Modeling of Portfolio Dependence in Terms of Copulas. A Rating-based Approach.

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Abstract

This paper is about the modeling of dependence in credit risk. In the recent times, this subject is the matter of concern in many papers. Due to the relation in the behavior of individual exposures, it is very important to take this fact into account by judgment of the risk magnitude. The direct way in this case is to specify the variable that describes the behavior of individual exposures and make the stochastic process of the random variables to be dependent on each other. This paper looks into this subject from a different point of view. We apply the portfolio view and assume that the composition of the portfolio in terms of ratings at a specific point of time is the result of the interactions of the individual obligors with each other in a prior time. In this paper, we specify the random variable as a composition of the portfolio in terms of ratings and look at the variation of this variable over the time. We assume that the composition of the portfolio is the result of the interaction of the exposures and can be used to study the dependence structure in credit risk. We assume that the time behavior of the portfolio has the Markov property but in contrast to the previous works, we apply the copula approach to model the Markov chain. It allows using the advantages of the copula framework to model the dependence structures.

1 Introduction

It is generally accepted, that the defaults are dependent. In the modeling literature you will find two approaches explaining the dependence between defaults. The first approach leads back the dependence to the relationship between the firms. According to this view, the default of the firm has influence on the survival of the linked firms. The dependence is based on the contagion. The second approach relates the survival probability of the individual firm to

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the general factors. The factors influence the firms in the same direction and cause the default accumulation. For this reason, a model of credit risk has to allow for the dependence between the individual exposures. Apart from the different notions about the source of dependence, there are different formal setups to implement this.

Of the crucial importance is the specification of random variable. In the recent time the models specify the random variable as "time until default" and not as default. The first definition does not link the dependence to the specific time horizon but gives the information regarding distribution of defaults along the time line. As far as the risk management is concerned about both the number of default in the specific time horizon and how close they are, it is a convenient approach. But the random variable refers to one specific exposure. In the first step you define the default process of the individual exposure. To switch to the portfolio view you have to link the random variable by the copula. At this stage one has to specify the dependence between the individual exposures. By this approach you have so many random variables as individual exposures. In order to keep the problem manageable, you have to make simplifying assumptions regarding the size and the creditworthiness of the exposure. You will assume that all exposures in the portfolio have the same size and bear the same credit risk

In this framework the time of default and the dependence between the individual times of default is what are you interested in. The default of one obligor causes the default intensities of the others jump. This makes the default of the others more probable and the stochastic processes dependent. On this way one models the dependence between the default times of the individual obligors. But it will be reasonable to assume, that the default of one obligor can result in the deterioration of the creditworthiness of the another. It is also possible, that the deterioration of the creditworthiness of one obligor results in the default of the other. The deterioration of the creditworthiness can be specified as the change in the rating grade. So there are two types of events: the default and the change in the rating grade. The model will be more realistic, if we extend our dependence concept to include the changes in the rating grades. This is also an important step in the direction to measure the market risk of the portfolio under the dependence between individual obligors.

If we focus on the stochastic behavior of the random variable that describes the default behavior of an individual obligor, we distinguish between two states of nature: default and non-default. As seen from this point of view, the rating system has two rating grades. The rating systems used by rating agencies have more than two rating grades in order to distinguish between the credit qualities of the obligors. The approach in this paper models the dependence of migrations in the rating system typical for rating agencies.

It is usual to model the rating process as a Markov chain. This is also the assumption of our paper. We assume that the rating process has the Markov property. In formal terms it can be expressed as

$$\Pr\{X_{t+1} = j \,| X_0 = i_0, \ldots, X_t = i\} = \Pr\{X_{t+1} = j \,| X_t = i\}$$

for all time points t and all states. The states of Markov chain represent the rating categories.

The Markov process is completely defined by the initial state and transition probability matrix. The initial state is generally the probability distribution X_0. In the literature the rating process are given by rating transition matrix, without defining the initial distribution. The reason is that, we understand the rating process on the individual level. We say, what is the probability to find obligor in the rating category AAA, say, in one year time, if we know, that he is in the category B at present time. In this case, the initial distribution will have the form: $X_0 = (0, \ldots, 0, 1, 0, \ldots, 0)$. But what will happen if we switch to the portfolio view. The units included in the portfolio can be distributed between the rating categories in many ways. The composition of the portfolio is given by the proportion of each rating category and can be seen as the initial distribution in terms of Markov chain. In this case by multiplying the initial distribution with the transition matrix one would not get the distribution of the individual obligor to be in a specific rating category at the end of the period given his present rating, but you get the composition of the portfolio specified by proportion of each rating at the end of specific time period, given the present day composition. This view is very essential for many problems in the finance, like for example the pricing of basket type instruments.

At the moment we say that the transition matrix can be also applied to describe the default process of the portfolio, one would like to know how it can handle the dependence between the individual default processes. If we assume, that the composition of the portfolio is essential for the dependence that we will have in the portfolio, then it will be wrong to apply the same transition matrix to the both portfolios to calculate the portfolio composition at the end of some specific period of time. There must be two transition matrices with different dependence structures. The empirical work in this area gives some hints that it is also plausible to differentiate between different composition of the portfolios regarding the industries, countries, etc.(Nelsen, 1998). Therefore, we will conclude, that the transition matrices published by rating agencies are good for the universe of obligors underlying their calculation. But one can say nothing about the dependence structure implied in these matrices and we do not have any clues about the strength of dependence.

If we are aware of this problem, the next step will be to find a framework, which can specify the dependence structure in the transition matrix of the

Markov chain. We use the idea of Darsow et al. (1992) to show, how we can use the copula approach to model the dependence in terms of rating process. This paper is organized as follows. First, we introduce the copula followed by the specification of Markov chain in terms of copula. Then, we transfer the idea of Darsow et al. (1992) on the rating process and show an example, how it works. We use the rating matrix from Standard and Poor's and hypothetical portfolios to show the implications for the dependence structure, if we use one of the portfolios in combination with this matrix.

2 Model Setup

2.1 Copulas[1]

Generally speaking, copulas are the multivariate distribution functions with uniform distributed margins: the random variables take values on $[0, 1]$ and they are linked to each other by the copula function C, the joint distribution function.

A two-dimensional copula (2-copula) is a function C from I^2 to I with the following properties:

(1) For every u, v in I

$$C(u, 0) = 0 = C(0, v) \tag{1}$$

and

$$C(u, 1) = u \quad \text{und} \quad C(1, v) = v \tag{2}$$

(2) For every u_1, u_2, v_1, v_2 in I such that $u_1 \leq u_2$ and $v_1 \leq v_2$

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0 \tag{3}$$

There are three special cases of copulas:

$$W(u, v) = max(u + v - 1, 0)$$
$$M(u, v) = min(u, v)$$
$$\Pi(u, v) = u \cdot v$$

[1] In the recent time copulas were broad used in the modeling literature. The presentation here fails briefly and is limited to the bivariate copula used in this paper. For the details please refer to Darsow et al. (1992); Nelsen (1998)

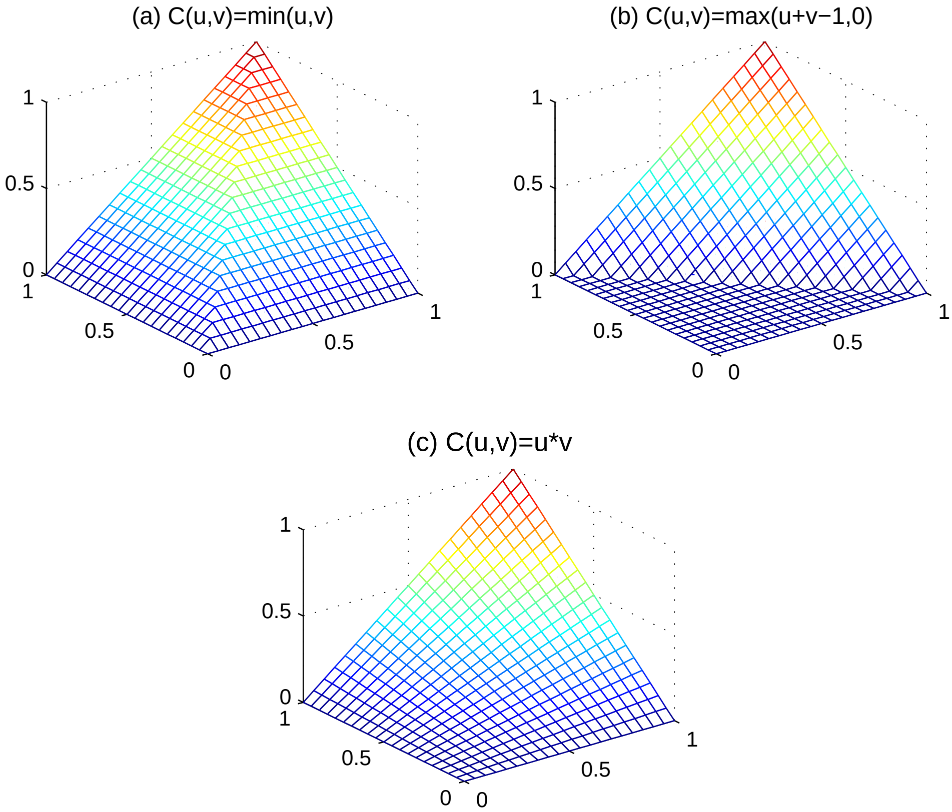

Figure 1. Graph of $M-$, $W-$ und $\Pi-$copulas

The figure 1 shows the graph of these copulas. For every copula C and every (u, v) in I^2:

$$W(u, v) \le C(u, v) \le M(u, v)$$

The copulas W and M form the lower and upper limit for all copulas and represent the perfect negative and positive dependence; Π stands for the independence.

Sklar's theorem: Let H be a joint distribution function of random variables x and y with margins $F(x)$ and $G(y)$. Then, there exist the copula C such that for all $x, y \in \bar{\mathbb{R}}$

$$H(x, y) = C(F(x), G(y)).$$

If F and G are continuous, then C is unique.

For the arbitrary continuous distribution F of the random variable x, the random variable u such that $u := F(x)$ is uniform distributed on $[0, 1]$. Hence, the distribution F is not reflected in the copula function. The copula gives the functional form of the joint distribution function $H(x, y)$ adjusted to the influence of the margins and is suited for the analysis of dependence.

Gauss copula

$$C_R^{Ga}(u, v) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\left(1 - R_{12}^2\right)^{1/2}} \exp\left(-\frac{s^2 - 2R_{12}st + t^2}{2\left(1 - R_{12}^2\right)}\right) dsdt \quad (4)$$

t-copula

$$C_{v,R}^{t}(u, v) = \int_{-\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_v^{-1}(v)} \frac{1}{2\pi\left(1 - R_{12}^2\right)^{1/2}} \left(1 + \frac{s^2 - 2R_{12}st + t^2}{v\left(1 - R_{12}^2\right)}\right) dsdt \quad (5)$$

Clayton copula

$$C(u, v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}} \quad (6)$$

Gumbel copula

$$C(u, v) = \exp\left(-\left((-\log u)^{\theta} + (-\log v)^{\theta}\right)^{\frac{1}{\theta}}\right) \quad (7)$$

Frank copula

$$C(u) = -\frac{1}{\theta} \log\left(1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1}\right) \quad (8)$$

The parameter of the copula function captures the dependence. For the specific parameter values the copulas converge to one of the specific cases: W, M or Π. Table 1 shows the range of the parameter, special and limiting cases for the four copulas presented above.

2.2 Markov chain in terms of copula[2]

Let X_t be a continuous random variable for all $t \in T$, where t is the time index. With Markov chain, we describe the real stochastic process of the random

[2] The people who first specified the Markov chain in terms of copula were Darsow et al. (1992, p. 604 et sqq). The brief presentation of the approach is given by Nelsen (1998, p. 195).

Table 1
The range of parameter and special cases.

	Range of parameter	Perfect negative dependence	Independence	Perfect positive dependence
Clayton	$[-1,\infty)\setminus\{0\}$	$C^{\{-1\}}=W$	$C^{\{0\}}=\Pi$	$C^{\{\infty\}}=M$
Gumbel	$[1,\infty)$	-	$C^{\{1\}}=\Pi$	$C^{\{\infty\}}=M$
Frank	$(-\infty,\infty)\setminus\{0\}$	$C^{\{-\infty\}}=W$	$C^{\{0\}}=\Pi$	$C^{\{\infty\}}=M$
Gauß	$[-1,1]$	$C^{\{-1\}}=W$	$C^{\{0\}}=\Pi$	$C^{\{1\}}=M$

$\Pi(u,v)=u\times v$; $W(u,v)=\max[u+v-1,0]$; $M(u,v)=\min[u,v]$.

variable X_t given by the sequence $X_t\in T$. We defined the 2-copula as a function of two uniformly on $[0,1]$ distributed continuous random variables: $C(u,v)$. Applying to the Markov chain, one variable of the copula stands for the initial distribution X_s, the other for the end distribution X_t of the same random variable X. If we denote with F_t the cumulative distribution function of X_t at time t und with F_{st} the joint cumulative distribution function of the variables X_t and X_s, we can link this notation to copula notation of the previous section:

$$F_{st}(x,y)=C_{st}(F_s(x),F_t(y)),$$

i.e., $C_{st}(u,v)$ gives us the joint probability for $F_s(x)<u$ und $F_t(y)<v$. In their work Darsow et al. defined the product of two 2-copulas and shown, that this is the continuous analog to the multiplication on matrices. They proved that the first derivation of the copula function gives the transition probabilities of the Markov chain. Formally,[3]

$$P(X_s<u\mid X_t=v)=C_{st,2}(F_s(x),F_t(y)) \tag{9}$$

with s, $t\in T$ such that $t<s$ and C_{st} copula function of random variables $X_s$and X_t.

In the common specification of the Markov chain we use the initial distribution and the transition matrix to define the stochastic process. It is obvious, that the changing the initial distribution and holding the transition matrix constant has an impact on the end distribution. The initial and the end distributions are linked to each other by transition matrix. Given the transition matrix in the absolute terms we do not know how strong the dependence of the end distribution on the initial distribution is. In the approach of Darsow et al., the Markov chain is specified by the initial and the end distributions as well as the

[3] See the proof of the theorem 3.1 in the essay of Darsow et al. (1992, pp. 608-609).

copula function. The copula function sets the dependence structure between the initial and the end distribution. The parameter of the copula function captures the dependence strength. The transition matrix in the absolute terms arises from the first derivation function of the copula.

2.3 Rating process in terms of copula

In this section, we apply the copula approach to the rating process and specify the transition probabilities in terms of copula function. Therefore, we define the random variable X_t as the rating grade of the obligor at a specific point of time t. Applying the rating grade system of Standard & Poor's, the state space of the random variable X_t will be $AAA,\ AA,\ A,\ BBB,\ BB,\ B,\ CCC\ and\ D$. Notice, that X_t is a discrete variable. For each point of time, we describe the portfolio of exposures by the proportion of each rating grade in the universe of exposures. For the purpose of this paper we assume, that the exposures have the same size given by the nominal amount. In this case the proportion of each rating grade is the number of exposures of the rating grade i dividing by the total number of exposures: $\frac{n_i}{\sum_i n_i}$, were n_i is the number of exposures of the rating grade i. We calculate the probability, that the exposure has the rating grade BB by $f(X_t = BB) = \frac{n_{BB}}{\sum_i n_i}$ (density function). Then, $F(X_t = BB)$ is the cumulative distribution function and gives the probability, that the exposure is of the rating grade BB or better. The index t refers to the specific point of time. $f_t(x)$ und $F_t(x)$are the density and the cumulative distribution function of portfolio at time t.

The transition matrix gives the joint distribution (pdf) of two variables $X_s,\ X_t$, where $t,\ s\,(s < t)$ are two points of time. In the previous notation the joint cumulative distribution function of $X_s,\ X_t$ is given by F_{st}. Since X_t is a discrete variable, we cannot apply the copula approach directly. In the proof of the theorem, Darsow et al. (1992) use the interpolation. We use the same interpolation to define the Markov chain in terms of copula. By the interpolation, we expand the rating matrix (after cumulating the row values) to copula. The formal expression for the interpolation is: [4]

$$P(x, s; y, t) = \frac{C(F(x), F(y)) - C(F(x-1), F(y))}{f(x)} \tag{10}$$

The example below illustrates, how the interpolation works. For this purpose we use as input the composition of portfolio at two points of time as given

[4] You can find this expression in book of Joe (1997, p. 245). In book of Nelsen (1998, p. 16) refer to Lemma 2.3.5. See also Darsow et al. (1992, pp. 609-610).

Table 2
The composition of the portfolio at the beginning and the end of the specific time period in terms of the density and cumulative distribution function ($f(x)$ and $F(x)$).

	AAA	AA	A	BBB	BB	B	CCC	D
$f_0(x)$	0.0250	0.1000	0.2500	0.2500	0.2500	0.1000	0.0200	0.0050
$F_0(x)$	0.0250	0.1250	0.3750	0.6250	0.8750	0.9750	0.9950	1.0000
$f_1(x)$	0.0200	0.1200	0.2000	0.2500	0.2000	0.1000	0.0600	0.0500
$F_1(x)$	0.0200	0.1400	0.3400	0.5900	0.7900	0.8900	0.9500	1.0000

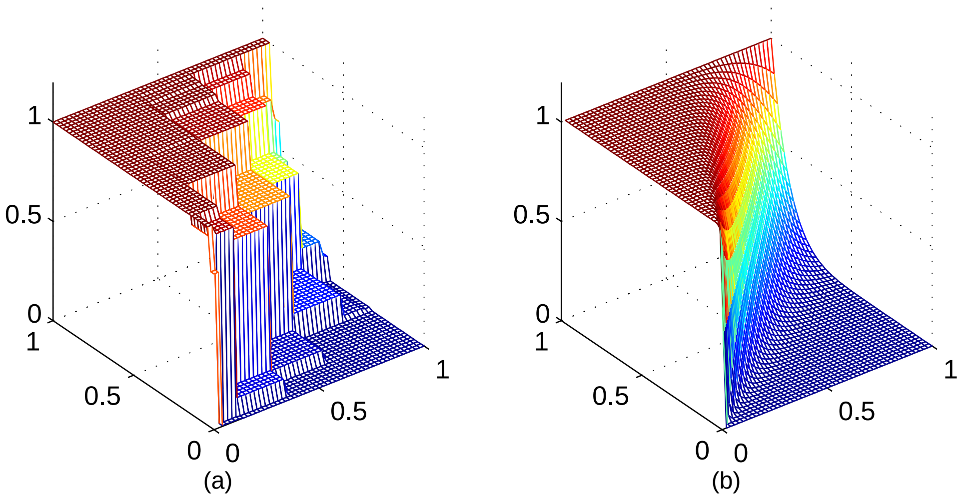

Figure 2. Migration matrix and function of the first derivation of the Clayton copula $\theta = 8$.

in table 2. Furthermore we use in this example the Clayton copula with the parameter $\theta = 8$.

First, we calculate $C(F_0(x), F_1(x))$ by setting the values of the cdf's in the copula function. We get a $8 * 8$ matrix. Then, we apply the equation 10 to get the transition matrix. We have to modify the matrix another time by subtracting from each column the previous one beginning with the last column. We can see the result of the modification in figure 2 (a). For comparison, the figure 2 (b) contains the first derivation function of the Clayton copula.

In the common specification of Markov chain we use the initial distribution and the transition matrix to define the Markov chain. It is obvious, that changing the initial distribution and holding the transition matrix constant has an impact on the end distribution. In the approach of Darsow et al. (1992), the Markov chain is specified by the marginal distributions and the copula function. By the copula function, we can control the dependence structure and leave the copula unchanged as long as the same dependence structure can be assumed. Given the marginal distributions and the transition rates for two portfolios that are different in composition regarding rating weights, we can investigate using copula whether the portfolios have different dependence structures due to their composition differences.

3 Example

The idea of this paper is based on the assumption that the interactions of the exposures in the portfolio will be different due to the differences in their composition, for example regarding the weights of rating grades. The interaction of exposures will make migrations, we observe, dependent. In so far as the migrations are influenced by the composition of the portfolio we can not assume the same migration matrix as given in the absolute terms for different portfolios. Based on the observed migrations the calculated transition matrix should be understood in terms of the portfolio (group of firms) underlying it.

In the example we proceed contrary to the reasoning underlying the approach above. We consider the 1-year and 10-year transition matrix in conjunction with different portfolios. The example is based on the 1-year transition matrix from Standard & Poor's given in table 3. The rating matrix for the 10-years period of time was calculated as 10-fold product of the one year transition matrix: $Q(10) = Q^{10}(1)$. Further, we assume 4 portfolios as given in table 4. For the purpose of this example it is essential, that they differ in the composition regarding the weights of rating grades. We notify the vector of weights of Portfolio a with $P^a(0)$ at the begin, $t = 0$, and apply the 1-year transition matrix to find the portfolio composition at the end of the first year $P^a(1)$: $P^a(1) = P^a(0) \times Q(1)$. In the same way we proceed to get the composition of the portfolios at the end of the tenth year: $P^a(10) = P^a(0) \times Q(10)$.

By means of this example we attempt to verify the following claims, which support the idea of the paper in the indirect way:

Claim 1 *The parameter of the copula underlying the transition matrix is sensitive to the composition of the portfolio.*

To proof this claim, we calculate parameter value for each portfolio implied by the transition matrix assuming the validity of the particular copula function. Since the parameter of the copula determines the dependence magnitude, differences in parameter value imply different dependence grade.

It is intuitive, that the dependence will decrease with time lag. Taken to the limit, $t \to \infty$, we will have independence between the composition of portfolio at the beginning $P(0)$ and the end $P(\infty)$ of time horizon. This intuition

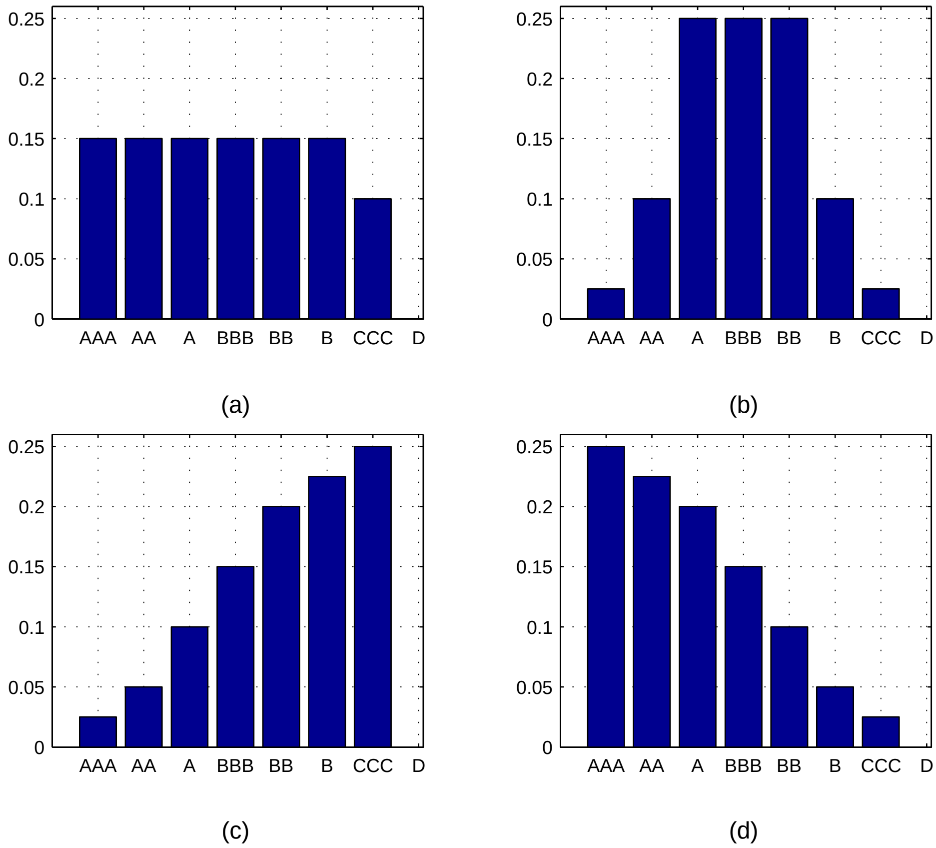

Figure 3. Composition of the hypothetical portfolios in term of rating grades at the initial point of time.

underlies claims 2 and 3.

Claim 2 *The parameter of copula relates to the period of time and for a given portfolio decreases for the time horizon increases.*

In this case, we compare the parameter value of copula for 1-year and 10-years time horizon as calculated for a particular portfolio. If this claim turns out to be true, we should observe the convergence in the parameter value of the portfolios with the increasing time horizon.

Claim 3 *The differences in the value of parameter for different portfolios must decrease as the time horizon increases.*

In this paper we can not investigate which copula function fits best the empirical data because we do not know the composition of firms underlying the transition matrix. But we can see how the different copulas perform on the data of this example.

In order to determine the parameter value for the selected copulas, we performed the optimization in Matlab. Given the copula function, we use the

Table 3
One-year rating transition frequencies of Standard and Poor's (in percentages) (Schönbucher, 2004, p. 227).

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.8910	0.0963	0.0078	0.0019	0.0030	0.0000	0.0000	0.0000
AA	0.0086	0.9010	0.0747	0.0099	0.0029	0.0029	0.0000	0.0000
A	0.0009	0.0291	0.8894	0.0649	0.0101	0.0045	0.0000	0.0009
BBB	0.0006	0.0043	0.0656	0.8427	0.0644	0.0160	0.0018	0.0045
BB	0.0004	0.0022	0.0079	0.0719	0.7764	0.1043	0.0127	0.0241
B	0.0000	0.0019	0.0031	0.0066	0.0517	0.8246	0.0435	0.0685
CCC	0.0000	0.0000	0.0116	0.0116	0.0203	0.0754	0.6493	0.2319
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Historical average (1981-1991), "no rating" eliminated.

Matlab-function "lsqnonlin" to find the parameter value which minimizes the distance between the corresponding elements of the migration matrices: the ones given in the example (see table 3) and ones arising from the copula for the parameter in question.

The results of the optimization are given in table 5 and table 6 for 1-year and 10-years period of time, respectively.

For 1-year period of time, we find the significant variation in the parameters across the portfolios disregarding the copula function we assume. This result is in accordance with the first claim. We also find that the variation of parameter for 10-years time horizon is less distinct as for 1-year. This finding maintains claim 3. By all portfolios we observe the decline in the value of the parameter over time (claim 2). The comparison between the one-year und ten-year parameter value for the particular portfolio points for all portfolios and all copula functions at the decline in dependence given by decline in parameter value.

Regarding the question which copula fits best the data of the example, the findings are not clear: for the 1-year period the Clayton and Gumbel copula turns out to have the smallest residual for all portfolios, whereas for the 10-year period these two copulas provide the best fit only for portfolio c. Regarding portfolios a and b the Frank and Gauss copula give the smallest residual for the 10-years period. By portfolio d the Gauss copula shows the best performance followed by Gumbel and Frank copula.

Table 4
The composition of the portfolio in terms of rating grades
at the beginning (t_0) and the end of the one year (t_1) and ten year (t_{10}) period.

	AAA	AA	A	BBB	BB	B	CCC	D
	Portfolio a							
$P^a(0)$	0.1500	0.1500	0.1500	0.1500	0.1500	0.1500	0.1000	0.0000
$P^a(1)$	0.1352	0.1552	0.1584	0.1508	0.1383	0.1504	0.0736	0.0379
$P^a(10)$	0.0577	0.1527	0.1962	0.1450	0.0909	0.1065	0.0203	0.2300
	Portfolio b							
$P^b(0)$	0.0250	0.1000	0.2500	0.2500	0.2500	0.1000	0.0250	0.0000
$P^b(1)$	0.0236	0.1016	0.2490	0.2469	0.2188	0.1158	0.0242	0.0200
$P^b(10)$	0.0161	0.1034	0.2290	0.1942	0.1176	0.1227	0.0220	0.1943
	Portfolio c							
$P^c(0)$	0.0250	0.0500	0.1000	0.1500	0.2000	0.2250	0.2500	0.0000
$P^c(1)$	0.0230	0.0519	0.1079	0.1522	0.1829	0.2282	0.1749	0.0790
$P^c(10)$	0.0126	0.0604	0.1334	0.1328	0.1020	0.1379	0.0286	0.3917
	Portfolio d							
$P^d(0)$	0.2500	0.2250	0.2000	0.1500	0.1000	0.0500	0.0250	0.0000
$P^d(1)$	0.2250	0.2336	0.2077	0.1499	0.0938	0.0575	0.0199	0.0125
$P^d(10)$	0.0937	0.2252	0.2482	0.1541	0.0788	0.0747	0.0129	0.1116

Table 5
Results of the optimization for the 1-year time horizon.

		Portfolio a	Portfolio b	Portfolio c	Portfolio d
Clayton	Parameter	42.514312	42.941125	19.520435	109.527114
	Residuum	0.053975	0.237859	0.032926	0.136884
Gumbel	Parameter	21.888600	19.043297	23.574518	17.272355
	Residuum	0.051853	0.581547	0.046097	0.131391
Frank	Parameter	35.211145	22.602965	36.333209	32.396330
	Residuum	0.168832	1.072155	0.667351	0.497427
Gauß	Parameter	0.974438	0.616887	0.704439	0.664971
	Residuum	1.995890	2.799504	2.682245	2.710970

Table 6
Results of the optimization for the 10-years time horizon.

		Portfolio a	Portfolio b	Portfolio c	Portfolio d
Clayton	Parameter	3.166551	1.844106	1.527723	4.959245
	Residuum	0.104420	0.248017	0.067062	0.261621
Gumbel	Parameter	2.446430	1.838830	2.740726	1.865865
	Residuum	0.084838	0.154729	0.094904	0.083952
Frank	Parameter	7.928077	6.577437	7.514955	8.528203
	Residuum	0.043987	0.141955	0.186208	0.104685
Gauß	Parameter	0.792962	0.543514	0.600013	0.733866
	Residuum	0.051547	0.125408	0.174038	0.047071

4 Conclusion

In this paper we have shown how we can specify the rating process in terms of copula. We defined the rating grade of the firm as random variable. The distribution of this random variable is then given by the composition of portfolio in terms of weights of rating grades. The migration matrix arises from the first derivation of the copula function in both variables. Due to discrete specification of random variables we have to use the interpolation to calculate the migration matrix.

We argued that the transition matrix calculated on the observed migration implies a specific dependence degree in the migration behavior as measured by the parameter of the copula function. Furthermore, we argued that the migration probabilities capture the dependence between migrations of the firms. We find reasonable to assume, that the interaction of the exposures in the portfolio and thereby the dependence in the migration behavior is influenced by the composition of the portfolio. Till now, the transition matrices of the rating process were never related to the composition of the firms in formal framework. Assuming the migrations dependent on the initial distribution, the approach presented here allows to calculate the migration matrix for the specific composition of the portfolio. As far as the dependence degree and therewith the magnitude of risk is determined by the composition of the portfolio, this approach is of great interest.

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