

# Why is the Index Smile So Steep?

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## Abstract

There is empirical evidence that the implied volatility smile for index options is significantly steeper than the smile for individual options. We propose a simple model setup that is able to explain this difference. When modelling the index, an aggregation restriction has to be taken into account. The index level is a weighted sum of individual stock prices, so that the distribution of the index is completely determined by the joint distribution of the component stocks. The difference between the index smile and the smiles for individual stocks is then determined entirely by the dependence structure among the stocks. Changing this dependence structure changes the implied volatility curve for the index, whereas individual smiles would remain unchanged.

We illustrate our basic idea in the context of a jump-diffusion model. The dependence among stocks is captured by decomposing both the jump and the diffusion terms into common and idiosyncratic parts. Special attention is paid to the dependence in a crash. In this situation stocks are supposed to move together more than during normal market periods, which causes the difference between the implied volatilities of at-the-money and out-of-the-money puts to be much larger for the index than for individual stocks.

Although the smile is explained exclusively by the risk-neutral distribution the relation between this distribution and the data-generating process is also of interest. It is an important feature of our model that large downward movements are caused by jumps, which behave quite differently from diffusions under a change of measure. While for purely diffusion-based models second moments are preserved under the new measure this is not necessarily true for models with jump components. Here a change of measure may also alter the dependence structure of the stocks.

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