An Empirical Analysis of Yield Curves across Euro and Non-Euro Countries Using Interbank Interest Rates

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Abstract

This paper studies the interrelations among yield curve factors, market expectations and monetary policy rates using interbank interest rates across Euro- and non-Euro countries. The term structure of interest rates can be summarized by the level and slope factor, whereas curvature is not a common feature of interbank rates. Interest rates are first modelled in an equilibrium framework using a two-factor CIR (1985) model, and Kalman filter is used to extract the two factors under the no–arbitrage restriction. Impulse response analysis shows that German factors and forecast errors have the biggest influence on those factors from other markets, and that yield slope is a useful variable for capturing market expectations. Based on the estimated factors, theoretcial yields implied by the Expectations Hypothesis match remarkably well the movements of monetary policy rates, providing a consistent link between yield curve factors and macro–economic variables and thus integrating the approaches between no–arbitrage yield curve modelling and macro–economic based Expectations Hypothesis.

JEL classification: E43; C33; C53
Keywords: term structure; CIR; Kalman filter; impulse response; yield slope; Expectations Hypothesis

1 Introduction

Movements in the short-end of the term structure suggest different monetary policy decisions and the expectations about which held by market participants. Following the introduction of euro, a single money market yield curve for the euro area was established. To avoid arbitrage opportunities, all countries adopting the euro should have similar yield curves, though there exist local differences in the structure of bonds. The need to compare yield curve data, especially at the very short end of the maturity spectrum, for different countries may have become unnecessary.

It is easier to define the short-end of term structure using interbank money market interest rates. The interbank market provides quotes every day for a wide range of short-end maturities, and participants in the market are top credit quality banks, this combined with the high liquidity of interbank market implies that the credit premiums in the rate quotes are small. Furthermore, interbank data are not subject to issues like specialness or favorable tax treatment that are persistent features of government bond markets (Malz (1998) [10]). These issues render otherwise identical interbank money market interest rates from different European countries more comparable for the purpose of extracting market expectations. Interbank money market interest rates have been used for their informational or predictive content in a number of studies (Gerlach and Smets (1996) [5], Malz (1998)), and have been suggested as a valuable source of information for market expectations and the stance of monetary policy (Malz (1998)).

The Expectations Hypothesis test in its simplest form uses current spread between long and short rates to predict a) future short rate changes over the long-rate horization, or b) long rate changes over the next holding period. The CIR (Cox, Ingersoll and Ross) (1985) [3] model proceeds to use the stochastic generalizations of equilibrium theory to explain the term structure of interest rates and to predict how changes in the underlying variables will affect the term structure. It has a number of practical properties: it can model some of the heteroskedasticity in the errors, have non-negative bound on the interest rates, can generate a hump-shaped curve, and is analytically tractable. The CIR (1985) model has its multi-factor extensions: Longstaff and Schwartz (1992) [9] analyze a two-factor model and interprete the factors as short-term interest rates and interest rates volatility. Chen and Scott (1993) [2] further estimate both two-factor and three-factor models, and give an alternative interpretation for the two factors: short-term rates and long-term rates; and take interest rate volatility as the third factor in their three-factor model. They also argue that the estimated three-factor model is able to capture three common features of the term structure—level, slope and curvature—that are able to characterize empirically most of the variations in the shapes of yield curve, as first documented in Litterman and Scheinkman (1991) [8].

This paper takes a step towards the interrelations among the shapes of yield curve, market expectations and monetary policy decisions, linked by the underlying predicting factors of yield curves, for Euro and non-Euro countries. This is done by first looking at how variations in the shapes of interbank money market yield curve are related to the underlying predicting factors, then computing theoretical yields based on the predicting factors under the assumption that the Expectations Hypothesis holds, and finally analyzing the relations between the theoretical yields and monetary policy rates.

Modelling yield curves using factor representation in the no–arbitrage framework gives us a way to make explicit the factors affecting the dynamics of the shapes of yield curves. If similiar monetary policy results in similiar yield curve behaviour, then yield curve can be useful information variable for monetary policy analysis. Turning to the modelling focus of macroeconomists, studies on yield curve models found that both the shapes of yield curve (see Mishkin, 1990a [12], 1990b [13], 1991 [14]) and some macro–economic variables (Bernanke and Blinder(1990) [1]) contain information about markets' expectations of future inflation and real economic activity, an empirical analysis of yield curve in this context can be used to evaluate jointly the effectiveness of central banks' communications with financial markets and the rationality of the expectations held by market participants. Empirically relating yield curve factors to macro–economic variables thus provides a link between the equilibrium and no–arbitrage yield curve modelling approach and the macro–economic based Expectations Hypothesis approach.

The comparisons across Euro and non-Euro yield curves help to assess the impact of European Monetary Union (EMU) on the short-end of the yield curve, if the impact is instantaneous and money market yield curves are the first to converge, then they could be useful benchmark for pricing and hedging short-term interest rates derivatives.

The empirical analysis of the yield curve uses a two-factor CIR (1985) model on a sample of interbank money market interest rates for two euro countries—Germany and Finland, and for two non-euro countries–UK and Sweden. The data is weekly and ranges from April 26, 1995 to May 4, 2005. It covers the period from stage two of EMU, the introduction of Euro to the post-Euro period. Since monetary policies for Euro-countries are staged and converging during the sample period, if long rates move in the direction as dictated by the expected future path of short-term interest rates, and market participants and central banks attentively 'read' the expectations from yield curve, then under constant term premia, the Expectations Hypothesis would hold in practice. Upon this note, we compute theoretical yields implied by the Expectations Hypothesis, then analyze the dynamic relations between the theoretical yields and monetary policy rates, in order to see whether the computed yields reflect market expectations for the immediate path of monetary policy rates.

In actual implementations, the dynamic evolutions of yield curve are assumed driven by two unobservable factors, whereas the cross-sectional relation between yield and factors are determined by the no-arbitrage condition. The state space form of the CIR model puts the panel structure in a theoretically consistent way, Kalman filter is used to estimate the model and extract the two factors. The two factors are found respectively related to yield level and slope—both determine the shape of yield curve.

The level and slope factors are also dynamically related across markets, with German factors having a larger influence than others. The two-factor model fits very well three of the yield curves, namely Germany, Finland, and UK, with the fitted and actual yield curves of Germany and Finland lying very close to each other. This confirms the notion that countries adopting similiar monetary policy have similiar yield curve behavior, and yield curves can be compactly summarized by two factors—level and slope. For non-Euro countries, the two-factor model describes UK yield curve considerably better than Sweden. The loose fit for Sweden yield curve may be a result of forcing independent error structure while in fact the two factors are correlated to a larger extent than others.

That yield curve can be well described by the level and slope factors suggests that changes in short-term interest rates are more predictable around the establishment of the European Central Bank (ECB) and the introduction of Euro. And the predicatability of short-term interest rates tends to validate the Expectations Hypothesis (Hardouvelis (1988) [6], Mankiw and Miron (1986) [11], Gerlach and Smets (1996)). This also suggests that longer-term (6- and 12-month) interest rates might be good candidates for gauging markets' expectations about the stance of monetary policy. To the extent that market expectations can be affected by the predictability of future short rates, the shape of yield curve as charaterized by the predicting factors can be altered in a manner more in line with market expectations.

Under the assumption that the Expectations Hypothesis holds, the implied theorectical yields, based on the estimated factors from the first stage equilibrium and no–arbitrage framework, match remarkably well the movements of monetary policy rates, suggesting that the factors underlying the movements of interbank yield curve are closely related to monetary policy rates, namely, there is a link between financial factors and macro–economic variables. This analysis also integrates the no–arbitrage yield cuve modelling approach with the macroeconomic based Expectations Hypothesis.

The structure of this paper is as follows. Section 2 briefly describes the CIR model in a multi-factor framework, section 3 casts the CIR model in state space form, and Kalman filter is used to estimate the state space model and the actual implementation procedure is presented in section 4. Section 5 gives the data and the estimation results for the two-factor CIR model, section 6 then analyses the impulse responses of the two factors across the four interbank money markets, section 7 relates the theoretical yields implied from the Expectations Hypothesis to monetary policy rates, and finally section 8 summarizes and concludes the paper.

2 The Multi-factor CIR Model

Assume the instantaneous short rate r_t is a linear combination of n independent state variables, or factors, denoted as $x_1(t), ..., x_n(t)$, that is, we have the following identity:

$$r_t = \Sigma_{i=1}^{n} x_i(t), \tag{1}$$

and the state variables follow the CIR(1985) square-root process:

$$dx_i(t) = k_i(\theta_i - x_i(t))dt + \sigma_i \sqrt{x_i(t)} dw_i(t), \ i = 1, ..., n, \tag{2}$$

where the parameter k_i determines the mean reversion speed of the state variable i to the long-run mean θ_i, and σ_i is the deviation from the mean. The terms $w_1, ..., w_n$ are independent standard scalar Wiener processes defined on the actual probability space $(\Omega, \mathcal{F}, \mathcal{P})$. The price for a risk-free zero-coupon bond at time t paying one unit currency at time T is the expected discounted payoff

$$P(t,T) = \mathrm{E}_t^{\mathcal{Q}}\left[\exp\left(-\int_t^T r_s ds\right)\right], \tag{3}$$

where the expectation is taken under the risk-neutral probability measure $\mathcal{Q}$. The solution for the zero-coupon bond price as a function of the state variables takes the following form:

$$P_t(\tau, x_1, ..., x_n) = \exp(\Sigma_{i=1}^n (A_i(\tau) - B_i(\tau)x_{i,t})),\ \tau = T - t, \tag{4}$$

$$A_i(\tau) = \ln\left(\frac{2\gamma_i e^{\frac{(\gamma_i+\kappa_i+\lambda_i)\tau}{2}}}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i\tau}-1)+2\gamma_i}\right)^{\frac{2\kappa_i\theta_i}{\sigma_i^2}},$$

$$B_i(\tau) = \frac{2(e^{\gamma_i\tau}-1)}{(\gamma_i+\kappa_i+\lambda_i)(e^{\gamma_i\tau}-1)+2\gamma_i},$$

where

$$\gamma_i = \sqrt{(\kappa_i+\lambda_i)^2 + 2\sigma_i^2},$$

and λ_i is the market price of risk for the ith factor, it is determined endogeneously from changing the underlying process from $\mathcal{P}$ to $\mathcal{Q}$. Due to this particular function form, the yields on the zero-coupon bond can be written as a linear function of the factors

$$Y_t(\tau) = -\frac{\ln P_t(\tau)}{\tau} = -\frac{A(\tau)}{\tau} + \frac{B(\tau)}{\tau}X_t, \tag{5}$$

where the coefficients $A(\tau)$ and $B(\tau)$ are time-invariant functions of time to maturity τ. As the maturity becomes longer and longer, the yield approaches a limit that is independent of time:

$$Y_t(\infty) = \frac{2\kappa\theta}{\gamma+\kappa+\lambda}. \tag{6}$$

3 The State Space Form of Multi-factor CIR Models

The CIR model casted in the state space form involves specification of the measurement system and the transition system in discrete time dimension. Evenly subdivide the time interval $[0, T]$ into M subintervals, let $t_j = j\frac{T}{M}$, $j = 1, \ldots, m$ as the corresponding time point, and denote each time step as $\Delta t = \frac{T}{M}$. This discretization allows us to represent equation (5) as the following measurement system:

$$Y_t = A + BX_t + \varepsilon_t, \tag{7}$$

where

$$Y_t = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}, \; A = -\begin{bmatrix} \frac{A(\tau_1)}{\tau_1} \\ \frac{A(\tau_2)}{\tau_2} \\ \vdots \\ \frac{A(\tau_m)}{\tau_m} \end{bmatrix}, \; B = \begin{bmatrix} \frac{B_1(\tau_1)}{\tau_1} & \frac{B_2(\tau_1)}{\tau_1} & \cdots & \frac{B_n(\tau_1)}{\tau_1} \\ \frac{B_1(\tau_2)}{\tau_2} & \frac{B_2(\tau_2)}{\tau_2} & \cdots & \frac{B_n(\tau_2)}{\tau_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{B_1(\tau_m)}{\tau_m} & \frac{B_2(\tau_m)}{\tau_m} & \cdots & \frac{B_n(\tau_m)}{\tau_m} \end{bmatrix},$$

and

$$\varepsilon_t \sim \mathcal{N}(0, H),$$

$$H = \begin{bmatrix} h_1^2 & 0 & \cdots & 0 \\ 0 & h_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & h_m^2 \end{bmatrix}.$$

In the measurement system (7), Y_t is an $m \times 1$ maturity vector of observed yields, and A is an $m \times 1$ vector of intercepts, the $m \times n$ coefficient matrix B is the corresponding factor loadings. The $m \times 1$ vector of measurement errors ε_t is normally distributed with mean 0 and variance matrix H, the errors are assumed cross-sectionally and serially uncorrelated, but with variances differing across maturities. In the state space formulation of interest rates models, we typically have more maturities of observed yields than the number of factors, i.e. $m>n$, so that the system is not exactly identified. The addition of the measurement error term is used to account for the overidentification problem as well as any data imperfections or model misspecificaton errors.

The transition system is a discretized representation of the continuous-time CIR model, which involves solving the stochastic differential equation (2) explicitly for the first two conditional moments, then discretizing over the time intervals using the Euler scheme. In matrix form, the transition equations take the following form:

$$X_t = C + DX_{t-1} + v_t, \tag{8}$$

where

$$X_t = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \; C = \begin{bmatrix} \theta_1 \left(1 - e^{-k_1 \Delta t}\right) \\ \theta_2 \left(1 - e^{-k_2 \Delta t}\right) \\ \vdots \\ \theta_n \left(1 - e^{-k_n \Delta t}\right) \end{bmatrix}, \; D = \begin{bmatrix} e^{-k_1 \Delta t} & 0 & \cdots & 0 \\ 0 & e^{-k_2 \Delta t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & e^{-k_n \Delta t} \end{bmatrix},$$

and

$$v_t | \, \mathcal{F}_{t-1} \sim \mathcal{N}(0, R_t),$$

$$R_t = \begin{bmatrix} \xi_1^2 & 0 & \cdots & 0 \\ 0 & \xi_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi_n^2 \end{bmatrix},$$

where

$$\xi_i^2 = \frac{\theta_i \sigma_i^2}{2\kappa_i}\left(1 - e^{-\kappa_i \Delta t}\right)^2 + \frac{\sigma_i^2}{\kappa_i}\left(e^{-\kappa_i \Delta t} - e^{-2\kappa_i \Delta t}\right) x_{i,t-1},$$

for $i = 1, \ldots, n$. The dimensions for X_t, C, and D are respectively $n \times 1$, $n \times 1$, and $n \times n$. Since the state variables are governed by the square-root processes, the conditional variances of the errors ξ_i^2 depend on $x_{i,t-1}$.

4 The Kalman Filter Implementation

The state space formulation of CIR model pools both cross-sectional and time series information in the interbank interest rates, it also uniquely idenfies the market price of risk λ that is not identified with other single-dimensioned estimation methods. The no-arbitrage condition also imposes restrictions on the form that the measurement equations can take and enforces the consistency of yield curve evolution over time. The fixed parameters of the state space models are typically estimated using the method of maximum likelihood for normally distributed errors, and Kalman filter then delivers the optimal filtered estimates of the unobserved state variables. In the CIR model, the errors are non-central χ^2 distributed, so exact maximum likelihood is not applicable. However, the method of quasi-maximum likelihood can be used; the method is based on the exact discrete form of the conditional mean and conditional variance functions derived from the CIR square-root stochastic process. The quasi-maximum likelihood estimator is not consistent, but Monte Carlo studies (Chen and Scott (1993), Zhou (2001) [15]) have shown that compared to other estimation methods like GMM (Generalized Method of Moments) and SMM (Simulated Method of Moments), the magnitude of biases is not large.

The actual implementation of the state space model begins with finding the appropriate initial values for the state variables, which are then used to start the recursive algorithm in the Kalman filter technique (see Harvey (1991) [7]). The unconditional mean and unconditional variance of the state vector are used to initialize the Kalman filter. The unconditional mean for the CIR model is

$$\mathrm{E}\left[\, X_1 \,|\, \mathcal{F}_0 \,\right]^T = \left[\, \theta_1 \quad \theta_2 \quad \cdots \quad \theta_n \,\right]^T, \tag{9}$$

where the superscript T denotes matrix transposition, $\mathcal{F}_0$ the Filtration at time 0, and θ_i the unconditional mean for factor i. The unconditional variance for the CIR model is

$$\operatorname{var}\left[X_1 \middle| \mathcal{F}_0\right] = \begin{bmatrix} \frac{\theta_1\sigma_1^2}{2\kappa_1} & 0 & \cdots & 0 \\ 0 & \frac{\theta_2\sigma_2^2}{2\kappa_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\theta_n\sigma_n^2}{2\kappa_n} \end{bmatrix}. \tag{10}$$

The conditional forecast of the measurement equation (7) takes the following form

$$\mathrm{E}\left[Y_t \middle| \mathcal{F}_{t-1}\right] = A + B\mathrm{E}\left[X_t \middle| \mathcal{F}_{t-1}\right], \tag{11}$$

and the corresponding conditional variance is

$$\operatorname{var}\left[Y_t \middle| \mathcal{F}_{t-1}\right] = B\operatorname{var}\left[X_t \middle| \mathcal{F}_{t-1}\right] B^T + H, \tag{12}$$

where the matrices A, B and H are defined in the measurement system (7). The errors for the conditional prediction in the measurement equations are denoted as

$$\varepsilon_t = Y_t - \mathrm{E}\left[Y_t \middle| \mathcal{F}_{t-1}\right], \tag{13}$$

where Y_t is the observed yield at time t. The prediction errors are then used to update the unobserved state variables

$$\mathrm{E}\left[X_t \middle| \mathcal{F}_t\right] = \mathrm{E}\left[X_t \middle| \mathcal{F}_{t-1}\right] + \mathrm{K}_t\varepsilon_t, \tag{14}$$

where

$$\mathrm{K}_t = \operatorname{var}\left[X_t \middle| F_{t-1}\right] B^T \operatorname{var}\left[Y_t \middle| F_{t-1}\right]^{-1}$$

is the Kalman gain matrix. The conditional variance of the state variables is also updated as

$$\operatorname{var}\left[X_t \middle| F_t\right] = \left(\mathrm{I} - \mathrm{K}_t B\right)\operatorname{var}\left[X_t \middle| F_{t-1}\right]. \tag{15}$$

Conditional on the updated values, the forecasts for the transition system are computed as

$$\mathrm{E}\left[X_{t+1} \middle| F_t\right] = C + D\mathrm{E}\left[X_t \middle| F_t\right], \tag{16}$$

$$\operatorname{var}\left[X_{t+1} \middle| F_t\right] = \operatorname{var}\left[X_t \middle| F_{t-1}\right] - D\operatorname{var}\left[X_t \middle| F_t\right] D^T + R_t, \tag{17}$$

where the matrices C and D are defined as in the transition system (8). Equations (11) to (17) are the recursive steps in the Kalman filter algorithm. At each time point, it generates a prediction error ε_t and a prediction error variance-covariance matrix $\operatorname{var}\left[X_t \middle| F_{t-1}\right]$, which are then used to maximize the likelihood function with respect to the parameter vector $\hat{\phi}$:

$$\ell(\hat{\phi}) = -\frac{Nm}{2}\ln\left(2\pi\right) - \frac{N}{2}\ln\left(\det\left(\operatorname{var}\left[X_t \middle| F_{t-1}\right]\right)\right) - \frac{N}{2}\varepsilon_t^T \operatorname{var}\left[X_t \middle| F_{t-1}\right]^{-1}\varepsilon_t, \tag{18}$$

where N is the total number of observations assuming equal discrete time steps, and m is the number of yield series. The maximization of the likelihood function is done using the iterative algorithms of Marquart and Berndt-Hall-Hall-Hausman, computing numerical derivatives, and setting a convergence criterion of 10^{-6} for the gradient of the estimated coefficients.

5 Estimation of the Two-Factor Models

5.1 Data

The interbank interest rates are obtained from DataStream for four European countries: Germany, Finland, UK and Sweden. They are respectively coded FIBOR, FNIBC, SIBOR and BBGBP in DataStream. The selected maturities are 1 month, 3 month, 6 month and 12 month. The observations are weekly, ranging from April 26, 1995 to May 4, 2005, for a total of 524 data points.

Descriptive statistics for yields and yield spreads are given in Table 1 Panel A and B, respectively. Panel A shows that at the short-end of the maturity spectrum average yield curves are increasing, convex functions of maturity. For all the four markets, average yield curves increase first from 1 month to 6 month then rise more sharply from 6 month to 12 month. This is confirmed by the average yield spreads in Panel B that shows more clearly that there is a sharpening of the slopes at the longer end of the selected maturities. In addition, yields are quite persistent, with first-order autocorrelations above 0.980 for all markets. Yields at 1-month and 3-month maturties are more persistent than yields at 6-month and 12-month maturities. Yield spreads, on the other hand, are less persistent than yields, but the extent of persistence increases with maturity.

The volatility curves for yield levels in Table 1 Panel A, however, show different patterns across the markets. The average volatilities for the German and Finnish interbank markets are increasing, convex functions of maturities, while the Swedish interbank volatility curve looks like an inverted wedge with a trough at the 6-month maturity, and the UK interbank volatility has a hump at 3-month and 6-month maturities. The volatilities for yield spreads on the other hand are monotone increasing functions of maturities, i.e. the spread between 12- and 1-month is more variable than the spread between 6- and 1-month, which is in turn more variable than the 3-month yield spread.

Litterman and Scheinkman (1991) shows that about 90% of the variability in bond yields can be captured by three factors—level, slope and curvature. The results from principal components analysis of weekly changes in interbank interest rates in Table 2 show that for all the interbank markets, the first factor explains over 98% of the variability in rate changes, the second factor explains a further 1% of the variability, and in total three factors are able to explain over 99.9% of the total variations in the changes of interbank interest rates.

Plots of the factor loadings in Figure 1 further show that loadings on the first factor are close to one for the four selected maturities, suggesting that shocks to the first factor result in parallel shifts in the yield curve. The loadings on the second factor are negative for 1-month and 3-month maturities but positive for 6-month and 12-month maturities, resembling a slant upward slope. Thus shocks to the second factor pull yields at the short- and longer-end of the interbank markets at different directions. Finally, loadings on the third factor are much smaller relative to others; they are slightly hump-shaped, positive for the 3-month and 6-month maturities but negative for the 1-month and 12-month maturities. This means that shocks to the third factor pulls the short- and long-end yields in one direction but the middle-ranged yields to the opposite direction.

5.2 Estimation Results

From the principal components analysis, a two-factor CIR model can be considered adequate to capture most of the variations in the shape of yield curves, as the contribution of the third factor is marginal—less than 0.1%. Chen and Scott (1993) compare the relative performance of two-factor and three-factor CIR models using model pricing errors for bonds of different maturities, and find that the two-factor model performs almost as well as the three-factor model.

The empirical implementation of the two-factor CIR model involves setting the number of maturities $m = 4$ and the number of factors $n = 2$ in systems (7) and (8). The estimation of the parameter set $\{\theta_1, \kappa_1, \sigma_1, \lambda_1, \theta_2, \kappa_2, \sigma_2, \lambda_2, h_1^2, h_2^2, h_3^2, h_4^2\}$ uses the quasi-maximum likelihood method mentioned in section 4. Maximization of the likelihood function starts with appropiate initial values for the parameter set, and the estimation results are given in Table 3.

The results show that the estimated two factors have very different properties (see Figure 2 for a time-series plot of the two factors). The mean reversions on the first factor are much slower than those on the second factor. The half-life[1] for the first factor ranges from 4.4 years for the Finnish interbank market up to 63 years for UK interbank market, while the half-life for the second factor has a value of 0.6 year to 1.8 years.

Figure 3 graphs the factor loadings $B(\tau)$ for the interbank markets. The loadings for the first factor $B_1(\tau)$ are slightly above one and increases slowly with maturity, while the loadings for the second factor $B_2(\tau)$ deline much faster with maturity for all interbank markets. Thus the first factor affects all rates and determines the general level of the interest rates, and the second factor has a stronger influence for the shorter-term rates and resembles the slope factor.

The exact interpretation of the factors can be more clearly seen from their comparisons with yield levels and yield spreads. The dynamic series of the two factors are extracted from the Kalman filter estimation process. The graphs of 3-month yield levels with the first factors is given in Figure 5, while yield spreads (approximated as the difference between 12 month and 1 month rates) with the second factor is given in Figures 6. As the graphs show, the first factor closely follows the movements of yield level while the second factor behaves like yield spread but with some time lag. Thus the two unobservable factors delivered by the Kalman filter can be respectively related to the level and slope of interbank rates.

The market price of risk estimates, λ, are all negative for the first factor (level), implying a positive risk premium for holding longer-term bonds. The second factor

[1]Half-life is calculated as $\frac{ln2}{\kappa}$.

(slope) has a positive market price of risk for all markets. The implied negative risk premium suggests that the market has expected that the inflation component of longer-term yields will fall (decreasing slope) as dictated by the central banks' monetary policy.

The forecast errors are on average close to zero for all maturities except for the 1-month rates. This could be because that the two-factor model cannot fully capture the variations in 1-month rates or that the observations are more noisy at the maturity of one month. The dynamic series of the forecast errors are given in Figures 7 and 8. For all the markets, forecast errors are small in scale and decreasing with maturity, in general they behave like white noise though there still seem to be some structure left in the series. This could partly be attributable to the potential cross–correlations or a third factor (curvature) that has not been accounted for.

To see how closely the forecast errors are related to the curvature factor, Table 4 gives the common sample correlations between the curvature factor[2] and the forecast errors, as well as the cross-sectional correlations among the forecast errors. As the table shows, the curvature factor is negatively correlated with the 1- and 12-month forecast errors but positively correlated with the 3- and 6-month forecast errors for all interbank markets.

The cross-sectional correlations among the forecast errors behave differently for Euro and non-Euro interest rates markets. For FIBOR and FNIBC, forecast errors at the shorter-end of maturities (i.e. 1- and 3-month) are both negatively correlated with forecast errors at the longer-end maturities (i.e. 6- and 12-month). The pattern is different, however, for SIBOR and BBGBP, where the forecast errors are all positively related except that the pairwise correlation between 3- and 12-month errors are negative. Regressions of the forecast errors on the curvature factor show that loadings are humped shaped for all markets, positive at 3- and 6-month and negative at 1- and 12-month maturities. The adjusted R-squares of the regressions for FIBOR, FNIBC, SIBOR and UK are 18.6%, 17.1%, 18.1%, and 16.7%, respectively. Thus the structure left in the forecast errors resemble the curvature factor but the relative extent is rather small, considering that on average forecast errors are about one-tenth the size of interbank rates.

To assess how well the two-factor model fits the observed yields, the implied model yields are calculated using the yield–and–factor relation as in Equation (5). Figure 4 shows model implied yields and actual yields for the four studied markets. For three of the interbank markets—UK, Germany and Finland, the model yields lie slightly below the actual yields, with almost no distinguishable gap between the two. Thus the two-factor CIR model provides an adequate characterization for these three money market yield curves—it is able to fit the levels and the slopes of the yield curves. For the Swedish interbank market, however, the model underestimates the yield curve by a paralell gap for all maturities. The different model performances could be partly attributable to the presence of correlations between the two factors, which are forced to be zero during

[2]The empirical proxy for curvature is approximated as the difference between two times the 6-month yield and the sum of 12-month yield and 1-month yield

the estimation process to avoid complex algorithm. The average correlations of the estimated factors for Germany, Finland, Sweden and UK are $0.149, 0.055, 0.353, 0.066$, respectively. The relatively higher correlation for the Swedish interbank market that is not accounted for by the model could be a cause for the loose fit of the yield curve.

On average, both the implied and actual yield curve are slightly upward sloping with little curvature. This general shape is also confirmed by the principal components analysis where the third factor (curvature) explains less than 0.1% of the variations in money market yield curve. Thus curvature is not a common feature of interbank term structure.

The implied long yield, as defined in Equation (6), ranges from 0.895 to 1.001 for the first factor and from 0.648 to 0.742 for the second factor. That the implied long yield falls within such a narrow range suggests that market expects in the future long rates will stand close to each other for the examined European markets. And this expectation seems rational in view of the long-run inflation target set by the European central banks.

6 Impulse Responses of the Two Factors

If monetary policy actions in one country leads to similiar reactions in other countries, it would be reasonable to expect that changes in yields and yield spreads are dynamically related to some extent, and if the linkage is governed by some factors then the factors may be used to predict changes of yields and yield spreads in other markets. This section uses impulse respose functions to explore this possibility. We will consider four groups of impulse responses in turn: 1) responses of factor 1 across markets, 2) responses of factor 2 across markets, 3) responses of forecast errors across markets, and 4) cross responses of factors, forecast errors and yield curve curvature within a single market.

The impulse reponse function is obtained by first estimating the following VAR system:

$$F_t \ = C(L) \ F_{t-1} \ + \ \varepsilon_t \ , \tag{19}$$

where F_t is a vector of the estimated first factors and second factors for the four interbank markets, the forecast errors and yield curve curvature. The term $C(L)$ is a matrix of second-order lagged polynomials, and ε_t is a vector of residuals of the estimated factors. Having obtained the vector of residuals ε_t, the inverse of the cholesky factor of the residual covariance matrix P is used to orthogonalize the impulses for easier interpretation:

$$\nu_t = P\varepsilon_t - (0, D) \, , \tag{20}$$

where D is a diagonal covariance matrix, and ν_t is a vector of uncorrelated innovations. The impulse response function then traces the effects of a one-time shock to one of the innovations ν_t on the current and future 13 periods (corresponding to one quarter) of the estimated factors F_t. Note that the estimated factors for German interbank market come first in the VAR ordering, and the cholesky transformation P attributes all of the effect of any common component to the German factors.

Figures 9 and 10 show that German factors mainly respond to their own shocks, lasting out to one quarter, the other three markets have less than one-six of the effects. The German factors also have the largest and longest lasting influence on the Finnish factors who in turn respond only secondarily to their own shocks. The UK and Swedish factors respond primarily to their own shocks, and secondarily to shocks of the German factors. The actual and implied yield curves for euro countries—Germany and Finland, lie close to each other and are both subject to the dominant influence of German factors in a similiar way. However, the Finnish factors behave differently in that they are also affected by local shocks. Non-Euro countries—Sweden and UK, have factors responding to shocks very differently: Swedish factors respond to shocks from other markets in a more dispersed way and to a larger extent than UK factors. On the whole, the effects of the German factors on other markets suggests that decisions by the ECB have a predominant role in the monetary policy decisions in Euro countries such as Finland and also have a big influence on other non-Euro European countries such as UK and Sweden.

For all the markets, the response of factor 1 to its own shocks reaches its maximum at about period 3 (3 weeks), then slowly declining though at different rates over the remaining periods. The response of factor 2 to its own shock, on the other hand, approaches its maximum limit at around period 6 (6 weeks), and remains at that level untill quarter-end. The different behavior of the two factors in response to the shocks can be better understood when relating to their interpretations respectively as level and slope factors. Thus, shocks to yield level have a shorter lagged response and less significant future effect than shocks to yield spread (slope). The lasting effect of one-time shocks on yield spread is consistent with the Expectations Hypothesis of the term structure of interest rates, according to which, current yield spread contains all information on future short rate changes. Additionally, changes in slopes of Finnish, Swedish and UK interest rates respond to changes in slopes of German interest rates, which can be useful information besides domestic data when predicting changes in their (Finnish, Swedish or UK) future interest rates.

Relative to the impulse responses of factor 1 and 2, the effects of a one-time shock on the forecast errors die out very quickly for all markets, as shown in Figure 11. This suggests that the information left in the forecast errors is short-lived and almost of no forecasting value beyond three months. The German forecast errors, however, still have the biggest influence on the others. This could be that other interbank markets respond to news originating from Germany or that there is some structure left in the errors related to the first two factors (such as cross–correlations).

To explore this possibility, the graph for the impulse responses of the following series: factor 1, factor 2, forecast errors, and a measure for curvature are presented in Figure 12. The empirical proxy for curvature is approximated as the difference between two times the six-month rates and the sum of two-month and twelve-month rates.

Since German interbank rates have the dominant effect on other interbank rates, Figure 12 only presents the four series for 3-month FIBOR. Though the graph is selective, it provides a good summary of the relationships among the factors and their respective informational role, which we document below. First of all, the current level of short rates responds most negatively to shocks on the yield spread, that is, a widening yield spread indicates lower current short rates. Secondly, yield spread is the only series that is less affected by others, it is hump-shaped and the response to a one-time shock is permant, so that when new information changes current short rates, it revises all future short rates and the expected level of long-term rates. Thus, future short rate changes are to a large extent predictable and yield spread is a useful measure for assessing markets' expectations. Thirdly, forecast errors repond mainly to idosyncratic shocks in the near term, up to two months. Beyond that, other factors like curvature and slope have dominant influence. Fourthly, in three out of four cases, the curvature measure and forecast errors are most closely related—they respond to shocks to a similiar extent, though forecast errors have some of its own idiosyncracies, possibly due to the missing cross–correlations or the presence of data imperfections and mispricing errors at the short-end of maturity.

Although the above analyses point to the possibility of including a third factor in the CIR (1985) model, actual implementations of a three-factor CIR model have a number of numerical difficulities associated with the estimation procedures. Firstly, the third factor as presumably interpreted as curvature is rather small in scale relative to the first two factors, including it makes the task of separating different components harder and causes non-invertability of the Hessian matrix. More parameters also renders optimization of the likelihood function rather difficult. Secondly, direct modelling of the forecast errors seems to be a better option at first, but even after some scaling of the data such that the Hessian matrix is invertable, the estimated results for the extracted factor—the curvature series—are in most cases insignificant and not interpretable. Furthermore, the forecast errors of the forecast errors (of the two-factor model) are almost indistinguishable from the original series, meaning that the model does not succeed in explaining any structure in the forecast errors. After such attempts, we conclude that there is insufficient variation in interbank term structure to require the use of a third factor and that a two-factor model is enough to capture most of the variations in interbank term structure of interest rates.

7 Relating Yield Curve Factors to Macroeconomic Variables

Given the optimal yield predictions delivered by Kalman filter and the good fit of the factors to yield curve, it is of interest to relate the estimated factors to the macroeconomic variables used in tests of the Expectations Hypothesis, and analyze their dynamic interactions.

The Expectations Hypothesis states that changes in long rates are sums of the expected changes in future short rates over the long-rate horizon, plus zero or constant term premium. Based on the optimally predicted yields, the theoretical yields consistent with the Expectations Hypothesis are given as

$$y_t(\tau)^{EH} \equiv \frac{1}{\tau}\sum\nolimits_{i=0}^{\tau-1} \mathrm{E}_t y_{t+i}(1) + c_\tau, \tag{21}$$

where c_τ is the constant term premium and may vary with maturity, but assumed to be zero in computing the theoretical yields $y_t(\tau)^{EH}$. The maturity of interest in Equation (21) is for 3-month, 6-month and 9-month horizon respectively. The one-period ahead predicted yields y_{t+i} are spaced at monthly intervals, computed as monthly averages of the weekly predicted yields for 3-month, 6-month and 9-month maturity from the first-stage estimation. Yields for the intermediate maturities are linearly interpolated between adjacent yields.

The theoretical yields are graphed in Figure 13, together with series of policy rates for the four markets. They are ECB target rates, average bank reference rates, Repo rates and Bank of England target rates for Germany, Finland, Sweden and UK, respectively. In macro–economic tests of Expectations Hypothesis, interest rates on federal funds have been shown (Bernanke and Blinder(1990) [1]) to have the most information content about future movements of the real economy than any other macro–economic variables such as industrial production, capacity utilization and monetary aggregates. Hence, central bank policy rates are good forecasting instruments of monetary policy decisions.

As Figure 13 shows, the theoretical yields match remarkably well the movements of policy rates, at a consistent gap though the extent varies with markets. The correlations between the theoretical yields and central bank policy rates are all above 0.90 for all markets. As theoretical yields are basically linearly weighted functions ($A(\tau)$ and $B(\tau)$) of the predicting factors, high correlations between theoretical yields and policy rates implies high correlations between the predicting factors and policy rates as well. Hence, there is a close link between the financial factors extracted from the equilibrium and no–arbitrage framework and the macro–economic factors used by macroeconomists in tests of the Expectations Hypothesis. The above analyses also suggest that the shapes of yield curve (as summarized by the predicting factors) embody market expectations about monetary policy decisions which are empirically proxied by central bank policy rates.

8 Summary and Conclusions

This paper studies the interrelations among shapes of yield curve, market expectations, and monetary policy decisions, linked by the underlying predicting factors of interbank yield curve.

The establishment of EMU and the convergence of monetary policies among Euro countries lend themselves to such an examination of the relations. Since monetary authorities determine very short-term interest rates and 'read'market expectations from the yield curve, the term structure of interbank money market interest rates can be seen as the best variable capturing markets' expectations about the stance of monetary policy. As monetary policy during the sample period is converging, the expected future path of interest rates is more predictable, if markets have rational expectations and term premia are constant, the Expectations Hypothesis should hold.

The main findings are as follows: firstly, the shapes of interbank yield curve are very well described by the level and slope factors with little curvature. Secondly, the level and slope factors for German interbank market have the biggest influence on the factors for Finnish, Swedish, and UK interbank markets. Thus when predicting future interest rates movements, German factors are useful information variable. Thirdly, the response of slope factors to a one-time shock is persistent, meaning that current yield spread contain all information about future short rate changes. Finally and more importantly, the two predicting factors can be linked to the central bank policy rates through the Expectations Hypothesis framework, confirming the notion that the shapes of yield curve embody market expectations of future monetary policy stance.

The empirical analysis assumes that the variations of yield curve are explained by two factors and uses the CIR model to describe the process followed by the two unobservable factors, the two factors are then extracted using Kalman filter and are respectively interpreted as level and slope factor. Based on the estimated factors, theoretical yields are derived under the assumption that Expectations Hypothesis holds, and are shown matching the movements of central bank policy rates remarkably well. The high correlation between theoretical yields and policy rates builds a link between financail factors and macroeconomic variables and a link between the no–arbitrage modelling approach and macroeconomic determinant Expectations Hypothesis approach.

Overall, in a period of monetary policy certainty, the expected future path of short-term rates are more predictable, market participants tend to hold rational expectations and the term premia behave more constant, yield curve will then move in line with market expectations. Therefore, it is only when markets are uncertain about the stance of monetary policy and hold inaccurate forecasts that the future short rates become unpredictable, hence failing the tests of Expectations Hypothesis.

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Table 1: Descriptive Statistics

This table presents the statistical summary of the interbank money market interest rates for four European countries—German, Finland, Sweden and UK, the interest rates series are respectively denoted as FIBOR, FNIBC, SIBOR, and BBGBP in Panel A of the table. The series in Panel B are the corresponding yield spreads calculated as the spread of the corresponding maturities over the one-month rates. The Jarque-Bera statistic is for normality test, and Auto denotes the first-order autocorrelation of the series.

Panel A	FIBOR				FNIBC				SIBOR				BBGBP			
Yields	1M	3M	6M	12M	1M	3M	6M	12M	1M	3M	6M	12M	1M	3M	6M	12M
Mean	3.285	3.315	3.347	3.451	3.334	3.386	3.452	3.598	4.247	4.288	4.350	4.543	5.505	5.585	5.647	5.796
StdDev	0.820	0.822	0.824	0.838	0.990	1.004	1.029	1.069	1.677	1.669	1.658	1.671	1.205	1.222	1.221	1.215
Max	5.046	5.137	5.195	5.325	5.975	6.125	6.375	6.825	9.030	9.300	9.530	10.040	7.766	7.875	7.938	8.000
Min	2.024	1.958	1.936	1.957	1.985	1.935	1.905	1.885	2.100	2.068	2.065	2.070	3.336	3.391	3.390	3.416
Skew	0.204	0.191	0.182	0.149	0.734	0.684	0.682	0.698	1.542	1.532	1.507	1.458	0.008	-0.008	-0.040	-0.100
Kurtosis	2.327	2.348	2.367	2.322	3.148	3.153	3.284	3.481	5.160	5.298	5.369	5.402	1.903	1.899	1.914	1.927
JB	13.542	12.466	11.640	11.976	47.528	41.338	42.407	47.661	309.526	320.426	320.905	311.531	26.304	26.466	25.907	25.996
Prob.	0.001	0.002	0.003	0.003	0.000	0.000	0.000	0.000	n	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Auto	0.992	0.993	0.993	0.991	0.990	0.991	0.990	0.988	0.991	0.990	0.988	0.986	0.997	0.997	0.996	0.994
Panel B	YSPFIBOR				YSPFNIBC				YSPSIBOR				YSPBBGBP			
YieldSpread		3M	6M	12M		3M	6M	12M		3M	6M	12M		3M	6M	12M
Mean		0.030	0.062	0.166		0.053	0.118	0.264		0.041	0.103	0.296		0.080	0.143	0.291
StdDev		0.102	0.180	0.288		0.111	0.205	0.331		0.134	0.237	0.386		0.128	0.233	0.387
Max		0.675	0.764	0.997		0.690	0.770	1.140		0.680	0.860	1.330		0.671	0.874	1.294
Min		-0.184	-0.332	-0.499		-0.200	-0.340	-0.490		-0.500	-0.660	-0.940		-0.288	-0.535	-0.873
Skew		2.247	0.649	0.270		1.591	0.365	-0.013		0.492	0.113	0.241		0.955	0.021	-0.219
Kurtosis		13.276	3.874	2.713		8.867	3.085	2.379		6.876	3.646	3.021		6.100	3.202	2.976
JB		2746.350	53.478	8.167		972.527	11.805	8.442		349.155	10.225	5.102		289.343	0.935	4.216
Prob.		0.000	0.000	0.017		0.000	0.003	0.015		0.000	0.006	0.078		0.000	0.627	0.121
Auto		0.913	0.961	0.970		0.925	0.954	0.967		0.926	0.962	0.971		0.897	0.947	0.965

Table 2: Principal Components Analysis of Changes in Interest Rates

This table displays the principal components output based on decomposing the sample correlation matrix of the interest rates series in each interbank market. The series FIBOR, FNIBC, SIBOR and BBGBP are for Germany, Finland, Sweden, and UK, respectively. The column 'Comp1' and 'Vector 1' corresponds to the first principal component, and so on. The row 'Eigenvalue' reports the eigenvalues of the correlation matrix in decending order. 'Variance Prop' denotes the proportional variance explained by each component. The second part of Panel A presents the eigenvectors corresponding to each eigenvalue. A linear combination of the series weighted by the first eigenvector gives the first principal component, and so on.

Panel A	FIBOR				FNIBC			
	Comp 1	Comp 2	Comp 3	Comp 4	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	3.929	0.066	0.003	0.001	3.943	0.054	0.003	0.001
Variance Prop.	0.982	0.017	0.001	0.000	0.986	0.014	0.001	0.000
Cum. Prop.	0.982	0.999	1.000	1.000	0.986	0.999	1.000	1.000
Eigenvectors	Vector 1	Vector 2	Vector 3	Vector 4	Vector 1	Vector 2	Vector 3	Vector 4
1M	-0.497	-0.636	-0.583	-0.090	-0.498	-0.640	-0.574	-0.114
3M	-0.503	-0.251	0.617	0.551	-0.502	-0.248	0.599	0.571
6M	-0.504	0.182	0.350	-0.769	-0.503	0.188	0.376	-0.755
12M	-0.496	0.707	-0.396	0.312	-0.497	0.702	-0.412	0.301
Panel B	SIBOR				UK			
	Comp 1	Comp 2	Comp 3	Comp 4	Comp 1	Comp 2	Comp 3	Comp 4
Eigenvalue	3.970	0.029	0.001	0.000	3.942	0.056	0.002	0.000
Variance Prop.	0.992	0.007	0.000	0.000	0.985	0.014	0.000	0.000
Cum. Prop.	0.992	1.000	1.000	1.000	0.985	0.999	1.000	1.000
Eigenvectors	Vector 1	Vector 2	Vector 3	Vector 4	Vector 1	Vector 2	Vector 3	Vector 4
1M	-0.499	-0.642	-0.575	-0.089	-0.498	-0.629	-0.582	-0.132
3M	-0.501	-0.233	0.607	0.571	-0.503	-0.245	0.555	0.616
6M	-0.502	0.166	0.369	-0.765	-0.503	0.156	0.428	-0.734
12M	-0.498	0.711	-0.406	0.285	-0.496	0.721	-0.412	0.253

Table 3: Estimated Parameters for the Two-factor CIR Models

This table presents the estimated parameters for state-space form of the two-factor CIR model using interbank interest rates. Kalman filter and quasi maximum likelihood method are used to estimate the parameters. The results in Panel A, B, C, and D are respectively for German, Finland, Sweden, and UK interbank market. The parameters θ_1, k_1, σ_1 and λ_1 denote respectively the estimated mean, mean-reversion, volatility and market price of risk parameter for the first factor, and so on. The estimated volatilities σ_{h1}, σ_{h2}, σ_{h3}, and σ_{h4} are for measurement errors of maturities 1-month, 3-month, 6-month, and 12-month. The row 'Half Life' reports the average time in years it takes for the factors revert to its long-run mean. And 'Long Yield' is the theorectical yield when the maturity approaches infinity. The parameters are in annual terms.

Panel A. FIBOR

Parameters	θ_1	k_1	θ_2	k_2	σ_1	σ_2	λ_1	λ_2	σ_{h1}	σ_{h2}	σ_{h3}	σ_{h4}
Estimates	1.255	0.102	1.799	0.848	0.216	0.200	-0.521	0.423	0.061	0.000	0.000	0.000
SE	(0.376)	(0.046)	(0.121)	(0.000)	(0.010)	(0.009)	(0.049)	(0.047)	(0.003)	(0.000)	(0.001)	(0.002)
Log Likelihood	2834.620											
Half Life	Factor1 =	6.776	Factor2 =	0.817								
Long Yield	Factor1 =	0.971	Factor2 =	0.718								

Panel B. FNIBC

Parameters	θ_1	k_1	θ_2	k_2	σ_1	σ_2	λ_1	λ_2	σ_{h1}	σ_{h2}	σ_{h3}	σ_{h4}
Estimates	1.284	0.157	1.560	1.147	0.225	0.261	-0.500	0.379	0.064	0.000	0.000	0.000
SE	(0.237)	(0.039)	(0.102)	(0.000)	(0.009)	(0.011)	(0.037)	(0.056)	(0.004)	(0.000)	(0.000)	(0.000)
Log Likelihood	2631.595											
Half Life	Factor1 =	4.415	Factor2 =	0.604								
Long Yield	Factor1 =	1.001	Factor2 =	0.648								

Panel C. SIBOR

Parameters	θ_1	k_1	θ_2	k_2	σ_1	σ_2	λ_1	λ_2	σ_{h1}	σ_{h2}	σ_{h3}	σ_{h4}
Estimates	1.688	0.147	1.655	0.375	0.188	0.223	-0.570	0.805	0.071	0.000	0.000	0.000
SE	(0.279)	(0.036)	(0.167)	(0.077)	(0.006)	(0.012)	(0.035)	(0.109)	(0.004)	(0.000)	(0.001)	(0.001)
Log Likelihood	2597.460											
Half Life	Factor1 =	4.709	Factor2 =	1.850								
Long Yield	Factor1 =	0.982	Factor2 =	0.742								

Panel D. UKINB

Parameters	θ_1	k_1	θ_2	k_2	σ_1	σ_2	λ_1	λ_2	σ_{h1}	σ_{h2}	σ_{h3}	σ_{h4}
Estimates	0.027	0.011	1.889	0.749	0.165	0.267	-0.282	0.483	0.085	0.000	0.000	0.023
SE	(0.000)	(0.010)	(0.204)	(0.058)	(0.007)	(0.017)	(0.019)	(0.079)	(0.004)	(0.000)	(0.000)	(0.006)
Log Likelihood	2325.119											
Half Life	Factor1 =	63.013	Factor2 =	0.925								
Long Yield	Factor1 =	0.895	Factor2 =	0.722								

Table 4: Variance Decomposition of the Interbank Rates

The entries are proportions of the forecast variances of the interbank rates explained by the factors at the forecast horizon (weeks ahead).

	Weeks	Factor1	Factor2		Weeks	Factor1	Factor2
FIBOR3M	4	35.41	64.56	FNIBC3M	4	46.02	53.82
	12	16.26	83.73		12	31.66	68.14
	20	7.92	92.06		20	20.75	79.07
FIBOR6M	4	46.35	53.65	FNIBC6M	4	50.03	49.42
	12	30.61	69.37		12	41.69	57.61
	20	19.97	80.00		20	31.67	67.56
FIBOR12M	4	30.70	69.07	FNIBC12M	4	39.25	59.87
	12	22.79	76.99		12	35.07	63.93
	20	16.28	83.52		20	28.65	70.37
BBGBP3M	4	30.93	67.42	SIBOR3M	4	48.88	50.21
	12	13.70	84.25		12	30.61	68.40
	20	6.30	91.43		20	20.07	79.12
BBGBP6M	4	38.20	60.99	SIBOR6M	4	49.32	50.19
	12	26.30	72.65		12	35.92	63.68
	20	18.77	80.15		20	26.49	73.23
BBGBP12M	4	31.91	67.97	SIBOR12M	4	43.65	56.34
	12	25.99	73.90		12	33.73	66.26
	20	22.09	77.85		20	27.34	72.65

Table 5: Correlation Matrix of the Forecast Errors with the Curvature Series

This table provides the common sample correlations between the curvature factor and the forecast errors of different maturities, as well as the cross-sectional correlations among the forecast errors. The forecast errors are the extracted Kalman filter prediction errors, they are denoted as $\hat{\varepsilon}_{1,t}$, $\hat{\varepsilon}_{3,t}$, $\hat{\varepsilon}_{6,t}$, and $\hat{\varepsilon}_{12,t}$ for maturities 1 month, 3 month, 6 month and 12 month, respectively. The curvature series (Curv) is proxied by the difference between two times the six-month rate and the sum of twelve- and one-month rates.

Panel A	FIBOR					FNIBC				
	Curv	$\hat{\varepsilon}_{1,t}$	$\hat{\varepsilon}_{3,t}$	$\hat{\varepsilon}_{6,t}$	$\hat{\varepsilon}_{12,t}$	Curv	$\hat{\varepsilon}_{1,t}$	$\hat{\varepsilon}_{3,t}$	$\hat{\varepsilon}_{6,t}$	$\hat{\varepsilon}_{12,t}$
Curv	1.000					1.000				
$\hat{\varepsilon}_{1,t}$	-0.392	1.000				-0.350	1.000			
$\hat{\varepsilon}_{3,t}$	0.519	0.142	1.000			0.500	0.210	1.000		
$\hat{\varepsilon}_{6,t}$	0.333	-0.281	-0.116	1.000		0.369	-0.123	0.094	1.000	
$\hat{\varepsilon}_{12,t}$	-0.444	-0.308	-0.428	0.064	1.000	-0.309	-0.252	-0.366	0.170	1.000

Panel B	SIBOR					UK				
	Curv	$\hat{\varepsilon}_{1,t}$	$\hat{\varepsilon}_{3,t}$	$\hat{\varepsilon}_{6,t}$	$\hat{\varepsilon}_{12,t}$	Curv	$\hat{\varepsilon}_{1,t}$	$\hat{\varepsilon}_{3,t}$	$\hat{\varepsilon}_{6,t}$	$\hat{\varepsilon}_{12,t}$
Curv	1.000					1.000				
$\hat{\varepsilon}_{1,t}$	-0.246	1.000				-0.141	1.000			
$\hat{\varepsilon}_{3,t}$	0.500	0.273	1.000			0.490	0.428	1.000		
$\hat{\varepsilon}_{6,t}$	0.206	0.138	0.227	1.000		0.304	0.148	0.340	1.000	
$\hat{\varepsilon}_{12,t}$	-0.389	0.112	-0.216	0.171	1.000	-0.274	0.095	-0.031	0.525	1.000

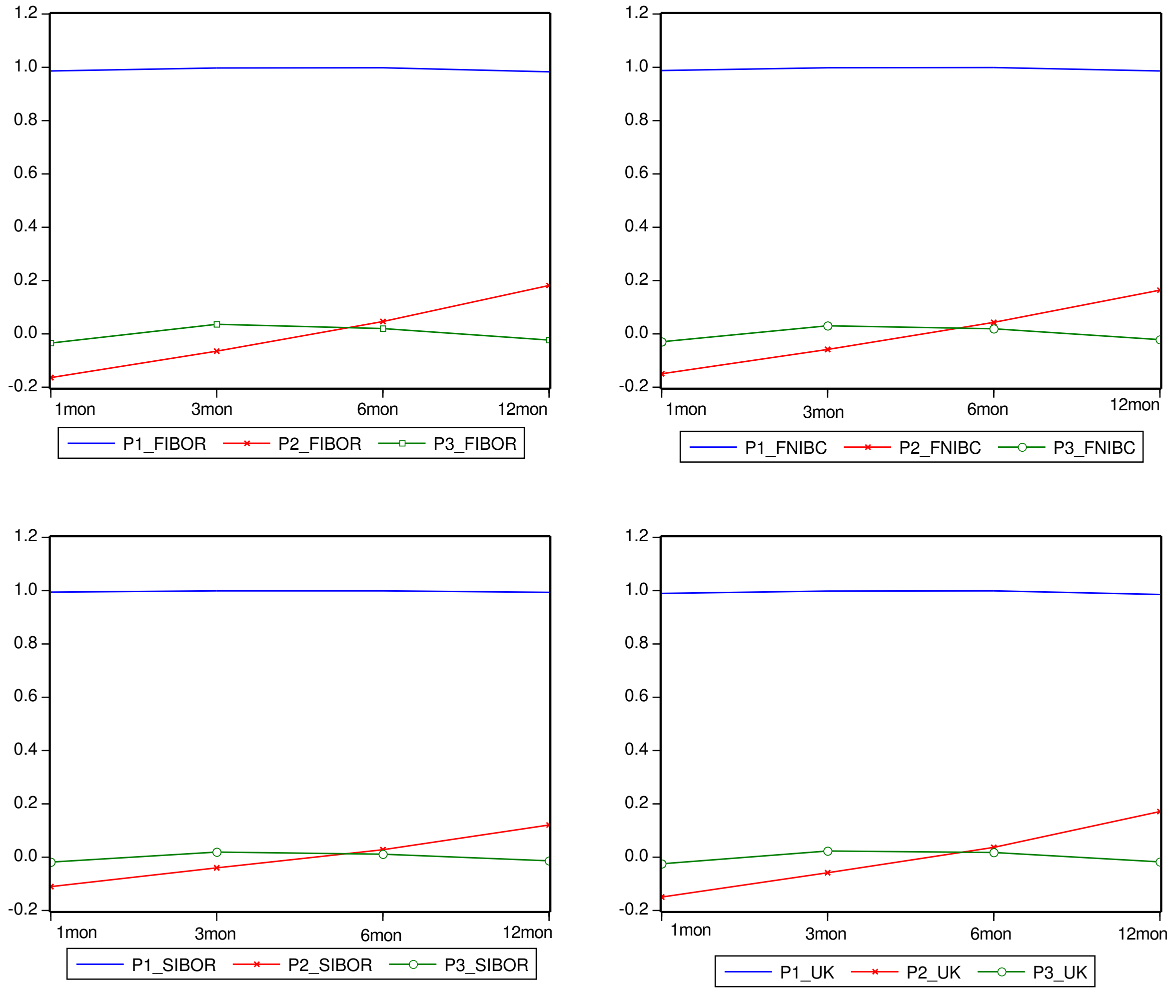

Figure 1: Factor Loadings from the Principal Components Analysis
The loadings are obtained from the regression of yield series on the first three principal components of the correlation matrix, with P1_, P2_, and P3_denote respectively loadings on the first, second, and third principal components, and the series ending with _FIBOR, _FNIBC, _SIBOR, and _UK are respectively for Germany, Finland, Sweden, and UK interbank markets.

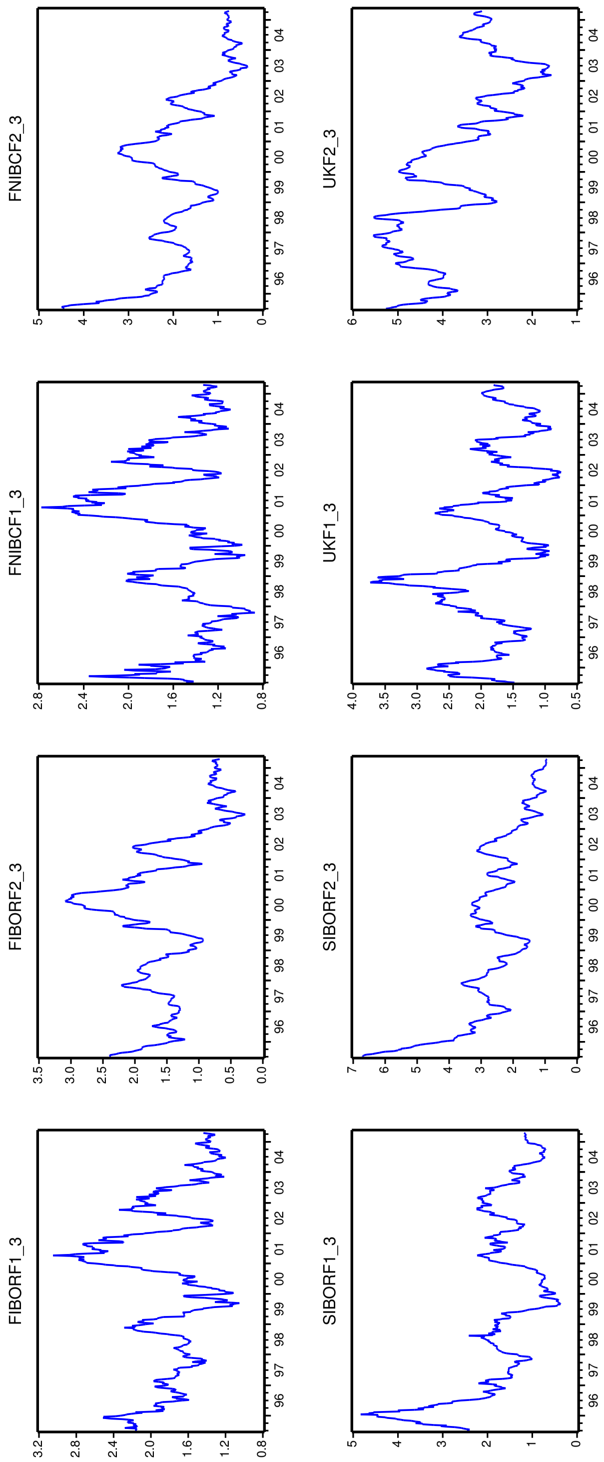

Figure 2: Extracted Factors for the Two-factor CIR Model
Note that the scales for the factors are different, they are presented in a manner more in line with each other, since in this case, the dynamics of the factors are more important than the scale.

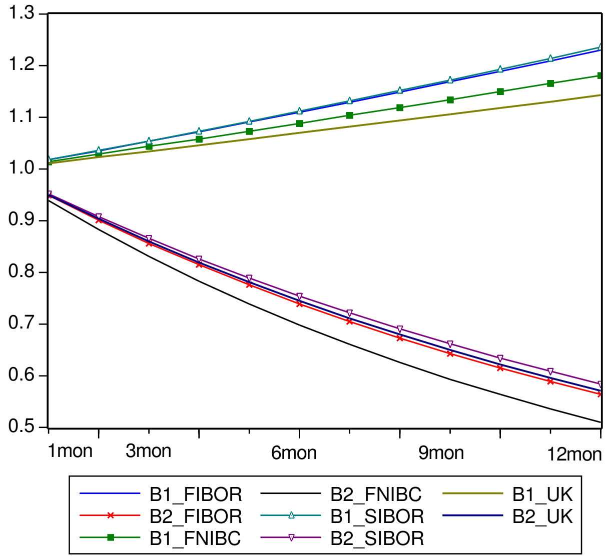

Figure 3: Factor Loadings from the Two-factor CIR Model
This figure shows the theorectical loadings of the two factors in the CIR model. The loadings are calculated using equations (4) and (5), with the estimated parameters in Table 3. The upward-sloping lines are the loadings on the first factors, and the downward-sloping lines are for the second factor.

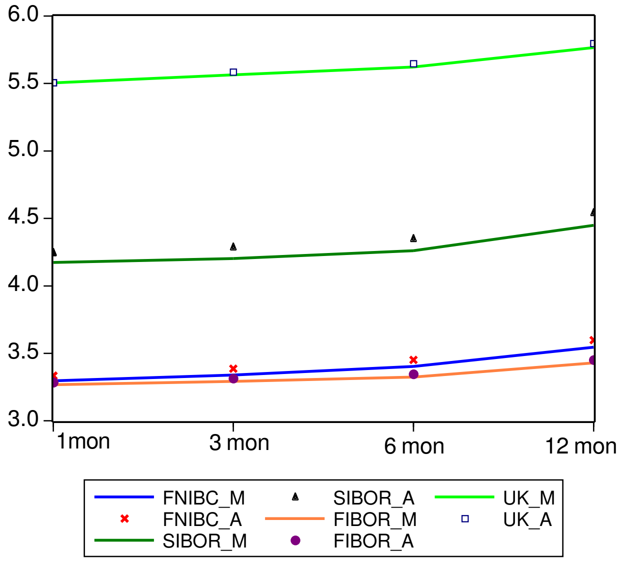

Figure 4: Actual and Fitted Term Structures
The actual yield are marked by symbols, and the model yields are denoted by lines only.

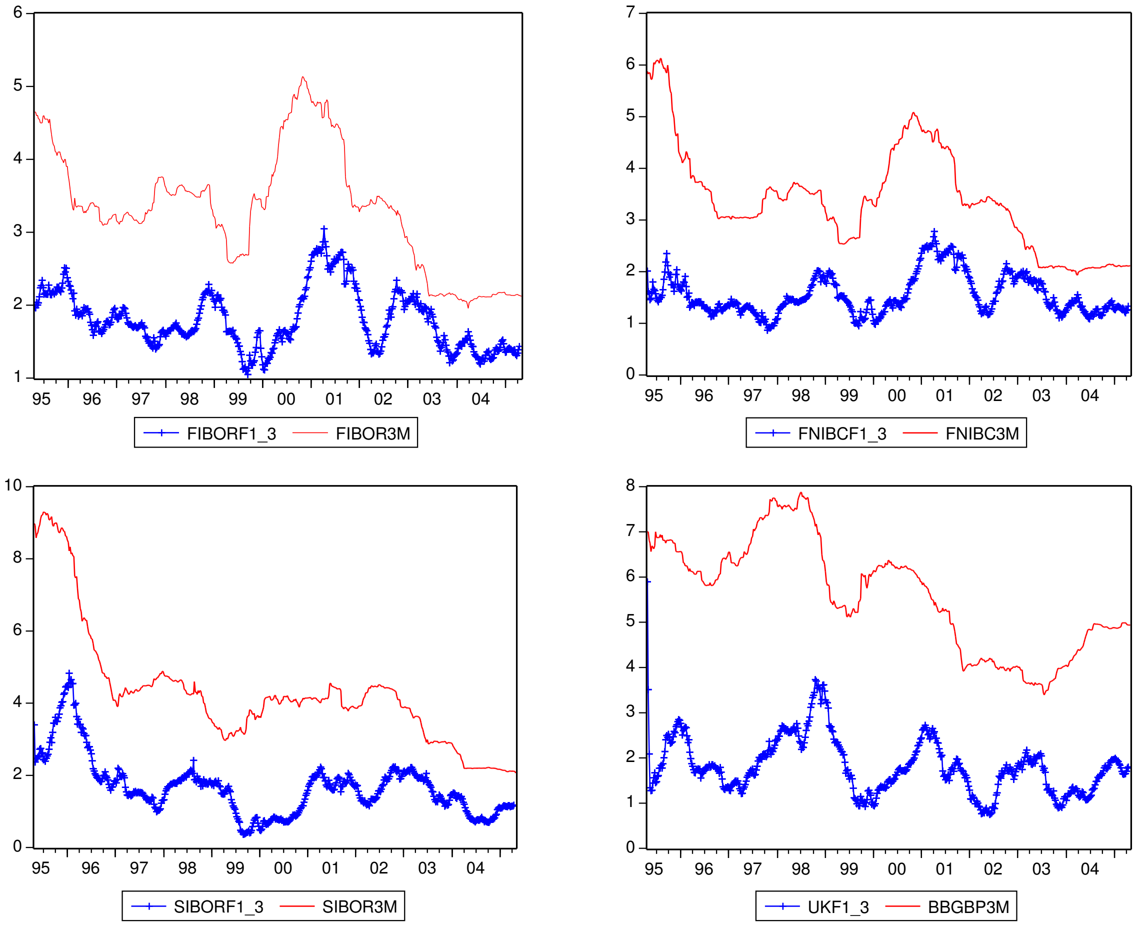

Figure 5: Extracted Factor 1 and Interbank Interest Rates for 3-month Maturity
The marked series are the first factors extracted from the state space formulation of the CIR process, and the plain series are the actual 3-month interbank rates.

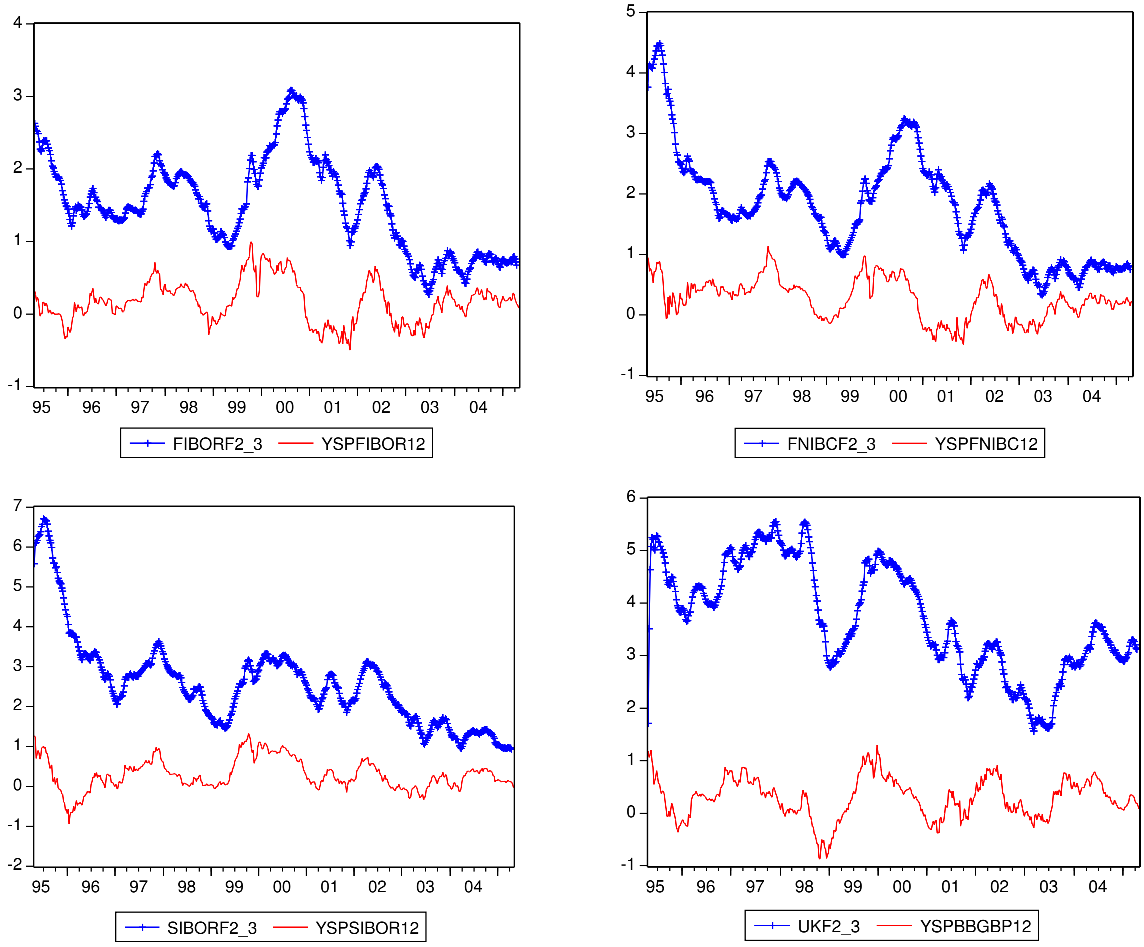

Figure 6: Extracted Factor 2 and Yield Spread
The marked series are the second factors extracted from the state space formulation of the CIR process, and the plain series are the spread of 12-month over 1-month interbank rates.

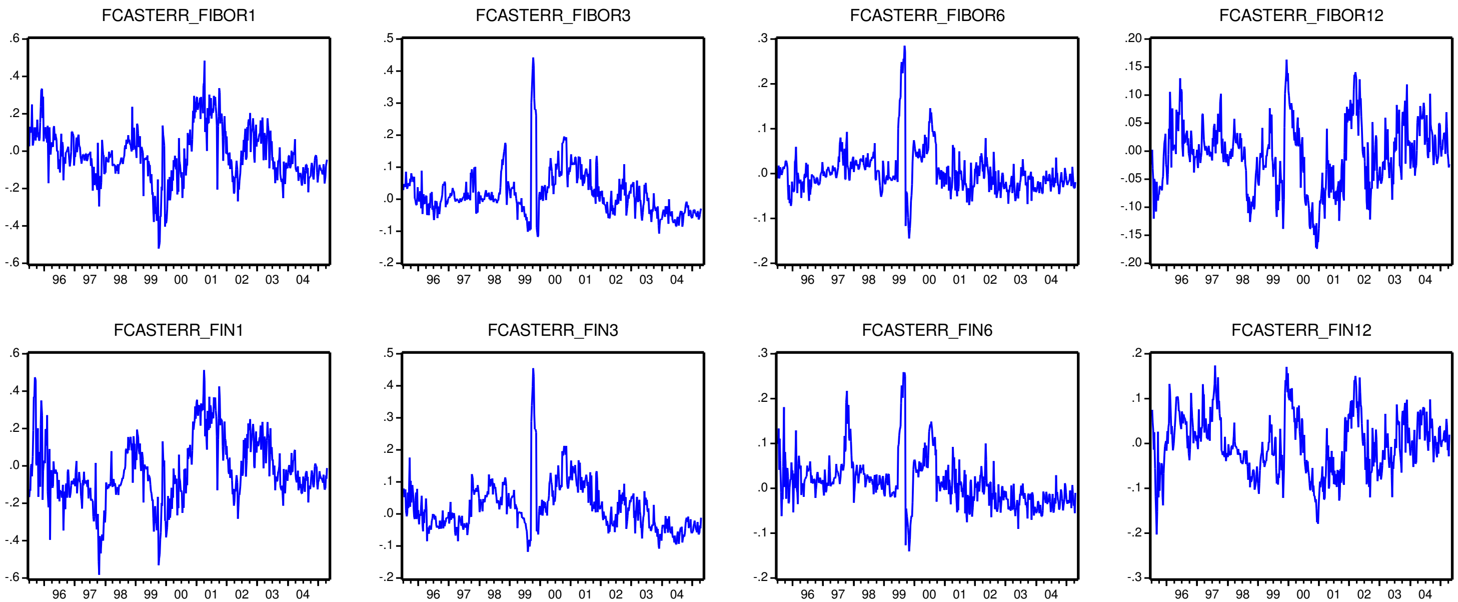

Figure 7: Forecast Errors for FIBOR and FNIBC

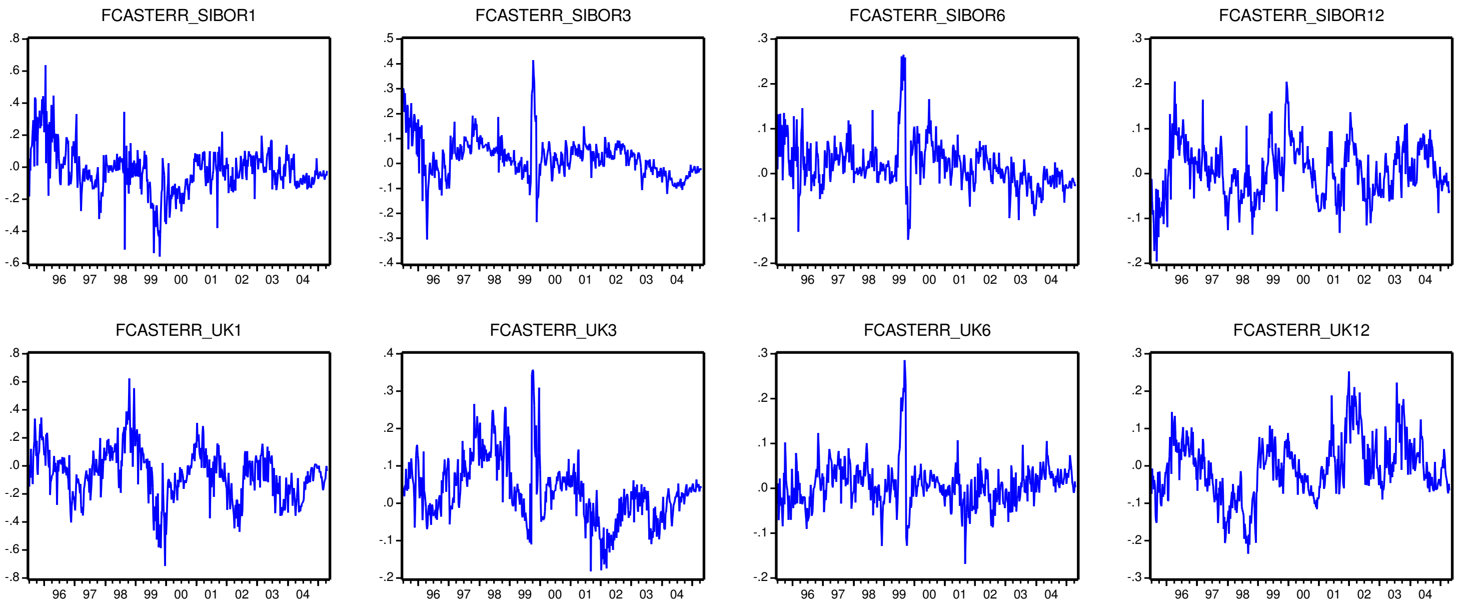

Figure 8: Forecast Errors for SIBOR and UK

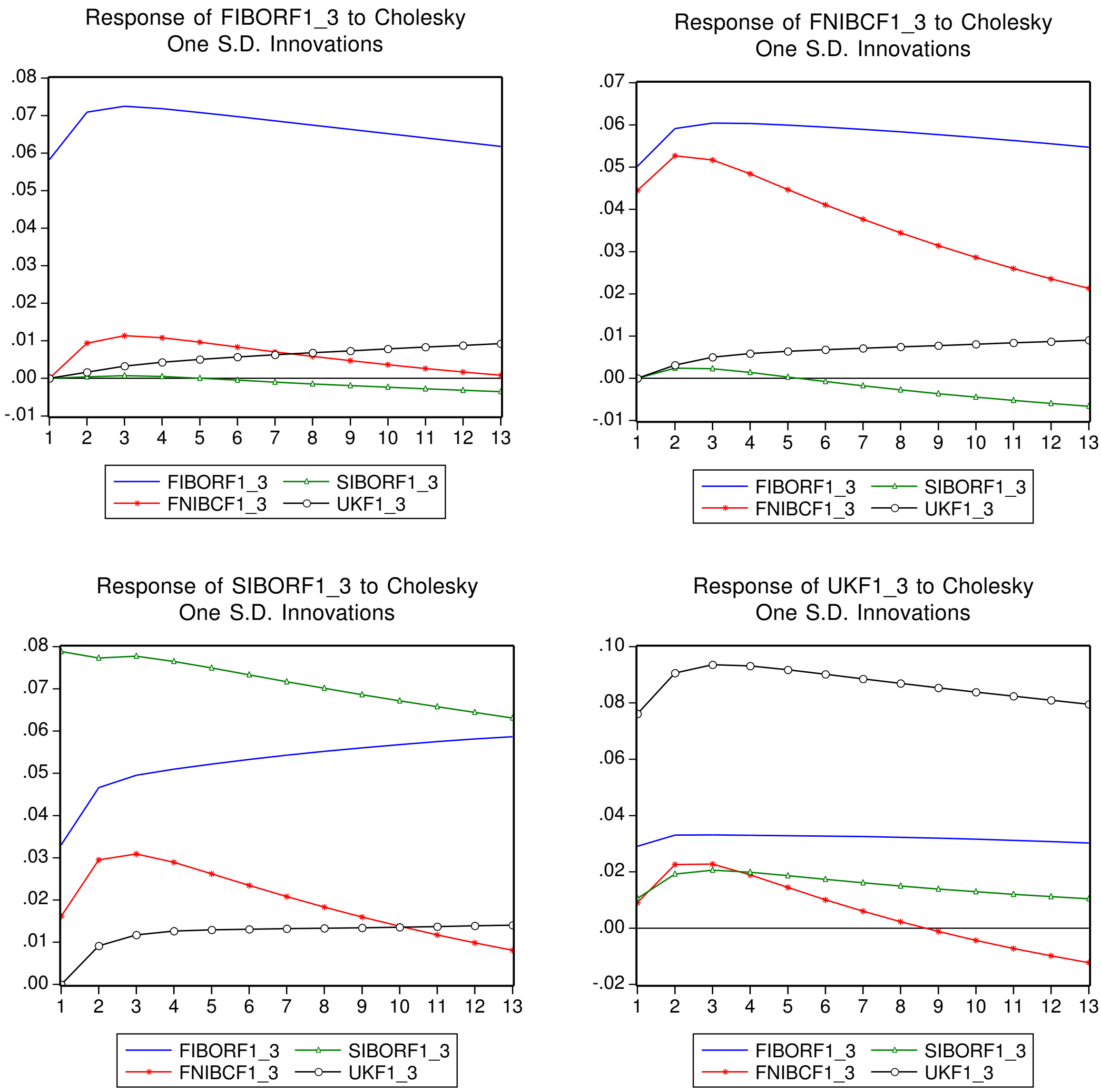

Figure 9: Impulse Responses of Factor 1 for 3-month Interest Rates

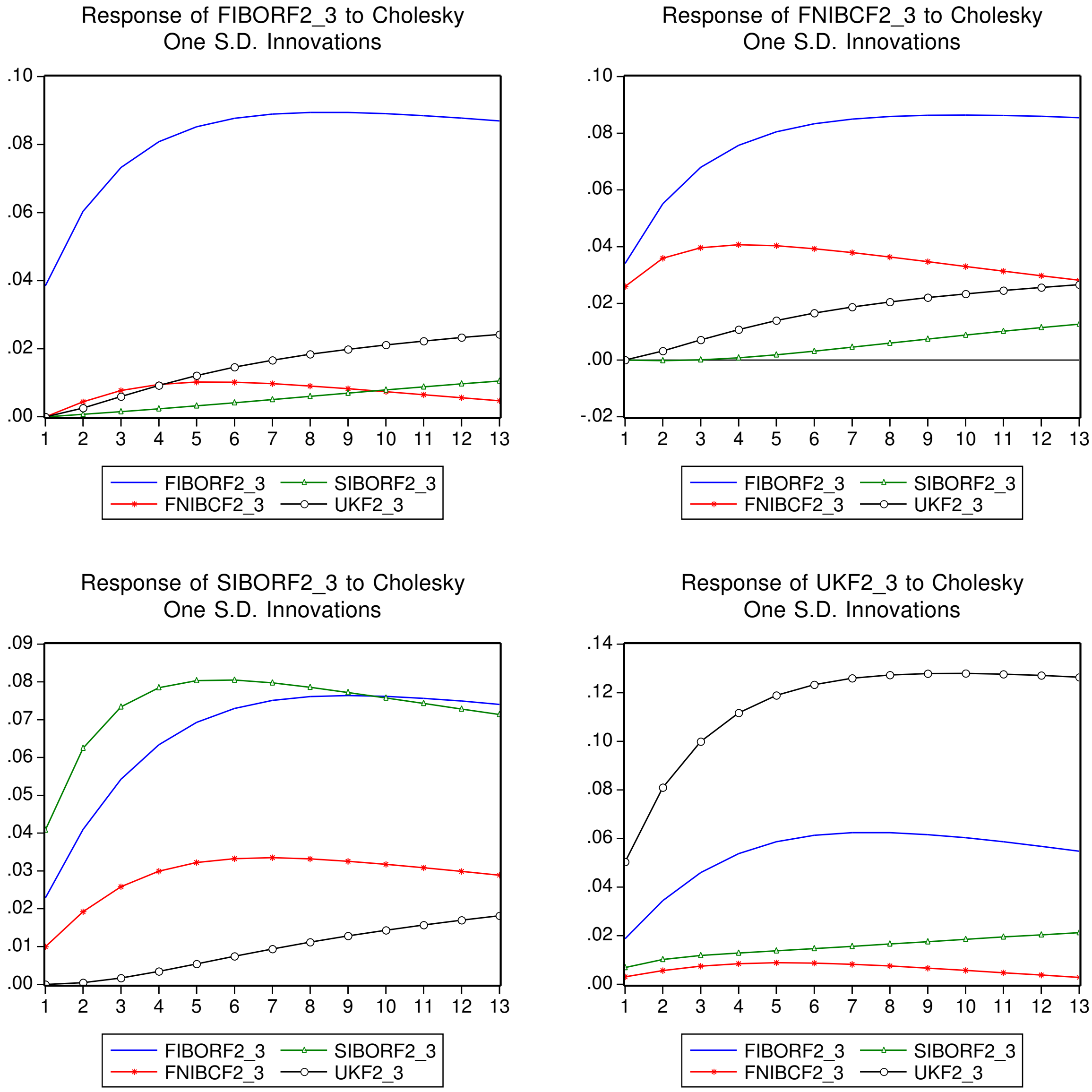

Figure 10: Impulse Responses of Factor 2 for 3-month Interest Rates

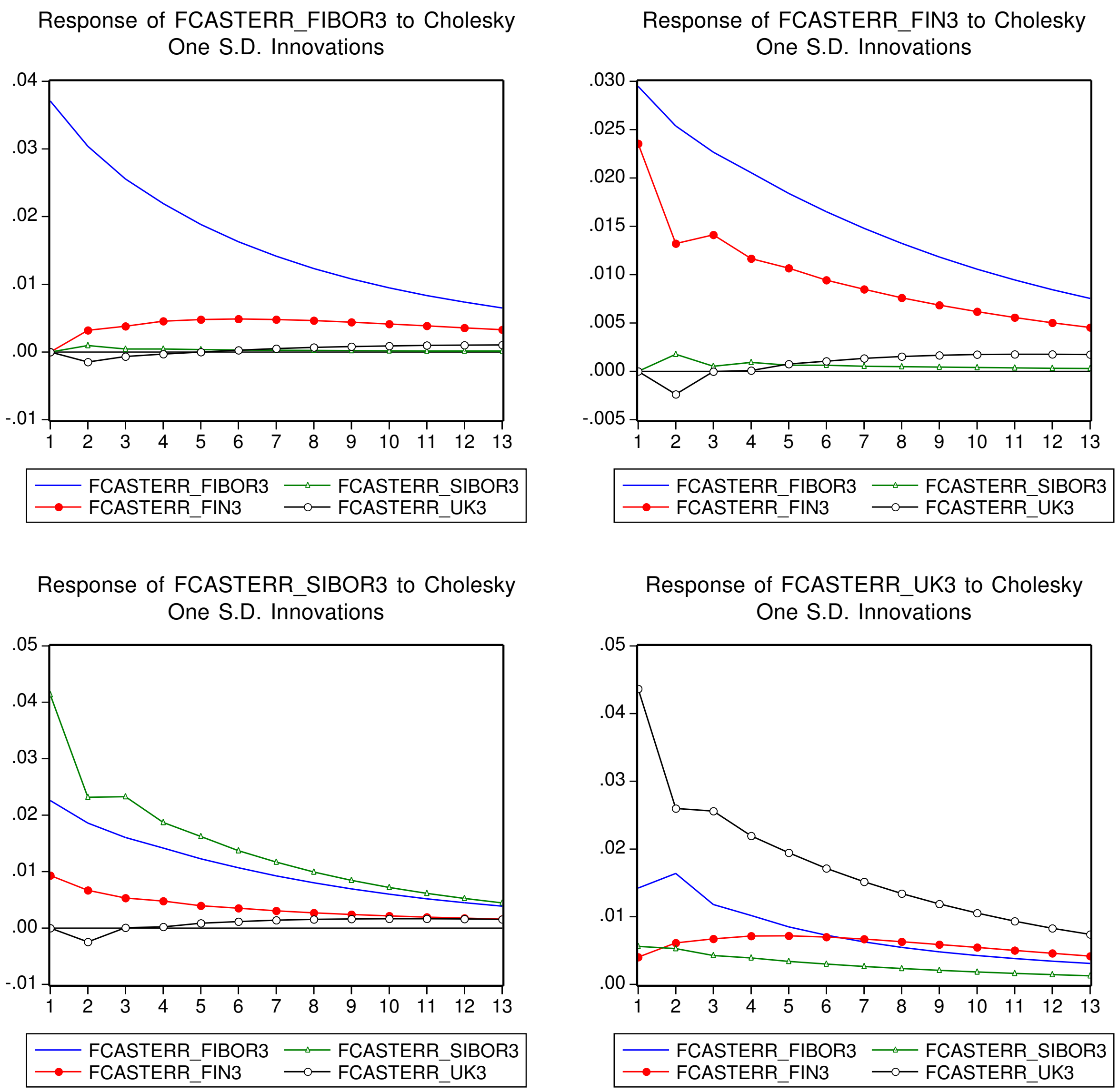

Figure 11: Impulse Responses of Forecast Errors for 3-month Interest Rates

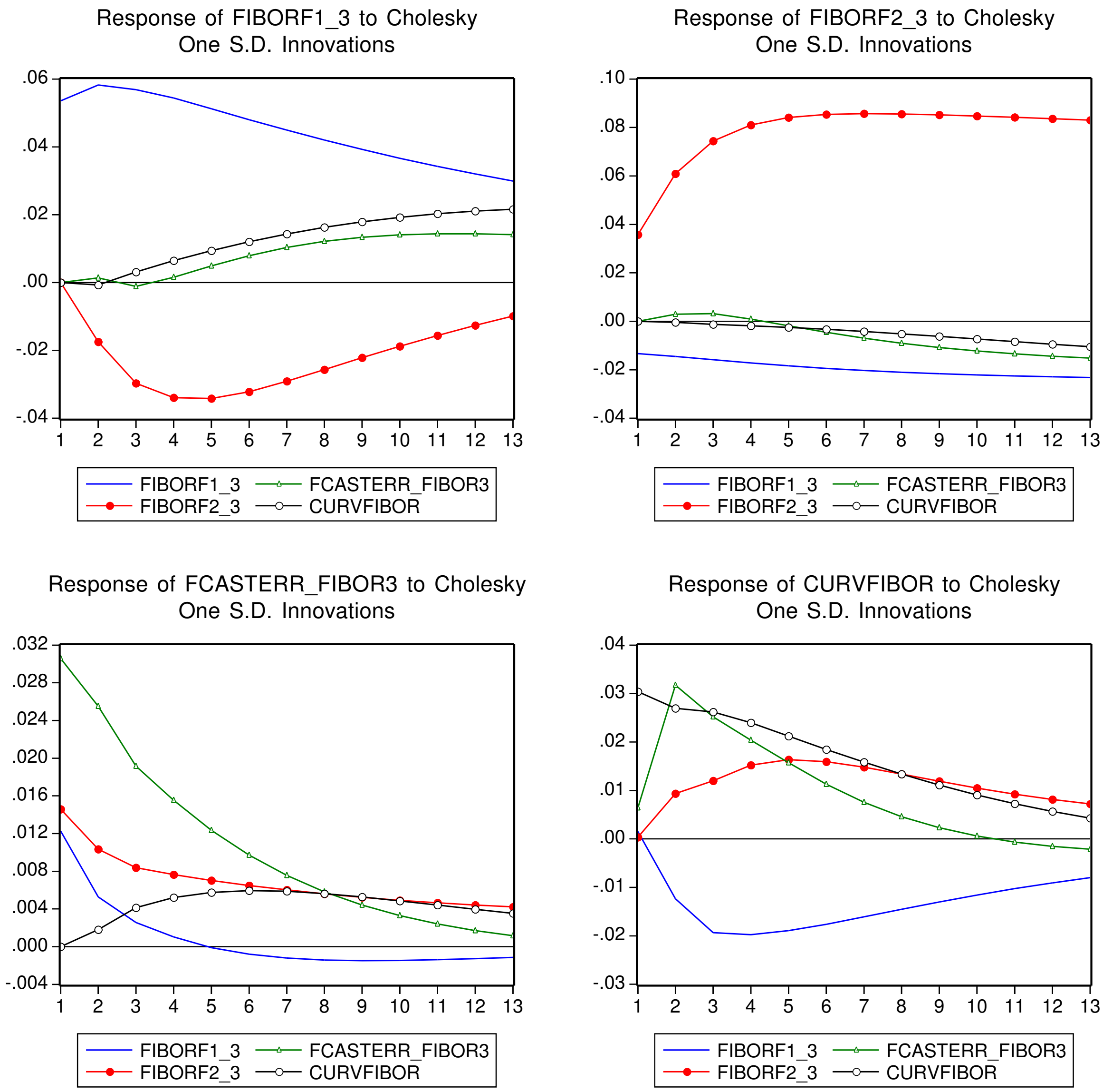

Figure 12: Impulse Responses of the Estimated Factors, Forecast Errors, and Curvature for 3-month FIBOR

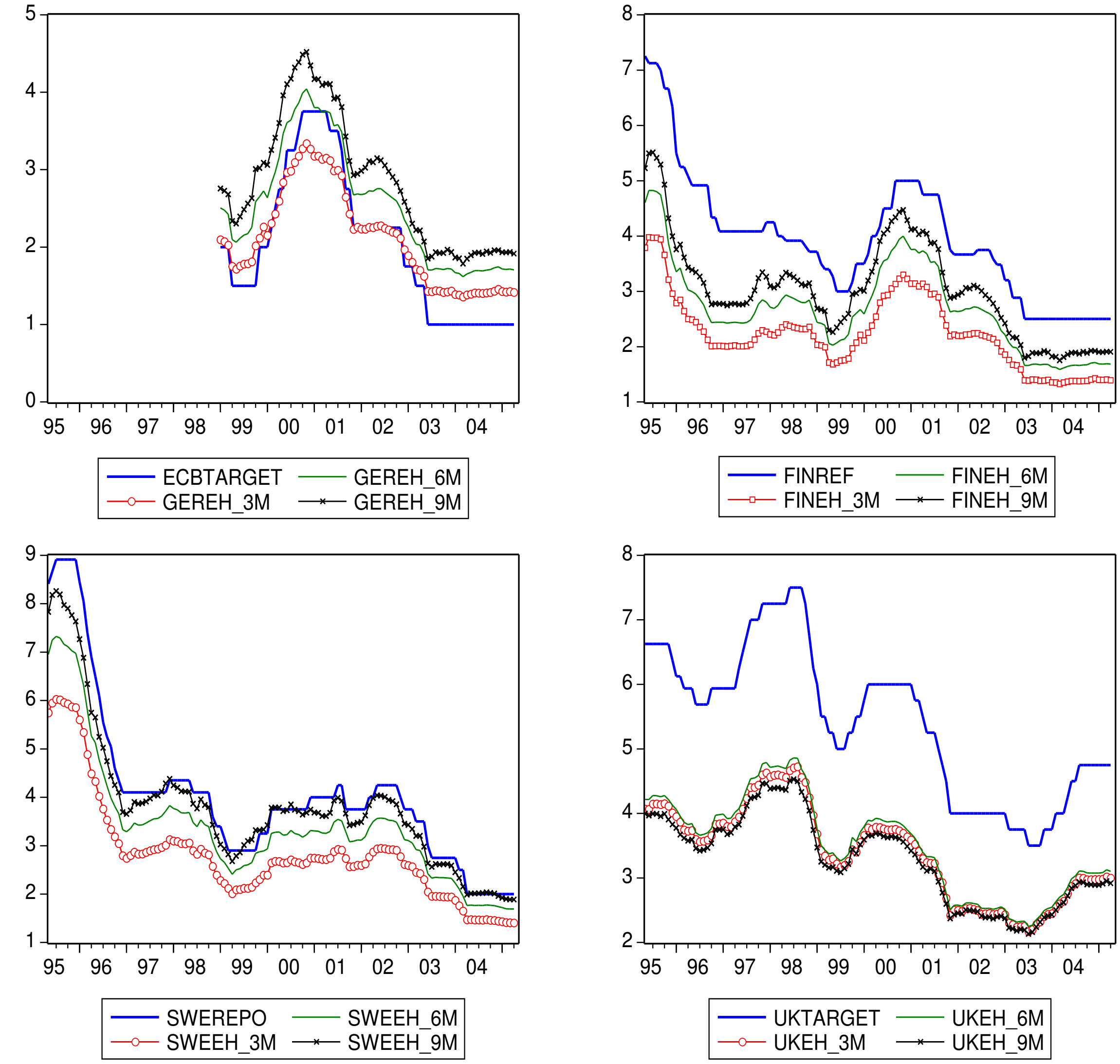

Figure 13: Policy Rates and Theoretical Bond Yields under Rational Expectations Hypothesis

The policy rates are respectively ECB target rates, average bank reference rates, repo rates and Bank of England target rates for Germany, Finland, Sweden and UK. The theoretical bond yields are derived under the assumption that Rational Expectations Hypothesis holds. The graph for Germany on the upper left starts from 1998, when the ECB target rates began to be available.