SHORT-RUN DEVIATIONS AND TIME-VARYING HEDGE RATIOS: EVIDENCE FROM AGRICULTURAL FUTURES MARKETS

By

TAUFIQ CHOUDHRY

School of Management
University of Bradford
Emm Lane
Bradford BD9 4JL
UK

Phone: (44) 1274-234363
Fax: (44) 1274-235680
Email: T.Choudhry@bradford.ac.uk

ABSTRACT

This paper investigates the hedging effectiveness of time-varying hedge ratios in the agricultural commodities futures markets based on four different versions of the GARCH models. The GARCH models applied are the standard bivariate GARCH, the bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X models are uniquely different from the other two models because they takes into consideration effect of the short-run deviations from the long-run relationship between the cash and the futures prices on the second conditional moments of the bivariate distribution of the variable. Futures data for corn, coffee, wheat, sugar and soybeans are applied. Comparison of the hedging effectiveness is done for the with-in sample period (1980-2004) and two out-of-sample periods performance. Overall results indicate that all GARCH models perform similar during the shorter out-of-sample period (2003-2004) while during the with-in sample period and the longer out-of-sample period (2002-2004) the standard GARCH model out performs the other methods.

JEL Classification: G1, G13, G15

Key Words: Hedge Ratio, GARCH, BEKK GARCH, GARCH-X, BEKK GARCH-X and Variance

1. Introduction

The rapid expansion of derivatives markets over the last twenty-five years has led to a corresponding increase in interest in the theory and practice of hedging. Numerous empirical and statistical methods are applied to estimate hedge ratios in the futures markets. The traditional constant hedge ratio obtained by means of the ordinary least square (OLS) has been discarded as being inappropriate, because it ignores the heteroskedasticity often encountered in price series. Baillie and Myers (1991) further claim that if the joint distribution of cash price and futures prices is changing over time, estimating a constant hedge ratio may not be appropriate. In other words, the hedge ratios will certainly vary over time as the conditional distribution between cash and futures prices changes. Recently, autoregressive conditional heteroskedastic (ARCH) and the generalized ARCH (GARCH) have been applied to estimate time-varying hedge ratios in the futures markets (see Choudhry (2004), Moschini and Myers (2002), Gagon et al. (1998), Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1990), Gagnon and Lypny (1995), Park and Switzer (1995) and Tong (1996)). The optimal hedge ratios estimated by means of the GARCH models is time varying, because these models take into consideration the time-varying distribution of the cash and futures price changes.

This paper investigates and compares the risk-reducing ability of different optimal time-varying hedge ratios for the futures of five agricultural commodities: corn, coffee, wheat, sugar and soybeans. An optimal hedge ratio is defined as the proportion of a cash position that should be covered with an opposite position on a futures market. When using a futures contract in order to hedge a portfolio of risky assets, the primary objective is to estimate the size of the short position that must be held in the futures market, as a proportion of the long position held in the spot market, that maximises the agent’s expected utility, defined over the risk and expected return of the hedged portfolio.

In this paper, the (time-varying) optimal hedge ratios are estimated using four different types of the generalized autoregressive conditional heteroscedasticity (GARCH) models: the standard bivariate GARCH, bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X models are different from the other two GARCH models because they takes into consideration the effects of the short-run deviations from the long-run relationship between the cash and futures prices on the conditional variance and covariance (second conditional moments of the bivariate distribution) of log difference of the cash and the futures prices. The BEKK GARCH and the BEKK GARCH-X models are also unique because they allow time variation in the conditional correlations as well as the conditional variance. All GARCH methods applied take into consideration the effects of the short-run deviations on the first moment (mean) of the bivariate distributions of the variables. The short-run deviations are represented by the error correction term from a cointegration relationship between the cash and the futures prices.[1]

In this paper long-run relationship between the commodities cash price and the futures price is conducted by means of the Engle and Granger (1987) cointegration test. Long-run stationary relationship (cointegration) between the cash price and the futures price has been extensively investigated.[2] Brenner and Kroner (1995) claim that cointegration between cash and futures prices is likely to hold in currency markets, but not in commodity markets. But, Yang et al. (2001) are able to show cointegration in the

1 Cointegration implies that in a long-run relationship between two or more non-stationary variables, it is required that these variables should not move too far apart from each other. Such nonstationary variables might drift apart in the short run, but in the long run they are constrained. Brenner and Kroner (1995) present a model and conditions under which spot and futures prices may be cointegrated. Yang et al. (2001) present a model and conditions under which spot and future prices of storable commodities may be cointegrated.

2 See Kroner and Sultan (1993), Brenner and Kroner (1995) and Yang et al. (2001) for citation of papers investigating cointegration between cash and futures prices. Baillie and Myers (1991), Covey and Bessler (1992), Fortenbery and Zapata (1993, 1997) provide a study of cointegration between commodities spot and future prices.

commodities' market. Yang et al. (2001) further claim that prevalent cointegration between cash and futures prices on commodity markets suggest that cointegration should be incorporated into commodity hedging decision.[3] Even when the GARCH effect is considered, allowance for the existence of cointegration is argued to be an indispensable component when comparing ex post performance of various hedging strategies.

The main contribution of the paper is to investigate the effects of short-run deviations from the long-run relationship between cash price and futures price on the second moment of the bivariate distributions and the optimal hedge ratio, by means of the GARCH-X and the BEKK GARCH-X models. To our knowledge no other paper applies the GARCH-X and/or the BEKK GARCH-X in the estimation and comparison of time varying hedge ratios for agricultural futures market. The risk-reducing effectiveness of the time-varying hedge ratios is investigated by checking the with-in sample period (1980-2004) and two out-of-sample periods performance of the ratios. The hedging effectiveness is estimated and compared by checking the variance of the portfolios created using these hedge ratios. The lower the variance of the portfolio, the higher is the hedging effectiveness of the hedge ratio.

2. Optimal Hedge Ratios

The following section describes the optimal hedge ratio, relying heavily on Cecchetti et al. (1988) and Baillie and Myers (1991). The returns on the portfolio of an investor trying to hedge some proportion of the cash position in a futures market can be represented by:

$$r_t = r_t^c - \beta_{t-1} r_t^f \tag{1}$$

3 Ghosh (1995), Ghosh and Clayton (1996) and Kroner and Sultan (1993) have shown that hedge ratios and hedging performance may change considerably if cointegration between the cash and futures prices is omitted from the statistical models and estimations.

Where r_t is the return holding the portfolio of cash and futures position between t-1 and t; r_t^c is the return on holding the cash position for the same period; r_t^f is the return on holding the futures position for the same period; and β_{t-1} is the hedge ratio. The variance of the return on the hedged portfolio is give by

$$Var(r_t/\Omega_{t-1}) = Var(r_t^c/\Omega_{t-1}) + \beta^2_{t-1}Var(r_t^f/\Omega_{t-1}) - 2\beta_{t-1}Cov(r_t^c,r_t^f/\Omega_{t-1}) \qquad (2)$$

where Ω_{t-1} presents the information available over the last period. As indicated by Cecchetti et al. (1988), the return on a hedged position will normally be exposed to risk caused by unanticipated changes in the relative price between the position being hedged and the futures contract. This 'basis risk' ensures that no hedge ratio completely eliminates risk. The hedge ratio that minimises risk may be obtained by setting the derivative of equation 2 with respect to β equal to zero. The hedge ratio β_{t-1} can then be expressed as:

$$\beta_{t-1} = Cov(r_t^c,r_t^f/\Omega_{t-1})/Var(r_t^f/\Omega_{t-1}). \qquad (3)$$

The value of β_{t-1}, which minimises the conditional variance of the hedged portfolio return, is the optimal hedge ratio (Baillie and Myers, 1991). Commonly, the value of the hedge ratio is less than unity, so that the hedge ratio that minimises risk in the absence of basis risk turns out to be dominated by β when basis risk is taken into consideration.[4]

4 According to Cecchetti et al. (1988), the optimal hedge ratio β can be expressed as $\rho\sigma^c/\sigma^f$, where ρ is the correlation between futures price and cash price, σ^c is the cash standard deviation and σ^f is the futures standard deviation. Thus, if the futures have the same or higher price volatility than the cash, the hedge ratio can be no greater than the correlation between them, which will be less than unity.

3. Bivariate GARCH, BEKK GARCH, GARCH-X and BEKK GARCH-X Models

3.1 Bivariate GARCH

As shown by Baillie and Myers (1991) and Bollerslev et al. (1992), weak dependence of successive asset price changes may be modelled by means of the GARCH model. The multivariate GARCH model uses information from more than one market's history. According to Engle and Kroner (1995), multivariate GARCH models are useful in multivariate finance and economic models, which require the modelling of both variance and covariance. Multivariate GARCH models allow the variance and covariance to depend on the information set in a vector ARMA manner (Engle and Kroner, 1995). This, in turn, leads to the unbiased and more precise estimate of the parameters (Wahab, 1995).

The following bivariate GARCH(p,q) model may be used to represent the log difference of the cash (spot) and futures prices:

$$y_t = \mu + \delta(z_{t-1}) + \varepsilon_t \tag{4}$$

$$\varepsilon_t/\Omega_{t-1} \sim N(0, H_t) \tag{5}$$

$$vech(H_t) = C + \sum_{j=1}^{p} A_j vech(\varepsilon_{t-j})^2 + \sum_{j=1}^{q} B_j vech(H_{t-j}) \tag{6}$$

Where $y_t = (r_t^{\,c}, r_t^{\,f})$ is a (2x1) vector containing the log difference of the cash (r_t^c) price and futures (r_t^f) prices; H_t is a (2x2) conditional covariance matrix; C is a (3x1) parameter vector (constant); A_j and B_j are (3x3) parameter matrices; and vech is the column stacking operator that stacks the lower triangular portion of a symmetric matrix. The error correction term (z_t) from the cointegration represents the short-run deviations from a long-run relationship between the cash price and the futures price.[5] A significant and positive coefficient (δ) on the error term implies an increase in short-run deviations raises the log difference of cash and/or future prices. Opposite is true if the error term coefficient is negative and significant. Thus the GARCH(1,1) model applied here models the first moment of the bivariate distributions of the variables with a bivariate error correction term (see Kroner and Sultan (1993)).[6] As advocated by Baillie and Myers (1991, p. 116), it is vital to let the conditional covariance be time-dependent, as in the bivariate GARCH model, rather than be constant. This ability of the bivariate GARCH model to have time-dependent conditional variance makes it ideal to provide a time-variant hedge ratio.

Given the bivariate GARCH model of the log difference of the cash and the futures prices presented above, the time-varying hedge ratio can be expressed as:

$$\beta_t = \hat{H}_{12,t} / \hat{H}_{22,t} \tag{7}$$

Where $\hat{H}_{12,t}$ is the estimated conditional variance between the log difference of the cash and futures prices, and $\hat{H}_{22,t}$ is the estimated conditional variance of the log difference of the futures prices from the bivariate GARCH model. Given that conditional covariance is time-dependent, the optimal hedge ratio will be time-dependent.

---

5 The following cointegration relationship is investigated by means of the Engle and Granger (1987) method:

$$S_t = \eta + \gamma F_t + z_t$$

where S_t and F_t are log of cash index and futures price index, respectively. The residuals z_t are tested for unit root(s) to check for cointegration between S_t and F_t. The error correction term, which represents the short-run deviations from the long-run cointegrated relationship, has important predictive powers for the conditional mean of the cointegrated series (Engle and Yoo, 1987). Cointegration is found between the log of cash and futures prices for all five commodities. These results are available on request.

6 Bera and Higgins (1993) and Engle and Kroner (1995) provide detailed analysis of multivariate GARCH models.

3.2 Bivariate BEKK GARCH

Lately, a more stable GARCH presentation has been put forward. This presentation is termed by Engle and Kroner (1995) the BEKK model; the conditional covariance matrix is parameterized as

$$\text{vech}(H_t) = C'C + \sum_{K=1}^{K} \sum_{i=1}^{q} A'_{Ki}\varepsilon_{t-i}\,\varepsilon'_{t-i}\,A_{ki} + \sum_{K=1}^{K} \sum_{i=1}^{p} B'_{Kj}\,H_{t-j}B_{kj} \qquad (8)$$

Equations 4 and 5 also apply to the BEKK model and defined as before. In equation 8 A_{ki}, $i=1,\ldots, q$, $k=1,\ldots K$, and B_{kj} $j=1, \ldots p$, $k = 1,\ldots, K$ are all $N \times N$ matrices. This formulation has the advantage over the general specification of the multivariate GARCH that conditional variance (H_t) is guaranteed to be positive for all t (Bollerslev et al. 1994). The BEKK GARCH model is sufficiently general that it includes all positive definite diagonal representation, and nearly all positive definite vector representation. The following presents the BEKK bivariate GARCH(1,1), with K=1.

$$H_t = C'C + A'\varepsilon_{t-1}\,\varepsilon'_{t-1}A + B'H_{t-1}B \qquad (8a)$$

Where C is a 2x2 lower triangular matrix with intercept parameters, and A, and B are 2x2 square matrices of parameters. The bivariate BEKK GARCH(1,1) parameterization requires estimation of only 11 parameters in the conditional variance-covariance structure, and guarantees H_t positive definite. Importantly, the BEKK model implies that only the magnitude of past returns innovations is important in determining current conditional variances and co-variances. The time-varying hedge ratio based on the BEKK GARCH model is also expressed as equation 7.

3.3 Bivariate GARCH-X

Lee (1994) provides an extension of the standard GARCH model linked to an error-correction model of cointegrated series on the second moment of the bivariate distributions of the variables. This model is known as the GARCH-X model. According to Lee (1994), if short-run deviations affect the conditional mean, they may also affect conditional variance, and a significant positive effect may imply that the further the series deviate from each other in the short run, the harder they are to predict. If the error correction term (short-run deviations) from the cointegrated relationship between cash price and futures price affects the conditional variance (and conditional covariance), then conditional heteroscedasticity may be modelled with a function of the lagged error correction term. If shocks to the system that propagate on the first and the second moments change the volatility, then it is reasonable to study the behaviour of conditional variance as a function of short-run deviations (Lee, 1994). Given that short-run deviations from the long-run relationship between the cash and futures prices may affect the conditional variance and conditional covariance, then they will also influence the time-varying optimal hedge ratio, as defined in equation 7.

The following bivariate GARCH(p,q)-X model may be used to represent the log difference of the cash prices and the futures prices:

$$\mathrm{vech}(H_t) = C + \sum_{j=1}^{p} A_j \mathrm{vech}(\varepsilon_{t-j})^2 + \sum_{j=1}^{q} B_j \mathrm{vech}(H_{t-j}) + \sum_{j=1}^{k} D_j \mathrm{vech}(z_{t-1})^2 \qquad (9)$$

Once again equations 4 and 5 (defined as before) also apply to the GARCH-X model. The squared error term (z_{t-1}) in the conditional variance and covariance equation (equation 9) measures the influences of the short-run deviations on conditional variance and covariance.

As advocated by Lee (1994, p. 337), the square of the error-correction term (z) lagged once should be applied in the GARCH(1,1)-X model. The parameters D_{11} and D_{33} indicate the effects of the short-run deviations between the cash and the futures prices from a long-run cointegrated relationship on the conditional variance of the residuals of the log difference of the cash and futures prices, respectively. The parameter D_{22} shows the effect of the short-run deviations on the conditional covariance between the two variables. As stated above, if short-run deviations between cash price and futures price affect the conditional variance of the log difference of the cash and futures prices, and the conditional covariance between the two variables, then optimal hedge, as defined in equation 7, will also be affected. In other words, if D_{33} and D_{22} are significant, then H_{12} (conditional covariance) and H_{22} (conditional variance of futures returns) are going to differ from the standard GARCH model H_{12} and H_{22}. For example, if D_{22} and D_{33} are positive, an increase in short-run deviations will increase H_{12} and H_{22}. In such a case, the GARCH-X time-varying hedge ratio will be different from the standard GARCH time-varying hedge ratio.

3.4 Bivariate BEKK GARCH-X

Similar extension can be made to the standard BEKK GARCH linked to an error-correction model of cointegrated series on the second moment of the bivariate distributions of the variables. Such a model is know as the the BEKK GARCH-X. The formulation of the BEKK GARCH(1,1)-X model is given by

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B + D'Dz^2_{t-1} \tag{10}$$

Equations 4 and 5 apply to this model also and the variables are as defined in the BEKK

GARCH section. Once again the z_t is the error term from the cointegration tests between the cash and futures prices and the D is the (1x2) matrix of coefficients. The analysis of the size and sign on the error term coefficients are the same as described in the bivariate GARCH-X section. The time varying hedge ratio from the BEKK GARCH-X should differ from the standard BEKK hedge ratio. If the four time-varying hedge ratios are different, then the interesting empirical question arises; which one is more effective? All the above methods of estimating the hedge ratios are applied, and their effectiveness is compared in this paper.

4. Data and Basic Statistics

Weekly log difference of the cash (spot) and the futures prices of corn, coffee, wheat, sugar and soybeans are used in the empirical tests. All the data range from August 1980 to July 2004. All futures price indices are continuous series.[7] All data are obtained from *Global Financial Data*. Table 1 (parts A, B and C) shows some of the basic statistics of the four series: log difference of the cash prices and the futures prices, square of the first two series and the cross product of the first two series. The basic statistics are provided for the with-in sample period (1980-2004) and the two out-of-sample periods, 1980-2002 and 1980-2003. Table 1 part A presents the total period statistics and almost all series are significantly skewed and, as expected, all series are found to have significant and positive kurtosis, implying higher peaks and fatter tails. Thus, the Jarque-Bera statistic shows all series to be non-normal. The statistics from the sub-periods table 1 parts B and C also show similar results. All series are found to be non-normal during the two sub-periods. The mean and variance of all four series seem to stay similar across the three periods. This may imply lack of structural breaks in the different series.

7 The continuous series is a perpetual series of futures prices. It starts at the nearest contract month, which forms the first values for the continuous series, until either the contract reaches its expiry date or until the first business day of the actual contract month. At this point, the next trading contract month is taken.

5. Empirical Results

5.1 Bivariate GARCH, BEKK GARCH and GARCH-X Results

Tables 2, 3, 4 and 5 shows the results from the standard bivariate GARCH(1,1), BEKK GARCH(1,1), GARCH-X(1,1) and BEKK GARCH-X(1,1) models for with-in sample period, respectively.[8] The results from these tests are quite standard. In most tests the ARCH coefficients are all positive (A_{11} and A_{33} in the GARCH and GARCH-X tests) and significant, thus implying volatility clustering both in the log difference of cash price and the futures price. The ARCH coefficients are also less than unity in all significant cases. The ARCH coefficients (A_{11} and A_{22}) from the BEKK model are close unity and higher than the other models. The smallest ARCH effects (A_{11} and A_{22}) are found in the BEKK GARCH-X tests. The sign and significance of the covariance parameters indicate positive and significant interaction between the two prices in most cases.

The short-run deviations from a long-run relationship between the cash price and future prices have significant effect on both the mean of cash returns (δ_1) and log difference of futures prices (δ_2) in most of the cases. For majority of the commodities the effect on the mean of the cash returns is negative and significant. In the case of log difference of futures prices, the effect is mostly positive and significant. Thus, an increase in short-run deviations raises the lowers the cash returns but increases the log difference of future prices.

The important part of the GARCH-X and BEKK GARCH-X results is the influence of the short-run deviations between the cash price and the futures price on the conditional

8 In these models, different combinations of p and q may be applied but, as indicated by Bollerslev et al. (1992, p. 10), p=q=1 is sufficient for most financial and economic series. Bollerslev (1988) provides a method of selecting the length of p and q in a GARCH model. Tests in this paper were also conducted with different combinations of p and q, with p=q=2 being the maximum lag length. Results based on log-likelihood function and likelihood ratio tests indicate that the best combination is p=q=1. These results are

variance and covariance. For GARCH-X the parameters measuring the effects of the short-run deviations on the conditional variance of cash returns (D_{11}) and log difference of the futures prices (D_{33}) are found to be positive and significant in all tests. A positive and significant effect of the short-run deviations on the conditional variance implies that as the deviation between the cash and future prices gets larger, the volatility of log difference of the cash and futures prices increases, and prediction becomes more difficult. Also, in the case of BEKK GARCH-X the significant parameters are found to be positive. The short-run deviation coefficients (D_{11} and D_{33}) are relatively small, as expected. The parameter D_{22} measures the affect of the short-run deviations on the conditional covariance between the two variables. For GARCH-X only in the case of sugar and corn, D_{22} is found to be significant and positive. The parameter D_{22} is not significant for any commodity using the BEKK GARCH-X. The question to be answered is whether these effects of the short-run deviations also influence the effectiveness of the time-varying hedge ratio.

To assess the general descriptive validity of the model, a battery of standard specification tests is employed. Specification adequacy of the first two conditional moments is verified through the serial correlation test of white noise. These tests employ the Ljung-Box Q statistics on the standardised (normalised) residuals ($\varepsilon_t/H_t^{1/2}$), standardised squared residuals (ε_t/H_t^2) and the cross standardised residuals. The cross standardised residuals is the cross product between the standardised residuals of cash and futures. All series are found to be free of serial correlation (at the 5% level). Absence of serial correlation in the standardised squared residuals implies the absence of need to encompass a higher order ARCH process (Giannopoulos, 1995). In other words, these residual based diagnostic tests lend support to the maintained specifications of the

available on request.

GARCH models employed.

5.2 With-in Sample Period Hedge Ratios Comparison Result

Comparison between the effectiveness of different hedge ratios is made by constructing portfolios implied by the computed ratios, and the change in the variance of these portfolios indicates the hedging effectiveness of the hedge ratios. The portfolios are constructed as (r^c_t - $\beta_t^* r^f_t$), where r^c_t is the log difference of the cash (spot) prices, r^f_t is the log difference of the futures prices, and β_t^* is the estimated optimal hedge ratio. The variance of these constructed portfolios is estimated and compared. For example, for comparison between the GARCH and GARCH-X-based portfolios, the change in variance is calculated as (Var_{GARCH} - Var_{GARCHX})/Var_{GARCH}. Comparison is also provided between the four time-varying hedge ratios-oriented portfolios and an unhedged portfolio. Variance of an unhedged portfolio is presented by the variance of the returns in the cash market.

Table 6 presents the variance of the portfolios and the comparison results for with-in sample period (January 1980-July 2004). The table shows the variance of the portfolios estimated using the different types of hedge ratios and the percentage change in the variance of the portfolios constructed. The top part of the table shows the actual variance of the time-varying hedge ratios-oriented portfolios and the unhedged portfolio. The second part shows the percentage change in the variance between GARCH-X and the other three methods-oriented portfolios. The third part presents the percentage change in the variance between BEKK GARCH-X and other methods-oriented portfolios (excluding the GARCH-X). The fourth part presents the percentage change in the variance between BEKK GARCH and other methods (excluding the GARCH-X and BEKK GARCH-X)-oriented portfolios. The fifth and last part shows the difference between the GARCH-oriented and unhedged portfolios.

Portfolios created using the hedge ratios from the GARCH-X model outperform all other portfolios for corn. The GARCH-X time-varying hedge-ratio portfolios provide the lowest variance and outperform the unhedged portfolio by 65.79%, the BEKK GARCH-X portfolio by 42.64%, the standard BEKK GARCH portfolio by 37.10% and the standard GARCH portfolio by 49.91%. In the case of wheat the standard GARCH and the BEKK GARCH-X portfolios do better than GARCH-X portfolio but by small margin (0.72%). There is no difference in the performance between the portfolios estimated by means of the GARCH-X and the unhedged but GARCH-X does better than the BEKK portfolio by a small margin (2.78%). For coffee the GARCH-X out performances all the other methods but by a small margins against the standard GARCH (0.69%) and the BEKK GARCH-X (3.69%). Sugar provides the weakest results, as far as GARCH-X is concern. All other methods perform better than the GARCH-X. For soybeans, results are more encouraging for the GARCH-X portfolio. It does better than the BEKK, the BEKK GARCH-X and the unhedged portfolios, but it performs the same as the standard GARCH. Overall, the GARCH-X only outperforms the standard GARCH for corn and coffee, but overall does a better job compared to the BEKK GARCH, BEKK GARCH-X and the unhedged portfolios.

The standard GARCH portfolio outperforms the BEKK GARCH portfolio in all cases, except corn. The GARCH also outperforms the BEKK GARCH-X in all cases except wheat. The BEKK GARCH-X does better than the standard BEKK (except for corn) and the unhedged for most commodities (except for sugar). Against the unhedged portfolios, the standard BEKK only does better in the cases of corn. But, many of the differences are quite small. The standard GARCH portfolios outperform the unhedged portfolios in all cases. Overall the standard GARCH oriented portfolios outperform other portfolios. The GARCH-X and BEKK GARCH-X provide similar performance while the standard BEKK performs the worst.

5.3 Out-of-sample Periods Hedge Ratios Comparison Result

Baillie and Myers (1991) and other papers further claim that the more reliable measure of hedging effectiveness is the hedging performance of different methods for out-of-sample periods. This paper compares the hedging effectiveness of the different methods during two different out-of-sample time periods. The out-of-sample periods used are from August 2002 to July 2004 (two years) and from August 2003 to July 2004 (one year). Two different lengths of out-of-sample periods are applied to check whether changing the length has any significant effect on the hedging effectiveness of the hedge ratios. In order to investigate the out-of-sample hedging effectiveness of the hedging methods, all GARCH models are estimated for the periods January 1980 to July 2002, and January 1980 to July 2003, and then the estimated parameters are applied to compute the hedge ratios and the portfolios for the two out-of-sample periods.[9] Once again, the variance of these portfolios is compared and the change in the variance indicates the hedging effectiveness of the hedge ratios.

Table 7 shows the variance of the out-of-sample portfolios and the percentage change in variance of the portfolios from August 2002 to July 2004. The set-up of table 7 is the same as for table 6. The GARCH-X portfolio performs better than the standard GARCH, the BEKK GARCH-X and the unhedged portfolios for corn. The BEKK oriented portfolio does better than the GARCH-X based portfolio by extensive margin of 500%. The GARCH-X also does better than standard BEKK GARCH for wheat, sugar and soybeans. The GARCH portfolio outperforms the GARCH-X portfolio for coffee, sugar and soybeans. The difference is relatively small, except in the case of sugar, where

9 The GARCH estimations for the period 1980-2002 and 1980-2003 are not provided, in order to save space but are available on request. These parameters are similar to the ones estimated for the whole sample period. Once again cointegration is also found during these periods.

GARCH does better by 17.32%. The GARCH-X portfolios performs better than the BEKK GARCH-X portfolios for all commodities except wheat. In the case of corn the difference is large (333.33%). The GARCH-X outperforms unhedged in majority of the cases.

The GARCH portfolios perform better than the BEKK GARCH portfolios for wheat, sugar and soybeans. In the case of soybeans the difference is substantial (279%). The GARCH portfolio also performs better than the BEKK GARCH-X portfolio for all cases except wheat. The standard BEKK GARCH hedge ratio does better than the BEKK GARCH-X ratio by large amount for corn (800%) and coffee (326%). The BEKK GARCH-X portfolios outperform the unhedged portfolios in majority of the case. Compared to the unhedged portfolio, BEKK GARCH does better in the cases of corn and coffee only. In the case of soybeans the unhedged performs better by a large margin (277%). The GARCH portfolio does better than the unhedged for all commodities. Overall, the GARCH and GARCH-X oriented hedge ratios seem to outperform BEKK and BEKK GARCH-X hedge ratios. During this out-of-sample period compared to with-in sample period there is a substantial improvement in the performance of the standard BEKK. There is no set pattern to the performance of the four GARCH ratios regarding each commodity futures.

Figure 1 presents the actual and the forecasted corn hedge ratios based on the four GARCH models over the out-of-sample period August 2002 to July 2004. The actual and forecasted hedge ratios based on the GARCH move together, and thus are very similar. The same is true of the GARCH-X actual and forecasted hedge ratio. Difference in the standard BEKK GARCH actual and forecasted is clearly visible. The BEKK GARCH-X tends to move together more closely than in the case of standard BEKK ratios. Also, the BEKK GARCH-based and the BEKK GARCH-X-based actual and forecasted ratios are different from the standard GARCH and GARCH-X ratios. This difference is also portrayed in the results. Graphs of other commodities are not provided to save space, but are available on request. These graphs also portray a similar story.

Table 8 shows the results from the shorter out-of-sample (August 2003-July 2004) period. The standard GARCH hedge ratio-based portfolios perform better than the GARCH-X portfolios for coffee, sugar and soybeans, the BEKK GARCH-X portfolios for wheat, coffee, and sugar and the standard BEKK GARCH based portfolios for wheat and soybeans. In the case of corn and coffee, the BEKK GARCH does better than the GARCH-X and BEKK GARCH-X by large margins. Compared to the unhedged portfolios, GARCH-X over-performs for all commodities except sugar. The BEKK GARCH-X based portfolio outperforms the standard BEKK GARCH portfolio for wheat, sugar and soybeans and outperforms the unhedged portfolios for all commodities except wheat. Once again the standard BEKK hedge ratio outperforms the BEKK GARCH-X hedge ratio by large margin for corn and coffee. The standard GARCH does better than the unhedged portfolios for all commodities.

Changing the length of the out-of-sample does somewhat affect the performance of the hedge ratios. The BEKK GARCH-X shows an improvement in the level of performance from the longer (two year) out-of-sample period to the shorter (one year) out-of-sample period. The GARCH-X and the BEKK GARCH-X provide similar performance. This result is similar to with-in sample period results. During the shorter sub-period the performance of the standard GARCH hedge ratio seem to decrease. Also, again there seems to be no set pattern in the results during the shorter out-of-sample.

Figure 2 presents the actual and the forecasted corn hedge ratios based on the four GARCH models over the shorter out-of-sample period August 2003 to July 2004. These graphs provide similar story as the longer out-of-sample graphs. The actual and forecasted hedge ratios based on the GARCH again move together, and similar is the case with GARCH-X. The actual and forecasted hedge ratios based on the BEKK GARCH model again tend not to move together. The BEKK GARCH-X provides a much improved forecast during the shorter period. Once again, graphs of other commodities are not provided to save space, but are available on request.

The standard bivariate GARCH generally performs better than the more sophisticated bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X during with-in sample period and the longer (two year) out-of-sample period. However, in most cases, the standard GARCH perform better by a small margin. During the shorter (one year) out-of-sample period the BEKK GARCH-X hedge ratio performs better than the other methods and as good as the standard GARCH. In few tests the standard BEKK and the BEKK GARCH-X methods over performance the other methods by a substantial amount. Of course, with any GARCH method, the hedge portfolio has to be rebalanced frequently. In this paper, the time-varying GARCH hedge ratio changed every week, which may not be too frequent for a short-term hedging strategy. The trade-off between the risk reduction and the transaction cost will determine the practically of the GARCH hedging method.[10] According to Myers (1991), since the different GARCH models are more complex to estimate, and since the continual futures adjustments that it requires entails extra commission charges, then the extra cost of working with any GARCH model may only be warranted if the investor is extremely risk averse.

6. Conclusion

It is a well-documented claim in the futures market literature that the optimal hedge ratio should be time-varying and not constant. Lately, different versions of the GARCH models have been applied to estimate time-varying hedge ratios for different futures

10 Park and Switzer (1995) suggest an alternate strategy method that involves less frequent rebalancing,

markets. This paper investigates the hedging effectiveness of GARCH estimated time-varying hedge ratios in five agricultural commodities futures: corn, wheat, coffee, sugar and soybeans. The time-varying hedge ratios are estimated by means of four different types of GARCH models: the standard bivariate GARCH, the bivariate BEKK GARCH, the bivariate GARCH-X and the bivariate BEKK GARCH-X. The GARCH-X and the BEKK GARCH-X are unique among the GARCH models in taking into consideration the effects of the short-run deviations from a long-run relationship between the cash and the futures price indices on the hedge ratio. The long-run relationship between the price indices is estimated by the Engle-Granger cointegration method. The hedging effectiveness is estimated and compared by checking the variance of the portfolios created using these hedge ratios. The lower the variance of the portfolio, the higher is the hedging effectiveness of the hedge ratio.

The empirical tests are conducted by applying weekly data. The effectiveness of the hedge ratio is investigated by comparing with-in the sample period (August 1980-July 2004) and out-of-sample period performance of the different hedge ratios for two periods, August 2002- July 2004 (two years) and August 2003-July 2004 (one year). The two different lengths of out-of-sample periods are applied to investigate the effect of changing the length on the hedging effectiveness of the hedge ratios.

What do the results show? During with-in sample period and the longer out-of-sample the standard GARCH oriented hedge ratio overall performs better than the other GARCH methods and the unhedged portfolio. During the shorter out-of-sample the four GARCH oriented ratios perform similar to each other. But, no set patter is present in the results. Also changing the length of the out-of-sample period does change the hedging effectiveness of the GARCH oriented hedge ratios. This is especially true in the case of

---

such as rebalancing only when the hedge ratio changes by a fixed amount.

standard GARCH and the BEKK GARCH-X method.

The inconsistent performance of the GARCH models ratios may be attributed to the complexity of the model (Baillie and Myers, 1991). With any GARCH method, the hedge portfolio has to be rebalanced frequently. In this paper, the time-varying GARCH hedge ratio changed every week, which may not be too frequent for a short-term hedging strategy. The trade-off between the risk reduction and the transaction cost will determine the practically of the GARCH hedging method. Results in this paper advocate further research in this field. Further research may be conducted using different frequency of the data, different method of estimation, time period, type of futures markets, etc.

References

Baillie, R., Myers, R., 1991. Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge. Journal of Applied Econometrics 6, 109-124.

Bera, A., Higgins, M., 1993. ARCH Models: Properties, Estimation and Testing. Journal of Economic Surveys 7, 305-366.

Bollerslev, T., Engle, R. and Nelson, D., 1994. ARCH Models. *Handbook of Econometrics*, Volume IV, Edited by R.F. Engle and D.L. McFadden, Elsevier Science.

Bollerslev, T., Chou, R., Kroner, K., 1992. ARCH Modeling in Finance. Journal of Econometrics 52, 5-59.

Bollerslev, T., 1988. On the Correlation Structure for the Generalized Autoregressive Conditional Heteroscedastic Process. Journal of Time Series Analysis 9, 121-131.

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31, 307-327.

Brenner, R., Kroner, K., 1995. Arbitrage, Cointegration, and Testing the Unbiasedness Hypothesis in Financial Markets. Journal of Financial and Quantitative Analysis 30, 23-42.

Cecchetti, S., Cumby, R., Figlewski, S., 1988. Estimation of the Optimal Futures Hedge. Review of Economics and Statistics 70, 623-630.

Choudhry, T., 2004. Time-Varying Distribution and Hedging Effectiveness of Three Pacific-Basin Stock Futures. International Review of Economics and Finance, 13, 371-385.

Covey, T., Bessler, D., 1992. Testing for Granger's Full Causality. Review of Economics and Statistics, 74, 146-153.

Engle, R., Kroner, K., 1995. Multivariate Simultaneous Generalized ARCH. Econometric Theory 11, 122-150.

Engle, R.,Granger, C., 1987. Cointegration and Error Correction: Representative, Estimation, and Testing. Econometrica 55, 251-276.

Engle, R., Yoo, B., 1987. Forecasting and Testing in the Cointegrated System. Journal of Econometrics 35, 143-159.

Fortenbery, T., Zapata, H., 1993. An Examination of Cointegration Relations Between Futures and Local Grain Markets. Journal of Futures Markets, 13, 921-932.

Fortenbery, T., Zapata, H., 1997. An Evaluation of Price Linkages Between Futures and Cash Markets for Cheddar Cheese. Journal of Futures Markets, 17, 279-301.

Gagnon, L., Lypny, G., 1995. Hedging Short-term Interest Risk Under Time-Varying Distribution. Journal of Futures Market, 15, 767-783.

Gagon, L., Lypny, G. and McCurdy, T., 1998. Hedging Foreign Currency Portfolios. Journal of Empirical Finance 5, 197-220.

Ghosh, A., 1995. The Hedging Effectiveness of ECU Futures Contracts. Forecasting Evidence from an Error Correction Model. Financial Review 30, 567-581.

Ghosh, A., Clayton, R., 1996. Hedging with International Stock Index Futures. An Intertemporal Error Correction Model. Journal of Financial Research 19, 477-491.

Giannopoulos, K., 1995. Estimating the Time-Varying Components of International Stock Markets' Risk. European Journal of Finance 1, 129-164.

Kroner, K., Sultan, J., 1993. Time-Varying Distribution and Dynamic Hedging with Foreign Currency Futures. Journal of Financial and Quantitative Analysis, 28, 535-551.

Lee, T., 1994. Spread and Volatility in Spot and Forward Exchange Rates. Journal of International Money and Finance 13, 375-383.

Moschini, G., Myers, R., 2002. Testing for Constant Hedge Ratios in Commodity Markets: A Multivariate GARCH Approach. Journal of Empirical Finance, 9, 589-603.

Myers, R., 1991. Estimating Time Varying Hedge Ratios on Futures Markets. Journal of Futures Markets 11, 39-53.

Park, T., Switzer, L., 1995. Time-Varying Distribution and the Optimal Hedge Ratios for Stock Index Futures. Applied Financial Economics 5, 131-137.

Tong, W., 1995. An Examination of Dynamic Hedging. Journal of International Money and Finance, 15, 19-35.

Wahab, M., 1995. Conditional Dynamics and Optimal Spreading in the Precious Metals Futures Markets. Journal of Futures Markets 15, 131-166.

Yang, J., Bessler, D. and Leatham, D., 2001. Asset Storability and Price Discovery in Commodity Futures markets: A New Look. Journal of Futures Markets 21, 279-300.

Table 1
Part A
Basic Statistics of the Total Period (1980-2004)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.0004	0.0011	4.5116[a]	-0.4042[a]	1095.08[a]
Wheat	-0.0002	0.0014	6.4784[a]	-0.4794[a]	2235.59[a]

Coffee	-0.0008	0.0030	9.7500[a]	0.5733[a]	5095.78[a]
Sugar	-0.0006	0.0029	4.9118[a]	-0.1643[b]	1287.39[a]
Soybeans	-0.0002	0.0011	7.8281[a]	-0.6752[a]	3244.51[a]
Log Difference of Futures Price					
Corn	-0.0003	0.0011	8.8837[a]	-0.6118[a]	4191.75[a]
Wheat	-0.0003	0.0012	4.6699[a]	-0.4323[a]	1175.73[a]
Coffee	-0.0008	0.0026	2.4230[a]	0.0692	311.437[a]
Sugar	-0.0008	0.0036	12.2957[a]	0.7665[a]	8156.48[a]
Soybeans	-0.00007	0.0011	6.8581[a]	-0.4797[a]	2465.57[a]
Square of Log Difference of Cash Price					
Corn	0.0011[a]	0.000008	87.5248[a]	7.9159[a]	412373.53[a]
Wheat	0.0014[a]	0.000016	214.858[a]	12.884[a]	2440910.74[a]
Coffee	0.0104[a]	0.00011	255.651[a]	13.421[a]	3493925.02[a]
Sugar	0.0029[a]	0.00006	195.5715[a]	11.239[a]	2058778.34[a]
Soybeans	0.0011[a]	0.000012	484.685[a]	18.624[a]	12150113.76[a]
Square of Log Difference of Futures Price					
Corn	0.0011[a]	0.00013	245.000[a]	13.0384[a]	3164264.81[a]
Wheat	0.0012[a]	0.00001	184.429[a]	11.3278[a]	1799736.61[a]
Coffee	0.0026[a]	0.00003	42.900[a]	5.4111[a]	103503.66[a]
Sugar	0.0036[a]	0.00018	484.4218[a]	18.8649[a]	12542173.75[a]
Soybeans	0.0011[a]	0.00001	237.501[a]	12.6537[a]	2933174.64[a]
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.00077[a]	0.000004	59.557[a]	6.546[a]	193825.25[a]
Wheat	0.00007	0.000002	52.765[a]	-1.0292[a]	145343.76[a]
Coffee	0.00059[a]	0.00003	326.627[a]	13.168[a]	5677643.65[a]
Sugar	0.00022[b]	0.00002	33.689[a]	0.2772[a]	60309.00[a]
Soybeans	-0.00006	0.000002	39.1500[a]	-2.5330[a]	80124.49[a]

Note:
a- implies significantly different from zero at 1% level.

Table 1
Part B
Basic Statistics of the Sub Period (1980-2002)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.0003	0.0011	5.1032[a]	-0.5326[a]	1298.86[a]
Wheat	-0.0003	0.0013	6.7910[a]	-0.7998[a]	2326.31[a]

Coffee	-0.0016	0.0029	11.5546[a]	0.7160[a]	6580.288[a]
Sugar	-0.0007	0.0030	4.9348[a]	-0.1789[b]	1184.437[a]
Soybeans	-0.0005	0.0010	2.4751[a]	-0.0479	288.880[a]
Log Difference of Futures Price					
Corn	-0.0002	0.0011	9.9107[a]	-0.7054[a]	4789.349[a]
Wheat	-0.0003	0.0011	5.7457[a]	-0.5253[a]	1630.54[a]
Coffee	-0.0012	0.0026	2.555[a]	0.0552	317.515[a]
Sugar	-0.0011	0.0032	3.564[a]	0.2434[a]	631.204[a]
Soybeans	-0.0002	0.00095	4.0720[a]	0.0545	781.240[a]
Square of Log Difference of Cash Price					
Corn	0.0011[a]	0.000008	90.1377[a]	8.1365[a]	400953.26[a]
Wheat	0.0013[a]	0.000015	269.454[a]	14.4980[a]	3510107.04[a]
Coffee	0.0029[a]	0.00011	255.170[a]	13.622[a]	3196667.06[a]
Sugar	0.0030[a]	0.00006	184.000[a]	10.950[a]	1675307.35[a]
Soybeans	0.0010[a]	0.000005	58.0387[a]	6.1662[a]	165760.30[a]
Square of Log Difference of Futures Price					
Corn	0.0010[a]	0.000014	238.213[a]	13.000[a]	2744271.84[a]
Wheat	0.0011[a]	0.000010	191.0386[a]	11.7876[a]	1770753.89[a]
Coffee	0.0026[a]	0.00003	41.507[a]	5.3475[a]	89183.48[a]
Sugar	0.0032[a]	0.00006	66.894[a]	6.7701[a]	227279.95[a]
Soybeans	0.0010[a]	0.000006	74.8317[a]	7.5229[a]	274314.71[a]
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.0007[a]	0.000004	62.448[a]	6.8000[a]	195216.98[a]
Wheat	0.0015[a]	0.000002	70.1271[a]	-1.9881[a]	235785.66[a]
Coffee	0.0006[a]	0.00003	323.822[a]	13.320[a]	5124589.60[a]
Sugar	0.0002[b]	0.000017	33.164[a]	0.0721	53665.54[a]
Soybeans	-0.00006	0.000002	33.468[a]	-2.2176[a]	53665.39[a]

Note:
a- implies significantly different from zero at 1% level.
b - implies significant differently from zero at 5% level.

Table 1
Part C
Basic Statistics of the Sub Period (1980-2003)

Variables	Mean	Variance	Kurtosis	Skewness	Jarque-Bera
Log Difference of Cash Price					
Corn	-0.0004	0.0011	4.9572[a]	-0.4376[a]	1265.97[a]
Wheat	-0.0002	0.0014	7.0326[a]	-0.1859[a]	2520.94[a]

Coffee	-0.0010	0.0030	10.283[a]	0.6158[a]	5438.884[a]
Sugar	-0.0008	0.0029	4.9252[a]	-0.1682[b]	1241.872[a]
Soybeans	-0.0004	0.0010	2.4579[a]	-0.0511	298.048[a]
Log Difference of Futures Price					
Corn	-0.0004	0.0010	9.6507[a]	-0.6669[a]	4741.80[a]
Wheat	-0.0002	0.0012	5.1786[a]	-0.4371[a]	1377.95[a]
Coffee	-0.0009	0.0026	2.4668[a]	0.0481	309.038[a]
Sugar	-0.0010	0.0031	3.5372[a]	0.2308[a]	648.431[a]
Soybeans	-0.00027	0.0010	4.0495[a]	0.0095	807.645[a]
Square of Log Difference of Cash Price					
Corn	0.0011[a]	0.000009	88.0227[a]	8.0082[a]	399892.89[a]
Wheat	0.0014[a]	0.000016	217.031[a]	13.0778[a]	2387342.45[a]
Coffee	0.0030[a]	0.00011	248.755[a]	13.293[a]	3173614.26[a]
Sugar	0.0029[a]	0.00006	190.001[a]	11.1033[a]	1864746.69[a]
Soybeans	0.0010[a]	0.000004	59.502[a]	6.2249[a]	181999.92[a]
Square of Log Difference of Futures Price					
Corn	0.0010[a]	0.000014	244.551[a]	13.1249[a]	3022202.96[a]
Wheat	0.0012[a]	0.00001	188.173[a]	11.5709[a]	1795742.16[a]
Coffee	0.0026[a]	0.00003	42.7828[a]	5.4094[a]	98749.996[a]
Sugar	0.0031[a]	0.00005	69.177[a]	6.8700[a]	253476.44[a]
Soybeans	0.00095	0.000005	73.625[a]	7.450[a]	277902.55[a]
Log Difference of Cash Price x Log Difference of Futures Price					
Corn	0.0007[a]	0.000004	62.0458[a]	6.7485[a]	201424.96[a]
Wheat	0.00009[b]	0.000002	58.667[a]	-1.0868[a]	172185.36[a]
Coffee	0.0006[a]	0.00003	318.877[a]	13.063[a]	5190773.75[a]
Sugar	0.00022	0.000017	33.921[a]	0.0744	58634.336[a]
Soybeans	-0.00006	0.000002	33.213[a]	-2.2011[a]	55281.25[a]

Note:
a - implies significantly different from zero at 1% level.
b - implies significant differently from zero at 5% level.

Table 2
Bivariate GARCH Results

	Corn	Wheat	Coffee	Sugar	Soybeans
$\mu_1 \times 10^{-4}$	8.600 (1.2109)	8.6338 (0.9183)	8.3446 (1.1744)	31.4310[b] (2.5245)	5.4992 (0.7205)
δ_1	-0.0943[a] (-5.5315)	-0.2155[a] (-20.0309)	-0.0676[a] (-12.4671)	-0.2186[a] (-18.9289)	-0.1246[a] (-5.9654)

μ_2 x 10^{-4}	2.6127 (0.3522)	-5.6617 (-0.6136)	-13.416 (-1.0808)	-22.5900 (-1.7792)	-25.1778[a] (-5.3123)
δ_2	0.1340[a] (6.8041)	0.0637[a] (5.3310)	0.1702[a] (12.0785)	0.0271[b] (2.3321)	0.6802[a] (69.7767)
C_1 x 10^{-4}	0.9941[a] (6.7625)	1.9687[a] (4.6526)	0.0077[a] (4.5760)	731.4500[a] (2.6269)	0.5893[a] (5.9099)
A_{11}	0.1637[a] (8.6631)	0.1854[a] (8.4601)	0.1500[a] (15.4832)	0.0261[a] (8.1131)	0.2050[a] (9.7902)
B_{11}	0.7484[a] (32.6863)	0.6648[a] (13.8098)	0.8746[a] (118.2633)	0.9702[a] (487.7795)	0.7668[a] (50.5591)
C_3 x 10^{-4}	2.1880[a] (11.1891)	1.2926[a] (3.0594)	0.0764[a] (4.4684)	2.0589[a] (4.6421)	0.2211[a] (7.3599)
A_{33}	0.1525[a] (11.1891)	0.0880[a] (4.7618)	0.0955[a] (10.2441)	0.1930[a] (10.6831)	0.3051[a] (9.8162)
B_{33}	0.6302[a] (27.8373)	0.8066[a] (15.7079)	0.8751[a] (68.1865)	0.7635[a] (34.9134)	0.6868[a] (39.8674)
C_2 x 10^{-4}	0.9839[a] (8.7211)	4.4538 (0.6991)	0.0106[a] (3.1715)	4.2866[a] (3.4020)	0.4984 (0.7170)
A_{22}	0.0977[a] (7.3605)	0.0096 (0.8670)	0.0667[a] (6.4584)	-0.0104 (-0.6504)	-0.0032 (-0.0986)
B_{22}	0.7631[a] (37.6241)	0.9506[a] (15.6341)	0.9126[a] (75.6198)	-0.9281 (-6.4654)	-0.7996 (-0.3241)
L	7971.100	7210.689	6765.884	6396.647	7966.792
LB(9) test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$- Cash	8.3950	6.5727	7.9533	8.6905	6.1176
ε_t^2/h_t – Cash	8.5702	8.8249	3.5935	2.7870	4.6240
$\varepsilon_t/h_t^{1/2}$- Futures	10.3396	5.7596	8.9429	10.1768	11.3274
ε_t^2/h_t – Futures	3.2792	2.5289	9.8289	5.2262	11.2984
CSR	8.0154	7.4190	2.9941	10.2601	6.0744

Notes:
a, b & c imply significance at the 1%, 5% & 10% level, respectively.
t-statistics in the parentheses; L=log likelihood function value.
LB=Ljung-Box statistics for serial correlation of the order 9.
ε_t^2/H_t = Standardized Squared Residuals
$\varepsilon_t/H_t^{1/2}$ = Standardized Residuals
Cross Standardized Residuals (CSR) = standardized residuals (cash) x standardized residuals (futures)

Table 3
Bivariate BEKK GARCH Results

	Corn	Wheat	Coffee	Sugar	Soybeans
μ_1 x 10^{-4}	16.0806[a] (2.8565)	6.9331 (0.5370)	5.1605 (0.4322)	3.4090 (0.2573)	0.8509 (0.0993)
δ_1	-0.1071[a] (-3.1711)	-0.1420[a] (-8.0081)	-0.0684[b] (-2.5752)	-0.1250[a] (-6.7210)	-0.1075[a] (-2.4924)

$\mu_2 \times 10^{-4}$	10.6644[c] (1.8333)	-1.7300 (-0.1458)	-14.0527 (-1.1857)	2.4845 (0.1270)	-0.0020[a] (-2.7836)
δ_2	0.1080[a] (3.5463)	0.0412[b] (2.2964)	0.1744[a] (9.9568)	-0.0154 (-0.6406)	0.6019[a] (14.6895)
C_{11}	0.0092[a] (5.7989)	0.0092[b] (2.2025)	0.0029[a] (2.9065)	0.0188[a] (3.8802)	0.0106[a] (3.6463)
A_{11}	0.9231[a] (41.0389)	0.5165 (3.8350	0.9282[a] (66.5587)	0.5180[a] (9.3875)	0.8796[a] (30.8263)
B_{11}	0.1998[a] (3.0473)	0.1148 (1.1870)	0.4292[a] (7.2684)	0.2240[a] (4.2092)	0.0775 (0.7292)
C_{22}	0.0082[a] (3.1727)	0.0043 (1.2666)	0.0046[a] (9.0449)	0.00004 (0.9973)	2.7285 (0.1669)
A_{22}	0.0640 (0.7249)	0.7914[a] (16.2060)	0.9122[a] (129.6998)	0.9661[a] (59.4700)	0.7271[a] (19.9833)
B_{22}	0.3259[a] (4.5358)	0.0834 (0.9092)	0.2711[a] (5.8875)	0.1993[c] (1.7081)	0.4415[a] (5.6747)
C_{12}	0.0207[a] (10.3554)	-0.01662[a] (-8.5684)	0.0114[b] (6.9474)	-0.0093[b] (-1.9166)	0.0047[a] (3.4278)
A_{12}	0.2917[a] (2.6654)	-0.3368[a] (-4.4950)	-0.0447[c] (-1.9950)	-0.2035 (-0.6545)	-0.14267[a] (-9.9049)
B_{12}	0.4163[a] (10.1446)	-0.2275[b] (-2.3243)	0.1819[b] (2.1965)	-0.0109 (-0.2659)	0.2859[a] (8.4138)
A_{21}	-0.1098[a] (-4.3700)	0.5446[a] (11.0265)	-0.0233[a] (-2.8209)	0.2129 (1.2653)	0.4106[a] (3.5691)
B_{21}	0.3334[a] (4.7799)	-0.5961[a] (-6.7884)	0.0331 (0.8412)	-0.7178[a] (-13.4566)	-0.2427[a] (-3.3407)
L	5679.892	4983.072	4430.182	4079.549	5720.832
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$- Cash	11.8471	4.5407	9.2151	6.0112	6.3027
ε_t^2/h_t – Cash	6.3690	9.6969	3.9478	5.9275	3.9895
$\varepsilon_t/h_t^{1/2}$- Futures	4.8336	2.4045	6.0162	9.5032	9.1288
ε_t^2/h_t – Futures	7.5644	10.1563	4.8686	9.9556	2.6607
CSR	10.0661	8.6274	11.4756	7.9082	2.2532

See notes at the end of table 2.

Table 4
Bivariate GARCH-X Results

	Corn	Wheat	Coffee	Sugar	Soybeans
$\mu_1 \times 10^{-4}$	3.5341 (0.4844)	13.7426 (1.5096)	2.8876 (0.3780)	1.5095 (0.1166)	5.0123 (0.6322)
δ_1	-0.0512[b] (-2.0166)	-0.2246[a] (-13.3221)	-0.0492[a] (-4.1970)	-0.2788[a] (-16.6351)	0.1136[a] (4.3190)
$\mu_2 \times 10^{-4}$	-3.6209	-6.5712	-18.1800	-23.9482[c]	-31.4713[a]

	(-0.4837)	(-0.7072)	(-1.4661)	(-1.8904)	(-6.3494)
δ_2	0.1692[a] (6.2466)	0.0610[a] (3.8000)	0.1851[a] (11.9733)	0.0205 (1.4000)	-0.7093[a] (-49.6800)
$C_1 \times 10^{-4}$	1.7075[a] (6.3869)	1.3492[a] (4.2467)	0.0055[a] (3.2119)	3.0345[a] (6.8259)	0.5353[a] (4.9505)
A_{11}	0.1730[a] (7.2818)	0.0876[a] (6.1585)	0.1447[a] (13.6516)	0.1750[a] (7.0599)	0.1757[a] (8.1613)
B_{11}	0.6004[a] (14.3987)	0.6910[a] (16.2601)	0.8704[a] (109.4624)	0.5854[a] (15.9719)	0.7693[a] (38.4160)
D_{11}	0.0502[a] (3.8449)	0.0300[a] (5.4243)	0.0016[a] (4.4714)	0.0303[a] (4.3159)	0.0212[c] (1.6311)
$C_3 \times 10^{-4}$	4.2288[a] (9.5419)	1.5134[a] (3.1017)	0.0870[a] (4.4083)	2.1123[a] (4.5025)	0.1786[a] (3.5333)
A_{33}	0.2277[a] (10.8218)	0.0551[a] (3.8292)	0.0964[a] (9.8741)	0.2061[a] (9.4644)	0.3258[a] (7.3654)
B_{33}	0.2703[a] (5.8809)	0.7665[a] (12.4331)	0.520[a] (52.8869)	0.7379[a] (30.5687)	0.5514[a] (14.5965)
D_{33}	0.0725[a] (4.5504)	0.0124[a] (3.4803)	0.0045[b] (2.3462)	0.0051[b] (2.1050)	0.03147[a] (6.6460)
$C_2 \times 10^{-4}$	2.4522[a] (7.1769)	1.4863 (1.4535)	0.0101[b] (2.2820)	1.6000 (1.3060)	0.0588 (0.2880)
A_{22}	0.1354[a] (6.8617)	-0.0005 (-0.0235)	0.0710[a] (6.2139)	0.0005 (0.0135)	0.0162 (0.3624)
B_{22}	0.4402[a] (7.9936)	-0.3884 (-0.5448)	0.8942[a] (55.1891)	-0.1187 (-0.2669)	0.5936 (0.5871)
D_{22}	0.0446[a] (2.9660)	0.0093 (1.1415)	0.0019 (1.3984)	0.0240[b] (2.4708)	0.0046 (0.4010)
L	7992.17	7251.88	6776.14	6397.84	8025.51
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$- Cash	11.4558	6.7586	5.4715	11.8532	4.1256
ε_t^2/h_t – Cash	11.4845	6.6448	6.3142	4.1816	4.9177
$\varepsilon_t/h_t^{1/2}$- Futures	7.1433	4.4237	4.9836	9.1564	8.6563
ε_t^2/h_t – Futures	8.0613	6.5191	8.5040	4.0087	5.4844
CSR	7.3456	10.1377	4.1684	5.9998	7.2583

See notes at the end of table 2.

Table 5
Bivariate BEKK GARCH-X Results

	Corn	Wheat	Coffee	Sugar	Soybeans
$\mu_1 \times 10^{-4}$	2.1899 (0.3134)	15.4876 (1.4084)	2.5318 (0.2612)	-11.7633 (-0.6494)	10.1051 (0.9904)
δ_1	-0.0581[b] (-2.0433)	-0.2274[a] (-10.6693)	-0.0453[a] (-3.8068)	-0.2822[a] (-8.2899)	-0.0700[b] (-1.9739)
$\mu_2 \times 10^{-4}$	-2.5844	-7.1964	-17.6787	-26.1344[c]	-29.1273[a]

	(-0.4011)	(-0.6378)	(-1.4568)	(-1.7686)	(-3.4666)
δ_2	0.1080[a] (3.5463)	0.0638[a] (3.5009)	0.1942[a] (6.3807)	0.0152 (0.6794)	0.7079[a] (36.6287)
C_{11}	0.0157[a] (22.1050)	0.0114[b] (2.4121)	0.0022[b] (2.4907)	0.0175[c] (1.7686)	0.0091[a] (4.6703)
A_{11}	0.4266[a] (8.2833)	0.2670[b] (2.1275)	0.3877[a] (7.2177)	0.2558 (1.6089)	0.1534 (0.7131)
B_{11}	0.7191[a] (29.2406)	0.8444[a] (7.9905)	0.9313[a] (58.0764)	0.8246[a] (5.4764)	0.8736[a] (31.2080)
D_{11}	0.0444[a] (10.8589)	0.0293[b] (2.4024)	0.0017 (1.2981)	0.0289[c] (1.8387)	0.1067[c] (1.7665)
C_{22}	0.0017 (0.4556)	0.0130[b] (2.0648)	0.0090[a] (3.5369)	0.0136[a] (4.5175)	0.0042[a] (5.1776)
A_{22}	0.4462[a] (6.6053)	0.1480 (0.8540)	0.2826[a] (4.9218)	0.4078[a] (6.6145)	0.5612[a] (11.4499)
B_{22}	0.2884[a] (5.2520)	0.8708[a] (10.1703)	0.9182[a] (22.1920)	0.8771[a] (29.0071)	0.7431[a] (24.6474)
D_{33}	0.0770[a] (8.1851)	0.0148 (1.0421)	0.0065 (0.9385)	0.0057 (0.8637)	0.0334[a] (4.6844)
C_{12}	0.0248[a] (41.7686)	0.0043[b] (2.5440)	0.0066 (1.5365)	0.0049[c] (1.9588)	0.0013 (0.8293)
A_{12}	0.0009[a] (2.6667)	0.0010[a] (4.9550)	0.0009[a] (3.4536)	0.0008[a] (3.7612)	0.0035[a] (9.1011)
B_{12}	0.0008[a] (10.1342)	0.0010[b] (2.3433)	0.0017[a] (3.9834)	0.0011[a] (11.3649)	0.2859[a] (8.4138)
A_{21}	0.0008[a] (5.6743)	0.0008[a] (11.3454)	0.0007[a] (2.8210)	0.0006[b] (2.1128)	0.0011[a] (3.6718)
B_{21}	0.0007[a] (4.8000)	0.0010[a] (6.7829)	0.0008[a] (3.3500)	0.0008[a] (13.6676)	-0.0019[a] (-3.4556)
D_{22}	0.0222 (1.1899)	-0.0030 (-0.6258)	0.0011 (0.9455)	0.0056 (1.0688)	0.0035 (0.3802)
L	5671.775	4963.852	4437.146	4052.447	5739.500
Test for Serial Correlation in the Residuals					
$\varepsilon_t/h_t^{1/2}$- Cash	12.0848	6.5561	9.7333	11.3605	6.8399
ε_t^2/h_t – Cash	8.5747	7.2606	6.6843	4.3337	10.0610
$\varepsilon_t/h_t^{1/2}$- Futures	8.7174	4.5871	8.9282	4.3033	5.7651
ε_t^2/h_t – Futures	8.4671	3.9979	11.0832	8.9613	8.9392
CSR	7.0319	12.1504	5.4720	5.4818	6.2687

See notes at the end of table 2.

Table 6
Total Period Portfolio Variance and Percentage Change in the Variance

Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.00066	0.00139	0.00289	0.00288	0.00112
BEKK GARCH	0.00062	0.00144	0.00345	0.00295	0.00128
GARCH-X	0.00039	0.00140	0.00287	0.00330	0.00112
BEKK-X	0.00068	0.00139	0.00298	0.00292	0.00117
No Hedge	0.00114	0.00140	0.00303	0.00290	0.00119
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	49.91	-0.72	0.69	-14.58	0.00
BEKK GARCH	37.10	2.78	16.81	-11.86	12.50
BEKK-X	42.64	-0.72	3.69	-13.01	4.27
No Hedge	65.79	0.00	5.28	-2.14	5.88
Percentage Change in the Portfolio Variance Between BEKK-X and other Methods (excluding GARCH-X)					
GARCH	-3.03	0.00	-3.11	-1.39	-4.46
BEKK GARCH	-9.68	3.47	13.62	1.01	8.59
No Hedge	40.35	0.72	1.65	-0.69	1.68
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCH-X and BEKK-X)					
GARCH	6.06	-3.60	-19.38	-2.43	-14.29
No Hedge	45.56	-2.86	-13.86	-1.72	-7.56
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
No Hedge	42.11	0.71	4.62	0.69	5.88

Notes:
The change in the variance between GARCH and GARCH-X is estimated as $(Var_{GARCH} – Var_{GARCHX})/Var_{GARCH}$.
The change in the variance between GARCH and BEKK GARCH is estimated as $(Var_{GARCH} – Var_{BEKK})/Var_{GARCH}$.
The change in the variance between GARCH and BEKK GARCH-X is estimated as $(Var_{GARCH} – Var_{BEKKX})/Var_{GARCH}$.
The change in the variance between GARCH-X and BEKK GARCH-X is estimated as $(Var_{BEKK\text{-}X} – Var_{GARCHX})/Var_{BEKK\text{-}X}$.
The change in the variance between GARCH-X and BEKK GARCH is estimated as $(Var_{BEKK} – Var_{GAARCH})/Var_{BEKK}$.
The change in the variance between BEKK GARCH and BEKK GARCH-X is estimated as $(Var_{BEKK} – Var_{BEKKX})/Var_{BEKK}$.

Table 7
Out-of-Sample Period (Aug 2002 – July 2004) Portfolio Variance and Percentage Change in the Variance

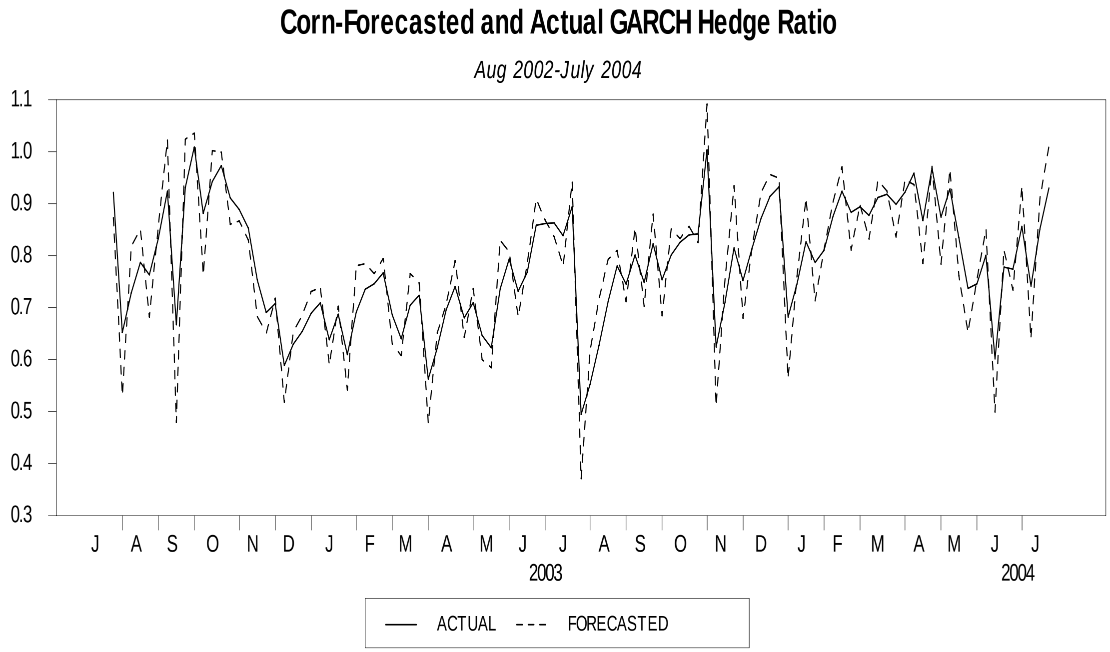
Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.00067	0.00266	0.00449	0.00179	0.00219
BEKK GARCH	0.00011	0.00332	0.00110	0.00220	0.00830
GARCH-X	0.00066	0.00262	0.00468	0.00210	0.00226
BEKK-X	0.00099	0.00257	0.00469	0.00225	0.00372
No Hedge	0.00152	0.00267	0.00474	0.00199	0.00220
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	1.49	1.50	-4.23	-17.32	-3.20
BEKK GARCH	-500.00	21.08	-3.25	4.55	72.77
BEKK-X	333.33	-1.95	0.21	6.67	39.25
No Hedge	56.58	1.87	1.27	-10.55	-2.72
Percentage Change in the Portfolio Variance Between BEKK-X and other methods (excluding GARCH-X)					
GARCH	-47.76	3.38	-4.45	-25.70	-69.86
BEKK GARCH	-800.00	22.59	-326.36	-2.27	55.18
No Hedge	34.87	3.75	1.05	-13.07	-69.09
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCH-X and BEKK-X)					
GARCH	83.58	-24.81	75.50	-17.32	-279.00
No Hedge	92.76	-24.34	76.79	-10.55	-277.00
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
No Hedge	55.92	0.37	5.27	10.05	0.45

See notes at the end of table 6

Table 8
Out-of-Sample Period (Aug 2003- July 2004) Portfolio Variance and Percentage Change in the Variance

Hedge Ratios	Corn	Wheat	Coffee	Sugar	Soybeans
GARCH	0.00060	0.00222	0.00362	0.00177	0.00338
BEKK GARCH	0.00010	0.00270	0.00100	0.00350	0.00580
GARCH-X	0.00058	0.00220	0.00370	0.00245	0.00343
BEKK-X	0.00058	0.00226	0.00365	0.00212	0.00247
No Hedge	0.00173	0.00222	0.00371	0.00218	0.00350
Percentage Change in the Portfolio Variance between GARCH-X and other methods					
GARCH	3.33	0.90	-2.20	-38.42	-1.50
BEKK GARCH	-427.27	18.52	-270.00	30.00	40.86
BEKK-X	0.34	2.66	-1.37	-15.56	-38.87
No Hedge	66.47	0.90	0.27	-12.39	2.00
Percentage Change in the Portfolio Variance Between BEKK-X and other Methods (excluding GARCH-X)					
GARCH	3.00	-1.80	-0.83	-19.77	26.93
BEKK GARCH	-482.00	16.30	-265.00	39.43	134.82
No Hedge	66.36	-1.80	1.62	2.75	1.03
Percentage Change in the Portfolio Variance between BEKK GARCH and other methods (excluding GARCHX)					
GARCH	83.33	-21.82	72.38	49.43	-71.60
No Hedge	94.22	-21.82	73.05	-60.55	-65.71
Percentage Change in the Portfolio Variance between Bi-GARCH and No Hedge					
No Hedge	65.32	0.00	2.42	18.81	3.43

See notes at the end of table 6.



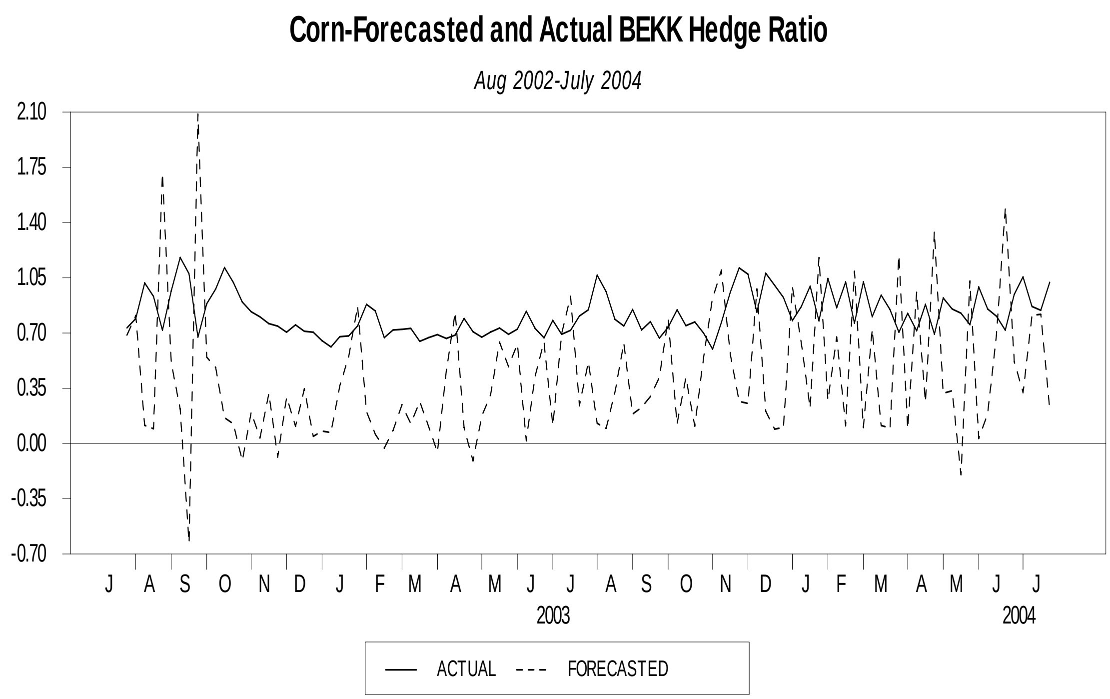
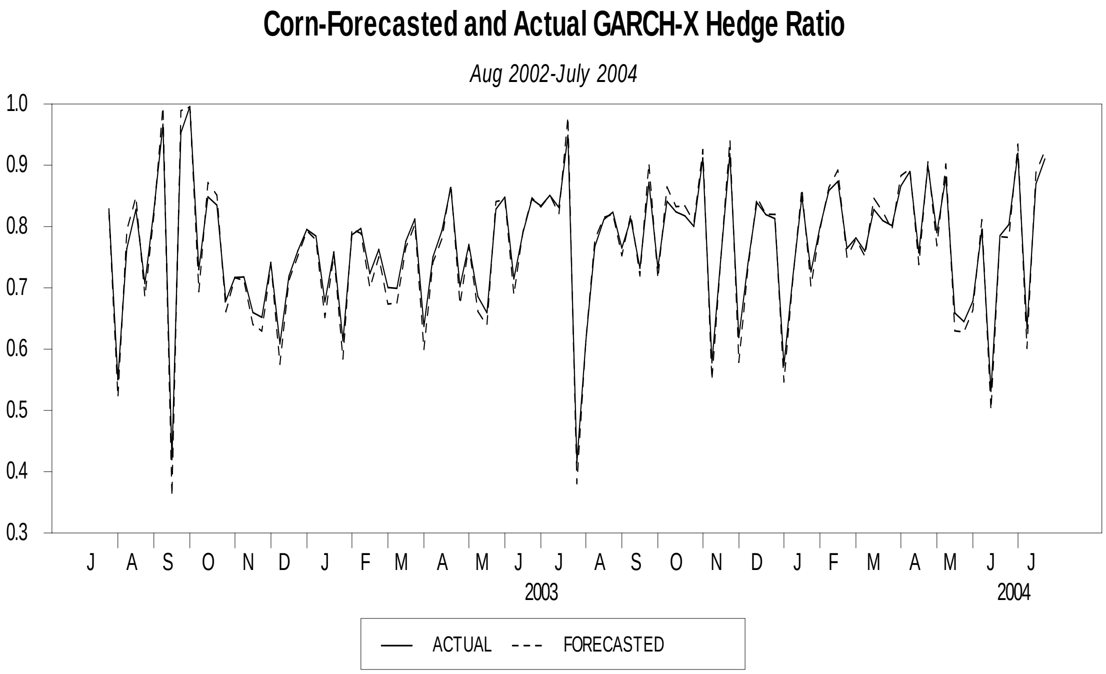


Figure 1
Forecasted and Actual Hedge Ratios (Aug 2002-July 2004)



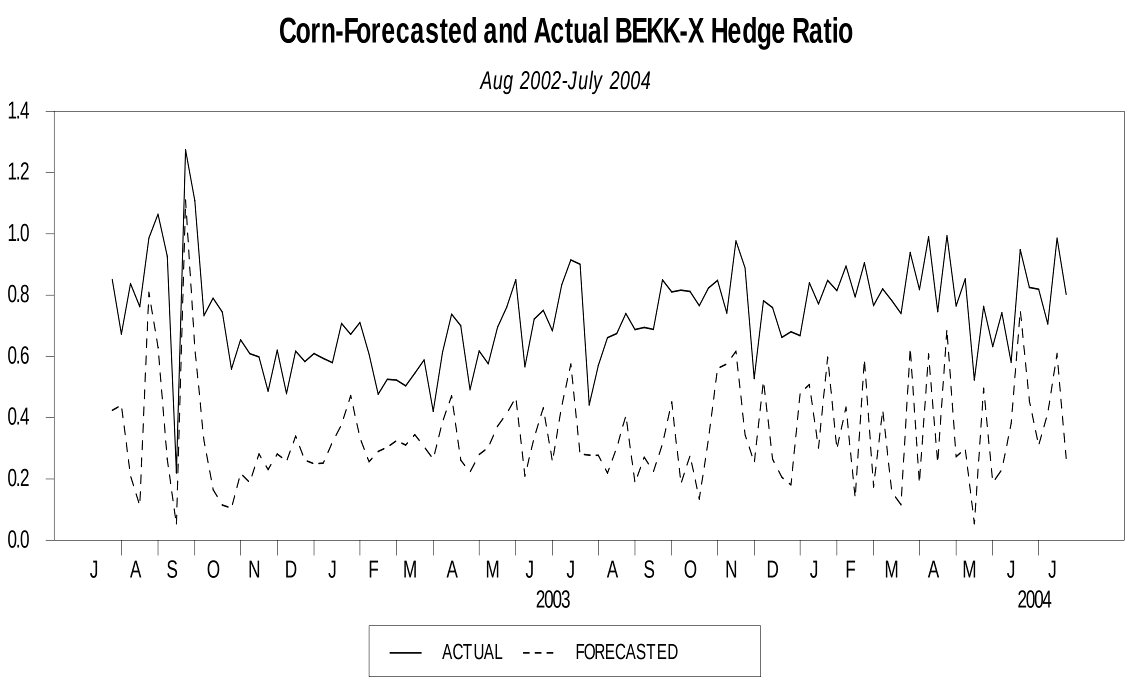
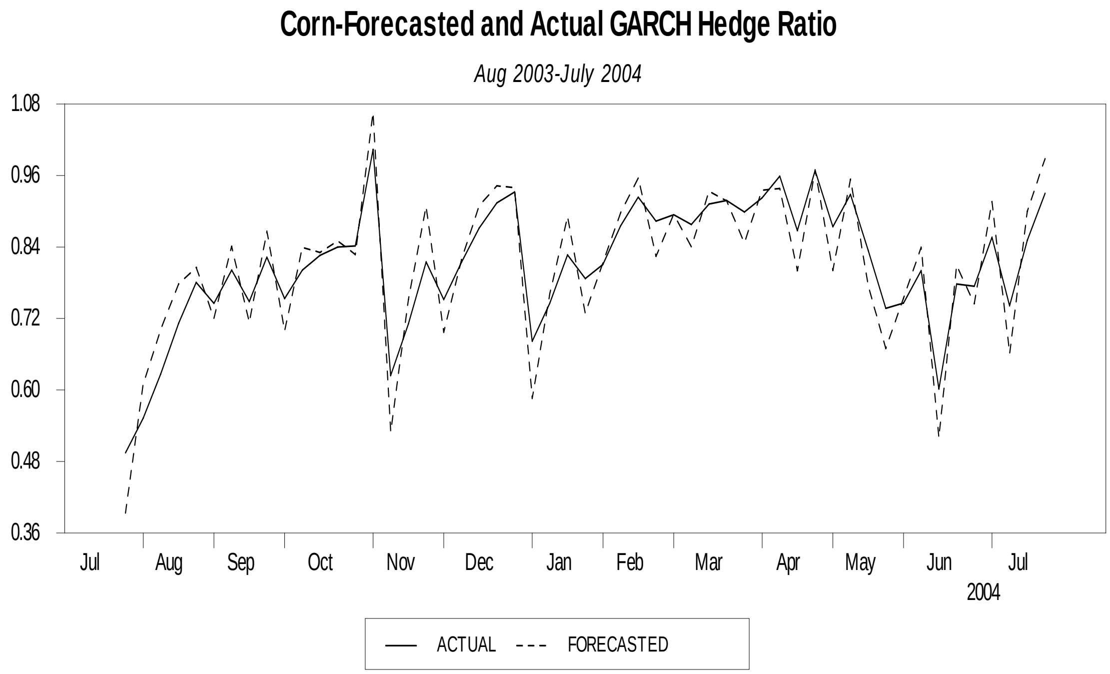


Figure 1
Forecasted and Actual Hedge Ratios (Aug 2002-July 2004)



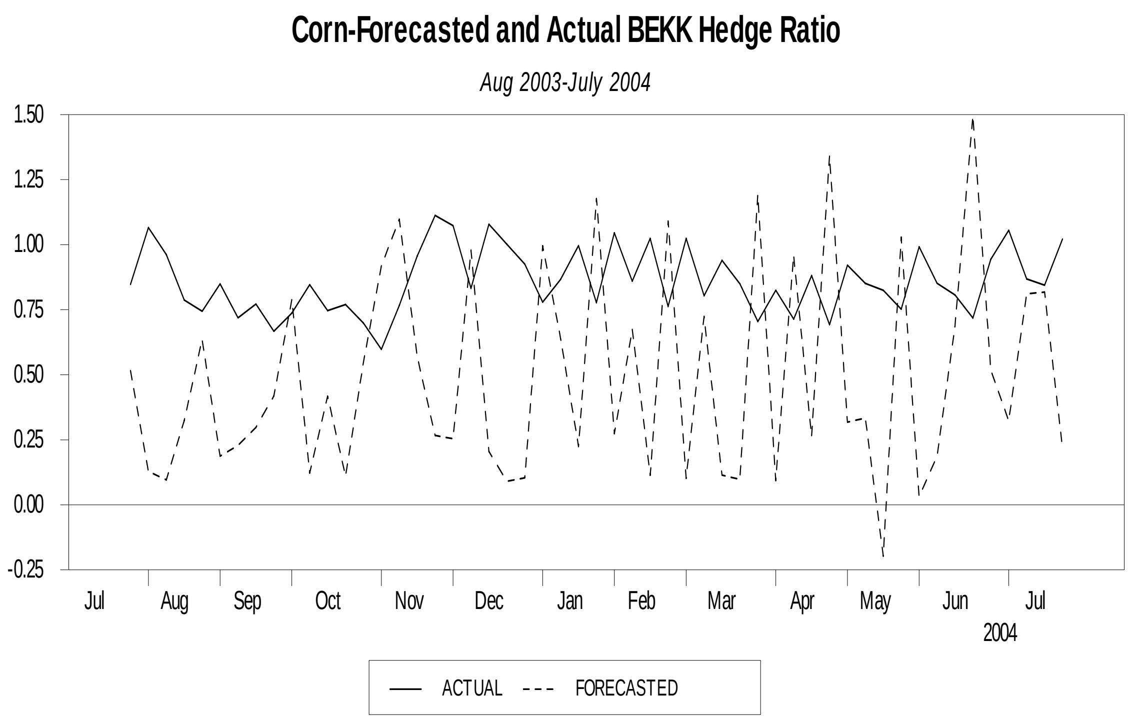
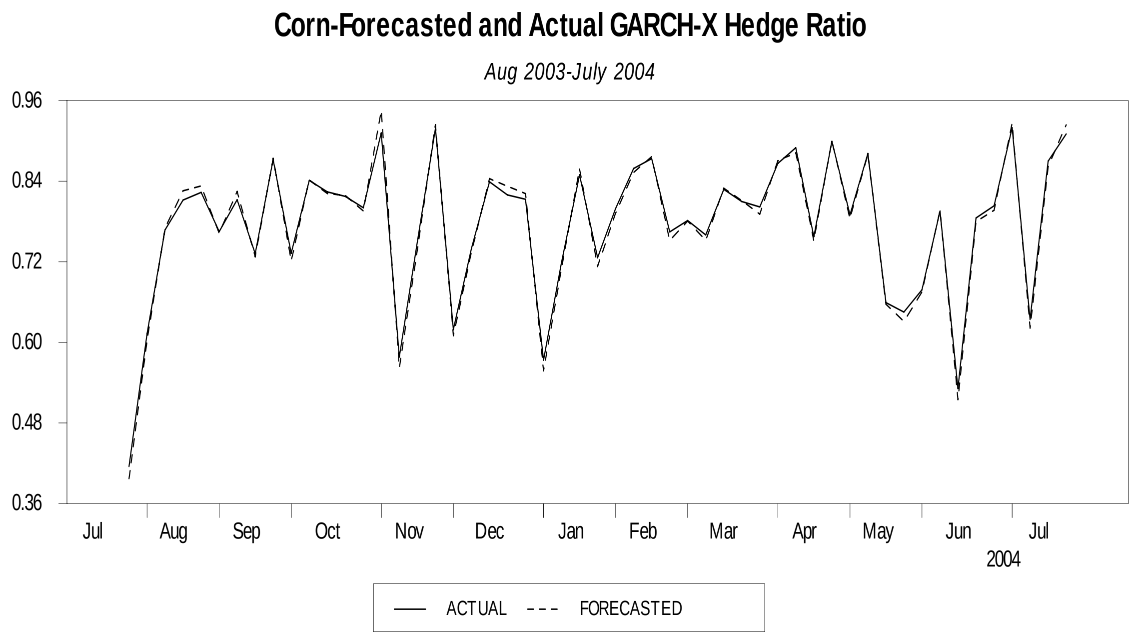


Figure 2
Forecasted and Actual Hedge Ratios (Aug 2003-July 2004)



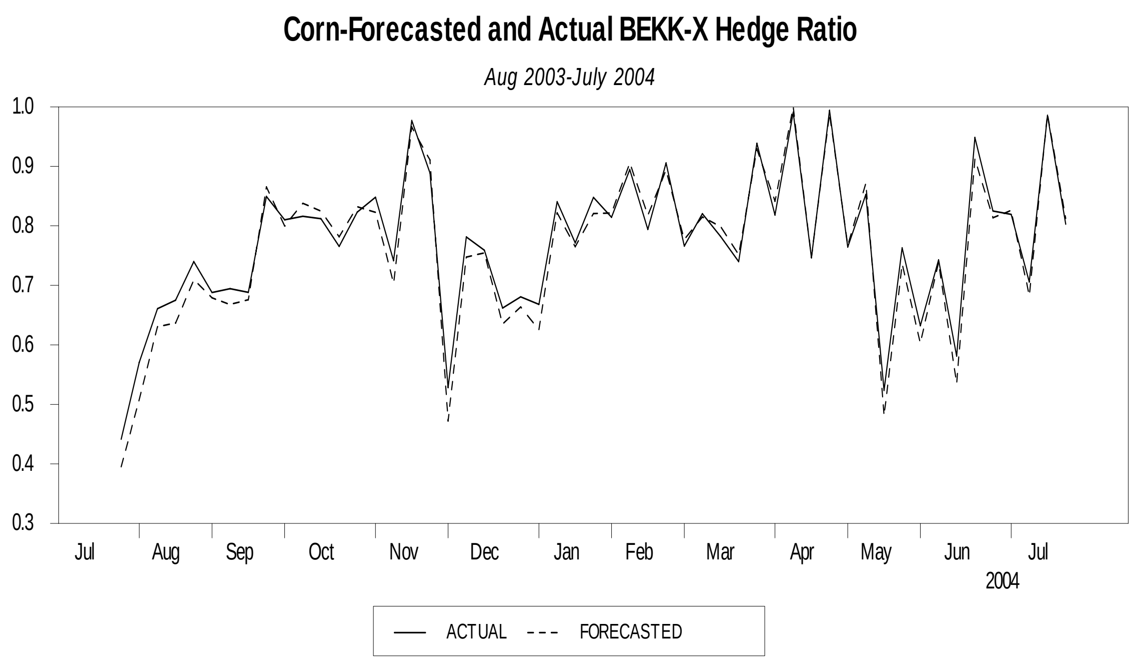


Figure 2
Forecasted and Actual Hedge Ratios (Aug 2003-July 2004)