Investment Frictions and Leverage Dynamics

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Abstract

I examine the effect of investment frictions on leverage dynamics, using a model of the firm whose investments are 1) indivisible and lumpy, and 2) subject to time-to-build, which is a time lag between when investment expenditures are made and when the investment begins to generate cash. The firm dynamically chooses the timing of its investments and the source of investment funding and can endogenously adjust its capital structure. The model predicts that investment frictions can have significant impact on leverage fluctuations with only limited impact on long-term leverage means. Model simulations predict that a firm whose investments are lumpier and subject to longer time-to-build 1) has more volatile leverage target and realized leverage ratios, 2) has faster reversion to its target, and 3) funds its new incremental investments with a greater fraction of equity. The model simulations also demonstrate that fluctuations in the target leverage ratio exhibit countercyclical behavior with equity returns where frictions amplify the countercyclical property of the target. Regression analysis on the model-simulated data demonstrates that investment frictions can provide alternative interpretations of the observed leverage dynamics documented in the recent empirical literature.

"Half finished ships and factories are not part of the productive capital stock."

Finn Kydland and Edward Prescott

Econometrica, 1982, Vol. 50, No. 6.

1 Introduction

In the Miller and Modigliani (1958) frictionless world, financing decisions are independent of capital investments. In the world with financial distress and costly default, investment and capital structure decisions are made jointly and, therefore, fluctuations in real investments and frictions associated with investments can influence financing choices. The objective of this paper is to study how investment frictions affect capital structure dynamics. I propose that investment frictions such as time-to-build and lumpiness of investments are important determinants of capital structure fluctuations. Specifically, I provide a theoretical framework for analyzing the role of these frictions and examine whether they can cause time-series and cross-sectional heterogeneity in leverage dynamics.

The role of investment frictions is a new question in the capital structure literature, but it has been well-recognized in the literature of real investments. Since Hayachi (1982), the real investments literature has moved away from conventional frictionless models towards models with frictions commonly including either time-to-build (or investments lags) or lumpiness of investments. Time-to build refers to a time lag required to complete new capital stock. Only after this time lag elapses can the new capital become a part of the productive capital that can generate cash.[1] The lumpiness of investments does not allow small incremental adjustments in the stock of capital because investments are indivisible.[2] Real world examples of

[1]There are many industries where there is a significant lag between the time investments are made and the time new projects are fully productive. Such industries include hi-tech and R&D industries, exploration and extraction of natural resources or ship manufacturing. For example, Koeva (2000) reports that the lag can be as long as 86 months for utilities and 23 months for chemical plants.

[2]The assumption of lumpy investments precludes infinitely small investment responses to infinitely small changes in business conditions. The causes of lumpy investments include the presence of fixed and/or non-convex adjustment costs associated with changes in capital stock as well as indivisibility and irreversibility of investment projects. See Abel and Eberly (1994), Caballero and Leahy (1996), Caballero and Engel (1999), Cooper, Haltiwanger, and Power (1999).

investment frictions may include an oil company building a new oil rig, or an aircraft manufacturer developing a new airplane. In both examples, these new investment expenditures are a substantial fraction of the firms' overall values, and are subject to a time lag between when investment expenditures are made and when these investments begin to generate cash. Kidland and Prescott (1982) show that the assumption of multiperiod construction is crucial in explaining aggregate economic fluctuations. Caballero and Engel (1999) report that models with lumpy investments perform better than standard linear models in predicting investments of US manufacturing firms.[3] If theories with investment frictions offer a better explanation of real investment behavior, they may also help explain capital structure choices.

Traditional capital structure literature, however, tends to ignore frictions on the real side of a firm. The models in this literature either assume exogenous investments or frictionless investments that are instantaneously incorporated in the capital stock. The literature distinguishes what is commonly referred to as tradeoff, pecking order and timing theories of capital structure. These three theories depart from the frictionless M&M environment and focus primarily on effects of capital market imperfections.[4] Pecking order theory stresses imperfections such as asymmetric information and the costs associated with the issuance of equity.[5] Bankruptcy, financial distress costs, taxes, as well as transaction costs, are the main factors determining the target debt ratios in static and dynamic trade-off theories. In timing theory, managers exploit inefficiency of the equity markets by timing the issuance of overpriced shares.

Empirical studies, in turn, interpret regression results through the prism of these three theories and attribute the differences in leverage behavior to the cross-sectional differences in financing imperfections and tacitly assume that firms' real sides are frictionless. Empirical studies demonstrate that these theories do a relatively good job in predicting cross-sectional variation in long-term leverage ratios, but not in explaining cross-sectional differences in leverage fluctuations over shorter periods. For example, recent empirical studies document

[3]Doms and Dunne (1993) document that on the firm level, relatively infrequent periods with investment spikes account for a sizable fraction of total investments. See also Nilsen and Schiantarelli (2003).

[4]See literature reviews on the role of capital market imperfections in Hubbard (1998) and Chirinko (1993). See also Long and Malitz (1985).

[5]Myers (1977) suggests that the conflict of interests between debtholders and equity holders may lead to extra costs of equity. Also, Myers and Majluf (1984) suggest that the cost of external equity is higher because of an adverse selection problem.

the following three facts about leverage fluctuations. First, the speed of adjustment to target leverage, while slow, appears to be faster for small firms than for large firms.[6] Second, there is evidence that changes in earnings and changes in financial deficit have stronger influences on capital structure changes for large (dividend paying) firms than small firms (non-payers).[7] Third, the tendency of firms to raise equity following increases in their stock prices is more pronounced for small firms than for large ones.[8] None of the theories can support these facts simultaneously.

In this paper, I examine the extent to which cross-sectional variations in investment frictions can provide economically plausible explanations for these three facts and other empirical observations. Specifically, I argue that if small firms have relatively lumpier investments and longer time-to-build, then the model can provide a theoretical foundation for the difference in leverage behavior between small and large firms. Intuitively, in my earlier examples of an oil company and an aircraft manufacturer making new large investments, it is plausible that for smaller firms, such a large investment expenditures will constitute a bigger fraction of the firms' existing assets, and hence a larger fraction of assets will remain as "assets-in-progress" for a duration of time-to-build.

Integration of investment frictions into the model of this paper extends the existing dynamic capital structure models.[9] The model is built in continuous time with an infinite horizon framework where the firm can sequentially increase its future output by making lumpy, incremental investments. Each new incremental investment is incomplete initially because it is subject to time-to-build, which implies that an incremental increase in the actual output and earnings will occur with the time-to-build lag. The uncertainty in the model comes from stochastic prices of the product the firm sells. The firm endogenously chooses the timing of its new investments and its funding, the source of which can be either debt, external equity or any mix of debt and equity. The firm endogenously adjusts its capital

[6] See Fama and French (2002), and Flannery and Rangan (2003).

[7] See Shyam-Sunder and Myers (1999), Fama and French (2002), and Frank and Goyal (2003).

[8] See, for example, Masulis and Korwar (1986), Asquith and Mullins (1986), Korajczyk, Lucas and McDonald (1991), Jung, Kim and Stulz (1996), and Hovakimian, Opler and Titman (2001).

[9] In dynamic models such as Brennan and Schwartz (1984) Fischer, Heinkel and Zechner (1989), Mauer and Triantis (1994), Leland (1998), Moyen (2000), Goldstein, Ju and Leland (2001), Childs, Mauer and Ott (2003), Strebulaev (2004), Hennesy and Whited (2005), and Titman and Tsyplakov (2005), investments are assumed to be either exogenous or frictionless.

structure over time and defaults as soon as its earnings fall below the interest payments. Thus, the firm chooses its leverage target by trading off the tax benefits of debt and costs associated with default.

Model simulations demonstrate that investment frictions have important implications for both the timing of investments and the timing of capital structure adjustments. The model predicts that investment frictions have a greater impact on the short-term dynamics of capital structure than on its long-term averages. Predicted target debt ratios are time-varying and more volatile for a firm whose incremental investments are lumpier and subject to longer time-to-build. The leverage target exhibits countercyclical behavior with equity returns. Within the cycle, target is temporarily lower following periods of stock price run-ups, as well as for the periods prior to and immediately after investments are made. The reason is consistent with the more general intuition from the real options literature that, given uncertainty, lumpy investments force the firm to delay investment and wait for a higher product price (and hence for a greater equity returns), which, in turn, leads to a decrease in target leverage. Time-to-build is the necessary condition for target ratios being lower for periods immediately following investments, because, during these periods, the firm has a greater fraction of incomplete incremental assets (assets-in-progress) that do not yet generate earnings. The degree of investment frictions enhances the cyclical property of the target ratio. Specifically, lumpier investments naturally cause less frequent investments, greater cycle and lumpier capital structure adjustments[10], whereas the longer time-to-build increases the expected duration of the cycle and its amplitude.

Because new investments do not immediately increase earnings, firms with greater investment frictions tend to fund new investments with a larger fraction of external equity than do firms with lesser frictions. Since investments are made after an increase in equity values, this tendency provides a rational explanation for two stylized facts that seem to contradict both trade-off and pecking order theories, but appear to be in line with timing theory. The first fact – highlighted by Fama and French (2005) – is that "[equity] issues are on average large and not typically done by firms under duress." The second fact is that periods of positive equity returns are often followed by new equity issuances.[11] I don't claim that market timing

[10] Bernanke (1983), among others, shows that irreversable investments under uncertainty can explain cyclical fluctuations in investments.

[11] See for example Masulis and Korwar (1986) and Korajczyk, et al (1990).

explanations of these facts are implausible; rather, I suggest that these observed dynamics of capital structures can be attributed to investment frictions in a fully rational economy without asymmetric information.

In order to compare the model-generated leverage dynamics with empirical observations, I analyze the model-simulated data using regressions that have appeared in several recent empirical studies. My regression results demonstrate that investment frictions can produce cross-sectional differences in leverage dynamics that are qualitatively and quantitatively similar to those documented in the literature. For example, I replicate regressions featured in Shyam-Sunder and Myers (1999) who argue that within the pecking order theory, financial deficit (the net amount raised externally) should be a dominant factor in explaining changes in debt. The simulated regressions suggest that firms with lesser investment frictions do behave according to pecking order even though the model doesn't assume asymmetric information – the necessary condition of adverse selection in the pecking order theory. Conversely, firms with greater investment frictions exhibit behavior that is less consistent with the behavior predicted by the pecking order theory. Frank and Goyal (2003) use regressions similar to those in Shyam-Sunder and Myers (1999) and find that small firms tend to use more equity to finance their financial deficits, which is opposite to the predictions of pecking order. The interpretation of their result may be consistent with the idea that investment frictions are a driving force behind such leverage behavior because small firms may be subject to greater investment frictions than are large firms. Also, regressions employed in Fama and French (2002) reveal a relatively slow speed of target adjustment and that changes in earnings and changes in assets have a strong positive influence on capital structure changes. These findings are less pronounced for non-dividend paying (presumably small) firms. Their regressions applied to the simulated data illustrate that the findings in Fama and French (2002) are less significant for firms with greater investment frictions, suggesting that less perfect investments of small firms can be a factor for their study.

To analyze the effect of investment frictions on the target adjustment rate, I estimate the standard partial adjustment regressions of Flannery and Rangan (2003). These authors find that small firms exhibit faster reversion to their targets than large firms. In support of findings in Flannery and Rangan (2003), my model predicts that the speed of adjustment is higher for firms that have greater investment frictions. The reason is that fluctuations in the market debt ratio, which are caused by fluctuations in equity values (stock returns), are stronger negatively correlated with changes in the target ratios for small firms. Intuitively, positive stock returns simultaneously decrease the firm's market debt ratio and increase the likelihood of investments, which in turn decreases its target debt ratio. Because of this stronger positive correlation between changes in leverage and changes in target leverage, the estimated target adjustment rate is faster for firms with greater investment frictions.

In summary, the model analysis suggests that investment frictions can cause the leverage dynamics to be distinctly different in the short run than in the long-run. In the short run, the dynamics is determined by the historic path of investments and target ratio. Empirically, this implies that firms with identical characteristics observed at the same points in time may have very different leverage in place because the observed firms may be in different cycles or because their investment histories are different. This analysis provides insights as to why incorporating information about the preceding firm history can lead to improved explanatory power of empirical tests.[12] For example, investment frictions can provide an explanation for how and why a variable that keeps track of past histories of market-to-book ratios (MB) and external financing is negatively related to observed leverage ratios. Baker and Wurgler (2000) claim that this negative relationship supports the evidence for "market timing" behavior. Applying their regressions to the data generated by firms with greater investment frictions, the model can generate leverage behavior that appears to be similar to "market timing."

The remainder of this paper is organized as follows. The next section overviews theories of capital structure and empirical tests. Section 3 develops the theoretical model of a firm. Sections 4 and 5 reports valuation and calibration of the model. Numerical results and the model simulations for firms with different levels of investment frictions are presented in Section 5. The last section concludes the paper. Some stochastic control problems of the firm valuation are formulated in Appendix A, and the technical details of the numerical algorithm are presented in Appendix B.

2 Capital Structure Theories and Empirical Tests

In this review, I pay particular attention to empirical studies that analyze changes in firms' leverage ratios. Earlier empirical studies concentrate on testing trade-off and pecking order

[12] See Kayhan and Titman (2005) and Liu (2005).

theories. According to trade-off theory, firms choose optimal target ratios that reflect the benefits and costs of debt financing. The benefits are typically tax savings, and the costs are typically bankruptcy, agency and financial distress costs. Studies that test trade-off theories focus on cross-sectional analysis and examine the relation between observed capital structures and various proxies that are likely to be related to the firms' target capital structures. For example, Titman and Wessels (1988) and Harris and Raviv (1991) document the evidence consistent with trade-off theories that variables such as volatility of earnings, level of tangible assets, MB ratios and other proxies for growth opportunities are negatively related to leverage ratios. They also find that firms with greater profit margins tend to have lower leverages– the finding that is contradicted by the static trade-off theory.

In contrast, the emphasis of pecking order models is on the costs associated with the issuance of equity. In the presence of these costs, which may arise because of asymmetric information, taxes, as well as debt holder/equity holder conflicts,[13] a firm's earnings play an important role in determining its financial structure. As a result, the firm will finance new investments first with retained earnings, then – if the firm has to rely on external financing – it will raise debt and then, only as a last resort, it will raise equity. Therefore, in the framework of pecking order theory, firms don't have targets, and the changes in leverages should be driven by changes in changes in earnings and financing deficit (i.e., the net amount that must be raised externally). Shyam-Sunder and Myers (1999) test pecking order theory by regressing debt changes on the level of financing deficit. Given the sample of large firms, their tests document that financial deficit is covered primarily with debt. Using broader sample of firms, Frank and Goyal (2003) report that financing deficit explains less than a third of changes in debt, which has an even smaller explanatory power for small firms. On the other hand, Fama and French (2002) empirically demonstrate that 1) changes in leverage are positively related to changes in earnings, supporting predictions of the pecking order theory, and at the same time 2) market leverages do move to their targets but the rate appears to be slow in the range of 7-15% per year, and 3) there is a cross sectional-difference in adjustment speed between dividend-paying and non-paying firms, where the latter adjust faster. Along the same lines, Flannery and Rangan (2003) document that target adjustment is greater for small firms and that small firms tend to finance their investments with greater fraction of equity, which is the opposite to what pecking order theory would predict.

[13] See Myers (1984) and Myers and Majluf (1984).

In summary, the conclusions of empirical studies is that even though pecking order and trade-off theories are based on very different assumptions, they are not mutually exclusive. Firms do choose target ratios that reflect the benefits and costs of debt financing put forth in the trade-off literature, but may deviate from their targets for the reasons described in the pecking order theory.

An alternative interpretation to slow target adjustment, described in the dynamic models of Fischer, Heinkel and Zechner (1989) and Leland (1998), is that transaction costs may keep a firm away from its target and lead to a wide range of observed debt ratios over time. Fischer, Heinkel and Zechner (1989) report that empirically observed leverage ranges are negatively related to firm size, suggesting that the adjustment rate is slower for small firms because small firms have proportionally greater transaction costs. However, transaction costs alone cannot explain the fact documented in Flannery and Rangan (2003) that small firms move to their targets faster. Another interpretation to observed target adjustment rates suggested by theories in Hennesy and Whited (2005) and Titman and Tsyplakov (2005) is that target debt ratios change over time, which can make the target adjustment rate difficult to measure.

There are still two empirical facts that appear to contradict both trade-off and pecking order theories. The first evidence is that firms tend to time equity raising following increases in their stock prices and buy back shares following stock price decline.[14] The second fact is that changes in equity returns play a significant role in the observed capital structures. The first evidence motivates Baker and Wurgler (2002) to suggest that the market timing has a persistent effect on observed capital structures. The conclusion of Baker and Wurgler (2002) is that firms' capital structures reflect cumulative ability to sell overpriced equity shares when firms' MB ratios are temporarily high. The interpretation of the second evidence is given by Welch (2004), who argues that fluctuations in the observed capital structures occur primarily due to fluctuations in equity values because managers take an "inactive" leverage strategy. The conclusions of these two studies have been questioned by a series of subsequent empirical papers including Leary and Roberts (2004), Hovakimian (2005), Kayhan and Titman (2005), and Liu (2005) that provide different interpretations to the regressions presented in Baker and Wurgler (2002) and Welch (2004).

I see three potential reasons why there is disagreement in interpretations of the empirical

[14] See, for example, Jung, Kim and Stulz (1996), Ikenberry, Lakonishok and Vermaelen (1995) and Hovakimian, Opler and Titman (2001).

results when analyzing the timing of leverage changes. First, there is no strong consensus of how to measure the capital market imperfections, such as asymmetric information and/or financial constraints across firms.[15] Second, the cross-sectional variations in frictions inherent in the investments are ignored. Third, most studies examine changes in actual debt ratios rather than changes in target ratios. In practice, changes in debt ratios can potentially occur because of changes in target ratios which, in turn, can be driven by investment fluctuations and the frictions associated with investments. Ignoring these issues can yield ambiguous interpretations.

3 Description of the model

3.1 Summary of the Model

In the model, the firm continuously generates earnings which depend on the price of the firm's product determined by the stochastic state variable, and the firm's output level. I assume that the firm can make up to N subsequent incremental investments, which will increase its future output. It can invest until its output reaches the maximum possible output level, beyond which the firm cannot increase its output anymore. The firm is constrained to invest in incremental, lumpy amounts only, and can endogenously determine the timing of its investments. Each incremental investment is subject to a predetermined time-to-build period. Therefore, an incremental increase in the actual output and earnings occurs with the time-to-build lag. The firm can make only one incremental investment at a time, and it can make the next investment only after the time-to-build elapses i.e., after its previous incremental investment becomes complete. One can view an incremental investment during the lag period as an incomplete asset (non-productive asset or asset-in-progress).[16] Only completed assets increase the firm's output and its earnings. I also assume that investments are irreversible, and once the firm makes an incremental investment it cannot undo it. Because of the assumption of lumpy incremental investments, the output function is a multi-step function of the firm's complete assets. Due to diminishing return to scale, the increase in

[15] Observations regarding the role of financial frictions allow ambiguous interpretations. For example, there is a disagreement between Fazzari, Hubbard, and Petersen (1988) and Kaplan and Zingales (1997).

[16] Throughout the remainder of the paper the terms assets-in-progress, incomplete assets, and non-productive assets are used interchangeably.

output in each subsequent step of the output function is smaller than the preceding increase.

When the firm chooses the timing of its investments, it also chooses its financing source between coupon debt, external equity, or any mix of debt and equity.[17] The firm can endogenously change its capital structure at any time including periods when the firm is not investing. The periodic coupon payments are tax deductible. The firm will default as soon as its earnings fall below periodic debt payments. Firm's decisions with respect to investment and capital structure choices are made from the objectives of the shareholders who maximize the value of their equity stake.[18]

3.2 The Firm's Earnings and Investment

The firm I examine continuously produces and sells a product whose unit market price, p, evolves through time in the manner described by the following stochastic process:

$$\frac{\mathrm{d}p}{p} = (r - \alpha)\mathrm{d}t + \sigma_p \mathrm{d}W_p, \tag{1}$$

where W_p is a Weiner processes under the risk neutral measure Q, σ_p is the instantaneous volatility coefficient, r is the risk free rate, which is assumed to be constant, and α ($\alpha > 0$) is the convenience yield. The drift μ under the real probability measure may be different from r.

The firm's instantaneous earnings before interest and taxes (EBIT) are assumed to equal the product $p \times c$, where c is the firm's current production volume or, as I will call it, the output, which is normalized to be between zero and one (maximum output). The output level of the firm is described by a strictly concave and increasing function $c(A) = 1 - e^{-\gamma \cdot A}$ of the value of the firm's completed (fully productive) assets A, where γ is constant parameter. The output function corresponds to a typical production function with diminishing marginal returns.[19] I assume that there are only N discrete levels of the firm's assets. $A \subseteq \{ A_0 < A_1 < ...A_i < A_{i+1} < ...A_N \}$, where $\underline{A} = A_0$ and $\overline{A} = A_N$ the lower and the upper bound

[17] The assumption that incremental investments are large enough to be financed internally is empirically justified. For example, Mayer and Sussman (2004) document that external finance plays an important role in funding large investments.

[18] For simplicity, I ignore the asset substitution problem discussed in Jensen and Meckling (1976) and Jensen (1986).

[19] Note, that $c(0) = 0$ and $c(A) \to 1$ as $A \to \infty$.

assets. I assume that the firm is at the the lower bound asset $\underline{A}$ initially. Since the output is a function of A, there are N discrete levels of output $c(A) \subseteq \{\, c(A_0) < c(A_1) < ...c(A_i) < c(A_{i+1}) < ...c(A_N)\,\}$, where $\underline{c} = c(A_0)$ is the lower bound (the initial) output, and $\overline{c} = c(A_N)$ is the upper bound (maximum) output. After the firm reaches its maximum output, it stays at this output level until it defaults. For simplicity I assume that discrete asset levels A_{i+1} are equally-spaced with the incremental asset ΔA, $\Delta A = (\overline{A} - \underline{A})/N$, and $A_{i+1} = A_i + \Delta A$, $0 \leqslant i < N$. The size of the incremental asset ΔA represents the lumpiness of investment: the larger ΔA (the smaller N) the greater the lumpiness.

If the firm's current assets are complete, the firm can choose to make an incremental investment ΔA. I assume that a new incremental investment ΔA doesn't instantaneously increase the firm's output and its earnings due to the time-to-build lag T, which is the lag between the time the incremental investment is made and the time the investment is complete. Specifically, if at time t the firm with complete assets A_i (the output $c(A_i)$) makes the incremental investment ΔA, its output will still be $c(A_i)$ until time $t + T$, and at time $t + T$, its output will increase instantly from $c(A_i)$ to $c(A_{i+1}) = c(A_i + \Delta A)$. The firm can make only one incremental investment at a time, and it can invest only after its previous incremental investment is complete. I also assume that investments are irreversible. Given that the firm starts from its initial output level, it has to make N incremental investment before it can reach the upper bound output. Because each incremental investment requires time-to-build T, it will take the firm at least $T \times N$ periods to reach its upper bound output level.

Given this structure, the firm can be in either of two states: 1) the state where all its assets A_i are complete and the firm's output is $c(A_i)$, and 2) the state where the firm has assets A_i that include an incomplete incremental investment ΔA with some positive time τ remaining until ΔA becomes complete, where $0 < \tau \leq T$. In state 2) the firm's output will remain $c(A_i - \Delta A)$ for the duration of the remaining lag τ. Thus, the firm's instantaneous earnings before interest and taxes equal either $p \cdot c(A_i)$, if the firm's assets are complete, i.e. if $\tau = 0$, or $p \cdot c(A_i - \Delta A)$ if $\tau > 0$, i.e., if there is some time lag τ remaining until an incremental asset ΔA becomes complete.

As I show later, incremental investments can be funded either by debt, by equity or any mix of debt and equity. This setup implies, that when the price increases, the firm will generally invest to increase its future output.

3.3 Corporate Taxes and Dividends

I assume that the firm's earnings after debt payments are taxed continuously at a constant corporate rate λ, and that the periodic debt coupon payments d are tax deductible. The firm uses its earnings to meet its debt obligations, pay taxes, with any residual being paid out as a dividend.[20] The firm's instantaneous tax obligation equals

$$(\lambda)\left[p \cdot c' - d\right], \tag{2}$$

and, the firm's instantaneous after tax dividends are:

$$(1 - \lambda) \cdot \left[p \cdot c' - d\right], \tag{3}$$

where d represents the periodic debt payments, and output c' equals either $c(A_i - \Delta A)$, if the firm has incomplete assets ($\tau > 0$), or $c' = c(A_i)$, otherwise ($\tau = 0$).

3.4 The Debt Structure and Debt Funding

Similar to assumptions in Fischer, Heinkel and Zechner (1989), Leland (1998) and Titman and Tsyplakov (2005), I assume that the firm issues perpetual coupon debt that can be called by the equityholders at market value D at any time. For simplicity, I assume that the face value of the debt is $F = \frac{d}{r}$. The face value is equal to the value of perpetual debt with periodic payment d discounted at the risk-free rate.[21] Following Fischer, Heinkel and Zechner (1989) and Leland (1998), I allow the firm to increase its debt ratio but not to decrease it.[22]

I assume that the firm can instantly increase debt ratio by simultaneously repurchasing its total outstanding debt at its market value and issuing new debt with greater face value and greater periodic coupon. This assumption preserves the rights of the current debtholders in the event that the firm increases its debt level. For simplicity, I assume that when the

[20] In practice, a firm may retain part of its cash flow and then use it for future debt service, which may affect the investment strategy and the valuation of equity. Although feasible, incorporating these features would lead to an increase in the dimensionality of the model and would further complicate the analysis.

[21] I effectively assume that the firm is not allowed to sell zero coupon bonds. Lack of zero coupon corporate bonds in practice justifies this assumption.

[22] Titman and Tsyplakov (2005) show that with absence of significant distress costs incurred by equityholders (not debtholders), the equityholders have no incentives to reduce debt size, because such a transaction will benefit debtholders at the expense of equityholders.

firm adjusts its debt level, it has to pay transaction cost described by constant parameter C_{Debt}, that doesn't depend on the level of debt.

When the firm increases its debt, it receives the net proceeds of the debt issue, which the firm can use to either repurchase equity, or use, the whole or any part of the proceeds to fund incremental investments (if the firm is investing).

3.5 Default

I assume, that the firm is in distress and is forced to bankruptcy if there is insufficient earnings to meet debt payments, i.e., if $p \cdot c' < d$, where $c' = c(A_i - \Delta A)$ if the firm has incomplete assets ($\tau > 0$), or $c' = c(A_i)$, otherwise ($\tau = 0$).[23] In the event of default, the equityholders get nothing, and the debtholders recover the liquidation value of the firm E^U minus default costs $C_{default}$ proportional to E^U. For simplicity, I assume that the liquidation value of the firm equals the value of the unlevered firm E^U, assuming that after the default the firm cannot recapitalize. Thus, at default, the debt value satisfies $D = (1 - C_{default}) \cdot E^U$, where $C_{default}$ is the constant parameter for the proportional default costs.

4 Valuation

The market value of the equity and debt depends on the equityholders' investment and financing strategy. Since I assume complete markets for the firm's product, the firm's debt and equity can be regarded as tradeable financial claims for which the usual pricing conditions must hold. The value of debt $D = D(p, A_i, \tau, d)$ and equity $E = E(p, A_i, \tau, d)$, i.e., the present value of the cash flows they generate, depend on the product price p, the value of the firm's assets A_i, the remaining lag time τ, and the level of the periodic coupon payment d. Values for D and E can be determined by solving stochastic control problems with boundary conditions, where the control variables are the investment and financing decisions.

[23] In reality, the firms are financially distressed when their cash flows are low relative to their debt obligations. In the event of financial distress, firms normally suffer a reduction in their cash flows due to difficulties that they face in dealing with customers and markets. See Opler and Titman (1994) and Titman (1984). Some firms may be forced to default. In this paper I assume an extreme case of financial distress that leads to an immediate default as soon as the firm's earnings fall below debt payments. Davydenko (2005) documents that a sizable fraction of defaults is triggered by liquidity shortages.

The following section discusses the valuation of equity for the levered firm and debt. The valuation of the unlevered firm is discussed in the appendix A.

4.1 Valuation of the Equity

The firm chooses its investment timing and its debt restructuring policy to maximize the market value of its equity $E(p, A_i, \tau, d)$. The values can be determined by solving stochastic control problems with free boundary and fix boundary conditions. The boundary conditions divide the state space (p, A_i, τ, d) into five regions: 1) the no recapitalization/no investment region, 2) the recapitalization/no investment region, 3) the recapitalization/investment region, 4) the investment/no recapitalization region, and 5) the default region. The free-boundary and smooth pasting conditions are different in states where $\tau > 0$, and there $\tau = 0$. In the states of the "no recapitalization/no investment" region, the firm optimally chooses not to invest and not to change the level of its debt. Using standard arbitrage arguments, in the "no recapitalization/no investment region" the value of the equity $E(p, A_i, \tau, d)$ is given by the solution to the following PDE:

$$
\begin{cases}
\frac{1}{2}\sigma_p^2 p^2 E_{pp} + (r - \alpha) p E_p - E_\tau - rE + (1 - \lambda)(p \cdot c(A_i - \Delta A) - d) = 0, \\
\qquad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\
\frac{1}{2}\sigma_p^2 p^2 E_{pp} + (r - \alpha) p E_p - rE + (1 - \lambda)(p \cdot c(A_i) - d) = 0, \\
\qquad \tau = 0, \text{ the firm has no incomplete assets,}
\end{cases}
$$

where subscripts denote partial derivatives. The term $-E_\tau$ in the first equation represents a linear decrease in the remaining lag time until an incremental investment becomes productive. Because it is an infinite horizon model, the value of equity is independent of time, i.e., $E_t(p, A_i, \tau, d) = 0$.

In the "recapitalization/no investment region", the firm optimally increases its debt level, i.e., replaces the existing debt that has a periodic payment d with a new debt that has a periodic payment $\widehat{d}$. In such a transaction, all net debt proceeds are paid to the equityholders for the repurchased shares. This transaction can be done by the firm that either has or has no incomplete assets, i.e., for any τ. If the firm increases its debt $(\widehat{d} > d)$, the value of the equity must satisfy the following free boundary condition:

$$E(p, A_i, \tau, d) = \max_{\widehat{d}>d}[E(p, A_i, \tau, \widehat{d}) + D(p, A_i, \tau, \widehat{d}) - D(p, A_i, \tau, d) - C_{Debt}], \quad (4)$$

$$\text{such that } E(p, A_i, \tau, \widehat{d}) > 0, \ 0 \leqslant i \leqslant N, \ \text{for any } \tau,$$

where C_{Debt} is constant transaction costs that are introduced earlier, $D(p, A_i, \tau, d)$ and $D(p, A_i, \tau, \widehat{d})$ are the market values of old and new debt respectively. The amount $D(p, A_i, \tau, \widehat{d}) - D(p, A_i, \tau, d)$ is paid to current equityholders for the portion of their shares that is repurchased. The last inequality prohibits the debt restructure that leads to an immediate default. Note that in the states of the "no recapitalization/no investment" region there is a strict inequality ($>$) for any $\widehat{d} > d$ in (4), implying that it is not optimal for the firm to increase its debt.

The regions where the firm invests include only states where all incremental assets are complete, i.e., there is no remaining time lag left, $\tau = 0$. In these regions, the the firm can choose to make an incremental investment ΔA and choose the source of financing. The funds can be raised either from external equity, debt or any mix of debt and equity. In the "investment/no recapitalization" region, the firm chooses to finance exclusively with new equity (no new debt), in which case it has to raise the amount ΔA of external equity. The firm's equity value must satisfy the following free boundary condition:

$$E(p, A_i, 0, d) = E(p, A_i + \Delta A, T, d) - \Delta A, \ 0 \leqslant i < N. \quad (5)$$

Note that any time the firm invests, it instantly transits from states $(p, A_i, 0, \cdot)$ to states $(p, A_i + \Delta A, T, \cdot\ )$, where the remaining time lag left τ increases instantly from 0 to T.

If the firm chooses to finance using the proceeds of the mix of new equity and new debt (new debt has coupon $\widehat{d}$), the value of the equity must satisfy the following free boundary condition:

$$E(p, A_i, 0, d) = \max_{\widehat{d} \geq d}[E(p, A_i + \Delta A, T, \widehat{d}) + D(p, A_i + \Delta A, T, \widehat{d}) - \quad (6)$$

$$D(p, A_i, 0, d) - \Delta A - C_{Debt}], \quad (7)$$

$$\text{such that } \ E(p, A_i + \Delta A, T, \widehat{d}) > 0.$$

The amount $D(p, A_i + \Delta A, T, \widehat{d}) - D(p, A_i, 0, d)$ is the net proceed of newly issued debt. If $D(p, A_i + \Delta A, T, \widehat{d}) - D(p, A_i, 0, d) < \Delta A$ the remaining portion of required financing of

$\Delta A - (D(p, A_i + \Delta A, T, \widehat{d}) - D(p, A_i, 0, d))$ is raised through new equity. The last inequality prohibits a situation in which the debt restructure would lead to an immediate default. In the no recapitalization/no investment region, strict inequality (>) holds (6).

In the states where an incremental investment is about to become complete, i.e., $\tau \xrightarrow{+} 0$, there is the following boundary condition:

$$E(p, A_i, \tau, d) \to E(p, A_i, 0, d), \text{ as } \tau \xrightarrow{+} 0, \text{ for any } p, A_i, \text{ and } d. \tag{8}$$

I also impose a fixed boundary describing the default region. The firm defaults as soon as its earnings fall below debt payments. The equity value at default is 0:

$$E(p, A_i, \tau, d) = 0, \text{ if } (p \cdot c' - d) < 0,$$

where c' equals either $c(A_i - \Delta A)$ if the firm has incomplete assets, i.e., $\tau > 0$; or $c\prime = c(A_i)$, otherwise.

4.2 Valuation of Debt

In this section I calculate the debt value $D(p, A_i, \tau, d)$. The debt entitles its holders to receive a periodic coupon payment d until the firm restructures or defaults. For each p, A_i, τ, and d, the value of debt $D(p, A_i, \tau, d)$ depends on the equityholders' investment and financial decisions. In the states of the "no recapitalization/no investment" region, the debt restructuring satisfy the following PDE:

$$\begin{cases} \frac{1}{2}\sigma_p^2 p^2 D_{pp} + (r-\alpha)pD_p - D_\tau - rD + d = 0, \\ \qquad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\ \frac{1}{2}\sigma_p^2 p^2 D_{pp} + (r-\alpha)pD_p - rD + d = 0, \\ \qquad \tau = 0, \text{ the firm has no incomplete assets.} \end{cases},$$

In the region where the firm invests and chooses to finance exclusively with new equity (no new debt), the firm's debt value must satisfy the following free boundary condition:

$$D(p, A_i, 0, d) = D(p, A_i + \Delta A, T, d), 0 \leqslant i < N.$$

In case of default, the ownership of the firm is transferred from the equityholders to the debtholders. The recovery value equals the value of the unlevered firm minus default costs $(1 - C_{default})E^U$. Thus, in the default region, the debt value satisfies

$$D(p, A_i, \tau, d) = (1 - C_{default})E^U, \quad \text{if} \quad E(p, A_i, \tau, d) = 0.$$

5 Numerical Results

5.1 Parameters and Variables of the Model

In the next 2 subsections, I describe the base case parameters of the model as well as the variables for 5 firms that have different degrees of investment frictions.

5.1.1 Base Case Parameters

The model parameters are chosen to roughly match empirical observations for commodity producing firm. The volatility of the product price σ is set at 10%. The convenience yield a is set at 7%. The risk free interest rate is set to $r = 7\%$, this means that the risk neutral drift of the product prices is zero. The drift (net of convenience yield) $\mu - a$ under the real probability measure is assumed to be 4%. Transaction costs of recapitalization C_{Debt} =1.

I assume that the time-to-build T and the number of discrete levels of assets N are such that $N \times T = 12$ years. $A_i \subseteq \{\, \underline{A} = A_0, A_1, ..A_i.., A_N = \overline{A}\,\}$, where $\underline{A} = A_0 = \$1500$ and $\overline{A} = A_N = \$4700$ the upper and low bounds for the firms assets and the incremental asset $A_i = \overline{A} + i \cdot \Delta A$. According to production function $c(A_i) = 1 - e^{-0.00034 \cdot A_i}$, there are N corresponding levels of output $c(A_i) \subseteq \{\, c(A_0), c(A_1), c(A_2), ..., c(A_N)\,\} = \{0.4, ..., 0.8\}$, where $\underline{c} = c(A_0) = 0.4$ is the initial output and $\overline{c} = c(A_N) = 0.8$ is the maximum output levels. Thus, the firm that starts at its initial output level can reach its upper bound output for period not shorter than $N \times T = 12$ years. All the initial parameter values are displayed in Table 1.

5.1.2 Different levels of Investment frictions

In order to measure the impact of frictions, I analyze firms that have different degrees of frictions but the same investment opportunities. I consider five firms that have different sizes of incremental investment and different time-to-build lags. Each firm j is characterized by a pair of parameters (N_j, T_j), where N_j is the total number of incremental investments the firm j has to make in order to move from the initial output to the upper bound output, N_j =$(\overline{A} - \underline{A})/\Delta A_j$, and T_j is the time-to-build for each incremental investment ΔA_j. Firms are described by the following pairs: $\{T = 2, N = 6\}$, $\{T = 3, N = 4\}$, $\{T = 4, N = 3\}$, $\{T = 6, N = 2\}$ and $\{T = 12, N = 1\}$, where T is measured in years, and the size of the incremental investment $\Delta A_j = \frac{(\overline{A}-\underline{A})}{N_j} = \frac{(\$4700-\$1500)}{N_j} = \frac{\$3200}{N_j}$. The remaining variables are as in the base case. I should emphasize that for these five cases, the firms are identical in terms of their investment opportunities. First, their initial and upper outputs are the same, and the aggregated investments that are needed to reach the upper bound $\overline{A}-\underline{A}$ are the same as well. Second, the total time-to-build required to move from the initial output level to its upper bound is the same for each firm $N_j \times T_j = 12$ years. The degree of investment frictions are introduced through parameter N_j; the lower value of parameter N_j implies lumpier incremental investment ΔA_j, and longer time-to-build for each incremental investment.[24] Figure 1 provides an illustration for output functions of two firms: $\{T = 2, N = 6\}$ and $\{T = 6, N = 2\}$. For each of these five firms I run simulations and generate time series data.

5.2 Simulation Results for Firms with Different Investment frictions

In the following subsections, I describe the simulation results. The purpose of the simulations is twofold. First, I want to verify whether the model generated dynamics resemble the real data. Second, I analyze whether investment frictions can provide alternative interpretations for regressions coefficients in the actual data.

In the next section, I describe the simulation approach. The subsequent sections present summary statistics for the simulated data separately for firms with different investment frictions and then analyze the model-generated time series using regressions featured in the recent empirical literature. I compare regression coefficients for the model generated data with those for the actual data.

5.2.1 Simulation Procedure

I use the time step $\Delta t = 0.1$ *years* and generate 100 simulated product price paths p_t, while recording the investment choice and capital structure for each price level. At each point on the simulated path, I incorporate investment and financing strategies calculated in

[24] An alternative approach for the comparative statics is to fix N and vary T only, or vice versa. Note, for this alternative comparative statics, the projects' profitability across firms will not be comparable because firms will have different time periods (total time-to-build) required to move from the initial output level to its upper bound output.

section 4.1 and keep track of the firm's assets A_i, its output level $c(A_i - \Delta A)$ or $c(A_i)$, the time remaining until incremental investments become productive τ, and the level of debt F. At time zero of each simulated path, the firm starts with the initial output level of $\underline{c} = 0.4$. I terminate each path as soon as the firm reaches its maximum output level $\overline{c}$ and the firm's last incremental investment becomes complete. I also terminate the path if the simulated price reaches the default boundary. The initial price level at which the path starts is $p = \$612$ and the firm starts with the initially optimal target capital structure.

For each year of the simulated path, I keep track of several annualized variables that are used in the regressions, such as: EBIT, leverage ratios, the target leverage ratio, the measure of financial deficit, and market-to-book ratio. I measure the firm's leverage as the ratio of the face value of debt divided by the market value of equity plus the face value of debt, $\frac{D}{V} = \frac{F}{(E+F)}$. At any given point of time on the simulated path, the target debt ratio is the debt ratio that maximizes the total market value of debt and equity. The MB ratio is $\frac{F+E}{A}$. Note, that by assumption, financial deficit variable FD, which measures the external funds raised, can take the value of either zero or ΔA, which is the amount needed for each incremental investment.

5.2.2 Summary statistics for simulated data

Summary Statistics for all periods Table 2 reports the summary statistics for each of the 5 different cases of simulated time series data. The table shows that firms whose investments are lumpier and subject to longer time-to-build 1) have more volatile leverage as well as target debt ratios, and 2) have greater and more volatile MB ratios. I should stress that the long-term target ratios, as well as the long-term realized leverage ratios, are almost the same across firms with different investment frictions. The result that firms with greater investment frictions have more volatile leverage ratios is consistent with the empirical observations in Fischer, Heinkel and Zechner (1989) who report that the observed leverage ranges (measured as the difference between the maximum and the minimum ratio) are negatively related to firm size. The authors interpret their findings as showing that small firms have greater (on a percentage basis) transaction costs of recapitalization which leads to wider swings in their debt ratios. My interpretation for the observed leverage ranges does not rely on adjustment costs and suggests that investment frictions may have an effect on the variability of leverage and its target. Moreover, the time-varying target in this model extends the dynamic models of capital structure in Fischer, Heinkel and Zechner (1989) and Leland (1998) that assume that target ratio is the same over time.

I also report the time-series correlation between equity returns and changes in target debt ratios. Firms with greater investment frictions have changes in target ratios that exhibit a significantly stronger negative correlation with equity returns. For example, the correlation for the firm with lesser frictions ($T = 2, N = 6$) is -7%, while this correlation increases in magnitude to -30% for the firm with greater investment frictions ($T = 12, N = 1$). The reason of stronger negative correlation is consistent with the more general intuition from the real options literature that it is optimal to delay the investment in an uncertain environment and the delay is longer for lumpier investments, which, in turn, leads to greater increases in equity values before the firm makes investments. This stronger negative correlation between changes in target and equity returns caused by frictions is the driving force for the countercyclical behavior of target, which is the issue I discuss in the following section.

Summary Statistics for periods prior and after investments Since investment timing is an important factor in determining how target debt ratios evolve over time, I pay particular attention to the ratios measured during the year before and the year after incremental investments are made. One of the important observations reported in Table 3 is that target leverage ratios for periods before and after investments are considerably lower compared to the average long-term target ratios for firms with greater investment frictions. For example, for the firm with greater frictions $\{T = 12, N = 1\}$, the target ratios for the year before and after investment periods are 46% and 39% respectively. For comparison, the average target for this firm is higher at 54%. This confirms the observation that changes in target exhibit countercyclical behavior with equity returns. Intuitively, within the cycle, the target leverages are temporarily lower following periods of stock price run-ups, after which the firm tend to invest, as well as for the periods after investments are made. As the table shows, the degree of investment frictions enhances the cyclical property of target ratio. Note that time-to-build is the necessary condition that target ratios to be lower than average during periods immediately following investments, because during these periods the firm has incomplete incremental assets that don't yet generate earnings.

The numbers in Table 3 also show that MB ratios are relatively higher for the year prior to investments compared to long-term averages of MB ratios. This is an empirically well-documented result that higher MB ratios are associated with a reduction in target debt ratios, because higher MB ratios proxy for greater investment opportunities.[25] The observed increase in MB ratios prior to periods with investments is significantly more pronounced for firms with greater investment frictions because those firms have to wait for sufficiently higher product prices before they make incremental investments.

Source of investment funding As shown in Table 4, firms which are subject to greater investment frictions tend to fund their new incremental investments with a larger fraction of equity than firms with lesser frictions. Specifically, the firm with greater frictions $\{T = 12, N = 1\}$ funds its new investments on average with 42% debt and with 58% equity, whereas the firm with lesser investment frictions $\{T = 2, N = 6\}$ funds its investments with 89% debt and 11% equity. Although the relation between investment frictions and funding choices has not been directly analyzed in the empirical literature, the study of Mayer and Sussman (2004) examines related issues. They document that during instances with investment spikes, equity accounts for about 60% of funding sources for small firms, whereas for larger and medium size firms, the fraction of equity declines to about 30%. The empirical literature tends to explain the fact that small firms use more equity as a source of investment funds, by pointing out that small firms have limited access to debt financing and/or that there is greater information asymmetry between markets and small firms.[26] My model provides an alternative interpretation of this fact by suggesting that small firms choose equity funding because small firms are likely to be subject to greater investment frictions.

Investment frictions can also provide a rational explanation for two stylized facts. The first fact, noted in Fama and French (2005), is that the equity issues are on average large and not typically done by firms in distress. Second fact is that periods of positive equity returns are often followed by new equity issuances.[27] These two empirical facts contradict both trade-off and pecking order theories, but appear to be in line with timing theory. The model suggests that these two facts can be explained by investment frictions. How do lumpiness of investments and time-to-build cause such a leverage behavior? The intuition is the following: An increase in product price leads to an increase in equity values (and positive stock returns)

[25] See Hovakimian, Opler, and Titman (2001) and Hovakimian Hovakimian and Tehranian (2004).

[26] See for example Lemmon and Zender (2004).

[27] See Asquith and Mullins (1986), Korajczyk, Lucas and McDonald (1991), Jung, Kim and Stulz (1996) and Hovakimian, Opler and Titman (2001).

and a decline in both the firm's leverage and its target. After the product price increases to a sufficiently high level, the firm finds it optimal to make an incremental investment. However, due to time-to-build, the firm's earnings will not immediately increase with investments. How should the firm fund such an investment: with new debt, new equity, or a mix of the two? If the firm decides to finance its investment entirely with new debt, it has to make sure that it will be able to cover future interest payments using internally generated earnings. If investments are sufficiently lumpy and time-to-build is long, an increase in interest payments due to new debt may be too high, so it can increase the risk that the firm's earnings may decline below debt payments. If this happens, the firm will be forced to default without being able to complete its new incremental assets. Because of this increased risk of default during the lag periods, the firm will tend to fund its incremental investment with a larger fraction of new equity. Note that we need the combination of both lumpiness of investments and time-to-build in order to generate such financing behavior.[28]

To conclude this section, I should emphasize that investment frictions cause the leverage dynamics to be distinctly different in the long-run compared to the short run. In the short run, the dynamics are determined by the historic path of investments and the target ratio. These findings are consistent with the empirical facts reported in Mayer and Sussman (2004) who show that financing role shifts across regimes with investment spikes and regimes with lower investment rates.

5.2.3 Target Adjustment Rate

In this section, I analyze the leverage behavior using regressions that measure the speed of adjustment to a firm's target. I start with a simple regression of the change in the debt ratio on the difference between the firm's actual and target debt ratio, similar to one in Flannery and Rangan (2003):

$$\left(\frac{D}{V}\right)_{t+1} - \left(\frac{D}{V}\right)_t = \lambda \cdot \left(TL_t - \left(\frac{D}{V}\right)_t\right) + \varepsilon_t,$$

[28] A firm whose investments instantly translate into productive capital is not subject to such a risk. The assumptions of recapitalization costs and/or instantaneous investments with adjustment costs alone, are not enough to generate such leverage patterns.

where TL_t and $(\frac{D}{V})_t$ are the target and the realized debt ratio at time t. In this regression, the coefficient λ measures the speed of adjustment. A coefficient of $\lambda = 1$ would imply a 100% adjustment within one year, while a coefficient of 0 would imply no adjustment. In Flannery and Rangan (2003), the target leverage ratios TL_t are the fitted values from the regression on various factors including R&D expenses, depreciation expenses, etc., whereas I use an exact measure of the target leverage in this regression. As reported in Table 5, the estimated speed of adjustment across all firms is within the range of the empirically estimated values reported in Flannery and Rangan (2005), but is somewhat faster than the estimates found in Kayhan and Titman (2005) and Fama and French (2002).

The speed of adjustment is greater for firms that have greater investment frictions. For example, for lesser investment frictions $\{N = 6, T = 2\}$, the adjustment speed is 0.21, while for the case with the greatest investment frictions $\{N = 1, T = 12\}$ the speed of adjustment increases to 0.39. Intuitively, what is happening is that changes in target ratios for firms with the greater investment frictions have stronger negative correlation with changes in their equity values. This, in turn, implies a stronger positive correlation between changes in target and changes in leverage caused by changes in equity values. This stronger positive correlation between changes in leverage and changes in target ratios leads to a greater estimate of adjustment rate.

Although the relation between investment frictions and capital structure changes has not been directly tested, Flannery and Rangan (2003) report that small firms exhibit faster adjustments to their leverage targets compared to large firms. Arguably, if small firms have lumpier investments and longer time-to-build, then the model suggests that these investment frictions may be a cause behind the faster target adjustment for small firms.[29]

5.2.4 Pecking Order Theory Tests

The pecking order theory predicts that due to information asymmetry, investments should first be financed with internally generated income, then with debt and, as a last resort, with equity. Shyam-Sunder and Myers (1999) argue that according to pecking order theory, a financial deficit should be a dominant factor in explaining changes in debt and estimate the

[29] Lemmon and Zender (2004) suggest that smaller firms, or more generally, firms without debt ratings, tend to raise debt capital from banks rather than the bond market. Since banks exert more control over the firm's financing decisions than outside bondholders, these firms tend to move faster towards their targets.

following regression:

$$\Delta D_t = const + a \cdot FD_t + \varepsilon_t,$$

where the dependent variable is the change in debt value over one year, and the financial deficit variable FD_t is the level of external finance (net debt plus net external equity) for year t. Both variables are scaled by assets A_t. The value of 1 for the coefficient a would imply that pecking order explains 100% of debt changes, while the smaller coefficient (or zero) would delegate less predictive power to this theory. As reported in Table 6, the estimated coefficients for the model generated data are relatively high ($a = 0.88$ and $a = 0.79$) for the 2 firms with lesser investment frictions $\{N = 6, T = 2\}$ and $\{N = 3, T = 4\}$. These estimated values are close in magnitude to the actual estimate of 0.75 reported in Shyam-Sunder and Myers (1999) for the sample of 157 firms. This result may appear to suggest that firms with lesser investment frictions behave according to pecking order. Note that even though the model does not assume asymmetric information – the necessary condition of adverse selection required for pecking order – it can still generate the behavior that is qualitatively and quantitatively similar to the pecking order behavior. The coefficient a for FD declines to 0.37 for the firm with greatest investment frictions, implying that such a firm exhibits behavior that is less consistent with the behavior predicted by the pecking order theory.

Frank and Goyal (2003) replicate the above regressions for a broader sample, which includes a larger percentage of small firms compared to the sample used in Shyam-Sunder and Myers (1999). Frank and Goyal (2003) find that given their bigger sample, the coefficient for FD declines significantly to the level of 0.28, thereby implying that the inclusion of small firms contributes to a lower coefficient for FD. Thus, the model implies that firms with greater investment frictions use less debt during periods with financial deficit. Thus, if small firms are subject to greater investment frictions, the model provides alternative explanations of the findings in Frank and Goyal (2003).[30]

[30] A study by Lemmon and Zender (2004) suggests that the positive relation between size and pecking order behavior is generated by the fact that smaller firms tend to be less likely to have a bond rating, and thus are likely to have less available debt to fund their investments.

5.2.5 Tests of Pecking Order Theory versus Tradeoff Theory

Fama and French (2002) test both the trade-off and pecking order theories in the same regression. The authors stress the relation between changes in earnings, and profitability with changes in leverage, and run the following regression:

$$(\frac{D}{V})_{t+1} - (\frac{D}{V})_t = const + \lambda \cdot TL_{t+1} + b \cdot (\frac{D}{V})_t + c \cdot \frac{EBI_{t+1} - EBI_t}{A_{t+1}} + d \cdot \frac{A_{t+1} - A_t}{A_{t+1}} + e \cdot \frac{EBI_t - EBI_{t-1}}{A_{t+1}} + \varepsilon_t,$$

where the dependent variable is the change in market leverage over one year and EBI_t is the earnings before interest but after taxes. Variables $\frac{EBI_{t+1}-EBI_t}{A_{t+1}}$ and $\frac{A_{t+1}-A_t}{A_{t+1}}$ measure the contemporaneous shocks to earnings and assets; the target leverage ratios TL_t are estimated from the regression on various factors. As reported in Table 7, most of the regression coefficients in the model generated data have the same signs as well as a comparable economic significance to coefficients reported in Fama and French (2002). The authors point out that the negative coefficient for contemporaneous changes in earnings $\frac{EBI_{t+1}-EBI_t}{A_{t+1}}$ implies that firms don't instantly adjust their leverages in response to changes in earnings, which is consistent with the pecking order theory, but contradicts trade-off theory. Similarly, for the model-simulated data, changes in earnings have negative impact on changes in leverage, with this relation being slightly weaker for firms with greater investment frictions. I claim that the reason for this negative relation in the model is that changes in earnings are proxy for changes in investment opportunities that, in turn, are affected by investment frictions.[31]

In this regression, the estimate of the adjustment rate λ for the model generated data varies between 27% and 69%, for firms with the least and the most investment frictions respectively. For comparison, empirically estimated values for this variable in Fama and French (2002) vary between 7% and 15%. The results pertaining to the effect of investment frictions on adjustment speed is consistent with the findings in Fama and French (2002) that for no-dividend paying firms – which are arguably small firms and firms that are subject to greater frictions— the actual speed of leverage adjustment is reported to be greater than for dividend paying firms.

[31] Changes in lagged earnings $\frac{E_t-E_{t-1}}{A_{t+1}}$ have significantly lesser economic impact and most cases in the simulations have either a positive sign or are insignificant.

5.2.6 The Market Timing and Effect of History

In this subsection, I analyze the model using regressions that test timing theory and analyze the role of history on the longer-term target adjustment rate. According to the timing theory tested in Baker and Wurgler (2003), the capital structure is determined by the cumulative outcome of past attempts to time the market. The authors report that firms' debt ratios, as well as changes in debt ratios, are negatively related to the variable ($EFWA\_MB$) that measures the external-finance weighted average of past market-to-book (MB) ratios. The regression that appeared in Baker and Wurgler (2003) is essentially the following:

$$(\frac{D}{A})_t - (\frac{D}{A})_{t-5} = const + b \cdot EFWA\_MB_{t,t-5} + c \cdot (\frac{M}{B})_{t-1} + e \cdot (\frac{EBIT}{A})_{t-1} + g \cdot (\frac{D}{A})_{t-5} + \varepsilon_t,$$

where $EFWA\_MB_{t,t-5} = \frac{\sum_{s=t-5}^{t-1} FD_S (\frac{M}{B})_s}{\sum_{r=t-5}^{t-1} FD_r}$ is the external finance weighted-average MB ratio over the past 5 years.[32] Table 8 reports regressions results for the simulated data. The regression shows that the firm with the greatest investment frictions have negative coefficient b, while firms with lesser investment frictions have positive coefficients b.

Baker and Wurgler (2003) claim that the negative sign for coefficient b provides evidence for "market timing" in leverage decisions, i.e., that managers tend to time issuing equity in order to take advantage of overvalued equity proxied by MB ratios. Yet the model suggests that this negative relation can be due to investment timing coupled with investment frictions. Why do firms with greater investment frictions have a negative relationship between changes in leverage and the external finance weighted-average MB ratio ($EFWA\_MB$)? The reason is the following: As reported in the summary statistics, firms with greater investment frictions make their new incremental investments at higher MB ratios and invest at the lower leverages, and fund their new investments with greater fraction of equity. The combination of these facts induces stronger negative relationship between changes in leverage and the external finance weighted-average MB ratio.[33] This fact suggests that the perceived "market timing" behavior can be consistent with firms' rational investment and financing behavior in the economy with investment frictions without asymmetric information.

[32] Baker and Wurgler (2000) also include a variable for logarithm of sales that controls for the firm's size. In my simulations, the firm starts with the same size, implying no need to control for size.

[33] Hennessy and Whited (2004) also demonstrate via simulations that the "market timing" result can be replicated in a model where the firm has equity flotation costs and can retain earnings.

6 Conclusion

The complexity of modeling firms' financing behavior arises from the numerous technological constraints, investment frictions, as well as from capital markets imperfections. The dynamic model developed in this paper extends existing dynamic models by incorporating investment frictions, such as time-to-build and lumpiness of investments as well as endogenous financing. By developing a fully dynamic model, I am able to generate insights that relate to the recent empirical capital structure literature. In particular, I am interested in the empirical literature that examines the relative importance of what has been referred to as the trade-off theory, pecking order, and timing theories of capital structure. Recent empirical literature reports the evidence of observed leverage fluctuations that cannot be fully explained in the context of these three theories.

As the model illustrates, some of the empirically observed leverage patterns are consistent with the behavior generated by the "investment frictions" model. For example, the model suggests that the empirically observed faster target reversion exhibited by small firms can be driven by their greater investment frictions. Analysis of the simulated data also provides a rational interpretation for the documented behavior that firms tend to issue equity following periods with run-ups in equity prices.

The model-implied leverage behavior suggests that the differences in financing behavior across industries and/or across firms in the same industry can, in part, be attributed to the differences in investment frictions. Ignoring these frictions can yield predictions that attribute too much to financing imperfections. Recognizing the impact of these frictions increases the importance of more careful modeling of the real side of firms. For example, models may be expanded by incorporating the microstructure of the production process and frictions associated with it. On the other hand, the power of the empirical tests can be improved by including variables that measure cross-sectional variations in investment frictions in addition to the variables that are normally included.[34] For example, variables that measure the degree of time-to-build and the lumpiness of investments can provide important cross-sectional controls for different industries. Such variables can include measures of the fraction of assets-in-progress and/or the age distribution of assets, because both these

[34] Traditionally, existing tests control for characteristics such as the volatility of cash flows (or sales), MB ratios and the tangibility of assets.

measures proxy for the amount of newly built assets that may not be fully productive.

Before concluding, I should stress that the model can be extended along a number of ways. One can relax the assumption that investment lumps are of the same size by assuming that the firm can choose the size endogenously. Also, I assume that the depreciation rate is zero, which implies that investments are not reversible. Relaxing this assumption can allow us to explore the cross-sectional variation in leverage dynamics for firms with different depreciation rates and different irreversibility constraints. Although these issues are outside the scope of the current paper, one can pursue them in the future research.

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A Appendix: Valuation

This appendix presents a detailed formulation for the valuation problem of the equity and of the debt as well as of the all-equity firm. The numerical algorithm that I use for solving stochastic control problems is described in Appendix B.

A.1 Valuation of the All-equity Firm

In this section I consider the valuation of an unlevered firm $E^U(p, A_i, \tau)$. At each state (p, A_i, τ), the all-equity firm selects its investment strategy that maximizes its value E^U. By Ito's lemma, in the "no investment region" the value of the all-equity firm E^U is given by the solution to the following PDE:

$$\begin{cases} \left[\frac{1}{2}\sigma_p^2 p^2 E_{pp}^U + (r-\alpha)pE_p^U - E_\tau^U - rE^U + (1-\lambda)(p \cdot c(A_i - \Delta A))\right] = 0, \\ \quad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\ \left[\frac{1}{2}\sigma_p^2 p^2 E_{pp}^U + (r-\alpha)pE_p^U - rE^U + (1-\lambda)(p \cdot c(A_i))\right] = 0, \\ \quad \tau = 0, \text{ the firm has no incomplete assets.} \end{cases}$$

If the firm chooses to invest, it has to raise the amount of ΔA of new external equity and the equity value must satisfy the following free boundary condition:

$$E^U(p, A_i, 0) = E^U(p, A_i + \Delta A, T) - \Delta A.$$

B Appendix: Numerical Algorithm

In this appendix I describe the numerical algorithm that I apply to solve stochastic control problems for the valuation of equity. The algorithm is based on the finite-difference method augmented by a "policy iteration."

Since this is an infinite horizon stochastic optimization problems, values are time independent. Therefore, numerical solutions are reformulated into finite horizon approximation.[35] I initialize the procedure by approximating (guessing) values in each node of the terminal

[35] See Kushner and Dupuis (1992), Barraquand and Martineau (1995) and Langetieg (1986) for the theory and applications numerical of methods of stochastic control problems. Flam and Wets (1987) and Mercenier and Michel (1994) discuss the approximation of infinite horizon problems in the deterministic dynamic programming models.

time. This reformulation effectively implies that a derivative with respect to time is added to equations of each optimal stochastic control problem. For example, in the valuation problem for the all-equity firm, a new term E_t^U is added to the left hand side. The errors that result from the approximation of functions at the terminal time can be reduced by increasing the length of the horizon of the problem and iterating until the derivative E_t^U is indistinguishable from zero for each node on the grid.

For each problem I use a discrete grid and a discrete time step Δt. The state space (p, A_i, τ, d) is discretized using a four-dimensional grid $N_p \times N_A \times N_\tau \times N_d$ with corresponding spacing between nodes in each dimension of Δp, ΔA, $\Delta\tau$ and Δd and where $\Delta X = \frac{X_{\max} - X_{\min}}{N_x}$ and $X \in \{p, A_i, \tau, d\}$; $X_{\max}$ and $X_{\min}$ are the upper and low boundaries. In each node on the grid (p, A_i, τ, d) the partial derivatives are computed according to Euler method. For example, the first and the second derivatives of the equity value with respect to p are

$E_p(p, A_i, \tau, d) = \frac{E(p+\Delta p, A_i, \tau, d) - E(p-\Delta p, A_i, \tau, d)}{2\Delta p}$, $E_{pp}(p, A_i, \tau, d) = \frac{E(p+\Delta p, A_i, \tau, d) - 2E(p, A_i, \tau, d) + E(p-\Delta p, A_i, \tau, d)}{\Delta p \Delta p}$, with appropriate modifications at the grid boundaries.

B.1 Calculation of the Value of the All-equity Firm

In this section, I describe the computation of the value of the all-equity firm. The values of the all-equity firm at each node of the terminal time t are assigned the values of the expected earnings assuming that the value of assets is kept constant at a given level, i.e. $E_{(t)}^U(p, A_i, \tau) = \frac{p \cdot c(A_i)}{\alpha}$, the subscript (t) denotes the time of the node and α is the convenience yield. This approximation tends to underestimate the firm value because the approximation ignores future investments. However, by running backward recursion long enough, the values on the grid converge to the steady state values since the initial mispecifications of the terminal values are "smoothed" away due to discounting. Thus, working backward in time for each node on the grid according to the explicit finite-difference scheme and taking into account the investment decision, the value of the all-equity firm $E_{(t-\Delta t)}^U$ at each node (p, A_i, τ) at time $t - \Delta t$ is determined as follows:

$$
\begin{cases}
E^U_{(t-\Delta t)}(p, A_i, \tau) = (1-\lambda)\cdot p \cdot c(A_i)\cdot \Delta t + e^{-r\Delta t}\cdot \mathbb{E}_Q\Big[E^U_{(t)}(p, A_i, \tau - \Delta t)\Big], \\
\qquad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\
E^U_{(t-\Delta t)}(p, A_i, 0) = (1-\lambda)\cdot p \cdot c(A_i)\cdot \Delta t + \\
e^{-r\Delta t}\cdot \max\Big\{\mathbb{E}_Q\Big[E^U_{(t)}(p, A_i+\Delta A, T) - \Delta A\Big], \mathbb{E}_Q\Big[E^U_{(t)}(p, A_i, 0)\Big]\Big\}, \\
\qquad \tau = 0, \text{ the firm has no incomplete assets,}
\end{cases}
$$

where

$$
e^{-r\Delta t}\cdot \mathbb{E}_Q\Big[E^U_{(t)}(p, A_i, \tau)\Big] = E^U_{(t)}(p, A_i, \tau) + \Delta t \cdot \mathcal{L}[E^U_{(t)}(p, A_i, \tau)]\Big], \tag{9}
$$

where $\mathcal{L}[E^U_{(t)}(p, A_i, \tau, \widehat{d})]$ is the differential operator applied to $E^U_{(t)}$ in the node (p, A_i, τ, d), in which all partial derivatives are computed according to Euler method:

$$
\begin{cases}
\mathcal{L}[Z] = \frac{1}{2}\sigma_p^2 p^2 Z_{pp} + (r-\alpha)Z_p - Z_\tau - rZ, \\
\qquad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\
\mathcal{L}[Z] = \frac{1}{2}\sigma_p^2 p^2 Z_{pp} + (r-\alpha)Z_p - rZ, \\
\qquad \tau = 0, \text{ the firm has no incomplete assets.}
\end{cases}
$$

This derivation comes from Euler decomposition of the equation for the valuation of the all-equity firm, in which a new term E^U_t is added, where $E^U_t = \frac{E^U_{(t)} - E^U_{(t-\Delta t)}}{\Delta t}$. I repeat this backward induction procedure for $t-2\Delta t, t-3\Delta t, .., t-n\Delta t$ until the value function $E^U_{(t)}$ reaches a steady state in each node on the grid, i.e., until $\max\limits_{(p,A_i,\tau,d)} |E^U_{(t)}(p, A_i, \tau) - E^U_{(t-\Delta t)}(p, A_i, \tau)| < \varepsilon$, where ε is the predetermined accuracy level.

B.2 Calculation of the Equity Values

In order to compute the value of equity, I extend the procedure described in the previous section by incorporating the recapitalization decisions. I first approximate the values of the equity for the "terminal" time t. In each node (p, A_i, τ, d) I set the terminal values $E_{(t)}(p, A_i, \tau, d)$=max[0, $E^U_{(t)}(p, A_i, \tau, d) - (1-\lambda)\cdot F)$] if $p \cdot c(A) > d$, and 0, otherwise. In the calculation of the equity values, I separate the decision on the investment decision and the decision on whether or not to increase the debt ratio of the firm.

The value of equity E in each node on the grid (p, A_i, τ, d) at time $t - \Delta t$ are determined by working backward in time:

$$\begin{cases} E_{(t-\Delta t)}(p, A_i, \tau, d) = (1-\lambda)\cdot p \cdot c(A) \cdot \Delta t + e^{-r\Delta t} \max\limits_{\widehat{d}>d} \mathbb{E}_Q\Big[E_{(t)}(p, A_i, \tau - \Delta t, \widehat{d})\Big], \\ \qquad 0 < \tau \leq T, \text{ the firm has incomplete assets.} \\ E_{(t-\Delta t)}(p, A_i, 0, d) = (1-\lambda)\cdot p \cdot c(A) \cdot \Delta t + \\ e^{-r\Delta t} \cdot \max\Big\{\max\limits_{\widehat{d}>d} \mathbb{E}_Q\Big[E_{(t)}(p, A_i + \Delta A, T, \widehat{d}) - \Delta A\Big], \max\limits_{\widehat{d}>d} \mathbb{E}_Q\Big[E_{(t)}(p, A_i, 0, \widehat{d})\Big]\Big\} \\ \qquad \tau = 0, \text{ the firm has no incomplete assets.} \end{cases}$$

The firm increases its debt from d to $\widehat{d}$ if the following condition is satisfied

$$E_{(t)}(p, A_i, \tau, d) < \max_{\widehat{d}>d}[E_{(t)}(p, A_i, \tau, \widehat{d}) + D_{(t)}(p, A_i, \tau, \widehat{d}) - \\ D_{(t)}(p, A_i, \tau, d) - \Delta A - C_{Debt}], \; E_{(t)}(p, A_i, \tau, \widehat{d}) > 0. \quad (10)$$

If inequality (10) is satisfied then it is optimal to recapitalize, and the value of the equity is set equal to the maximum over all $\widehat{d}$ in the right-hand side of (10).[36] If the nodes where earnings are below debt payments, $p \cdot c(A) < d$, the default occurs and I set the value of the equity to 0. Similar to the computation of the all-equity firm, I repeat this iteration until values of the equity and debt reach the steady states in each node, i.e. until $\max\limits_{(p,A_i,\tau,d)}\{|E_{(t)}(p, A_i, \tau, d) - E_{(t-\Delta t)}(p, A_i, \tau, d)|\} < \varepsilon$.The procedure for the computation of the debt value is similar.

[36] Since the optimal value of $\widehat{d}$ does not necessarily fall exactly on a node, I use an interpolation technique in order to find the optimal debt level.

TABLE 1. Base Case Parameters and Variables Value	
p , the initial product price	\$612
σ_p volatility of the price	10%
α, convenience yield	7%
r, risk-free rate	7% per year
$c(A) = 1 - e^{-0.00034 \cdot A}$, initial and upper bound output	0.4 and 0.8
A_0 and A_N, value of initial and upper bound assets	\$1500 and \$4700
λ, corporate tax	35%
C_{Debt}, fixed transaction costs for issuing debt	\$1
$C_{Default}$	50%

Table 2. Summary statistics

This table presents summary statistics for the model generated annual data for five firms with different degrees of investment frictions, where MB is market-to-book ratio, $(TL)_t$ is the target leverage at time t, $(D/V)_t$ is the leverage ratio.

Summary statistics are reported for all time periods and for the periods before and after the firm makes investments.

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.
Market-to-Book, MB	3.44	0.80	3.60	0.88	3.72	0.94	3.99	1.07	4.70	1.77
Debt Ratio, $(D/V)_t$	0.61	0.07	0.61	0.07	0.59	0.07	0.57	0.08	0.56	0.11
Target Ratio, $(TL)_t$	0.58	0.02	0.58	0.03	0.57	0.04	0.55	0.05	0.54	0.08
Annual Correlation between Stock Return and Changes in Target	-0.07		-0.12		-0.17		-0.27		-0.30	

Table 3. Summary statistics

Summary statistics during the year before and the year after investments are made.

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.
Target before investments are made	0.58	0.02	0.55	0.04	0.56	0.01	0.52	0.02	0.46	0.03
Target after investments are made	0.58	0.02	0.57	0.02	0.55	0.04	0.50	0.05	0.39	0.01
MB before investments are made	3.65	0.59	3.94	0.62	4.19	0.68	4.86	0.73	8.10	0.53
MB after investments are made	3.85	0.70	3.92	0.77	3.94	0.83	3.96	0.91	4.19	1.28

Table 4. Investment Funding choice.

This table presents summary statistics for funding choices for five firms with different degrees of investment frictions.

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions				
	{T=2, N=6}	{T=3, N=4}	{T=4, N=3}	{T=6, N=2}	{T=12, N=1}
	Mean	Mean	Mean	Mean	Mean
$ Volume of Equity as % of total external Funding	0.11	0.14	0.18	0.26	0.58
$ Volume of Debt as % of total external Funding	0.89	0.86	0.82	0.74	0.42
Prob of Funding with Debt only	0.76	0.71	0.54	0.51	0.00
Prob of Funding with Equity only	0.01	0.06	0.08	0.00	0.00
Prob of Funding with both Debt and Equity	0.22	0.23	0.38	0.49	1.00

Table 5. Speed of Adjustment

These table reports the speed of adjustment to a firm's target for the partial adjustment model both estimated for model generated data of five firms with different degrees of investment frictions.

The regression appeared in Flannery and Rangan (2003).

$(D/V)_{t+1} - (D/V)_t = \lambda^*(TL_t - (D/V)_t) + e_t$,

where TL_t is the target leverage, and $(D/V)_t$ is the leverage ratio at time *t* .

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
$TL_t - (D/V)_t$	0.21	77.43	0.25	85.56	0.30	99.88	0.33	124.39	0.39	112.61
Adjusted R-squared		0.16		0.18		0.22		0.29		0.23

Table 6. Pecking Order Theory Tests

This table reports regressions on simulated data that are similar to regressions in Shyam-Sunder and Myers (1999) and in Frank and Goyal (2003).

$\Delta D_t = c + a \cdot FD_t + e_t,$

where ΔD is the change in debt, and FDt is the level of financial deficit at time t, both scaled by assets.

Regressions are run on the model generated annual data and are reported for five different levels of investment frictions.

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
C	0.05	78.06	0.06	87.53	0.07	98.04	0.07	91.58	0.06	85.92
FD_t	0.88	160.80	0.79	188.68	0.60	172.47	0.45	162.99	0.37	197.42
Adjusted R-squared		0.38		0.46		0.41		0.38		0.47

Table 7. Pecking Order Theory vs. Trade-off theory Tests

This table reports regressions on simulated data that are similar to the regression in Fama and French (2002).

$(D/V)_{t+1} - (D/V)_t = C + \lambda * TL_t + b*(EBI_{t+1}-EBI_t)/A_{t+1} + d*(A_{t+1}-A_t)/A_{t+1} + e*(EBI_t-EBI_{t-1})/A_{t+1} + f*(A_t-A_{t-1})/A_{t+1} + e_t$

where TL_t is the target leverage, $(D/V)_t$ is the leverage ratio, A_t is book value of assets and EBIT are earnings before interest but after taxes at time t.

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
C	-0.10	-25.36	-0.05	-15.04	-0.14	-47.24	-0.11	-51.81	-0.02	-15.09
TL_t	0.27	72.11	0.40	67.68	0.65	105.85	0.68	137.42	0.69	107.47
$(D/A)_t$	-0.29	-109.13	-0.30	-107.19	-0.36	-131.52	-0.46	-157.33	-0.61	-119.64
$(EBI_{t+1}-EBI_t)/A_{t+1}$	-0.93	-147.66	-0.75	-125.29	-0.68	-117.58	-0.61	-92.36	-0.44	-155.86
$(A_{t+1}-A_t)/A_{t+1}$	0.00	-0.15	0.01	6.11	-0.01	-2.97	-0.01	-6.68	-0.05	-29.55
$(EBI_t-EBI_{t-1})/A_{t+1}$	0.01	1.02	0.04	5.59	-0.02	-4.10	-0.06	-11.70	-0.04	-8.26
$(A_t-A_{t-1})/A_{t+1}$	0.00	-0.49	-0.02	-9.13	0.02	11.40	0.02	9.81	0.01	5.28
Adjusted R-squared		0.49		0.43		0.47		0.47		0.54

Table 8. Market Timing Theory

The effect of the external finance weighted-average market-to-book ratio (EFWA_MB) on leverage changes.

The regression appeared in Baker and Wurgler (2000).

$(D/A)_t - (D/A)_{t-5} = a + b \cdot EFWA\_MB_{t,t-5} + c \cdot (M/B)_{t-1} + e \cdot (EBIT/A)_{t-1} + g \cdot (D/A)_{t-5} + \varepsilon_{t,}$

where $(D/V)_t$ is the leverage ratio, M/B is market-to book ratio; A_t is book value of assets and EBIT are earnings before interest and taxes at time t and t-1; EFWA_MB is the external finance weighted-average market-to-book ratio over past 5 years.

$$EFWA\_MB_{t,t-5} = \sum_{s=t-5}^{t-1} \frac{FD_s}{\sum_{r=0}^{t-1} FD_r} \left(\frac{M}{B}\right)_s$$

{T is time-to-build, N is number of incremental investments}	Lesser Investment Frictions --------->----->----->------> Greater Investment Frictions									
	{T=2, N=6}		{T=3, N=4}		{T=4, N=3}		{T=6, N=2}		{T=12, N=1}	
	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat	Coeff	t-stat
C	0.54	167.13	0.51	164.88	0.52	173.86	0.54	171.29	0.42	153.90
$EFWA\_MB_{t,t-5}$	0.005	3.148	0.004	19.436	0.001	27.291	0.000	1.229	-0.009	-75.429
$(M/B)_{t-1}$,	-0.11	-54.66	-0.10	-60.50	-0.12	-82.90	-0.10	-90.38	-0.06	-75.65
$(EBIT/A)_{t-1}$	0.90	40.69	0.77	45.16	1.01	69.84	0.91	77.22	0.57	54.24
$(D/A)_{t-5}$	-0.62	-147.42	-0.60	-148.12	-0.64	-158.01	-0.72	-172.58	-0.55	-141.27
Adjusted R-squared		0.41		0.40		0.42		0.42		0.45

Figure 1. Illustration of output functions c(A) for two firms with different degrees of investment frictions {T=2, N=6} and {T=6, N=2}.