past 1-month Winners	Past 1-month Losers	Reversal
Low TCR	0.5500	0.5727	0.5641	0.6631	0.5246	0.7127	0.6732	0.4685	0.5864
2	0.6714	0.6772	0.6929	0.7406	0.6597	0.7650	0.7592	0.6612	0.7244
3	0.7722	0.7876	0.8145	0.8309	0.7455	0.8510	0.8683	0.7405	0.7717
4	0.8212	0.8127	0.8402	0.8887	0.7829	0.8880	0.9174	0.7605	0.8635
High TCR	0.8202	0.8077	0.8393	0.9339	0.7487	0.9307	0.9408	0.7391	0.8253
High TCR – Low TCR	0.2702	0.2350	0.2752	0.2707	0.2241	0.2180	0.2675	0.2706	0.2388
t-statistic	(2.98)	(2.56)	(3.00)	(3.19)	(2.43)	(2.38)	(2.88)	(2.83)	(2.67)
FF-3 Alpha Difference	0.2407	0.2043	0.2490	0.2492	0.2191	0.1724	0.2563	0.2510	0.2119
t-statistic	(2.94)	(2.50)	(3.02)	(3.24)	(2.62)	(2.13)	(3.03)	(2.91)	(2.51)

* After excluding stocks with the highest 10% volatility, the results in this table are different from those in Panel B of Table 5. This is because in Table 5 total volatility is computed as the standard deviation of daily returns over the past one year, whereas in this table, total volatility is calculated as the standard deviation of daily returns over the past one month.

Table 8. Portfolios of Tail Covariance Risk Including Additional Controls for Size/Book-to-Market and Momentum, Reversal, Liquidity, Volatility, and Co-Skewness

In column labeled “Size/BM adjusted,” we report the average returns in excess of the size and book-to-market matched benchmark portfolios similar to Daniel and Titman (1997). In the next five columns, we include additional controls for momentum (as measured by past 12-month returns), short-term reversal (as measured by past 1-month returns), illiquidity measure of Amihud (2002), total volatility, and co-skewness. For each additional control, we first perform a quintile sort based on the characteristic and then on tail covariance risk (TCR). Then, we average the TCR quintiles across the characteristic quintiles and report size and book-to-market matched returns within each TCR quintile. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) tail covariance risk. This table reports the average returns in monthly percentage terms. The row “High TCR – Low TCR” refers to the difference in monthly returns between portfolios 5 and 1. The row “FF-3 Alpha Difference” reports Jensen’s alpha with respect to the 3-factor Fama-French (1993) model. Newey-West (1987) adjusted t-statistics are reported in parentheses.

	Size/BM Adjusted	Including Additional Controls for				
		Momentum	Reversal	Illiquidity	Volatility	Co-skewness
Low TCR	-0.1613	-0.1457	-0.1698	-0.1505	-0.1298	-0.1527
2	-0.0333	-0.0218	-0.0410	-0.0385	-0.0553	-0.0429
3	0.0734	0.0519	0.0700	0.0582	0.0706	0.0806
4	0.0680	0.0633	0.0848	0.0687	0.0559	0.0614
High TCR	0.0530	0.0521	0.0558	0.0622	0.0589	0.0539
High TCR – Low TCR	0.2143	0.1978	0.2256	0.2127	0.1887	0.2066
t-statistic	(2.96)	(2.75)	(3.06)	(2.71)	(3.21)	(2.84)

Table 9. Firm-Level Cross-Sectional Regressions

This table presents firm-level Fama-MacBeth cross-sectional regression results for the sample period July 1962 to December 2006. The time-series average slope coefficients are reported in each row. Newey-West (1987) adjusted t-statistics are given in parentheses. Panel A reports the average slope coefficients from the univariate regressions of one-month ahead returns on downside risk (value at risk, expected shortfall) and tail covariance risk (TCR). Panel B presents results from the bivariate regressions of returns on TCR after controlling for downside risk. Panels C and D report results from the bivariate and multivariate regressions of returns on TCR after controlling for size, book-to-market, momentum, short-term reversal, illiquidity, co-skewness, total volatility, and idiosyncratic volatility.

Panel A. Univariate Regressions: Downside and Tail Risk Measures

TCR	1% VaR	5% VaR	10% VaR	1% ES	5% ES	10% ES
0.1509 (2.07)	-0.0164 (-0.42)	-0.0245 (-0.34)	-0.0372 (-0.39)	-0.0158 (-0.55)	-0.0265 (-0.51)	-0.0312 (-0.47)

Panel B. Bivariate Regressions: TCR with Downside Risk Measures

TCR	1% VaR	5% VaR	10% VaR	1% ES	5% ES	10% ES
0.1281 (2.30)	-0.0212 (-0.55)					
0.1390 (2.44)		-0.0312 (-0.43)				
0.1482 (2.54)			-0.0454 (-0.49)			
0.1399 (2.37)				-0.0207 (-0.73)		
0.1371 (2.46)					-0.0322 (-0.63)	
0.1395 (2.50)						-0.0377 (-0.58)

Table 9 (continued)

Panel C. Bivariate Regressions: TCR with Control Variables

TCR	SIZE	BM	MOM	REV	ILLIQ	COSKEW	TVOL	IVOL
0.1284	-0.1152							
(1.98)	(-2.43)							
0.1464		0.2393						
(2.07)		(3.37)						
0.1514			0.8624					
(2.14)			(3.84)					
0.1684				-0.0412				
(2.32)				(-8.02)				
0.1578					0.1131			
(2.22)					(2.45)			
0.1528						-0.0359		
(2.02)						(-0.73)		
0.1341							-0.0364	
(2.39)							(-0.15)	
0.1301								-0.0172
(2.16)								(-0.07)

Panel D. Multivariate Regressions: TCR with Control Variables

TCR	SIZE	BM	MOM	REV	ILLIQ	COSKEW	TVOL	IVOL
0.1555	-0.0926	0.1271	0.9263	-0.0519	0.0336	-0.0095		
(2.49)	(-2.34)	(2.09)	(3.71)	(-10.89)	(0.81)	(-0.24)		
0.1299	-0.1051	0.1312	1.0403	-0.0578	0.0370	0.0094	-0.2058	
(2.41)	(-3.26)	(2.24)	(4.46)	(-12.92)	(0.89)	(0.30)	(-0.91)	
0.1280	-0.1152	0.1298	1.0388	-0.0578	0.0360	0.0153		-0.2658
(2.22)	(-3.64)	(2.20)	(4.40)	(-12.93)	(0.86)	(0.47)		(-1.24)

