THE EFFECT OF MARKET STRUCTURE ON COUNTERPARTY RISK

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Abstract. Two network structures of derivative contracts are explored in a two-period model. The structures represent a bilaterally-cleared OTC market and a centrally-cleared market. An initial bankruptcy induces counterparties to trade with price impact. The two market structures yield different price impact and volatility. A large market-induced bankruptcy yields two destabilizing phenomena in bilateral markets: checkmate and hunting. Checkmate occurs when a counterparty cannot expect to prevent impending bankruptcy. Hunting occurs when counterparties push markets further than necessary, inducing further bankruptcies which may yield profits. The results suggest that bilateral OTC markets have larger externalities (distress volatility) which can be priced relative to centrally-cleared markets. This may suggest when and how to encourage markets to transition from bilateral OTC to central clearing. The results also suggest that limiting leverage ratios may reduce distress, that leverage limits may not vary linearly with capital, and that in times of distress coordination by market authorities has value. (*JEL: G01, G28, D49*)

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1. Introduction

Systemic crisis is a recurring theme in finance. In the past fifteen years, crises at Askin Funds, Long-Term Capital Management, Bear Stearns, and Lehman Brothers have spread beyond those firms to affect markets and firms worldwide. Of particular interest is the epidemic nature of these crises: trouble at one firm may spread to other firms. This leads to the idea of counterparty risk. In the strictest sense, this is the risk to an institution due to a counterparty defaulting on a contract with that institution. In a broad/systemic sense, counterparty risk includes how these situations affect the overall market.

Counterparty risk might seem to be a feature of over-the-counter (OTC) markets; however, many bonds are traded OTC without any worry of counterparty risk. Derivatives might seem to create counterparty risk since they are agreements between two parties; however, futures and options are derivatives and the CME and CBOE clearinghouses have never defaulted.

Refco was one of the largest US futures brokers when it went bankrupt in 2005. Such a bankruptcy might induce some anxiety among counterparties; however, the increase in volatility around Refco's bankruptcy was small. Similarly, the near-bankruptcy of Bear Stearns and bankruptcy of Lehman Brothers in 2007 caused chaos and interruptions in many OTC markets; however, the CME and CBOE continued to trade without interruption[1].

Many derivatives exchanges (including the CME and CBOE) use a centralized counterparty while swaps markets do not. That these markets continued trading while many OTC markets did not suggests counterparty risk

[1] Melamed (2009) gives the notional of CME contracts held at the time of these incidents as $761 billion (Bear) and $1.15 trillion (Lehman).

is a feature of market structure. Specifically: differences in network structure connecting counterparties may multiply or reduce the systemic effects of bankruptcy.

I use a two-period model to study the effects of a financial institution bankruptcy. The approach is general enough to be applied to any network structure and extended to multiple periods. Thus the approach could be used to design markets which most reduce the undesirable effects (externalities) studied here.

The model is applied to two market-based network structures which represent markets with and without a central counterparty. However, the model excludes the effects of adverse selection on price discovery. This allows us to study differences in volatility and follow-on bankruptcies due strictly to market structure.

For these network structures, the model suggests that a central counterparty stabilizes the market by reducing post-bankruptcy volatility and follow-on bankruptcies. The model also allows us to predict what these differences would be for a given initial bankruptcy. This can even be extended to characterize a market's susceptibility to follow-on bankruptcies (*i.e.* market "criticality").

2. The Two-Period Network Model

The economy we study has one risky underlying asset. Financial institutions (counterparties) trade OTC swaps on this asset. The only risk to the n counterparties is that changes to the risky asset price affect the worth of their swap contracts. The risk-free rate is assumed to be 0.

To simplify, we assume there is at most one contract between any two counterparties. (This is akin to netting contracts between counterparties.) The collection of counterparties and contracts defines a network: counterparties comprise the nodes of the network and contracts define its edges.

Counterparties are endowed with capital and risk aversion. For simplicity, all counterparties start with the same capital K and risk aversion λ. Contracts are endowed with signed sizes. These sizes may be constrained to give the network a certain topology.

Counterparties begin in equilibrium, holding their desired exposure to the risky asset, and will seek to return to this exposure if perturbed from it. Contracts are continually marked-to-market: All gains and losses are realized after each trade in the market. Thus any cashflow which exceeds the remaining capital results in bankruptcy.

What the exposures mean is important. While we examine only one risky asset, a more complex world would have multiple risky assets. In that situation, we could view these exposures in two ways. If they are overall exposures to risk, counterparties might not seek to return to equilibrium. If they are positions in only one risky instrument, however, counterparties might hold counterbalancing positions; in that case, returning to equilibrium might be more natural. However, neither of these cases diminishes the seriousness of a capital-depleting cashflow. Even a firm with a counterbalancing position would surely see the dangers in matching the timing of such large cashflow movements.

Each trade affects the market by moving prices. This is modeled by a price impact model with linear permanent component. A counterparty will therefore trade strategically given expected price impact, trading costs, and variance reduction. In the extreme, we can think of this as a *no-seppuku rule*: a counterparty will not rehedge completely if that rehedging would push it into bankruptcy

Trading occurs in a random sequence within a period. Price impact implies counterparties do not all rehedge at the same price. This leads to high and low prices "during" each period as well as an increase in market volatility. These price movements may cause some of the initially-living $n-1$ counterparties to go bankrupt.

All trading is done with a counterparty outside the network who has no concerns about risk or bankruptcy. One could appeal to an influx of liquidity providers in a crisis as justifying this approach. While that may be true, this assumption is unsatisfactory and a possible weakness of the model.

The model is a two-period model with trading in periods 1 and 2.

At time $t = 0$, bankruptcy of the n-th counterparty occurs; $n-1$ counterparties survive. Some or all of the living counterparties may have one of their contracts (connected edges) eliminated.

At time $t = 1$, each counterparty trades to maximize mean-variance utility given its desired exposure, the volatility of the risky asset, and expectations of others' actions. Follow-on bankruptcies may occur in period 1.

At time $t = 2$, all remaining exposures due to the bankruptcy are hedged with trading again occurring in a random order. While follow-on bankruptcies may occur in period 2, these may result from the constraints inherent to a two-period model.

2.1. **Notation.**

We introduce notation to express the dynamics of this model:

p_t = price of the risky asset at end of period t;

r_t = return of the risky asset in period t;

K = capital of each counterparty at start of period 0;

σ = volatility per period of the risky asset price; and,

q_{ij} = exposure of counterparty i via contract with counterparty $j \neq i$.

Worth noting is that the contract notation implies direction: $q_{ij} = -q_{ji}$.

The price impact model is linear in trade size and posits only permanent price impact. This is done in keeping with Huberman and Stanzl (2004) to ensure that the model is as simple as possible and arbitrage-free. If we assume price innovations are iid and have mean zero, we get the expected price for a trade by counterparty i (absent other trading) as a function of the quantity rehedged x_i:

$$E(p(x_i)) = p_0 + \underbrace{\pi x_i}_{\text{permanent}} . \tag{1}$$

The price p_1 at the end of period 1 is:

$$p_1 = p_0 + \sigma Z_1 + \pi \sum_{j=1}^{n-1} x_j \tag{2}$$

where $Z_{t\in\{1,2\}} \overset{iid}{\sim} (0, 1)$.

While the end-of-period price is unaffected by the ordering of trades within the period, bankruptcies do depend on the path of prices in a period.

2.2. **Network Topologies.** While any network topology could be studied, we consider two market-based extremes. A star network with n contracts represents a market with a central counterparty; a fully-connected network with $n(n-1)/2$ contracts represents a bilateral OTC market. Examples for four counterparties are shown in Figure 1.

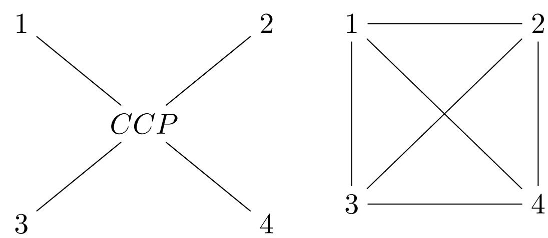

Figure 1. The two network structures considered shown for $n = 4$ counterparties: a star network connected via a central counterparty (left) and a fully-connected network (right).

The bankruptcy at $t = 0$ affects these two topologies in different ways. For the star (centrally-cleared) network, the initial bankruptcy only invalidates one contract with the central counter party (CCP). For a fully-connected (bilateral OTC) network, the initial bankruptcy invalidates contracts with $n - 1$ living counterparties.

3. Two-Period Analysis

With a few assumptions we can analyze the effect of the initial bankruptcy for these two network types. Contract sizes are assumed to have a zero mean and finite variance: $q_{ij} \overset{iid}{\sim} (0, \eta^2)$ for $i < j$ in the fully-connected network. The star network has contract sizes equal to net exposures in the fully-connected network. Counterparty i has net exposure of $Q_i = \sum_j q_{ij} = q_{i,CCP}$. Net exposures have expectation 0 and variance $(n-1)\eta^2$.

After the initial bankruptcy ($t = 1$), living counterparties in a star network have no unwanted exposure; only the central counterparty has unwanted exposure to the risky asset. For a fully-connected network, each living counterparty i has unwanted exposure of $-q_{in}$ reflecting the subtraction of the invalidated contract with the bankrupted counterparty.

Since the informational implications are different, we examine two cases: small and large initial bankruptcies.

3.1. **Small Bankruptcy.** We first consider the bankruptcy of a small financial firm. The small size suggests other counterparties have less information about the bankrupted firm. Thus bankruptcy may be due to market risk or idiosyncratic (management-related) factors. This is manifested in the capital at the start of period 1 being K for each living firm. The only information living counterparties have about counterparty n's market exposure is their individual contracts with the bankrupted.

3.1.1. *Star Network.* In a star network, none of the living counterparties has a broken contract. Therefore none of the living are directly affected by counterparty risk nor need they rehedge. Since the living counterparties' contracts incur no default, there is no early signal that any counterparty has gone bankrupt.

The central counterparty (CCP) takes on the bankrupted's exposure at time $t = 1$. If the CCP rehedges immediately, the permanent price impact will be for a $-Q_n$-sized trade: $\Delta p = -\pi Q_n$.

The CCP has advantages over other counterparties: It knows all counterparties' positions and trades; and, it would have immediate evidence of predatory trading by a living counterparty. This lets the CCP rehedge to reduce price impact and avoid causing further bankruptcies.

Also relevant (but not in the model) are the CCP's operating and contractual agreements. The CCP might have a contractual claim against counterparties if it goes bankrupt[2]. The CCP can also dictate margin and mark-to-market requirements to penalize large or risky positions. With these agreements,

[2]The CME clearinghouse uses such a structure.

living counterparties have even stronger incentives not to move the market against the CCP. Thus while these details are outside the model, they further justify the CCP's trading to reduce price impact and contagion[3].

Thus the CCP trades to maximize mean-variance utility:

$$U_{CCP}(x) = \underbrace{-\pi x^2}_{\substack{\text{period 1}\\ \text{impact}}} - \underbrace{\lambda \frac{\sigma^2}{2}[Q_n^2 + (Q_n + x)^2]}_{\text{variance penalty}} - \underbrace{\pi Q_n(Q_n + x)}_{\text{period 2 impact}} \tag{3}$$

This yields an optimal period 1 trade size of:

$$x_{CCP} = \frac{-(\pi + \lambda\sigma^2)Q_n}{2\pi + \lambda\sigma^2}. \tag{4}$$

Note that without price impact, the optimal policy is to rehedge completely in period 1 ($x_{CCP} = -Q_n$). As linear price impact π increases, the optimal trade tends toward an equal split of rehedging between periods 1 and 2. As volatility (σ) increases, the optimal trade tends to rehedge completely in period 1.

3.1.2. *Fully-Connected Network.* In a fully-connected network, bankruptcy by counterparty n invalidates $n - 1$ contracts. Each counterparty trades to rehedge their eliminated contract by the end of period 2. Since the bankruptcy is small, we ignore high and low prices triggering bankruptcies.

Each living counterparty chooses x_i to maximize Markowitz mean-variance utility. This is the same as minimizing price impact (affecting exposure and the traded amount) plus the penalized variance of the unhedged exposure

[3]Further research should examine when these incentives break down and members act against a CCP.

$(x - q_{in})$:

$$U_i(x) = \underbrace{-\pi x^2}_{\substack{\text{period 1}\\ \text{impact}}} - \underbrace{\lambda \frac{\sigma^2}{2}[q_{in}^2 + (x - q_{in})^2]}_{\text{variance penalty}} - \underbrace{\pi q_{in}(q_{in} - x)}_{\text{period 2 impact}} \tag{5}$$

When each counterparty knows nothing about the other counterparties' exposures, the optimal period 1 trade size is

$$x_i = \frac{(\pi + \lambda\sigma^2) q_{in}}{2\pi + \lambda\sigma^2}. \tag{6}$$

The sequencing of trades in periods 1 and 2 increases price volatility of the risky asset. The price volatility in periods 1 and 2 comes from equation (1) and the variation in contract sizes:

$$\mathrm{Var}(p_{t\in(0,1]}) = \sigma^2 + \underbrace{\pi^2 (n-1) \left(\frac{\pi + \lambda\sigma^2}{2\pi + \lambda\sigma^2} \right)^2 \eta^2}_{\text{added variance}}; \tag{7}$$

$$\mathrm{Var}(p_{t\in(1,2]}) = \sigma^2 + \overbrace{\pi^2 (n-1) \left(\frac{\pi}{2\pi + \lambda\sigma^2} \right)^2 \eta^2}. \tag{8}$$

3.2. **Large Market-Induced Bankruptcy.** While the bankruptcy of a large financial firm could come from mismanagement or fraud, we consider bankruptcies known or suspected to arise from market risk. Bankruptcies suspected of arising from market risk are also considered because strategic issues in exiting a large position cast doubt on legitimate claims of mismanagement. (Living counterparties may suspect market risk is to blame despite legitimate claims of mismanagement.)

3.2.1. *Initial Bankruptcy.* To study the effect of a market-induced bankruptcy, we impose an exogenous market return shock in period 0, r_0, such that bankruptcy occurs for the most exposed counterparty (labeled counterparty n for convenience). Mathematically, this means: $K + Q_n r_0 \leq 0$ where $Q_n r_0 < Q_i r_0$ for all $i < n$. For ease of exposition, we assume Q_n is positive and r_0 is negative.

While living counterparties may not know Q_n, they can infer it from the market return preceding counterparty n's bankruptcy[4]. With initial capital of K, the living counterparties estimate Q_n as $\hat{Q}_n = E(Q_n | K + Q_n r_0 \leq 0)$. Evaluating this expectation requires weak distributional assumptions and extreme value theory.

For iid normal Q_i's, we estimate the initial large bankruptcy exposure (see Appendix A.1) as:

$$\hat{Q}_n = E(Q_n | K + Q_n r_0 \leq 0) \tag{9}$$

$$= \frac{-K}{r_0} + \frac{\eta\sqrt{n-1}}{c_n(1 - e^{-e^{-c_n \kappa_1 - d_n}})} \sum_{k=1}^{\infty} \frac{(-1)^{k+1} e^{-k(c_n \kappa_1 + d_n)}}{k k!} \tag{10}$$

where $c_n = \frac{1}{\sqrt{2\log(n)}}$, $d_n = \sqrt{2\log(n)} - \frac{\log\log(n) + \log(16\tan^{-1}(1))}{2\sqrt{2\log(n)}}$, and κ_1 is the standardized maximum possible exposure of a living counterparty. If Q_n is not known, $\kappa_1 = \frac{-K}{r_0 \eta \sqrt{n-1}}$.

Note that $\hat{Q}_n$ is a function of r_0. Even if we know Q_n, analysis involving both Q_n and r_0 should use $\hat{Q}_n$ to be consistent with r_0 and to allow for comparisons between different network structures.

3.2.2. *Follow-On Bankruptcies.* Since the broken contract is large, trading in periods 1 and 2 might cause follow-on bankruptcies. All broken contracts must be (in expectation) rehedged, and counterparties' mark-to-market payments depend on price impact and their positions subject to that impact.

[4]Till (2006) infers Q_n, albeit using different methods.

Follow-on bankruptcies come from counterparties with exposures less than $\kappa_1 = \frac{-K}{r_0\eta\sqrt{n-1}}$ and greater than $\kappa_2(\hat{Q}_f)$ which depends on the market structure. (Details are in Appendix A.2.)

We then solve for the equilibrium follow-on exposure $\hat{Q}_f$:

$$\hat{Q}_f = E(Q_f | K + Q_n r_0 \leq 0) \tag{11}$$

$$= \frac{(n-1)^{3/2}\eta}{\Phi(\kappa_1)}(\phi(\kappa_2(\hat{Q}_f)) - \phi(\kappa_1)) \tag{12}$$

where Φ and ϕ are the standard normal cdf and pdf.

A natural question is how sensitive follow-on bankruptcies are to an initial bankruptcy. One measure of the market's fragility or susceptibility to distress is the *elasticity of distress exposure*, $\frac{\partial \log(\hat{Q}_f)}{\partial \log(\hat{Q}_n)}$; another measure is the *elasticity of distress pervasiveness*, $\frac{\partial \log(\hat{b})}{\partial \log(\hat{Q}_n)}$.

A valid criticism of these elasticities is that counterparties going bankrupt with very little exposure would require a very large drop in the price of the risky asset — an unlikely event. To account for this, we can look at elasticities weighted by the likelihood of such a precipitating r_0. Thus we would examine the likelihood-weighted elasticity of distress exposure to see what size of initial bankruptcy is of greatest likely concern.

3.2.3. *Star Network.* For a star network, only the CCP holds a broken contract. Thus only the CCP learns immediately of default. While such a default would become public knowledge quickly, the CCP also sees predatory trading immediately and can punish it.

Since only the CCP must rehedge, we can ignore low prices due to rehedging volatility. The additional unwanted exposure incurred by all counterparties due to follow-on bankruptcies, $\hat{Q}_f$, then results from counterparties with exposures between κ_1 and $\kappa_2(\hat{Q}_f)$ where

$$\kappa_2(\hat{Q}_f) = \frac{-Kp_0}{\eta\sqrt{n-1}(p_0 r_0 - \pi(\hat{Q}_n + \hat{Q}_f))}. \tag{13}$$

Since there are no worries about low prices and the CCP trades a fraction of the total trade $\nu = \frac{\pi+\lambda\sigma^2}{2\pi+\lambda\sigma^2} \in [0,1]$ in period 1, we may ignore the period 1 versus 2 distinction.

3.2.4. *Fully-Connected Network.* In a fully-connected network, each living counterparty immediately detects default. Thus each living counterparty must trade to rehedge. This not only pushes the market further; the variation in rehedging trades also increases the volatility of the risky asset. That volatility also creates price extremes that are likely to be greater than the trading range from the rehedging of a CCP. A comparison of possible price paths (Figure 2) shows the difference in range.

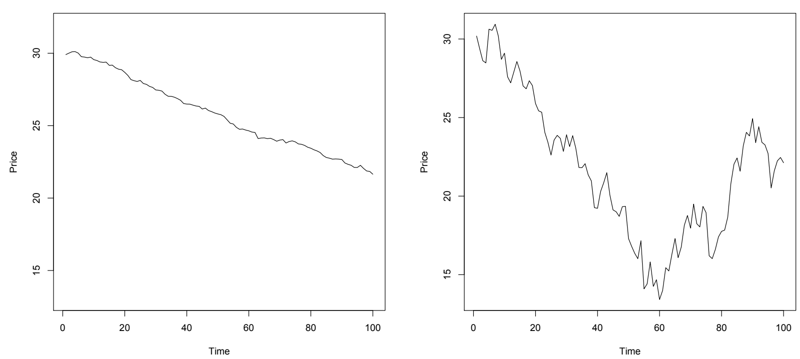

FIGURE 2. Possible price paths from \$30 to \$24 due to rehedging by a centralized counterparty (left) and OTC market participants (right).

We should expect more follow-on bankruptcies in fully-connected networks for three reasons. First, extreme prices beyond the CCP rehedging price range will result in more counterparties being (expectationally) unable to pay mark-to-market. Second, counterparties with a small exposure will be driven by unwanted variance and may hedge completely in period 1. Third, the fraction of hedging in period 1 ν enters κ_2 and thus the follow-on exposure equation (12) in a way that creates a Prisoner's Dilemma situation.

If the overall fraction of the total rehedge traded in period 1 is ν, the overall impact incurred for OTC markets will have three components. Two components, $\nu/2$ and $(1-\nu)/2$, are due to the random sequence of trading in periods 1 and 2 implying that each counterparty expects half of the other trades in that period to occur first. The third component, $(1-\nu)\nu$, is due to the position to be hedged in period 2 which incurs all impact of period 1 trading. The total impact is then $\frac{1}{2}+\nu-\nu^2$ which varies between 1/2 ($\nu=$ 0 or 1) and 3/4 ($\nu = 1/2$). If we assume risk aversion ($\lambda > 0$), we would hope to be able to restrict our attention to the sub-interval $\nu \in [\frac{1}{2}, 1]$.

Trading by multiple counterparties with differing exposures creates high and low prices. Since we consider rehedgers who (net) sell, we estimate the low price and see how that effects follow-on bankruptcies. The low price is driven by the running sum of trades. That is approximated by a Brownian bridge, a Brownian motion tied to end at $-(\hat{Q}_n + \hat{Q}_f)$ and can be handled by time inversion. (See Appendix A.3 for details.)

Thus the expected low quantity $\underline{S_{n-1}}$ of cumulated trades over trading periods 1 and 2:

$$\begin{aligned}(14)\qquad & E(\underline{S_{n-1}}|S_{n-1} = -(\hat{Q}_n + \hat{Q}_f)) = \\ & -(\hat{Q}_n + \hat{Q}_f) - \eta\sqrt{n-1}2\tan^{-1}(1)\phi\left(\frac{\hat{Q}_n + \hat{Q}_f}{\eta\sqrt{n-1}}\right),\end{aligned}$$

and $\kappa_2(\hat{Q}_f)$ for this market structure:

$$\kappa_2(\hat{Q}_f) = \frac{-Kp_0/[\eta\sqrt{n-1}]}{p_0 r_0 - \pi E(\underline{S_{n-1}}|S_{n-1} = -(\hat{Q}_n + \hat{Q}_f))}. \tag{15}$$

This allows us to determine $\hat{Q}_f$ from (12).

If we were to solve the $n-1$-player game, we would have to solve an optimization of each player's mean-variance utility given the others' trades. This optimization then yields the period 1 fraction of trade ν. Initial simulations suggest that heterogeneous rehedging needs — in particular the presence of long and short rehedgers — can yield values of ν in excess of 1 and even of the order $\nu = 1.5 - -2$.

3.2.5. *Destabilizing Phenomena.* Such multi-player games reveal two phenomena of interest. While one phenomenon is unfortunate, the other is undesirable and may greatly destabilize the market. While these phenomena are always possible, they may easily affect or become the equilibrium solution in the large market-induced bankruptcy case.

The first of these phenomena is *checkmate*: when any action (or inaction) by a counterparty cannot be expected to avoid bankruptcy. Checkmate is unfortunate for the ensnared counterparty; they cannot hedge in such a way as to expect to stay in business by the end of period 2. Since the checkmated counterparty cannot expect to stay in business, it may be in their interest to do nothing in hopes of randomly avoiding period 2 bankruptcy[5]. A necessary condition for checkmate is that closing the position would result in bankruptcy:

$$-\pi Q_n^2/p_0 + K < 0 \Leftrightarrow Q_n > \sqrt{Kp_0/\pi}. \tag{16}$$

Proposition 1 (Checkmate)**.** *In a fully-connected network, there is a $Q_n \in (0, \infty)$ such that for some $k < n$ and any finite x_k we expect bankruptcy in period 1: $E(\pi \sum_{j<n} x_j Q_k | \mathcal{F}_1) > K - Q_k r_0$.*

[5]A checkmated counterparty might even seek to become "Too Big to Fail," in effect taking the market hostage to seek more favorable liquidation terms.

What Proposition 1 means is that a large enough initial bankruptcy may result in an expected follow-on bankruptcy in period 1 despite the best efforts of the checkmated counterparty. This implies that policies restricting or taxing excess leverage might reduce distress (in this case, the number of market participants operating in checkmate). Note, however, that the leverage ratio implying checkmate varies with $1/\sqrt{K}$.

The second phenomenon is *hunting*: when other counterparties expect to profit by inducing follow-on bankruptcies. This means some counterparties act to push prices further in a particular direction to make money. Normally, this is not possible in a market with price impact as we assume here. The invalidation of contracts with the bankrupted counterparty, however, makes profits possible.

Proposition 2 (Hunting)**.** *In a fully-connected network of 3 or more counterparties, there is a $Q_n \in (0, \infty)$ such that for all exposures of Q_n or greater, bankruptcy has a positive expected payoff for two or more other counterparties.*

A sketch of the proof for $n = 3$ offers insight into how hunting works.

Proof. Assume counterparty 3 is checkmated. Let $Q_1, Q_2 < 0 < Q_3$ be such that $Q_1 + Q_2 = -Q_3$. Without loss of generality, we assume $q_{13} = Q_1$, *i.e.* counterparties 1 and 2 have no exposure to one another. For $\pi > 0$, counterparties 1 and 2 trade Q_1 and Q_2 to replicate their counterparty 3 exposure, causing losses to counterparty 3: $\pi(Q_1 + Q_2)Q_3/p_0 + K < 0$. Note that the market impact of trading by counterparties 1 and 2 is expected to bankrupt counterparty 3.

The positive expected profit from such a strategy is due to the random ordering of trading. The first "hunter" to trade receives a mark-to-market profit due to the second hunter's trading. The expected profit is $E(PL_i) = \pi\frac{Q_1+Q_2}{2p_0}Q_i$. If counterparty 3 goes bankrupt, all contracts with counterparty 3 are canceled. The exposure of the hunting counterparties reverts from $2Q_i$ back to Q_i.

If counterparty 3 does not go bankrupt, the hunting may continue or hunters may unwind their trades at no cost (since the market impact model is arbitrage-free). □

3.2.6. *A Separating Equilibrium?* We can also examine a fully-connected network from another perspective. Since we have multiple players, we can expect that there are multiple equilibria for how people would trade. One possible equilibrium is for rehedgers to separate themselves with buyers and sellers trading in different periods. We can see this by considering a large market drop which induces those who are long to sell.

Since the sellers are at risk of going bankrupt, they will sell in period 1 hoping to trade at the beginning of the period and not going bankrupt. This generates the typical "race for the exits." However, with only sellers in period 1, those whose sales are executed at the end of the period are likely to go bankrupt. Meanwhile, buyers wait to trade in period 2 at the lowest prices after sellers have bankrupted one another and pushed the market down.

In the extreme case, the buyers in period 2 would not even need to trade: bankruptcies in period 1 would annul their contracts and leave them flat. This possibility alone might support such an equilibrium.

With these sorts of dynamics, the period 2 trade is difficult to calculate. However, the result is the maximum distress in terms of volatility, low price, and follow-on bankruptcies. Thankfully, we can find the low price with only the net period 1 trade (which is easier to calculate).

We can think of the trades from buyers and sellers x_i as adding up to some total trade quantity $\bar{Q} < 0$. The question then is, what is the sum of all the sell trades? Mathematically, we want to know:

$$E(\sum_{i=1}^{n-1}[x_i]^- | \sum_{i=1}^{n-1} x_i = \bar{Q} < 0). \tag{17}$$

Unfortunately, this is a tricky question to answer. However, we can take an approximate answer by finding the expected sum of the absolute value of $n-1$ standard normal variables for a distribution with mean $\mu = -\bar{Q}/((n-1)^{3/2}\eta)$.

This is just (see Appendix A.5):

$$E(\sum_{i=1}^{n-1}[x_i]^- | E\sum_{i=1}^{n-1} x_i = \bar{Q} < 0). = \tag{18}$$

$$=(n-1)^{3/2}\eta\phi(-\mu) + \bar{Q}(1 - \Phi(-\mu)). \tag{19}$$

4. Examples

To get an idea of what different bankruptcies look like, we consider a market with $n = 10$ counterparties, each having capital K of \$1 million. Note that for a large bankruptcy these are very conservative assumptions: a market with counterparties having large exposures is likely to have more than 10 well-connected counterparties and, we generally expect trades to incur temporary impact — which would exacerbate the price extremes.

The risky asset has a price of \$50, daily price volatility of \$0.95 (equivalent to a 30% annual return volatility), and trades 5 million units daily. For the risky asset's price impact, we have that $\pi = 2 \times 10^{-6}$. Risk aversion is a daily $\lambda = 1 \times 10^{-6}$. These parameter values are in line with examples in Almgren and Chriss (2001).

4.1. **Small Bankruptcy Example.** To see the effect of a small bankruptcy, we consider a market where counterparties hold contracts with exposure standard deviations of \$100,000.

In this case, the period 1 price impact is −\$0.20 and the period 2 price impact is −\$0.17. The price volatility increases to \$1.30 in period 1 and \$1.11 in period 2 . These are equivalent to return volatility increasing from 30% annualized to 41% and 35% annualized.

4.2. **Large Bankruptcy Example.** To get an idea of what a large financial bankruptcy looks like, we consider an example for $n = 10$ counterparties, each having capital K of \$1 million. The counterparties hold contracts equivalent to OTC contracts with a standard deviation η of (also) \$1 million. We assume the CCP trades $\nu = 0.5$ of the expected rehedge in period 1; however, for the CCP market structure, the results are not sensitive to ν.

The risky asset has a price of \$50 and daily price volatility of \$0.95 (equivalent to a 30% annual volatility). For the risky asset's cash market liquidity, we have that $\pi = 2 \times 10^{-6}$ and $\lambda = 1 \times 10^{-6}$. These parameter values are in line with examples in Almgren and Chriss (2001).

This gives us the plot of follow-on bankruptcy exposure $\hat{Q}_f$ due to initial bankruptcy size $\hat{Q}_n$ shown in Figure 3.

We first consider the convex hull enclosing the central clearing market line C and the bilateral OTC market lines P and S. If buyers and sellers in the bilateral OTC market traded together and split their trades over periods

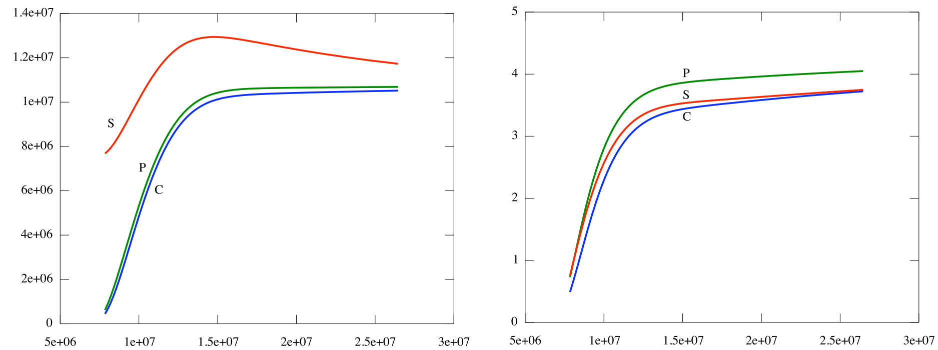

FIGURE 3. Follow-on bankruptcy exposure $\hat{Q}_f$ (left) and count $\hat{b}$ (right) versus initial bankruptcy size $\hat{Q}_n$ for $n = 10$ counterparties, each with capital $K = \$1$ million, holding equivalent contracts with sd(exposure) $= \$1$ million. Line S is for an OTC market separating equilibrium where sellers and buyers trade in different periods. Line P is for an OTC market with pooled buying and selling and overtrading by an amount typical from simulations ($1.75\times$). Line C is for a market with central clearing.

1 and 2 without trying to profit from one another (as for line P), their behavior would yield a line identical to C. However, simulations yielded an equilibrium where individuals behaved in a way that would yield line P. Thus we can think of the convex hull generated by lines C, P, and S as the *envelope of distress* defining the space of possible distress equilibria.

Since P results from one equilibrium and S results from another, we cannot ignore the difference between bilaterally-cleared (OTC) and centrally-cleared markets. Further, that C lies at the bottom of the envelope makes clear that distress is more likely and more destructive in markets without a central counterparty.

We can also note that the expected notional of contracts anulled by follow-on bankruptcies, $\hat{Q}_f$, is not monotonically increasing for the separating equilibrium S. This is because the total amount to be rehedged (*i.e.* including Q_n)

is monotonically increasing; but, for larger bankruptcies, the initial failure dominates the total annulled exposure. It also suggests that for mid-sized bankruptcies, the uncertainty about the effect may increase the expected amount to rehedge; however, for very large bankruptcies the uncertainty about the net rehedge decreases.

While the separating equilibrium S yields much greater follow-on bankruptcy exposure, it may well yield fewer expected follow-on bankruptcies than for pooled trading P. This suggests a "boiled frog" scenario: traders who panic may lose more in total; but, they may slightly increase their probability of survival. Traders who panic less (trading in periods 1 and 2; line S) incur mark-to-market losses from period 1 trading and incur half the losses (expectationally) in period 2. In other words, more traders may find themselves checkmated in period 2. This seeming paradox also lends support to the possibility of an equilibrium as for line S.

We can also see the elasticities of distress exposure and pervasiveness: the percentage changes in $\hat{Q}_f$ and $\hat{b}$ for a percentage change in $\hat{Q}_n$ (see Figure 4).

From a policy perspective, we should consider the absolute sensitivity of follow-on bankruptcy exposure $\hat{Q}_f$ to the initial bankruptcy $\hat{Q}_n$: *i.e.* $\partial\hat{Q}_f/\partial\hat{Q}_n$. Figure 5 shows that both OTC market lines (pooled buyers and sellers, line P; separated buyers and sellers, line S) and the central counterparty line (C) have intervals where they are greater than 1. In these intervals, a counterparty who could affect the initial bankruptcy size would generate rehedging (due to follow-on bankruptcies) in excess of their effect on $\hat{Q}_n$. This suggests that hunting is more than just an abstract or theoretical concept.

This plot suggests that distress (follow-on bankruptcies) are most likely to be destructive for initial bankruptcies by counterparties having exposure of

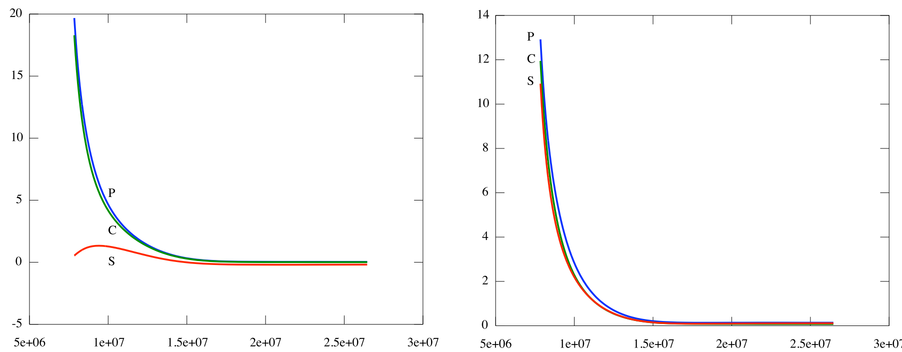

FIGURE 4. Elasticities of distress exposure $\hat{Q}_f$ (left) and distress pervasiveness $\hat{b}$ (right) with respect to an initial bankruptcy of size $\hat{Q}_n$. Plots are for $n = 10$ counterparties each with capital $K = \$1$ million and holding positions equivalent to OTC contracts with sd(exposure) $= \$1$ million. Plots are for a central counterparty, pooled OTC buyers and sellers, and separated buyers and sellers (lines C, P, and S).

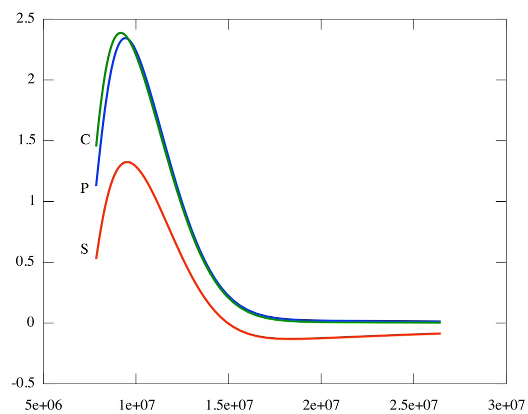

FIGURE 5. Sensitivity of distress exposure $\hat{Q}_f$ to an initial bankruptcy of size $\hat{Q}_n$, *i.e.* $\partial\hat{Q}_f/\partial\hat{Q}_n$. Plot is for $n = 10$ counterparties each with capital $K = \$1$ million and positions equivalent to OTC contracts with sd(exposure) $= \$1$ million. Lines C, P, and S correspond to a market with a central counterparty, pooled OTC buyers and sellers, and separated buyers and sellers.

\$11–\$14 million to one risky asset with a capital base of \$1 million. While few firms hold one asset, anecdotal accounts and conditional correlations from Pesaran and Pesaran (2010) suggest some banks in the credit crisis saw the correlation of their assets reach levels of 0.7 or more. A correlation of 0.7 implies we can explain $R^2 = 0.7^2 = 0.49$ (49%) of the variance. Thus one risky asset having \$11–\$14 million of exposure could be thought of as similarly destructive to assets of \$22–\$28 million which become correlated at a level of 0.7. This suggests leverage ratios of 22–28 may be modal levels at which financial distress occurs. That is especially troubling since most investment banks have leverage ratios at least this great.

This suggests that policies which restrict or tax leverage ratios beyond 22 (or so) may reduce distress and volatility externalities — even in markets with a central counterparty. Another possibility might be to auction permits to exceed some base leverage ratio and then allow financial companies to trade these permits (as is done with emissions permits). Aggressive investments banks with leverage ratios of 30–35 would thus be penalized relative to banks which earn similar profits on lower leverage.

5. Conclusion

We have shown that different network structures of exposures and mark-to-market payments can yield different market effects when an exogenous shock is introduced and trades have price impact. These effects are apart from any concerns about adverse selection and are due strictly to market structure.

The bankruptcy of a small non-financial firm increases the volatility of a risky asset held by the failed firm. Further, we can model this increased volatility as a function of exposures to the failed counterparty and market impact parameters. The bankruptcy of a large financial firm is shown to be more destructive: counterparties may be checkmated (unable to avoid expected bankruptcy), and counterparties may hunt the weak (seek to bankrupt counterparties) for positive expected profit. In the extreme case, buyers and sellers may separate when they trade, causing greatly increased follow-on bankruptcies, market swings, and volatility.

Both of these cases, small and large bankruptcies, have policy implications for market structures. In the large bankruptcy case, one example suggests that leverage ratios in the mid-20's may be modal for distress and may cause the most destruction relative to the size of the initial bankruptcy. This is especially troubling since many investment banks have leverage ratios in the 20's to low 30's.

These models suggest that distress and rehedging leads to increased volatility. That increased volatility is clearly an externality. Using an aggregate risk-aversion parameter, that externality may be priced to estimate the cost of differing market structures under stress.

The results also suggest there is a benefit to trading on centrally-cleared (and perhaps even exchange-traded) markets versus bilaterally-cleared OTC markets. However, bilateral markets have small startup costs and are thus important for financial innovation. If we could price the evolution of that flexibility, we might know more about when to offer incentives for trading to migrate from bilaterally-cleared to centrally-cleared markets. Given that the recently-passed Dodd-Frank financial reform bill encourages such transitions, these findings may help policymakers determine when is the best time to move markets to central clearing.

These results also suggest that monitoring exposures and capital levels is critical. The monitoring allows us to see how many players are in a network and how connected the network is. That lets us infer when checkmate and hunting are likely or possible.

Finally, hunting poses a Prisoner's Dilemma situation: counterparties collectively hurt themselves by rehedging to avoid losses from other rehedgers. This suggests a possible reason for concerted action by market authorities in periods of sever distress. One could view the Federal Reserve's forcing 15 consortium banks to collectively takeover LTCM as such an action.

More work should be done to study counterparty risk. Having all trading occur with an external counterparty outside of the network is unsatisfactory and various fixes could be explored. In particular, we should study the marginal effect of adding a market maker concerned about adverse selection when trading.

We might also consider analyzing other network structures, especially "small world" networks. We should explore the possibility of counterparties colluding to trade with one another and trigger mark-to-market profits. The game could be changed to end when follow-on bankruptcies cease. We could study what happens if counterparties' risk aversions change as follow-on bankruptcies occur. We could also allow new counterparties to enter the network if certain return or price levels are breached.

More thinking is also needed on the risk of any swap position. This paper does not allow counterparties to go bankrupt because of mark-to-market payments on their desired exposures (swap positions). However, the timing of cashflows matters; and, mark-to-market payments can induce bankruptcy, even for pure hedgers.

Appendix A. Derivations for Large Market-Induced Bankruptcy

A.1. Expected Exposure for First Bankruptcy. We assume the Q_i are normally distributed and use the Gumbel extreme value distribution for the maximum (Q_n) distribution. Since the Q_i's are iid we have

$$E(Q_n|K + Q_n r_0 \leq 0) = \eta\sqrt{n-1}\frac{\int_{\kappa_1}^{\infty} x c_n e^{-c_n x - d_n} e^{-e^{-c_n x - d_n}} dx}{\int_{\kappa_1}^{\infty} c_n e^{-c_n x - d_n} e^{-e^{-c_n x - d_n}} dx} \tag{20}$$

$$= \eta\sqrt{n-1}\frac{\int_{c_n\kappa_1 + d_n}^{\infty} \frac{u - d_n}{c_n} e^{-u} e^{-e^{-u}} du}{1 - e^{-e^{-c_n\kappa_1 - d_n}}} \tag{21}$$

$$= \frac{\eta\sqrt{n-1}}{c_n}\left(\frac{\int_{c_n\kappa_1 + d_n}^{\infty} u e^{-u} e^{-e^{-u}} du}{1 - e^{-e^{-c_n\kappa_1 - d_n}}} - d_n\right). \tag{22}$$

where $\kappa_1 = \frac{-K}{r_0\eta\sqrt{n-1}}$, $c_n = \frac{1}{\sqrt{2\log(n)}}$, and[6] $d_n = \sqrt{2\log(n)} - \frac{\log\log(n) + \log(16\tan^{-1}(1))}{2\sqrt{2\log(n)}}$.

The integral of u over the partial domain of the Gumbel distribution cannot be found in closed form. Therefore, we note that

$$\int_a^{\infty} u e^{-u} e^{-e^{-u}} du = \gamma - \int_{-\infty}^{a} u e^{-u} e^{-e^{-u}} du = \gamma + \int_{e^{-a}}^{\infty} \log(v) e^{-v} dv \tag{23}$$

$$= \gamma - \log(v)e^{-v}\,\Big]_{e^{-a}}^{\infty} + \int_{e^{-a}}^{\infty} \frac{e^{-v}}{v} dv \tag{24}$$

$$= \gamma - a e^{-e^{-a}} + E_1(e^{-a}) \tag{25}$$

$$= \gamma - a e^{-e^{-a}} + (-\gamma - \log(e^{-a}) - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-ka}}{k k!}) \tag{26}$$

$$= a - a e^{-e^{-a}} - \sum_{k=1}^{\infty} \frac{(-1)^k e^{-ka}}{k k!}. \tag{27}$$

[6]The arctangent function is used to preserve π for the permanent impact parameter. A single appearance of a transcendental number should not curtail a convenient consonance.

This then gives us

$$E(Q_n|K+Q_nr_0\le 0)=\frac{\eta\sqrt{n-1}}{c_n}\left(\frac{\int_{c_n\kappa_1+d_n}^{\infty} ue^{-u}e^{-e^{-u}}du}{1-e^{-e^{-c_n\kappa_1-d_n}}}-d_n\right) \tag{28}$$

$$=\frac{\eta\sqrt{n-1}}{c_n}\left(\frac{a-ae^{-e^{-a}}+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}e^{-ka}}{kk!}\,]_{c_n\kappa_1+d_n}}{1-e^{-e^{-c_n\kappa_1-d_n}}}-d_n\right) \tag{29}$$

$$=\eta\sqrt{n-1}\left(\kappa_1+\frac{\sum_{k=1}^{\infty}\frac{(-1)^{k+1}e^{-c_n\kappa_1-d_n}}{kk!}}{c_n(1-e^{-e^{-c_n\kappa_1-d_n}})}\right) \tag{30}$$

$$=\frac{-K}{r_0}+\frac{\eta\sqrt{n-1}}{c_n(1-e^{-e^{-c_n\kappa_1-d_n}})}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}e^{-k(c_n\kappa_1+d_n)}}{kk!}. \tag{31}$$

A.2. **Exposure to Follow-On Bankruptcies.** The number of follow-on bankruptcies b has expectation $\hat{b}=E(b|K+Q_nr_0\le 0)$ of:

$$\hat{b}=(n-1)\frac{\int_{\kappa_2}^{\kappa_1}\phi(z)dz}{\int_{-\infty}^{\kappa_1}\phi(z)dz}=(n-1)\left(1-\frac{\Phi(\kappa_2)}{\Phi(\kappa_1)}\right) \tag{32}$$

where $\kappa_2=\frac{-K/[\eta\sqrt{n-1}]}{r_0-\pi(\hat{Q}_n+\hat{Q}_f)(\frac{1}{2}+\nu-\nu^2)}$, and ϕ,Φ are the standard normal pdf and cdf.

Note that the bounds κ_1 and κ_2 are assumed to be of the same sign. This fails if the direction of aggregate trading is opposite that needed for aggregate rehedging. This situation is ignored since trading in that manner would be suboptimal.

We also have that the expected exposure of a bankrupted counterparty is

$$E(Q|K+Q_nr_0\le 0)=\frac{\eta\sqrt{n-1}\int_{\kappa_2}^{\kappa_1}z\phi(z)dz}{\int_{\kappa_2}^{\kappa_1}\phi(z)dz}. \tag{33}$$

The additional unwanted exposure, $\hat{Q}_f=E(Q_f|K+Q_nr_0\le 0)$, due to follow-on bankruptcies then follows from integration by parts and Wald's

Formula:

$$\hat{Q}_f = E(Q|K + Q_n r_0 \leq 0)E(b|K + Q_n r_0 \leq 0) \tag{34}$$

$$= \frac{\eta\sqrt{n-1}\int_{\kappa_2}^{\kappa_1} z\phi(z)dz}{\int_{\kappa_2}^{\kappa_1}\phi(z)dz}(n-1)\frac{\int_{\kappa_2}^{\kappa_1}\phi(z)dz}{\int_{-\infty}^{\kappa_1}\phi(z)dz} \tag{35}$$

$$= \frac{(n-1)^{3/2}\eta}{\Phi(\kappa_1)}\int_{\kappa_2}^{\kappa_1} z\phi(z)dz = \frac{(n-1)^{3/2}\eta}{\Phi(\kappa_1)}(\phi(\kappa_2) - \phi(\kappa_1)). \tag{36}$$

A.3. **Expectation of Minimum Price.** To find the maximum amount sold, we note that each trade is normally-distributed in size. Also, the order of trading is random, *i.e.* uniformly distributed across the $(n-1)!$ different permutations. Let x'_i be the i-th trade, or $x'_i = x_{\rho(i)}$ if ρ is a permutation operator. Then, we let $S_{n'} = \sum_{i=1}^{n'} x'_i$, $\underline{S_n} = \min_{n' \in \{1,\ldots,n\}} S_{n'}$, and $\overline{S_n} = \max_{n' \in \{1,\ldots,n\}} S_{n'}$.

Since the beginning and ending positions are tied down, we can model the sum of trades S_n as a Brownian bridge and use time inversion to handle our ending position. Starting from Karatzas and Shreve (1991), equation (4.3.40), we get

$$P(\overline{S_{n-1}} \geq m | S_{n-1} = \hat{Q}_n + \hat{Q}_f) = e^{-2\frac{m(m-(\hat{Q}_n+\hat{Q}_f))}{(n-1)\eta^2}}. \tag{37}$$

Integrating this gives us the expected exceedance of the high beyond an ending trade of $\hat{Q}_n + \hat{Q}_f$. (This is the opposite of what we are doing; but, the development here eases comparison with Karatzas and Shreve's formula.)

Thus we have that

$$E(\overline{S_{n-1}}|S_{n-1} = \hat{Q}_n + \hat{Q}_f) - (\hat{Q}_n + \hat{Q}_f) = \tag{38}$$

$$= \int_{\hat{Q}_n+\hat{Q}_f}^{\infty} e^{-2\frac{m(m-(\hat{Q}_n+\hat{Q}_f))}{(n-1)\eta^2}} dm \tag{39}$$

$$= \int_{\hat{Q}_n+\hat{Q}_f}^{\infty} e^{-2\frac{m^2-m(\hat{Q}_n+\hat{Q}_f)+(\hat{Q}_n+\hat{Q}_f)^2/4-(\hat{Q}_n+\hat{Q}_f)^2/4}{(n-1)\eta^2}} dm \tag{40}$$

$$= \eta\sqrt{n-1}e^{-\frac{1}{2}\left(\frac{\hat{Q}_n+\hat{Q}_f}{\eta\sqrt{n-1}}\right)^2} \int_{\hat{Q}_n+\hat{Q}_f}^{\infty} e^{-2\left(\frac{m-(\hat{Q}_n+\hat{Q}_f)}{\eta\sqrt{n-1}}\right)^2} dm \tag{41}$$

$$= \eta^2(n-1)e^{-\frac{1}{2}\left(\frac{\hat{Q}_n+\hat{Q}_f}{\eta\sqrt{n-1}}\right)^2} \int_0^{\infty} e^{-2v^2} dv. \tag{42}$$

This implies, via symmetry, that

$$\begin{aligned} &E(\underline{S_{n-1}}|S_{n-1} = -(\hat{Q}_n + \hat{Q}_f)) = \\ -(\hat{Q}_n + \hat{Q}_f) - (n-1)\eta^2 2\tan^{-1}(1)&\phi\left(\frac{\hat{Q}_n + \hat{Q}_f}{\eta\sqrt{n-1}}\right). \end{aligned} \tag{43}$$

A.4. **Derivatives of the Expected Utility Function.** We first note that

$$\frac{\partial \kappa_2}{\partial x_i} = \frac{K\pi}{(n-1)\eta(r_0 + \pi(x_i + \hat{y}_i))^2}, \quad \text{and} \tag{44}$$

$$\frac{\partial \kappa_2}{\partial \hat{y}_i} = \frac{K\pi}{(n-1)\eta(r_0 + \pi(x_i + \hat{y}_i))^2} \tag{45}$$

We also note that

$$\frac{\partial \hat{b}}{\partial x_i} = -(n-1)\frac{\phi(\kappa_2)}{\Phi(\kappa_1)}\frac{\partial \kappa_2}{\partial x_i} = \frac{-K\phi(\kappa_2)}{\eta\pi x_i^2}, \quad \text{and} \tag{46}$$

$$\frac{\partial \hat{b}}{\partial \hat{y}_i} = -(n-1)\frac{\phi(\kappa_2)}{\Phi(\kappa_1)}\frac{\partial \kappa_2}{\partial \hat{y}_i} = \frac{-K\phi(\kappa_2)}{\eta\pi \hat{y}_i^2}. \tag{47}$$

For mathematical ease, we assumed $Q_n > 0$ and $r_0 < 0$. Thus rehedgers must sell to recreate canceled positions. Therefore, the sign of the $\hat{b}$ derivatives makes sense: more selling will increase the expected number of follow-on bankruptcies.

We can now find derivatives for $\hat{Q}_f$:

$$\frac{\partial \hat{Q}_f}{\partial x_i} = -\frac{(n-1)^2\eta}{\Phi(\kappa_1)}\kappa_2\phi(\kappa_2)\frac{\partial \kappa_2}{\partial x_i} \tag{48}$$

$$= \frac{\pi K^2 \phi(\kappa_2)}{\eta\Phi(\kappa_1)(r_0 + \pi(x_i + \hat{y}_i))^3} \tag{49}$$

and

$$\frac{\partial \hat{Q}_f}{\partial \hat{y}_i} = -\frac{(n-1)^2\eta}{\Phi(\kappa_1)}\kappa_2\phi(\kappa_2)\frac{\partial \kappa_2}{\partial \hat{y}_i} \tag{50}$$

$$= \frac{K^2 \phi(\kappa_2)\pi}{\eta\Phi(\kappa_1)(r_0 + \pi(x_i + \hat{y}_i))^3}. \tag{51}$$

Since $r_0 < 0$, these derivatives are negative. Thus more selling will increase the total exposure cancelled by follow-on bankruptcies.

Finally, we can differentiate player i's expected utility function with respect to period 1 trade x_i:

$$\begin{aligned}
\frac{\partial \hat{U}_i}{\partial x_i} = &-\lambda\sigma^2\left(\frac{\hat{Q}_f}{n-\hat{b}-1} - q_{in} + x_i\right)\left(\frac{\frac{\partial \hat{Q}_f}{\partial x_i} + \frac{\hat{Q}_f}{n-\hat{b}-1}\frac{\partial \hat{b}}{\partial x_i}}{n-\hat{b}-1} + 1\right) \\
&-\pi\left(\frac{\hat{y}_i}{2} + 2x_i\right) \\
&-\frac{\pi}{2}\left(q_{in} + \hat{y}_i - \hat{Q}_n - \hat{Q}_f\frac{n-\hat{b}}{n-\hat{b}-1}\right)\left(\frac{\frac{\partial \hat{Q}_f}{\partial x_i} + \frac{\hat{Q}_f}{n-\hat{b}-1}\frac{\partial \hat{b}}{\partial x_i}}{n-\hat{b}-1} + 1\right) \\
&-\frac{\pi}{2}\left(-\frac{\partial \hat{Q}_f}{\partial x_i} - \frac{\frac{\partial \hat{Q}_f}{\partial x_i} - \frac{\hat{Q}_f}{n-\hat{b}-1}\frac{\partial \hat{b}}{\partial x_i}}{n-\hat{b}-1}\right)\left(\frac{\hat{Q}_f}{n-\hat{b}-1} - q_{in} + x_i\right).
\end{aligned} \tag{52}$$

Then we differentiate the expected other players' utility with respect to the expected period 1 net trade of others, $\hat{y}_i$:

(53)
$$\frac{\partial \hat{U}_{(i)}}{\partial \hat{y}_i} = -\lambda\sigma^2 \left(\hat{Q}_n + \hat{Q}_f \frac{n-\hat{b}-2}{n-\hat{b}-1} + q_{in} + \hat{y}_i\right) \cdot \left(\frac{\partial \hat{Q}_f}{\partial \hat{y}_i}\frac{n-\hat{b}-2}{n-\hat{b}-1} - \frac{\hat{Q}_f}{(n-\hat{b}-1)^2}\frac{\partial \hat{b}}{\partial \hat{y}_i} + 1\right) - \pi\left(\frac{x_i}{2} + 2\hat{y}_i\right) - \pi\left(-\frac{\partial \hat{Q}_f}{\partial \hat{y}_i}\frac{n-\hat{b}-\frac{3}{2}}{n-\hat{b}-1} + \frac{\hat{Q}_f}{2(n-\hat{b}-1)^2}\frac{\partial \hat{b}}{\partial \hat{y}_i}\right)\left(\hat{Q}_n + q_{in} + \hat{Q}_f\frac{n-\hat{b}-2}{n-\hat{b}-1} + \hat{y}_i\right) - \pi\left(\frac{x_i - q_{in}}{2} - \hat{Q}_n - \hat{Q}_f\frac{n-\hat{b}-\frac{3}{2}}{n-\hat{b}-1}\right)\left(\frac{\partial \hat{Q}_f}{\partial \hat{y}_i}\frac{n-\hat{b}-2}{n-\hat{b}-1} - \frac{\hat{Q}_f}{(n-\hat{b}-1)^2}\frac{\partial \hat{b}}{\partial \hat{y}_i} + 1\right).$$

A.5. **Expectation of One-Sided Trade Quantities.** We start with the idea of finding the expected sell trades after rehedging a large bankruptcy by a counterparty who was long the market. This can be thought of as

$$E\Big(\sum_{i=1}^{n-1}[x_i]^- \,\Big|\, \sum_{i=1}^{n-1} x_i = \bar{Q} < 0\Big) \tag{54}$$

where x_i is the amount traded by counterparty i in period 1 and $\bar{Q}$ is the amount which would be rehedged if buyer and seller both traded anticipating bankruptcies.

Instead of solving this, we substitute a similar problem and find the solution to

$$E\Big(\sum_{i=1}^{n-1}[x_i]^- \,\Big|\, E\sum_{i=1}^{n-1} x_i = \bar{Q} < 0\Big). \tag{55}$$

This is much easier if we let $\mu = -\frac{\bar{Q}}{(n-1)^{3/2}\eta}$:

$$E(\sum_{i=1}^{n-1}[x_i]^-|E\sum_{i=1}^{n-1}x_i = \bar{Q} < 0). = \tag{56}$$

$$=(n-1)\sqrt{n-1}\eta\int_0^\infty \frac{z}{\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2}}dz \tag{57}$$

$$=(n-1)^{3/2}\eta\int_{-\mu}^\infty \frac{w+\mu}{\sqrt{2\pi}}e^{-w^2/2}dw \tag{58}$$

$$=(n-1)^{3/2}\eta\left(\frac{1}{\sqrt{2\pi}}e^{-\mu^2/2}+\mu(1-\Phi(-\mu))\right). \tag{59}$$

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