Liquidation Triggers and the Valuation of Equity and Debt

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Abstract

Many bankruptcy codes implicitly or explicitly contain net-worth covenants, which provide the firm's bondholders with the right to force reorganization or liquidation if the value of the firm falls below a certain threshold. In practice, however, default does not necessarily lead to immediate change of control or to liquidation of the firm's assets by its debtholders. To consider the impact of this on the valuation of corporate securities, we develop a model in which liquidation is driven by a state variable that accumulates with time and severity of distress. Recent or severe distress events may have greater impact on the liquidation trigger. Our model can be applied to a wide array of bankruptcy codes and jurisdictions.

Keywords: bankruptcy, liquidation trigger, debt pricing, Assets pricing

JEL codes: G12; G32; G33

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I. Introduction

Modeling of default is instrumental in determining the values of corporate securities, since market participants agree to finance a company on terms that reflect the possible outcomes, which may include the reorganization or liquidation of the company's assets. A common assumption in current pricing models of corporate securities is that defaults and financial distress lead to immediate liquidation of firm assets. This assumption is supported by the nature of net-worth covenants, which provide the firm's bondholders with the right to force reorganization or liquidation if the firm's value falls below a certain threshold. In practice, however, default does not necessarily lead to immediate change of control or to liquidation of the firm's assets by its debtholders. This fact is confirmed by numerous empirical studies showing that in the US, the average interval between the indication of financial distress and its resolution ranges from two to three years.

The finance literature contains different explanations for this gap between default and liquidation or reorganization. Hotchkiss (1995) explains it as resulting from inefficient design of bankruptcy laws, which favor firm continuation even at the expense of some violations of the lenders' contractual rights. On the other hand, Kahl (2002) claims the bankruptcy process to be efficient, since creditors and court accumulate information and learn about a firm's viability by observing its performance over time. Be it as it may, it is well documented by empirical research that firms who improve their operating performance when still in financial distress usually survive, while those who keep presenting poor operating performance eventually lose their independence in a liquidation process or in acquisition (e.g., Wruck (1990), White (1996), Kahl (2001) and Morrison (2003)).

The criteria for liquidation of a firm after the onset of financial distress vary substantially across countries and regimes. The UK insolvency law, for example, is characterized by strict enforcement of creditors' contractual rights, including the liquidation rights of secured creditors. In contrast, Chapter-11 of the US bankruptcy codes enables prolonged operation of firms in financial distress (e.g., Weiss and Wruck (1998)). Thus, a general model that can express the different relationships between default and liquidation is needed.

To capture the effects of the difference between default and liquidation or reorganization on the valuation of corporate securities, we present a general and adjustable valuation model in which liquidation is triggered according to the accumulated distress record of the firm. More precisely, we build on the framework of the structural modeling of debt (Merton (1974); Black and Cox (1976)), which allows us to determine the overall firm value, along with the values of equity and debt. However, while in the standard structural model default leads to immediate liquidation of the firm's assets and to assets distribution among the various claimants, our modeling approach allows us to draw a clear distinction between the notions of default and liquidation. [1]

The viability of a firm in our model is determined in each distress event according to the accumulated information about the firm's records of bankruptcy. This record is captured by a new state variable that accumulates the weighted distress events, which are defined as any period spent by the value of the firm's assets below a predetermined distress threshold. Liquidation is executed once this state variable exceeds a certain value. The effect of each distress event on the liquidation state variable, and thus on the liquidation decision, may depend on the severity of the distress period, on its length and on its distance

[1] In our model, liquidation may occur in or outside bankruptcy proceedings. We refer to liquidation and reorganization interchangeably.

from the present. Put differently, whenever the firm enters into default an additional "grace period", in which the debtholders cannot extract value, is granted. However, the length of this grace period is not constant, and the decreased tolerance of debtholders and court to repeated default events is reflected by a shorter grace period. The effect of each past distress event on the length of the grace period is derived ~~by~~ from its distance from the present and from the severity of the distress event. By applying this process, we can capture the following two common features of bankruptcy procedures:

[1] Recent distress events may have a greater effect on the decision to liquidate a firm's assets than old distress events. A distress event that occurred a long time ago may have a light effect on the liquidation decision, since the nature of the firm might have been completely changed during that time (management replacement, nature of activity etc.). In such a case, the willingness of court and debtholders to avoid immediate liquidation would be stronger than in a case where the distress event occurred recently.

[2] Severe distress events may have greater effect on the decision to liquidate a firm than mild distress events. A mild financial distress does not necessarily lead to immediate liquidation of the firm assets.

By controlling the parameters that affect the liquidation state variable, our model can be easily adjusted to multiple bankruptcy procedures and contractual agreements. At one extreme, we can exclude or significantly reduce the impact that distress episodes in the distant past have on the liquidation trigger. At the other extreme, we can weigh each distress observation equally.

With our approach, we can directly value different types of corporate securities and analyze complex capital structure scenarios for various bankruptcy procedures. We provide numerical examples to investigate how the length of the grace period, liquidation decay factors, the distress severity, the leverage ratios and the firm's asset volatility affect both asset prices and credit spreads. Although the model presented in this paper assumes a simple capital structure with one type of zero-coupon debt, it can be extended to cases in which the firm has issued senior and junior debt, convertible bonds or warrants.

The remainder of the paper is organized as follows: Section (II) reviews the previous theoretical research in relation to the topic of financial distress and debtholders' ability to extract value. Section (III) specifies our assumptions and describes the liquidation procedure. Section (IV) derives the valuation of equity and debt. Section (V) is devoted to explaining bankruptcy procedures of previous pricing models for corporate securities, and highlights the advantage of our model over these models. Section (VI) contains a numerical analysis of the main implications of our assets pricing model. Section (VII) is the conclusion.

II. Review of Literature

In their seminal papers, Black and Scholes (1973) and Merton (1974) offer the insight that equity value is identical to the price of a standard European call option on the total market value of the firm's assets, with an exercise price equal to the promised payment of corporate liabilities. Since the price of a standard call option is path-independent, default can only occur at maturity if the total value of the firm is less than the contractual payment due on the debt. The inadequacy of this basic method arises because it ignores the consequences of bankruptcy at all points in time except maturity. Many bankruptcy codes implicitly or explicitly contain positive net-worth agreements whereby creditors have the right to force the firm into bankruptcy whenever the value of assets falls below a predetermined threshold.

To cope with the possibility of early default prior to bond maturity, Black and Cox (1976, hereafter BC) developed a “first passage” model, in which default can occur at any time. The bondholders are protected by net worth covenants that provide the right to force bankruptcy or reorganization at the first instance where the market value of assets falls below a specified distress threshold. This distress threshold marks the trigger for the liquidation of firm assets in subsequent structural models as well (see Brennan and Schwartz (1978), Longstaff and Schwartz (1995), Leland (1994), Ericsson and Reneby (1998) and others).

According to the BC model, the relationship between a default event and the creditors’ ability to extract value is restricted to one specific regime, in which default leads to immediate liquidation of the firm’s assets and to assets distribution among the different claimants. In practice, the onset of financial distress does not necessarily lead to immediate change of control or liquidation of the firm’s assets by its debtholders. For example, empirical studies have shown that the average interval between the indication of financial distress and its resolution ranges from two to three years, for firms that renegotiate their claim under Chapter 11 of the U.S. Bankruptcy Code (See Franks and Torous (1989), Betker (1995), Gilson (1997) and Helwege (1999)).

To address the discrepancy between default event and the liquidation of the firm’s assets, recent works on capital structure and securities valuation suggest that liquidation occurs only when the value of the firm’s assets has reached the distress threshold and remained below this threshold for a prolonged period of time. Fan and Sundaresan (2000) suggest that when the firm is in default, borrowers stop making the contractual coupon and start servicing the debt strategically until the firm's asset value returns to a level above the distress threshold.

In this spirit, a valuation model developed by François and Morellec (2002, hereafter FM)~~,~~ assumes that the firm issues perpetual debt with contractual coupon payments, and that liquidation occurs when the value of its assets dips below the distress threshold and remains below that level for an interval exceeding a pre-determined 'grace' period. If the value of the firm's assets rebounds and rises above the distress threshold before the pre-determined grace period ends, the procedure is discontinued and the invisible "distress clock" is reset to zero. According to this approach, while debt is automatically serviced strategically after the value of the firm crosses the distress threshold, liquidation is declared only after the predetermined grace period has elapsed. As opposed to previous pricing models of the firm capital structure, FM's model can adjust itself to a variety of bankruptcy procedures by varying the pre-determined grace period

Moraux (2002) points out that according to FM, each time the firm value falls below threshold level an additional grace period is granted without reference to previous instances of insolvency, and thus FM implicitly assume that the bankruptcy procedure is automatically stopped at each time that the value of the firm's assets rebounds above the threshold level. Thus asset value could theoretically remain below the threshold level for the majority of the duration of debt, without the firm being liquidated. To overcome this disadvantage, Moraux (2002) proposes that liquidation is triggered when the *total* time that the firm's asset value spends under the distress threshold ("excursion time") exceeds a pre-determined grace period. Consequently, the previous "distress clock" is not reset to zero when the value of the firm's assets rebounds above the threshold. In this manner, the liquidation decision becomes highly path-dependent, since it accumulates the entire history of a firm's financial distress. Put differently, whenever the firm enters into financial distress the initial grace period is shortened by the total length of all past distress events. However, by simply accumulating excursion time, this model gives a company's history of financial distress an equal weight in triggering liquidation. This description of the bankruptcy process implicitly assumes that the procedure is never stopped, even if the value of the firm's assets has rebounded above the distress threshold for a prolonged consecutive period.

Both models (Moraux and FM) do not distinguish between cases in which the firm value is below but close to the distress threshold and cases in which the firm value falls far below the threshold level. The severity of the distress event has no influence on the decision to liquidate a firm. In contrast, our model takes into account the effect of the severity of the distress event on the decision to liquidate a firm's assets. Moreover, all the models mentioned above (Merton (1974), BC, FM, Moraux (2002), Fan and Sundaresan (2001)), as well as other models ((Leland (1994) Anderson and Sundaresan (1996)), are special cases of our general model. Its generality comes at the cost of being able to solve the model only numerically but not analytically.

III. Pricing Model for Corporate Securities with Adjustable distress Memory

In this section, we construct a general pricing model with adjustable distress memory to estimate the value of various corporate securities under a wide array of bankruptcy procedures. According to our model, liquidation is triggered when the weighted cumulative time that firm value has spent under the distress threshold exceeds a fixed exogenous amount of time. We rely on standard structural approach assumptions: assets are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate r. The instantaneous standard deviation of the firm's rate of return, σ, is constant; the value of the firm's assets, V_t, is independent of the firm's capital structure, and is well described under the risk neutral measure Q, by the following stochastic differential equation:

$$dV_t = (r - \delta)V_t dt + \sigma V_t dW_t \tag{1}$$

where W is a standard Brownian motion and δ is the firm's payout ratio.

We suppose that the firm has only equity and a single bond issue with a promised final payment of P and maturity T outstanding. The firm goes bankrupt in one of two ways: either the value of the firm's assets falls below a time dependent threshold level, denoted by K_t, at any time prior to debt maturity, or the value of the assets is less than some constant F at debt maturity.[2]

According to the BC model, the default event allows the creditor to force immediate liquidation through the bond's safety covenant. In our model, as in FM and Moraux (2002) models, distress and liquidation might diverge. We assume that liquidation is declared when the liquidation state variable (i.e., the "weighted cumulative distress time") exceeds a

[2] Usually this parameter is set to equal the principal of debt as in Merton (1974) and in subsequent models. However, if liquidation costs are incurred at maturity, as in Anderson and Sundaresan (1996), this may not accurately reflect the value of debt.

pre-determined grace period, denoted by d. In order to determine the liquidation state variable, we define the time-dependent threshold level K_t according to BC:

$$K_t = \omega F e^{-r(T-t)} \quad \text{where}: 0 \le \omega \le 1 \tag{2}$$

In this exponential form, the reorganization/liquidation value specified in the safety covenants is a fraction of the promised final payment. Under the BC assumption of immediate liquidation of the firm's assets at the threshold level, as ω goes to one the debt becomes riskless. Let us define the following random variable:

$$g_t^K = \sup\left\{ s \le t \middle| V_s = K_s \right\} \tag{3}$$

when g_t^K is the last time before t that the firm value crossed the then-prevailing threshold K_s. The state variable for the liquidation trigger is I_t^K, which is defined at date t as:

$$I_t^K = \int_0^{g_t^{\mathrm{K}}} e^{-\beta(t-s)} f(V_s) ds + \int_{g_t^{\mathrm{K}}}^{t} e^{-\gamma(t-s)} f(V_s) ds \tag{4}$$

where β is the decay factor for previous distress periods and γ is the decay factor of the last distress period. As β and γ increase, the impact of past distress events become decreasingly meaningful for the decision to liquidate the firm. For example, when $\beta = 3$ the effect of a distress event that occurred a year ago on the liquidation state variable is negligible (has a weight of 0.05). However, when $\beta = 0.5$, this effect is more than twelve times larger. These parameters enable us to adjust the survey period and the length of the bankruptcy procedures to the multiplicity of legal regimes and contractual agreements.

The function $f(V_t)$ defines the impact of the severity of the distress event on the liquidation state variable. We model $f(V_t)$ as follows:

$$f(V_t) = \begin{cases} e^{\alpha\left[\left(\frac{K_t - V_t}{K_t}\right) - \lambda\right]} & V_t \le K_t \\ 0 & V_t > K_t \end{cases} \tag{5}$$

where α determines the slope of the function and the parameter λ determines the point of intersection with the value of one. To ensure that the liquidation state variable would increase with the severity of the distress event we set $\alpha \ge 0$, and to ensure that the function would intersect the value of one we set $0 \le \lambda \le 1$.

Accordingly, the decision to liquidate a firm's assets does not depend solely on the duration of the distress events or on its continuity, as described in Moraux (2002) and FM (2002) respectively, but also on the distance of past distress events from the present and on the severity of distress, i.e., the degree to which firm value falls below the threshold. Liquidation occurs at the first time when the cumulative distress time extends beyond d. The liquidation time is denoted by θ^K, and is defined mathematically by:

$$\theta^K = \inf\left\{ t > 0 \,\middle|\, I_t^K \ge d, V_t \le K_t \right\} \tag{6}$$

In the particular case where α=0, the severity of the distress event has no impact on the liquidation decision and the liquidation state variable can be calculated by the expression:

$$I_t^K = \int_0^{g_t^K} e^{-\beta(t-s)} 1_{\{V_s \leq K_s\}} ds + \int_{g_t^K}^{t} e^{-\gamma(t-s)} 1_{\{V_s \leq K_s\}} ds \tag{7}$$

where $1_{\{V_S \leq K_B\}}$ is the characteristic function that receives the value of one if firm value is below the distress threshold level, and zero otherwise.

An alternative interpretation of (7) is to look at the remaining grace period as the difference between the initial grace period (*d*) and the weighted cumulative distress events up to the beginning of the current distress period. This interpretation of our general model is consistent with the more stylized model of Kahl (2002), in which reentering financial distress revealed the firm's lack of viability and led to a higher degree of liquidation.

In our setup, shareholders have a residual claim on the cash flows generated by the firm's assets unless the cumulative distress time reached the pre-determined grace period, *d*. The bondholders receive the debt's face value at maturity if liquidation has not previously occurred. In the event of liquidation, the debtholders would receive the remaining assets of the firm. The following examples are of special interest since they pertain to previous contributions of the literature. In all of these examples α=0 and λ=0.

Example 1: *When* $\beta \to +\infty$ *and* $\gamma = 0$*, the liquidation procedure occurs at the first point in time when the firm value process has spent consecutively more than the pre- specified* *grace period below the threshold* K_t*. Thus, when* $\beta \to +\infty$ *and* $\gamma = 0$*, we get the François and Morellec (2002) bankruptcy procedure.*

In this example, the liquidation state variable is accumulated only during the current distress period, where past distress periods do not influence the liquidation state variable. Put differently, the length of the grace period is always constant. At the one extreme, when $d = 0$, the FM model is similar, as a special case, to the standard modeling of default and liquidation [see Leland (1994)]. At the other extreme, when $d > (T - t)$, i.e. the grace period is longer than the maturity of debt, default never leads to liquidation before debt maturity and the FM (2002) model takes on, as a special case, the standard model for default and reorganization [see Anderson and Sundaresan (1996) or Fan and Sundaresan (2000)].

Example 2: *When* $\beta = 0$ *and* $\gamma = 0$*, liquidation occurs the first time the firm value spends a total time greater than the prespecified grace period below* K_t*. Thus, when* $\beta = 0$ *and* $\gamma = 0$*, we have the Moraux (2002) bankruptcy procedure.*

Under this parameterization, each distress period is weighted equally and each period has the same influence on I_t^K. At the one extreme, when $d = 0$, no extra survival time below the distress threshold is allowed, default leads to immediate liquidation of the firm's assets and we get, as a special case, the BC liquidation model. At the other extreme, when $d \geq (T - t)$, liquidation can occur only at debt maturity, and the model collapses to the basic structural approach introduced by Merton (1974) and extended by Galai and Masulis (1976) to equity valuation.

IV. The Valuation of Defaultable Bonds

In this section we evaluate the various corporate securities by considering the simple case of a firm with market value of assets V_t, which is financed by equity S_t, and one debt obligation, maturing at time T, with par value F, and market value B_t. The bond contract gives debtholders, under a *protective covenant*, the right to force liquidation at any time $t \in [0, \mathrm{T}]$, if asset value equals or falls lower than an exogenous threshold level K_t. However, the debtholders succeed to force liquidation only when the liquidation trigger I_t^K, exceeds the predetermined grace period d. At liquidation, debtholders would receive V_{θ^K} at time θ^K; equityholders would receive nothing. At debt maturity, T, assuming no early liquidation has been declared, equityholders would receive the maximum between zero and the difference between the firm's assets value V_T, and the promised face value F. Using the indicator function $1_{\{\theta^K > T\}}$ the equityholders payoff is given by the following function:

$$\mathrm{S}(V_T, T, I_T^K) = (V_T - P)^+ 1_{\{\theta^K > T\}} = \begin{cases} V_T - F & \text{if } V_T > F \text{ and } \theta^K > T \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

The value of the equityholders claim at any time prior to debt maturity $t \in [0, T]$, provided that default has not occurred by time t, is expressed by:

$$S(V_t, t, I_t^K) = e^{-r(T-t)} E_t^Q[(V_T - F)^+ 1_{\{\theta^K > T\}}] \tag{9}$$

where $E_t^Q[\cdot]$ denotes the conditional expectation under a risk neutral measure Q, given the information available at time t.

The value of the zero-coupon bond is composed of two sources of value: first, its value at maturity, assuming the firm is not prematurely liquidated, and second, its value if the firm is liquidated before debt maturity T, since the pre- determined grace period d was exceeded by the cumulative distress period. As noted by BC, although those two components are mutually exclusive, they are both possible outcomes. Accordingly, each of them contributes to the present value of both equity and debt.[3] The price of a zero coupon bond B, with maturity $T > t$, is given by the expected discounted cash flows under the risk-neutral probability measure:

$$B(V_t, t, I_t^K) = \mathrm{E}_t^Q[\min(V_T, F) e^{-r(T-t)} 1_{\{\theta^K > T\}}] + \mathrm{E}_t^Q[V_{\theta^K} e^{-r(\theta^K - t)} 1_{\{\theta^K \le T\}}] \tag{10}$$

Roughly speaking, the payoff at time $\theta_d \wedge T$ is given according to the no-liquidation scenarios (the left expression at the RHS of the equation), in which debtholders receive the

[3] Black and Cox (1976) decompose firm value into two additional components: the upper boundary of the security value if the firm is reorganized and the value of the payouts it will potentially receive.

minimum between the value of the firm's assets and the par value of debt, or alternatively, should early liquidation take place (the right expression at the RHS of the equation), debtholders receive the value of the firm's assets at that time.

The next step in evaluating the firm's capital structure is to calculate the zero-coupon yield spread. Practitioners typically quote corporate bond prices in terms of the spread of their yield-to-maturity over the riskless interest rate. The firm credit spread at time t, denoted by sp_t, is calculated as:

$$sp_t = \left[\ln\left(\frac{B_t}{P}\right) \Big/ -(T-t) \right] - r \qquad (11)$$

Given the above assumptions, we can derive the governing partial differential equations and boundary conditions that should be solved to value the firm's stocks and bonds as a function of the three state variables $V, t,$ and I.

The relevant form of the valuation equation for the stock, S, will be:

$$\frac{\partial S}{\partial t} + \frac{\sigma^2 V^2}{2}\frac{\partial^2 S}{\partial V^2} + (r-\delta)V\frac{\partial S}{\partial V} - rS + \frac{\partial S}{\partial I} = 0 \qquad (12)$$

The boundary conditions are as follows:

$$S(V_T, T, I_T^K, T) = \max\,(V_T - F, 0) \qquad for\;\; 0 \le I_T^K < d \qquad (13)$$

$$S(V_t, t, d, T) = 0 \qquad (14)$$

The relevant form of the valuation equation for the bond, B, will be:

$$\frac{\partial B}{\partial t}+\frac{\sigma^2 V^2}{2}\frac{\partial^2 B}{\partial V^2}+(r-\delta)V\frac{\partial B}{\partial V}-rB+\frac{\partial B}{\partial I}=0 \tag{15}$$

The boundary conditions are as follows:

$$B(V_T,T,I_T^K,T)=\min(V_T,F) \qquad for\ 0\le I_T^K<d \tag{16}$$

$$B(V_t,t,d,T)=V_t \tag{17}$$

Bergman (1985) has developed a general procedure for pricing path-dependent contingent claims, and applied the procedure to the case of averaging claims. A new term, that is proportional to the rate of change of the average, is introduced in the Black-Scholes equation. Haber, Schönbucher and Wilmott (1999) have used this extension for the pricing of Parisian options, where a new state variable I_t^K gives rise to a modified form of the Black-Scholes equation. In a standard Parisian option, the knocked-out feature is activated if the value of the firm's assets has spent a prespecified consecutive time below K_t, and thus the clock variable $I_t^{K_B}$ is reset to zero once the asset value rises above K_t. However, in a Parasian contract the knocked-out feature is activated only if the cumulative time spent below K_t exceeds some prescribed value. The type of option that we value in our model can be considered as a hybrid of the Parasian contract and the Parisian options.

V. A comparison with Past Liquidation Models Based on the Excursion Time

In this section we describe the two existing pricing models for corporate securities that are based on excursion time as developed by FM and Moraux (2002). By using two numerical examples and a simple discrete version of the model, we illustrate the anomalous behavior that may stem from each of them, and demonstrate how, through the parameters' determination our general model can prevent such anomalies.

In both examples, we consider a leveraged firm that issues only one stock and one zero-coupon bond maturing in 10 years. The debtholders are protected by a safety covenant that allows them to force liquidation when the value of the firm's assets becomes less than the distress threshold K_t. For each of the models, the distress period before liquidation lasts at most one year, so $d = 1$. However, the state variable that triggers liquidation, I_t^K, is treated differently in each model. According, to Moraux's (2002) *cumulative excursion method*, liquidation occurs when the value of the firm's assets accumulates more than one year under the threshold level, even with interruptions, and thus in our setting equation (7) is parameterized as follow: $\beta = \gamma = 0$. According to FM's *consecutive excursion method*, liquidation occurs when the value of the firm's exceeds uninterruptedly a consecutive one-year period under the distress threshold, and thus: $\gamma = 0$ and $\beta \to \infty$. To illustrate our *weighted excursion method* we have chosen a third set of parameters: $\gamma = 0,$ and $\beta = 0.5$. This parameterization constitutes a particular case of our method.

In the first example, depicted in Figure 1 and 2, the value of the firm's assets between the middle of the second year and the beginning of the seventh year has accumulated three and half years under the distress threshold. Figure 2 shows the value of the state variable that triggers liquidation according to each method. The length of the grace period according to cumulative excursion method decreases linearly after each distress event, and thus after two years and nine months, the cumulative distress time is greater than a year and liquidation is triggered. Our *adjustable excursion method* reduces the impact of previous distress periods and liquidation is postponed by seven months, since the excursion periods are not consecutive. In the *consecutive excursion method*, the value of the firm's assets value falls below the threshold level during eleven non-consecutive periods, which means that default occurs no less than eleven times, however, liquidation is avoided since none of these comprise a consecutive twelve-month period and the bankruptcy procedure is stopped at each time that the firm's assets value rebounds above the threshold. The safety covenant is not respected despite the fact that the firm has been in dire financial straits for a prolonged period of time.

In the second example, as described in Figures 3 and 4, the value of the firm's assets crosses the distress threshold at the end of the third year for the first time and stays there for a consecutive time of ten months until firm value rebounds above the threshold. Liquidation is not triggered under any of the three models since the liquidation state variable I_t^K is less than one (10/12). In the middle of the ninth year, the value of the firm falls below the threshold level once again, and stays there for two consecutive months. According to the *consecutive excursion method,* the distress clock is reset and liquidation procedures are not initiated after two months. We receive similar results for the chosen parameterization of the *adjusted excursion method,* since the liquidation state variable has fallen to the value of 0.05 from 0.83 given the fact that firm value remained above the threshold for more than five years. In contrast, according to the *cumulative excursion method,* liquidation is warranted. The distress clock is not reset or "moved back", and liquidation is declared immediately after two months. The state variable that triggers default has not forgotten or reduced the impact of the distress period that occurred in the distant past. Since the sensitivity of the liquidation state variable in this framework exaggerates distress experienced in the previous episodes, one can conclude that the consecutive excursion model may have too strong a memory.

VI. Numerical Implementation and Sensitivity Analysis

We now turn to the implementation of the model for calculating bond prices, equity prices and the credit spread of a levered firm. Since in most cases an analytical solution is not available, we need to employ a numerical solution. We follow the Monte-Carlo simulation approach, since it is easy to implement and applicable to a wide range of problems presented in this paper. We consider some examples and perform a sensitivity analysis of the bond price, the equity price and the credit spread with respect to a number of parameters. In order to emphasize the impact of our method on the value of the various corporate claims, we also compare our results to existing structural methods for modeling credit spread.

As the base case, we assume a firm with capital structure comprised of one stock and one zero-coupon bond with $F = 109.926$ and $T = 5$. The firm value is 100 and as a result the quasi leverage ratio, which is defined as $LR = Fe^{-rT} / V_t$, is equal to $LR = 0.9$. The risk free interest rate is $r = 4\%$, the initial pre-determined grace period is $d = 0.25$, the firm asset volatility is 30%, and no payout is expected $(\delta = 0)$. The parameter α is set to zero, and as a result the distress severity has no influence on the liquidation state variable (no matter which value λ receives). The parameter γ equals zero as well, which means that each observation on the last distress period has an equal impact on the decision to liquidate the firm's assets. Bondholders hold a contract that enables them to take over the firm at a time when the value of the firm's assets is smaller than the discounted face value of debt, and as a result the distress threshold parallels the secured discounted balance and equals $K_t = Fe^{-r(T-s)}$. To isolate the impact of deviations from the provisions of the bondholder's contract on claim value, we assume the absence of costs pertaining to liquidation and bankruptcy.

We now analyze the determinants of the level of credit spreads and corporate securities values. Table 1 lists the numerical estimates of corporate securities within various structural frameworks of default and liquidation. The credit spread according to the Merton model comes to 5.1%. This high spread stems from the model's underlying assumption that neither liquidation nor default can occur before the contractual maturity of debt, and thus, in instances of financial distress, debtholders cannot extract value from the firm prior to maturity. At the other extreme, BC assume that the firm's assets are immediately liquidated upon hitting the distress threshold. If this threshold is equal to the secured discounted debt balance, there is no effective credit risk and the credit spread is equal to zero. Figure 5 and table 1 show that as the decay factor of previous distress events increases, spreads fall and the value of debt decreases. At the extreme, as in FM, when $\beta \to \infty$, the distress clock is reset whenever the firm's assets value crosses the distress threshold. When the grace period is shorter and equal to one month, the gap between the credit-spread according to the *cumulative excursion method* $(\beta = 0, \gamma = 0)$ and the *consecutive excursion method* $(\beta \to \infty, \gamma = 0)$ is relatively small and equal 46 basis points. However, when the grace period is prolonged to three months, and the violation of terms of the safety covenant is more severe, the gap grows larger, reaching 70 basis points. In this case, modeling the true nature of the bankruptcy procedure becomes of the essence; intermediate values of β may capture the true nature of bankruptcy procedure more appropriately. When the grace period is increased to one year, the gap between the two extreme cases declines to 67 basis points, since the probability of early liquidation by any method is much reduced.

Figure 6 and table 2 show that credit spreads increase and bond values decrease with asset volatility. As volatility increases (potentially hurting bondholders), having a liquidation procedure with long memory, i.e., better bondholder protection (small β), has greater impact: when β is equal to zero, an increase from 30% to 40% in asset's volatility leads to an increase of the spread by 77 basis points, while a similar increase of the volatility when $\beta \to \infty$ causes an increase of 106 basis points.

Figure 7 and table 2 provide estimated credit spreads for a combination of financial quasi-leverage ratios (LR) and decay factors for previous distress periods (β). As β increases, the gap between the credit-spreads of the two leveraged firms increases. When β=0, the credit spread of a firm with quasi leverage ratio of 90% is equal to 1.84% and the credit spread of a firm with quasi leverage ratio of 95% is equal to 1.97%. However, when $\beta \to \infty$, the credit spreads amount to 2.54% and 3.76% respectively. The gap between the credit spreads of the two leveraged firms according to the Merton model comes to 2.87%. This important outcome emphasizes the fact that as the liquidation state variable is less sensitive to the impact of past distress periods, an increase in financial leverage has a greater impact on debtholders protection.

Table 3 shows the influence of the degree of insolvency $f(V_t)$, which is a function of α and λ, on the value of the firm's capital structure. When $\lambda = 0.2$, the value of f is less than unity, for default events where the value of assets is less than 20% below the liquidation threshold. As a result, the observed credit spreads are higher than in the base case.

Figure 8 shows the value of the equity as a function of the volatility of the firm's assets, σ, and the decay of previous distress periods (β). The value of equity goes up with σ , as is well known. However, it has a greater effect in the Merton (1974) model than in the Moraux (2002) model. The sensitivity of the stock value to σ increases with β. Merton's model yields the highest equity value and the greatest sensitivity to σ given similar parameters, since it assumes away the probability of early liquidation. In Merton's model the incentive of stockholders to assume more unexpected volatility is highest, and thus we may observe a strong incentive for assets substitution *a la* Jensen and Meckling (1976). Therefore, if two companies, identical in all parameters, operate under two different liquidation regimes, we would expect to find higher sensitivity of stock volatility under a regime in which creditor's rights are less protected.

VI. Conclusion

We present a simple and general structural model for the valuation of corporate securities, where the bondholders' right to force immediate reorganization or liquidation of a distressed firm may consume time. To evaluate the impact on the firm's capital structure, we develop a general and adjustable pricing model for corporate securities driven by a liquidation state variable. Unlike other models, our state variable, which may trigger liquidation of the firm's assets, accommodates a greater number of scenarios and yields a more accurate assessment of financial distress. The liquidation trigger accumulates over distress time, but is also dependent on the degree to which the threshold is violated. Also, recent distress episodes can carry a higher influence than older episodes.

We show that by applying the appropriate liquidation parameters, our model converges to the François and Morellec (2002) bankruptcy model, in which liquidation is triggered if the value of the firm's assets exceeds a consecutive excursion time. Moreover, our general model also accommodates Moraux's (2002) bankruptcy model, which assumes that liquidation occurs when total excursion time exceeds a pre-determined grace period. While these two models may accurately describe the bankruptcy procedure for a specific set of legal regime and law enforcement, our model, as illustrated in this paper, covers a wider array of legal precepts and contractual arrangements. All of these liquidation models may be viewed as a middle ground approach between Merton's framework, in which liquidation occurs only upon debt maturity, and the Black-Cox model, in which reorganization of the firm's assets is invoked when a minimum threshold is violated during the lifetime of the debt.

We illustrate the applicability of our model for the valuation of firms with simple capital structures, and we present both comparative statics and sensitivity analysis of the various corporate claims for different legal regimes and corporate capital structures.

A natural direction for future research is to apply the model to environments characterized by empirically supported dynamics of risk-free short rates and observed credit spreads. Additional features such as interim payments, taxes, liquidation costs, debt subordination and alternative bond indentures can be incorporated as well. Though not a trivial task, exploring these directions may be rewarding in providing new guidance for risk measurement and pricing, as well as for supporting empirical findings and observed behavior patterns in the fixed income and equity markets.

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Table 1

Corporate credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various grace periods and past period decay factors. Parameters for the base case are the risk-free interest rate $r = 4\%$, the volatility of the firm's assets volatility $\sigma = 30\%$, $P = 109.926$ and $T = 5$. The firm asset value equals 100, and as a result the leverage ratio, which is defined as $LR = Fe^{-rT}/V_t$, equals $LR = 0.9$. The pre-determined grace period: $d = 0.25$, no payout is delivered $(\delta = 0)$. The liquidation model parameters α and γ are set at zero.

Scenario	*β*	*Equity value*	*Debt Value*	*Credit spread*
Base case	β=0	*17.93*	*82.07*	*1.84%*
	β=1.5	*19.31*	*80.69*	*2.19%*
	β=3	*20.03*	*79.97*	*2.36%*
	β→∞	*20.74*	*79.26*	*2.54%*
d=0.083 (One month)	β=0	*14.46*	*85.54*	*1.02%*
	β=1.5	*15.01*	*84.99*	*1.14%*
	β=3	*15.27*	*84.73*	*1.21%*
	β→∞	*16.41*	*83.59*	*1.48%*
d=1 (One year)	β=0	*24.17*	*75.83*	*3.43%*
	β=1.5	*26.53*	*73.47*	*4.06%*
	β=3	26.66	73.34	4.09%
	β→∞	26.67	73.33	4.10%
d=T (Merton 1974)		30.25	69.75	5.10%
d=0 (BC 1976)		10.00	90.0	0.0%

Table 2

Credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various past period decay factors, asset volatilities and leverage ratios. All other parameter values are the same as in Table 1.

Scenario	*β*	*Equity value*	*Debt Value*	*Credit spread*
Base case	β=0	*17.93*	*82.07*	*1.84%*
	β=1.5	*19.31*	*80.69*	*2.19%*
	β=3	*20.03*	*79.97*	*2.36%*
	β→∞	*20.74*	*79.26*	*2.54%*
d=T (Merton 1974)		30.25	69.75	5.10%
d=0 (BC 1976)		10.0	90.0	0.0%
σ=40%	β=0	*21.02*	*78.98*	*2.61%*
	β=1.5	*22.90*	*77.10*	*3.09%*
	β=3	*23.89*	*76.11*	*3.35%*
	β→∞	*24.82*	*75.18*	*3.60%*
d=T (Merton 1974)		36.21	63.78	7.97%
LR=0.95 (*F=116.0 3*)	β=0	*13.93*	*86.07*	*1.97%*
	β=1.5	*15.46*	*84.54*	*2.33%*
	β=3	*16.27*	*83.73*	*2.53%*
	β→∞	21.28	78.72	3.76%
d=T (Merton 1974)		28.18	71.82	5.60%
d=0 (BC 1976)		5.00	95.00	0.00%

Table 3

Corporate credit spread and the value of the firm's capital structure

This table presents the corporate credit spread and the value of the firm's capital structure for various α and λ, parameters that determine the impact of the severity of the distress event on the liquidation trigger. All other parameter values are the same as in Table 1.

Scenario		*β*	*Equity value*	*Debt Value*	*Credit spread*
λ=0.2	α=0.5	β=0	*18.42*	*81.58*	*1.90%*
		β=1.5	*19.85*	*80.15*	*2.26%*
		β=3	*20.56*	*79.44*	*2.44%*
		β→∞	*21.12*	*78.88*	*2.58%*
	α=1.5	β=0	*18.96*	*81.04*	*2.04%*
		β=1.5	*20.56*	*79.44*	*2.44%*
		β=3	*21.20*	*78.80*	*2.60%*
		β→∞	*21.60*	*78.40*	*2.71%*
λ=0.4	α=0.5	β=0	*18.79*	*81.21*	*2.00%*
		β=1.5	*20.35*	*79.65*	*2.39%*
		β=3	*21.08*	*78.92*	*2.57%*
		β→∞	*21.58*	*78.42*	*2.70%*
	α=1.5	β=0	*20.18*	*79.82*	*2.34%*
		β=1.5	*22.06*	*77.94*	*2.82%*
		β=3	*22.69*	*77.31*	*2.98%*
		β→∞	*22.86*	*77.14*	*3.03%*

Figure 1

Example 1: Simulation of the firm's asset value and the distress threshold.

In Figure 1 we simulate one path of the distress threshold and firm value over a ten- year period, as discussed in example 1 in section VI. The distress threshold is worth $K_t = Fe^{-rt}$, where $r = 0.04$ and $F = 100$.

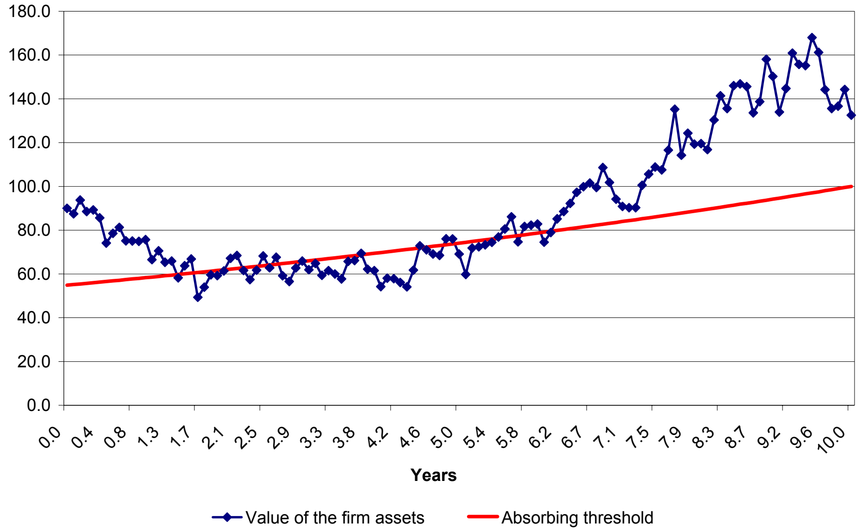


Figure 2

Example 1: Simulation of excursion time according to alternative trigger models.

In Figure 2 the weighted excursion period is calculated for the value of the firm's path presented in Figure 1. The initial grace period is set at $d = 1$. Liquidation is not triggered according to the FM model, while according to Moraux (2002) liquidation is triggered after two years and nine months. With $\beta = 0.5$ and $\gamma = 0$ the liquidation is triggered after three years and four months.

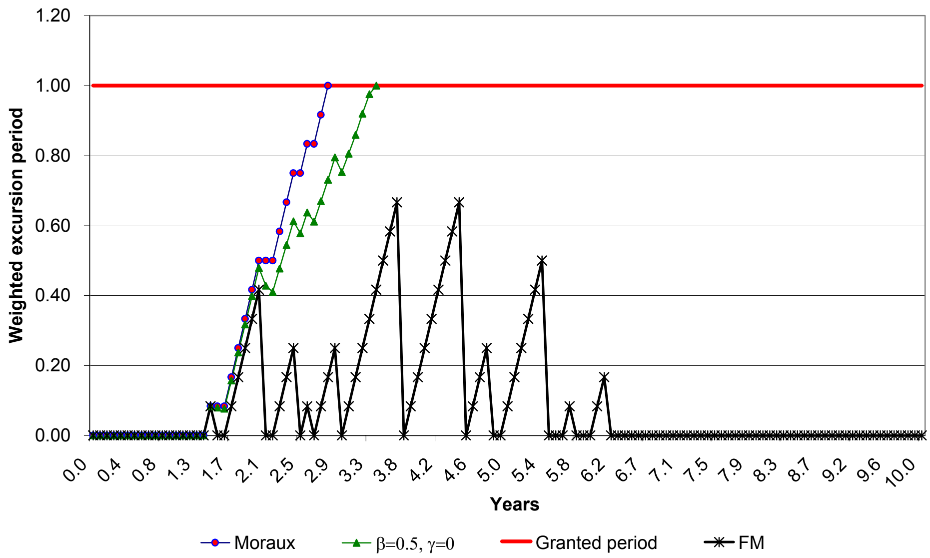


Figure 3

Example 2: Simulation of firm value and the distress threshold.

In Figure 3 we simulate one path of the distress threshold and the value of the firm's asset over a ten- year period, as discussed in Example 2 in Section VI. The distress threshold is $K_t = Fe^{-rt}$, with $r = 0.04$ and $F = 100$.

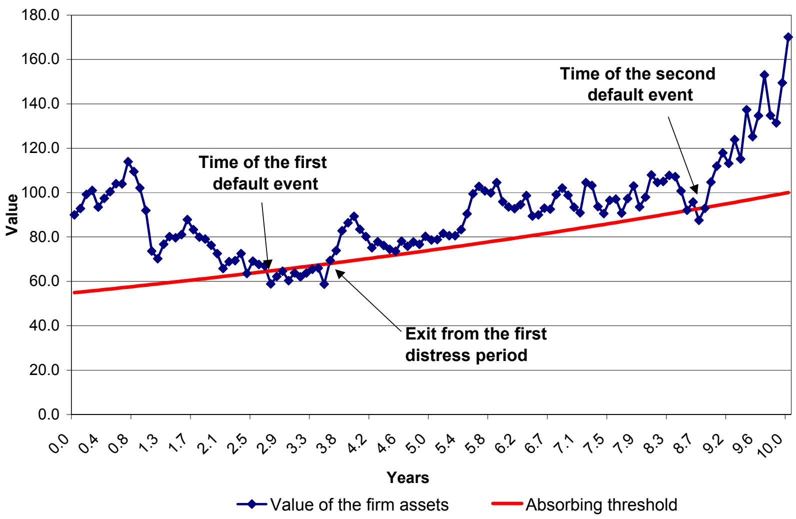


Figure 4

Example 2: Simulation of excursion time according to alternative trigger models.

In Figure 4, the weighted excursion period is calculated for the firm's path value presented in Figure 3. The initial grace period is set at $d = 1$. Liquidation is not triggered according to the FM model, while according to Moraux (2002) liquidation is triggered eight years and eleven months. With $\beta = 0.5$ and $\gamma = 0$ liquidation is not triggered too.

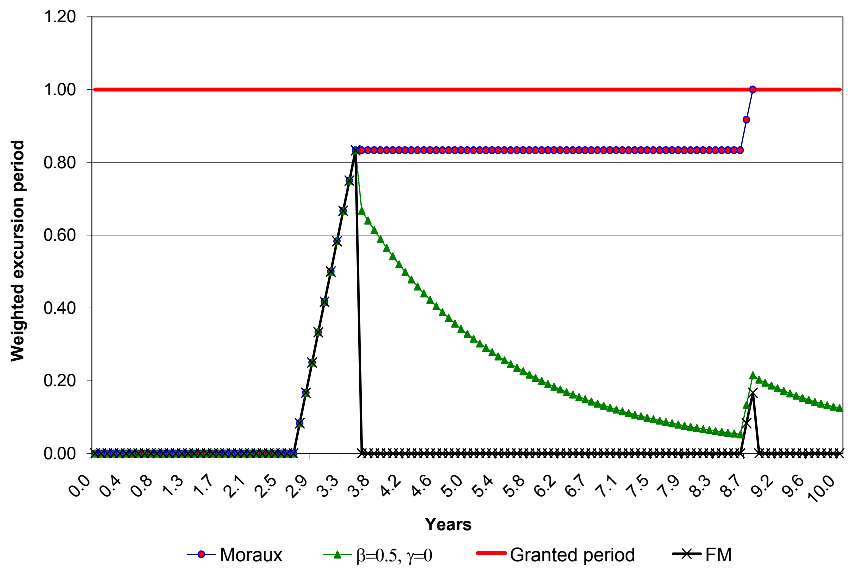
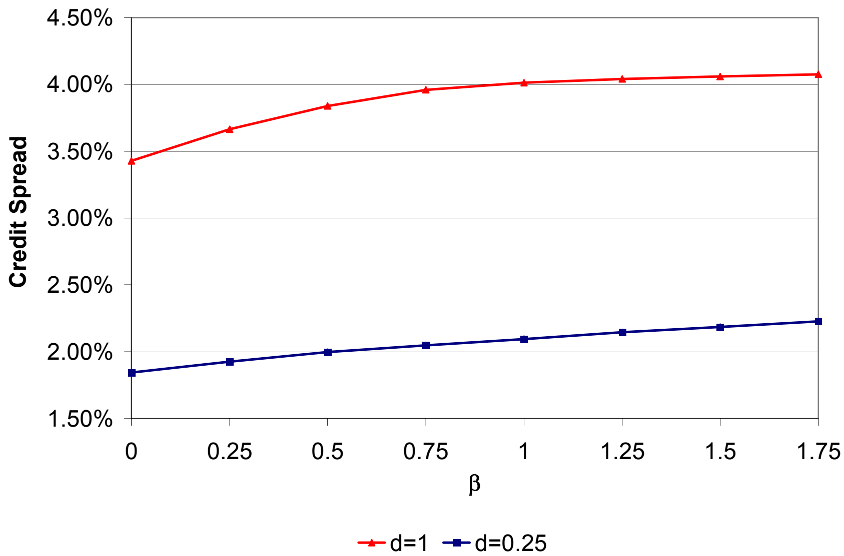


Figure 5

Corporate credit spread as a function of past distress decay factor (β) and grace period (d).

Parameters: See table 1.



Parameters: See table 1.

Figure 6

Corporate credit spread as a function of past distress decay factor (β) and asset volatility (σ).

Parameters: See table 1.

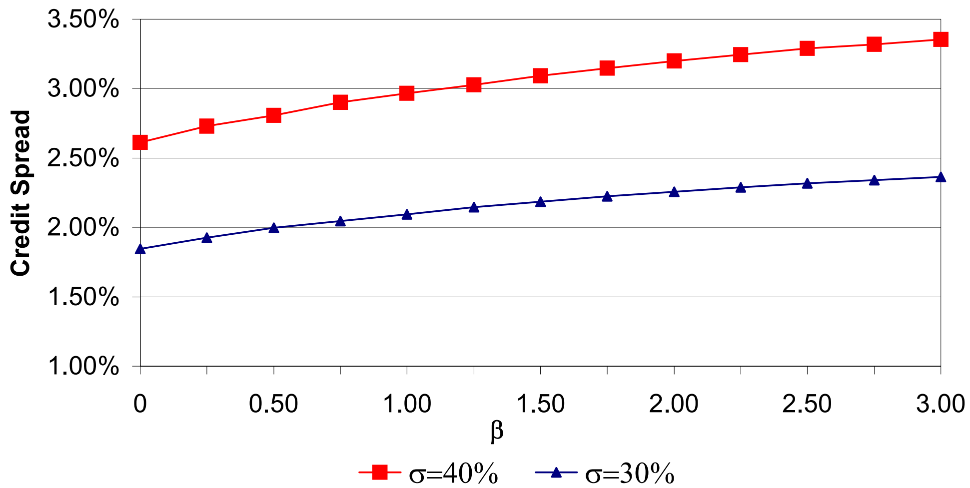
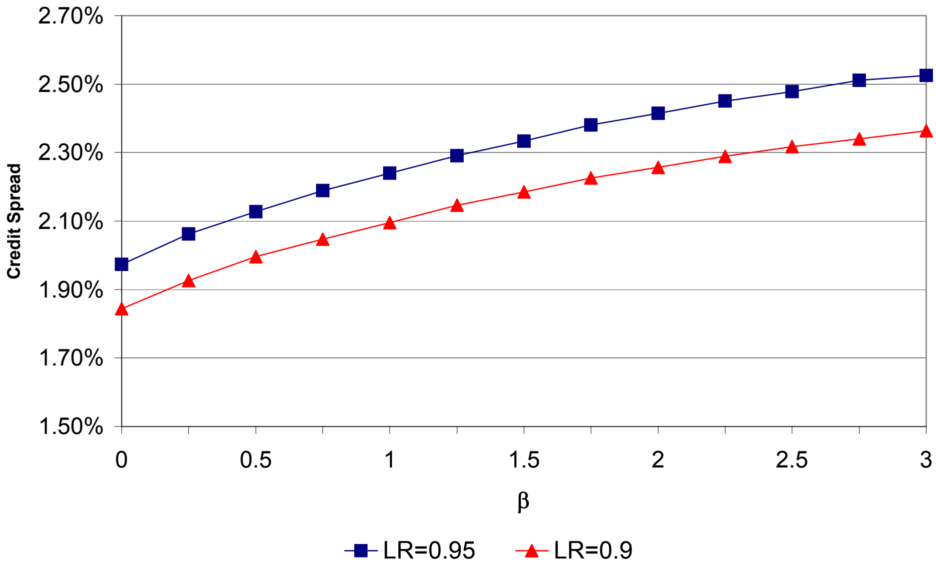


Figure 7

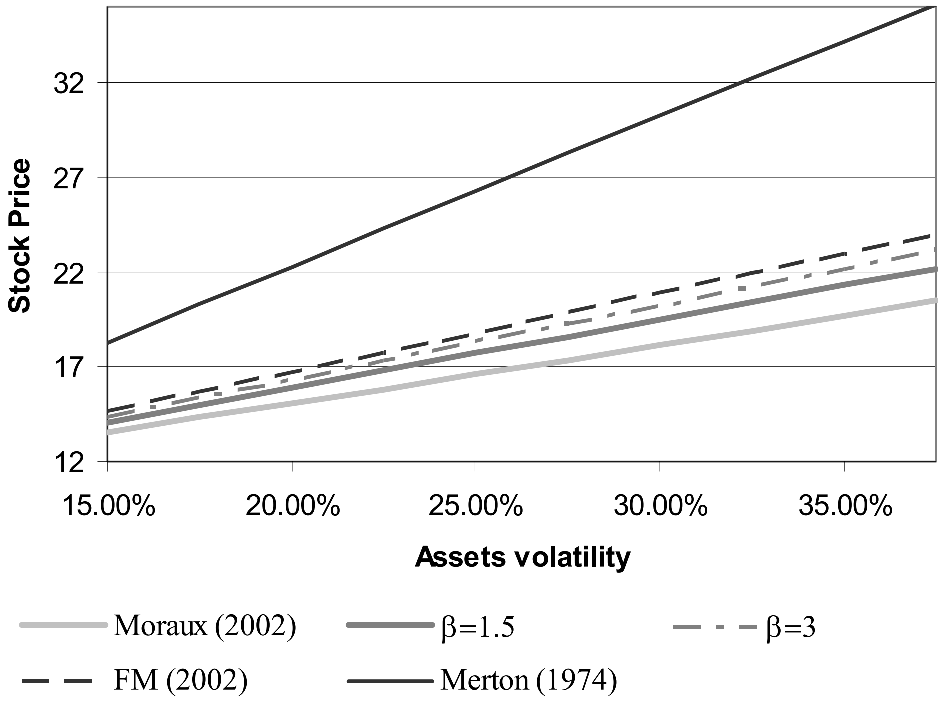
Corporate credit spread as a function of past distress decay factor (β) and leverage ratio (LR).



Parameters: See table 1.

Figure 8

Stock price as a function of assets' volatility and past distress decay factor (β)



Parameters: See table 1.