Hidden Limit Orders and Liquidity in Limit Order Markets.¹

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Abstract

Many limit order markets allow limit order traders to submit “hidden” orders (also called “iceberg” or “undisclosed” orders). Liquidity suppliers thus have the possibility to post a quote and either display none or only a fraction of their order’s quantity to the market. Some recent empirical studies show that such orders represent a large proportion of the market liquidity. Building on the assumption that informed agents may act as liquidity suppliers, we analyze the hidden orders’ submission strategies of informed and uninformed limit order traders. We first show that the informed liquidity supplier adopts a “camouflage” equilibrium strategy. She tries to mimic the behaviour of an uninformed liquidity supplier, by replicating its limit order submission’s strategy. This prevents the liquidity demander from detecting her presence. Since this uninformed liquidity demander takes into account the signal revealed by the depth supplied in the limit order book, the submission of a hidden order may indeed enable the informed liquidity supplier to post a large limit order without decreasing the probability of execution of this order. Our model enables us to propose predictions that may guide an empirical analysis of an opaque limit order book. In particular, it leads to new hypothesis on the probability of informed trading (“PIN”), on the informational content of the limit order book or on the frequency at which hidden depth is supplied at the best quotes. Finally, we compare the quality of a transparent and an opaque order-driven markets. We show that the introduction of hidden orders improves efficiency. Their impact on liquidity and on the agents’ welfare is however ambiguous. Counter-intuitively, informed agents may be better off in a transparent market.

Keywords: Limit Order Book, Transparency, Hidden orders, Informed limit order trading.
1 Introduction

The recent development of electronic trading systems has drawn attention to “order driven markets”. In those markets, liquidity is provided by investors submitting limit orders. These orders queue in the limit order book, and are then matched and executed against market orders submitted by other investors, who act as liquidity demanders. Regulators of those markets have adopted heterogenous rules regarding the disclosure of information concerning the full limit order book. In order to facilitate liquidity demand, most of these systems provide to their members some information on the characteristics of at least the five best limits on each side of the limit order book. However, some markets allow agents to submit “hidden” limit orders (also called “iceberg” or “undisclosed” orders). Liquidity suppliers thus have the possibility to post a quote and either display only a fraction of their order to the market (as in Euronext for instance), or fully hide the quantity of their limit order and display the price only (as in the Australian Stock Exchange). This raises the question of knowing who gains and looses from the authorization of hidden orders. Providing an answer to this question would help institutions and regulators in designing the optimal limit order market organization.

The submission of hidden limit orders has long been neglected, since such an opportunity was considered as a simple tool used to automate splitting orders’ strategies. In a continuous setting, Esser and Moench (2005) determine the limit price and the optimal size of a hidden order for a static liquidation strategy. To our knowledge, there exists no other theoretical paper on hidden orders. Recent empirical studies have however highlighted the importance of hidden depth in limit order markets. Since researchers have had access to proprietary data from exchanges that have enabled them to reconstitute the full limit order book, including its hidden part, they have indeed shown that hidden orders represent a large proportion of the market liquidity. Hasbrouck and Saar (2002) report that hidden orders account for 12% of all order executions in Island. On the Nasdaq, Tuttle (2002) finds that hidden liquidity represents 20% of the inside depth in Nasdaq 100 stocks. Even more strikingly, in Euronext Paris, the hidden depth accounts for 45% of the total depth available at the best five quotes, and 55% of the total depth at the best limit according to D’Hondt, De Winne and François-Heude (2004). Counter-intuitively, the authors also show that hidden

1“Order-driven markets” (like Euronext or Electronic Communication Networks such as Inet ATS) differ from the more traditional “quote-driven markets” (like the NYSE or the LSE) where dealers first set their quotes, and then investors (brokers) willing to trade the security submit an order at the best price.
orders are themselves managed with attention: they are often split and cancelled. This suggests that there is more at stake with hidden orders than the only automation of liquidation strategies.

Intuitively, it seems that hidden orders would reduce the exposure of liquidity suppliers to the adverse selection risk. Aitken et al. (2001) suggest that this risk would be less severe for undisclosed than for disclosed limit orders, “since picking off is more complicated than simply hitting the stale limit order for a known total number of disclosed shares”. Their empirical findings, as those of Pardo and Pascual (2003), provide support for this economic intuition. Besides, Harris (1998) argues that hidden orders may be used by large traders as a defensive strategy against quote-matchers, by limiting the price impact of their order, and thus their execution cost. By providing to limit order traders a trade-off between liquidity and transparency, that would allow them to reduce their risk exposure, hidden orders would thus encourage liquidity provision. A natural experiment seems to confirm the intuition that decreasing the degree of transparency on the quantities supplied in the limit order book would improve market liquidity. Anand and Weaver (2004) analyze the reintroduction of hidden orders in the Toronto Stock Exchange in 2002, and show that it increased the total depth of the limit order book. Both intuitions rely on the idea that hidden orders may influence the cost of liquidity provision through their impact on the informational content of the limit order book. Market participants in opaque markets, which authorize hidden orders, are indeed unable to observe the total depth supplied at the best quotes of the limit order book. In our paper, we build on this reasoning to propose a signaling model, based on the idea that hidden orders introduce some opacity in the limit order book’s information disclosure.

Different recent studies, either empirical (Kaniel and Liu (2005), Aitken et al. (2001) or Anand et al. (2005)) or theoretical (Goettler, Parlour and Rajan (2005)) show (differering from the traditional paradigm) that speculators, who possess some private information on the true value of the security, do not only submit market orders that would be executed against the best limit orders standing in the limit order book, but may themselves supply liquidity by submitting limit orders. According to Kaniel and Liu (2005), such a behavior may indeed increase their expected profits thanks to a better execution price, when their information is long-lived. Does the introduction of some opacity in the limit order book still improve market liquidity, as suggested by the intuitions described above?

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In order to investigate this issue, we propose a theoretical model of trading in a limit order book, where a limit order trader possesses a private information on the realization of the value of the security. Limit order traders may submit small or large orders. In the opaque market, they may also submit a large order, but only display a fraction of it to the market, so that participants observe a visible depth that is inferior to the total depth of the order book, since the hidden depth is undisclosed.

The informed limit order trader’s transaction volume depends on her limit order’s size, and on the size of the market order submitted by the liquidity demander. Since the later is uninformed, but rational and Bayesian, his demand is conditional on the state of the limit order book. When he observes the depth at the best quotes, he updates his beliefs on the value of the security by taking into account two elements. First, the direction of the limit order he observes is informative. A sell order signals him that the security is likely to be overvalued. The impact of this signal on the uninformed agent’s beliefs depends on the probability of facing an informed limit order trader. Second, the size of the limit order allows him to better assess his adverse selection risk, since he takes into account the fact that an informed agent is induced to submit a large limit order to increase her expected profit.

Consequently, the informed agent’s limit order submission strategy plays a crucial role. When the informed liquidity supplier submits a limit order, she faces a trade-off. On the one hand, she is induced to submit a large order, since she would voluntarily restrict her transaction volume by submitting a small order. But on the other hand, displaying a large order would impact the beliefs of the uninformed liquidity demander more strongly, which may decrease the probability of execution of her limit order. The authorization of hidden orders could enable an informed liquidity supplier to overcome this trade-off.

We first show that the informed liquidity supplier adopts a “camouflage” equilibrium strategy. She tries to mimic the behaviour of an uninformed liquidity supplier, by replicating its limit order submission’s strategy. This prevents the liquidity demander from detecting her presence. Since the uninformed agent takes into account the signal revealed by the depth supplied in the limit order book, the submission of a hidden order may indeed enable the informed liquidity supplier to post a large limit order without decreasing the probability of execution of this order.

However, the use of hidden orders is not systematic for the informed agent. When the probability of an information event is extreme, then the reaction of the liquidity demander does not, in equilibrium, depend on the observation of the signal contained in depth of the limit order book. Whatever the size of the limit
order submitted and displayed by the informed liquidity supplier, the adverse selection risk faced by the
uninformed liquidity demander is either too low to prevent him from submitting a large order, or too high
to induce him to submit an order. Consequently, it is not necessary, for the informed agent, to submit a
hidden order, even if she has such an opportunity.

Our model thus enables us to propose predictions that may guide an empirical analysis of an opaque
limit order book. In particular, it leads to new hypothesis on the probability of informed trading (“PIN”),
on the informational content of the limit order book or on the frequency at which hidden depth is supplied
at the best quotes.

Finally, we compare market efficiency and the welfare of agents in a transparent market and in an
opaque market. In practice, there exists a typology of orders according to their aggressivity (cf. Biais,
Hillion and Spatt (1995)). The most aggressive orders are supposed to be submitted by liquidity demanders,
while the least aggressive orders would be submitted by liquidity suppliers. Two types of orders are usually
said to consume liquidity, namely the marketable limit orders, and the market orders. We show that the
relative quality of an opaque market, as compared to a transparent market, may rely on the type of orders
authorized by market regulators.

Whatever the market organization, efficiency is stronger in the opaque market. On the one hand,
when marketable limit orders are authorized, the total depth is smaller in the transparent market. In the
opaque market, the informed liquidity supplier is induced to submit large limit orders (hidden or fully
disclosed) more often than in the transparent, market, since she finds it easier not to reveal her presence.
The informational content of the limit order book is thus larger. On the other hand, when marketable limit
orders are not authorized, a larger visible depth in the transparent market enables the informed liquidity
supplier to better implement her camouflage equilibrium strategy, so that market efficiency is also stronger
in the opaque market.

The improvement of market efficiency does however not leads to lower trading costs for the uninformed
liquidity demander in the opaque market. We show that when marketable limit orders are authorized, the
increase of the adverse selection risk faced by the liquidity demander overcomes his benefits linked with an
increase in the trading volume. When marketable limit orders are not authorized, a potential decrease in
the total depth in the transparent market does not impact the liquidity demander’s transaction costs, since
it is only observed for adverse selection’s degrees that are so high that it deters this agent from submitting
an order. Consequently, his expected profits are also higher in the transparent market.

Counter-intuitively, the informed liquidity supplier may also be better off in the transparent market. Admittedly, when marketable limit orders are authorized, the liquidity suppliers’ expected profits are higher in the opaque market, since whatever the adverse selection, exchanges between market participants are maximal in this market. When these orders are not authorized however, the informed liquidity supplier beneficiates from a higher probability of execution in the transparent market. The increase in the visible depth in this market indeed enables her to implement a more efficient camouflage strategy, which compensates in this case the absence of hidden orders. Finally, the authorization of hidden orders does not always enable the informed agent to increase her expected profits: their use is simply part of her camouflage strategy in the opaque market.

The paper is organized as follows. Section 2 introduces the theoretical model. In Section 3, we describe the equilibrium’s strategies in a partial equilibrium. Section 4 derives some comparative statics, which are used to reconcile our theoretical findings with the empirical literature, and to draw some new empirical predictions of our model. In Section 5, we finally present the equilibrium, and we study the impact of transparency on market quality. Section 6 concludes. The proofs that do not appear in the text are collected in the appendix.

2 The model

In this section, we introduce a model of trading in a limit order book, in a market for a single risky security.

2.1 Timing and market structure

Risky asset - The liquidation value of the security is a random variable \( \tilde{v} \). The realization of the random variable becomes publicly known at the end of the trading session. For simplicity, we assume that its final value can take two values with equal probability \( \frac{1}{2} \), \((v_H, v_L)\) with \(v_H = v_0 + \sigma\) and \(v_L = v_0 - \sigma\), where \(v_0\) denotes the unconditional expected value of the asset, i.e. \(v_0 = E(\tilde{v})\). The distribution of the liquidation value of the security is common knowledge to all market participants.
**Timing** - Trading occurs in a sequential game with three periods. At date 0, limit orders to buy or to sell the asset stand in the initial limit order book, waiting for execution. At date 1, one agent submits a limit order to buy or sell one or two units of the risky asset. At date 2, after having observed the depth at the best quotes in the order book, one agent may submit an order to buy or sell one or two units of the security, which is immediately executed against the opposite order(s) standing in the limit order book. Price then time priority are enforced. At date 3, the game ends, and the value of the security is revealed.

**Price grid** - Limit orders of liquidity suppliers are submitted on a price grid \( \{p_k, k \in [-n, n]\} \). We assume that the minimum tick size between two consecutive prices, is \( \Delta \), so that \( p_{k+1} = p_k + \Delta \). For simplicity, we assume that \( p_0 = v_0 \) and that \( \Delta \leq \frac{1}{2} \sigma \). In this case, orders submitted at prices \( p_k \) for \( 1 \leq k \leq n \) (resp. \( -n \leq k \leq -1 \)) represent sell (resp. buy) orders. Therefore, to avoid confusion, we define \( p_k \) for \( k \geq 1 \) (resp. \( k \leq -1 \)) as an ask price \( A_k \) (resp. a bid price \( B_k \)), and we note \( A_k = v_0 + k\Delta \) (resp. \( B_k = v_0 - k\Delta \)). Because of the symmetry of the model, we concentrate our analysis on the ask side.

**Initial limit order book** - To understand how limit order traders choose their order size and the quantity they display to the market, we fix the prices, and we focus on the quantity submitted and displayed at these prices. We do so by assuming that at date 0, the book is filled in with a sell order at price \( A_2 \). This ask price \( A_2 \) can be viewed as the minimum price at which investors currently in the market are willing to sell two units of the security, given their (unknown) characteristics and their expectations on the final value of the security. Because of the priority rules defined above, for a limit order trader entering the market at date 1, only the limit orders undercutting the current quotes by (at least) one tick have a non-zero probability of being executed.

**Market transparency** - We shall distinguish two different trading systems: (i) the transparent limit order market and (ii) the opaque limit order market. In the transparent limit order market, agents are able to observe the total depth supplied at the best quotes in the limit order book. In the opaque limit order market however, liquidity suppliers have the opportunity to hide a part of the total quantity of their limit order. Thus, before submitting market orders, agents are only able to observe the visible depth at the best quotes, but not the hidden depth.

Figure 1 depicts the timing of the game and of the limit order book information disclosure.
2.2 Agents and information structure

We assume that agents differ in their motive for trading and in their degree of impatience.

**Liquidity suppliers** - At date 1, one agent submits a limit order. This agent can either be willing to trade to speculate on her private information, or for liquidity reasons.

1) An uninformed liquidity supplier (with probability $1 - \pi$)

With probability $1 - \pi$, an uninformed agent ("it") is randomly selected to submit a limit order. This agent acquires no information about the realized value of the security, but trades for liquidity motives. With an equal probability $\frac{1}{2}$, it is a net seller or a net buyer, and we assume that it submits a limit order undercutting the current quotes by one tick (*i.e.* at price $A_1$).\(^3\) Let $D^t$ be the “total depth”, *i.e.* the quantity submitted at price $A_1$ at date 1, $D^v$ the “visible depth”, *i.e.* the quantity displayed to the market and observed by market participants, and $D^h$ the “hidden depth”, *i.e.* the quantity undisclosed to market participants. Notice that by definition, $D^t = D^v + D^h$. The uninformed liquidity supplier has the choice between the three following offers:

- **Offer L**: it may submit and display a large limit order for two units, so that $D^t = D^v = 2$ and $D^h = 0$,
- **Offer S**: it may submit a small limit order for one unit, so that $D^t = D^v = 1$ and $D^h = 0$,
- **Offer H**: in the opaque market and only in this market, it may submit a large limit order $D^t = 2$, disclose a fraction of it, $D^h = 1$, and display one unit to the market, so that $D^v = 1$.

2) An informed liquidity supplier (with probability $\pi$)

We assume that with probability $\pi$, there is an “information acquisition” at date 0, so that one agent (“she”) obtains private information about the realized value of the security before trading occurs. She submits a limit order undercutting the current spread by one tick (*i.e.* at price $A_1$).\(^4\) Like the uninformed liquidity supplier, the informed limit order trader has the choice between the three following offers:

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\(^3\)Allowing agents to undercut the current spread by more than one tick would not change the nature of the results, so for simplicity, we focus on this simple case.

\(^4\)Due to time priority, a limit order at price $A_2$ would have a zero-probability to get executed. Besides, we show below in Section 2.4 that she has no incentives to submit a limit order at a lower price $A_0$. 

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• Offer $L'$: she may submit and display a large limit order for two units, so that $D^t = D^v = 2$ and $D^h = 0$,

• Offer $S'$: she may submit a small limit order for one unit, so that $D^t = D^v = 1$ and $D^h = 0$,

• Offer $H'$: in the opaque market and only in this market, she may submit a large limit order $D^t = 2$, disclose a fraction of it, $D^h = 1$, and display one unit to the market, so that $D^v = 1$.

Since we decided to focus on the ask side, we assume that this agent receives the signal $\tilde{v} = v_L$.

One liquidity demander- At date 2, one Bayesian liquidity demander (“he”) enters the market. We assume that this agent is uninformed and trades for liquidity reasons. He buys or sell the asset with equal probabilities. We assume that he is ready to pay higher transaction costs in order to get his order executed, therefore, he only trades using an aggressive order at date 2. To represent his impatience, we assume that a buyer has a marginal private value from trading $\beta \in [\Delta, \sigma]$, that may either be due to an inventory cost or to a difference in the valuation of the security (cf. Foucault (1999) or Parlour (1998)).

Our assumptions reverse the traditional paradigm, according to which (and differently from the liquidity supplier) the liquidity demander is not exposed to an adverse selection risk. In our model, the liquidity demander fears that his order would be matched with a limit order submitted by an informed agent. Consequently, he takes into account in his expected profits the asset’s expected value, conditional on the observation of the limit order book. Thus each unit he trades when he observes a visible depth $D^v$ leads to an expected gain of $E(\tilde{v}|D^v) + \beta$.

Finally, the liquidity demander’s expected profit depends on the trade size and on the trade price. Both elements are linked to a second aspect of market structure, i.e. the orders’ typology.

Remark 1 According to the market organization, an aggressive buyer may either submit a “market order”, or a “marketable limit order”. A market order guarantees a full execution, but it may have a price impact by walking up or down the limit order book. Conversely, a marketable limit order has no price impact, but if its size is strictly larger than the total depth available at the best limit price, then its non-executed part automatically becomes a limit order at the opposite side of the book.

In this paper, we study both market organizations. The difference between a large (resp. a small)
market order and a large (resp. a small) marketable limit order is non-existent when the visible depth is large (resp. small), i.e. when $D^v = 2$ (resp. $D^v = 1$). In both cases, the state of the limit order book enables the liquidity demander to get a transaction price $A_1$. His expected profit is thus as follows:

$$E\Pi_u (M|D^v = 2) = (E(\tilde{v}|D^v = 2) + \beta - A_1) \times M \text{ for } M \in \{0, 1, 2\},$$

$$E\Pi_u (M|D^v = 1) = (E(\tilde{v}|D^v = 1) + \beta - A_1) \times M \text{ for } M \in \{0, 1\}.$$

When the visible depth is small however, i.e. when $D^v = 1$, a large market order or a large marketable limit order differently impact: i) the beliefs of the agent relative to his execution price and ii) his beliefs relative to the expected value of the asset, depending on the presence of hidden depth. If he submits a large marketable limit order ($M = 2^{MLO}$), he is sure that any unit of his order will be executed at price $A_1$. But he does not know his transaction size. If actually $D^f = 2$ (i.e. if there is hidden depth at price $A_1$), then the liquidity demander trades a large volume (which is good news since he is willing to trade). However, due to the presence of an informed liquidity supplier, who is willing to trade a larger volume as possible, revealing the presence of hidden depth may be bad news, since it may be informative about the fact that the security is overvalued. Therefore, if he submits a large marketable limit order $M = 2^{MLO}$, his expected profit writes:

$$E\Pi_u (M = 2^{MLO}|D^v = 1) = \left( E\left(\tilde{v}|D^v = 1 \cap D^h = 0\right) + \beta - A_1\right) \times \Pr\left(D^v = 1 \cap D^h = 0\right)$$

$$+ 2 \times \left( E\left(\tilde{v}|D^v = 1 \cap D^h = 1\right) + \beta - A_1\right) \times \Pr\left(D^v = 1 \cap D^h = 0\right).$$

In the same situation, a large market order ($M = 2^{MO}$) would be fully executed, but it may have a price impact. The uninformed liquidity demander’s expected profit is then:

$$E\Pi_u (M = 2^{MO}|D^v = 1) = 2 \times (E(\tilde{v}|D^v = 1) + \beta) - A_1 - E\left(\hat{A}(2)|D^v = 1\right),$$

where $\hat{A}(2)$ is the price paid for the second unit of the order. If the limit order he observes is actually a hidden order (i.e. if $D^v = 1$ and $D^h = 1$), then the second marginal unit of his order will be executed at price $A_1$, otherwise (i.e. if $D^v = 1$ and $D^h = 0$), it will be executed at price $A_2$.

We propose to characterize equilibria in both market structures. Agents’ decision planning on the ask side of the limit order book is depicted in Figure 2.
2.3 Equilibrium’s definition

Our model actually represents a signaling game between a liquidity supplier whose type is “informed” with an \textit{ex ante} probability \(\pi\), moving first, and an uninformed liquidity demander, moving second. In this dynamic game with incomplete information, we look for a perfect Bayesian equilibrium.

We do not exclude mixed strategies. At such an equilibrium, if agents are indifferent between different types of orders, then their strategies’ set is not the set including their possible orders, but a set of probability vectors, which describe for each possible order the probability with which the agent chooses this order. To define such an equilibrium, we therefore adopt the following notations.

i) We write \(\mu_M(D^v)\) the probability with which the liquidity demander submits an order \(M \in \{0, 1, 2^{M0}, 2^{MLO}\}\) when he observes the depth \(D^v\).

ii) We write \(l\) the probability with which the uninformed liquidity supplier submits and displays a large limit order (\(L\)), \(h\) the probability with which he submits a hidden order (\(H\)) and \(s\) the probability with which he submits a small limit order (\(S\)).

iii) We write the corresponding probabilities \(\lambda\), \(\chi\) and \(\xi\) for the informed liquidity supplier (resp. for \(L', H'\) and \(S'\)).

Table 1 summarizes the liquidity suppliers’ strategies, \textit{i.e.} the probabilities with which different offers may be observed.

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Limit order’s size</th>
<th>Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>submitted</td>
<td>hidden</td>
</tr>
<tr>
<td>Small limit order</td>
<td>(D^L = 1)</td>
<td>(D^h = 0)</td>
</tr>
<tr>
<td>Large hidden order</td>
<td>(D^L = 2)</td>
<td>(D^h = 1)</td>
</tr>
<tr>
<td>Large unhidden order</td>
<td>(D^L = 2)</td>
<td>(D^h = 0)</td>
</tr>
</tbody>
</table>

Remark 2 We first consider the probabilities \((l, h, s)\) as given, \textit{i.e.} tat the uninformed liquidity supplier strategy is exogenous. This indeed enables us, in Sections 3 and 4, to analyze the behavior of the informed liquidity supplier in a partial equilibrium’s setting. In Section 5, we relax this assumption to study the equilibrium of our model, and we then describe the profit function of this agent.
We first assume that the uninformed liquidity supplier’s strategy is exogenous. An equilibrium is then defined as follows.

**Definition 1** An equilibrium in mixed strategies in the opaque market is defined by (i) a limit order submission strategy for the informed liquidity supplier \((\lambda, \chi, \xi)\); (ii-a) an expectation on the value of the risky asset \(E(\tilde{v}|D^v)\) and (ii-b) an order submission strategy \((\mu_0(D^v), \mu_1(D^v), \mu_{2MO}(D^v), \mu_{2MLO}(D^v))\) for the uninformed liquidity demander, both conditional on the depth \(D^v \in \{1, 2\}\) he observes; such that whatever \(D^v \in \{1, 2\}\):

a. \(E(\tilde{v}|D^v)\) follows from Bayes’ rule and depends on the informed liquidity supplier’s strategy \((\lambda, \chi, \xi)\);

b. Given the informed liquidity suppliers strategy \((\lambda, \chi, \xi)\) and his belief \(E(\tilde{v}|D^v)\), the liquidity demander is indifferent between all orders he submits with a strictly positive probability; c. Given the uninformed liquidity supplier’s strategy \((\mu_0(D^v), \mu_1(D^v), \mu_2(D^v))\), the informed liquidity supplier is indifferent between all limit orders he submits with a strictly positive probability.

An equilibrium in the transparent market (resp. without marketable limit orders) is defined similarly. The difference lies in the fact that in this case \(h = \chi = 0\) (resp. \(\mu_{2MLO}(D^v) = 0\)), i.e. the strategies’ space of the liquidity suppliers is restricted.

**Remarks on the beliefs of the uninformed liquidity demander** For clarity of the presentation, we replaced the beliefs of the second mover on the probability to face an agent whose type is “informed” by his expectations on the value of the asset in our definition of the equilibrium.\(^5,6\) A perfect Bayesian equilibrium is indeed characterized by a Bayesian beliefs’ update of the uninformed liquidity demander, when he observes a signal that potentially arises from an informed liquidity supplier.

Besides, we notice that in our model, the liquidity demander does not only update his beliefs on the value of the security by observing the visible depth displayed in the limit order book, but also by analyzing his execution conditions. In particular, observing a small limit order \(D^v = 1\) is a noisy signal of the total depth supplied at price \(A_1, D^f\), since there may be hidden depth at the best quotes. Given the strategies

\(^5\)We are allowed to do so because there is a one-to-one relation between his beliefs and his expectations.

\(^6\)For clarity, we will not report the equilibrium beliefs of the uninformed liquidity demander in our Propositions in Section 3.
summarized in Table 1, the probability with which there is hidden depth at price $A_1$ is as follows:

$$\Pr(D^h = 1|D^v = 1) = \Pr(D^t = 2|D^v = 1) = \frac{\pi \chi + (1 - \pi) h}{\pi (\chi + \xi) + (1 - \pi) (h + s)}.$$  

This probability may be interpreted in terms of correlation between the total depth and the visible depth.

### 2.4 Parametrization of the model

Our model is discrete and in incomplete information. As in many similar models, the optimality of agents’ strategies depends on the subset of the parameters’ space. Optimal strategies are thus discontinuous. In order not to obscure our results with the presentation of uninteresting cases, we propose to parametrize the model before looking for its equilibria. We therefore suggest to restrict our attention to the case where:

$$\Delta (1 + s) < \beta < 2\Delta.$$  

This parametrization is obtained by the analysis of the benchmark cases reported below.

**Benchmark model with no information** - We first present a benchmark model of trading in an opaque limit order market, when there is no asymmetric information, i.e. $\pi = 0$. In this case, observing the visible depth, $D^v$, is not informative about the value of the security, because there is no adverse selection. Consequently, when he observes a large limit order, the uninformed liquidity demander’s expected profit from trading $M$ units of the security is simply:

$$E\Pi_u(M|D^v = 2) = (v_0 + \beta - A_1) \times M.$$  

For clarity, let us define:

$$\gamma = v_0 + \beta - A_1 = \beta - \Delta$$

We refer to this parameter $\gamma$ as to his “gains from trade” at price $A_1$, since the market order trader obtains a surplus $\gamma$ for each unit traded at this price, in the absence of adverse selection. If this parameter was negative, he would never submit a market order, whatever the size of the depth available at price $A_1$, since $E\Pi_u(M|D^v = 2) = \gamma \times M$. We assume that $\beta \geq \Delta$, so that $\gamma \geq 0$, and that the liquidity demander submits a large order when he observes $D^v = 2$.

**Axiom 1** $\beta \geq \Delta$ (i.e. $\gamma \geq 0$).
Besides, when $D^v = 1$, the uninformed market order trader’s expected cost of trading is random, since there may be, or not, hidden depth at the best quote. In particular, if he cannot submit a marketable limit order, his market order may have a price impact. Thus, the price the agent expects to pay for his second unit is $A_1$ if the limit order is actually a hidden order (i.e. if $D^f = 2$), and $A_1 + \Delta$ otherwise (i.e. if $D^f = 1$). When there are only uninformed liquidity suppliers, his expected profit from submitting a large market order, while observing a limit order of size $D^v = 1$, is thus:

$$E\Pi_u (M = 2^{MO}|D^v = 1) = E\Pi_u (M = 1|D^v = 1) + \gamma - \frac{s}{s+h}\Delta$$

The uninformed agent faces a trade-off. If the second unit of his market order is executed, he receives a net gain $\gamma$, otherwise he must pay an additional cost $\Delta$ since this unit is executed at price $A_2$. The outcome of this trade-off is described in Lemma (1).

**Lemma 1** When there is no asymmetric information ($\pi = 0$), when marketable limit orders are not authorized, the optimal strategy of the uninformed liquidity demander at date 2 when he observes $D^v = 1$ is to submit a large market order $M = 2^{MO}$ if $\gamma > \left(\frac{s}{s+h}\right)\Delta$ and a small market order $M = 1$ if $\gamma \leq \left(\frac{s}{s+h}\right)\Delta$.

The proof is straightforward, comparing the expected profits of the uninformed agent for each limit order observed $D^v \in \{1, 2\}$, and each market order submission $M \in \{0, 1, 2^{MO}\}$. If the condition $\gamma \leq \left(\frac{s}{s+h}\right)\Delta$ holds, then even when there is no adverse selection, the uninformed liquidity demander never submits a larger market order than the quantity he observes at price $A_1$. His gains from trade are indeed too low in this case to cover the extra-cost of $\Delta$ he needs to pay for his second marginal unit, if the limit order is unhidden. In this case, we induce that *a fortiori* in the presence of adverse selection, the liquidity demander would not dare to submit a large market order when $D^v = 1$. Consequently, for $\pi > 0$, the submission of a hidden order would be perfectly equivalent to the submission of a small limit order for liquidity supplier, in terms of trading volume and in informational content. We therefore decide to restrict our attention to the case where the liquidity demander submits a large market order when he observes $D^v = 1$ in the absence of adverse selection.

In parallel, if $\gamma \geq \Delta$, the liquidity demander would always submit a large order, even in the transparent market where he is sure to get a price $A_2$ for his second marginal unit (*i.e.* if $h = 0$). He would thus be ready to trade at this price in the absence of adverse selection, which characterizes a very high impatience. In
this case, when marketable limit orders are authorized, he would rather submit a market than a marketable limit order. The informed liquidity supplier could only benefit from such an aggressivity. We rather focus to the case where $\gamma < \Delta$. Under this condition, the liquidity demander is willing to trade, but not at any price.

Both conditions yield:

**Axiom 2** $\Delta s < \gamma < \Delta$ (i.e. $\Delta (1 + s) < \beta < 2\Delta$).

The assumption $\Delta s < \gamma < \Delta$ enables us to focus on the most interesting case. On the one hand,

On the one hand, the informed liquidity supplier wants to submit large limit orders, in order to increase her trade size. But on the other hand, she cares about the impact of her strategy on the uninformed market order trader’s beliefs, since these beliefs influence her probability of execution. Since the liquidity demander is rational and Bayesian, he is conscious that his adverse selection risk increases with the depth supplied at price $A_1$. Submitting a hidden order could enable the informed liquidity supplier to submit a large order, without signalling her presence to the uninformed liquidity demander.

**Informed agent’s strategies’ space** - We now show that submitting a sell limit order at a lower price than $A_1$ is suboptimal for the informed liquidity supplier. Assume that she submits an order for $D_0$ shares at price $A_0$. Since the uninformed limit order trader never submits an order at this price, the informed agent perfectly reveals her presence. Thus the uninformed liquidity demander infers that the value of the security is low, i.e. $E(\tilde{v}|D_0) = v_0 - \sigma$. Consequently, his expected profit from buying $M$ units of the asset at this price is as follows:

$$E\Pi_u (M|D_0 \text{ units at } A_0) = (\gamma + \Delta - \sigma) \times M$$

Since we assumed that $\beta \leq \frac{1}{2}\sigma$, the condition $\gamma < \Delta$ guarantees that this expected profit is negative. The uninformed liquidity demander’s optimal strategy is to submit no order. Consequently, if the informed liquidity supplier submits an order at price $A_0$, her expected profit is equal to zero.

**Marketable limit orders versus market orders**- If $\gamma < \Delta$, submitting a marketable limit order dominates submitting a large market order when $D^v = 1$, even out of the equilibrium path. This result is based on the following intuition. In the absence of adverse selection, if the liquidity demander submits
a market order when he observes $D^v = 1$, and if there is no hidden depth at the best quotes, then his
gain from trade $\gamma$ is strictly inferior to the additional cost $\Delta$ paid to get this second unit executed. In a
similar situation, if he submits a marketable limit order, he avoids incurring this loss. We show in appendix
that this intuition is robust when we take into account the impact of adverse selection on the liquidity
demander’s expected profits.

3 Partial equilibrium and the informed liquidity supplier’s strategies

In this section, we look for partial equilibria of our model in the opaque market, when the strategy of the
uninformed liquidity supplier is supposed to be exogenous. We first determine the uninformed liquidity
demander’s beliefs and reaction at date 2, for each possible observation of the depth at price $A_1$ in the
limit order book. Given his reaction, we then find the informed liquidity supplier’s optimal limit order
submission strategy at date 1.

3.1 Beliefs of the uninformed liquidity demander

At date $t = 2$, the behavior of the uninformed impatient agent depends on his expectations on the value
of the asset, that he updates after observing the visible depth displayed in the limit order book at date 1.
At this stage, as illustrated in Table 1 above, there are three possible states of the best quotes in the limit
order book. The parameter $\pi$ measures the degree of adverse selection faced by an uninformed liquidity
demander at date 2. Following Bayes’ rule:

$$E(\tilde{v}|D^v = 1) = v_0 - \sigma \pi \left(\frac{\xi + \chi}{\pi (\xi + \chi) + (1 - \pi) (s + h)}\right),$$

$$E(\tilde{v}|D^v = 2) = v_0 - \sigma \left(\frac{\pi \lambda}{\pi \lambda + (1 - \pi) l}\right).$$

Observing the limit order book enables the uninformed liquidity demander to update his beliefs on
the true value of the security, due to the presence of an informed liquidity supplier. For this reason, the
magnitude of the beliefs’ update increases with $\pi$, and with the asset volatility $\sigma$.

But it also depends on $\lambda$, $\chi$ and $\xi$, i.e. on the strategies played by the informed liquidity supplier.
Suppose for instance that she always submits small orders, i.e. $\xi = 1$. Then, for the uninformed liquidity
demander, observing a large order $D^v = 2$ is not informative about the value of the security. Suppose in an opposite case that the uninformed liquidity supplier never submits large orders, i.e. $l = 0$. Then observing a large order $D^v = 2$ perfectly reveals the presence of the informed liquidity supplier, and her private information that the value of the security is low. The conditional expectation of the value of the security becomes in this case $E(\tilde{v}|D^v = 2) = v_0 - \sigma = v_L$.

The magnitude of the beliefs’ update therefore also relies on the informed agent’s strategy and on the trading “noise” created by the uninformed liquidity supplier’s behavior. We show in Section 5 that in equilibrium, the magnitude of the beliefs’ update is higher when the uninformed liquidity demander observes a large order, $D^v = 2$, than when he observes a small order, $D^v = 1$.

3.2 Reaction of the uninformed liquidity demander

Given his beliefs’ update described above, we study the the uninformed liquidity demander’s optimal reaction at date 2. This reaction is conditional on the depth he observes, for two reasons. First, given the presence of an informed liquidity supplier, it contains information on the value of the security. Second, it is informative about the trade conditions.

3.2.1 Reaction of the uninformed liquidity demander when he observes $D^v = 2$

Observing a large limit order at price $A_1$ allows the liquidity demander to perfectly know his transaction price, namely $A_1$, and the fact that a large order would be fully executed. He is thus induced to submit a large order. However, due to the adverse selection risk, he may not be ready to trade if he fears to face an informed limit order trader. His optimal reaction when he observes $D^v = 2$ thus results from a trade-off between his gain from trade and his adverse selection risk. It is described in Lemma (2).

Lemma 2 The uninformed liquidity demander’s strictly dominant strategy when he observes $D^v = 2$ is to submit a large market or marketable limit order $M = 2^{MO}$ or $M = 2^{MLO}$ if $\lambda < \frac{\gamma}{(\sigma - \gamma)\pi}(1 - \pi)l$. If $\lambda > \frac{\gamma}{(\sigma - \gamma)\pi}(1 - \pi)l$, his strictly dominant strategy is to submit no order $M = 0$. Finally, if $\lambda = \frac{\gamma}{(\sigma - \gamma)\pi}(1 - \pi)l$, he is indifferent between any size of order, $M \in \{0, 1, 2^{MO}, 2^{MLO}\}$. 

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The reaction of the uninformed liquidity demander at date 2 depends on the probability to face an informed limit order, given the depth displayed in the limit order book, and thus on the strategy of large order submission of the informed agent at date 1, $\lambda$. The higher $\lambda$, the larger is the belief’s update of uninformed liquidity demander. For $\lambda < \gamma (\sigma - \gamma) (1 - \pi)^l$, it keeps sufficiently low to induce him to submit a large (market or marketable limit) order when he observes $D^v = 2$. Otherwise, it is so high that he submits either a small market order, or no order at all.

The cutoff value that determines the move from a large order to no order is such that $\pi \lambda = \gamma (\sigma - \gamma) (1 - \pi)^l$. This value illustrates the liquidity demander’s trade-off between adverse selection and gains from trade. The adverse selection component appears as he compares the probability that an informed liquidity supplier submits a large limit order, $\pi \lambda$, with the probability to observe a large uninformed limit order, $(1 - \pi)^l$. When $l$ increases or when $\pi$ decreases, the adverse selection risk faced by the liquidity demander decreases, therefore he is induced to submit larger orders. At the same time, the later takes into account his private benefit from trading, in the component $\frac{\gamma}{\sigma - \gamma}$. This component is increasing in $\gamma$. The larger this parameter, the more the uninformed liquidity demander is likely to submit a large order.

### 3.2.2 Reaction of an uninformed liquidity demander when he observes $D^v = 1$

Similarly, the uninformed liquidity demander’s reaction when he observes $D^v = 1$ is the outcome of a trade-off between his desire to trade and his risk of adverse selection. This trade-off is however more complex than in the case where he observes a large limit order. Even if the visible depth in the limit order book is small, it may well be that its total depth is large, due to the potential presence of a hidden order. This possibility may induce him to submit an order that would be larger than the visible depth. Such a strategy enables him to benefit from gains from trade, $\gamma$. It is however linked with two types of risk.

First, he faces adverse selection. This risk is evaluated by the difference between the conditional expectation of the asset and its trading price, i.e. $E(\tilde{v}|D^v = 1) - A_1$. If there is hidden depth at this price, such that $D^h = 1$, then his adverse selection risk may increase, since the informed liquidity supplier is induced to submit large order. Indeed, the following necessary and sufficient condition holds:

$$|E(\tilde{v}|D^v = 1 \cap D^h = 1) - v_0| \geq |E(\tilde{v}|D^v = 1 \cap D^h = 0) - v_0| \Leftrightarrow \chi s \geq h \xi$$

In this case, executing a large order thanks to the presence of a hidden order is “bad news”. It leads
the uninformed liquidity demander to update his beliefs more strongly when he discovers a hidden order.

Second, the execution conditions of a large order are random when $D^v = 1$, and depend on the market structure.

**Reaction of the uninformed liquidity demander at date 2 when he observes $D^v = 1$, if marketable limit orders are authorized** When marketable limit orders are authorized, we show in Appendix that for $\gamma < \Delta$, submitting a large market order $M = 2^{MO}$ is a dominated strategy for the uninformed liquidity demander. If he submits a large marketable limit order instead, he is sure that his order will not be (partially) executed at price $A_2$. He however still faces an adverse selection risk. We introduce the dummy function $\delta_{xz}$, that takes the value 1 if the condition $\{x\}$ holds, and 0 otherwise.

**Lemma 3** Let conditions (a) and (b) be such that:

\[
\begin{align*}
\chi &< \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} \ h + \min \left( \frac{1}{2} \left( \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} s - \xi \right); 0 \right) \quad (a) \\
\chi &> \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} h + \frac{1}{2} \times \left( \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} s - \xi \right) \times \left( \delta_{\frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} s - \xi > 0} + 1 \right) \quad (b)
\end{align*}
\]

When marketable limit orders are authorized, the uninformed liquidity demander’s strictly dominant strategy is to submit a large marketable limit order $M = 2^{MLO}$ if Condition (a) holds. If Condition (b) holds, his strictly dominant strategy is to submit no order $M = 0$. If neither Condition (a) nor Condition (b) holds, then his dominant strategy is either to submit a small order $M = 1$, or he is indifferent between different order’s sizes.

The condition $\lambda > \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} l$ which according to Lemma 2 determines the uninformed liquidity demander’s reaction when he observes $D^v = 2$ here translates into Condition (b) when he observes $D^v = 1$. We indeed notice that if $\frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} s - \xi > 0$, Condition (b) becomes $\chi + \xi > \frac{\gamma}{\sigma - \gamma} \frac{(1 - \pi)}{\pi} (h + s)$. This threshold corresponds to a cutoff value which determines liquidity demand. When the probability with which the informed liquidity supplier displays $D^v = 1$ (i.e. $\chi + \xi$) is too high, then the uninformed liquidity demander is deterred from submitting any order. His adverse selection risk is so high that he is not willing to trade, even at price $A_1$.

Differently from Lemma 2, the uninformed liquidity demander’s reaction is driven by a second condition when $D^v = 1$. Two offers indeed lead to the observation of $D^v = 1$, namely a small limit order ($S$ or $S'$)
and a hidden order ($H$ or $H'$). But the informational content of both limit orders is different, what the liquidity demanders takes into account. We have suggested that the revelation of a hidden order could be “bad news”. This is what Condition (a) accounts for: if the informed liquidity supplier submits a hidden order with a probability $\chi$ that is sufficiently low, then revealing a hidden order has low informational content, therefore the liquidity demander is ready to submit a larger order than the depth displayed at price $A_1$. Conversely, if $\chi$ is too high, he either submits a small order, or no order.

Reaction of the uninformed liquidity demander when he observes $D^v = 1$, if marketable limit orders are not authorized When marketable limit orders are not authorized, the liquidity demander must not only take into account his adverse selection risk, but also the price impact of a large market order. If there is no hidden depth at price $A_1$, i.e. if $D^l = D^v = 1$, then the second unit of his market order executes at price $A_2 \equiv A_1 + \Delta$. The price impact’s risk may be evaluated by $E(A(2) | D^v = 1) - A_1$. The liquidity demander’s expected profit thus depends on the difference between his gains from trade, $\gamma$, and the tick size $\Delta$. Lemma 4 describes his reaction.

**Lemma 4** Let Conditions (1) and (2) be such that:

\[
\chi + \left(\frac{\sigma - \gamma + \Delta}{\sigma - \gamma}\right) \xi < \frac{\gamma}{(\sigma - \gamma)} \frac{1 - \pi}{\pi} \left( h + \left(1 - \frac{\Delta}{\gamma}\right) s \right) \quad \text{(Condition 1)}
\]

\[
\xi + \chi > \frac{\gamma}{(\sigma - \gamma)} \frac{1 - \pi}{\pi} (s + h) \quad \text{(Condition 2)}
\]

The uninformed liquidity demander’s strictly dominant strategy is to submit a large market order $M = 2$ if Condition (1) holds. If Condition (2) holds, his strictly dominant strategy is to submit no market order $M = 0$. If neither condition holds, his strictly dominant strategy is to submit a small market order $M = 1$, or he is indifferent between different market order’s sizes.

We find two conditions that are similar as Conditions (a) and (b) in Lemma 3. On the one hand, if the informed liquidity supplier displays too often a small limit order, i.e. if $\chi + \xi$ is relatively high, then the adverse selection risk is such that the liquidity demander do not want to trade when he observes $D^v = 1$ (Condition (2)). On the other hand, he may submit a larger order than the depth displayed at the best quotes. In this case, he also accounts for the potential price impact of his order. Condition (1) determines the optimality of such a strategy. Two cases emerge.
If $h + \left(1 - \frac{A_1}{A}\right) s \leq 0$ then according to Lemma 1, the liquidity demander never submits a large market order when he observes $D_v = 1$, even if there is no asymmetric information (thus a fortiori when there is some). Consequently, submitting a small or a hidden limit order become perfectly equivalent offers for the liquidity demanders (in terms of maximum trading volume and informational content).

In the opposite case, if $h + \left(1 - \frac{A_1}{A}\right) s > 0$, then the liquidity demander always submit a large order when he observes $D_v = 1$, in the absence of asymmetric information. In this benchmark case, revealing a hidden order is good news: it indeed guarantees a complete execution at price $A_1$. With the introduction of asymmetric information though, the liquidity demander updates his beliefs on the value of the security after revealing a hidden order. In this case, the impact of this revelation on his beliefs may be such bad news, that the increase in the adverse selection risk overcomes his gains from trade at price $A_1$. The relative strength of both effects depends on the value of the fraction of the informed liquidity supplier’s strategies $\chi$ with respect to the similar fraction for the uninformed liquidity supplier, i.e. $h/s$.

We study both cases separately.

3.3 The informed liquidity supplier’s optimal limit order submission strategy

When the uninformed liquidity demander observes the depth of the limit order book, he updates his beliefs on the value of the security by taking into account two elements. First, the direction of the limit order, a sell order in our case, signals him that the security is likely to be overvalued. The impact of this signal on the uninformed agent’s beliefs depends on the probability of information acquisition, $\pi$.

Second, the size of the limit order displayd enables him to better assess his adverse selection risk. The informed liquidity supplier’s limit order submission strategy has no impact on the first signal, but plays a crucial role with respect to this second signal.

Actually, for a fixed level of adverse selection, there exists a multiplicity of equilibria, due to the discretization of our model. In the absence of any clear criteria to select equilibria, we present here all equilibria.
3.3.1 Equilibria with no signaling

For extreme values of the parameter $\pi$, the uninformed liquidity demander’s optimal reaction is independent from the informed liquidity supplier’s limit order submission strategy, in equilibrium. The liquidity demander indeed focuses on the information content of the direction of the limit order, rather than on its size.

Assume first that $\pi$ is very low. Then the uninformed liquidity demander is induced to submit a large order, whatever the informed liquidity supplier’s strategy. Therefore, even if she always submits and displays a large limit order $D^v = 2$, she can expect to see it getting fully executed. Formally, Proposition 1 shows that there indeed exists such an equilibrium in pure strategies when $\pi < \frac{\gamma_l}{\sigma - \gamma + \gamma_l}$.

**Proposition 1** Whatever the market organization, there exists an equilibrium in pure strategies, in which:

1. The informed liquidity supplier submits and displays a large unhidden order, such that $D^h = 0$, with a probability $\lambda^* = 1$,

2. The uninformed liquidity demander updates his beliefs and submits a large market or marketable limit order, $M = 2^{MO}$ or $M = 2^{MLO}$, whatever the limit order $D^v$ he observes,

if and only if $\pi < \frac{\gamma_l}{\sigma - \gamma + \gamma_l}$.

The condition $\pi < \frac{\gamma_l}{\sigma - \gamma + \gamma_l}$ indeed ensures that $\frac{\gamma_l(1-\pi)}{\pi(\sigma-\gamma)} < 1$, which, according to Lemma 2, guarantees that the uninformed liquidity demander is willing to trade two units when he observes $D^v = 2$, whatever the informed agent’s strategy. This condition thus naturally determines the cutoff value of the adverse selection’s degree, $\pi$, such that an equilibrium in pure strategies with a large trading volume exists.

By contrast, assume that adverse selection is very high, for instance, $\pi = 1$. If the informed liquidity supplier submits a small or a large hidden order with a strictly positive probability, then the uninformed liquidity demander submits no order when he observes $D^v = 1$, as stated in Lemmas 3 and 4. Similarly, according to Lemma 2, if she submits $D^v = 2$ with a strictly positive probability, the liquidity demander submits no order when he observes $D^v = 2$. Consequently, if the informed liquidity supplier submits $D^v = 1$ or $D^v = 2$ with strictly positive probabilities, there is no trade in equilibrium. Assume now that

$\pi < \frac{\gamma_l}{\sigma - \gamma + \gamma_l}$, we show in the Appendix that there exist other equilibria, that we characterize.
the informed agent plays a pure strategy, say $\lambda^* = 1$ for instance. Then her expected profit would be zero when she displays $D^v = 2$ but it would be strictly positive if she sometimes submitted $D^v = 1$. Thus there exists no equilibrium in pure strategies, since she would have incentives to deviate. Using the same reasoning, Proposition 2 shows that for $\pi > \frac{2}{\sigma}$, this type of equilibrium with no execution arises.\(^8\)

**Proposition 2** Whatever market organization, if $\pi > \frac{2}{\sigma}$, there exists multiplicity of equilibria. These equilibria are characterized by no trading, i.e. the uninformed liquidity demander submits no order whatever the visible depth $D^v$ he observes.

The condition $\pi > \frac{2}{\sigma}$ indeed implies that the uninformed liquidity demander so strongly updates his beliefs after observing a sell limit order, that whatever the informed supplier’s equilibrium strategy, his gains from trade $\gamma$ do not compensate the fact that he is about to buy an asset that is clearly overvalued. Thus he does not trade.

In both cases analyzed above, we notice that there exist equilibria in the opaque market such that the informed liquidity supplier do not submit any hidden order. This finding is not surprising. Intuitively, the informed agent may submit a hidden order to moderate the impact of her large order on the uninformed liquidity demander’s beliefs. But when adverse selection is extreme, this uninformed agent does not account for the informational content of the limit order book’s depth to update his beliefs. Consequently, there is no need for the informed liquidity supplier to submit a hidden order, since he cannot manipulate the uninformed agent’s beliefs in any case.

### 3.3.2 Equilibria with signaling

For intermediate levels of $\pi$ however, the uninformed liquidity demander gives credit to his observation of the depth at the best prices in his trading decision. The informed agent’s limit order strategy therefore influences his reaction. While choosing the size of her limit order, the informed liquidity supplier faces the following trade-off. On the one hand, if she submits a small limit order, her maximum transaction volume is only one unit, so she is induced to submit a larger limit order. But on the other hand, if she displays a large order too frequently, the probability of execution of her order decreases due to its impact on the

\(^8\)See the Appendix for a complete description of such equilibria.
uninformed liquidity demander’s beliefs.

In the opaque market, she may therefore submit a large hidden order, to mitigate the impact of her order on the uninformed agent’s beliefs on the value of the security, without restricting her trade size. We show that the optimality of such a strategy depends on the market organization, i.e. on the authorization of marketable limit orders. We first present the case where marketable limit orders are not authorized. We then deduce from this the case where such orders are authorized.

**Opaque market without marketable limit orders**  We restrict the uninformed liquidity demander’s strategies’ space to $\mu_{MLO} = 0$. In this case, this agent must take into account his adverse selection risk, and the price impact of his market order. As shown in Lemma 4, his reaction depends on Condition $h + \left(1 - \frac{\Delta}{\gamma}\right) s > 0$. We study both cases separately.

**Informed liquidity supplier’s optimal strategy in the opaque market when $h + \left(1 - \frac{\Delta}{\gamma}\right) s > 0$.**

In the case where $h + \left(1 - \frac{\Delta}{\gamma}\right) s > 0$, Proposition (3) shows that the informed liquidity supplier submits hidden orders in equilibrium when

$$\pi \in \left[\frac{\gamma}{\sigma - \gamma}, \frac{\gamma - \Delta}{\sigma - \Delta}\right].$$

**Proposition 3** When marketable limit orders are not authorized, and when $h + \left(1 - \frac{\Delta}{\gamma}\right) s > 0$:

a) If $\pi \in \left[\frac{\gamma}{\sigma - \gamma}, \frac{\gamma - \Delta}{\sigma - \Delta}\right]$, there is a multiplicity of equilibria such that:

1. The informed liquidity supplier submits a large limit order $(D^l = 2)$, fully displayed $(D^h = 0)$ with a probability $\lambda^* \in \left[\frac{\gamma}{\sigma - \gamma}, \frac{\gamma - \Delta}{\sigma - \Delta}\right]$, and hidden $(D^h = 1)$ with a probability $\chi^* = 1 - \lambda^*$, with $0 < \lambda^* < 1$.

2. The uninformed liquidity demander updates his beliefs and submits a large market order $M = 2^{MO}$ whatever the depth he observes.

3. In particular, $\chi^* > 0$.

b) If $\pi = \frac{\gamma - \Delta}{\sigma - \Delta}$, then there exist other equilibria, but the equilibrium characterized in (a) is Pareto-dominant.\(^9\)

\(9\)For $\pi = \frac{\gamma - \Delta}{\sigma - \Delta}$, the equilibrium characterized in (a) is unique, since $\frac{(\gamma - \Delta\gamma) - (\sigma - \Delta\gamma)\pi}{\pi(\sigma - \gamma)} = 0$. In equilibrium, $\lambda^* = \frac{\gamma}{\sigma - \Delta} = 1 - \chi^*$. However, there exist other equilibria. We show in Appendix that the informed liquidity supplier’s expected profit is strictly superior for $\lambda^* = \frac{\gamma}{\sigma - \Delta} \frac{\gamma - \Delta}{\sigma - \Delta} l$, while the uninformed liquidity demander is indifferent between all equilibria.
For $\pi > -\frac{\gamma}{\sigma - \gamma}$, if the informed liquidity supplier systematically displays a large limit order (i.e. if he plays the pure strategy $L'$), the liquidity demander do not trade when he observes $D^v = 2$, due to a high adverse selection risk. When the adverse selection degree is sufficiently low however, i.e. for $\pi \leq -\frac{\Delta_s}{\sigma - \Delta_s}$, the informed liquidity supplier succeeds in manipulating the uninformed liquidity demander’s beliefs and thus in getting her large order fully executed. This success is due to the use of hidden orders in equilibrium. These hidden orders enable her to diminish the uninformed liquidity demander’s inference capacities on the value of the security. In order to induce the liquidity demander to submit a large market order when he observes $D^v = 2$ and when he observes $D^v = 1$, the informed liquidity supplier (who always submits large orders $D^t = 2$) plays a mixed strategy: she sometimes fully displays her order (i.e. $D^h = 0$), and sometimes hides it (i.e. $D^h = 1$).

The cutoff value $\frac{\gamma}{\sigma - \Delta_s}$ corresponds to the maximal adverse selection degree such that the informed liquidity supplier can expect a large trading volume (using hidden orders). For a higher $\pi$, there is no equilibrium such that the uninformed liquidity demander submits a large order, as shown in the following Proposition.

**Proposition 4** When marketable limit orders are not authorized, and when $h + \left(1 - \frac{\Delta_s}{\sigma - \Delta_s}\right)s > 0$:

a) If $\pi \in \left[\frac{-\Delta_s}{\sigma - \Delta_s}, \frac{\gamma}{\sigma}\right]$, there exists a multiplicity of equilibria such that:

1. The informed liquidity supplier submits a large unhidden limit order (i.e. $D^t = 2$ and $D^h = 0$) with a probability $\lambda^* = \frac{\gamma}{\sigma - \gamma} - \frac{\pi}{\sigma - \gamma} - 1$, and either a large hidden order (i.e. $D^t = 2$ and $D^h = 1$) or a small limit order (i.e. $D^t = 1$) with a probability $(\chi + \xi)^*$ such that $(\chi + \xi)^* = 1 - \lambda^*$.

2. The uninformed liquidity demander updates his beliefs and submits a small market order $M = 1$ when he observes $D^v = 1$, and a large (resp. small, no) market order with a probability $\mu_{2, \text{MO} |1}$ (resp. $\mu_{1, \text{MO} |2}$, $\mu_{0, \text{MO} |2}$) when he observes $D^v = 2$, such that $2\mu_{2, \text{MO} |2} + \mu_{1, \text{MO} |2} = 1$.

3. In particular, there exist equilibria such that the informed liquidity supplier does not submit a hidden order (i.e. $\chi^* = 0$).

b) If $\pi = \frac{\gamma}{\sigma}$, then there exist other equilibria, but the equilibrium characterized in (a) is Pareto-dominant.

For $\pi \in \left[\frac{-\Delta_s}{\sigma - \Delta_s}, \frac{\gamma}{\sigma}\right]$, the uninformed liquidity demander faces a too high adverse selection risk to submit a large market order whatever the depth he observes. Besides, we have seen that there exists no equilibrium
in pure strategies for this level of $\pi$. At the equilibrium in mixed strategies, the liquidity suppliers’ expected trade size must be equal whatever the offer submitted with a strictly positive probability. For these reasons, we show in Appendix that there exists no equilibrium such that the liquidity demander submits a large market order.

Consequently, the informed liquidity supplier is perfectly indifferent between submitting a small limit order, or a hidden order. Both types of orders indeed have the same conditions of execution (i.e., same maximum trading volume and same informational content). Thus there exists no economic rationale for the use of hidden orders by the informed liquidity supplier.

Besides, at the cutoff value $\pi = \frac{\gamma}{\sigma}$, there is a pooling equilibrium, meaning that the uninformed liquidity demander’s conditional expectations are identical whatever the depth he observes, i.e. $E(\tilde{v}|D_v = 1) = E(\tilde{v}|D_v = 2)$. By mimicking the uninformed liquidity supplier’s behavior and by choosing $\lambda^* = l$, the informed liquidity supplier perfectly “hides” in the trading noise, so that the beliefs of the liquidity demander are only affected by the direction of the limit order he observes, but not by its size. This characteristic illustrates the role of the threshold value $\frac{\gamma}{\sigma}$ for the adverse selection degree, $\pi$. For $\pi$ strictly higher than this value, whatever the depth he observes, the uninformed liquidity demander is not willing to exchange, because with an expected value of the asset strictly inferior to $v_0 - \gamma$, his expected profit from exchange would be strictly negative.

The main results of Propositions 3 and 4 are reported in Table 2.

**Informed liquidity supplier’s optimal strategy in the opaque market when $h + \left(1 - \frac{\Delta s}{\gamma}\right)s \leq 0$.**

We now consider the case where $h + \left(1 - \frac{\Delta s}{\gamma}\right)s \leq 0$. This condition implies that even in the absence of adverse selection, the liquidity demander never submits a larger order than the depth he observes (cf. Lemma 1). It can be easily shown that for $\pi > \frac{\gamma - \Delta s}{\sigma - \Delta s}$, Propositions 2 and 4 are still valid in this case. In equilibrium indeed, the liquidity demander never submits a large market order $M = 2^{MO}$ when he observes $D_v = 1$. In the opposite case, for $\pi < \frac{l}{\sigma - \Delta s}$, Proposition 1 shows that there exists an equilibrium in pure strategies, without hidden order. For $\pi \leq \frac{\gamma - \Delta s}{\sigma - \Delta s}$ however, in the equilibria described in Proposition 3, the informed liquidity supplier submits hidden orders to manipulate the uninformed liquidity demander’s beliefs. This cannot be the case when the probability with which the uninformed liquidity supplier submits hidden orders, i.e., $h$, is too small, because the adverse selection risk faced by the
uninformed liquidity demander who would reveal a hidden order prevents him from trading. For clarity, equilibria when \( h + \left(1 - \frac{\Delta}{\gamma} \right) s \leq 0 \) are reported in Table 3. We only report Pareto-dominant equilibria in the following Proposition.

**Proposition 5** When marketable limit orders are not authorized, and when \( h + \left(1 - \frac{\Delta}{\gamma} \right) s \leq 0 \), then for \( \frac{\gamma}{\sigma - \gamma + 1} < \pi \leq \frac{\gamma - \Delta}{\sigma - \Delta} \), there exists a multiplicity of equilibria, in which the uninformed liquidity demander submits a small market order \( M = 1 \). The equilibrium that maximizes the liquidity demander’s expected profit is such that the informed liquidity supplier displays a large limit order (i.e. \( D^\nu = 2 \)) with a probability \( \lambda^* = \frac{\gamma - 1}{\sigma - \gamma} \), and submits either a small limit order or a hidden order (i.e. \( D^\nu = 1 \)) with a probability \( (\chi + \xi)^* = 1 - \lambda^* \).

Again, as in the previous case, there exists an equilibrium in which the informed liquidity supplier do not use hidden orders.

**Transparent market** In the opaque market, liquidity suppliers have the opportunity to hide a part of the total quantity of their limit order. We assumed that the uninformed liquidity supplier submits a large hidden order with a probability \( h \). In the transparent market however, participants are able to observe the full quantity supplied at the best quotes in the limit order book, so that the strategies’ space is restricted to \( h = 0 \).

This implies that \( h + \left(1 - \frac{\Delta}{\gamma} \right) s \leq 0 \) (since we assumed that \( \gamma < \Delta \)). But in this case, whatever the level of adverse selection, there exist equilibria in which the informed liquidity supplier do not submit hidden orders. Propositions 1, 2, 4 and 5 apply to the transparent market with \( \chi^* = h = 0 \).

**Opaque market with marketable limit orders** We now turn to the case where marketable limit orders are authorized. In such markets, liquidity demanders willing to exchange a larger quantity than the depth displayed may submit a marketable limit order to “test” the presence of hidden depth. Since they do not incur any additional cost in the case where there is no hidden depth, it seems hard to understand the reason why liquidity supplier would submit hidden orders.

The following Proposition shows however that in this case, the informed liquidity supplier is induced to submit hidden orders.
Proposition 6  When marketable limit orders are authorized,

a) If \( \pi \in \left[ \frac{\sigma - \gamma + \sigma s}{\sigma - \gamma + \sigma s}, \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} \right] \), there is a multiplicity of equilibria such that:

1. The informed liquidity supplier submits a large limit order \( (D^l = 2) \), fully displayed \( (D^h = 0) \) with a probability \( \lambda^* \in \left[ \max \left( 1 - \frac{1}{2} \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} (s + 2h); 0 \right), \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} \right] \), and hidden \( (D^h = 1) \) with a probability \( \chi^* = 1 - \lambda^* \), with \( 0 \leq \lambda^* < 1 \).

2. The uninformed liquidity demander updates his beliefs and submits a large marketable limit order \( M = 2^\text{MLO} \) whatever the depth he observes.

3. In particular, \( \chi^* > 0 \).

b) If \( \pi = \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} \), then there exist other equilibria, but the equilibrium characterized in (a) is Pareto-dominant.

Actually, submitting a large marketable limit order is less costly for the uninformed liquidity demander than submitting a large market order when he observes \( D^v = 1 \), since the former has no price impact. In the absence of adverse selection, he would be ready to systematically submit such orders, whatever the visible depth. In return, this could increase the benefits of the use of hidden orders for the informed liquidity supplier. Since \( \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} > \frac{\gamma - \Delta s}{\sigma - \Delta s} \), submitting hidden order indeed enables her to trade a larger volume for \( \pi \in \left[ \frac{\gamma - \Delta s}{\sigma - \Delta s}, \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} \right] \) when marketable limit orders are authorized.\(^{10}\) What happens when the adverse selection degree is higher than \( \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s} \)?

Proposition 7  When marketable limit orders are authorized,

a) If \( \pi \in \left[ \frac{\gamma - \gamma s}{\sigma - \gamma + \sigma s}, \frac{\gamma}{\sigma} \right] \), there exists a multiplicity of equilibria such that:

1. The informed liquidity supplier submits a large unhiddem limit order (i.e. \( D^l = 2 \) and \( D^h = 0 \)) with a probability \( \lambda^* = \frac{\gamma}{\sigma - \gamma + \sigma s} \), and either a large hidden order (i.e. \( D^l = 2 \) and \( D^h = 1 \)) with a probability \( \chi^* \) or a small limit order (i.e. \( D^l = 1 \)) with a probability \( \xi^* \) such that \( \chi^* + \xi^* = 1 - \lambda^* \) and \( \chi^* \geq \frac{\gamma}{\sigma - \gamma + \sigma s} h \).

2. The uninformed liquidity demander updates his beliefs and submits a small market order \( M = 1 \) when he observes \( D^v = 1 \), and a large (resp. small, no) marketable limit order with a probability \( \mu_{2^\text{MLO}|1} \) (resp. \( \mu_{1|2}, \mu_{0|2} \)) when he observes \( D^v = 2 \), such that \( 2\mu_{2^\text{MLO}|2} + \mu_{1|2} = 1 \).

\(^{10}\)By assumption, \( \gamma < \Delta \) which implies this inequality.
3. In particular, $\chi^* \geq \gamma \frac{1 - \pi h}{\sigma - \gamma} \gamma^1 - \pi^h$.

b) If $\pi = \gamma^2$, then there exist other equilibria, but the equilibrium characterized in (a) is Pareto-dominant.

As in the case where marketable limit orders are not authorized, the informed liquidity supplier cannot expect a large trading volume when adverse selection is too high. However, in this case, he does submit hidden orders in equilibrium.

Formally, this condition is necessary in equilibrium to guarantee that she is indifferent between all the offers that she submits with a strictly positive probability. This condition does not appear in Proposition 4 when marketable limit orders are not authorized. In the later case, for $\pi \in [\gamma^2, \frac{\gamma}{2}]$, the uninformed liquidity demander is not ready to submit a large market order due to adverse selection but also to the price impact of his order. Even if hidden orders are not informative (i.e. if $\chi = 0$), he does not take the risk to execute his order at price $A_2$, because he still strongly update his beliefs on the value of the security while observing a sell limit order.

Economically, it seems that since his order has no price impact, the liquidity demander pays more attention to the informational content of the limit order book. Consequently, the informed liquidity supplier must imitate the behavior of the uninformed liquidity supplier more precisely in order not to reveal her presence. In particular, she must also submit hidden orders.

The main results of Propositions 6 and 7 are reported in Table 4.

3.4 Equilibrium’s expected profits

One of the objectives of this paper is to compare the welfare of participants in the opaque and in the transparent market. To this end, we determine the agents’ expected profits in both market structures.

The uninformed liquidity demander’s expected profit when he observes $D^v$ is defined in Section 2. Let us note $E\Pi^*_u (D^v)$ his equilibrium expected profit when he observes $D^v$. We compute his $ex \ ante$ expected profit as follows:

$$E\Pi^{ante}_u = (\pi \lambda + (1 - \pi) l) E\Pi^*_u (D^v = 2) + (\pi (1 - \lambda) + (1 - \pi) (1 - l)) E\Pi^*_u (D^v = 1) \quad (3)$$

The informed liquidity supplier’s $ex \ ante$ expected profit relies on two elements. First, she enters the
market with a probability \( \pi \), and meets a buyer with a probability \( \frac{1}{2} \). Second, her expected profit is linear in the equilibrium trading volume.

Let \( TS^* \) be this transaction size. It is equal to zero if her limit order does not get execution, to one if she submits a small limit order that is fully executed or if she submits a large limit order that is only partially executed, and to two if she submits a large limit order that is fully executed. Her expected trade size thus depends on one hand on the size of her limit order, and on the other hand on the uninformed liquidity demander’s reaction. In equilibrium, both elements are determined by adverse selection.

\[
E\Pi_{\text{ante}}^i (\pi | \tilde{v} = v_L) = \pi \times (A_1 - v_L) \times TS^* (\pi) \times \frac{1}{2} \tag{4}
\]

As we have seen, depending on the level of adverse selection, we find different types of equilibria. For each equilibrium, we compute the strategic agents’ expected profit as described above. Details of these computations are provided in the Appendix. Table 2, 3 and 4 also report the expected profits in the different cases.

4 Hidden orders and the probability of informed trading: some empirical predictions

The objective of this section is to compare our conclusions with recent empirical results, and to draw new predictions to be tested in opaque markets. We first analyze the informational content of the limit order book, then the determinants of the submission of informed hidden orders. We focus on the case where \( \pi \leq \frac{\gamma}{\sigma} \), so that there are exchanges in equilibrium.

4.1 The multiple equilibria issue

In the previous section, we have shown that there often exist multiple equilibria. Many of these are “bootstrap” equilibria: they occur only because they are expected to occur. There is indeed no economic intuition for such a multiplicity. Driskill (2006) reviews and compares all the different criteria that may be used to select a unique equilibrium in dynamic rational expectations models. Our model is not dynamic, thus these criteria cannot not apply here. However, their analysis provides us with some tools that help us
restricting the set of admissible equilibria. In particular, we exploit two conditions quoted in this paper. First, according to McCallum (1983), “solution formulae must be valid for all admissible parameter values”. Second, Driskill (2006) suggest to select the solution that would arise in a limit model, that would be very similar to the model studied, but that would be characterized by a unique solution (in his case, the author’s limit model to an infinite-horizon model would be the similar finite-horizon model).

Both arguments induce us to select equilibria such that the informed liquidity suppliers adopts a perfect “camouflage” strategy, i.e. such that $\lambda^* = \lambda^c = \frac{\gamma}{\sigma - \gamma} \frac{1 - \pi_l}{\pi_l}$. This equilibrium exists for all $\pi$ and whatever market organization. Besides, it is the unique equilibrium in the transparent market for $\pi \in \left[ \frac{\gamma}{\sigma - \gamma + \gamma}, \frac{\gamma}{\sigma} \right]$ and in the opaque market without (resp. with) marketable limit orders for $\pi \in \left[ \frac{\gamma - \frac{\Delta s}{\sigma - \gamma} \sigma - \frac{\Delta s}{\sigma}, \frac{\gamma}{\sigma} \right]$ (resp. for $\pi \in \left[ \frac{\gamma - \frac{\Delta s}{\sigma} \sigma - \frac{\Delta s}{\sigma}, \frac{\gamma}{\sigma} \right]$). Notice though that this does not necessarily lead to a unique equilibrium. In particular, we cannot use any existing selection criterion to determine the frequency at which the informed liquidity supplier submits a hidden or a small limit order, when he is indifferent between both types of orders (see Propositions 4 or 5 for instance).

Finally, our empirical predictions are similar whether marketable limit orders are authorized or not. For clarity, we only present the case where they are not authorized. The following Corollaries apply to the case where marketable limit orders are authorized by replacing condition $\pi \leq \frac{\gamma - \Delta s}{\sigma - \Delta s}$ by condition $\pi \leq \frac{\gamma - \frac{\Delta s}{\sigma} \sigma - \frac{\Delta s}{\sigma}}{\sigma}$.  

### 4.2 The informational content of the limit order book in the opaque market

Our model shows that the informed liquidity supplier may benefit from the submission of a hidden order. Such a strategy allows her to trade a large volume, without revealing her presence. This is the case because the uninformed liquidity demander updates his beliefs less strongly on observing a small than on observing a large limit order at price $A_1$, as shown in Corollary (1).

**Corollary 1** At the camouflage equilibrium, the price impact of the observation of a large limit order $D^v = 2$ is larger than the price impact of the observation of a small limit order $D^v = 1$, i.e. $|E(\tilde{\nu}/D^v = 2) - \nu_0| \geq |E(\tilde{\nu}/D^v = 1) - \nu_0|$.

What is the impact of hidden orders on market efficiency? Does the limit order book contain more information on the value of the security in a transparent or in an opaque market? To provide an answer
to this question, empirical researchers must condition their analysis on the same information space as the market participants. The later indeed do not have access, in the opaque market, to the total limit order book, but only to its visible part. However, if someone submits a market or a marketable limit order which size exceeds the depth displayed in the order book, and if a larger quantity of this order is executed than this visible depth, then it means that the visible depth is lower than the total depth. By convention, we say that such an order “tests” the presence of hidden depth, and that this hidden depth is then “revealed”.

We now study the impact of a hidden order’s revelation on the uninformed liquidity demander’s beliefs. We define the expected value of the security, conditional on a hidden order to have been submitted and revealed, as $E(\tilde{v}|D^h_{rev} = 1)$. Our model enables us to shed light on the existing empirical debate relative to hidden orders: do hidden orders contain information on the future value of the security?

Tuttle (2003) seems to indicate that the hidden depth has an informational content. She finds a significant temporary price impact of the reserve size (i.e. the hidden depth) at the inside market in the Nasdaq. This result is coherent with the following Corollary of our model.

**Corollary 2** If $h + \left(1 - \frac{\Delta}{\gamma}\right)s > 0$ and if $\frac{\Delta}{\gamma\gamma - 1} < \pi \leq \frac{\gamma - \Delta s}{\gamma - \Delta s}$, then the revelation of a hidden order has a price impact, i.e. $|E(\tilde{v}/D^h_{rev} = 1) - v_0| > 0$.

Indeed, Tables 2, 3 and 4 report that under the conditions quoted above, we have $\chi^* > 0$. Since it is likely that the informed liquidity supplier has submitted a hidden order in equilibrium, such an order may contain information as any order potentially posted by an agent who is better informed.

But on the other hand, according to Pardo and Pascual (2003), revealing a hidden order has no permanent price impact, relative to the price impact of an equally sized and matched ordinary trade. Our model however shows that this result do not contradict the previous one: it does not imply that hidden orders would not be informative. The magnitude of the belief’s update when the uninformed liquidity demander detects the presence of a large hidden order, relative to what it would be if he observed a fully displayed limit order of the same size, indeed depends on the equilibrium strategy of the informed liquidity supplier. Our model yields the following Corollary.

**Corollary 3** At the camouflage equilibrium, the informational content of a revealed hidden order is smaller than the information content of a non-hidden order of the same size, i.e. $|E(\tilde{v}/D^h = 2 \cap D^h = 1) - v_0| \leq$
Our model is thus coherent with the results of Tuttle (2003) and Pardo and Pascual (2003).

4.3 The informed liquidity supplier’s hidden order submission

We have suggested that the informed liquidity supplier could benefit from the submission of a hidden order, due to the impact of the observation of a large visible depth on the uninformed liquidity demander’s beliefs. Corollary 2 however shows that the necessary conditions for informed liquidity supplier to submit hidden orders in equilibrium are
\[ h + \left( 1 - \frac{\Delta}{\gamma} \right) s > 0 \quad \text{and} \quad \frac{\gamma}{\sigma - \gamma + \gamma l} < \pi \leq \frac{\gamma - \Delta l}{\sigma - \Delta l}. \]
Otherwise, there exist equilibria such that she perfectly displays her limit orders.

The informed liquidity supplier is induced to submit large limit orders in order to extract the highest possible profit from her information. However, since the liquidity demander is conscious about this incitement, she must take into account the impact of a large order’s submission on his beliefs. The following Corollary shows that the highest the adverse selection degree, the less frequently does the informed liquidity supplier display a large limit order, in order to avoid being detected.

**Corollary 4** At the camouflage equilibrium, the frequency with which the informed liquidity supplier submits and displays a large limit order, i.e. \( \lambda^c \), decreases with the adverse selection degree \( \pi \).

Indeed, \( \frac{\gamma}{\sigma - \gamma + \gamma l} \frac{\pi}{1 - \pi} \) decreases with \( \pi \). Conversely, when the informed liquidity supplier submits hidden order, the frequency at which the later are submitted increases with \( \pi \). Recall however that for \( \pi \leq \frac{\gamma}{\sigma - \gamma + \gamma l} \) and \( \pi > \frac{\gamma - \Delta l}{\sigma - \Delta l} \), the informed liquidity supplier does not need to submit hidden orders.

5 Equilibria and market transparency

In this section, we analyze the uninformed liquidity supplier’s behavior. We now relax the constraint according to which its strategy is exogenously given (cf. Remark 2). We have two objectives: i) proposing more general empirical predictions on the observation of hidden orders, and ii) studying the impact of market transparency on market quality in equilibrium.
5.1 The uninformed liquidity supplier’s strategy

In the model presented in Section 2, we assumed that the uninformed liquidity supplier’s limit order submission was exogenous. In Section 3, we have then analyzed the determinants of the strategies chosen by the informed liquidity supplier and by the uninformed liquidity demander. We now relax this constraint to study the uninformed liquidity supplier’s strategy. We first introduce assumptions on its expected profit.

5.1.1 New assumptions

At date 1, we assume that limit orders are submitted with a probability \((1 - \pi)\) by an uninformed liquidity supplier, willing to trade for liquidity reasons. These agents differ from liquidity demanders in their impatience degree: they are ready to postpone the execution of their trade in exchange from a benefit \(A_1 - v_0\). We do not study here the determinants of the choice between limit and market orders for uninformed agents (see Parlour (1998), Foucault (1999) or Foucault, Kadan and Kandel (2005) for an analysis of these determinants).

We distinguish two types of uninformed liquidity suppliers, depending on their liquidity needs:

i) With a probability \(\theta\), the uninformed liquidity supplier has small liquidity needs, and he always submits a small limit order (offer \(S\), i.e. \(D^f = D^v = 1\)).

ii) With a probability \(1 - \theta\), the uninformed liquidity supplier has large liquidity needs, so that its objective is to trade two units of the asset. It may either submit a small limit order (offer \(S\), i.e. \(D^f = D^v = 1\)), or a large hidden order (offer \(H\), i.e. \(D^f = 2\) and \(D^v = 1\)), or a large unhidden order (offer \(L\), i.e. \(D^f = D^v = 2\)).

In parallel, based on the intuitions developed by Harris (1998), we assume that revealing its large liquidity needs is costly for an uninformed liquidity supplier. This assumption is translated into a fixed cost \(c > 0\), incurred by the uninformed liquidity supplier when he displays a large limit order. We further assume that \(c < \Delta\), i.e. that this agent is better off displaying its large limit order if it ensures its full execution, rather than hiding it but only get a partial execution. The expected profit of this uninformed

\footnote{Harris (1998) indeed sheds light on the existence of parasitic agents, namely front-runners and quote-matchers, who take advantage of the presence of large limit orders in the limit order book.}
liquidity supplier with large liquidity needs writes:

\[ E\Pi_{ul}(S) = (A_1 - v_0) \times \left( \mu_{2MO|1} + \mu_{2MLO|1} + \mu_{1|1} \right) \]

\[ E\Pi_{ul}(H) = (A_1 - v_0) \times \left( 2\mu_{2MO|1} + 2\mu_{2MLO|1} + \mu_{1|1} \right) \]

\[ E\Pi_{ul}(L) = (A_1 - v_0) \times \left( 2\mu_{2MO|2} + 2\mu_{2MLO|2} + \mu_{1|2} \right) - c \]

We look for perfect Bayesian equilibria. Again, we do not exclude mixed strategies, and we therefore introduce the following notations. Let \( l' \) be the probability with which the uninformed liquidity supplier with large liquidity needs displays a large limit order \((L)\), \( h' \) the probability with which it submits a hidden order \((H)\) and \( s' \) the probability with which it submits a small limit order \((S)\). Notice that by definition:

\[ s \equiv \theta + (1 - \theta) s', \]
\[ h \equiv (1 - \theta) h', \]
\[ l \equiv (1 - \theta) l'. \]

This agent would like to always submit hidden orders to avoid revealing its large liquidity needs. However, because of the potential price impact of his large market order, the liquidity demander may not want to submit a larger order than the visible depth. Consequently, a hidden order may not be fully-executed. The large uninformed liquidity supplier therefore faces a trade-off between the cost it incurs to display a large limit order, and its probability of (full) execution.

### 5.1.2 Equilibrium strategies

We now characterize the uninformed liquidity supplier’s equilibrium strategies, in the opaque and in the transparent market, when marketable limit orders are authorized and when they are not. The equilibrium reads from the Propositions derived in Section 3.

**Equilibrium in the opaque market** The large uninformed liquidity supplier is better off disclosing its large limit order than displaying it if both order types have the same probability of execution. But whatever adverse selection, the Propositions in Section 3 show that the liquidity demander’s reaction is identical, whatever the visible depth. Consequently, in equilibrium, the uninformed liquidity demander never displays a large limit order, so that \( l'^* = 0 \).
In parallel, in the opaque market, submitting a small limit order with a strictly positive probability is a dominated strategy for the large uninformed liquidity supplier. If he submits a hidden order instead, he may expect a larger trading volume, without incurring any revelation’s cost. Thus in equilibrium in the opaque market, \( s^* = 0 \).

Finally, whatever adverse selection, the large uninformed liquidity supplier always submits hidden order, i.e. \( h^* = 1 \). Consequently, in equilibrium:

\[
\begin{align*}
    s^* &= \theta, \\
    h^* &= (1 - \theta), \\
    l^* &= 0.
\end{align*}
\]

The following Proposition describes the uninformed liquidity demander’s reaction according to the Propositions derived in Section 3.

**Proposition 8** In equilibrium, if marketable limit order are (resp. not) authorized,

a) If \( \pi > \frac{\gamma}{\sigma} \), then the liquidity demander submits no order \( M = 0 \).

b) If \( \frac{\gamma - \gamma \theta}{\sigma - \frac{\gamma \theta}{\sigma}} < \pi \leq \frac{\gamma}{\sigma} \) (resp. \( \frac{\gamma - \gamma \theta}{\sigma - \frac{\gamma \theta}{\sigma}} < \pi \leq \frac{\gamma}{\sigma} \)), then the liquidity demander submits a small market order \( M = 1 \).

c) If \( \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \frac{\gamma \theta}{\sigma}} \) (resp. \( \pi \leq \frac{\gamma - \gamma \Delta \theta}{\sigma - \Delta \gamma \theta} \)), then the liquidity demander submits a large marketable limit order \( M = 2^{MLO} \) (resp. market order \( M = 2^{MO} \)).

In any case, in equilibrium, the informed liquidity supplier never displays a large limit order.

**Equilibrium in the transparent market** In the transparent market, the uninformed liquidity supplier’s strategies space is restricted to \( h^* \equiv 0 \). Its optimal strategy is thus to display a large limit order if he expects a large trading volume, and a small limit order otherwise. Proposition 1 shows that (whatever market organization) the uninformed liquidity demander submits a large order in the transparent market if and only if \( \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} \). Consequently, the uninformed liquidity supplier submits a large order with a probability \( l^* = 1 \) if and only if \( \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} \), and otherwise submits a small limit order with a probability \( s^* = 1 \).
Table 5 reports the equilibria in the transparent and in the opaque market, when marketable limit orders are authorized and when they are not.

5.2 Analysis of hidden depth

Given the exogeneity of the uninformed liquidity supplier’s strategy, our empirical predictions in Section 4 only applied to the informed liquidity supplier. Relaxing this constraint enables us to extend our analysis to the hidden depth in the limit order book. Again, we focus on the case where \( \pi \leq \frac{3}{4} \), so that there is trading in equilibrium. We assume that when the use of a hidden order is not economically justified, the informed liquidity supplier submits a small limit order.

We first analyze the presence of hidden depth in the limit order book, depending on market conditions. Corollary (5) analyses the probability with which there is hidden depth at the best quotes, when the liquidity demander observes a small limit order, i.e. \( \Pr (D^h = 1 | D^v = 1) \).

**Corollary 5** The frequency with which there is hidden depth at the best quotes, conditional on the visible depth being small increases with adverse selection \( \pi \) and decreases with the asset volatility \( \sigma \).

When the asset volatility \( \sigma \) increases, the liquidity demander is less induced to submit large orders when he observes \( D^v = 1 \), since his adverse selection risk increases. Consequently, submitting a hidden order is less profitable for the informed liquidity supplier. Conversely, when \( \pi \) increases, the informed liquidity supplier enters the market more frequently. She submits a hidden order more often than uninformed liquidity suppliers (due to the presence of uninformed agents with small liquidity needs).

We have seen that liquidity demanders could detect the presence of hidden depth. Our model draws new empirical predictions on the conditions for the liquidity demander to test the presence of hidden depth.

**Corollary 6** The frequency with which the uninformed liquidity demander submits a larger order than the visible depth, i.e. \( \Pr (M = 2 | D^v = 1) \), decreases with the probability of informed trading \( \pi \), with the tick size \( \Delta \) and with the volatility of the asset \( \sigma \).

In our model, it may be costly for the uninformed liquidity demander to submit a large order when he observes \( D^v = 1 \). In equilibrium, he tests the presence of a hidden order when he observes a small
limit order if \( \pi < \frac{\gamma-\Delta^s}{\sigma} \) when marketable limit orders are not authorized, and if \( \pi < \frac{\gamma-\Delta^s}{\sigma} \) when they are. Therefore, when the adverse selection level \( \pi \), when the asset volatility \( \sigma \), or when the tick size \( \Delta \) increase, this condition may become obsolete.

This prediction is partly consistent with the findings of Aitken et al. (2001), who find a negative correlation between the hidden order use and the relative tick size.

Finally, the frequency with which a hidden order is revealed is:

\[
\Pr \left( M = 2 | D^h = 1 \right) = \Pr (M = 2 | D^u = 1) \times \Pr \left( D^h = 1 | D^u = 1 \right)
\]

Combining corollaries (5) and (6) shows that the adverse selection degree differently impact the frequency with which hidden orders are submitted (\( i.e. \) \( \Pr (D^h = 1 | D^u = 1) \)) and the frequency with which they are tested (\( i.e. \) \( \Pr (M = 2 | D^u = 1) \)). The impact of adverse selection on this probability \( \Pr (M = 2 | D^h = 1) \) is thus ambiguous. However, it decreases with asset volatility.

5.3 The impact of market transparency on market quality

We finally study the impact of the move from a transparent market to an opaque market.

5.3.1 Impact of market transparency on market efficiency

We compare the uninformed liquidity demander’s beliefs update in both market organizations. In many papers related to market transparency, an increase in market liquidity goes often along with a decrease in market efficiency (see for instance Bloomfield and O’Hara (2000)). To study efficiency, we compute the difference of the liquidity demander’s conditional expectations on the value of the security in the opaque and in the transparent market. Let \( E_x (\tilde{v} | D^x) \) denote the conditional expected value of the security, when the market is transparent \( (x = t) \) or opaque \( (x = o) \). Surprisingly here, a decrease in the \textit{ex ante} transparency of the market increases its efficiency.

**Corollary 7** Whatever the level of adverse selection \( \pi \), the expected value of the security, conditional on the observation of the visible depth \( D^v \), is closer to its true value \( v_L \) in the opaque market. Indeed,

\[
|E_t (\tilde{v} | D^v = 1) - v_0| \leq |E_o (\tilde{v} | D^v = 1) - v_0|.
\]
When marketable limit orders are authorized, the informed liquidity supplier is induced to submit large orders (hidden or not) more frequently in the opaque market than in the transparent market (indeed $\frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} < \frac{\gamma - \frac{\gamma \theta}{\sigma - \gamma \theta}}{\sigma - \frac{\gamma \theta}{\sigma - \gamma \theta}}$). She thus reveals her presence.

In parallel, since $\frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} < \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta}$, the informed liquidity supplier submits large orders less frequently in the opaque market than in the transparent market when marketable limit orders are authorized, which could lead to an opposite effect of the market transparency on its efficiency. However, since in this case the uninformed liquidity supplier submits large undisclosed limit order more frequently in the transparent market, the informed liquidity supplier is able to better hide her presence in the transparent market.

5.3.2 Impact of market transparency on market liquidity

We now analyze the impact of the move to a transparent market on market liquidity. Since we have assumed that the prices were fixed to focus on the size of the limit order submitted and displayed at price $A_1$, the best measure of market liquidity in our model is the depth at this price. Actually, there exist two measures of this depth, namely the visible depth, $D^v$, and the total depth, $D^t$. The move to an opaque market may have different effects on both measures.

On the one hand, the proportion of large undisclosed limit orders decreases in the opaque market: the large uninformed liquidity supplier never submits such orders if it has the opportunity to hide its large order for the same probability of execution. But this decrease of the visible depth does not necessarily mean that the total depth decreases in the opaque market. The uninformed liquidity supplier who submits a small limit order in the transparent market may indeed decide to submit a large hidden order in the opaque market, where it does not incur a cost of revelation. Differently from the previous Corollaries, the authorization of marketable limit orders has an impact on this decision.

**Corollary 8** Visible depth $D^v$ decreases in the move to an opaque market. The impact of the move to an opaque market on the total depth $D^t$ however depends on adverse selection and on market organization.

i) If marketable limit orders are authorized, the total depth $D^t$ increases in the opaque market.

ii) If marketable limit orders are not authorized, the total depth $D^t$ strictly decreases in the opaque market if $\frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} < \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta}$, and strictly increases if $\frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} < \pi \leq \frac{\gamma}{\sigma}$. 

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The informed liquidity supplier mimics the uninformed limit order trader’s behavior. In equilibrium, the proportion of large undisclosed orders submitted by this uninformed agent decreases in the opaque market. Consequently, the informed liquidity supplier must submit the same type of orders less frequently in the opaque market. Both effects lead to a decrease in the visible depth.

The total depth however is composed of the visible plus the hidden depth, i.e. \( D^t = D^v + D^h \). Since \( D^h \) is equal to zero in the transparent market, the total depth may only be higher than in the opaque market if the increase in the visible depth, discussed above, is sufficiently high to compensate the absence of hidden depth. When this is the case, surprisingly, market liquidity may be reduced in the opaque market.

If marketable limit orders are authorized, then since \( \frac{\gamma - \gamma_0}{\sigma - \gamma_0} < \frac{\gamma - \gamma_0}{\sigma - \gamma_0} \), the size of the uninformed liquidity demander’s order is maximal for larger levels of adverse selection in the opaque market than in the transparent market. This in turn induces liquidity suppliers to submit large orders (hidden or not) more frequently, which leads to an increase of the total depth in the opaque market.

The case where marketable limit orders are not authorized is more complex. On the one hand, when \( \pi \in [\frac{\gamma - \gamma_0}{\sigma - \gamma_0}, \frac{\gamma - \gamma_0}{\sigma - \gamma_0}] \), the liquidity demander submits a large market order in the transparent market but a small order in the opaque market. Consequently, liquidity suppliers are induced to submit a large limit order in the transparent market, but not in the opaque market. In the later, they are indifferent between submitting a small or a hidden order which both get partially executed. Total depth strictly decreases in the opaque market when these agents decide to submit small orders when they are indifferent.

On the other hand, when \( \pi \in [\frac{\gamma - \gamma_0}{\sigma - \gamma_0}, \frac{\gamma}{\sigma}] \), the liquidity demander submits a small order in both market structures. The uninformed liquidity supplier is thus induced to submit a small limit order in the transparent market in order to avoid the cost of revelation, while it may submit a large hidden order in the opaque market instead. It is imitated by the informed liquidity supplier, so that the total depth is larger in the opaque market.

5.3.3 Impact of market transparency on the agents’ expected profits

The agents’ expected profits strongly depend on the total depth of the limit order book. In our model indeed, all strategic agents initially want to trade the maximal volume of two units. Since the total depth depends on the type of aggressive orders that are authorized, the impact of market transparency on their
expected profits is linked to the market structure. Figures 3, 4 and 5 (resp. 6, 7 and 8) represent the agents’ expected profits in the transparent and in the opaque market, when marketable limit orders are not authorized (resp. are authorized).

**Corollary 9** When marketable limit orders are authorized,

a) The expected profits of both the informed and the uninformed large liquidity suppliers are larger in the opaque market.

b) The expected profits of the uninformed liquidity demander are strictly smaller in the opaque market if \( \pi \in ]\frac{\gamma - \gamma_0}{\sigma - \gamma}, \frac{\gamma - \gamma_0}{\sigma - \gamma}]. \)

When marketable limit orders are authorized, for a given level of adverse selection, the liquidity demander has the opportunity to trade a larger volume at price \( A_1 \) in the opaque market than in the transparent market (increase in market liquidity). However, he is more exposed to the adverse selection risk in the opaque market, since the informed liquidity supplier hides her presence by using hidden orders (increase in market efficiency). This second effect dominates. The uninformed liquidity supplier though benefits from the opaque market, since it does not incur any cost of revelation while still getting a large trading volume.

When marketable limit orders are not authorized though, the liquidity demander takes into account the potential price impact of his order, and is therefore less induced to submit a large market order. The following Corollary holds.

**Corollary 10** When marketable limit orders are not authorized,

a) The expected profits of the informed liquidity supplier and those of the and the uninformed liquidity demander are smaller in the opaque market,

b) The expected profits of the uninformed large liquidity supplier are strictly smaller in the opaque market if \( \pi \in ]\frac{\gamma - \Delta_0}{\sigma - \Delta_0}, \frac{\gamma - \gamma_0}{\sigma - \gamma_0} \) and strictly higher if \( \pi \in ]\frac{\gamma - \gamma_0}{\sigma - \gamma_0}, \frac{\gamma - \Delta_0}{\sigma - \Delta_0} \).

Thus opacity counter-intuitively decreases the informed agent’s profits. The negative effect on her profits of the decrease in the visible depth in the opaque market overcomes the positive effect of the
increase of the total depth. Although she may use hidden orders in this market, the decrease of the visible depth makes her camouflage strategy more difficult to implement.

The impact of transparency on the uninformed liquidity supplier is more ambiguous. On the one hand, it is less exposed to “parasitic traders” in the opaque market. On the other hand, when marketable limit orders are not authorized, the probability of execution of its order decreases in the opaque market. Depending on adverse selection, either of both effect may dominate.

6 Conclusion

In this paper, we present a simple theoretical model of trading in a limit order book, to investigate the impact of the authorization of hidden orders on agents’ strategies and expected profits. This model builds on the assumption that there exists an informed agent who submits limit orders. We show that in equilibrium, the informed liquidity supplier has a camouflage strategy: she tries to mimic the uninformed liquidity supplier’s behavior in order not to be detected. Besides, this informed agent uses hidden orders, but only when some conditions are fulfilled. When these conditions hold, submitting a hidden order enables her to trade a large volume, without signalling her presence.

Our model draws new empirical predictions on the presence of hidden depth.

Finally, we find that the impact of market transparency on market quality depends on the authorization of marketable limit orders. When those orders are authorized, hidden orders not only increase market efficiency, but also the total depth in the limit order book. This however has a negative effect on the liquidity demander’s expected profits, since he is more exposed to adverse selection in the opaque market. In the opposite case, the informed liquidity supplier’s expected profit is counter-intuitively lower in the opaque market.
7 References

References


8 Proofs

Proof of Lemma 2: Liquidity demander’s reaction when \( D^v = 2 \)

When an uninformed buyer observes \( D^v = 2 \), he may either submit no order, or a small order, or a large market or marketable limit order. His corresponding expected profit writes:

\[
E\Pi_u(0 | D^v = 2) = 0
\]
\[
E\Pi_u(1 | D^v = 2) = E(v | D^v = 2) - A_1 + \beta = \gamma - \frac{\pi \lambda}{\pi \lambda + (1 - \pi)l}
\]
\[
E\Pi_u(2^MO | D^v = 2) = E\Pi_u(2^{MLO} | D^v = 2) = 2E\Pi_u(1 | D^v = 2).
\]

The proof immediately follows. ■

Proof of Lemma 3: Liquidity demander’s reaction when \( D^v = 1 \) with marketable limit orders

• Dominated strategy

When he submits a market order, the liquidity demander knows his trading volume, but not his transaction price:

\[
E\Pi_u(M = 2^{MLO} | D^v = 1) = (2 \times E(v | D^v = 1 \cap D^h = 0) + 2\beta - A_1 - A_2) \times \Pr(D^v = 1 \cap D^h = 0) + 2 \times (E(v | D^v = 1 \cap D^h = 1) + \beta - A_1) \times \Pr(D^v = 1 \cap D^h = 1)
\]

Which rewrites:

\[
E\Pi_u(M = 2^{MLO} | D^v = 1) = 2 \times \left( \gamma - \sigma - \frac{\pi (\xi + \chi)}{\pi (\xi + \chi) + (1 - \pi) (s + h)} \right) - \Delta \times \frac{(\pi \xi + (1 - \pi) s)}{\pi (\xi + \chi) + (1 - \pi) (s + h)}
\]
If he submits a marketable limit order, he knows his transaction price, but not his trading volume:

\[
\text{EII}_u \left( M = 2^{MLO} \mid D^v = 1 \right) = \left( E \left( \tilde{v} \mid D^v = 1 \cap D^b = 0 \right) + \beta - A_1 \right) \times \text{Pr} \left( D^v = 1 \cap D^b = 0 \right) + 2 \times \left( E \left( \tilde{v} \mid D^v = 1 \cap D^b = 1 \right) + \beta - A_1 \right) \times \text{Pr} \left( D^v = 1 \cap D^b = 1 \right)
\]

Which rewrites:

\[
\text{EII}_u \left( M = 2^{MLO} \mid D^v = 1 \right) = \gamma \pi (\xi + 2\chi) + (1 - \pi) (s + 2h) \pi (\xi) \pi (\xi + \chi) + (1 - \pi) (s + h) - \sigma \pi (\xi + 2\chi) + (1 - \pi) (s + h) - \sigma \pi (\xi + 2\chi) \pi (\xi + \chi) + (1 - \pi) (s + h)
\]

When \( \gamma < \Delta \), submitting a marketable limit order is a dominant strategy when \( D^v = 1 \), even out of the equilibrium path. If we compare the expected profits of both strategies:

\[
(\Delta - \gamma) \times \left( \frac{\pi \xi + (1 - \pi) s}{\pi (\xi + \chi) + (1 - \pi) (s + h)} \right) > \frac{-\sigma}{\pi (\xi + \chi) + (1 - \pi) (s + h)}
\]

Under our assumptions, for \( \gamma < \Delta \) and \( s > 0 \), the strategy \( M = 2^{MO} \) is strictly dominated for the liquidity demander. We therefore exclude this strategy when marketable limit orders are authorized.

- **Liquidity demander’s reaction when \( D^v = 1 \)**

When an uninformed buyer observes \( D^v = 1 \), he may either submit no order, or a small order, or a marketable limit order (since submitting a large market order is a strictly dominated strategy in this case). His corresponding expected profit writes:

\[
\begin{align*}
\text{EII}_u \left( 0 \mid D^v = 1 \right) &= 0 \\
\text{EII}_u \left( 1 \mid D^v = 1 \right) &= \gamma - \sigma \frac{\pi (\xi + \chi)}{\pi (\xi + \chi) + (1 - \pi) (s + h)} \\
\text{EII}_u \left( 2^{MLO} \mid D^v = 1 \right) &= \text{EII}_u \left( 1 \mid D^v = 1 \right) - \frac{(\sigma - \gamma) \pi \chi - \gamma (1 - \pi) h}{\pi (\xi + \chi) + (1 - \pi) (s + h)}
\end{align*}
\]
Let \( x \succ y \) mean that strategy \( x \) strictly dominates strategy \( y \). Since we assume that \( 0 < \gamma < \sigma \), the following conditions hold:

\[
M = 1 \succ M = 0 \iff \xi + \chi < \frac{\gamma}{\sigma - \gamma} \frac{1 - \pi}{\pi} (s + h)
\]

\[
M = 2^{MLO} \succ M = 0 \iff \xi + 2\chi < \frac{\gamma}{(\sigma - \gamma)} \frac{1 - \pi}{\pi} (s + 2h)
\]

\[
M = 2^{MLO} \succ M = 1 \iff \chi < \frac{\gamma}{\sigma - \gamma} \frac{1 - \pi}{\pi} h
\]

\[\Box\]

**Proof of Lemma 4:** Liquidity demander’s reaction when \( D^v = 1 \) without marketable limit orders

- **Liquidity demander’s reaction when \( D^v = 1 \)**

When an uninformed buyer observes \( D^v = 2 \) submits a large market order, his expected profit writes:

\[
\begin{align*}
E\Pi_u(2^{MO} | D^v = 1) &= 2 (E(\tilde{v} | D^v = 1) - A_1 + \beta) - (A_2 - A_1) \Pr(D^t = 1 | D^v = 1) \\
&= 2 \left( \gamma - \sigma \pi \frac{(\xi + \chi)}{(\xi + \chi) + (1 - \pi) (s + h)} \right) \\
&\quad - \Delta \left( \frac{\pi \xi + (1 - \pi) s}{(\xi + \chi) + (1 - \pi) (s + h)} \right).
\end{align*}
\]

Since we assume that \( 0 < \gamma < \sigma \), the following conditions hold:

\[
M = 1 \succ M = 0 \iff \xi + \chi < \frac{\gamma}{\sigma - \gamma} \frac{1 - \pi}{\pi} (s + h)
\]

\[
M = 2^{MO} \succ M = 0 \iff \chi + \frac{\Delta}{2(\sigma - \gamma)} \xi < \frac{\gamma}{\sigma - \gamma} \frac{1 - \pi}{\pi} \left( h + \left( 1 - \frac{\Delta}{2\gamma} \right) s \right)
\]

\[
M = 2^{MO} \succ M = 1 \iff \chi + \frac{\Delta}{(\sigma - \gamma)} \xi < \frac{\gamma}{(\sigma - \gamma)} \frac{1 - \pi}{\pi} \left( h + \left( 1 - \frac{\Delta}{\gamma} \right) s \right).
\]

Notice that in some cases, the fact that one condition holds guarantees that the other two also hold. For instance, if \( M = 2^{MO} \succ M = 1 \), then \( M = 1 \succ M = 0 \). If \( M = 0 \succ M = 1 \), then \( M = 1 \succ M = 2^{MO} \).

This leads to Lemma 4. \[\Box\]
• **Liquidity demander’s ex ante expected profit**

Accounting for the liquidity demander’s expected profit corresponding to each strategy and computed above, his *ex ante* expected profit writes:

\[
E\Pi_{ante}^u = \Pr (D^u = 2) \times \left( \gamma - \sigma \frac{\pi \lambda}{\pi \lambda + (1 - \pi) l} \right) \times (2\mu_{2|2} + \mu_{1|2})
+ \Pr (D^u = 1) \times \left( 2 \left( \gamma - \sigma \frac{\pi (\xi + \chi)}{\pi (\xi + \chi) + (1 - \pi) (s + h)} \right) - \Delta \left( \frac{\pi (\xi + (1 - \pi) s)}{\pi (\xi + (1 - \pi) (s + h))} \right) \right)
\]

Finally,

\[
E\Pi_{ante}^u = \left( (1 - \pi) \gamma l - (\sigma - \gamma) \pi \lambda \right) \times \left( 2\mu_{2|2} + \mu_{1|2} - \mu_{1|1} - 2\mu_{2MO|1} - 2\mu_{2MLO|1} \right)
+ \left( (\gamma - \sigma \pi) \times \left( \mu_{1|1} + 2\mu_{2MO|1} + 2\mu_{2MLO|1} \right) \right)
- \Delta \left( \pi \xi + (1 - \pi) s \times \mu_{2MO|1} \right)
+ \left( (\sigma - \gamma) \pi \xi - \gamma (1 - \pi) s \times \mu_{2MLO|1} \right)
\]

**Proof of Proposition 1**

• If \( \pi < \frac{\gamma l}{\sigma - \gamma + \pi} \), then \( \frac{\gamma}{\sigma - \gamma} (1 - \pi) l > 1 \) implies that there exists an equilibrium in pure strategies such that \( \lambda^* = 1 \).

• If \( \pi = \frac{\gamma l}{\sigma - \gamma + \pi} \), the the liquidity demander is indifferent between \( M = 2^{MO} \) (or \( M = 2^{MLO} \)), \( M = 1 \) and \( M = 0 \) when he observes \( D^u = 2 \), if \( \lambda^* = 1 \). Thus such an equilibrium exists. Let us check whether this equilibrium may be Pareto-ranked.

Assume first that marketable limit orders are not authorized. If \( \lambda = 1 \), the liquidity demander submits
an order $M = 2^{M_0}$ when $D^v = 1$ (see Lemma 4). His \textit{ex ante} expected profit is thus

$$E\Pi_{\text{ante}} = (\sigma - \gamma) \frac{\gamma l}{\sigma - \gamma + \gamma l} (1 - \lambda) \times (2\mu_{2|2} + \mu_{1|2} - 2)$$

$$+ 2 (\sigma - \gamma) \left( \frac{\gamma (1 - l)}{\sigma - \gamma + \gamma l} \right) - \Delta s (1 - \pi)$$

- If $\mu_{2|2} = 1$ then $\lambda = 1$ and his profit becomes:

$$E\Pi_{\text{ante}} = \left( \frac{\sigma - \gamma}{\sigma - \gamma + \gamma l} \right) (2\gamma (1 - l) - \Delta s)$$

- If $\mu_{2|2} = 1 - \varepsilon$ and $\mu_{1|2} = \varepsilon$, then $\lambda = 1 - \varepsilon$ and is profit writes:

$$E\Pi_{\text{ante}} = \left( \frac{\sigma - \gamma}{\sigma - \gamma + \gamma l} \right) \gamma l \times (-\varepsilon) + \left( \frac{\sigma - \gamma}{\sigma - \gamma + \gamma l} \right) (2\gamma (1 - l) - \Delta s)$$

Consequently, submitting $\mu_{2|2} = 1$ is \textit{ex ante} an optimal strategy for the liquidity demander. It can be similarly shown that $\mu_{2|2} = 1$ is also a dominant strategy when marketable limit orders are authorized.

\textbf{Proof of Proposition 2}

If $\pi > \frac{\gamma}{\sigma}$, the only sustainable equilibrium is such that there is no trading in equilibrium. In this case indeed, conditions $\lambda < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l$ and $\xi + \chi < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} (s + h)$ cannot simultaneously hold, since they imply:

$$1 < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} (l + s + h) \Rightarrow \pi < \frac{\gamma}{\sigma}.$$ 

Thus in equilibrium $M = 0$. The equilibrium is characterized by the following conditions, which guarantee that no deviation is profitable:

$$\xi + \chi \geq \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} (s + h)$$

$$\lambda \geq \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l$$

$$\mu_{0|1} = \mu_{0|2} = 1.$$ 

Thus in equilibrium:
\[
\frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} I \leq \lambda^* \leq \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} I + \frac{\pi \sigma - \gamma}{\pi (\sigma - \gamma)} \\
\chi^* + \xi^* = 1 - \lambda^* \\
\mu_{0|1} = \mu_{0|2} = 1
\]


\textbf{Proofs of Propositions 3 and 4}

When the liquidity demander is \textit{ex post} indifferent between several strategies when he observes \(D^v\), we look for the strategy that maximizes his \textit{ex ante} expected profit. We propose to focus our analysis on the opaque market, when marketale limit orders are not authorized, and when \(h + \left(1 - \frac{\Delta}{\gamma}\right) s > 0\). Equilibria for other market organizations and other parameter values can be easily deduced from those. We now study the different cases.

- 1st case: \(\pi < \frac{\gamma - \Delta s}{\sigma - \Delta s}\)

We first look for equilibria that would be characterized by a large trading volume. Many cases may appear:

a) The liquidity demander is strictly better off submitting \(M = 2^{MO}\) when \(D^v = 1\), and \(M = 2^{MO}\) when \(D^v = 2\).

\[
\lambda < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} I \text{ and } \chi < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} \left(h + \left(1 - \frac{\Delta}{\gamma}\right) s\right) \text{ and } \chi = 1 - \lambda \text{ (a)}
\]

Since in equilibrium \(\chi + \lambda = 1\), the existence of at least one strict inequality in the first two conditions implies that:

\[
\pi < \frac{\gamma - \Delta s}{\sigma - \Delta s}.
\]

b) The liquidity demander is indifferent between \(M = 2^{MO}\), \(M = 1\) and \(M = 0\) when \(D^v = 2\) but chooses \(\mu_{2^{MO}|2} = 1\), and is strictly better off submitting \(M = 2^{MO}\) when \(D^v = 1\).

\[
\lambda = \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} I \text{ and } \chi < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} \left(h + \left(1 - \frac{\Delta}{\gamma}\right) s\right) \text{ and } \chi = 1 - \lambda \text{ and } \mu_{2^{MO}|2} = 1
\]

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The condition $\pi < \frac{\gamma - \Delta s}{\sigma - \Delta s}$ guarantees that if $\lambda = \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) l$, then $\chi < \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) \left( h + \left( 1 - \frac{\Delta}{\gamma} \right) s \right)$. Let us check whether condition $\mu_{2MO|2} = 1$ holds in this case. The liquidity demander’s *ex ante* expected profit does not depend on his reaction when $D^v = 2$:

$$E\Pi_{u}^{\text{ante}} = 2(\gamma - \Delta s - (\sigma - \Delta s) \pi).$$

Thus such an equilibrium exists and is not Pareto suboptimal.

c) The liquidity demander is indifferent between $M = 2^{MO}$ and $M = 1$ when $D^v = 1$ but chooses $\mu_{2MO|1} = 1$, and is strictly better off submitting $M = 2^{MO}$ when $D^v = 2$.

$$\lambda < \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) l \text{ and } \chi = \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) \left( h + \left( 1 - \frac{\Delta}{\gamma} \right) s \right) \text{ and } \chi = 1 - \lambda \text{ and } \mu_{2MO|1} = 1$$

If $\chi = \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) \left( h + \left( 1 - \frac{\Delta}{\gamma} \right) s \right)$, then $\lambda < \frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) l$. Let us check whether condition $\mu_{2MO|1} = 1$ holds in this case. In this case, the liquidity demander’s *ex ante* expected profit does depend on his reaction when $D^v = 1$:

$$E\Pi_{u}^{\text{ante}} = \left( (1 - \pi) \gamma - \Delta s - (\sigma - \pi) \pi \right) \times \left( 2 - \mu_{1|1} - 2\mu_{2MO|1} \right)$$

$$+ (\gamma - \sigma \pi) \times \left( \mu_{1|1} + 2\mu_{2MO|1} \right) - \Delta \left( (1 - \pi) s \right) \times \left( \mu_{2MO|1} \right).$$

Since $\frac{\partial E\Pi_{u}^{\text{ante}}}{\partial \mu_{2MO|1}} = (1 - \pi) \Delta s > 0$, such an equilibrium exists, characterized by $\mu_{2MO|1} = 1$, and this equilibrium is Pareto-optimal.

Finally, if $\pi < \frac{\gamma - \Delta s}{\sigma - \Delta s}$, there exists a multiplicity of equilibria such that:

$$\frac{\gamma}{(\sigma - \gamma)} \left( \frac{1 - \pi}{\pi} \right) l - \frac{(\gamma - \Delta s) - (\sigma - \Delta s) \pi}{\pi (\sigma - \gamma)} \leq \lambda^* \leq \frac{\gamma}{\sigma - \gamma} \left( \frac{1 - \pi}{\pi} \right) l$$

$$\chi^* = 1 - \lambda^*$$

$$\mu_{2|2} = \mu_{2|1} = 1.$$
• 2nd case: $\pi = \frac{\gamma - \Delta s}{\sigma - \Delta s}$

When $\pi \geq \frac{\gamma - \Delta s}{\sigma - \Delta s}$, we have proved that there exists no equilibrium in which the trading volume would be equal to two units, and in which a liquidity demander’s strictly dominant strategy would be to submit a large order $M = 2$, either when he observes $D^v = 2$, or when he observes $D^v = 1$. A necessary condition for the liquidity suppliers to trade two units in equilibrium is thus that the liquidity demander is indifferent between different strategies when he observes $D^v = 1$ and when he observes $D^v = 2$, which implies that in equilibrium:

$$\lambda = \frac{\gamma}{\gamma - \Delta s}$$
$$\chi = 1 - \lambda.$$

Besides, such an equilibrium may be sustained iff the informed liquidity supplier gets the same trading volume when she hides her large order:

$$2\mu_{2\text{MO}|2} + \mu_{1|2} = 2\mu_{2\text{MO}|1} + \mu_{1|1}.$$  

In equilibrium, whatever $\mu_{2\text{MO}|1}^*$,

$$E\Pi_u^{\text{ante}} = (\gamma - \sigma\pi) \times (1 - \mu_{0|1}).$$

Since the liquidity demander is indifferent between different strategies as long as the condition $\mu_{0|1} = 0$ holds, but since the informed liquidity supplier is better off trading a large volume, there exists a Pareto-dominant equilibrium which characterized by:

$$\mu_{2\text{MO}|2} = \mu_{2|1} = 1$$

• 3rd case: $\frac{\gamma - \Delta s}{\sigma - \Delta s} < \pi < \frac{\gamma}{\sigma}$

When $\pi > \frac{\gamma - \Delta s}{\sigma - \Delta s}$, there exists no equilibrium characterized by a large trading volume. We look for equilibria characterized by a small trading volume. The necessary and sufficient conditions for such an
equilibrium to arise are:

\[
\lambda = \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l
\]

\[
\chi^* + \left(\frac{\sigma - \gamma}{\sigma - \gamma} + \Delta\right) \xi \geq \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} \left(h + \left(1 - \frac{\Delta}{\sigma}\right) s\right)
\]

\[
\xi + \chi < \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} (s + h)
\]

\[
2\mu_{2|2} + \mu_{4|2} = 1
\]

\[
\mu_{4|1} = 1.
\]

We must therefore have \(\frac{\gamma - \Delta s}{\sigma - \Delta s} < \pi < \frac{\gamma}{\sigma}\). Besides, for \(\frac{(\gamma - \Delta s) - (\sigma - \Delta s)\pi}{\Delta\pi} < 0\), if condition (1) holds, then condition (2) is always strict. Thus if the two following conditions hold simultaneously:

\[
\lambda^* = \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l
\]

\[
\chi^* + \xi^* = 1 - \lambda^*
\]

the liquidity demander is indifferent between \(M = 2^{MO}\), \(M = 1\) and \(M = 0\) when \(D^{v} = 2\), and he submits an order of size \(M = 0\) or \(M = 1\) when \(D^{v} = 1\) (i.e. \(\mu_{2^{MO}|1} = 0\)). His \textit{ex ante} expected profit writes:

\[
E\Pi_{2^{MO}}^{ante} = (\gamma - \sigma\pi) \times \mu_{4|1}.
\]

Choosing \(\mu_{4|1} = 1\) is indeed a dominant strategy.

Finally if \(\frac{\gamma - \Delta s}{\sigma - \Delta s} < \pi < \frac{\gamma}{\sigma}\), there exists a multiplicity of equilibria such that:

\[
\lambda^* = \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l
\]

\[
\chi^* + \xi^* = 1 - \lambda^*
\]

\[
2\mu_{2^{MO}|2} + \mu_{4|2} = \mu_{4|1} = 1
\]

In particular, there exists an equilibrium such that \(\chi^* = 0\).

\[4\text{th case: } \pi = \frac{\gamma}{\sigma}\]

For \(\pi \geq \frac{\gamma}{\sigma}\) there exists no equilibrium in which it would be strictly optimal for the liquidity demander to submit demand at least one unit when he observes \(D^{v} = 1\). The only case where there is trading in
equilibrium is thus if the liquidity demander is simultaneously indifferent between \( M = 0, M = 1, M = 2^{MO} \) when \( D^v = 2 \), and between \( M = 0, M = 1 \) when \( D^v = 1 \), which writes:

\[
\begin{align*}
\lambda^* &= l \\
\xi + \chi &= (s + h).
\end{align*}
\]

In equilibrium, we must have:

\[
2\mu_{2|2} + \mu_{1|2} = \mu_{1|1}.
\]

Since:

\[
E\Pi_{ante}^u = \gamma - \sigma \pi - \Delta \sigma (s + \gamma (h - \chi)) \times 2\mu_{2MO|1}.
\]

The equilibrium such that \( \mu_{1|1} = \mu_{1|2} = 1 \) is Pareto-dominant: the ex ante expected profit of the liquidity demander is maximal for \( \mu_{2MO|1} = 0 \), and the informed liquidity supplier is strictly better off when her limit order is executed.

- **Equilibrium expected profits**

Finally, when marketable limit orders are not authorized:

\[
\begin{align*}
E\Pi_{ante}^{ante} \left( \pi < \frac{\gamma - \Delta s}{\sigma - \Delta s} \right) &= 2(\gamma - \sigma \pi) - (1 - \pi) \Delta s \\
E\Pi_{ante}^{ante} \left( \frac{\gamma - \Delta s}{\sigma - \Delta s} \leq \pi \leq \frac{\gamma}{\sigma} \right) &= (\gamma - \sigma \pi) \\
E\Pi_{ante}^{ante} \left( \pi > \frac{\gamma}{\sigma} \right) &= 0.
\end{align*}
\]

Besides, for the informed liquidity supplier:

\[
\begin{align*}
E\Pi_{i}^{ante} \left( \pi \leq \frac{\gamma - \Delta s}{\sigma - \Delta s} \bar{v} = v_L \right) &= \pi \times (\Delta + \sigma) \\
E\Pi_{i}^{ante} \left( \frac{\gamma - \Delta s}{\sigma - \Delta s} < \pi \leq \frac{\gamma}{\sigma} \bar{v} = v_L \right) &= \pi \times (\Delta + \sigma) \times \frac{1}{2} \\
E\Pi_{i}^{ante} \left( \pi > \frac{\gamma}{\sigma} \bar{v} = v_L \right) &= 0
\end{align*}
\]

**Proof of Proposition 5**

The reasoning is similar as the Proof of Proposition 3 and 4.
Proofs of Propositions 6 and 7

The reasoning is similar as the Proof of Proposition 3 and 4. Complete proofs are available upon request.

Proof of Corollary 1

We have

\[ |E (\tilde{v}/D^v = 1) - v_0| \leq |E (\tilde{v}/D^v = 2) - v_0| \iff l \leq \lambda. \]

Since \( \pi \leq \frac{\gamma}{\sigma} \) implies \( \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} \geq 1 \), if \( \lambda^c = \frac{\gamma}{(\sigma - \gamma)} \frac{(1 - \pi)}{\pi} l \), then this inequality holds.

Proof of Corollary 2

We have:

\[ |E \left( \tilde{v}/D^t = 2 \cap D^h = 1 \right) - v_0| \leq |E \left( \tilde{v}/D^t = 2 \cap D^h = 0 \right) - v_0| \iff \chi l \leq \lambda h. \]

Since \( \chi^* = 1 - \lambda^c \), the inequality becomes:

\( \chi l \leq \lambda^c h \iff \frac{l}{(h + l)} \leq \lambda^c \iff \pi \leq \frac{\gamma - 2 \theta}{\sigma - \gamma^s} \)

Proof of Corollary 5

According to Bayes’ rule:

\[ \text{Pr} \left( D^h = 1 | D^v = 1 \right) = \frac{\pi \chi + (1 - \pi) h}{\pi (\chi + \xi) + (1 - \pi) (h + s)}. \]

In equilibrium in the opaque market, whatever \( \pi \), \( h = 1 - s \) and \( \chi + \xi = 1 \), thus:

\[ \text{Pr} \left( D^h = 1 | D^v = 1 \right) = \pi \chi + (1 - \pi) (1 - s). \]

If \( \pi \leq \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} \) when marketable limit orders are not authorized, or if \( \pi \leq \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} \) when they are, then \( \chi^* = 1 \) so that:

\[ \text{Pr} \left( D^h = 1 | D^v = 1 \right) = 1 - (1 - \pi) \theta. \]

This probability is increasing in \( \pi \).

Otherwise:
This probability is increasing in \( \pi \) if \( \xi \leq \theta \). Besides, when marketable limit orders are not authorized:

\[
\Pr \left( D^h = 1 \mid D^v = 1 \right) \kappa_{\pi > \frac{\gamma - \Delta s}{\sigma - \Delta s}} < \Pr \left( D^h = 1 \mid D^v = 1 \right) \kappa_{\pi \leq \frac{\gamma - \Delta s}{\sigma - \Delta s}} \Leftrightarrow \pi (\xi) < 0.
\]

Thus if \( \xi \leq s \), \( \Pr \left( D^h = 1 \mid D^v = 1 \right) \) increases with \( \pi \).

When \( \sigma \) increases, \( \frac{\gamma - \Delta s}{\sigma - \Delta s} \) and \( \frac{\gamma}{\sigma} \) decrease. Thus whatever the market organization, the \( \pi \)-interval for which it is strictly optimal for the informed liquidity supplier to submit a hidden order shrinks. So \( \int_{0}^{\frac{\gamma}{\sigma}} \Pr \left( D^h = 1 \mid D^v = 1 \right) \, dx \) decreases.

**Proof of Corollary 7**

We have:

\[
E_t (\tilde{v}|D^v = 1) - E_o (\tilde{v}|D^v = 1) = \frac{\sigma \pi (1 - \pi)}{[\pi (1 - \lambda^o) + (1 - \pi) (1 - \lambda^t)] \times [\pi (1 - \lambda^t) + (1 - \pi) (1 - \lambda^o)]}
\times \left( (1 - \lambda^o) (1 - \lambda^t) - (1 - \lambda^o) (1 - \lambda^t) \right) (5)
\]

\[
E_t (\tilde{v}|D^v = 2) - E_o (\tilde{v}|D^v = 2) = \frac{\sigma \pi (1 - \pi)}{[\pi \lambda^o + (1 - \pi) \lambda^t] \times [\pi \lambda^t + (1 - \pi) \lambda^o]} \times \lambda^o \lambda^t - \lambda^o \lambda^t (6)
\]

It can be easily shown that:

i) If \( \pi \leq \frac{\gamma - \lambda^\theta}{\sigma - \Delta \sigma} \), then:

\[
E_t (\tilde{v}|D^v = 1) - E_o (\tilde{v}|D^v = 1) = \sigma \pi \geq 0
\]

ii) If \( \frac{\gamma - \lambda^\theta}{\sigma - \Delta \sigma} < \pi \leq \frac{\gamma}{\sigma} \), then:

\[
E_t (\tilde{v}|D^v = 1) - E_o (\tilde{v}|D^v = 1) = 0
\]

In both cases:

\[
|E_t (\tilde{v}|D^v = 1) - v_0| \leq |E_o (\tilde{v}|D^v = 1) - v_0|
\]

In equilibrium, the informational content of \( D^v = 2 \) in the opaque market cannot be evaluated since it is out of the equilibrium path.

**Proof of Corollary 8**
A) Visible depth:

\[ D^v_t \geq D^v_o \iff \left( \frac{1 - \pi}{\pi} \right) (l^t - l^o) \geq (\lambda^o - \lambda^t). \]

This condition holds whether marketable limit orders are authorized or not, for all \( \pi \leq \frac{\gamma}{\sigma} \). This is due to \( l^o = 0 \) and \( \lambda^o = 0 \).

B) Total depth:

\[ D^t_t \geq D^t_o \iff \left( \xi^o - \xi^t \right) > \left( \frac{1 - \pi}{\pi} \right) (1 - \theta) (s'' - s'^o). \]

Since in the opaque market, \( s' = 0 \),

- if \( \pi \leq \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} \) then in the transparent market, \( s'' = 0 \), and \( \xi^t = 0 \). In the opaque market, \( s'^o = 0 \) and \( \xi^o = 0 \), so that:

\[ D^t_t = D^t_o \]

- if \( \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} \leq \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} \) then in the transparent market, \( s'' = 0 \), and \( \xi^t = 0 \). In the opaque market, \( s'^o = 0 \) but \( \xi^o + \chi^o = 0 \), so:

\[ D^t_t \geq D^t_o \iff 1 - \chi^o \geq 0 \]

- if \( \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} < \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} \) then in the transparent market, \( s'' = 0 \), and \( \xi^t = 0 \). In the opaque market, \( s'^o = 0 \) but \( \xi^o = 0 \) if marketable limit orders are authorized and \( \xi^o + \chi^o = 0 \) otherwise, so:

i) if marketable limit orders are authorized,

\[ D^t_t = D^t_o \]

ii) if marketable limit orders are not authorized,

\[ D^t_t \geq D^t_o \iff 1 - \chi^o \geq 0 \]

- if \( \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} < \pi \leq \frac{\gamma}{\sigma} \) then in the transparent market, \( s'' = 1 \), and \( \xi^t = 1 \), while \( s'^o = 0 \). \( \xi^o = 0 \) if marketable limit orders are authorized and \( \xi^o + \chi^o = 0 \) otherwise, so:

\[ D^t_t \leq D^t_o \]

\[ \blacksquare \]
Proof of Corollaries 10 and 9

If marketable limit orders are authorized, for \( \pi \in [0, \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta}] \):

\[
DL^O_{OM} \leq DL_t \iff (2\gamma - \Delta \theta) - (2\sigma - \Delta \theta) \pi \leq \gamma (2 - \theta) - (2\sigma - \gamma \theta) \pi \iff 1 \leq \pi
\]
\[
OLNI^O_{OM} > OLNI_t \iff \frac{1}{2} (1 - \pi) \times 2\Delta > \frac{1}{2} (1 - \pi) \times (2\Delta - c)
\]
\[
OLI^O_{OM} = OLI_t \iff \pi (\Delta + \sigma) = \pi (\Delta + \sigma)
\]

For \( \pi \in [\frac{\gamma}{\sigma - \Delta \theta}, \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta}] \):

\[
DL^O_{OM} \leq DL_t \iff \gamma - \sigma \pi \leq \gamma (2 - \theta) - (2\sigma - \gamma \theta) (1 - \theta) \pi \iff \pi \leq \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta}
\]
\[
OLNI^O_{OM} \leq OLNI_t \iff \frac{1}{2} (1 - \pi) \Delta \leq \frac{1}{2} (1 - \pi) \times (2\Delta - c) \iff c \leq \Delta
\]
\[
OLI^O_{OM} \leq OLI_t \iff \frac{1}{2} \pi (\Delta + \sigma) \leq \pi (\Delta + \sigma)
\]

For \( [\frac{\gamma - \gamma \theta}{\sigma - \gamma \theta}, \frac{\gamma}{\sigma - \Delta \theta}] \):

\[
DL^O_{OM} = DL_t \iff \frac{1}{2} \pi (\Delta + \sigma) = \frac{1}{2} \pi (\Delta + \sigma)
\]
\[
OLNI^O_{OM} = OLNI_t \iff \frac{1}{2} (1 - \pi) \Delta = \frac{1}{2} (1 - \pi) \Delta
\]
\[
OLI^O_{OM} = OLI_t \iff \gamma - \sigma \pi = \gamma - \sigma \pi
\]

We find similar results when if marketable limit orders are not authorized, if we replace \( \frac{\gamma - \Delta \theta}{\sigma - \Delta \theta} \) by \( \frac{\gamma - \gamma \theta}{\sigma - \gamma \theta} \).
The limit order book is filled in with buy and sell limit orders.

The best quotes are \((A_2, B_2)\).

Agents observe the best quotes \((A_2, B_2)\).

One agent submits a limit order that undercuts the best quotes by one tick (at price \(A_1\) for a sell order or \(B_1\) for a buy order).

Agents observe the depth of the limit order book at price \((A_1, B_1)\).

One liquidity demander enters the market and potentially submits a market or a marketable limit order to buy or sell the asset.

The final value of the asset is revealed.
Information acquisition: **INFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$

Uninformed liquidity demander, observes $D^v=2$, Submits $M(D^v=2)$

No information acquisition: **UNINFORMED** liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=1$, displays $D^v=1$

Uninformed liquidity demander, observes $D^v=1$, Submits $M(D^v=1)$

Uninformed liquidity supplier.

Limit order at price $A_1$.

Submits $D^t=2$, displays $D^v=2$
**Figure 3** Informed liquidity supplier’s expected profit with market orders

The uninformed liquidity supplier submits large orders in the transparent market, which encourages liquidity demand.

**Figure 4** Uninformed liquidity supplier’s expected profit with market orders.

Information revelation is costly. But larger probability of execution in the transparent market.
The liquidity demander’s large market order may have a price impact in the opaque market.
Figure 4.6 : Profit espéré de l'offreur de liquidité informé sur les marchés opaque et transparent lorsque les ordres à la meilleure limite sont autorisés.

Figure 6 : Informed liquidity supplier’s expected profit with marketable limit orders.

Submitting hidden orders enables the informed agent not to reveal her presence: for the liquidity demander, no price impact so larger orders’ submission.

Opaque market
Transparent market
\[
\frac{\pi}{\gamma} \frac{(\gamma - \gamma \theta)}{\sigma - \gamma \theta} \frac{(\gamma - \gamma \theta/2)}{\sigma - \gamma \theta/2} \frac{\gamma}{\sigma} \frac{(\gamma - \Delta \theta)}{\sigma - \Delta \theta}
\]

**Figure 7**: Uninformed liquidity supplier’s expected profit with marketable limit orders.

No cost of information revelation **and** higher probability of execution in the opaque market.

**Opaque market**

**Transparent market**

**Figure 8**: Liquidity demander’s expected profit with marketable limit orders.

Total depth is larger in the opaque market, but the adverse selection risk is also higher.

**Opaque market**

**Transparent market**
Table 1: Probabilities with different offers are submitted

<table>
<thead>
<tr>
<th>Offer</th>
<th>Depth</th>
<th>Probabilities</th>
<th>Informed (π)</th>
<th>Uninformed (1-π)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small limit order</td>
<td>$D^t=1$</td>
<td>$D^h=0$</td>
<td>$D^v=1$</td>
<td>$ξ$</td>
</tr>
<tr>
<td>Large hidden limit order</td>
<td>$D^t=1$</td>
<td>$D^h=1$</td>
<td>$D^v=1$</td>
<td>$χ$</td>
</tr>
<tr>
<td>Large unhidden limit order</td>
<td>$D^t=2$</td>
<td>$D^h=0$</td>
<td>$D^v=2$</td>
<td>$λ$</td>
</tr>
</tbody>
</table>
### Table 2: Equilibrium Profits in the Opaque Market Without MLO for $h + (1 - \frac{\Delta}{s}) < 0$

<table>
<thead>
<tr>
<th>Adverse Selection $\pi$</th>
<th>$[0, \frac{\gamma + \Delta}{s - \Delta}]$</th>
<th>$[\frac{\gamma + \Delta}{s - \Delta}, \frac{\gamma}{s}]$</th>
<th>$[\frac{\gamma}{s}, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order submission’s strategies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\lambda^* \in [\frac{1 - \pi}{\sigma - \gamma}, \frac{1 - \pi}{\sigma - \gamma}]$</td>
<td>$\lambda^* = \frac{(1 - \pi)(\sigma - \gamma)}{\sigma}$</td>
<td>$\lambda^* \in [\frac{\gamma}{s - \Delta}, \frac{\gamma}{s - \Delta}]$</td>
</tr>
<tr>
<td>Liquidity demander</td>
<td>$\mu_{2</td>
<td>2} = 1$</td>
<td>$\mu_{2</td>
</tr>
<tr>
<td>Equilibrium expected profits</td>
<td>$\pi(\Delta + \sigma)$</td>
<td>$\frac{1}{2}\pi(\Delta + \sigma)$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

### Table 3: Equilibrium Profits in the Opaque Market Without MLO for $h + (1 - \frac{\Delta}{s}) > 0$

<table>
<thead>
<tr>
<th>Adverse Selection $\pi$</th>
<th>$[0, \frac{\gamma - \Delta}{s - \Delta}]$</th>
<th>$[\frac{\gamma - \Delta}{s - \Delta}, \frac{\gamma}{s}]$</th>
<th>$[\frac{\gamma}{s}, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order submission’s strategies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\lambda^* = 1$</td>
<td>$\lambda^* = \frac{(1 - \pi)(\sigma - \gamma)}{\sigma}$</td>
<td>$\lambda^* \in [\frac{\gamma}{s - \Delta}, \frac{\gamma}{s - \Delta}]$</td>
</tr>
<tr>
<td>Liquidity demander</td>
<td>$\mu_{2</td>
<td>2} = 1$</td>
<td>$\mu_{2</td>
</tr>
<tr>
<td>Equilibrium expected profits</td>
<td>$\pi(\Delta + \sigma)$</td>
<td>$\frac{1}{2}\pi(\Delta + \sigma)$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

### Table 4: Equilibrium Profits in the Opaque Market With MLO

<table>
<thead>
<tr>
<th>Adverse Selection $\pi$</th>
<th>$[0, \frac{\gamma + \Delta}{s - \Delta}]$</th>
<th>$[\frac{\gamma + \Delta}{s - \Delta}, \frac{\gamma}{s}]$</th>
<th>$[\frac{\gamma}{s}, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order submission’s strategies</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\lambda^* \in [1 - \frac{\gamma + \Delta}{2(\sigma - \gamma)}, \frac{1 - \pi}{\sigma}]$</td>
<td>$\lambda^* \geq \frac{\gamma \sigma}{\sigma - \gamma}$</td>
<td>$\lambda^* \in [\frac{\gamma}{s - \Delta}, \frac{\gamma}{s - \Delta}]$</td>
</tr>
<tr>
<td>Liquidity demander</td>
<td>$\mu_{2</td>
<td>2} = 1$</td>
<td>$\mu_{2</td>
</tr>
<tr>
<td>Equilibrium expected profits</td>
<td>$\pi(\Delta + \sigma)$</td>
<td>$\frac{1}{2}\pi(\Delta + \sigma)$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

For $\gamma > \frac{\gamma}{s}$, cf. appendix.
**TABLE 5: EQUILIBRIUM PROFITS IN THE OPAQUE AND IN THE TRANSPARENT MARKETS**

<table>
<thead>
<tr>
<th>Adverse Selection $\pi$</th>
<th>$[0, \frac{2 - \Delta}{\mu}]$</th>
<th>$[2 - \frac{\Delta}{\mu}, \frac{2 - \Delta}{2\mu}]$</th>
<th>$[\frac{2 - \Delta}{2\mu}, \frac{1}{2\mu}]$</th>
<th>$[\frac{1}{2\mu}, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order submission’s strategies in the opaque market, without marketable limit orders</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\lambda^* = 0$</td>
<td>$\lambda^* = 0$</td>
<td>$2\mu_{2\text{MLO}</td>
<td>2} + \mu_{1</td>
</tr>
<tr>
<td></td>
<td>$\lambda^* = 1$</td>
<td>$\lambda^* + \xi^* = 1$</td>
<td>$\mu_{0</td>
<td>1} = 1$</td>
</tr>
<tr>
<td>Uninformed liquidity supplier</td>
<td>$h^* = 1$</td>
<td>$h^* = 1$</td>
<td>$\gamma^* = 1$</td>
<td></td>
</tr>
<tr>
<td>Liquidity demander</td>
<td>$\mu_{2\text{MLO}</td>
<td>2} = 1$</td>
<td>$\mu_{2\text{MLO}</td>
<td>1} = 1$</td>
</tr>
<tr>
<td>Equilibrium expected profits in the opaque market, without marketable limit orders</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\pi (\Delta + \sigma)$</td>
<td>$\frac{1}{2} \pi (\Delta + \sigma)$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
</tr>
<tr>
<td>Uninformed liquidity supplier</td>
<td>$\frac{1}{2} (1 - \pi) \times 2\Delta$</td>
<td>$\frac{1}{2} (1 - \pi) \Delta$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
</tr>
<tr>
<td>Liquidity demander</td>
<td>$(2\gamma^* + \Delta \theta) - (2\sigma^* - \Delta \theta) \pi$</td>
<td>$\gamma = \sigma \pi$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
</tr>
<tr>
<td>Order submission’s strategies in the opaque market, with marketable limit orders</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Informed liquidity supplier</td>
<td>$\lambda^* = 0$</td>
<td>$\lambda^* = 0$</td>
<td>$2\mu_{2\text{MLO}</td>
<td>2} + \mu_{1</td>
</tr>
<tr>
<td></td>
<td>$\lambda^* = 1$</td>
<td>$\lambda^* + \xi^* = 1$</td>
<td>$\mu_{0</td>
<td>1} = 1$</td>
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<tr>
<td>Offre de liquidité non-informé</td>
<td>$h^* = 1$</td>
<td>$h^* = 1$</td>
<td>$\gamma^* = 1$</td>
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<tr>
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<td>$\mu_{2\text{MLO}</td>
<td>2} = 1$</td>
<td>$\mu_{2\text{MLO}</td>
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<tr>
<td>Equilibrium expected profits in the opaque market, with marketable limit orders</td>
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<tr>
<td>Informed liquidity supplier</td>
<td>$\pi (\Delta + \sigma)$</td>
<td>$\frac{1}{2} \pi (\Delta + \sigma)$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
</tr>
<tr>
<td>Uninformed liquidity supplier</td>
<td>$\frac{1}{2} (1 - \pi) \times 2\Delta$</td>
<td>$\frac{1}{2} (1 - \pi) \Delta$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
</tr>
<tr>
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<td>$(2\gamma^* + \Delta \theta) - (2\sigma^* - \Delta \theta) \pi$</td>
<td>$\gamma = \sigma \pi$</td>
<td>$\gamma^* - \theta^*$</td>
<td>$\sigma^* \pi$</td>
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<td>Order submission’s strategies in the transparent market</td>
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<tr>
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<td>$\lambda^* = 0$</td>
<td>$\lambda^* \in [0, \frac{\mu^* - \gamma^<em>}{\mu^</em> - \gamma}]$</td>
<td>$\xi^* = 1 - \lambda^*$</td>
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<tr>
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<td>Equilibrium expected profits in the transparent market</td>
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<td>$\frac{1}{2} \pi (\Delta + \sigma)$</td>
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<td>$\gamma = \sigma \pi$</td>
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<tr>
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<td>$\frac{1}{2} (1 - \pi) \Delta$</td>
<td>$\gamma^* - (\theta^* - \sigma^* \pi)$</td>
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<td>$\gamma - \sigma \pi$</td>
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