Regimes of Index Out-Performance:

A Markov Switching Model of Index Dispersion

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Abstract

This paper investigates the case of time-variability in the performance of the cointegration-based index tracking, which is shown to produce consistent excess return during some time periods. The only information used to construct the trading strategy is the history of stock prices, and the cause of the index out-performance should be linked to their time-variability characteristics. We define a new measure of prices' cohesion within the market index called 'index dispersion'. The relationship between index dispersion and fund out-performance is found to depend on stock market regimes. A Markov switching model estimated for this relationship indicates the presence of two regimes having very different characteristics. The first regime, associated with more volatile market conditions and prevailing during the last few years, is responsible for the entire excess return generated from index tracking. In this regime, an increase in dispersion is followed by a relative gain from index tracking. The second regime, less volatile but with no significant out-performance of the index tracking strategy, is prevailing at the beginning of the data sample, and indeed during most of the 1990's. In this regime, it is a decrease in dispersion which is followed by a relative gain from index tracking. We relate these observations to the long-run equilibrium relationships identified by cointegration and show that the tracking portfolio disregards temporary deviations of stock prices from the equilibrium levels which occur in regime two, and tracks the index very accurately. However, when disequilibria in stock prices are no longer temporary, but instead represent transitions towards new equilibrium prices, the cointegration-based tracking portfolio generates consistent excess returns, and this is what is happening in regime one.

1. Introduction

The issue of time-variability in the performance of funds has recently attracted considerable academic interest. One particularly important question is whether the observed time-variability can provide additional insight on the factors influencing fund returns. Generally, hedge funds and mutual funds have been found to perform better in recession periods than in boom periods. This time-variability has been associated with informational asymmetries (Shin, 2002) and changes in the investment environment, according to the phase of the business cycle (Moskowitz, 2000; Kosowski, 2001). A separate line of research concerns trend following strategies, which have been found to generate returns similar to a lookback straddle paying the owner the difference between the highest and the lowest price of the underlying asset over the observation period (Fung and Hsieh, 1997 and 2001). Trend followers appear to perform best in extreme up or down markets, and less well during calm markets. Even without highlighting the cause of this behaviour, there is considerable interest from the investment community in the fact that these funds provide a partial hedge against general market conditions (Fung and Hsieh, 1997).

This paper investigates one particular case of time-variability in fund performance and identifies a highly significant leading indicator for it. Both the investment strategy and the leading indicator are generated through a purely statistical methodology. The fund is assumed to operate solely on the cointegration-based index tracking methodology introduced by Alexander (1999). This strategy is known to have some interesting features, such as the fact that the tracking portfolio comprising the same stocks as the market index produces a consistent excess return during some time periods (Alexander and Dimitriu, 2002). This is a rather counterintuitive result, as one would expect the most complete combination of stocks available, which is found to be cointegrated with the index, to produce always a stationary tracking error, even out of sample. Nevertheless, the pattern of the excess return exhibits a pronounced time-variability: periods of stationary excess return are alternating with periods during which the excess return is accumulated consistently. Moreover, the periods during which most of the excess return is produced appear to coincide with the main market crises during the sample period.

The only information used to construct the tracking strategy is the history of the stock prices, and therefore, the cause of the index out-performance should be linked to the time-variability characteristics of the stock prices in the system. We define a new measure of prices' cohesion within the market index called 'index dispersion'. The relationship between index dispersion and fund out-performance is found to depend on stock market regimes. A Markov switching model estimated for this relationship identifies two, very distinct regimes, with almost all out-performance being attributed to just one of these regimes.

Throughout the analysis we justify the conclusions drawn from real-world and simulated stock and index prices. Beginning with the simplest two-stock scenario, we illustrate the connection between stock prices, portfolio weights and index out-performance. The relationship between excess return from index tracking and stock price dispersion is then examined cross-sectionally, by simulating stock prices and controlling for their dispersion in the index, and in a time series context. Both methods document a significant linear relationship which, however, has a considerable time-variability in parameters. To address this issue, a Markov switching model for the excess return from index tracking is estimated using the real-world universe of the Dow Jones Industrial Average (DJIA). We find strong evidence of a latent state variable, which determines the form of the linear relationship between the excess return from index tracking and the stock prices dispersion.

Belonging to a very general class of time series models, which encompasses both non-linear and time-varying parameter models, the regime switching models provide a systematic approach to modelling multiple breaks and regime shifts in the data generating process.

Increasingly, regime shifts are considered to be governed by exogenous stochastic processes, rather than being singular, deterministic events. When a time series is subject to regime shifts, the parameters of the statistical model will be time varying, but in a regime-switching model the process will be time-invariant conditional on a state variable that indicates the regime prevailing at the time.

The importance of these models has long been accepted, and the pioneering work of Hamilton (1989) has given rise to a huge research literature (Hansen, 1992 and 1996; Kim, 1994; Filardo, 1994; Diebold, Lee and Weinbach, 1994; Garcia, 1998; Psaradakis and Sola, 1998). Hamilton (1989) provided the first formal statistical representation of the idea that economic recessions and expansions influence the behaviour of economic variables. He demonstrated that real output growth might follow one of two different auto-regressions, depending on whether the economy is expanding or contracting, with the shift between the two states generated by the outcome of an unobserved Markov chain. Subsequent research in macroeconomics and business cycles includes several Markov switching models of GNP (Hansen, 1992 and 1996), interest rates (Gray, 1996; Sola and Driffill, 1994) and foreign exchange rates (Engel and Hamilton, 1990).

In finance, the applications of Markov switching techniques have been many and very diverse: from modelling state dependent returns (Perez-Quiros and Timmermann, 2000) and volatility regimes (Hamilton and Lin, 1996), to option pricing (Aingworth, Das and Motwani, 2002), to detecting financial crises (Coe, 2002), bull and bear markets (Maheu and McCurdy, 2000) and periodically collapsing bubbles (Hall, Psaradakis and Sola, 1999), or to measuring mutual fund performance (Kosowski, 2001). Despite their limited forecasting abilities (Dacco and Satchell, 1988), Markov switching models have been successfully applied to constructing trading rules in equity markets (Hwang and Satchell, 1999), equity and bond markets (Brooks and Persand, 2001) and foreign exchange markets (Dueker and Neely, 2002).

In this paper we find that, when applied to modelling the excess return from index tracking, the Markov switching model indicates the presence of two regimes having very different characteristics. The first regime, associated with more volatile market conditions, is responsible for the entire excess return generated from index tracking. This regime occurs much more frequently during the last few years: since 1999, even though stock markets have been excessively volatile, the prevailing regime has been the one associated with positive excess return from cointegration based index tracking. The second regime, less volatile but with no significant out-performance of the index tracking strategy, is prevailing at the beginning of the data sample, and indeed during most of the 1990's.

Additionally, the explanatory variable, the lagged change in index dispersion, has a different effect on the excess return in the two regimes: in the first regime an increase in dispersion is followed by a relative gain from index tracking in the next period, while in the second regime it is a decrease in dispersion that is associated with relative gains from index tracking in the next period, although in the second regime such gains are only temporary. We relate these observations to the long-run equilibrium relationships identified by cointegration and show that the tracking portfolio disregards the temporary deviations of stock prices from the equilibrium levels which occur in regime two, and tracks the index very accurately. However, when disequilibria in stock prices are no longer temporary, but instead represent transitions towards new equilibrium prices, the cointegration-based tracking portfolio generates consistent excess returns, and this is what is happening in regime one.

The remainder of the paper is organised as follows: section two reviews the cointegration-based index tracking methodology, defines the index dispersion and motivates a possible relationship between dispersion and tracker fund performance; section three uses simulated indices with different dispersion characteristics to document the relationship between dispersion and index tracking out-performance; section four examines the real world relationship between excess return from index tracking and lagged dispersion in the Dow Jones Industrial Average (DJIA) and motivates the need for a Markov switching framework; section five introduces the Markov switching model of index tracking performance and makes statistical inferences on its relationship with index dispersion; section six constructs trading rules based on the Markov switching model that demonstrate its predictive power; and finally, section seven summarises and draws the main conclusions.

2. Cointegration and Index Dispersion

The rationale for constructing tracking portfolios based on a cointegration relationship with the market index rests on the following features of cointegration: the price difference between the index and the tracking portfolio (i.e. the tracking error) is, by construction, stationary; the stock weights, being based on a large amount of history, have an enhanced stability; and, there is a full use of the information comprised in level variables such as stock prices. Moreover, cointegration relationships between the market index and portfolios comprising all or only part of their stocks should be easy to find since market indexes, either equally weighted or capitalisation weighted, are just linear combinations of stock prices.

Following Alexander and Dimitriu (2002), we assume that the fund uses a natural logarithmic formulation[1] of the cointegration-based index tracking model. The basic model for a tracking portfolio comprising all the stocks included in the market index at a given moment is a cointegrating regression of the form:

[1] Hendry and Juselius (2000) show that if level variables are cointegrated, so will be their logarithms. The level variables are cointegrated by definition, since the current weighted index is a linear combination of the stock prices.

$$\ln(\text{index}_t) = c_1 + \sum_{k=1}^{n} c_{k+1} \ln(P_{k,t}) + \varepsilon_t \qquad (1)$$

where the index is reconstructed historically based on the current membership of the market index, and *n* is the total number of stocks included in the market index. The specification of the model in natural log variables has the advantage that, when taking the first difference, the expected returns on the tracking portfolio will equal the expected returns on the market index, provided that the tracking error is a stationary process.

We note that the application of ordinary least squares (OLS) to non-stationary dependent variables such as ln(index) is only valid in the special case of a cointegration relationship. The residuals in (1) are stationary if, and only if, ln(index) and the tracking portfolio are cointegrated. If the residuals from the above regressions are non-stationary, the OLS coefficient estimates will not be consistent and no further inference will be valid. Testing for cointegration is, therefore, essential in constructing cointegration optimal tracking portfolios. The Engle-Granger (1986) methodology for cointegration testing is particularly appealing in this respect for its intuitive and straightforward implementation. Moreover, the well-known limitations (small sample problems, asymmetry in treating the variables, at most one cointegration vector) are not effective in the construction of tracking portfolios. The estimation sample is typically set to at least three years of daily data, there is a strong economic background to treat the market index as the dependent variable, and identifying only one cointegration vector is sufficient for our purposes. Further to estimation, the OLS coefficients in model (1) are normalised to sum up to one, thus providing the composition of the tracking portfolio.

Alexander and Dimitriu (2002) present an exhaustive analysis of the performance characteristics of cointegration optimal tracking portfolios, together with various other statistical arbitrage strategies derived from them, for different model parameters. Such model parameters include the number of stocks in the tracking portfolios, the stock selection method, the spread between the benchmarks tracked and the calibration period. The out-of-sample performance of all fully funded and self-financing portfolios, in the DJIA stock universe, was measured based on a commonly used rebalancing method: every 10 trading days the optimal weights of the stocks are rebalanced based on the new OLS coefficients of the cointegration regression. For each re-balancing, the cointegration regression (1) is re-estimated over a fixed-length rolling calibration period, ranging from 3 to 5 years of daily data. The number of shares held in each stock is further determined by the previous portfolio value, the current stock prices and the stock weights. In between re-balancings, the portfolios are left unmanaged, i.e. the number of stocks is kept constant.

From all the combinations investigated in Alexander and Dimitriu (2002), the case of the tracking portfolio comprising all stocks in the market index displayed a very interesting feature, the simple index tracking producing a positive excess return in certain market conditions. This is a rather counterintuitive result, as one would expect the most complete combinations of stocks, very strongly cointegrated with the reconstructed index, to produce out of sample returns with zero mean. To investigate whether this result can be replicated in other equity markets, we have constructed random subsets of stocks in the FTSE100, CAC40 and SP100 universes. For each index we have set up 100 random portfolios comprising a fixed number of stocks (50 for FTSE, 25 for CAC and 80 for SP100) and determined an equal holding index for each portfolio, as a simple average of the stock prices. Each of the 300 indexes was tracked with a cointegration-optimal portfolio comprising all the stocks included in that particular index. Based on the rebalancing strategy detailed above, we have determined and reported in Table 1 the average annual excess return from index tracking over the period 1997 to 2001, together with its standard deviation. For comparison, we have also reported the excess return from index tracking in the real-world DJIA universe.

[*Insert Table 1 here*]

The first observation is that, for all four cases, DJIA, FTSE, CAC and SP100, there is a positive average excess return over the index tracked, when measured over the entire data sample. This result is highly relevant, showing that excess return can be obtained in different markets, with largely different numbers of stocks (from 25 in CAC to 80 in SP100). Regarding the time distribution of the excess returns for the simulated indices, the last years in the data sample are responsible for most of this excess return, as it is also the case with the DJIA index tracking. The time variability is less evident in the CAC case, and most evident in the case of FTSE simulated indexes for which the excess return in 2000 is almost 5%, while in case of SP100 simulated indexes, the largest excess return occurs during 2001, amounting to 4.6%. Given the consistency of this pattern, it is expected that the same type of out-performance occurs also for other stock markets.

Considering the above, we conclude that there is clear evidence of excess return from cointegration-based index tracking, and this out-performance has an obvious time-variability pattern. This provides us with the motivation to investigate further the pattern of the index tracking out-performance, restricting the analysis, for reasons of space, to the DJIA universe.

Using daily close prices for the thirty stocks in the DJIA as of 31-Dec-01 and a sample period from 01-Jan-90 to 31-Dec-01, we have estimated the performance of a fund applying model (1) with a rolling 3-year calibration period and 10-day recalibration/rebalancing frequency. The cumulative daily excess return (calculated as the difference between the return on the tracking portfolio and the return on the index) during the entire sample period is shown in Figure 1. A very noticeable feature is the time variability of the excess return, which is far from being uniformly accumulated throughout the data sample. Periods of stationary excess returns alternate with periods during which there is consistently positive excess return. Moreover, the periods during which most of the excess return is accumulated coincide with the main market crises during the sample period: the Asian crisis, the Russian crisis and the technology market crash.

[Insert Figure 1 here]

The question arising is what causes this excess return, and there is no obvious answer. Although the periods with excess return are associated, on a longer time horizon, with market downturns, on a daily basis there is no significant negative correlation with the market returns. Neither is there a significant positive correlation with the change in market volatility (the unconditional correlation coefficients estimated over the entire sample using daily data are very close to zero: 0.01 and –0.03 respectively). Moreover, these low correlation coefficients are not an artefact of the simple (equally weighted) measure of association, since also the conditional correlation estimates remain low throughout the entire data sample.[2]

Considering that neither market returns nor market volatility manage to explain the excess return from index tracking, another potential cause of the excess return from index tracking could be the 10-day no-rebalancing period. However, a portfolio constructed based on market weights and held constant for 10-day intervals produces a zero-mean stationary excess return in out of sample backtests. Therefore, the 10-day no-trading interval is not responsible for the excess return from index tracking.

[2] Exponentially weighted moving average (EWMA) correlation between the excess return from index tracking and the market returns generally remain between -0.2 and +0.2, although this does depend to some extent on the choice of smoothing constant. The same can be observed for the EWMA correlation between excess return and the first difference in market volatility: this too generally falls between -0.2 and +0.2.

Having eliminated these potential causes, it follows that the out-performance of the cointegration tracking portfolio must be connected to the portfolio weighting system and its relationship with stock price dynamics. In order to understand better the relation between the market index and the tracking portfolio weights based on the normalised cointegration regression coefficients estimated using model (1), we investigate the simplest case of an index comprising only 2 stocks. The index, I_t, is computed as the average of the two stock prices. For constructing a portfolio tracking I_t from the two stocks, according to model (1) we need the weight w such that

$$\ln(I_t) = w\ln(P_{1,t}) + (1-w)\ln(P_{2,t}) \qquad (2)$$

It follows that $w = (\ln(1+a) - \ln(2))/\ln(a)$ where $a = P_1 / P_2$. Therefore $w > 0.5$ if and only if $P_1 > P_2$, meaning that in (2), the stock with the higher price will also have a higher weight in the tracking portfolio. The difference between the tracking portfolio weights in model (2) and the market weights, $w^* = a/(1+a)$, will increase with the spread between the stock prices. The further away is a from unity, the more significant will be the over-weighting of the stock with the higher price in the tracking portfolio, and, also, the larger the dispersion between the two stock prices. Therefore, a significant difference between the stock index weights and tracking portfolio weights occurs when the dispersion of stock prices increases.

This clear-cut result motivates our study of *index dispersion*, i.e. the cross sectional standard deviation of the prices across their mean (which is the reconstructed index), defined as:

$$d_t = \sqrt{\sum_{k=1}^{n} ((P_{k,t} - I_t)/I_t)^2 / n} \qquad (3)$$

For computing the time series of index dispersion, all stock prices are rescaled to be equal to 100 at the beginning of the period, the series starting therefore from zero. In (6), the index is reconstructed as an equally weighted average of the stock prices, which represents just a scale adjustment to the DJIA case, based on the value of the latest index divisor.

In Figure 2 the bold line represents the time series of dispersion in the DJIA. After a steady increase, the dispersion indicator increased substantially at the beginning of the technology sector boom, due to the sharp increase in the price of technology stocks, and a relative decline in price of other sectors. The highest dispersion occurred at the beginning of 2000, but since then the dispersion has decreased, most obviously during the crash of the technology bubble. We note that index dispersion in most major equity markets (whether capitalisation[3] or equally weighted) follows a similar pattern.

3. Cross-sectional Analysis of the Excess Return - Simulation Results

In this section we use simulation to verify that different degrees of dispersion in stock indexes give rise to different excess returns from index tracking. To this end, we need to construct equity indices which are similar in all respects except for their dispersion. In a classical simulation framework (i.e. based on a stock price model such as, for example, a lognormal diffusion with Poisson jumps), there is no straightforward way to allow for different degrees of dispersion in different sets of stock prices, since the dispersion is not an explicit parameter

[3] The dispersion in a capitalisation weighted index is computed as:

$$d_t = \sqrt{\sum_{k=1}^{n} (((P_{k,t} * n_{k,t} / \sum_{k=1}^{n} P_{k,t} * n_{k,t}) - I_t)/I_t)^2 / n}$$

in the model. Generally, dispersion in a stock prices system can be obtained by using different drifts in the individual stock returns, but this would infringe the requirement of having the systems of stock prices similar in all respects, except for the dispersion.

Another solution is to use the actual DJIA system of stock prices as reference, and further reduce/increase its dispersion, by preserving the general pattern of stock prices and having exactly the same index for all the systems. To this end, at every point in time, the stocks are split into two groups by the prices' median. We finely adjust the next-period return, r, for each stock with price above the median. For a particular stock, the adjusted return, r* is given by:

- *reduced dispersion simulations* $$r^* = \begin{cases} (1-\alpha)r \text{ if } r > 0 \\ (1+\alpha)r \text{ if } r < 0 \end{cases}$$

- *increased dispersion simulations:* $$r^* = \begin{cases} (1-\alpha)r \text{ if } r < 0 \\ (1+\alpha)r \text{ if } r > 0 \end{cases}$$

Thus the high price stock group will move closer to the median in the reduced dispersion simulations and away from the median in the increased dispersion simulations. The level of α should be set reasonably low, and in our case α was set to 0.01. In order to keep the index value unaffected by these adjustments, the price of each stock below the median must be increased (respectively decreased) proportionally to the difference between the adjusted prices mean and the index, which is also the mean of the unadjusted prices. This difference is uniformly distributed between the stocks, according to their price. The result is three systems of stock prices: the historical prices, a simulated system with reduced dispersion and a simulated system with increased dispersion. All three systems have the same index value at every point in time, computed as the simple average of the stock prices. The individual stock returns have slightly different means and volatility in the three systems, but the differences are very small indeed.

[Insert Figure 2 here]

Using the DJIA stocks as reference, the time series of the dispersion in the three systems are compared in Figure 2. The differences in dispersion are significant and uniformly accumulated. Since the systems are similar in all other features, we should be able to quantify the impact of dispersion on the excess return from index tracking. For each system we construct cointegration-optimal tracking portfolios comprising all stocks, and rebalance them every 10 days as explained in section 2. The excess return from index tracking is determined in each system as the difference between the tracking portfolio returns and the index returns. The cumulative excess return from index tracking in the original DJIA framework, accumulated over the period Jan-92 to Dec-01, is 11.30%, while in the reduced, respectively increased dispersion system it is 11.53%, respectively 9.97%. Overall there appears to be a negative relationship between dispersion and the excess return from index tracking. However, if up to the end of 2000, the excess return from index tracking is negatively related to dispersion, after that point the relationship changes: the highest excess return (6.9%) is generated by the system with the highest dispersion, and the lowest excess return (3.1%) is generated in the system with the reduced dispersion. Such change in the sign provides us with additional motivation to investigate, in a time series context, the relationship between index dispersion and the cumulative excess return from index tracking.

4. A Basic Time Series Analysis

Both cumulative excess return and index dispersion prove to be I(1) variables.[4] Thus a basic stationary specification of their relationship will relate the excess return from index tracking to the change in index dispersion, including also the lagged excess return and some lagged changes in dispersion:

$$\text{xs\_return}_t = \alpha + \beta_1 * \text{xs\_return}_{t-1} + \beta_2 * \Delta\text{disp}_t + \beta_3 * \Delta\text{disp}_{t-1} + \beta_4 * \Delta\text{disp}_{t-2} + \varepsilon_t \qquad (7)$$

The simple regression estimation results based on the DJIA sample from Jan-92 to Dec-01 are presented in Table 2. Statistically significant coefficients are associated with the lag of the excess return and the first lag of the change in dispersion. The contemporaneous and the second lag of the change in dispersion are not statistically significant. The positive coefficient of the lagged excess return accounts for the autocorrelation in the excess return from index tracking. As indicated also by simulation analysis, there is a negative, significant relationship between the excess return from index tracking and the lagged change in dispersion. Thus following an increase in dispersion, there will be a relative loss in the tracking portfolio compared with the market. The fact that the excess return from index tracking is determined by the lagged change in dispersion rather than by a simultaneous variable will be of further use in constructing trading rules.

[Insert Table 2]

We also need to test the structural stability of this relationship, as we have noticed in the cross-sectional framework a potential change at the end of year 2000. To this end, we run a standard Chow structural break test, with the breakpoint set at various dates in the sample. The results are shown in Figure 3. According to these results, the null hypothesis of no-structural break is most significantly rejected on 16th October 2000. The two regressions estimated before and after this date give quite different results (Table 3). The main difference between them is the sign of the coefficient of the lagged dispersion. As displayed also in Figure 3 by the rolling window estimates of the slope coefficient, until October 2000 there is, almost always, a negative relationship between excess return from index tracking and the lagged change in dispersion, but after October 2000, the relationship between the two variables becomes positive. Additionally, when the impact of the change in dispersion is separated in the two samples, the lagged dependent variable becomes insignificant.

[*Insert Figure 3 and Table 3 here*]

Rationale for the Relationship between Index Dispersion and Tracking Performance

Provided that the weights in the tracking portfolio are based on a long-run equilibrium relationship identified by cointegration, the negative relationship between the excess return from index tracking and the lagged change in dispersion therefore has a strong rationale. The dispersion can be interpreted of a measure of (dis)equilibrium – when prices diverge from long-run equilibrium levels, the dispersion in the entire stock prices system increases.

To illustrate this, we use the example of a stock from the upper part of the index stock prices. Figure 4(a) shows a smooth line representing the long-run equilibrium price of this stock and a wavy line representing the actual price of this stock. If the price of the stock increases, its weight in the market index will also increase (remember that our reconstructed index is an equally weighted average of stock prices). However, its weight in the tracking portfolio,

[4] Considering the ADF statistics for the dispersion, respectively cumulative excess return, of –0.81 and 0.18, we cannot reject the null hypothesis of a unit root in the series. However, the ADF tests on the first difference of the series clearly reject the null hypothesis of unit root (ADF statistics of –22.11 and -23.11) at any standard significance level.

being based on a long history of prices, is not likely to react immediately to the increase in the stock price, which could be just noise from a long-run equilibrium perspective. Therefore, the tracking portfolio will be relatively under-weighted on this particular stock while its price is increasing, and it will realise relative losses compared to the market index, during a period when dispersion is increasing. However, when the price of this stock returns towards its long-run equilibrium level, and consequently the dispersion in the system decreases, the tracking portfolio will make a relative profit compared with the market index, because it is still under-weighted (relative to the index) on a stock whose price is declining.

[*Insert Figure 4 here*]

In this framework, the positive relationship between the excess return and the change in dispersion after October 2000 is rather puzzling. The only feasible explanation is that the cointegration relationship identifies new equilibrium prices, which are even further dispersed. This case is illustrated in Figure 4(b) where now the high price stock has a long-run equilibrium price well above the actual stock price shown by the wavy line. The tracking portfolio will realise a relative profit when the stock price is increasing because it is over-weighted (relative to the index) on the high-price stock. Similarly when the high price stock declines and the dispersion decreases, the tracking portfolio, which is relatively over-weighted in this stock, will make a relative loss. Thus, when cointegration identifies a new equilibrium, where the stock prices are even further dispersed, positive excess returns will be associated with increasing dispersion.

Returning to the Chow test results, in order to explain the significant change in the behaviour of the excess return from index tracking, we take a closer look at the markets during the period September-December 2000. We examine a three-month period rather than only October 16th because the exact date indicated by the tests as having the highest likelihood of a structural break can be an artefact of the estimation method used. This three-month period is the time of the second great fall in the Nasdaq composite index - index volatility reached 47.59% and the index fell 48.25%, i.e. another 745.83 points, having already fallen 425 points from March 2000. Therefore, it is reasonable to infer that October 2000 marked the end of the technology bubble.

However, up to now, we have only found that there can be two different specifications of the relationship between the excess return from index tracking and the lagged change in dispersion. There is an obvious time-variability in the parameters of the estimated regressions, which cannot be accounted for with simple tools like Chow structural break tests without inducing a significant degree of arbitrariness. There appear to be some grounds for a structural break in the relationship between the excess return and dispersion in October 2000, but this does not ensure that the break identified is unique. To address these issues, we propose a Markov switching modelling alternative.

5. A Markov Switching Model

To specify and make further inference on the time-variability pattern identified in Figure 1, we have estimated a Markov switching model for the excess return from index tracking. The model assumes the presence of a latent variable (state variable) which determines the form of linear relationship between the excess return from index tracking and the lagged dispersion in stock prices. The advantages of using a latent variable approach instead of a pre-defined indicator have been long documented. For example, when analysing business cycles, the Markov switching model produces estimates of the state conditional probabilities, which contain more precise information about the states that are driving the process than a simple binary indicator of the states, which is prone to significant measurement errors. The estimates of the conditional probability of each state allow more flexibility in modelling the switching process. An additional motivation for using a latent variable approach in this case, is the fact that there is no obvious indicator of the states of the process generating the excess return from index tracking.

For the Markov switching model of excess return from index tracking, the intercept, regression slope and the variance of the error terms are all assumed to be state-dependent. If we let s_t denote the latent state variable which can take one of K = 2 possible values (i.e. 1 or 2), then the regression model can be written as:

$$\mathbf{y_t} = \mathbf{z_t}'\boldsymbol{\beta}_{\mathbf{s_t}} + \boldsymbol{\varepsilon}_{\mathbf{s_t}} \tag{8}$$

where $\mathbf{y_t}$ is the (T x 1) vector of the excess return from index tracking; $\mathbf{z_t} = (\mathbf{1}\ \mathbf{x_t})$ is the (T x 2) matrix of explanatory variables, with $\mathbf{x_t}$ denoting the lagged change in the prices dispersion; $\boldsymbol{\beta}_{\mathbf{s_t}} = (\mu_{s_t}\ \gamma_{s_t})$ is the vector of state dependent regression coefficients; $\boldsymbol{\varepsilon}_{\mathbf{s_t}}$ is the vector of state dependent disturbances, assumed normal with state dependent variance $\sigma^2_{s_t}$

The transition probabilities for the two states are assumed to follow a first-order Markov chain and to be constant over time:

$$P\{s_t = j \mid s_{t-1} = i, s_{t-2} = l, \ldots\} = P\{s_t = j \mid s_{t-1} = i\} = p_{ij} \tag{9}$$

The matrix of transition probabilities can be written:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix} = p_{ij} \tag{10}$$

If we let ξ_t represent a Markov chain, i.e. a random (2x1) vector whose element j is 1 if $s_t = j$ and zero otherwise:

$$\boldsymbol{\xi}_\mathbf{t} = \begin{cases} (1,0)' & \text{when } s_t = 1 \\ (0,1)' & \text{when } s_t = 2 \end{cases} \tag{11}$$

then the conditional expectation of ξ_{t+1} given $s_t = i$ is given by:

$$E(\boldsymbol{\xi}_{\mathbf{t+1}} \mid s_t = i) = \begin{bmatrix} p_{i1} \\ p_{i2} \end{bmatrix} = \mathbf{P}\boldsymbol{\xi}_\mathbf{t} \tag{12}$$

The conditional densities of $\mathbf{y}_t$, assumed to be Gaussian, are collected in a 2x1 vector:

$$\boldsymbol{\eta}_t = (\eta_{1t}, \eta_{2t})$$

$$\boldsymbol{\eta}_t = \begin{cases} f(\mathbf{y_t} \mid s_t = 1, \mathbf{z_t}; \boldsymbol{\alpha}) \\ f(\mathbf{y_t} \mid s_t = 2, \mathbf{z_t}; \boldsymbol{\alpha}) \end{cases} = \begin{cases} \dfrac{1}{\sqrt{2\Pi}\sigma_1} \exp\left\{\dfrac{-(\mathbf{y_t} - \mathbf{z_t}'\boldsymbol{\beta}_1)^2}{2\sigma_1^{\,2}}\right\} \\ \dfrac{1}{\sqrt{2\Pi}\sigma_2} \exp\left\{\dfrac{-(\mathbf{y_t} - \mathbf{z_t}'\boldsymbol{\beta}_2)^2}{2\sigma_2^{\,2}}\right\} \end{cases} \tag{12}$$

where α is the vector of parameters characterising the conditional density.

The conditional state probabilities can be obtained recursively:

$$
\begin{aligned}
\hat{\boldsymbol{\xi}}_{\mathbf{t|t}} &= \frac{\hat{\boldsymbol{\xi}}_{\mathrm{t|t\text{-}1}} \otimes \boldsymbol{\eta}_{\mathrm{t}}}{\mathbf{1}'(\hat{\boldsymbol{\xi}}_{\mathrm{t|t\text{-}1}} \otimes \boldsymbol{\eta}_{\mathrm{t}})} \\
\hat{\boldsymbol{\xi}}_{\mathrm{t+1|t}} &= \mathbf{P}\hat{\boldsymbol{\xi}}_{\mathrm{t|t}}
\end{aligned}
\tag{13}
$$

where $\hat{\boldsymbol{\xi}}_{\mathbf{t|t}}$ represents the vector of conditional probabilities for each state estimated at time t, based on all the information available at time t, while $\hat{\boldsymbol{\xi}}_{\mathbf{t+1|t}}$ represents the forecast of the same conditional probabilities based on the information available at time t for time t+1. The symbol $\otimes$ denotes element-by-element multiplication.

The i[th] element of the product $\hat{\boldsymbol{\xi}}_{\mathrm{t|t\text{-}1}} \otimes \boldsymbol{\eta}_{\mathrm{t}}$ can be interpreted as the conditional joint distribution of $\mathbf{y_t}$ and $s_t = i$. The numerator in expression (13) represents the density of the observed vector $\mathbf{y_t}$ conditional on past observations.

Provided the assumptions made on the conditional density of the disturbances, the log likelihood function can be written as:

$$
L(\boldsymbol{\alpha}, \mathbf{P}) = \sum_{t=1}^{T} \log f(\mathbf{y_t} \mid \mathbf{z_t}; \boldsymbol{\alpha}; \mathbf{P}) = \sum_{t=1}^{T} \log \mathbf{1}'(\hat{\boldsymbol{\xi}}_{\mathrm{t|t\text{-}1}} \otimes \boldsymbol{\eta}_{\mathrm{t}}) \tag{14}
$$

This approach allows the estimation of two sets of coefficients for the regression and variance of the residual terms, together with a set of transition probabilities.

Considering the complexity of the log likelihood function and the relatively high number of parameters to be estimated, the selection of starting values is critical for the convergence of the likelihood estimation. To reduce the risk of data mining, we have not used any state-dependent priors as starting values. Instead, we have used the unconditional estimates of the regression coefficients and the standard error of the residual term. Additionally, we have arbitrarily set $\xi_{1|1}$ to (1 0). A number of restrictions needed to be imposed on the coefficient values, in order to ensure their consistency with model assumptions. The transition probabilities were restricted to be between 0 and 1, while a non-negativity constraint was imposed on the standard deviation of the residuals in both states.

In Markov switching models it is essential to ensure a sufficiently long data sample for correctly identifying the time-variability of parameters. The data sample covered 10 years of daily data from 1992 to 2002. In a correctly specified switching model, i.e. one in which the entire time-variability of the parameters is captured by the regime switching and within each regime the parameter estimates are time-invariant, the use of such a long data sample should not create any difficulties.

[*Insert Table 4 here*]

Table 4 reports the Markov switching model estimation over the entire data sample. The only coefficients statistically non-significant at 1% are the regression intercepts, for both states. A noteworthy difference between the two regimes concerns the coefficient of the lagged change in dispersion: in the first state, the coefficient is positive, while in the second state it is negative, thus the lagged change in dispersion having a different effect on the excess return in the two regimes. In the first regime an increase in the index dispersion is followed by a relative gain from index tracking in the next period, while in the second regime, a decrease in dispersion is the one associated with relative gains. Additionally, the standard deviation of the residuals is higher in the first regime, but as we show below, so is the excess return generated during this regime. Regarding the transition probabilities, the second regime appears to be more persistent than the first one: the probability of staying in regime two at time t+1 provided that at time t the process was in regime two is 0.98, while the probability of remaining in regime one once there is 0.88.

If we split the sample observations between the two regimes based on the criterion of estimated probability[5], we can determine the excess return associated to each regime. Based on this procedure, the number of observations in regime two is almost three times the number of observations in regime one. Also, the cumulative excess return generated during regime one turns out to be even higher than the excess return generated by the entire process, because the second regime generates a relative loss, even if not very significant.

Figure 5 shows the cumulative excess return from each regime: there is a consistent excess return produced in regime one, while the second regime produces a slightly negative excess return. Apart from higher returns, and higher volatility, regime one returns have a positive skewness (0.77) and higher excess kurtosis (2.99). Regime two, as well as having negative mean returns, also exhibits a negative skewness (-0.11), which indicates a higher probability of returns below the average, but relatively low excess kurtosis (0.11).

[Insert Figure 5]

Based on the same separation procedure as above, we observe that, as opposed to the tracking portfolio, the market index generated smaller returns in regime one than in regime two (the equivalent of 8.75% p.a. as opposed to 12.93% p.a.). The notable difference concerns, however, the volatility of these returns: regime one returns are associated with an annual index volatility of 19%, while the returns in regime two have only 13% annual index volatility. Therefore, the tracking over-performance occurs in periods with lower returns and higher volatility for the market.

The time distribution of the states is an important feature to investigate. From Figure 6, which plots the estimated probability of the first regime, it becomes clear that in the first half of the sample regime two is the one prevailing, while towards the end of the sample, regime one becomes predominant. Over the entire data sample, observations in regime one represent 25% of the total number of observations. However, this distribution is far from being time invariant, since in the first half of the data sample, regime one accounts for only 7% of the total number of observations, while in the last two years of the data sample, i.e. 2000 and 2001, regime one occurs 87% of time.

Our main conclusion is that the two regimes have very distinctive characteristics: regime one, which occurs less frequently, but is predominant during the last few years, is responsible for producing the entire excess return from index tracking. This regime occurs in more volatile market conditions and the over-performance follows an increase in the index dispersion. In the second regime there is a negative, but not significant, excess return from index tracking. Any positive excess return from index tracking in regime two occurs following a decrease of the index dispersion.

Considering the consistent excess return generated in regime one, and its relationship with the lagged change in dispersion, it follows that, in this regime, when the stock prices become more dispersed, the index tracking is in a relative profit position. This can occur if the tracking portfolio is over-weighted on stocks having higher than average prices which are further increasing, and/or under-weighted on stocks having lower than average prices which

[5] If the estimated conditional probability of regime one at time t is above 0.5, we say that the process was in regime one at time t. Alternatively, the process will be in regime two.

are further decreasing. If these prices, after diverging, would return towards their previous levels, then the initial relative profit of the tracking portfolio would be reversed, and there would not be any consistent excess return. However, since in this regime the observed excess return is consistent, it follows that after diverging, the prices are not returning to their previous levels. Instead, they are actually moving towards new equilibrium levels, levels that are pre-identified by the cointegration relationship and accounted for in the composition of the tracking portfolio. Therefore, the tracking portfolio is in a relative profit position during such transition periods, which, in regime one, are not likely to be reversed and consequently the profit to be eroded.

When there is an increase in dispersion in regime two, i.e. when the prices move away from equilibrium, the tracking portfolio is in a relative loss position compared to the market index. However, since there is no consistent loss on the tracking portfolio during regime two, it means that these stock price movements are only temporary disequilibria, and the cointegration is treating them accordingly. In regime two, the drifting of stock prices around their long-run equilibrium results in a stationary tracking error in the cointegration process.

Testing the null hypothesis of no-switching

In order to validate the above inferences about the two-state process driving the excess return from index tracking, one needs to test and reject the null hypothesis of no switching. Even if there is evidence that the excess return has different patterns in the two regimes, this does not imply that the asymmetries between the two states are also statistically significant.

Standard testing methods such as likelihood ratio tests are not applicable to Markov switching models due to the presence of nuisance parameters under the null hypothesis of linearity, or no switching. The presence of nuisance parameters gives the likelihood surface sufficient freedom so that one cannot reject the null hypothesis of no switching, despite the fact that the parameters are apparently significant.

A formal test of the Markov switching models against the linear alternative of no-switching, which is designed to produce valid inference, has been proposed by Hansen (1992, 1996). This method implies the evaluation of the log likelihood function for a grid of different values for the regression coefficients, standard deviation and the transition probabilities. Therefore, for each set of parameters, a constrained optimisation takes place. However, depending on the complexity of the log likelihood function and the number of parameters, this method can become computationally burdensome.

Following Hamilton (1996), we let $\boldsymbol{\alpha} = (\mu_1\text{-}\mu_2, \gamma_1\text{-}\gamma_2, \sigma_1\text{-}\sigma_2, p_{11}, p_{22})'$ denote the regime switching parameters of model (8) and $\boldsymbol{\lambda} = (\mu_1, \gamma_1, \sigma_1)'$ denote the remaining parameters, which are not state dependent. The conditional log likelihood function for the parameters will be written as $L_t(\boldsymbol{\alpha},\boldsymbol{\lambda}) = \log f(y_t|y_{t-1}, y_{t-2}, \ldots y_1; \boldsymbol{\alpha},\boldsymbol{\lambda})$.

The null hypothesis of no switching can be written as $\boldsymbol{\alpha} = \boldsymbol{\alpha_0}$, where $\boldsymbol{\alpha_0} = (0, 0, 1, 0)'$. To represent the alternative hypothesis, we have constructed a grid of 1,125 possible values for α, with A denoting the set comprising all values of α. For any α, $\hat{\boldsymbol{\lambda}}(\boldsymbol{\alpha})$ denotes the value of λ that maximises the likelihood taking α as given. Hamilton (1996) defines the time series of the difference between each constraint log-likelihood function for the grid of alternatives and the constraint log-likelihood function estimated for the null hypothesis as:

$$q_t(\alpha) = l_t[\alpha, \hat{\lambda}(\alpha)] - l_t[\alpha_0, \hat{\lambda}(\alpha_0)] \tag{15}$$

The likelihood ratio statistic is:

$$L\hat{R} = \max_{\alpha \in A} T\overline{q}(\alpha) \div \{\sum_{t=1}^{T} [q_t(\alpha) - \overline{q}(\alpha)]^2\}^{1/2} \quad (16)$$

If the null hypothesis is true, then, for large samples, the probability that the above statistic exceeds a critical value z is less than the probability that the following statistic exceeds the same value z:

$$\max_{\alpha \in A} (1 + M)^{-1/2} \sum_{k=0}^{M} \sum_{t=1}^{T} [q_t(\alpha) - \overline{q}(\alpha)] u_{t+k} \div \{\sum_{t=1}^{T} [q_t(\alpha) - \overline{q}(\alpha)]^2\}^{1/2} \quad (17)$$

Following Hamilton (1996), we have generated Hansen's statistic for M values of 0-4 and found that the null hypothesis is strongly rejected with a p-value of 0.0000. The estimated Hansen statistic is of 5.58, while the upper bound of the simulated distribution is 2.82.

An alternative approach to the Hansen statistic uses a classical log likelihood ratio test for estimating (a) the asymmetries in the conditional mean, assuming the existence of two states in the conditional volatility, and (b) the asymmetries in the conditional volatility, assuming the existence of two states in the conditional mean. Such test follows the standard chi-squared distribution. Table 5 reports the log likelihood estimates for the restricted and unrestricted models, the log likelihood ratios and the associated p values.

[*Insert Table 5 here*]

We have tested the following hypotheses: (1) the intercept and slope coefficients are not significantly different between the two states, and (2) the standard deviations of the residuals of the two states are not significantly different. As shown by the results in Table 5, both tests turned out to be statistically significant, and the null hypotheses were rejected. Therefore, we conclude that there is clear evidence of the fact that the asymmetries between the two regimes identified by the model are not only economically, but also statistically significant.

6. Trading Rules

In this section we examine two market neural strategies which attempt to exploit the regime dependent relationship between the index tracking out-performance and the stock prices dispersion. The construction of trading rules is facilitated by the fact that the Markov switching model is using the lag of the change in dispersion to explain excess return. Also, forecasts of the latent state conditional probability can be produced for a number of steps ahead by using the unconditional transition probabilities and the current estimate of the conditional probabilities of the latent states.

The portfolio generating the excess return from index tracking, P, is defined as the difference between the tracking portfolio holdings and the market holdings in each stock. Both trading rules assume active trading, with daily rebalancing according to a trading signal.

The first trading rule ensures that P is held only if there is a buy/hold signal from the Markov switching model. In the second strategy, P is held if there is a buy/hold signal, and is shorted otherwise. The 'buy/hold' signal occurs either after an increase in the dispersion, if the forecast of the conditional probability of the latent state indicates that the process is currently in regime one, or after a decrease in dispersion, if the forecast of the conditional probability indicates that the process is currently in regime two. As the excess return from index tracking is not correlated with the market returns, both strategies will inherit market neutral characteristics. Moreover, they are self-financed, as the sum of all stock weights in P is, by construction, zero.

For pure out-of-sample tests, we have extended our initial database up to Nov-02, and the trading rules were implemented for the period Dec-01 to Nov-02. In order to obtain the signal for a given date, we have used only the information available at the moment of the signal estimation (the sign of the lagged change in dispersion and the one-period ahead forecast of the conditional probability of the regimes).

The returns of the trading strategies are plotted in Figure 7. Over the 11-months testing period, the first trading rule produced a cumulative return of 3.9%, with an average annual volatility of 2.2%. This translates into an average annual information ratio of 1.89, again, for a self-financing strategy. The second trading rule produced over the same time interval a cumulative return of 9.3%, with a slightly higher average annual volatility, i.e. 3.2% p.a. The average annual information ratio for this strategy is of 3.15.

However, these results need to be interpreted with caution. First, 11 months is a rather short sample, and secondly, the very high profitability of the trading rules during the last part of the data sample can be the result of the predominance and persistence of regime two during this period. Therefore, to estimate the impact of these limitations, we have also performed in-sample tests, for the period Dec-91 to Dec-01. The results are very similar: over the 10-year testing period, the first trading rule produced a cumulative return of 51.5%, with an average annual volatility of 1.9%. This translates into an average annual information ratio of 2.65, again, for a self-financing strategy. The second trading rule produced over the same time interval a cumulative return of 91.9%, with a slightly higher average annual volatility, i.e. 2.5% p.a. The average annual information ratio for this strategy is of 3.56.

However, the trading rules, as they are designed, require daily rebalancing, which can result in significant transaction costs. In our analysis we have not accounted for potential transaction costs, as we only aimed to test the efficiency of the model forecasts with real trading rules rather than with statistical tools. However, the problem of potentially high transaction costs in trading rules based on Markov switching forecasts is not new and has been dealt with either by reducing the frequency of trades, or by imposing some filtering of the signals, when trades take place only if the signal exceeds a given threshold (Dueker and Neely, 2001).

7. Summary and Conclusions

The cointegration-based tracking over-performance has been documented in several major real-world and simulated stock market indexes. The excess return from index tracking displays time-variability characteristics which are uncorrelated to general market conditions, such as market returns or volatility. The aim of this paper was to identify and model the cause of cointegration-based tracking over-performance.

Using a simple two stock scenario, we have related the cointegration-optimal portfolio weights to the level of stock prices, suggesting a relationship between excess return from index tracking and the dispersion of stock prices within the index. We examined this relationship cross-sectionally, by simulating stock prices and controlling for their dispersion in the index, and in a time series context. Both methods documented a strong relationship between the cumulative excess return from index tracking and the index dispersion, but with significant time-variability in the parameters. To account for this, we have specified a Markov switching model for the excess return from index tracking, assuming the presence of a latent state variable which determines the form of linear relationship between the excess return and the lagged change in index dispersion.

The results of estimating this Markov switching model indicate the presence of two regimes having very distinctive characteristics: the first regime, associated with higher index volatility and which occurs more frequently towards the end of the data sample, is responsible for the entire excess return generated from index tracking, while in the second regime, associated with less index volatility and prevailing at the beginning of the data sample, the excess return is marginally negative. The lagged change in dispersion has a different effect on the excess return in the two regimes: in the first regime an increase in the index dispersion is followed by a relative gain from index tracking in the next period, while in the second regime, a decrease in dispersion is associated with relative gains from index tracking in the next period.

We have related these observations to the long-run equilibrium relationships identified by the cointegration regression and showed that, disregarding temporary disequilibrium in stock prices, the tracking portfolio produces a stationary tracking error in regime one. However, in regime two, when the disequilibrium is no longer temporary, and represents instead a transition towards new equilibrium prices, the cointegration-based portfolio consistently out-performs the market index.

The information revealed by the Markov switching model was exploited by constructing simple trading rules. Such rules managed to enhance significantly the characteristics of simple index tracking, producing steady returns with low volatility and average information ratios of 1.89 to 3.15 during 2002.

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Figure 1 Cumulative excess return from index tracking

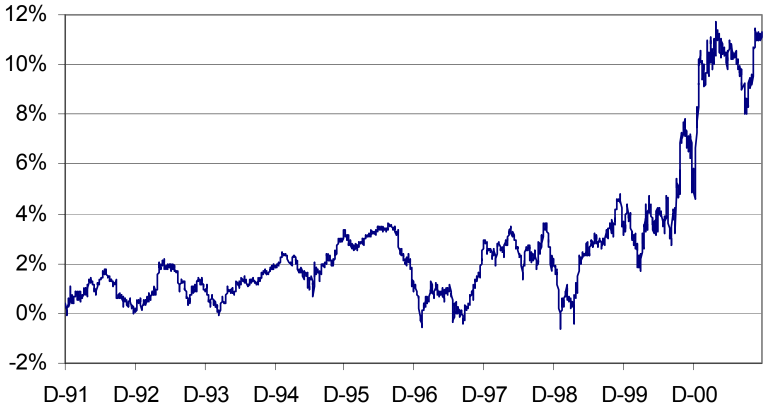


Figure 2 Price Dispersion in DJIA and Simulated Indices

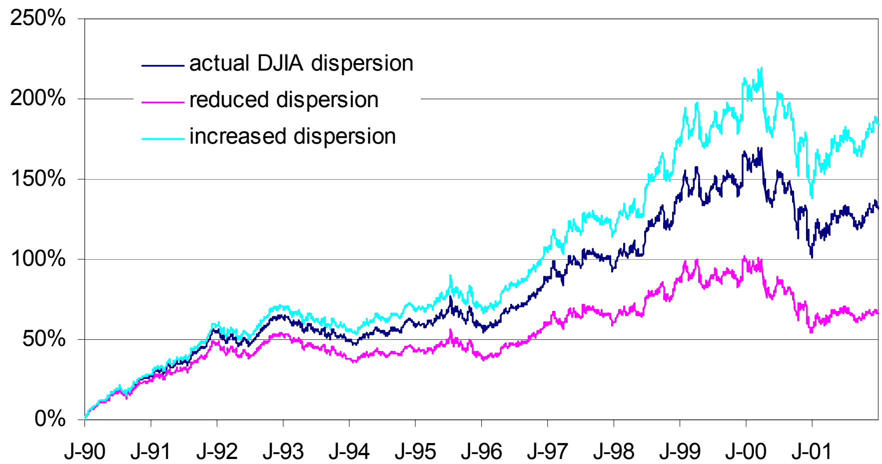


Figure 3 F-statistic Chow structural stability test and the coefficient of the lagged change in dispersion

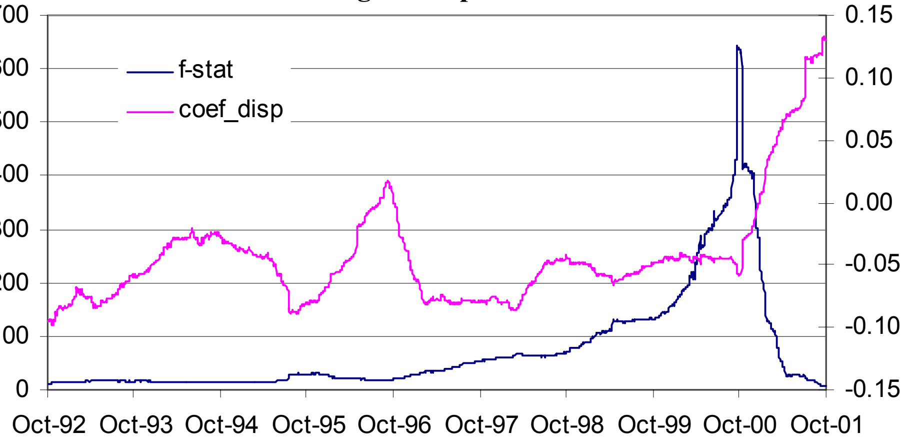


Figure 4(a) Stock prices movements in regime two

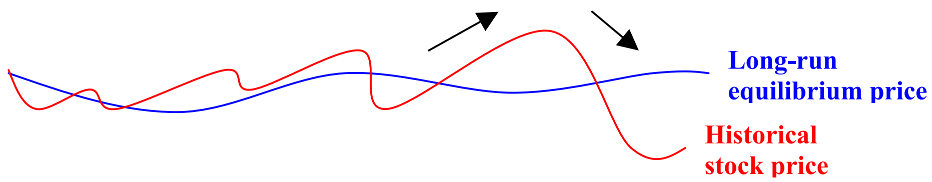


Figure 4(b) Stock prices movements in regime one

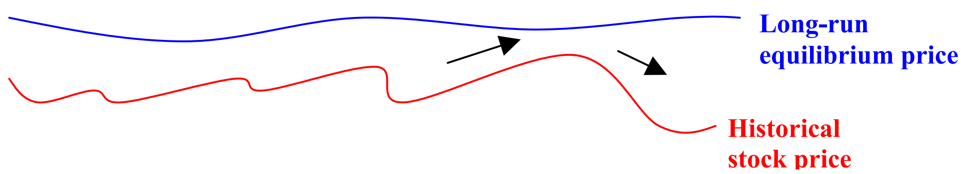
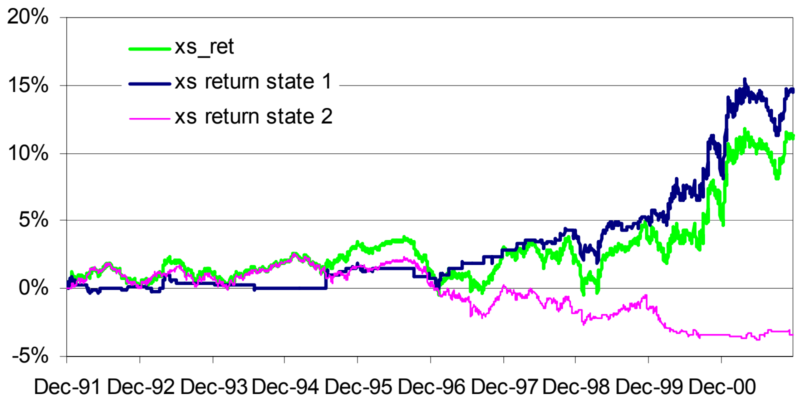


Figure 5 Regime conditional cumulative excess return from index tracking



	Regime 1	Regime 2
Mean	2.32E-04	-1.80E-05
Max	0.0154	0.0036
Min	-0.0094	-0.0038
Stdev	0.0028	0.0009
Skewness	0.77	-0.11
Kurtosis	5.99	3.84
No of obs	632	1888

Figure 6 Estimated probability of regime one

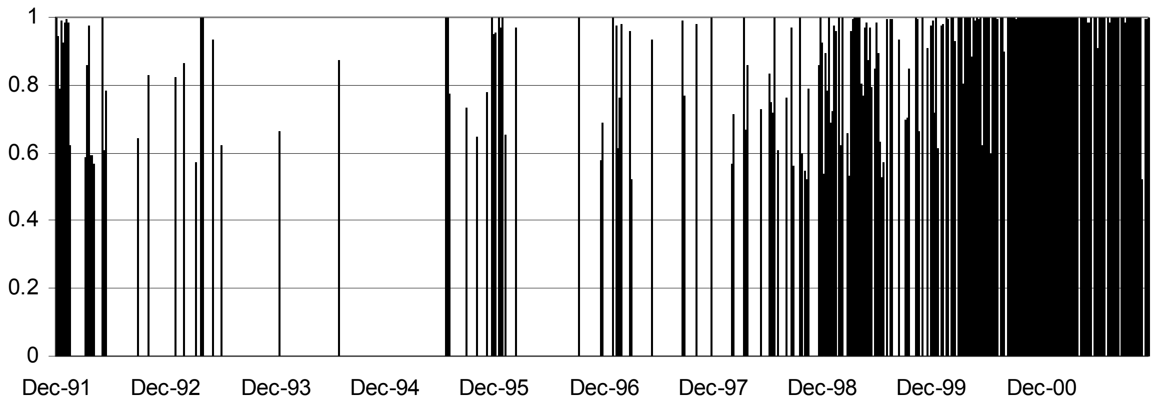


Figure 7 Cumulative returns produced by the trading rules

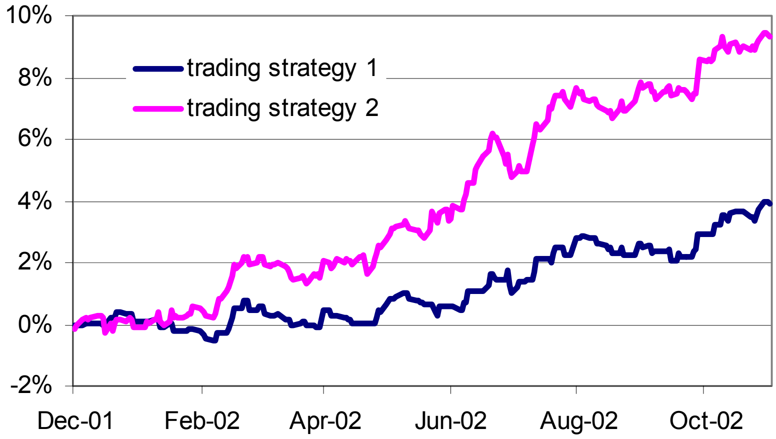


Table 1. A. Excess return from index tracking

		1997	1998	1999	2000	2001	overall
DJIA		1.71%	-1.00%	1.46%	2.08%	5.61%	7.82%
FTSE simulated	mean	-0.18%	-0.64%	0.92%	4.77%	0.34%	5.21%
indexes	stdev	0.0055	0.0101	0.0136	0.0137	0.0114	0.0227
CAC simulated	mean	1.19%	1.16%	-0.04%	1.22%	-0.13%	3.41%
indexes	stdev	0.0045	0.0045	0.0071	0.0123	0.0100	0.0232
SP100 simulated	mean	0.73%	-1.68%	-1.84%	1.00%	4.59%	2.79%
indexes	stdev	0.0033	0.0071	0.0125	0.0173	0.0139	0.0280

Table 1. B. Index returns

		1997	1998	1999	2000	2001
DJIA		20.41%	14.93%	22.49%	-6.37%	-7.37%
FTSE simulated	mean	19.86%	8.24%	8.35%	7.32%	-11.71%
CAC simulated	mean	8.21%	27.26%	38.44%	0.45%	-21.79%
SP100 simulated	mean	26%	19%	18%	1%	-13%

Table 2 Estimated coefficients of model (7)

	α	β_1	β_2	β_3	β_4
Coefficient	4.43E-05	0.075924	0.001347	-0.020175	0.005545
Standard error	3.25E-05	0.019892	0.002362	0.002361	0.002393
t-statistic	1.362186	3.816796	0.570434	-8.543316	2.317162
P-value	0.1733	0.0001	0.5684	0.0000	0.0206

Table 3 Stability test for model (7)

Chow Breakpoint Test: 2227 (October 16, 2000)			
F-statistic	638.1618	Probability	0.000000

Sample Jan-92 to Oct-00			
	α	β_1	β_3
Coefficient	3.92E-0.5	0.0025	-0.0567
Standard error	2.45E-0.5	0.0179	0.0018
t-statistic	1.60	0.14	-29.92
P-value	0.1094	0.8880	0.0000

Sample Oct-00 to Dec-01			
	α	β_1	β_2
Coefficient	1.30E-0.4	0.0599	0.1068
Standard error	1.33E-0.4	0.0435	0.0070
t-statistic	0.97	1.37	15.25
P-value	0.3313	0.1695	0.000

Table 4 Estimation output for model (8)

	μ_1	μ_2	γ_1	γ_2	σ_1	σ_2	p_{11}	p_{22}
Coefficient	2.68E-04	2.63E-05	0.0163	-0.056	0.0029	0.0006	0.88	0.98
Standard error	1.25E-04	1.48E-05	2.82E-03	1.14E-03	1.02E-05	1.58E-06	0.087	0.073
Z-statistic	2.15	1.78	5.77	-49.01	-286.01	-403.08	10.03	13.35
P-value	0.031	0.074	0.000	0.000	0.000	0.000	0.000	0.000

Table 5 Log likelihood ratio tests for identical mean and volatility

	Unrestricted log likelihood	Restricted log likelihood	LR statistic	P-value
H_0: $\mu_1 - \mu_2 = 0$, $\gamma_1 - \gamma_2 = 0$	13707.64	13666.05	82.24	0.0000
H_0: $\sigma_1 - \sigma_2 = 0$	13707.64	13057.31	1300.66	0.0000