Market Index Creation by Value-at-Risk Minimization. A Methodological and Empirical Proposal.

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Abstract:

Is it possible to create new Market Indices that are less risky than current ones? We propose a methodological approach to deal with this question using Value-at-Risk Minimization on the parametric VaR method. With this approach we can obtain the optimal weights each share must have in the Index to minimize Risk measured by VaR. We apply our method to three different stock markets and estimate Covariance matrices by different length moving averages. We would like to point out two innovations in our paper. First, an error dimension has been included in the backtesting and, second, the Sharpe's Ratio has been used to select the 'best' model from all models presented. Although the estimation methods used are very simple, our results seem very interesting. All our indices are less risky than the Spanish IBEX 35® and the Argentinian Merval (current Market Index) and, surprisingly, more profitable; this does not happen in the American DowJones^{SM} .This highlights two points. First, our indices could manage market risk without the problems of current risk measures [Basak and Shapiro (2001)]. Second, similar investment strategies could beat the market in some cases, thus questioning the Efficient Market Hypothesis. The possible applications of our Minimum Risk Indices are clear: they could reduce the risk assumed by institutional and mutual funds that nowadays follow Market Indices (these institutions could follow indices such as ours if it is confirmed that they are more profitable and less risky than some market indices). They could also be used as a benchmark for risky assets or as a basis for developing derivatives.

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1. Introduction.

Can we create new Market Indices that are less risky than current ones? Could these Indices beat market yield? The results of this paper, which deals with these questions, seem to imply that the answer to both questions could be affirmative in some cases.

There are two important reasons for creating new Market Indices. Firstly, there is a huge interest in market risk analysis and management. This is clear from the Basel agreements and other documents[3], or from the preoccupation derived from a bearish market context and financial bankruptcies such as the Long Term Capital Management case. That interest is perhaps bigger when we speak about emergent markets because of their special characteristics. Secondly, a lot of money is invested by following the market because of the traditional Fama's idea of efficient markets, and models as the CAPM that uses market yield as an essencial parameter. But what we mean with Market? Is the weighting criterion used in a Market Index especially important? Are the capitalization-weighted Indices better than price-weighted Indices? Could active investment strategies consistently beat the market contradicting the Efficient Market Hypothesis (EMH)? [Fama (1970)]. The results of our paper seem to point out that possibility, al least, in the Spanish and Argentinian market. As an example, if Spanish mutual funds had followed our investment strategy or similar during 2000-2004 they would have increased profits by €58.5 billion[4].

Financial risk has historically been analyzed by multiple measures[5] and models[6]. However, the increasing volatility in financial markets, derivatives and technological advances force us to now treat market risk from another perspective [Simons (1996) and Hendricks (1996)]. One emerging perspective is Value-at-Risk [Riskmetrics (1995)]—a measure of risk that has been rapidly and widely accepted since it was introduced in 1995. There are four VaR calculation methods: (1) the Parametric Method, with the original Riskmetrics (1995) approach or similar approximations such as Jorion's [Jorion (2001)]; (2) Historical Simulation and its evolutions [Boudoukh *et al.* (1998) and Hull and White (1998)]; (3) Stochastic Simulation (also known as the MonteCarlo Method) and its evolutions[7]; (4) Hibrid Methods such as Weighted Historical Simulation or mixed Stochastic and Historical Simulation [Boudoukh *et al.*

[3] See the Group of Thirty (1993), the documents of the Basel Committee on Banking Supervision or England (1997).
[4] Around €196,000 milion invested in Spanish mutual funds in 2003 according to FEFSI (Federación Europea de Fondos y sociedades de Inversión). Turnover and transaction costs are not included in the calculations.
[5] See Stone (1973), Pedersen and Satchell (1998) or Nawrocki (1999).
[6] See Markowitz (1952), Sharpe (1964) or Merton (1973).
[7] See Pearson and Smithson (2002), Frye (1996), or empirical studies in Pritsker (1997), Abken (2000), or Gibson and Pritsker (2000).

(1998)]. Each VaR calculation method has its pros and cons and provides quite different VaR measures [Hendricks (1996)]. Also, after a method has been applied it is necessary to check its reliability using a backtesting process. All the methods discussed above have several problems regarding leptokurtosis or skewness, non-linear positions or extreme returns. Other complementary techniques have therefore been developed to study extreme returns (Stress testing [Robinson (1996)], Conditional VaR or Extreme Value Theory with the Expected Shortfall method[8]). VaR has several problems, but if we can determine a controlled scenario with some interesting conditions, traditional VaR methods are reliable enough [Danielsson *et al.* (1998)] and easier to calculate than extreme value methods.

VaR and complementary methods have grown rapidly over the last few years. However, no attention has been paid to VaR as a possible tool for market risk management[9] or portfolio optimization[10] [Froot *et al.* (1994)]. Our study is therefore involved in these areas because, in our opinion, portfolio optimization using VaR or Conditional VaR is a natural evolution of Modern Portfolio Management Theory. On the other hand, latelly has appeared the idea of active indexing [Schoenfeld (2004)] as a framework to unify the traditional antagonic perspective of indexing and active investment. In this paper we propose the creation of new Market Indices through VaR minimization as another step in the direction of active indexing. These new Indices (created using VaR minimization) may be interesting for controlling market risk because better and more optimal instruments could then be used by institutions to reduce market risk. Moreover, the new indices could be used to estimate more stable Betas in the CAPM model, or as a reference for active investment strategies. In that line, mutual funds willing to beat the current Market Indices, would have with this methodology a more clear and transparent active management benchmark to be compared with, because is known active management performance is nowadays difficult and criticized for their the opacity.

We first discuss the theoretical framework of Minimum Market Risk Indices and then apply it to the Spanish, American and Argentinian Stock Market, using weekly logarithmic returns to determine the optimal weight each share must have within the index to minimize risk. Using historical data from the period 1999–2004 where we can find bearish and bullish markets, we reconstruct the performance of our Minimum Risk Indices for the 2000-2004 period. Despite the simple methods used to solve the problem, which can be easily improved by more complex econometric methods such as GARCH, our main objective has been achieved. Our Indices have less risk and in the

[8] See Pearson and Smithson (2002) or Neftci (2000).
[9] Only a few examples can be found in Garman (1996), (1997a,c) or in Aragall (2002).
[10] Only a few authors, such as Sentana (2001) or Rockafellar and Uryasev (2000), have dealt with this question.

Spanish and Argentinan markets present higher returns than the current Market Indices.

This paper is organized as follows. In Part 2 we describe the theoretical framework of Minimum Risk Market Indices using VaR minimization. In part 3 we discuss the results for the three Stock Markets taken as applications of our methodology. In part 4 we draw conclusions. In part 5 we outline future research. Finally, in part 6 we provide an Appendix and in part 7 the References.

2. Theoretical framework for Minimum Risk Market Indices using VaR minimization. A methodological proposal.

If we want to use VaR as a risk management tool, we have to find a method that institutions and investors find easy to follow. These characteristics mean that parametric VaR is the most suitable method for our objectives. However, it is necessary to take into account the weaknesses of the parametric approach, which are basically related to its principal hypothesis of normality. If this Hypothesis is not fulfilled, the VaR measure will be not coherent [Artzner, *et al.* (1999)]. In our study, we should be optimistic because the Central Limit Theorem should make Market Index returns similar to a Gaussian distribution if the number of shares forming the Index is high enough[11]. Indeed, as in our portfolio we do not include non-linear positions, and we use weekly data at a 5% significance level to calculate the VaR, the parametric Gaussian approach is considered reasonably good [Hendricks (1996), Danielsson *et al.* (1998)].

The problem to solve can be written as follows:

$$\begin{aligned} & Min\left\{ \ Z_{\alpha}\sqrt{x'\Sigma x} \ \right\} \\ & s.a. \\ & \qquad x_i \geq 0;\ i = 1..N \\ & \qquad \sum_{i=1}^{N} x_i = 1 \end{aligned} \qquad (1)$$

where Z_{α} is the Normal distribution value at the desired significance level, the x vector contains the weight of each share within the alternative Market Index we are trying to build, N is the number of shares forming the Index, and Σ is the logarithmic return Covariance matrix, which is assumed to be a Multivariate Normal. This problem is easy to simplify [Peña(2002)] using a Lagrangian optimization (see Appendix). The optimization result will provide the optimal weight each share must have within the Index to minimize risk

[11] See Appendix for further information.

using the estimated Covariance matrix. The literature contains several methods for estimating the Covariance matrix:

a) The *Historical Volatility Method:* empirical studies show that this method is not very good because it pays no attention to time-varying volatility.

b) The *Moving Average Method*: this method provides better estimations but also has some problems [Alexander (1996)].

c) The *Exponential Weighted Moving Average Method* [Riskmetrics (1995)]. With this method, the last observations in the data receive a higher weight, which solves the problems of the Moving Average Method. The decay factor election is critical [Hendricks (1996)]

d) The *GARCH and E-GARCH Methods*[12]*.* With these models [Engle (1982), Bollerslev (1986) and Nelson (1991)] it is possible to deal with heteroskedastic time dependent variance. GARCH Covariance estimations are clearly better than those from simple methods, but GARCH is not very used in the professional world.

e) Variance and covariance estimations by Implied Volatility and other less well-known methods such as those using the expectatives of experts in the financial field.

After the Covariance matrix has been estimated using one of these methods, the minimization method can be used to obtain the optimal weights each share must have within the Index to minimize the Index's market risk. With the historical data available, we can reconstruct the performance and evolution of Minimum Risk Indices to compare returns and risks between Minimum Risk Indices and current Market Indices.

Once the reconstruction of Minimum Risk Indices is available for a certain market, the validity of each approximation must be checked by a backtesting process. This process will establish how well the model applied to the data fits the real market. The more our main objective is to analyse and control risk by VaR techniques, the more backtesting is needed to study market risk not controlled by the VaR model and extreme losses within the significance level.

Finally, there is more than one Minimum Risk Index. With each Covariance matrix estimation method, we can build a different Minimum Risk Index. There is therefore a great number of these Indices, depending on the Covariance matrix estimation method and different parameters used by each method (data availability, moving average length, decay factor, etc.) Here we can see the importance of selecting the 'best' Index from all the approximations. Not too much work is available here and we only need to point to the paper written by Sarma *et al.* (2003). In our opinion the 'best' model should be selected in accordance with two key ideas:

[12] See extended models in Johnson (2001).

(1) The model's capacity to explain reality or, in other words, the model's capacity to be accepted by a periodic backtesting process. Risk not controlled by the model appears in extremely negative returns, but so far only the frequency of these extreme returns has been usually used for backtesting[13]. In this study we present a very simple way of incorporating the size of error in the traditional backtesting using the Excess Total Loss (ETL) measure . ETL is an *ex-post* measure that gives Total Losses beyond the VaR.

(2) It is important to establish a relationship between return and risk in VaR measures [Dembo and Freeman (1998)]. To do so, we use Sharpe's Ratio as a first approach and leave more appropriate tools such as the Reward-to-VaR Ratio [Alexander and Baptista (2003)] for future research.

Selecting the 'best' model is a very complex process. Here we only provide some simple orientative approximations to the problem. Further research on this issue will be developed soon.

3. Minimum Risk Indices in real markets. Some examples.

Using the theoretical framework developed in part 2, we are able to generate Minimum Risk Indices for each Stock Market we chose. As an example of how our methodology reacts to different Market Indices, in this section we apply it to the Spanish Stock Market (developing some Minimum Risk Indices for the IBEX35®), to the American Stock Market (developing some Minimum Risk Indices for the Dow Jones Industrial Average[SM]), and finally to the Argentinian Stock Market (developing some Minimum Risk Indices for the MERVAL). The objective of these examples is double. Firstly, is interesting to prove how our Minimum Risk Indices work in Stocks Markets with different volatilities and characteristics. Secondly, each of the Indices that have been chosen, represents a different way to built a Market Index[14] and using them in our approximation is a first step to notice the importance of different weightings in the market index calculation.

The objetive is to create Minimum Risk Indices based in the historical composition of the IBEX35®, the Dow Jones Industrial Average[SM] and the MERVAL for the 2000-2004 period. To say it more simply: our Minimum Risk Indices would be developed taking into account only the shares that composed each Index in each period, so, in that way, is possible to determine if a different weighting in the components of the actual Indices using a VaR Minimization criterion can reduce risk and how this affects the profitability of Market Indices. We call our Indices IndexVaR35 (IVaR35) in the case of the Spanish Market, IndexVaR30 (IVaR30) in the case of the American

[13] Few authors have tried to incorporate size of error in the backtesting process [Blanco and Oks (2004) or Lopez (1999)]

[14] See Appendix for more information.

Market, meaning 35 and 30 the number of traditionally components of the IBEX35® and Dow Jones Industrial Average^{SM} (DJIA^{SM}), and finally, we call IndexVaRM (IVaRM) the Minimum Risk Index in the case of the Argentinian Market. As we have mentioned, there is not just one Minimum Risk Index for each market, because with each estimation criterion we can create a Minimum Risk Index. The Covariance matrix was estimated in all the markets by the simplest estimation methods (the Historical method[15] and the Moving Average Method using lengths of 4, 8, 10, 15, 25, 30, 40, 52, 60, 70, 78, 85 and 100 weeks) in order to explain our method's potential benefits, although we know these estimates can be improved by more complex methods. In the end we decided to present IVaR35, IVaR30 and IVaRM Indices calculated only by some of these Moving Averages as being representative of the short, medium and long terms.

Covariance matrices estimated with a few data (4, 10, 15, 20, 30, etc. weeks) are problematical because the minimization process is difficult or rather unstable in some cases. Short-length Moving Averages change quickly in response to financial data but consistently underestimate the VaR value and provoque problems inside the minimization process due to the positive and semi-defined Variance-Covariance Matrix condition is sometimes not fullfilled, which has an impact on the backtesting process. Medium-length Moving Averages (25, 30, 40, 52 etc. weeks) are more stable and VaR measures closer to real values. Finally, long-length Moving Averages (e.g. 60, 70, 78, 85, 100 weeks) are the most stable but are less able to adapt to volatile short-term changes[16]. Despite the limited prediction capacity of Moving Averages'[17], our results with these approximations are quite interesting.

3.1 Volatility analysis of different Moving Average approximations.

The basic objective of our study was, by VaR minimization, to create Minimum Risk Market Indices that are less risky than current ones. Table 1 shows clearly how our Indices are less risky than the current ones in each market because overall volatility is lower. Paying atention to the data, is easy to see how the reduction of volatitily is more important in the Spanish Market than in the American and Argentinian Market. It also shows that, in general, the longer the moving average, the less volatile meaning risk is reduced. This seems not to be true in all the cases with the longest moving averages (52 and 78 in the IVaR35, 78-100 in the IVaR30 and 52-78 in the IVaRM) for which volatility is more or less the same or increases slightly. As with longer lengths it is more difficult to estimate short changes in volatility, this could mean that there is an optimal moving average length beyond which it is impossible to reduce risk using the moving average method. Improved Moving Averages (in

[15] Results in the 2002-2004 period were poor, similar to other empirical studies, so we decided not to generate a IVaR35 using this estimation method.

[16] See Appendix.

[17] Hopper (1996) shows that more complex estimation methods such as GARCH do not provide better results than simple estimation methods with long-term data.

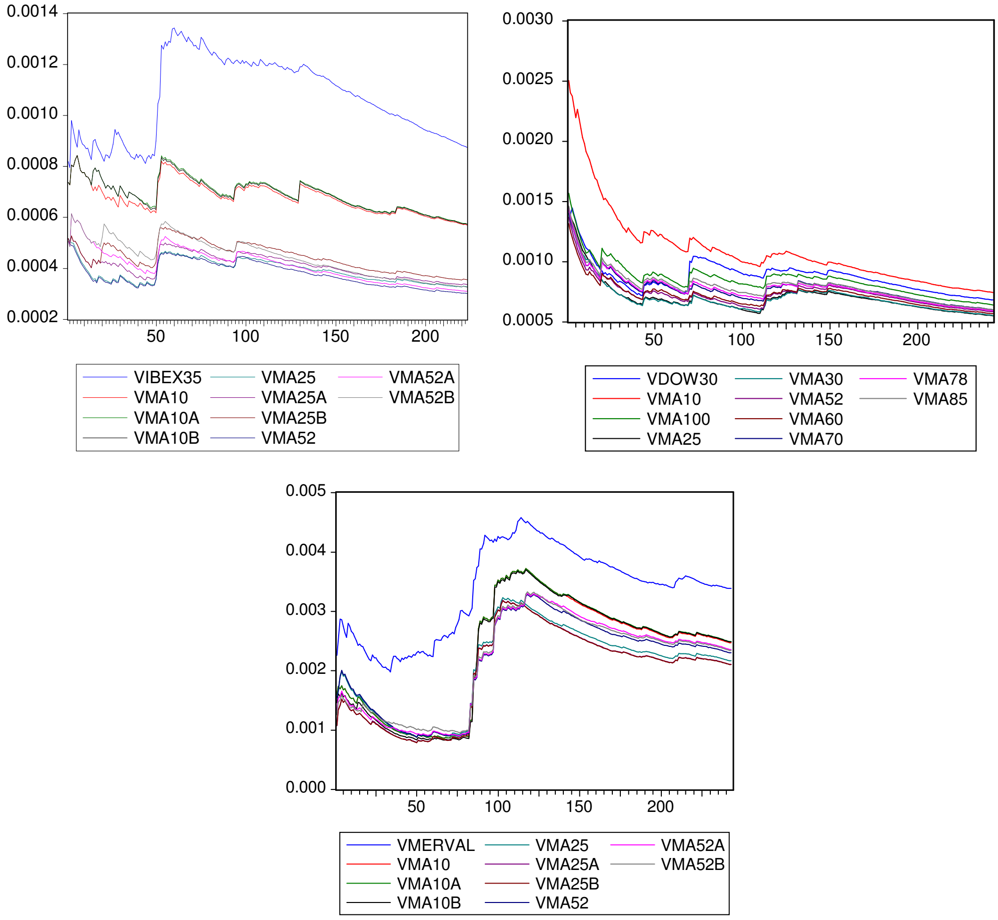
the IVaR35 and in IVaRM) are a little more risky than those with no improvements. This result is rational because, firstly, multiple-step estimation was applied to avoid underestimating the risk, and secondly, the 0.01% weighting restricts one asset, so that the portfolio can be less diversified, and risk rises.

Volatility reduction is clearer in Figure 1, which shows cumulative volatilities. In the first picture is possible to observe the Spanish Market. The first line represents the riskiest Index, which in our case was the current Market Index (IBEX 35®). The second group of lines is made up of the MA10, MA10a and MA10b approximations. The third group, with half the IBEX 35® risk, is made up of the MA25, MA52, MA78 approximations and all their modifications. The most stable approximations are the modifications a, especially MA52a, which is less risky than MA25a and MA78a. This again indicates the existence of an optimal length for moving averages beyond which it is impossible to better estimate the covariance matrix and reduce risk with moving average methods. In the second picture is possible to observe the American Market. The first line represents again the riskiest Index that now is the MA10 approximation due to problems with the positive and semi-defined covariance matrix condition. After that and before observation 75, the second riskiest Index is the current Dow Jones Industrial Average℠ . Below the current Market Index, and with less risk, is possible to find all the other MA approximations. The less risky are MA25 and MA30, and the more data is used to build the MA the more risky the Moving Average approximation seems to be, suporting the idea of the optimal length for moving averages. Finally, in the third picture is represented the Argentinian Market. The first line (the riskiest Index) is again the current Market Index (MERVAL). Below it is possible to find the MA10,a,b as the second group of riskiest approximations, and below, with less risk, the other MAs. Again, is possible to establish the idea of optimal lengh for moving averages because MA25 and MA30 have lower risk than MA52, MA60, MA70, MA78, MA85 and MA100.

Table 1
Market Indices' Standard Deviation

Approximation		Standard Deviation		Standard Deviation		Standard Deviation
Current Market Index	IBEX35®	0.02958	DJIA^{SM}	0.02610	MERVAL	0.05817
Minimum Risk Index	IVaR35		IVaR30		IVaRM	
MA10	MA10	0.02387		0.02727		0.04970
	MA10a	0.02397				0.04989
	MA10b	0.02393				0.04983
MA25	MA25	0.01811		0.02347		0.04655
	MA25a	0.01835				0.04591
	MA25b	0.01884				0.04587
MA30				0.02354		
MA52	MA52	0.01736		0.02415		0.04796
	MA52a	0.01760				0.04855
	MA52b	0.01807				0.04838
MA60				0.02383		
MA70				0.02452		
MA78	MA78	0.01742		0.02433		0.04805
	MA78a	0.01835				0.04822
	MA78b	0.01867				0.04971
MA85				0.02451		
MA100				0.02533		

Figure 1
Market Indices' cumulative volatility (IVaR35, IVaR30, IVaRM)



Note: here volatility is variance. In the first, second and third graph VIBEX35, VDOW30 and VMERVAL are the cumulative volatilities of the IBEX35®, the DowJones Industrial Average[SM] and the MERVAL respectivelly, and VMA are the cumulative volatilities of each moving average approximation used for each market.

3.2. Analysis of extreme losses in VaR minimization .

Basak and Shapiro (2001) and Larsen *et al.* (2002) show that by not allowing agents to assume more risk than a certain VaR value or to develop VaR minimizations can increase extreme losses, especially when return distributions are very different from Normal distributions. Following these ideas, Basak and Shapiro (2001) do not think it is useful to set limits on VaR values in institutions to control risk and Larsen *et al.* (2002) propose Conditional VaR minimizations. This results appear basically when distributions are heavily skewed or have long fat tails. In our case, the problems noticed by above authors were not excessively important[18] (see Table 2). For the

[18] See appendix for further information.

shortest moving average, extreme loses were similar to those of IBEX 35® and lower in the American and Argentinian market, and decreased when we increased the moving average length. There is a determinated moving average lenght when extreme losses start to raise again (Ma78 in IVaR35, MA70 in IVaR30 and more difficult to define in IVaRM), which again supports the existence of an optimal length moving average.

Table 2
Extreme Losses

		Highest Extreme Loss (%)			
IBEX35®	11.1	DJIA℠	15.4	MERVAL	15.3
IVaR35		IVaR30		IVaRM	
MA10	11.2		10.8		12.1
MA10a	11.2				12.1
MA10b	11.2				12.1
MA25	6.6		9.2		11.4
MA25a	6.6				9.8
MA25b	6.6				9.8
MA30			8.4		
MA52	6.3		10.4		15.3
MA52a	6.8				15.3
MA52b	6.8				15.3
MA60			9.2		
MA70			10.6		
MA78	6.4		10.5		11.4
MA78a	7.6				10.9
MA78b	7.6				10.0
MA85			10.7		
MA100			10.9		

3.3. VaR analysis of Moving Average approximations

Each approximation has a different VaR measure that evolves along time. In the figures shown in the Appendix is possible to see how great changes in volatility that are common in moving average approximations are greater in short length moving averages than in long length ones. On the other hand, as these kind of averages do not attach different weights to more recent data than to older data, moving averages are indicators of 'past' volatility, regarding inappropriate Covariance estimations when price tendency change[19]. This problem decreases when the lengths are longer [20]. Finally, we should point out that short moving averages usually underestimate VaR, so the

[19] Beginning of 2000 and 2002-2003.
[20] Short moving averages collect short-term volatility better but face tendency changes more often; long moving averages collect long-term tendencies and only have problems when facing long-term changes.

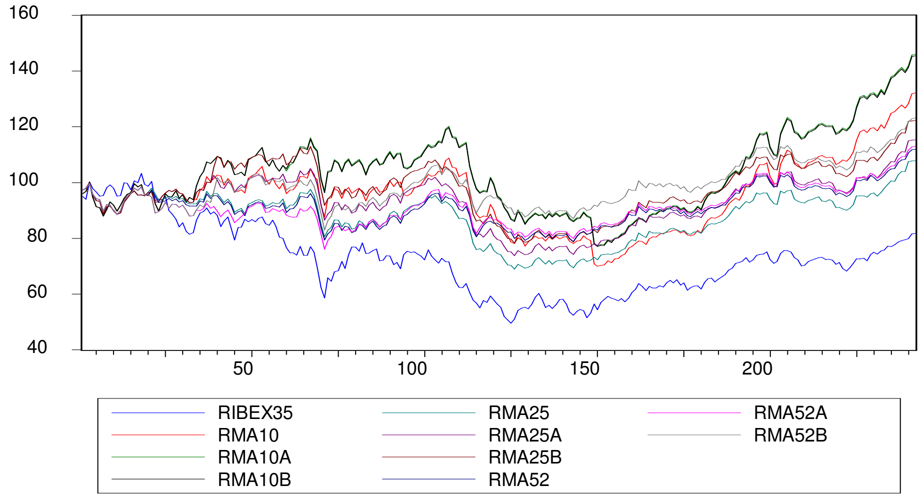
losses beyond the VaR will be more frequent in that cases. The longer the moving average length, the less underestimation of the VaR measure.

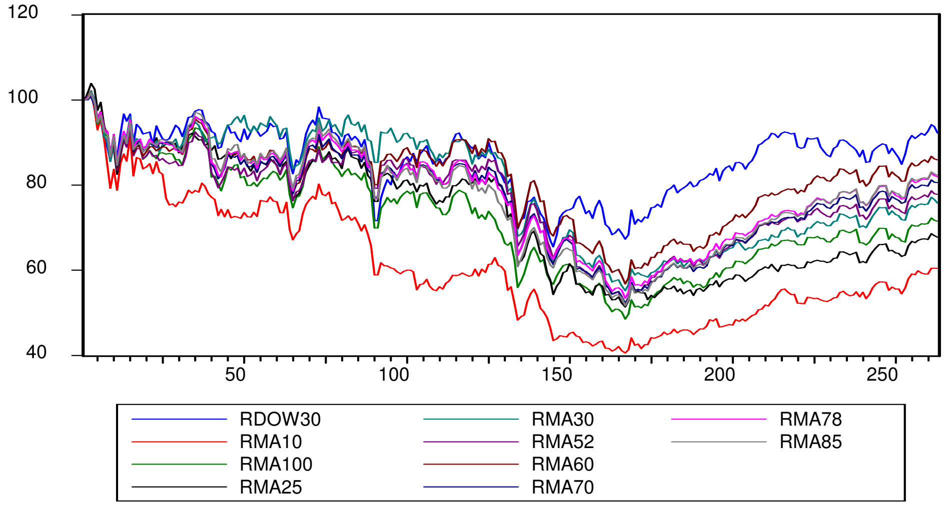
3.4. Return analysis of Moving Average approximations.

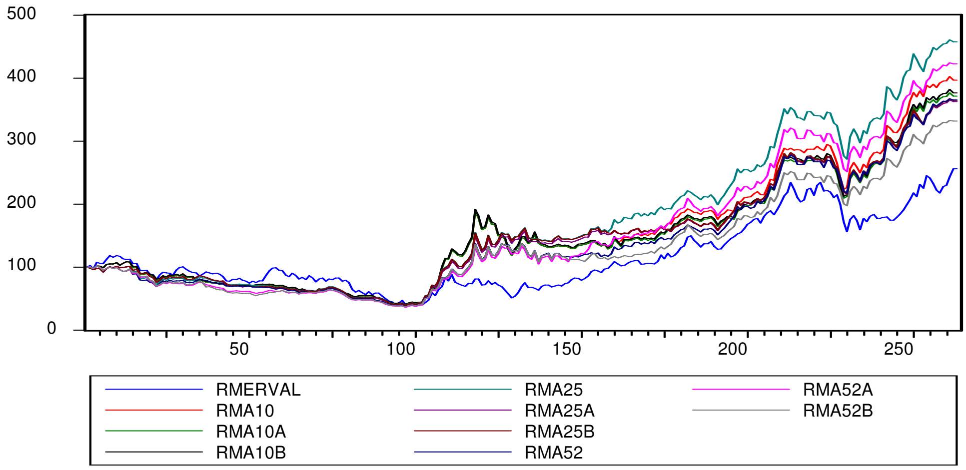
Figure 2 shows all the returns of Moving Average approximations. In the Spanish market, all our Indices, surprisingly, had higher returns than the IBEX 35®. There are two reasons for that data. The first one is Spanish Stock Market is not efficent, because is possible to built a porfolio more profitable than the market. The second one is that perhaps the way the IBEX35® is constructed (using market capitalization) could assure the benchmark represents the market but perhaps is easier to beat than other market index weightings. In contrast, in the American Stock Market, there is no Minimum Risk Index able to beat the market. In the IVaR30 the approximations with the worst returns are MA10. The other approximations performed quite good during the bearish market, being near the actual index or beating it in some periods, but performed worse thant the current index during the Irak war and after it, in the bullish market. At the end, the best approximation in terms of profitability is the MA60 with 20% lower return than the Dow Jones Industrial Average^{SM}. The same two reasons could be exposed in that case but, the important question outlined here is: Is the American Stock Market more efficient, or the price weighting more difficult to beat? Finally, in the Argentian Market, all approximations (except 78b) are able to beat the market. In that case, the same reasons explained in the case of the IBEX 35® are valid here.

Reaching that point one important issue has appeared. It seems is important to stablish firstly how much efficient is a market, and secondly, how the way of weighting Indices affects his profitability and the possibilities of beating it throught active strategies for example a Minimum Risk Index. This interesting question is keep to solve in future research.

Figure 2
Evolution of the indices (IVaR35, IVaR30, IVaRM)







Note: 100 based. In the first, second and third graph RIBEX35, RDOW30 and RMERVAL are the evolution of IBEX35®, DJIA^{SM} and MERVAL, and RMA are the evolutions of each moving average approximation in each market.

3.5. *IVaR35, IVaR30 and IVaRM Composition.*

Each Minimum Risk Index has a different optimal composition. With short-term Moving Averages, the optimal composition changes frequently over the weeks, whereas it is more stable with long-term Moving Averages. It is relativelly important if we have in mind our index approximation needs a weekly ajustment, meaning higher turnover and transaction costs with shorter moving averages. As optimal compositions are calculated using a risk measure and the Covariance matrix estimation, they are unlike current compositions of Market Indices.

3.6. *Normality Analysis and Backtesting .*

Normality analysis of logarithmic returns is not very positive, as is shown in Table 3, 4 and 5. In all cases, the Normality Hypothesis has been rejected excepts in the case of Merval. In the case of IVaR35 the distributions have leptokurtosis and are slightly negatively skewed. In the case of IVaR30 the distributions are more leptokurtical and negative skeweness is especially important, afecting as it has said, to the probitability results. Finally, in the case of IVaRM distributions are extremelly leptokutical and skeweness is positive due to the evolution of Merval during the analysed period. If we look at the backtesting results, though real errors are more frequent than the 5% significance level expected, they are not very large (around 2% higher than the VaR value in the IVaR35, 2.80% in the IVaR30, and 2% in the IVaRM). Errors are more controlled in terms of frecuency in the case of IVaR30 than in the IVaR35 and IVaRM, but are less controlled in terms of magnitude (Mean error in the American and Argentinian case is higher than in the Spanish case). Basak and Shapiro (2001) observed that setting VaR limits on institutions could lead to higher extreme loses than when these limits are not set. With our results is possible to see this theoretical result is not clear in the data.

Table 3
Backtesting process in the IVaR35

	Normality		Backtesting			
	Jarque-Bera	Probability	Mean VaR(%)	Errors	% Errors	Mean Error(%)
IBEX35®	12.22	0.002				
MA10	102.4	0.000	0.654	75	31	1.95
MA10a	101.4	0.000	0.671	75	31	1.89
MA10b	103.3	0.000	0.665	75	31	1.89
MA25	32.9	0.000	1.446	43	18	1.35
MA25a	40.86	0.000	1.557	37	15	1.35
MA25b	48.3	0.000	1.582	36	15	1.37
MA52	43.3	0.000	1.746	33	13	1.41
MA52a	65.8	0.000	1.943	28	11	1.34
MA52b	56.9	0.000	1.996	24	10	1.52
MA78	45.6	0.000	1.843	33	13	1.41
MA78a	116.2	0.000	2.136	25	10	1.46
MA78b	124.95	0.000	2.192	22	9	1.63

Note: VaR value calculated at 5% significance level using data available from 242 weeks. Errors: losses worse than the VaR value. % Errors: 'real' significance level. Mean Error (%) shows the mean loss exceeding the VaR value.

Table 4
Backtesting process in the IVaR30

	Normality		Backtesting			
	Jarque-Bera	Probability	Mean VaR(%)	Errors	% Errors	Mean Error(%)
DJIA^{SM}	271.4	0.000				
MA10	85.85	0.000	1.208	67	25.5	2.23
MA25	62.09	0.000	2.221	38	14.5	2.03
MA30	49.04	0.000	2.365	33	12.6	2.05
MA52	124.2	0.000	2.825	22	8.3	2.78
MA60	75.17	0.000	2.944	23	8.7	2.39
MA70	123.32	0.000	3.062	20	7.6	2.80
MA78	117.74	0.000	3.141	19	7.2	2.81
MA85	130.40	0.000	3.205	22	8.3	2.41
MA100	150.14	0.000	3.329	24	9.1	2.45

Note: VaR value calculated at 5% significance level using data available from 262 weeks. Errors: losses worse than the VaR value. % Errors: 'real' significance level. Mean Error (%) shows the mean loss exceeding the VaR value.

Table 5
Backtesting process in the IVaRM

	Normality		Backtesting			
	Jarque-Bera	Probability	Mean VaR(%)	Errors	% Errors	Mean Error(%)
MERVAL	4.28	0.111				
MA10	664.02	0.000	3.08	61	23.4	1.98
MA10a	648.56	0.000	3.10	60	22.9	2.07
MA10b	652.00	0.000	3.12	60	22.9	2.03
MA25	534.82	0.000	4.47	38	14,5	2.02
MA25a	619.22	0.000	4.64	33	12.64	2.04
MA25b	623.62	0.000	4.69	31	11.87	2.02
MA52	292.01	0.000	4.57	39	14.94	1.92
MA52a	243.82	0.000	5.24	29	11.11	1.98
MA52b	264.28	0.000	5.48	26	9.96	1.95
MA78	276.73	0.000	4.38	42	16.09	1.85
MA78a	207.70	0.000	5.59	29	11.11	1.90
MA78b	177.41	0.000	6.15	26	9.96	1.99

Note: VaR value calculated at 5% significance level using data available from 261 weeks. Errors: losses worse than the VaR value. % Errors: 'real' significance level. Mean Error (%) shows the mean loss exceeding the VaR value.

3.7. *Model Selection. The best IVaR35, IVaR30 and IVaRM.*

We have seen how parametric VaR minimization could create Minimum Risk Indices with less risk and in the Spanish and Argentinian case with greater profitability than current market indices. In this paper we constructed 12 approximations using Moving Averages of different lengths for the Spanish and Argentinian market and 9 approximations for the American market. Now we need to decide which is the 'best' approximation for use in each market from all the approximations presented. We think this selection should be done in accordance with two ideas:

(1) the model's capacity to explain reality or, in other words, its capacity to be accepted by the backtesting process. After determining the number of returns lower than the VaR value (classic backtesting), it is important to also measure the error magnitude. Until now this type of backtesting has not yet been developed, and here we only propose a very simple method that deals with error magnitude using the Excess Total Loss (ETL) measure, which is defined as the total sum of all returns lower than the VaR value over the studied period. We will choose those approximations with the lowest ETL taking into account that way the risk 'out of the model'. After chosing with the criterion of ETL, is necessary to select that approximations with less VaR, or with less risk 'within the model'. As we can see in Table 6, in the Spanish market, using ETL the best approximations are MA52a,b and MA78a,b. Moreover, studying the 'controlled' risk within the model, we

conclude that the MA52a,b approximations are the least risky. In the American Market, as it could be seen in Table 7, the best approximations using the ETL are MA60, MA70, MA78, MA85 and MA100. Using the 'controlled' risk we conclude the best approximations are MA60, MA78 and MA85. Finally, in the Argentinian market, the best approximations by ETL are MA52a,b and MA78a,b, and after using the 'controlled risk' measured by the VaR we can conclude the best approximation are MA52a and MA52b.

(2) the relationship between return and risk, since Dembo and Freeman (1998) criticize not attaching importance to that point in VaR calculations. Here we use Sharpe's ratio to analyse this relationship.

In the Spanish market, Sharpe's ratio in the MA52b approximation is bigger than in the MA52a approximation so we can conclude that MA52b is the best approximation with which to construct the Spanish Minimum Risk Index. In the American market, Sharpe's ratio in the MA60 approximation is the lower among the selected MAs, so MA60 is the best approximation with with to construct the American Minimum Risk Index. Finally, in the Argentinian market, Sharpe's ratio in MA52a is higher than in MA52b, so is reasonable to conclude MA52a is the best approximation to construct the Argentinian Minimum Risk Index.

Table 6
Model Selection in the Spanish Stock Market

	Backtesting			Mean VaR (%)	Sharpe's Ratio
	Errors	Mean Error(%)	ETL in 2000-2004		
MA10	75	1.95	146.25	0.654	11.71
MA10a	75	1.89	141.75	0.671	15.83
MA10b	75	1.89	141.75	0.665	15.69
MA25	43	1.35	58.05	1.446	4.18
MA25a	37	1.35	49.95	1.557	7.75
MA25b	36	1.37	49.32	1.582	10.70
MA52	33	1.41	46.53	1.746	6.52
MA52a	28	1.34	37.52	1.943	6.96
MA52b	24	1.52	36.48	1.996	11.56
MA78	33	1.41	46.53	1.843	8.03
MA78a	25	1.46	36.5	2.136	6.25
MA78b	22	1.63	35.86	2.192	10.54

Note: in Sharpe's ratio non-risk return has been considered equal to zero.

Table 7

Model Selection in the American Stock Market

	Backtesting			Mean VaR (%)	Sharpe's Ratio
	Errors	Mean Error(%)	ETL in 2000-2004		
MA10	67	2.23	149.42	1.208	-18.46
MA25	38	2.03	77.14	2.221	-16.58
MA30	33	2.05	67.80	2.365	-11.75
MA52	22	2.78	61.21	2.825	-10.39
MA60	23	2.39	55.03	2.944	-6.34
MA70	20	2.80	56.18	3.062	-8.80
MA78	19	2.81	53.56	3.141	-8.17
MA85	22	2.41	53.14	3.205	-7.94
MA100	24	2.45	59.02	3.329	-13.22

Note: in Sharpe's ratio non-risk return has been considered equal to zero.

Table 8

Model Selection in the Argentinian Stock Market

	Backtesting			Mean VaR (%)	Sharpe's Ratio
	Errors	Mean Error(%)	ETL in 2000-2004		
MA10	61	1.98	121.1	3.08	27.73
MA10a	60	2.07	124.2	3.10	26.31
MA10b	60	2.03	122.2	3.12	26.62
MA25	38	2.02	77.1	4.47	32.67
MA25a	33	2.04	67.4	4.64	28.08
MA25b	31	2.02	62.7	4.69	28.22
MA52	39	1.92	75.2	4.57	27.03
MA52a	29	1.98	57.7	5.24	29.69
MA52b	26	1.95	50.8	5.48	24.78
MA78	42	1.85	77.8	4.38	27.02
MA78a	29	1.90	55.2	5.59	27.67
MA78b	26	1.99	51.7	6.15	17.92

Note: in Sharpe's ratio non-risk return has been considered equal to zero.

4. Conclusions.

In this article we propose using the VaR as an active risk measure to construct Minimum Risk Market Indices. We have used the parametric VaR approach to construct a very simple minimization problem in which the Covariance matrix among asset returns has to be estimated. Covariance matrix estimation can be done using many methods. There are therefore many ways of constructing a Minimum Risk Index—one for each way of estimating the Covariance matrix—so a method of selecting the best model is needed. This selection method must be based on the model's capacity to be accepted by the backtesting process, (taking into account error frequency and error magnitude) and the return-risk relationship.

We applied this method to the Spanish, American and Argentinian market to create different Minimum Risk Indices (IVaR35, IVaR30 and IVaRM) for the 2000-2004 period. Using the simplest Covariance matrix estimation methods, we achieved interesting results: our indices are less risky than the current ones (half risk in the Spanish Market). Also, thanks to their optimal portfolio characteristics, in the Spanish and Argentinian case they are able to achieved bigger returns than those obtained by the current market Indices against what is expected from the Efficient Market Hypothesis. That results point out a interesting discusion that must be treated carefully in our future research that must be based in the following ideas. First, the capability of moving average approximations to estimate future covariance matrices and the possibility of obtaining better results with more complex estimation methods. Second, the influence of the weighting process in the market index construction and their profitability. Third, the Minimum Risk Index aproximation in order to prove the eficience of a market, and how efficience affects our results. And finally if is possible to obtain better results not limiting our Minimum Risk Index shares to the nowadays Market Index components and to the particular and 'legal' timing of changes in components.

The potential uses of our indices are clear. Firstly, they are less risky and in some cases more profitable than current ones, which makes them a suitable benchmark of risky assets for mutual funds that currently follow market indices or a suitable base for derivatives. Secondly, our Minimum Risk Indices may generate more stable Betas in the CAPM model, possibility that must be developed in the future.

5. Future lines of research.

The results achieved by very simple methods in the examples presented are interesting, but it must also be said that there is a lot still to do. Firstly, we need to determine whether better Covariance estimations using EWMA or GARCH methods can achieve better results in terms of risk and profitability, and secondly, if the weighting process in market index construction affects the eficience of the market and the possibility of beating it. Thirdly, the methods of selecting the 'best' model must be further developed since here we have only provided some general guidelines.

6. Appendix.

6.1. *Minimum VaR Indices..*

The problem to solve can be written as follows:

$$\begin{aligned} &Min\left\{ Z_\alpha \sqrt{x'\Sigma x} \right\} \\ &s.a. x_i \geq 0; i = 1..N \\ &\sum_{i=1}^{N} x_i = 1 \end{aligned} \qquad (1)$$

where Z_α is the Normal distribution value at the desired significance level. The x vector contains the weights of each share within the alternative Market Index we are trying to build, N is the number of shares that make up the Index, and Σ is the logarithmic return Covariance matrix assumed to be a Multivariate Normal. This problem is easy to simplify [Peña(2003)]:

$$\begin{aligned} &Mín\ x'\Sigma x \\ &s.a. \\ &x'\vec{1} = 1 \end{aligned} \qquad (2)$$

where $\vec{1}$ is a Nx1 vector all of whose values are equal to one.

Using a Lagrangian optimization (3) in which the optimal weights are the objective of our study and λ is a positive constant:

$$MínL = x'\Sigma x + \lambda(1 - x'\vec{1}) \qquad (3)$$

As the Covariance matrix is positive and semi-defined if the number of observations is bigger than the number of assets, first order conditions are enough for a minimum[21].

$$\frac{\partial L}{\partial x} = \Sigma x - \lambda \vec{1} = \vec{0} \qquad (4)$$

[21] Second-order conditions are also necessary in the other case.

$$\frac{\partial L}{\partial \lambda} = 1 - x'\vec{1} = 0 \qquad (5)$$

To solve the solution:

$$x^{*} = \lambda(\Sigma^{-1}\vec{1})$$
$$\lambda = (\vec{1}'\Sigma^{-1}\vec{1})^{-1} \qquad (6)$$

The solution gives us the optimal weight each share must have within the index to minimize market risk.

In the empirical case for the Spanish, American and Argentinian Stocks Markets, we used an iterative algorithm based on Newton's method to make the minimization process and the command of the quadratic programing problem in Gauss with similar results.

6.2. *The Normality Hypothesis in the parametric VaR and the 'best' model selection process.*

The return distributions of the indices are similar to those of a Normal distribution by the Central Limit Theorem, weekly data and VaR at a 5% significance level. However, when Normality is rejected, this affects extreme loses and the 'best' model selection process.

If return distributions are Normal, error risk is equal to the significance level. In the best model selection, it should be enough to choose the approximation with the biggest mean and the lowest volatility or the one with an interesting combination of mean and volatility to allow this approximation to suffer fewer loses.

If return distributions are not normal, extreme loses will not be controlled by the model. In this case, the main problem is negative skeweness because leptokurtosis makes little gains and loses more probable. The importance of negative skewness should be determined and backtesting of the model should be performed in order to detect extreme loses that are not controlled by the model. In the 'best' model selection, error frequency, error magnitude and the return-risk relationship must be taken into account.

6.3. *Market Indices and their Minimum Risk Indices.*

In this part, is necessary to explain shortly how the Spanish IBEX35®, the American Dow Jones Industrial Average℠ and the Argentinian MERVAL are built. Is is important to explain too, some characteristics and problems we have found and solved in a determinated way in creating the Minimum Risk Index for each Market.

The Spanish IBEX35®:

The IBEX35® is built using the 35 greatest companies in the Spanish Stock Market taking into account market capitalization and liquidity. Every six months the components of the Index are checked and some shares are included or excluded maintaining the total number of assets. The Index is calculated using a market capitalization weighting criterion.

The Minimum Risk Indices we create for this market received the name of IvaR35, and comprise the 35 shares of the IBEX 35® at each moment with the optimal weight established by the VaR minimization process. We must point out one problem with the IBEX 35® Spanish Market Index. In the six-month revision of composition of the IBEX 35® it is normal to include shares and companies with very little history on the Stock Exchange because it is relatively easy to be new and one of the biggest 35 companies in the Spanish Market. During the period of our analysis we sometimes encountered this problem—especially in 1999-2000 because of the Internet and .com companies that grew quickly at that time. This makes it difficult to obtain complete data of all the IBEX 35® components in some periods and has important consequences in Covariance matrix estimation. After April 2000 we solved this problem with the following techniques:

a) *Covariance matrix estimation using a multiple-step method:* when we did not have complete data of the 35 shares, the Covariance matrix estimation was done using a multiple-step method, estimating each individual value in the covariance matrix with all the available data.

b) *0.01% Weighting*: the above solution improved the results, but shares with short historical data tended to underestimate risk and therefore received high weights because of their 'artificial' low risk. With this approximation we forced these shares to have the minimum weight accepted for our study.

The approximations we developed finally are represented in Table 9:

Table 9
Approximations used for Covariance matrix estimation in the IVaR35

Approximation	Method	Length	Improvements Applied.
MA10	Moving Average	10 weeks	None
MA10a	Moving Average	10 weeks	Multiple-step Method
MA10b	Moving Average	10 weeks	Multiple-step Method and 0.01% Weighting
MA25	Moving Average	25 weeks	None
MA25a	Moving Average	25 weeks	Multiple-step Method
MA25b	Moving Average	25 weeks	Multiple-step Method and 0.01% Weighting
MA52	Moving Average	52 weeks	None
MA52a	Moving Average	52 weeks	Multiple-step Method
MA52b	Moving Average	52 weeks	Multiple-step Method and 0.01% Weighting
MA78	Moving Average	78 weeks	None
MA78a	Moving Average	78 weeks	Multiple-step Method
MA78b	Moving Average	78 weeks	Multiple-step Method and 0.01% Weighting

The American Dow Jones Industrial Average^{SM}:

The DJIA^{SM} is built using the 30 greatest companies in the American Stock Market and for the sake of continuity, composition changes are rare. So, shares' inclusions and exclusions are not very usual and basically related to corporate acquisitions or dramatics business events. The Index is calculated using a price weighting criterion.

The available data for the DJIA^{SM} allow us to avoid the necessity of applying improvements to the Covariance matrix estimation.The good quality of these data has permited us to use our methodology with a greater number of moving average lenghts. The Minimum Risk Index we have created to this market has the name of IVaR30.

The approximations we developed are in Table 10:

Table 10
Approximations used for Covariance matrix estimation in the IVaR30

Approximation	Method	Length	Improvements Applied.
MA10	Moving Average	10 weeks	None
MA25	Moving Average	25 weeks	None
MA30	Moving Average	30 weeks	None
MA52	Moving Average	52 weeks	None
MA60	Moving Average	60 weeks	None
MA70	Moving Average	70 weeks	None
MA78	Moving Average	78 weeks	None
MA85	Moving Average	85 weeks	None
MA100	Moving Average	100 weeks	None

The Argentinian MERVAL®:

The MERVAL is built using the most negotiated companies in the Argentinian Stock Market. The weights of each share in the index are calculated using the number of transactions of these shares in the Stock Market and the Volume of these transactions, so the Index is calculated using a negotiation weighting criterion.

The Minimum Risk Indices we create for this market received the name of IVaRM, and comprise the shares of the Merval at each moment with the optimal weight established by the VaR minimization process. Every three months, the Merval composition aren changed, and is possible, as we found in the case of the IBEX 35® to find companies with very little historical data. In that case is necessary to improve calculations using the same techniques we used in the IBEX 35®. In table 11 we present the approximations we have used in this paper. Is important to point out that approximation b is especially influenciated in the Merval by the fact there are a lot of stocks with shot history or not history in some cases when they enter the Index. If we 'eliminate' in some cases a great number of possible investments obliging them to take a 0.01% weighting this affects to the performance and backtesting of the approximation.

Table 11
Approximations used for Covariance matrix estimation in the IVaRM

Approximation	Method	Length	Improvements Applied.
MA10	Moving Average	10 weeks	None
MA10a	Moving Average	10 weeks	Multiple-step Method
MA10b	Moving Average	10 weeks	Multiple-step Method and 0.01% Weighting
MA25	Moving Average	25 weeks	None
MA25a	Moving Average	25 weeks	Multiple-step Method
MA25b	Moving Average	25 weeks	Multiple-step Method and 0.01% Weighting
MA52	Moving Average	52 weeks	None
MA52a	Moving Average	52 weeks	Multiple-step Method
MA52b	Moving Average	52 weeks	Multiple-step Method and 0.01% Weighting
MA78	Moving Average	78 weeks	None
MA78a	Moving Average	78 weeks	Multiple-step Method
MA78b	Moving Average	78 weeks	Multiple-step Method and 0.01% Weighting

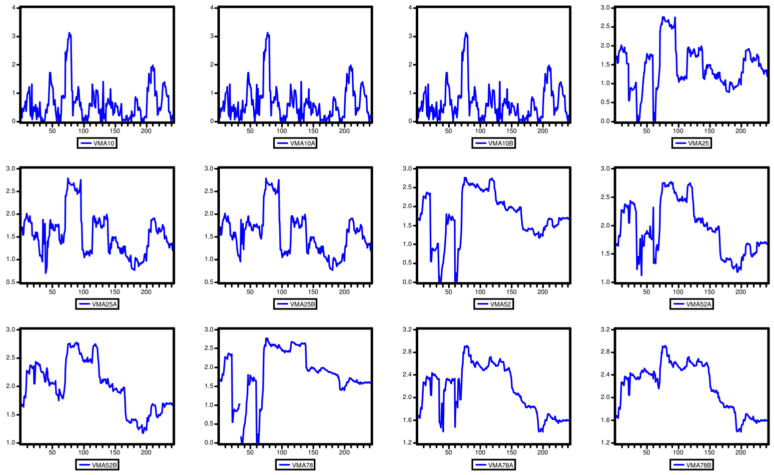
6.3. Market Indices and their Minimum Risk Indices.

In Figure 3 is possible to see the evolution of VaR in each approximation used to estimate the Covariance matrix for each period in the differents markets analysed.

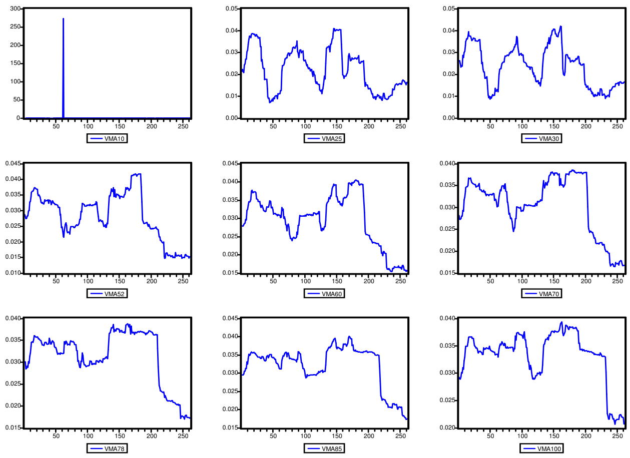
Figure 3

VAR

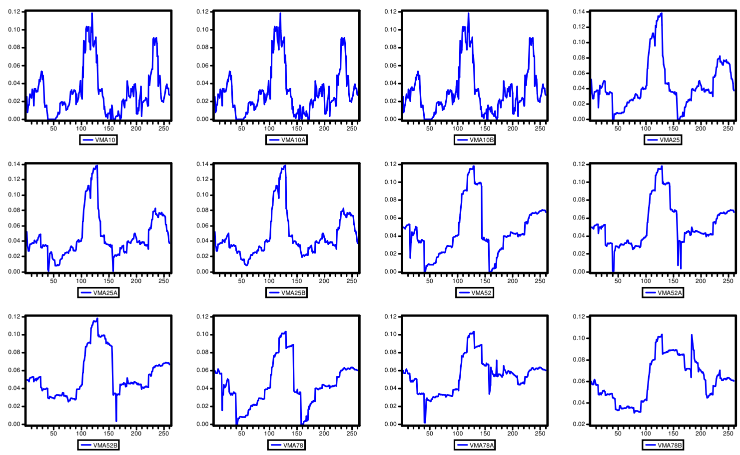
IVaR35



IVaR30



IvaRM



Note: The diferents figures in IVaR35 are the VaR evolution (left to righ and up to down): MA10, MA10a, MA10b, MA25, MA25a, MA25b, MA52, MA52a, MA52b, MA78, MA78a, MA78b. In IVaR30 we have represented the VaR evolution for MA10, MA25, MA30, MA52, MA60, MA70, MA78, MA85, MA100. And finally in IVaRM the VaR evolution for the same MA than in the IBEX35®

7. References.

Abken, P. A. (2000); "An empirical evaluation of value at risk by scenario simulation". *Journal of Derivatives*, pp.12-30

Ackerman, C.; McEnally, R.; Ravenscraft, D. (1999): "The Performance of Hedge Funds: Risk, Return and Incentives", *Journal of Finance,* v.54, iss.3, pp.833-874.

Alexander, C. (1996): "Volatility and Correlation Forecasting", *Handbook of Risk Management and Analysis*, John Wiley & Sons.

Alexander, J.; Baptista, M. (2003): "Portfolio Performance Evaluation Using Value at Risk: The Reward-to-VaR Ratio", *Journal of Portfolio Management*, v. 29, iss. 4, pp. 93-102.

Aragall, E. (2002): "Valor en risc. Mètodes de càlcul. Crítiques i alternatives: Introducció a la Teoria del Valor Extreme". *Estudis sobre el mercat de Valors*, n.30, Servei d'Estudis, Borsa de Barcelona.

Artzner, P.; Delbaen, F.; Eber, J.M.; Heath, D. (1999): "Coherent Measures of Risk". *Mathematical Finance*, v.9, n.3, pp.203-228, July.

Basak, S.; Shapiro, A. (2001): "Value-at-risk based risk management: optimal policies and asset prices". *The Review of Financial Studies*, v.14 (2), pp. 371-405.

Blanco, C.; Oks, M. (2004): "Backtesting VaR models: Quantitative and Qualitative Tests". *Risk Desk*, v. IV, n. I.

Bollerslev, T. (1986): "Generalized Autoregressive Conditional Heteroskedasticity*". Journal of Econometrics*, v. 31, pp.307-327.

Boudoukh, J.; Richardson, M.; Whitelaw, R. (1998): "The Best of Both Worlds". *Risk*, v.11(5), pp.64-66, May.

Danielsson, J.; De Vries, C. G.; y Jorgensen, B. N. (1998): " The value of Value at Risk: Statistical, Financial, and Regulatory Considerations". *FRBNY Economic Policy Review*, Oct.

Dembo, R.; Freeman, A. (1998): *Seeing Tomorrow: rewriting the Rules of Risk*. Wiley.

England, C. (1997): *Identify Critical Gap Between Risk Management Objectives and Practice*. Barra y RogersCasey. PR Newswire.

Engle, R. F. (1982): "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation". *Econometrica*, v.50, n.4, pp. 987-1008, Jul.

Fama, E. (1970): "Efficient capital markets: A review of theory and empirical work". *Journal of finance*, v.25, pp.383-417.

Froot, K.; Scharfstein, D.; Stein, J. (1994): " A Framework for Risk Management". *Harvard Business Review*, v.72, pp. 59-71.

Frye, J. (1998): "Monte Carlo by day", *Risk*, v.11(11), pp.66-71.

Garman, M. (1996): "Improving on VaR", *Risk,* v.9, iss.5, pp.61-63.

Garman, M. (1997a): "Ending the Search for Component VaR", *Financial Engineering Associates, Inc.*

Garman, M. (1997b): *Value at Risk: Evolving Topics*, (Conferencia) Organizada por Financial Engineering Associates, Inc, Berkeley, 3-4 Nov.

Garman, M. B. (1997c): "Taking VaR to pieces". *Risk*, v.10, iss.10, pp.70-71. Oct.

Gibson, M. S.; Pritsker, M. (2000): "Improving grid-based methods for estimating value at risk fixed-income portfolios". *Federal Reserve Board Finance and Economics Discussion Series*, 2000-25.

Group of Thirty (1993): *Derivatives: Practices and Principles*. Global Derivatives Study Group. Washington, D. C.

Hendricks, D. (1996): "Evaluation of VAR models using historical data". *Federal Reserve Bank of New York Economic Policy Review*, v.2, iss.1, pp.39-69.

Hopper, G. P. (1996): "Value at risk: a new methodology for measuring portfolio risk". *Federal Reserve Bank of Philadelphia Business Review,* v.0, iss.0, pp.19-39.

Hull, J. C.; White, A. W. (1998): "Incorporating volatility up-dating into the Historical Simulation method for value at risk." *Journal of Risk,* v.1(1), pp.5-19, Fall.

Johnson, C. A. (2001): "Value at Risk: Teoria y aplicaciones". *Estudios de Economía*, v.28, n.2, pp.217-247. Dec.

Jorion, P. (2001): *Value at risk*. New York: McGraw-Hill.

Larsen, N.; Mausser, H.; Uryasev, S. (2002): "Algorithms for Optimization of Value-at-Risk". *Financial Engineering, e-Commerce and SupplyChain*. Kluwer Academic Publishers, pp.129-157.

López, J. (1999): "Methods for evaluating Value at Risk estimates". *Federal Reserva Bank of San Francisco Economic Review*, v.2, pp. 3-17.

Markowitz, H. M. (1952): "Portfolio selection". *Journal of Finance*, 25(1), pp. 71-79.

Merton, R. (1973): "An Intemporal Capital Asset Pricing Model". *Econometrica*, 41, pp. 767-887.

Nawrocki, D. N. (1999): "A Brief History of Downside Risk Measures". *Journal of Investing*, v.8, n.3, pp.9-26, Fall.

Neftci, S. N. (2000): "Value at Risk calculations, extreme events, and tail estimation". *Journal of Derivatives*, v.7, iss.3, pp.23-37.

Nelson, D. B. (1991): "Conditional heteroskedasticity in asset returns: A new approach". *Econometrica*, n.59, pp.347-370.

Pearson, N. D.; Smithson, C. (2002): "VaR, the state of play". *Review of Financial Economics*, v.11, pp.175-189.

Pedersen, C. S.; Satchell, S. E. (1998): "An Extended Family of Financial-Risk Measures". *Geneva Papers on Risk and Insurance Theory*, v.23, iss.2, pp. 89-117.

Peña, J. I. (2002): *La gestión de riesgos financieros*. Madrid: Financial Times-Prentice Hall, Pearson Educación.

Pritsker, M. (1997): "Evaluating value at risk methodologies: accuracy versus computational time". *Journal of Financial Services Research*, v.12(2/3), pp. 201-242.

Riskmetrics (1995): *Technical Document*. New York: Morgan Guarantee Trust Company, Global Research.

Robinson, G. (1996): "More Haste, Less precision". *VaR: Understanding and applying Value-at-risk*. KPMG.

Rockafellar, R. T. ; Uryasev, S. (2000): "Optimization of Conditional Value at Risk". *The Journal of Risk*, v.2, n.3, pp.21-41.

Sarma, M.; Thomas, S.; Shah, A. (2003): "Selection of Value-at-Risk Models". *Journal of Forecasting*, v.22, iss.4, pp.337-358.

Schoenfeld, S. A. (2004): *Active Index Investing*. Hooboken, New Jersey: Wiley.

Sentana, E. (2001): "Mean-Variance Portfolio Allocation with a Value at Risk Constraint". *Working paper*, n.105, CEMFI.

Sharpe, W. G. (1964): "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk". *Journal of Finance*, 19, pp. 425-442.

Simons, K. (1996): "VaR-New approaches to Risk Management". *New England Economic Review*, v. 0, iss. 0, pp. 3-13, Sept.-Oct.

Stone, B. K. (1973): "A General Class of Three-Parameter Risk Measures". *The Journal of Finance*, v.28, iss.3, pp.675-685.