Relationship between downside beta and CAPM beta

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Abstract

This paper derives the relationship between the CAPM beta and three measures of downside beta. Assuming the market model and a downside variant of the market model as data generating processes restrictions on the risk measures are derived. The restrictions are used to establish the relationships between the betas. We argue that restrictions such as the ones that we have derived on the risk measures may be used to explain differing conclusions in comparable empirical studies. Through an empirical example we highlight how such relationships may reveal the underlying characteristics of different risk measures with respect to an assumed data generating process. Restrictions such as those we have derived could provide more insights on empirical validity of risk measures and hence may lead to proper conclusions in empirical investigations of competing asset pricing models.

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Key words: CAPM beta, downside risk, data generating process, asset pricing

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1. Introduction

There is strong evidence that the mean-variance CAPM performs poorly. A criticism of the mean-variance CAPM is its disregard to up and down movements of asset returns. The concept of downside risk is considered as an alternative. However, only a few studies compare the performance of the mean-variance CAPM and the pricing models in a downside framework. A study of UK equity markets by Pedersen and Hwang (2003) is one example. Data generating processes are widely used in designing asset pricing tests. In the popular Fama-MacBeth (1973) two-stage estimation and testing of equilibrium pricing models erroneous conclusions is possible due to high collinearity of estimated parameters. Data generating processes could to a certain extent arrest this problem. Barone-Adesi (1985) argue that when the Kraus and Litzenberger (1976) three-moment asset pricing model is reformulated as a quadratic market model parameter estimation in the quadratic market model is less likely to be affected by multicollinearity than when estimating the three-moment model itself. Studies that employ data generating processes to test the empirical validity of asset pricing models often report differing conclusions.

In this paper we establish restrictions on the downside beta and the CAPM beta in two assumed data generating processes. Through the relationships that we develop we highlight the conditions under which systematic risk captured by the CAPM beta may be numerically equal to the systematic risk estimated by the downside risk measure. We argue that such information may be useful to explain results in empirical studies that investigate competing asset pricing models through their equivalent data generating processes.[1]

We consider two data generating processes. The market model is considered as one data generating process. The other is a variant of the market model that accounts only for the downside movements of the market. These are the two commonly discussed pricing models

[1] Hwang and Satchell (1999) considered the linear market model, the quadratic market model and the cubic market model as the data generating processes that has been shown to be consistent with their equivalent CAPM versions. They derive restrictions for the systematic risk/s implied by the data generating processes.

under the mean-variance and mean semi-variance frameworks. We illustrate the relationships that we derive through an empirical example.

The paper is organized as follows. The downside risk measures are defined in the next section followed by data generating processes in the mean-variance framework and in a downside framework. There after the relationships derived are illustrated via an empirical example. The paper finishes with some concluding remarks.

2. Downside risk measures

In this study we focus only on the risk associated with the second moment. Therefore our discussion is limited to the CAPM beta and some measures of downside beta. There are several measures of downside risk proposed in the literature. We consider three such measures. Bawa and Lindenberg (1977) in their interpretation of the CAPM in a downside framework defined downside beta (BL-beta) which we denote by $\beta_{im}^{(BL)}$ as

$$\beta_{im}^{(BL)} = \frac{E\left[\left(R_i - R_f\right)\min\left(R_m - R_f, 0\right)\right]}{E\left[\min\left(R_m - R_f, 0\right)\right]^2} \tag{1}$$

where R_i is the return on security *i*, R_m is the return on market portfolio and R_f is the risk-free rate. The numerator in (1) is referred to as the co-semi-variance of returns below R_f on the market portfolio with returns in excess of excess of R_f on security *i*.

Harlow and Rao (1989) developed a pricing model citing the earlier work of Hogan and Warren (1974) and Bawa and Lindenberg (1977). They revealed that market participants appear to characterize risk as downside deviations below a target that is related to equity market mean returns rather than to the risk-free rate. In that case the expression for the downside beta (HR-beta) becomes

$$\beta_{im}^{(HR)} = \frac{E\left[\left(R_i - \mu_i\right)\min\left(R_m - \mu_m, 0\right)\right]}{E\left[\min\left(R_m - \mu_m, 0\right)\right]^2} \tag{2}$$

where μ_i and μ_m are security *i* and market average returns respectively.

Estrada (2002) defines an asset *i*'s covariance with the market portfolio in a downside framework as $E\left[\min\left(R_i-\mu_i,0\right)\min\left(R_m-\mu_m,0\right)\right]$ leading to a measure of systematic downside beta-risk (E-beta) given by

$$\beta_{im}^{(E)}=\frac{E\left[\min\left(R_i-\mu_i,0\right)\min\left(R_m-\mu_m,0\right)\right]}{E\left[\min\left(R_m-\mu_m\right),0\right]^2}.^2 \tag{3}$$

3. Mean-variance framework

In the mean-variance framework the data generating process consistent with the CAPM may be expressed as

$$R_{it}-R_f=b_{i1}+b_{i2}\left(R_{mt}-R_f\right)+\varepsilon_{it} \tag{4}$$

where $E\left(\varepsilon_{it}\right)=0$.[3] In this section assuming (4) holds we derive expressions for the CAPM beta and the downside betas in terms of the parameters of the data generating process. Also we discuss the conditions under which downside beta may be approximated numerically by the CAPM beta.

3.1 CAPM beta

Taking the expectation of (4),

$$E\left(R_i\right)-R_f=b_{i1}+b_{i2}E\left(R_m\right)-b_{i2}R_f \tag{5}$$

and subtracting (5) from (4) we obtain

$$R_{it}-E\left(R_i\right)=b_{i2}\left(R_{mt}-E\left(R_m\right)\right)+\varepsilon_{it} \tag{6}$$

Multiplying (6) by $\left(R_{mt}-E\left(R_m\right)\right)$ and applying the expectation operator follows

$$E\left[\left(R_{it}-E\left(R_i\right)\right)\left(R_{mt}-E\left(R_m\right)\right)\right]=b_{i2}E\left[\left(R_{mt}-E\left(R_m\right)\right)\right]^2 \tag{7}$$

[2] While aggregation applies for a threshold or target of zero (or the risk-free rate when using excess returns) it generally does not apply for other thresholds. This calls into question the theoretical basis for the HR-beta. On the other hand the E-beta cannot be linked to a well-behaved utility function due to the focus only on negative returns for the evaluated asset. Notwithstanding these shortcomings we consider all three measures due to their presence in the literature.

[3] A pricing model equivalent to this data generating process is $E\left(R_{it}\right)=R_f+b\left[E\left(R_m\right)-R_f\right]$. The data generating process defined in (4) is also referred to as the market model.

This reduces to

$$\frac{E\left[\left(R_{it}-E\left(R_i\right)\right)\left(R_{mt}-E\left(R_m\right)\right)\right]}{E\left[\left(R_{mt}-E\left(R_m\right)\right)\right]^2}=\beta_{im}=b_{i2} \tag{8}$$

where β_{im} is the CAPM beta.

3.2 Harlow and Rao beta

Now writing $R_{mt}-E(R_m)$ as $\left[\min\left(R_{mt}-E\left(R_m\right),0\right)+\max\left(R_{mt}-E\left(R_m\right),0\right)\right]$ in (6)

$$R_{it}-E\left(R_i\right)=b_{i2}\left[\min\left(R_{mt}-E\left(R_m\right),0\right)+\max\left(R_{mt}-E\left(R_m\right),0\right)\right]+\varepsilon_{it} \tag{9}$$

Multiplying (9) by $\min\left(R_{mt}-E\left(R_m\right),0\right)$, substituting

$\min\left(R_{mt}-E\left(R_m\right),0\right)\max\left(R_{mt}-E\left(R_m\right),0\right)=0$ and taking the expectation follows

$$E\left[\left(R_{it}-E\left(R_i\right)\right)\min\left(R_{mt}-E\left(R_m\right),0\right)\right]=b_{i2}E\left[\min\left(R_{mt}-E\left(R_m\right),0\right)\right]^2 \tag{10}$$

This reduces to

$$\frac{E\left[\left(R_{it}-E\left(R_i\right)\right)\min\left(R_{mt}-E\left(R_m\right),0\right)\right]}{E\left[\min\left(R_{mt}-E\left(R_m\right),0\right)\right]^2}=\beta_{im}^{(HR)}=b_{i2} \tag{11}$$

The above result indicates that when (4) adequately describes the data generating process the downside risk measure defined by Harlow and Rao is equal to the slope parameter of the market model and is therefore numerically equal to the CAPM beta.

3.3 Estrada beta

Writing $R_{it}-E(R_i)$ as $\left[\min\left(R_{it}-E\left(R_i\right),0\right)+\max\left(R_{it}-E\left(R_i\right),0\right)\right]$ in (9) we obtain

$$\begin{aligned}\min\left(R_{it}-E\left(R_i\right),0\right)+\max\left(R_{it}-E\left(R_i\right),0\right)&=\\ b_{i2}\left[\min\left(R_{mt}-E\left(R_m\right),0\right)+\max\left(R_{mt}-E\left(R_m\right),0\right)\right]&+\varepsilon_{it}\end{aligned} \tag{12}$$

Multiplying (12) by $\min\left(R_{mt}-E\left(R_m\right),0\right)$ and taking the expectation

$$\begin{aligned}&E\left[\min\left(R_{it}-E\left(R_i\right),0\right)\min\left(R_{mt}-E\left(R_m\right),0\right)\right]\\ &+E\left[\max\left(R_{it}-E\left(R_i\right),0\right)\min\left(R_{mt}-E\left(R_m\right),0\right)\right]=b_{i2}E\left[\min\left(R_{mt}-E\left(R_m\right),0\right)\right]^2\end{aligned} \tag{13}$$

This reduces to

$$\beta_{im}^{(E)} = b_{i2} - \frac{E[\max(R_{it} - E(R_i),0)][\min(R_{mt} - E(R_m),0)]}{E[\min(R_{mt} - E(R_m),0)]^2} \tag{14}$$

This result indicates that if we assume (4) as the asset price generating process the slope parameter under-estimates Estrada beta.[4] In other words when the mean-variance CAPM applies, the systematic risk measured by the CAPM beta is lower than the risk measured by the Estrada beta. Further, when the asset follows the market $\beta_{im}^{(E)}$ approximates the CAPM beta. In particular E-beta differs numerically from the CAPM beta only when the asset realises upward movement (asset return in excess of the asset mean return is positive) while the market experiences downward movement (market return in excess of the mean market return is negative).

3.4 Bawa and Lindenberg beta

Writing $R_{mt} - R_f$ as $\left[\min(R_{mt} - R_f,0) + \max(R_{mt} - R_f,0)\right]$ in (4) we obtain

$$R_{it} - R_f = b_{i1} + b_{i2}\left[\min(R_{mt} - R_f,0) + \max(R_{mt} - R_f,0)\right] + \varepsilon_{it} \tag{15}$$

Multiplying (15) by $\min(R_{mt} - R_f,0)$ and taking the expectation

$$\begin{aligned} E\left[(R_{it} - R_f)\min(R_{mt} - R_f,0)\right] &= b_{i1}E\left[\min(R_{mt} - R_f,0)\right] \\ &\quad + b_{i2}E\left[\min(R_{mt} - R_f,0)\right]^2 \end{aligned} \tag{16}$$

Hence follows

$$\frac{E\left[(R_{it} - R_f)\min(R_{mt} - R_f,0)\right]}{E\left[\min(R_{mt} - R_f,0)\right]^2} = \beta_{im}^{(BL)} = b_{i1}\frac{E\left[\min(R_{mt} - R_f,0)\right]}{E\left[\min(R_{mt} - R_f,0)\right]^2} + b_{i2} \tag{17}$$

In this case $\beta_{im}^{(BL)} = \beta_{im}$ when $b_{i1} = 0$.[5] If the CAPM is not valid and the market model is assumed as the data generating process Bawa and Lindenberg beta will be numerically higher (lower) than the CAPM beta when $b_{i1} < 0$ ($b_{i1} > 0$).

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[4] Note that $[\max(R_{it} - E(R_i),0)][\min(R_{mt} - E(R_m),0)]$ is non-positive.

[5] A condition that should be satisfied for validity of the CAPM is $b_{i1} = 0$. Alternatively when returns are spherically symmetric or quadratic utility is assumed, $\beta_{im}^{(BL)} = \beta_{im}$.

4. Downside framework

Here we assume that risk-free rate is the threshold in the market and that the data generating process is given by[6]

$$R_{it} - R_f = b_{i1}^d + b_{i2}^d \min(R_{mt} - R_f, 0) + \varepsilon_{it} \tag{18}$$

Equation (18) is a downside variation of the market model. We refer to (18) as the downside market model. We derive expressions for the CAPM beta and the three downside betas in terms of the model parameters.

4.1 CAPM beta

Subtracting the expectation of (18) from (18) we obtain

$$R_{it} - E(R_i) = b_{i2}^d \{\min(R_{mt} - R_f, 0) - E[\min(R_{mt} - R_f, 0)]\} + \varepsilon_{it}$$

Writing $\min(R_{mt} - R_f, 0)$ as $R_{mt} - R_f - \max(R_{mt} - R_f, 0)$

$$= b_{i2}^d \min(R_{mt} - R_f, 0) - b_{i2}^d E[(R_{mt} - R_f) - \max(R_{mt} - R_f, 0)] + \varepsilon_{it}$$

$$= b_{i2}^d \min(R_{mt} - R_f, 0) - b_{i2}^d E(R_{mt}) + b_{i2} R_f + b_{i2}^d E[\max(R_{mt} - R_f, 0)] + \varepsilon_{it}$$

$$= b_{i2}^d \min(R_{mt} - R_f, 0) + b_{i2}^d [R_{mt} - E(R_{mt})] - b_{i2}^d [R_{mt} - R_f] + b_{i2}^d E[\max(R_{mt} - R_f, 0)] + \varepsilon_{it} \tag{19}$$

Multiplying (19) by $R_{mt} - E(R_m)$, taking the expectation and dividing by $E[R_{mt} - E(R_m)]^2$

$$\frac{E[(R_{it} - E(R_i))(R_{mt} - E(R_m))]}{E[(R_{mt} - E(R_m))]^2} = \\ b_{i2}^d \frac{E[(R_{mt} - E(R_m))\min(R_{mt} - R_f, 0)]}{E[(R_{mt} - E(R_m))]^2} + b_{i2}^d - b_{i2}^d \frac{E[(R_{mt} - E(R_m))(R_{mt} - R_f)]}{E[(R_{mt} - E(R_m))]^2} \tag{20}$$

$$= b_{i2}^d - b_{i2}^d \frac{E[(R_{mt} - E(R_m))\max(R_{mt} - R_f, 0)]}{E[(R_{mt} - E(R_m))]^2} \tag{21}$$

Hence follows

[6] A pricing model equivalent to this data generating process is $E(R_{it}) = R_f + b^d E[\min(R_m - R_f, 0)]$.

$$\beta_{im} = b_{i2}^{d}\left\{1 - \frac{E\left[\left(R_{mt} - E\left(R_m\right)\right)\max\left(R_{mt} - R_f, 0\right)\right]}{E\left[\left(R_{mt} - E\left(R_m\right)\right)\right]^2}\right\} \tag{22}$$

4.2 Bawa and Lindenberg beta

Multiplying (18) by $\min\left(R_{mt} - R_f, 0\right)$, taking the expectation and dividing by $E\left[\min\left(R_{mt} - R_f, 0\right)\right]^2$ we obtain

$$\frac{E\left[\left(R_{it} - R_f\right)\min\left(R_{mt} - R_f, 0\right)\right]}{E\left[\min\left(R_{mt} - R_f, 0\right)\right]^2} = \beta_{im}^{(BL)} = b_{i1}^{d}\frac{E\left[\min\left(R_{mt} - R_f, 0\right)\right]}{E\left[\min\left(R_{mt} - R_f, 0\right)\right]^2} + b_{i2}^{d} \tag{23}$$

Equation (23) reveals that the slope parameter of the downside market model equals the Bawa and Lindenberg beta when the intercept is zero. In that case the asset pricing model in the downside framework may be expressed as

$$E\left(R_i\right) = R_f + \beta_{im}^{(BL)}E\left[\min\left(R_{mt} - R_f, 0\right)\right] \tag{24}$$

4.3 Harlow and Rao beta

Multiplying (19) by $\min\left(R_{mt} - E\left(R_m\right), 0\right)$, applying the expectation operator and dividing by $E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]^2$ we obtain

$$\begin{aligned}
&\frac{E\left[\left(R_{it} - E\left(R_i\right)\right)\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]}{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]^2} = \\
&\qquad b_{i2}^{d} - b_{i2}^{d}\frac{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\max\left(R_{mt} - R_f, 0\right)\right]}{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]^2} \\
&\qquad\qquad + b_{i2}^{d}\frac{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]E\left[\max\left(R_{mt} - R_f, 0\right)\right]}{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]^2}
\end{aligned} \tag{25}$$

Hence follows

$$\beta_{im}^{(HR)} = b_{i2}^{d}\left\{1 - \frac{Cov\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\max\left(R_{mt} - R_f, 0\right)\right]}{E\left[\min\left(R_{mt} - E\left(R_m\right), 0\right)\right]^2}\right\} \tag{26}$$

4.4 Estrada beta

Writing $\left(R_{it} - E\left(R_i\right)\right)$ as $\min\left(R_{it} - E\left(R_i\right), 0\right) + \max\left(R_{it} - E\left(R_i\right), 0\right)$ in (25) we obtain

$$\frac{E\left[\min\left(R_{it}-E(R_i),0\right)\min\left(R_{mt}-E(R_m),0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2}+\frac{E\left[\max\left(R_{it}-E(R_i),0\right)\min\left(R_{mt}-E(R_m),0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2}$$
$$=b_{i2}^d-b_{i2}^d\frac{E\left[\min\left(R_{mt}-E(R_m),0\right)\max\left(R_{mt}-R_f,0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2}+b_{i2}^d\frac{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]E\left[\max\left(R_{mt}-R_f,0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2} \quad (27)$$

Hence follows

$$\beta_{im}^{(E)}=b_{i2}^d\left\{1-\frac{Cov\left[\min\left(R_{mt}-E(R_m),0\right)\max\left(R_{mt}-R_f,0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2}\right\}-\frac{E\left[\max\left(R_{it}-E(R_i),0\right)\min\left(R_{mt}-E(R_m),0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2} \quad (28)$$

Equations (26) and (28) give the relationship between $\beta_{im}^{(E)}$ and $\beta_{im}^{(HR)}$ as

$$\beta_{im}^{(E)}=\beta_{im}^{(HR)}-\frac{E\left[\max\left(R_{it}-E(R_i),0\right)\min\left(R_{mt}-E(R_m),0\right)\right]}{E\left[\min\left(R_{mt}-E(R_m),0\right)\right]^2} \quad (29)$$

A summary of the relationships between the CAPM beta and the downside betas are given in Table 1.

5. An illustrative example

5.1 Data

The data used here is from the MSCI database on emerging market monthly indices. We investigate emerging markets for several reasons: (i) the traditional CAPM has failed to explain the variation in equity prices (Harvey, 1995), (ii) return distributions are found to be non-symmetric (Susmel, 2001; Hwang and Pedersen, 2002) and (iii) returns are highly volatile (Bekaert and Harvey, 2003).[7] Moreover, downside risk in emerging markets has recently been investigated (Estrada, 2002). Estrada (2002) reports evidence that supports E-beta over the CAPM beta. We consider the same 27 emerging markets that Estrada (2002) considered- 10 Asian, 7 Latin American and 10 African, Middle-Eastern and European. The sample period is from January 1995 to December 2004. The returns are computed as the

[7] See also Hwang and Satchell (1999) for more features of emerging markets which suggests that the mean-variance CAPM might not be applicable for emerging markets.

difference in two consecutive monthly log returns. The proxy used for the market index is the world index available in the MSCI database and the proxy for the risk-free rate is the 10-year US Treasury bond rate.

The complete list of the markets and some summary statistics is given in Table 2. Entries in Table 2 reveal that for markets, the minimum return ranges from –93.1 percent to –8.5 percent while the maximum varies between 54.4 percent and 8.5 percent. Excess kurtosis can be as high as 2.4 with the minimum being –3.6. Excess kurtosis is positive in eight markets. The skewness ranges from –1.2 to 0.6 with twenty markets with negative skew. The world market return distribution is negatively skewed and has -2.1 excess kurtosis and 0.5 percent mean return.

5.2 Discussion of the results

For each emerging market we compute the CAPM beta, and the downside betas using equations (1), (2) and (3). We then estimate the parameters of the two data generating processes (4) and (18) and estimate the CAPM beta and the downside betas according to the relationships identified in Sections (2) and (3). We plot these estimates to illustrate the relationships between the betas. The graphs are in Figure 1. The results are presented in four panels, one for each of the betas considered. In each panel of Figure 1 the beta estimated using the formula, the market model parameters and the downside market model parameters and the estimated slope coefficient of the market model and the downside market model for the 27 emerging markets are illustrated. In the graphical illustration the markets are sorted in ascending order according to the beta estimated using the relevant formula. We refer to the beta estimated using the formula as the true beta.

The graph in panel (a) where the CAPM betas are illustrated reveals that the beta estimated in the market model (using (8)) and in the downside market model (using (22)) are similar in magnitude for emerging markets with low true CAPM betas. In other words if the sample consist of emerging markets with low true CAPM betas it makes no difference in using the market model or the downside market model as long as in the downside market model the CAPM beta is estimated using (22). For the emerging markets with high true CAPM betas the CAPM beta estimated in (22) is lower than the true CAPM beta. This highlights the problems that may be encountered in empirical studies as the sample characteristics could easily lead to erroneous conclusions. As expected the estimated slope of the downside market model b_{i2}^{d} is a very poor estimator of the true CAPM-beta. Further, b_{i2}^{d} overestimates the downside market model CAPM beta (formula 22) in all emerging markets by a common fraction (0.407) of b_{i2}^{d}.[8] So the overestimation is higher for emerging markets with high b_{i2}^{d}.

Panel (b) of Figure 1 shows that the E-beta estimated in the market model (using (14)) and using the downside market model (using (28)) underestimate the true E-beta. In this case we see no pattern in the plot mainly because estimation of E-beta using (14) and (28) requires information on asset return as well.

The graphs of the estimated BL-betas and HR-betas are shown in panels (c) and (d) respectively. As shown in panel (d) the estimated slope in the downside market model overestimates the HR-beta estimated in the market model (using (11) as well as using the formula. The HR-beta estimated in the downside market model (using 26) is similar to the true estimate obtained via the formula. BL-betas plotted in panel (c) of Figure 1 displays patterns similar to those in panel (d).

In general for emerging markets considered here the estimated slope of the market model is a better estimator of Bawa and Lindenberg and Harlow and Rao downside betas compared to the estimated slope of the downside market model. Both data generating processes do not estimate the Estrada beta well. This is not surprising given that Estrada beta considers only the downside movements of asset returns.

6. Concluding remarks

[8] Pederson and Hwang (2003) derive restrictions on the systematic CAPM higher-order co-moments in terms of the parameters of the data generating process equivalent to the higher-order CAPM.

We consider CAPM beta and downside beta risk measures due to Bawa and Lindenberg, Harlow and Rao and Estrada. Assuming the market model and a downside variant of the market model as data generating processes we drive restrictions on the risk measures. The restrictions are then used to establish the relationships between the betas. We argue that restrictions such as the ones that we have derived on the risk measures may be used to explain differing conclusions in comparable empirical studies. Through an empirical example we highlight how such relationships may reveal the underlying characteristics of different risk measures with respect to an assumed data generating process. Restrictions such as those we have derived could provide more insights on the empirical validity of risk measures and hence may lead to proper conclusions in empirical investigations of competing asset pricing models.

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Table 1. Relationship between downside beta and CAPM beta

Pricing model: $E(R_{it}) = R_f + b\left[E(R_m) - R_f\right]$	
Equivalent DGP: $R_{it} - R_f = b_{i1} + b_{i2}\left(R_{mt} - R_f\right) + \varepsilon_{it}$	
$\beta_{im} = b_{i2}$	Equation (8)
$\beta_{im}^{(HR)} = \beta_{im}$	Equation (11)
$\beta_{im}^{(BL)} = \beta_{im} + b_{i1}K_1(R_m)$	Equation (17)
$\beta_{im}^{(E)} = \beta_{im} - K_2(R_i, R_m)$	Equation (14)
Pricing model: $E(R_{it}) = R_f + b^d E\left[\min\left(R_m - R_f, 0\right)\right]$	
Equivalent DGP: $R_{it} - R_f = b_{i1}^d + b_{i2}^d \min\left(R_{mt} - R_f, 0\right) + \varepsilon_{it}$	
$\beta_{im} = K_3(R_m) b_{i2}^d$	Equation (22)
$\beta_{im}^{(HR)} = K_4(R_m)\beta_{im}$	Equation (26)
$\beta_{im}^{(BL)} = K_5(R_m)\beta_{im} + b_{i1}^d K_6(R_m)$	Equation (23)
$\beta_{im}^{(E)} = K_7(R_m)\beta_{im} - K_8(R_i, R_m)$	Equation (28)

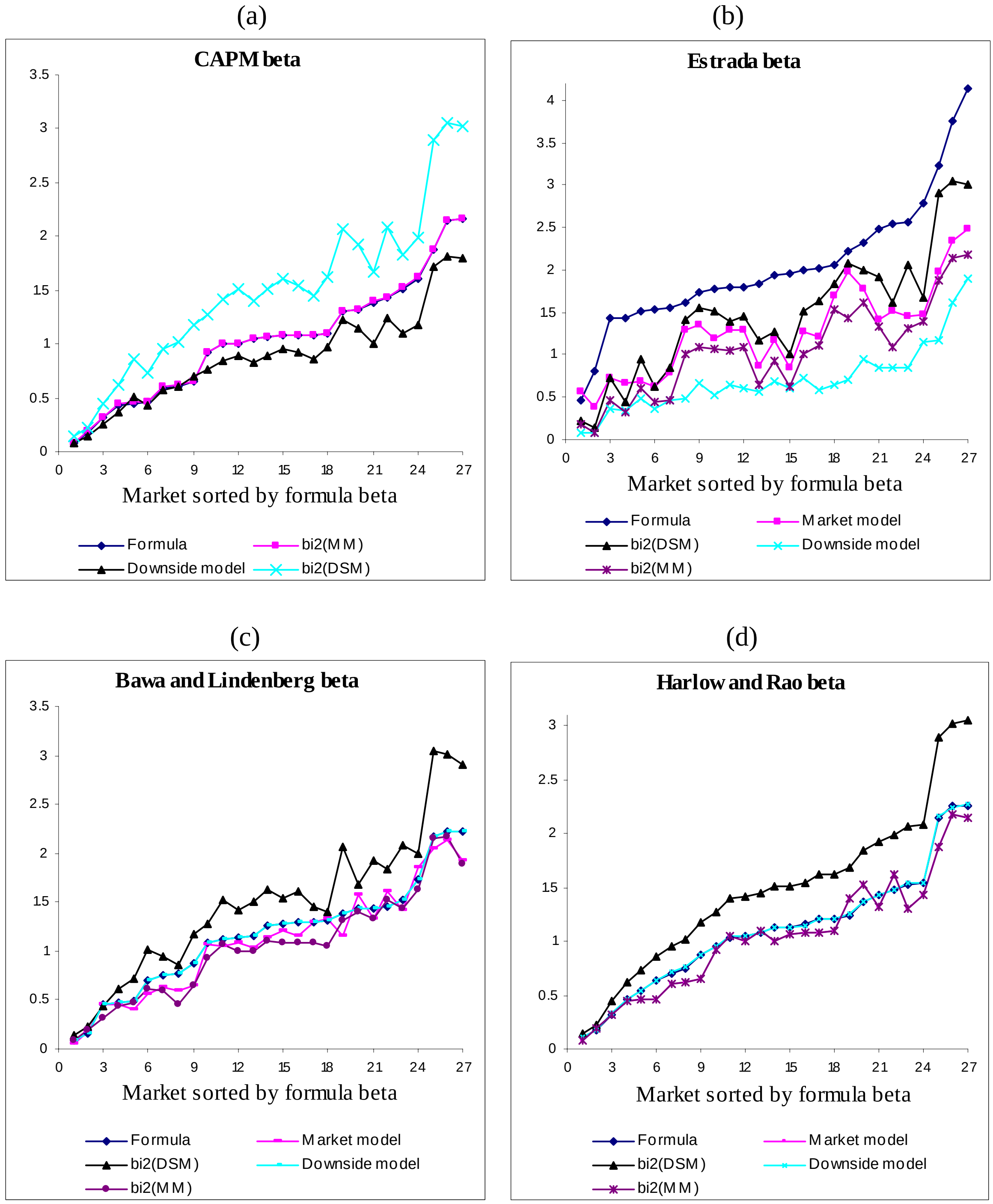
Notes: DGP= data generating process, $K_j(R_m)$ denotes a function of the market returns and $K_j(R_i, R_m)$ denotes a function of the asset and market returns. $K_1(R_m) = K_6(R_m) < 0$. $K_2(R_i, R_m) = K_8(R_i, R_m)$.

Table 2. Summary statistics of emerging market monthly log return

Country	Min	Max	Mean	SD	Skew	Kurtosis
Argentina	-38.43	42.47	0.017	11.558	-0.353	2.338
Brazil	-49.44	31.12	0.295	12.345	-1.013	2.531
Chile	-34.40	18.28	0.030	7.155	-0.956	3.835
China	-32.40	38.18	-0.858	11.363	0.332	1.755
Colombia	-27.59	26.47	0.395	9.774	-0.191	0.791
Czech Republic	-32.40	26.30	0.898	8.667	-0.497	1.661
Egypt	-15.11	28.04	0.869	8.261	0.615	0.634
Hungary	-49.09	37.96	1.575	10.596	-0.783	4.733
India	-19.53	19.89	0.241	8.494	-0.076	-0.560
Indonesia	-52.47	44.20	-0.581	15.852	-0.325	1.487
Israel	-20.94	23.86	0.660	7.855	-0.374	0.584
Jordan	-8.48	15.58	0.568	4.417	0.460	0.156
Korea	-37.48	53.41	0.062	13.036	0.390	2.546
Malaysia	-36.11	40.51	-0.347	10.532	-0.040	3.323
Mexico	-41.95	17.42	0.716	9.474	-1.186	3.016
Morocco	-16.49	16.28	0.597	5.045	-0.036	0.917
Pakistan	-47.62	31.68	-0.399	12.269	-0.261	1.879
Peru	-40.98	30.44	0.532	8.404	-0.902	5.390
Philippines	-34.65	36.01	-1.219	10.103	0.101	2.044
Poland	-42.98	33.93	0.561	10.808	-0.262	2.110
Russia	-93.07	47.71	1.307	19.519	-1.001	4.154
South Africa	-36.88	17.73	0.282	8.231	-1.218	3.409
Sri Lanka	-28.97	39.49	-0.391	10.446	0.459	2.511
Taiwan	-24.68	25.64	-0.285	9.271	0.129	0.225
Thailand	-41.63	35.90	-0.988	13.784	-0.219	1.161
Turkey	-53.18	54.41	0.895	17.449	-0.184	1.150
Venezuela	-63.77	48.04	0.350	14.800	-0.908	5.040
World market	-14.45	8.53	0.531	4.233	-0.777	0.921

Notes: The returns are expressed as a percentage. Sample period is from January 1995 to December 2004.

Figure 1. Graphs of estimated CAPM beta and downside beta



Notes: bi2(MM) = estimated slope in the market model for emerging market *i*. bi2(DSM) = estimated slope in the downside market model for emerging market *i*. Formula = beta estimated using the relevant formula, market model = beta estimated in the market model and downside model = beta estimated in the downside market model. Prior to graphing the emerging markets are sorted by the beta estimated by using the relevant formula.