

Pricing Derivative Securities Using Cross-Entropy: An Economic Analysis

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Abstract

This paper analyses two implied methods to determine the pricing function for derivatives when the market is incomplete. First, we consider the choice of an equivalent martingale measure with minimal cross-entropy relative to a given benchmark measure. We show that the choice of the numeraire has an impact on the resulting pricing function, but that there is no sound economic answer to the question which numeraire to choose. The ad-hoc choice of the numeraire introduces an element of arbitrariness into the pricing function, thus contradicting the motivation of this method as the least prejudiced way to choose the pricing operator. Second, we propose two new methods to select a pricing function: the choice of the stochastic discount factor (SDF) with minimal *extended* cross-entropy relative to a given benchmark SDF, and the choice of the Arrow-Debreu (AD) prices with minimal extended cross-entropy relative to some set of benchmark AD prices. We show that these two methods are equivalent in that they generate identical pricing functions. They avoid the dependence on the numeraire and replace it by the dependence on the benchmark pricing function. This benchmark pricing function, however, can be chosen based on economic considerations, in contrast to the arbitrary choice of the numeraire.

JEL: C61, G12, G13

Key Words: Equivalent Martingale Measure, Stochastic Discount Factor, Cross-Entropy, Implied Distributions, Option Pricing

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