Forward-looking Information Disclosure and Share
Return Volatility in a Multiperiod Model

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Incomplete and Preliminary.
Comments are Welcome!

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Abstract

Can a commitment to disclose forward-looking information reduce a firm’s stock return volatility? This paper addresses this issue. Assume that a firm has private information about the next period dividend, it can decide how to disclose the forward-looking information by adopting full disclosure, no disclosure, or partial disclosure policy. I show that while the expected holding period return is not affected, the \textit{ex ante} volatility of the holding period return is very sensitive to the disclosure policy adopted by the firm. The volatility is calculated for both \textit{unconditional} and \textit{conditional} on the disclosure, where the distinction between the two is based on whether the denominator in the calculation of the holding period return is the price \textit{before} or \textit{after} the disclosure. Such a distinction clarifies whether the holding period includes one or two disclosures, therefore makes a difference for the calculations.
1 Introduction

In competitive capital markets, prices of financial securities reflect all available information about the future cash flows and risks involved. The amount of information released to the market as well as the timing of the information disclosure affect the change of prices, thus affect the volatility of the holding period returns for such securities. While a large body of studies has been devoted to the abnormal returns associated with earnings announcement or other required and discretionary disclosure based on the event study methodology, relatively little attention has been paid to the relationship between disclosure policy and the volatility of holding period returns to shareholders.

Can disclosing more information reduce share return volatility? In a discussion at the year 2000 Journal of Accounting Research Conference on Accounting Information and the Economics of the Firm, Venkatachalam (2000) comments on the lack of theory that supports the direct effect of disclosure practices on volatility. In a recent survey of theoretical models of disclosure, Verrecchia (2001) has not focused on this relationship. Empirical results on this relationship so far are mixed: while Lang and Lundholm (1993) and Healy, Hutton, and Palepu (1999) find a negative relationship between the level of disclosure and stock return volatility, Bushee and Noe (2000) find no such a link. Leuz and Verrecchia (2000) even find that the increased reporting requirement may actually increase volatility, at least among those small less frequently traded stocks. It should be noted that these empirical studies discuss the level of disclosure, without distinguishing whether the information is current or forward-looking.

This paper attempts to establish the direct link between share return volatility and disclosure policy in a simple multi-period setting. Within the semi-strong form Efficient Market framework, it can be shown that although the expected holding period return is not affected, the ex ante volatility of the holding period return is very sensitive to the disclosure policy adopted by the firm.

Why should managers care about the volatility of its shares? First, managers may
have substantial holdings of their personal wealth in the firm’s shares, and cannot fully diversify their portfolio holdings. Thus, excessive volatility of its share may reduce the utility of a risk averse manager or less-diversified shareholders who may have liquidity needs\(^1\). Second, litigation risk is related to the large stock price drop, a special form of share price volatility, and therefore, also affects managerial decision how to disclose information to the market. Class action shareholder lawsuits are particularly a problem for the U.S. high technology companies because their share prices are more volatile and easily become targets for the professional litigants for litigation purposes. Third, the value of equity resembles a call option on the firm’s value (see Jensen and Meckling 1976), and the value of the option is related to the underlying volatility based on the Black-Scholes (1973) option pricing models. The first two considerations may lead to the manager to minimize the volatility, while the third motive leads him to increase the volatility because the option value increases as the volatility increases, according to the option pricing formula.\(^2\) In this paper, I do not intend to fully endogenize the managerial incentive whether to increase or decrease the volatility. Instead, I will simply study the impact of the forward-looking information disclosure policy on the volatility of holding period returns for shares.

In practice, firms may choose to disclose forward looking information when they report earnings or dividends. As a matter of fact, investors and analysts expect managers to share their outlook for the firm’s future outlook. For example, in the recent reporting season, some companies choose to disclose a quite specific forecast of next period profits\(^3\)

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\(^1\)For example, see Stulz (1985) and DeMarzo and Duffie (1991)

\(^2\)However, shareholders’ preference for risky investments are complicated once the bankruptcy risk is explicitly modelled. Under certain circumstances, shareholders may be risk avoiding, see Leland (1994) and Gong (2004).

\(^3\)For example, Bendigo Bank declared it could continue its phenomenal profit growth as its branch network slowly came of age. The bank announced a 36 per cent higher interim profit of $34.6 million, with its generally stronger second half likely to take the full year to $74 million. Chief executive Rob Hunt said hitting the full-year target would amount to 25 per cent growth for the fiscal year. See
, while the others choose to make a less specific disclosure.4

The key assumption in the model is the information asymmetry between outside shareholders and managers. Following Myers and Majluf (1985), I assume that the manager of a publicly traded firm has better information than the outside investors about the realization of a random variable that is the basis for setting the firm’s share price. This variable can be interpreted quite broadly as anything that goes into a standard valuation model, such as “dividends,” “free cash flows,” or “economic earnings.” For modelling purposes, let us refer it as “dividends.”

When a manager reports the current period dividend, the uncertainty associated with the current period operations is resolved. At the mean time, the manager has the option to disclose the information for the next period dividend to a certain degree. He can choose to disclose the information fully, partially, or none at all.

I then compare volatilities of the holding period returns under these three alternative disclosure policies. The volatilities are calculated for both unconditional and conditional on the disclosure, where the distinction between the two is based on the whether the denominator in the calculation of the holding period return is the price before or after the disclosure. Such a distinction clarifies whether the holding period includes one or two disclosure, therefore makes a difference for the calculations.

The rest of the paper is organized as follows. Section 2 describes the model and states the return processes under alternative disclosure policies. Section 3 presents the results of comparisons of both conditional and unconditional volatility for holding period returns. Discussions and conclusions are in Sections 4.


4For example, the Australian Financial Review reported that “ (the CEO) Mr Murray warned shareholders not to expect similar rates of growth in the second half. He said the home loan market would slow after the bank increased its mortgage book by 12 per cent since June 30, 2003, to $112 billion at December 31. See “CBA Customer Focus Pays Dividends,” 12 February 2004, Australian Financial Review.
2 The Model

We will adopt a simple model which crystalizes the role of disclosing forward-looking information on the volatility of the firm’s share return. For this purpose, let us assume that the market is semi-strong form efficient. That is, the market price reflects all publicly available information.

Consider the following dividend process. Given the dividend announcement at time \( t \) at the level of \( D_t \), the dividend at the subsequent period will take three values with equal probability: either move up to \((1 + x)D_t \) \((x > 0)\), or move down to \((1 - x)D_t \), or remain the same at the level of \( D_t \). We term the three states as “G”, “B”, and “M”, respectively. That is, the dividend process follows a trinomial process, which is a discrete version of the arithmetic Brownian Motion process with a drift of zero and variance of \( \frac{2}{3}x^2 D_t^2 \):

\[
D_{t+1} = \begin{cases} 
(1 + x)D_t & \text{in state } G; \\
D_t & \text{in state } M; \\
(1 - x)D_t & \text{in state } B. 
\end{cases}
\]  

The manager of the firm has better information than outside investors. Let us assume that the manager has precise private information and knows the exact dividend payment next period but he has no information with regard to the dividend payment beyond the next one. Should he disclose this information or not? To what extent? And what is the impact of the disclosure to the holding period return volatility? These are the questions we would like to answer in this paper.

Let us start with an arbitrary dividend announcement date, \( T = t \). Without disclosing forward-looking information, the \textit{ex-dividend} price of the share at \( T = t \) is the discounted expected future dividend payments:

\[
P_{t}^{ex} = \sum_{\tau=1}^{\infty} \frac{E(D_{t+\tau} \mid D_t)}{(1 + r)^\tau} = \frac{D_t}{r},
\]

which is the result that the share price is a Martingale. \( r \) is the discount rate.
In the paper, we adopt the notation $t_-$ to mean time $t$ right before the disclosure and the notation $t_+$ to denote the time $t$ right after the disclosure. These two prices can be interpreted as the left limit and right limit when time $T$ approaches $t$, respectively. Therefore, $P_{t_+}^{ex}$ will reflect the dividend announcement $D_t$ as well as the information set about the next period dividend $D_{t+1}$.

The timeline of the event is in the Table 1.

**Table 1: Timeline**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_-$</td>
<td>right before the announcement at $t$</td>
<td>$P_{t_-}^{ex}$</td>
</tr>
<tr>
<td>$t$</td>
<td>the announcement</td>
<td></td>
</tr>
<tr>
<td>$t_+$</td>
<td>right after the announcement at $t$</td>
<td>$P_{t_+}^{ex}$</td>
</tr>
</tbody>
</table>

The manager has three alternative strategies for forward-looking information disclosure about $D_{t+1}$ at time $t$:

- **full disclosure** — reveals the true state of nature for $D_{t+1}$ at time $t$;
- **partial disclosure** — reveals the true state of nature for $D_{t+1}$ either in the upper half of the range, $\{G, M\}$, or the lower range $\{M, B\}$ at time $t$;
- **no disclosure** — does not reveal any information about $D_{t+1}$ at time $t$.

Such a classification follows the tradition of information economics: the partial disclosure policy represents a finer partition of the information set, which includes the true state of nature. Furthermore, this classification excludes the situation where the manager can make a deceitful statement, which is against disclosure regulations. Following the partial information disclosure strategy, the manager will indicate the upper range if
the true signal indicates a state of $G$ in the next period. Similarly, he will indicate the lower range if the true signal indicates a state of $B$ in the next period. However, when he receives a signal of $M$ for the next period, he should use a randomized strategy and indicate the upper range and the lower range with an equal probability.

Now let us study the impact of these alternative disclosure strategies on the volatility of the holding period returns, respectively.

### 2.1 Full Disclosure

Suppose that the manager receives “good news” that the dividend will move up in the next period. If the manager discloses the information truthfully and promptly that the next period dividend will be at the level of $(1 + x)D_t$, then the ex-dividend price at $t$, conditional on the disclosure of good news about dividend, is $P_{t+1}^{ex} = (1 + x)D_t/r$. The cumulative price of the share at $t + 1$ will be

$$D_{t+1} + P_{t+1}^{ex} = \begin{cases} (1 + x)D_t + \frac{(1+x)^2D_t}{r}, & D_{t+2} \text{ in state } G; \\ (1 + x)D_t + \frac{(1+x)D_t}{r}, & D_{t+2} \text{ in state } M; \\ (1 + x)D_t + \frac{(1-x)(1+x)D_t}{r}, & D_{t+2} \text{ in state } B, \end{cases}$$

which reflects the fact that $D_{t+2}$ again follows the dividend process (1). Note that $P_{t+1}^{ex}$ is the expected ex-dividend price at $t+1$. For simplicity, I omit the expectations operator throughout the paper.

Thus, the holding period rate of return for the share post disclosure and conditional on the state of $D_{t+1}$ being in state $G$ is

$$r_{t+1} = \frac{D_{t+1} + P_{t+1}^{ex}}{P_{t+1}^{ex}} - 1 = \begin{cases} \frac{(1+x)D_{t+1} + (1+x)^2D_t}{(1+x)D_t} - 1 = r + x & D_{t+2} \text{ in state } G; \\ \frac{(1+x)D_{t+1} + (1+x)D_t}{(1+x)D_t} - 1 = r & D_{t+2} \text{ in state } M; \\ \frac{(1+x)D_{t+1} + (1-x)(1+x)D_t}{(1+x)D_t} - 1 = r - x & D_{t+2} \text{ in state } B; \end{cases}$$

The holding period rate of return calculated this way contains one set information: the next period dividend payment.
Similarly, suppose the manager receives a signal of a flat dividend for the next period and disclose it. Then the next period dividend will remain at the level of \( D_t \) and the *ex dividend* price at \( t \) will be \( P_{t+1}^{ex} = D_t / r \). The cumulative price of the share at \( t + 1 \) conditional on the disclosure of state \( M \) will be

\[
D_{t+1} + P_{t+1}^{ex} = \begin{cases} 
D_t + \frac{(1+x)D_t}{r}, & \text{if } D_{t+2} \text{ in state } G; \\
D_t + \frac{D_t}{r}, & \text{if } D_{t+2} \text{ in state } M; \\
D_t + \frac{(1-x)D_t}{r}, & \text{if } D_{t+2} \text{ in state } B.
\end{cases}
\]  

Thus, the holding period rate of return for the share *post disclosure* and conditional on the state of \( D_{t+1} \) being in state \( M \) is

\[
r_{t+1} = \frac{D_{t+1} + P_{t+1}^{ex}}{P_{t+1}^{ex}} - 1 = \begin{cases} 
\frac{D_{t+1} + (1+x)D_t}{(1-x)D_t} - 1 = r + x & \text{if } D_{t+2} \text{ in state } G; \\
\frac{D_{t+1} + D_t}{D_t} - 1 = r & \text{if } D_{t+2} \text{ in state } M; \\
\frac{D_{t+1} + (1-x)D_t}{(1-x)D_t} - 1 = r - x & \text{if } D_{t+2} \text{ in state } B.
\end{cases}
\]  

In the case the manager gets a “bad” signal and discloses it, then the next period dividend will be at the level of \( (1-x)D_t \) and the *ex dividend* price at \( t \), conditional on the disclosure of bad news, will be \( P_{t+1}^{ex} = (1-x)D_t / r \). The cumulative price of the share at \( t + 1 \) will be

\[
D_{t+1} + P_{t+1}^{ex} = \begin{cases} 
(1-x)D_t + \frac{(1-x)(1+x)D_t}{r}, & \text{if } D_{t+2} \text{ in state } G; \\
(1-x)D_t + \frac{(1-x)D_t}{r}, & \text{if } D_{t+2} \text{ in state } M; \\
(1-x)D_t + \frac{(1-x)^2D_t}{r}, & \text{if } D_{t+2} \text{ in state } B.
\end{cases}
\]  

Thus, the holding period rate of return for the share *post disclosure* and conditional on the state of \( D_{t+1} \) being in state \( B \) is

\[
r_{t+1} = \frac{D_{t+1} + P_{t+1}^{ex}}{P_{t+1}^{ex}} - 1 = \begin{cases} 
\frac{(1-x)D_t + (1-x)(1+x)D_t}{(1-x)D_t} - 1 = r + x & \text{if } D_{t+2} \text{ in state } G; \\
\frac{(1-x)D_t + (1-x)D_t}{(1-x)D_t} - 1 = r & \text{if } D_{t+2} \text{ in state } M; \\
\frac{(1-x)D_t + (1-x)^2D_t}{(1-x)D_t} - 1 = r - x & \text{if } D_{t+2} \text{ in state } B.
\end{cases}
\]  

We see that regardless of the signal of the dividend at \( t + 1 \), if the manager receives it at \( t \) and discloses it right away, the holding period return for shareholders can take
three values, \(r + x, r\), and \(r - x\) with equal probability. The mean of the holding period return is \(r\) and the conditional variance is \(\frac{2}{3} x^2\).

Alternatively, let us exam the holding period rate of return based on the price before the disclosure taking place and calculate the unconditional variance of the share return. Note that

\[
P_{t-} = D_{t-1} + \frac{D_{t-1}}{r} = D_{t-1}(1 + \frac{1}{r}).
\]

We assume that the dividend received in \(T = t\) will be reinvested in the same company’s shares. Therefore the rate of return with reinvestment of dividend is

\[
r_{t+1} = \left( \frac{P_{t+} + D_t}{P_{t-}} \cdot \frac{P_{t+1} + D_{t+1}}{P_{t+}} \right) - 1.
\]

With full disclosure, the holding period of return can be calculated as \(^5\)

\[
r_{t+1} = \begin{cases} 
(1 + x)r + (1 + x)^2 - 1 & \text{with } p_1 = 1/9; \\
(1 + x)r + (1 + x) - 1 & \text{with } p_2 = 1/9; \\
(1 + x)r + (1 + x)(1 - x) - 1 & \text{with } p_3 = 1/9; \\
r + x & \text{with } p_4 = 1/9; \\
r - x & \text{with } p_5 = 1/9; \\
(1 - x)r + (1 - x)(1 + x) - 1 & \text{with } p_7 = 1/9; \\
(1 - x)r - (1 - x) - 1 & \text{with } p_8 = 1/9; \\
(1 - x)r + (1 - x)^2 - 1 & \text{with } p_9 = 1/9;
\end{cases}
\]

\(^5\)The return process of (9) is derived by comparing the cumulative price of the share at \(t + 1\) with the share price before the disclosure, \(P_{t-}\). To see the calculation more clearly, let us work out the first scenario. At time \(t\), the dividend will have a probability of 1/3 at the level of \((1 + x)D_{t-1}\). At this level of dividend, the manager only has another 1/3 probability to receive a good signal and disclose it. The dividend at \(t + 1\) will be \((1 + x)^2D_{t-1}\). Because of the full disclosure, \(\frac{P_{t+} + D_t}{P_{t-}} = 1 + r\), and

\[
\frac{P_{t+} + D_t}{P_{t-}} = \frac{D_{t-1}(1 + x)^2/r + D_{t-1}(1 + x)}{D_{t-1}(1 + 1/r)},
\]

the holding period rate of return under this scenario will be \((1 + x)^2 + (1 + x)r - 1\) with a probability of 1/9. The calculations for the remaining scenarios follow the same steps.
Again the mean of the holding period return is $r$. I have verified in all of the
remaining calculations that the mean of the holding period return remains $r$, regardless
of the disclosure policy, and thus will omit this result in the remaining of this paper.

The *unconditional* variance of the holding period return following the full disclosure
policy is

$$
\sigma^2_{t+1 | \text{full}} = x^2 \left( 1 + \frac{2}{3} r \right)^2 + \frac{4}{9} x^4 + \frac{1}{3} x^2 + \frac{2}{9} r^2.
$$

### 2.2 No Disclosure

Suppose the manager does not disclose the signal of about the next period dividend to
the public. The *ex dividend* price at $t$ will be $P_{t+1}^{ex} = D_t/r$, but the cumulative price of
the share at $t + 1$ will be

$$
D_{t+1} + P_{t+1}^{ex} = \begin{cases} (1 + x) D_t + \frac{(1 + x) D_t}{r}, & D_{t+2} \text{ in state } G; \\ D_t + \frac{D_t}{r}, & D_{t+2} \text{ in state } M; \\ (1 - x) D_t + \frac{(1 - x) D_t}{r}, & D_{t+2} \text{ in state } B. \end{cases}
$$

Thus, the *conditional* holding period rate of return for the share is

$$
r_{t+1} = \frac{D_{t+1} + P_{t+1}^{ex}}{P_{t+1}^{ex}} - 1 = \begin{cases} \left( 1 + x \right) \frac{D_t}{r} + \frac{(1 + x) D_t}{r} - 1 = (1 + x) r + x & \text{in state } G; \\ \frac{D_t + \frac{D_t}{r}}{r} - 1 = r & \text{in state } M; \\ \frac{(1 - x) D_t + \frac{(1 - x) D_t}{r}}{r} - 1 = (1 - x) r - x & \text{in state } B; \end{cases}
$$

We see that regardless of the signal of the dividend at $t + 1$, if the manager receives
it at $t$ and does not disclose it, the holding period return for shareholders can take three
values with equal probability as in Equation (11). The variance is $\frac{2}{3} x^2 \left( 1 + r \right)^2$.

If the firm commits a no-disclosure policy, then the *unconditional* rate of return can
be expressed as

\[
    r_{t+1} = \begin{cases} 
        (1 + x)^2(1 + r) - 1 & \text{with } p_1 = 1/9; \\
        (1 + x)(1 + r) - 1 & \text{with } p_2 = 1/9; \\
        (1 + x)(1 - x)(1 + r) - 1 & \text{with } p_3 = 1/9; \\
        (1 + x)(1 + r) - 1 & \text{with } p_4 = 1/9; \\
        r & \text{with } p_5 = 1/9; \\
        (1 - x)(1 + r) - 1 & \text{with } p_6 = 1/9; \\
        (1 - x)(1 + x)(1 + r) - 1 & \text{with } p_7 = 1/9; \\
        (1 - x)(1 + r) - 1 & \text{with } p_8 = 1/9; \\
        (1 - x)^2(1 + r) - 1 & \text{with } p_9 = 1/9; 
    \end{cases}
\] (12)

The unconditional variance of \( r_{t+1} \) can be calculated as

\[
    \sigma^2_{r_{t+1}} |_{\text{no disclosure}} = \frac{4}{9} x^2 (x^2 + 3)(1 + r)^2. \quad (13)
\]

### 2.3 Partial Disclosure

When the manager receives a signal, he may choose to disclose the information partially. His disclosure strategy is to indicate that dividend is either in the upper range \( \{G, M\} \), or in the lower range \( \{M, B\} \). If the signal is \( G \), he discloses it to be in the set of \( \{G, M\} \). If the signal is \( B \), he discloses it to be \( \{M, B\} \). If the signal is \( M \), he randomizes and indicates the upper range or the lower range of dividend with equal probability.

The market, updating its belief of the share value following the Bayesian rule, figures out that the expected dividend at \( t + 1 \) is in state \( G \) with a probability of \( 2/3 \) and \( M \) with a probability of \( 1/3 \) when the manager indicates the upper range. Similarly, the market believes that the true dividend will be in the state of \( M \) with a probability of \( 1/3 \) and \( B \) with a probability of \( 2/3 \).

Given this information,

\[
    P_{t+1}^{ex} |_{\{G, M\}} = \frac{\frac{2}{3}(1 + x)D_t + \frac{1}{3}D_t}{r} = \frac{3 + 2x D_t}{3r}, \quad (14)
\]
and

\[ P_{t+1}^{ex} |_{\{M,B\}} = \frac{\frac{2}{3}(1 - x)D_t + \frac{1}{3}D_t}{r} = \frac{3 - 2x}{3} \frac{D_t}{r}. \] (15)

The cumulative price process at \( t + 1 \) given the upper range disclosure is

\[
D_{t+1} + P_{t+1}^{ex} |_{\{G,M\}} = \begin{cases} 
(1 + x) D_t + \frac{3+2x (1+x)}{3} \frac{D_t}{r} & \text{with } p_1 = 1/3; \\
(1 + x) D_t + \frac{3-2x (1+x)}{3} \frac{D_t}{r} & \text{with } p_2 = 1/3; \\
D_t + \frac{3+2x}{3} \frac{D_t}{r} & \text{with } p_3 = 1/6; \\
D_t + \frac{3-2x}{3} \frac{D_t}{r} & \text{with } p_4 = 1/6. 
\end{cases}
\] (16)

The rate of return conditional on the disclosure of the upper range is

\[
r_{t+1} |_{\{G,M\}} = \begin{cases} 
\frac{3x(1+x)}{3+2x} + x & \text{with } p_1 = 1/3; \\
\frac{3x(1+x) - x - 2x^2}{3+2x} & \text{with } p_2 = 1/3; \\
\frac{3x}{3+2x} & \text{with } p_3 = 1/6; \\
\frac{3x^4 - 16x^3}{3+2x} & \text{with } p_4 = 1/6. 
\end{cases}
\] (17)

The variance of the holding period rate of return, conditional on the firm’s disclosure of the upper range for the next period dividend, is

\[
\sigma_{t+1}^2 |_{\{G,M\}} = \frac{2x^2 r^2 + 4x^2 r + 6x^2 + \frac{16}{3} x^3 + \frac{8}{3} x^4}{(3 + 2x)^2}
\]

Similarly, the cumulative price process at \( t + 1 \) given the lower range disclosure is

\[
D_{t+1} + P_{t+1}^{ex} |_{\{M,B\}} = \begin{cases} 
(1 - x) D_t + \frac{3+2x (1-x)}{3} \frac{D_t}{r} & \text{with } p_1 = 1/3; \\
(1 - x) D_t + \frac{3-2x (1-x)}{3} \frac{D_t}{r} & \text{with } p_2 = 1/3; \\
D_t + \frac{3+2x}{3} \frac{D_t}{r} & \text{with } p_3 = 1/6; \\
D_t + \frac{3-2x}{3} \frac{D_t}{r} & \text{with } p_4 = 1/6. 
\end{cases}
\] (18)

The rate of return conditional on the disclosure of the lower range is

\[
r_{t+1} |_{\{M,B\}} = \begin{cases} 
\frac{3(1-x)r + x - x^2}{3-2x} & \text{with } p_1 = 1/3; \\
\frac{3(1-x)r - x}{3-2x} & \text{with } p_2 = 1/3; \\
\frac{3r+4x}{3-2x} & \text{with } p_3 = 1/6; \\
\frac{3r}{3-2x} & \text{with } p_4 = 1/6. 
\end{cases}
\] (19)
The variance of the holding period rate of return, conditional on the firm’s disclosure of the lower range for the next period dividend, is

\[
\sigma_{t+1}^2 \mid \{M,B\} = \frac{2x^2 r^2 - 4x^2 r + 6x^2 - \frac{16}{3}x^3 + \frac{8}{3}x^4}{(3 - 2x)^2}
\]

**Note:** The unconditional volatility for the holding period return based on pre-disclosure share price will be completed shortly.

### 3 Comparision of Volatilities

In all scenarios discussed so far, the expected holding period returns are exactly the same, namely, \(r\). However, the disclosure policies have resulted in different volatilities.

In summary, for unconditional variance,

\[
\sigma_{t+1}^2 \mid \text{full disclosure} = x^2(1 + \frac{2}{3} r)^2 + \frac{4}{9}x^4 + \frac{1}{3}x^2 + \frac{2}{9}r^2 \\
\sigma_{t+1}^2 \mid \text{no disclosure} = \frac{4}{9}x^2(x^2 + 3)(1 + r)^2 \\
\sigma_{t+1}^2 \mid \text{partial disclosure} = \text{to be completed}
\]  

**Proposition 1:** Under the assumptions made in this paper, we can rank the unconditional variance,

\[
\sigma_{t+1}^2 \mid \text{full disclosure} < \sigma_{t+1}^2 \mid \text{no disclosure}
\]

**Proof:** By inspection of Equations (9) and (12), we see the result.

Note that, if \(r = 0\), then the unconditional volatilities calculated under both no disclosure and full disclosure policy will be the same.

For conditional variance, we have obtained the following relationships:

\[
\sigma_{t+1}^2 \mid \text{full disclosure} = \frac{2}{3}x^2
\]
\[ \sigma_{T+1}^2 \mid \text{no disclosure} = \frac{2}{3} x^2 (1 + r)^2 \]  
\[ \sigma_{T+1}^2 \mid \text{partial disclosure upper} = \frac{2x^2 r^2 + 4x^2 r + 6x^2 + \frac{16}{3} x^3 + \frac{8}{3} x^4}{(3 + 2x)^2} \]  
\[ \sigma_{T+1}^2 \mid \text{partial disclosure lower} = \frac{2x^2 r^2 - 4x^2 r + 6x^2 - \frac{16}{3} x^3 + \frac{8}{3} x^4}{(3 - 2x)^2} \]  

It is obvious that the full disclosure policy will lead to a smaller conditional volatility than a no-disclosure policy. Furthermore, the comparison between the volatility under either no disclosure policy, or full disclosure policy with the volatility under partial disclosure policy yields no clear ordering. The exact comparison depends on the parameter values of \( r \) and \( x \).

It can be easily verified that if

\[ 3r^2 + 6r - 4x > 0, \]

then the conditional volatility under partial disclosure of upper range will be larger than that under the full disclosure policy. For example, this occurs when \( r = 0.1 \) and \( x < 0.1575 \). A similar relationship can be derived for the comparison between the conditional volatility under partial disclosure of lower range and that under the full disclosure policy.

Table 2 lists the results by assuming \( r = 0.1 \) and various values for \( x \).
Table 2: Summary of Conditional Variance under Alternative Disclosure Policy. \( r = 0.1 \).

<table>
<thead>
<tr>
<th>Disclosure</th>
<th>( x = 0.1 )</th>
<th>( x = 0.3 )</th>
<th>( x = 0.5 )</th>
<th>( x = 0.7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>0.006667</td>
<td>0.06</td>
<td>0.166667</td>
<td>0.326667</td>
</tr>
<tr>
<td>No</td>
<td>0.008067</td>
<td>0.0726</td>
<td>0.201667</td>
<td>0.395267</td>
</tr>
<tr>
<td>Partial Up</td>
<td>0.006816</td>
<td>0.057361</td>
<td>0.152396</td>
<td>0.290052</td>
</tr>
<tr>
<td>Partial Down</td>
<td>0.006522</td>
<td>0.066563</td>
<td>0.22625</td>
<td>0.611224</td>
</tr>
</tbody>
</table>

Table 3 lists the results by assuming \( x = 20\% \) and various values for the discount rate \( r \).

Table 3: Summary of Conditional Variance under Alternative Disclosure Policy. \( x = 20\% \).

<table>
<thead>
<tr>
<th>Disclosure</th>
<th>( r = 0.05 )</th>
<th>( r = 0.075 )</th>
<th>( r = 0.1 )</th>
<th>( r = 0.125 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full</td>
<td>0.026667</td>
<td>0.026667</td>
<td>0.026667</td>
<td>0.026667</td>
</tr>
<tr>
<td>No</td>
<td>0.0294</td>
<td>0.030817</td>
<td>0.032267</td>
<td>0.03375</td>
</tr>
<tr>
<td>Partial Up</td>
<td>0.025531</td>
<td>0.025898</td>
<td>0.026275</td>
<td>0.026659</td>
</tr>
<tr>
<td>Partial Down</td>
<td>0.028669</td>
<td>0.028114</td>
<td>0.027574</td>
<td>0.027049</td>
</tr>
</tbody>
</table>

These results show that the conditional volatility for the partial disclosure is \textit{path dependent}. Comparing Equation (25) with Equation (26) does not give us an unambiguous result which one is larger.

To be more precise about the conditional volatility in response to the change of the discount rate and the dispersion in the dividend process, the partial derivatives are
obtained. Tables 2 and 3 simply give snapshots of the comparative static results.

**Proposition 2:** The comparative static analysis reveals that for the *unconditional* volatility, all of the first-order partial derivatives with respect to \( r \) and \( x \) are positive. However, for the *conditional* volatility,

\[
\frac{\partial \sigma^2_{T+1} |_{fuil}}{\partial x} > 0 \quad (28)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{no}}{\partial x} > 0 \quad (29)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{p, up}}{\partial x} < 0 \quad (30)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{p, dn}}{\partial x} \quad (31)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{fuil}}{\partial r} = 0 \quad (32)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{no}}{\partial r} > 0 \quad (33)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{p, up}}{\partial r} > 0 \quad (34)
\]
\[
\frac{\partial \sigma^2_{T+1} |_{p, dn}}{\partial r} < 0 \quad (35)
\]

From Proposition 2, it is clear that with an increased dispersion in the dividend process, the *conditional* volatility of the holding period return will increase as well under either full or no-disclosure policy, but it will decrease under partial disclosure policy when the firm indicates an upper range for the next dividend. The volatility under partial disclosure policy when the firm indicates a lower range has an ambiguous relationship with respect to the increased dispersion in the dividend process. In general, while it is true that the increased volatility in the dividend process and the increased level of the discount rate will lead to an increased holding period *conditional* volatility, we cannot make a similarly broad statement for the *unconditional* volatility for the holding period.
return. The second half of Proposition 2 is a result that, under the consideration, we have the resolution of two combined sources of uncertainty: one is for the current dividend process, the other is for the next one.

4 Conclusions

Traditional economic theory believes that, by mitigating information asymmetry, increased level of disclosure reduces the magnitude of surprises about a firm’s performance and makes its stock return less volatile. In this paper, I have examined this proposition for a specific class of information, i.e., forward-looking information.

Assuming that a firm has private information about its next period dividend, I have calculated the volatility of the holding period rate of return under three alternative disclosure policies. The volatilities are calculated for both unconditional and conditional on the disclosure, where the distinction between the two is based on whether the denominator in the discounting is the price before or after the disclosure. Such a distinction clarifies whether the holding period includes one or two disclosures, therefore makes a difference for the calculations and results in different ordering relationships of the volatilities among the alternative disclosure policies.

The results show that, although the disclosure polices does not affect the expected return for the shareholders, the volatility of the holding period returns will be quite different under alternative disclosure strategies adopted by the firm. Some of these results are not intuitive. For example, I have shown that the full disclosure information policy may not lead to a lowest stock return volatility as in the simulated results contained in Tables 2 and 3, contrary to the popular beliefs.

For under-diversified investors and managers, their utility functions are affected by the volatilities of stocks they hold. Understanding the volatility of the holding period rate of return for the stocks is important. This paper shows the complexity of the volatility calculation, whether it is based on before or after the disclosure, and how it is
impacted by the disclosure policy adopted by the firm. Clearly, further works need to be done in this area.
References


