Value-at-Risk and Extreme Value Distributions for Financial Returns of French Firms

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ABSTRACT

The ability of the Generalised Extreme Value, Generalised Logistic and Generalised Pareto distributions to fit extreme financial returns in the French stock market is assessed. The results indicate that the GEV is not the most appropriate model for the data since the fatter tailed GL is found to provide better descriptions of the extreme minima. Extreme Value Theory based VaR estimates are then derived and compared to those generated by traditional methods. The results show that when the focus is on the really ruinous events which are located deep into the tails of the returns distribution, the Extreme Value Theory methods used in this study can be particularly useful since they produce estimates that outperform those derived by traditional methods at high confidence levels. However, these estimates were found to be considerably higher than those derived by traditional VaR models; consequently leading to higher Minimum Capital Requirements.

Keywords: Extreme Value Theory, Value-at-Risk, L-moments, Probability Weighted Moments, Anderson-Darling goodness of fit test, Generalised Extreme Value distribution, Generalised Logistic distribution.

1. INTRODUCTION

Value-at-Risk (VaR) is the maximum potential loss of a portfolio over a particular time horizon at a certain confidence level. During the last decade it has become an industry standard and it is now routinely used by financial firms when estimating a capital cushion against potential financial catastrophes; indeed, it is known as Minimum Capital Requirement (MCR)[1]. Statistically, VaR is defined as one of the lower quantiles of the distribution of returns that is only exceeded by a certain probability (e.g. 5% or 1%). Therefore, it is argued that accurate VaR estimates imply accurate descriptions of the tails of the distribution of financial returns. A convenient assumption usually made is that returns follow a normal distribution. However, empirical research suggests that the actual distribution of returns has a fatter tail than that suggested by the normal[2]. One implication of this feature is that the probability of large losses is much greater than implied by the normal distribution; in such a case, VaR models are prone to fail when they are needed most; i.e. where a financial institution may suffer enormous losses because of an extreme fall in share prices.

A branch of statistics, named Extreme Value Theory (EVT), focuses exclusively on these extremes and their associated probabilities by directly studying the tails of probability distributions. Applications in finance include, among others, Longin (1996) who investigated the limiting distribution of extremes in the US stock market, Lux (2001) who applied EVT to German data, Jondeau and Rockinger (2003) who analysed the daily extreme returns of 27 stock markets and Gençay and Selçuk (2004) who applied EVT to emerging markets. They all found that extremes of financial returns could be adequately characterised by the Fréchet distribution; a member of the Generalised Extreme Value (GEV) family. The role of EVT as an input in VaR estimation has been examined by Pownall and Koedijk (1999) who used data from Asian stock markets and compared VaR estimates generated by the normal distribution and the RiskMetrics

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[1] See Jorion (2001) for a thorough overview of VaR and the models used for its estimation. The use of VaR by financial firms as an input when calculating MCR was proposed by the Bank for International Settlements (BIS, 1996).

[2] See Aparicio and Estrada (2000) for a broad review of the literature regarding the empirical distributions of financial returns.

model[3] of JP Morgan with estimates generated using EVT. They found that the EVT-based VaR significantly outperformed the other two models and attributed this to the ability of EVT to fit fat tailed time series. Similar results were obtained by Neftci (2000) for the case of eight major exchange and interest rates. He also found that EVT-based VaR estimates were 20% to 30% larger than those generated by the normal distribution. Bali (2003) used daily observations of the annualised yield of the 3-month, 6-month, 1-year and 10-year US treasury securities from 1954 to 1998. He rejected the normality hypothesis and found that the GEV and Generalised Pareto (GP) distributions could lead to very precise VaR results. He also found that EVT-based VaR estimates were on average 24% to 38% larger than those generated by the normal distribution. Based on this finding, he argued that the multiplication factor that the BIS uses to adjust the VaR estimates of banks which employ their own internal models is rather too high and should be reduced[4]. Recently, Danielson (2002) used US data to compare daily VaR estimates at the 99% confidence level derived from the variance-covariance, historical simulation, GARCH, EWMA and EVT methods. He found that the EVT-based VaR provides more accurate VaR estimates than all the other models.

The literature which explores EVT applications in finance has a number of similarities. Firstly, in most studies the GEV and GP are the only distributions used to fit the extremes. Secondly, the Maximum Likelihood (ML) parameter estimation method tends to be used. Notable exceptions are Gettinby et al. (2004) who investigated the distribution of extreme share returns in the UK from 1975 to 2000 and found that the Generalised Logistic (GL) distribution describes better than the GEV both the minima and maxima data. Another exception is Da Silva and Mendes (2003) who used Probability Weighted Moments (PWM) to estimate the parameters of the limiting distribution of extremes in 10 Asian stock markets. However, they focused solely on the GEV distribution which was found to provide an adequate fit to the data. Recently,

[3] The RiskMetrics model is based on Exponential Weighted Moving Average (EWMA) estimates of volatility.
[4] When calculating MCR, a financial institution can adopt the standard approach proposed by the BIS or use its internal VaR models. In the latter case, however, regulators require that VaR estimates should be multiplied by a factor of at least 3.

Tolikas and Brown (2005) considered the GL, GEV and GP distributions and investigated the distribution of the extreme daily share returns in the Greek stock market. Their results added further support to the ability of the GL to fit extreme data and illustrated that the GL provides more accurate VaR estimates compared to the GEV and the normal distribution. Therefore, there are reasons to believe that there is scope for improvement and this is what this paper attempts to do by employing EVT methods whose use in finance has not yet been fully investigated.

The first aim of this paper is to describe the distribution of the extreme minima for daily returns in the French stock market[5]. The second aim of this paper is to assess whether this EVT approach can be useful for risk measurement purposes by deriving VaR estimates and comparing to those generated by traditional approaches. The remainder of the paper is set out as follows. Section 2 introduces the EVT methodology adopted in this paper, section 3 describes the data and section 4 contains the results of the analysis of the extremes. In section 5, VaR estimates generated by the EVT and traditional approaches are presented and compared. Section 6 discuses the implications of the results for both regulators and financial institutions and finally, section 7 concludes the paper.

2. APPLYING EXTREME VALUE THEORY TO ESTIMATE VALUE-AT-RISK

EVT is the statistical study of the extremal behaviour of random variables and its role is to develop procedures which are scientifically appropriate for describing and estimating their behaviour. Extremes of financial returns are defined as the minimum of the daily (or weekly, monthly or larger time periods) logarithmic returns over a given period (known as the selection interval). To illustrate this point, let us denote the time series of an index daily log-returns with the variable $Y_1, Y_2,...,Y_n$. If the length of the selection interval is m, we divide the series into non-overlapping time intervals of length m. The time series of the extreme minima will be $X_1 = \min(Y_1,\dots,Y_m)$, $X_2 = \min(Y_{m+1},\dots,Y_{2m}),\dots,X_{n/m} = \min(Y_{n-m},\dots,Y_n)$. The problem is then to find a

[5] The focus is kept on describing the lower tail of the returns distribution since this is where the big losses of a long position are located. However, similar analysis can be applied to the upper tail for the case of a short position.

probability distribution that adequately describes their behaviour. VaR estimates can then be calculated as certain lower quantiles of this distribution. Applying EVT to financial data involves a number of steps. Firstly, the length of the minima selection period must be chosen. Secondly, distributions that are likely to model adequately the empirical extreme minima returns should be identified. Thirdly, the parameters of these distributions should be estimated and the goodness of fit of these distributions to the data should be tested to choose the one that best fits the empirical data. In the following paragraphs these steps are analytically presented.

The number of extremes available for analysis depends on the length of the extremes selection interval. A longer interval will result in fewer extremes and thus, a lower level of efficiency when estimating a distribution's parameters. To some extent this is an arbitrary decision and in this paper it was decided to use extremes defined over weekly time spans (5 trading days)[6]. The behaviour of the extremes distribution over time aggregation is also studied by dividing the series of weekly extremes into 10 and 30 sub-periods.

Under the assumption that returns are independent and identically distributed (*iid*), Gnedenko (1943) showed that the limiting distribution of the extremes ought to be the GEV. The GEV is a three parameter distribution and its probability density function (pdf) is given by:

$$f(x) = a^{-1} e^{-(1-k)y} e^{-e^{-y}} \text{, where } y = \begin{cases} -k^{-1} \log\left\{1 - k(x-b)/a\right\}, & k \neq 0 \\ (x-b)/a, & k = 0 \end{cases} \tag{1}$$

the parameters a, b and k are called scale, location and shape, respectively. The first parameter is analogous to the standard deviation and high values imply that the distribution of extremes is widely spread out while the second is analogous to the mean and high values imply large extremes. The third governs the shape of the distribution and it is probably the most important parameter since larger values correspond to fatter tailed distributions. The Weibull distribution is the special case of the GEV when $k > 0$ and the range of x is

[6] A monthly selection interval was also employed but the results were not very different from those reported here. Hence, they are not included in the current paper.

$-\infty < x \le b + a/k$. The Gumbel distribution is obtained for $k = 0$ and the range of x is then $-\infty < x < \infty$ while when $k < 0$ the Fréchet distribution is obtained and the range of x is $b + a/k \le x < \infty$. The cumulative distribution function (cdf), $F(x)$, and the quantile function, $X(F)$, of a GEV distributed variable X are given in the Appendix (together with their counterparts for the GL and GP distributions).

However, although the GEV enjoys theoretical support there is strong evidence that financial returns exhibit heteroscedasticity and serial correlation. Kearns and Pagan (1997) used simulations to show that the shape parameter estimates can be exaggerated when the *iid* assumption is violated. On the other hand, Leadbetter et al. (1983) showed that EVT is valid for data structures with weak dependence. Therefore, the *iid* assumption was relaxed but at the same time, the GL and GP distributions were also included, accepting a trade off between being theoretically correct and empirically convincing[7].

The pdf of the GL is given by:

$$f(x) = a^{-1} e^{-(1-k)y} / (1 + e^{-y})^2 \text{, where } y = \begin{cases} -k^{-1} \log\{1 - k(x-b)/a\}, k \ne 0 \\ (x-b)/a, \qquad k = 0 \end{cases} \tag{2}$$

the logistic distribution is the special case of the GL when $k = 0$ and x is in the range $-\infty < x < \infty$, while when $k > 0$, x belongs to $-\infty < x \le b + a/k$ and when $k < 0$, x belongs to $b + a/k \le x < \infty$.

The pdf of the GP is given by:

[7] With respect to VaR estimation the series of the data will be divided into sub-periods and moving window techniques will be used to estimate the parameters. This can be reasonably assumed to capture some of the non-stationarity of the data thus, reducing the non-*iid* data problem. Another alternative would be to fit the tail of the conditional distribution of returns by using an autoregressive volatility model (e.g. GARCH), standardise the returns by the estimated conditional volatility and proceed in EVT analysis. This approach has received attention by McNeil and Frey (2000) and Byström (2004). However, additional parameters have to be estimated which make this approach subject to increased estimation standard error and model risk. Additionally, the non-constant variance of returns feature would tend to diminish if lower frequency data were to be used. However, the size of the dataset will also decrease significantly raising concerns for the soundness of the estimation procedures.

$$f(x) = a^{-1} e^{-(1-k)y}, \text{ where } y = \begin{cases} -k^{-1}\log\{1-k(x-b)/a\}, k \neq 0 \\ \\ (x-b)/a, \qquad\qquad k = 0 \end{cases} \qquad (3)$$

the exponential and the uniform distributions are the special case of the GP when $k = 0$ and $k = 1$, respectively, on the interval $0 \le x \le a$. The range of x when $k \le 0$ is $0 \le x < \infty$ while when $k > 0$ is $0 \le x < a/k$.

The detection of the best candidate distributions to fit the data is accomplished using L-moment diagrams. L-moments are linear combinations of ordered data which, like the conventional moments, provide a set of summary statistics for probability distributions[8]. Hosking (1990) defined the r^{th} L-moment, l_r, for any random variable X which has a finite mean as:

$$l_r \equiv r^{-1} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} EX_{(r-k:r)}, \qquad r = 1,2,\cdots \qquad (4)$$

where $EX_{(r-k:r)}$ is the expectation of the $(r-k)^{th}$ extreme order statistic. The first two such statistics, l_1 and l_2, are measures of location and scale and the two L-moment ratios, $t_3 = l_3/l_2$ and $t_4 = l_4/l_2$ are measures of skewness and kurtosis, respectively. An L-moment diagram contains the curves or points of the theoretical distributions whose ability to fit adequately the empirical data is examined[9]. The identification of the best candidate distributions is achieved by plotting the estimated t_3 and t_4 and choosing the distribution whose L-skewness and L-kurtosis theoretical curve is closest to the plotted point.

The next step is to estimate the parameters of the selected distribution/s. For moderate to large samples, the most widely used method is the ML method. However, its asymptotic properties are open to doubt in the case of small samples where convergence of the likelihood

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[8] The most important feature of the L-moments is that they are more robust to the presence of outliers than conventional moments. This is because the calculations of conventional moments involve powers which give greater weight to outliers that can lead to considerable bias and variance.

[9] On such diagram, a three-parameter distribution (e.g. the GL) is represented by a curve whereas a two-parameter distribution (e.g. the normal) is represented by a single point.

function is not always guaranteed to be at the global maximum (Hill, 1963). For small samples, which are the norm in EVT, the PWM is considered to be more efficient than the ML[10]. Hosking (1990) defined the PWM of a random variable X with a finite mean and a distribution function F as:

$$b_r = E\left[X\{F(X)\}^r\right], \qquad r = 0,1,\cdots \tag{5}$$

where $E[X(\cdot)]$ is the expectation of the quantile function of X. Although, PWM may be sensitive to outliers, Hosking (1990) demonstrated that there exist linear relationships between the PWM and the more robust L-moments, given by:

$$l_{r+1} = \sum_{k=0}^{r} P^*_{r,k} b_k, \qquad r = 0,1,\cdots, \quad \text{where } P^*_{r,k} = (-1)^{r-k}\binom{r}{k}\binom{r+k}{k} \tag{6}$$

this method involves estimating parameters by equating sample moments to those of the chosen distribution. For the GEV, GL and GP the solutions for the shape (k), scale (a) and location (b) parameter estimates can be found in the Appendix.

After fitting a distribution, it is important to assess how good the fit is[11]. For this reason the Anderson-Darling goodness of fit test is used (Anderson and Darling, 1954). This is an especially designated test to measure discrepancies in the tails and it has been found to be the most powerful among a wide set of available tests for small samples (Choulakian and Stephens (2001); Stephens (1976)). A tractable expression is given in d'Agostino and Stephens (1986):

$$A_n^2 = -n - (1/n)\sum_{i=1}^{n}\left[(2i-1)\log z_i + (2n+1-2i)\log(1-z_i)\right] \tag{7}$$

[10] Hosking et al. (1985) showed that for the GEV distribution, parameters and quantiles made using the PWM method are estimated with at least 70% efficiency. For example, when the shape parameter of the GEV is -0.2, the asymptotic bias of the 0.01 quantile estimated by the PWM and ML methods is found to be -0.2 and 1.6, respectively. In addition, for shape parameter values in the range -0.5 to 0.5 and samples of up to 100 observations, PWM estimates have lower root-mean square error than estimates generated by the ML method. Similar results are reported in the literature for the GEV (Landwehr et al. (1979); Smith (1987)) and for the GP (Hosking and Wallis (1987); Ro?tzen and Tajvidi (1997)).

[11] Formally, the goodness of fit problem can be stated as follows. We wish to test the null hypothesis, H_0, that the sample X of size n, with order statistics $x_1 \le x_2 \le \cdots \le x_n$, is generated by a particular distribution. The detailed steps for testing the H_0 involve (i), the arrangement of data in ascending order, (ii) the calculation of the statistic $z_i = F(x_i)$, $i = 1,\cdots,n$ where $F(x)$ is given in the Appendix by using the estimated parameters and (iii) the calculation of the A_n^2 statistic from equation 7. Finally, if the estimated statistic exceeds the critical value of the A_n^2 statistic at a particular significance level the null hypothesis can be rejected.

where, $z_i = F(x_i)$, $i = 1, \cdots, n$ is the empirical distribution function of a variable X of size n.

Once the empirical distribution of extremes has been adequately modelled, VaR estimates for the daily returns distribution can be derived from estimates of lower quantiles of the extremes distribution[12]. VaR models can only be useful if their forecasts are sufficiently accurate and this is why any VaR model should be validated. Backtesting is the task of systematically comparing the VaR forecasts with the actual returns using historical data. Thus, the number of times that the VaR forecasts are violated by the actual returns can be counted and this serves as an indication of how well calibrated a VaR model is[13]. The statistical test employed is the test proposed by Christoffersen (1998) which assesses whether the VaR forecasts overestimate or underestimate risk and in addition whether the VaR violations occur in clusters[14].

3. DESCRIPTION OF THE DATA

The dataset used to describe the behaviour of the French stock market consists of 7253 daily logarithmic returns that cover the 29 year period from the 2nd of January 1973 to the 28th of December 2001[15, 16]. Table 1 contains descriptive statistics of the CAC-DS index daily returns.

[12] However, as extremes are collected over non-overlapping periods of a certain length, the probability of a return not exceeding VaR, needs to be adjusted as to convert the EVT-based VaR to a VaR figure which corresponds to the returns distribution of the desired frequency. If, for example, the frequency of returns is daily and p_{ext} is the probability that an extreme return collected over a period of m daily returns will not exceed VaR and p is the probability that a daily return will not exceed VaR, then the probability that a daily return will not exceed the VaR is $p_{ext} = 1 - (1 - p)^m$.

[13] The basic idea is that if a model is perfectly specified then the number of reported violations over a time period should be in line with the confidence level.

[14] This is particularly important because a VaR model might generate acceptable estimates on average while it is possible that the majority of VaR violations occur within a short time interval.

[15] The corresponding Datastream code is TOTMKFR, the index is composed of 250 of the most heavily traded shares that aim to cover the 70% to 80% of the total market capitalisation and prices take account of capital changes. The main reason for using this index instead of the CAC40 is that it is available for a much longer period. In order to retain some familiarity throughout the paper this index is denoted as CAC-DS

[16] This time span contains the rather volatile periods of 1973 to 1975, 1978 to 1982, 1986 to 1988, 1990 to 1992 and 1997 to 2001 where some of the lowest daily returns occurred: -6.09% (10/10/79), -7.89% (12/5/81), -9.89% (19/10/87), -7.89% (26/10/87), -8.43% (28/10/87), -7.86% (10/11/87) and -7.36% (11/9/01). The first two periods, probably reflect the negative effects of the oil crises in 1973 and 1979 that impacted upon all industrialised countries. The second period may also reflect the market's expectations that the likely election of the socialist party would result in the nationalisation of the main private industrial groups and banks. The third period coincides with the collapse of international stock markets in 1987, as well as the loss of the elections by the socialist party and the

The daily mean return was 0.04% and the daily standard deviation 1.13%. The minimum daily return was -9.89% and occurred on the 19th of October 1987 while the maximum was 7.97% and occurred on the 12th of November 1987. The skewness value of -0.425 implies that negative returns were larger than their positive counterparts while the kurtosis value of 4.829 implies that the empirical distribution of daily returns is fat tailed. This result was also confirmed by the Shapiro-Wilk test which rejected the normality assumption at the 5% significance level.

INSERT TABLE 1 ABOUT HERE

The CAC-DS index daily returns can be further examined by standardising them, computing the pairs of empirical percentiles (1%, 99%) and (5%, 95%) and comparing these with those of a standard normal distribution, i.e. (-2.326, 2.326) and (-1.644, 1.644), respectively. The pairs of empirical percentiles were found to be larger, ((-2.755, 2.533)), and smaller, ((-1.568, 1.523)), for the (1%, 99%) and (5%, 95%), respectively, confirming the presence of fat tails in the empirical distribution of the daily returns. Furthermore, under the normality assumption only 20 of the 7253 observations would be expected to be outside the range plus or minus 3 standard deviations away from the mean; 10 in each tail. However, 84 observations were outside this range; 52 in the left and 32 in the right tail. Hence, the hypothesis that the daily returns of the CAC-DS index are generated by the normal distribution can be rejected. In this case it is the extremes that mainly contribute to the non-normality of the daily returns distribution.

4. ANALYSIS OF THE EXTREME RETURNS IN THE FRENCH STOCK MARKET

Weekly minima extremes were collected over the 29 years period under examination. Section 4.1 describes the identification of the appropriate distribution/s and section 4.2 details the estimation of parameters and the goodness of fit test.

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decision of the newly elected government to privatise 13 financial institutions. The fourth period contains the Gulf crisis and the collapse of the Exchange Rate Mechanism while the fifth period, reflects the global turbulence due to the Asian and the Russian financial crises and the terrorist attack on the US in September 2001.

4.1 Identifying the distribution of the extreme minimum daily returns

The L-skewness (t_3) and L-kurtosis (t_4) were calculated for the weekly minima divided into 10 and 30 sub-periods and were plotted on an L-moments diagram. Figure 1 contains the t_3 and t_4 for the series of the weekly minima divided into 30 sub-periods. From an initial inspection of this figure, it seems that all the distributions can be excluded except for the GL and the GEV. This is because the points of the t_3 and t_4 are mainly dispersed around the theoretical curves of the GL and the GEV distributions. The corresponding L-moment plot was also generated for the 10 sub-divisions of the weekly minima and a similar pattern appeared[17]. However, in order to choose between the GL and the GEV, further analysis is required and a more formal test of goodness of fit of these two distributions should be applied.

INSERT FIGURE 1 ABOUT HERE

4.2 Parameter estimates and goodness of fit test

The GL and GEV distributions were fitted to the weekly minima for the whole interval and for 10 and 30 sub-periods with the parameters being estimated by the PWM method. The parameter estimates and the *p*-value of the Anderson-Darling goodness of fit test are contained in Table 2. When the weekly minima for the whole interval were fitted by either the GL or GEV distributions, the Anderson-Darling goodness of fit test had a rather low *p*-value in both cases indicating a bad fit. One possible explanation is that the nature of the distribution of the extremes was changing over time and therefore, when long time-periods were used the data came from a mixture of distributions; thus, a single distribution was likely to be a bad fit. For the case of 10 sub-periods the GEV distribution performed as well as the GL distribution and they both fitted adequately in nine sub-periods. The minimum *p*-values for both distributions occurred in the turbulent period 5/8/87 to 25/6/90 which contains some of the largest negative

[17] In the interest of brevity this diagram is not included in the paper; however, it is available upon request.

daily returns in the French stock market[18]. For the case of 30 sub-periods, the GL distribution fitted the empirical data adequately in 28 of the 30 sub-periods with *p*-values within the range 0.037 to 0.901, while the GEV fitted 25 of them with *p*-values ranging from 0.002 to 0.769. In comparison to the GEV, the GL fitted better in 20 of the 30 sub-periods. Again the effect of international events on the ability of the GL and GEV distributions to fit the weekly extremes adequately becomes apparent in sub-periods 16, 20, 25 and 30 which correspond to the time periods that contain the stock market crash in 1987, the European Exchange Rate Mechanism crisis in 1992, the Asian crisis in 1997 and the terrorist attack on the US in 2001, respectively.

INSERT TABLE 2 ABOUT HERE

In summary, both the GEV and GL distributions appeared to be able to model adequately the extreme minima of the CAC-DS index daily returns over the period 1973 to 2001. However, the GL distribution provided a better fit than the GEV, especially when several sub-periods were used. This is an important result because current applications of EVT in finance focus on either the GEV or GP distributions and since these are less fat tailed than the GL there is considerable chance that the probabilities of extreme events are underestimated. Additionally, it seems that the nature of extremes changed over time since the behaviour of the shape parameter for both distributions varied substantially across different sub-periods[19]. In particular, volatile sub-periods which contained large negative daily returns tended to result in higher shape values than periods of low volatility which contained fewer and smaller negative daily returns. This is expected to have a significant effect upon VaR estimates and one would expect VaR estimates to be higher when the shape parameter values were higher. Such a result would naturally lead to larger MCR for banks if they were to be protected against large negative price movements. It

[18] For example, -3.54% in 15/10/87, -9.89% in 19/10/87, -3.43% in 22/10/87, -7.89% in 26/10/87, -8.43% in 28/10/87, -7.86% in 10/11/87, -3.51% in 4/12/87, -3.02% in 28/12/87 and -3.12% in 20/1/88 which can be attributed to the end of 1987 turbulent period and the loss of the election by the Socialist party and the decision of the newly elected government to privatise 13 financial institutions and -6.24% in 16/10/89 which can be attributed to the negative market sentiment regarding the fall of the Berlin Wall.

[19] For the GEV distribution it is also noticeable that the sign of the shape parameter estimates changed over time indicating that there is no unique distribution within this family that describes the empirical data well. This is at variance with Gettinby et al. (2004) and Longin (1996), who detected no sign changes when they fitted the GEV distribution to the extreme daily share returns of the UK and US stock markets, respectively.

seems that parameter estimates which correspond to short sub-periods should be used in VaR estimation since this would allow VaR to respond quickly to the changes of the macro and micro economic conditions prevailing in the market place.

5. ESTIMATING VALUE-AT-RISK USING EXTREME VALUE THEORY

VaR estimates for the CAC-DS index daily returns distribution were derived from lower quantiles of the distribution of extremes using the parameter estimates associated with the GEV and GL distributions. The 30 sub-divisions of the extremes time series were chosen to allow the parameters to change relatively frequently (static approach). However, the indication that the nature of extremes distribution is time variant suggests that more frequent updating of parameters might be more realistic. Consequently the parameters for the weekly minima were also estimated using moving windows of lengths 50, 100, 200 and 300 (moving window approach)[20, 21]. The set of confidence levels used comprise 90.00%, 95.00%, 99.00%, 99.50%, 99.75% and 99.90%[22]. For comparison, VaR estimates generated by traditional methods such as the variance-covariance, historical simulation, EWMA and the monte carlo simulation based on the normal distribution were also derived. For these methods 250, 500, 1000 and 1500 past daily returns were used for model calibration[23]. In order to examine the performance of each approach

[20] The underlying principle behind the choice of the number of daily returns used for the traditional methods, the number of sub-periods into which the minima were divided in the static approach and the length of the moving window used, was to compare VaR results based on the same informational time periods. For example, 250 daily returns correspond to about one trading year. When the series of weekly minima is divided into 30 sub-periods the parameters derived correspond also to about one year. The same is true when a moving window of length 50 weekly minima is used.

[21] Gençay and Selçuk (2004) argued that the usefulness of EVT methods in VaR estimation can be enhanced by allowing for the possibility that the parameters may change over time. There have been attempts to take into account the time varying distributional characteristics of the extremes by using autoregressive processes (McNeil and Frey (2000); Pownall and Koedijk (1999)) or quantile regression techniques (Engle and Manganelli (2003)). However, these approaches introduce yet more parameters in to the modelling procedure and this is likely to result in larger estimation errors and possibly even more inaccurate VaR estimates.

[22] The VaR estimates at these confidence levels can be interpreted as the maximum loss of a position in this index which is expected to occur in 100, 50, 10, 5, 2.5 and 1 days, respectively, out of 1000 days (4 trading years).

[23] In order for the EWMA method to effectively take account of 250, 500, 1000 and 1500 past daily returns in the estimation of volatility, the parameter ? was set to 0.996, 0.998, 0.999 and 0.9993333332, respectively. For the monte carlo simulation method the normal distribution was assumed and for each daily VaR estimate 10,000 random scenarios were generated. Use of fewer than 250 and 500 past daily returns for the historical simulation method makes it impossible to generate estimates at some of the highest confidence levels because the dataset becomes too small. For example, the calculation of VaR at the 99.90% confidence level requires at least 1000 daily returns.

the results were backtested over the two time periods from 2/1/87 to 31/12/91 and from 2/1/97 to 28/12/01; these periods contain some of the largest negative daily returns in the French stock market. In addition, they contain 1253 and 1265 daily returns, respectively, sizes which can be considered adequate for statistical evaluation. Tables 3 and 4 contain the VaR backtesting results which are presented in terms of the number of VaR forecasts violations by the actual returns followed by the corresponding Christoffersen test statistic *p*-values.

INSERT TABLE 3 ABOUT HERE

The time period 2/1/87 to 2/1/92 is a volatile one (standard deviation is 1.19%) with negative skewness (-1.103) and high kurtosis (12.719). Unsurprisingly, the variance-covariance method underestimated risk by a considerable amount since it gave more violations that would be expected from an accurately calibrated model; an exception to this generalisation relates to the 90.00% and 95.00% confidence levels where the model overestimated the lower tail. The inability of this model to describe adequately the tails of the returns distribution is rather serious; for example, at the 99.90% confidence level only 1 violation was expected but the variance-covariance method provided 10 to 12 violations[24]. The historical simulation method, on the other hand, gave better results, especially at high confidence levels where the *p*-values of the Christoffersen test statistic were acceptable. As the number of past daily returns used increased, the historical simulation provided very good results at the higher confidence levels of 99.75% and 99.90%, although its accuracy decreased at lower confidence levels. Finally, the EWMA and monte carlo simulation methods were the least accurate models at all confidence levels. This was probably because the EWMA tends to react quickly to volatility changes but only after the event, while for the monte carlo simulation it is probable that the normal distribution was not a good model for the daily returns over this particular time period.

The VaR results generated using EVT methods were examined next, starting with the static approach. It is noticeable that both the GEV and GL distributions underestimated risk with

[24] The 11 largest unexpected VaR violations from the VC250 were -3.59% (15/5/87), -9.89% (19/10/87), -7.89% (26/10/87), -8.43% (28/10/87), -7.86% (10/11/87), -6.24% (16/10/89), -2.55% (27/10/89), -4.73% (6/8/90), -3.22% (20/8/90), -3.62% (21/8/90) and -6.77% (19/8/91).

more violations being recorded than would be expected. A possible explanation is that volatility was changing quickly during this time period while the parameters of the distributions were changing only once every year and so, the parameters did not adequately reflect current market conditions. The moving window approach was better than the static approach although there was little difference between the two distributions. The only substantial advantage that accrued from the use of the GEV and GL distributions was the accurate prediction of the tail event at the 99.90% confidence level.

The period 2/1/97 to 28/12/01 is the next period over which the VaR models were evaluated[25]. The daily standard deviation was high (1.34%) with a skewness value of -0.313 and a kurtosis value of 1.753. As seen in Table 4, although the variance-covariance model when 250 past daily returns were used (VC250) predicted well at the 90.00% confidence level, it became seriously inaccurate at higher confidence levels. For example, at the 99.75% confidence level the expected number of VaR violations was 3 but 17 were observed; when additional past daily returns were used the model became even more inaccurate. The historical simulation method tended to give better results than the variance-covariance when 250 and 500 past daily returns were used. When 1000 and 1500 past daily returns were used this method gave a relatively good prediction at the 99.90% confidence level but the performance at lower confidence levels was very poor. The EWMA, on the other hand, was not particularly accurate at any confidence level. As one would expect, the performance of the monte carlo simulation method was also poor, especially at high confidence levels. This could be attributed to the inability of the normal distribution to provide a good description of the daily returns distribution over this volatile time period.

INSERT TABLE 4 ABOUT HERE

The performance of the GEV and GL distributions based on the static approach was far from being accurate, since at all confidence levels the number of VaR violations were greater

[25] This is a period in which the stock markets were affected by the Asian crisis, the turbulence surrounding the Long Term Capital Management collapse in 1998 and the sentiment of the market after the terrorist attack on the US in 2001.

than expected. The single exception was the 99.90% confidence level for both the GEV and GL distributions. The use of moving windows of weekly minima of length 50 resulted in a slight improvement in the results for both distributions; this was particularly noticeable deep in the tails of the empirical distribution of returns. In the prediction of the lower tail of the returns distribution the GL performed better than the GEV distribution. For example, at the 99.75% and 99.90% confidence levels one would expect 3 and 1 VaR violations, respectively, and the GL-MW-W50 gave 6 and 1, respectively while the GEV-MW-W50 gave 10 and 2, respectively.

In summary, EVT-based VaR estimates were found to be more accurate at high confidence levels compared with methods which assume that returns are normally distributed. The only other method which performed well was the historical simulation; however, for estimates deep into the tail of the returns distribution, data availability might be an issue when using the historical simulation approach. At low confidence levels, EVT-based VAR did not offer any benefits over less sophisticated methods but this to be anticipated since EVT focus on the tails of the returns distribution and not its central part.

7. IMPLICATIONS FOR REGULATORS AND FINANCIAL INSTITUTIONS

The fundamental objectives of financial regulators and financial institutions are quite different. Regulators are mainly interested in reinforcing the stability of the financial system and therefore, would tend to favour the most conservative VaR model; the one which results to the highest MCR. On the other hand, the profitability of financial firms is directly linked to the use of VaR models since the MCR is non-investable. Therefore, as Danielson et al. (2001) argued, investment banks have incentives to favour VaR models which result to lower MCR, thus exposing financial firms to the really ruinous events located deep into the tail of the returns distribution. According to the BIS guidelines (BIS, 1996) a financial institution can choose between the standard approach proposed by the BIS and their internal VaR models when calculating MCR. However, for those who choose to use in-house models, regulators require that VaR estimates[26] should be multiplied by a factor of at least 3. Based on the backtesting evaluation of a bank's VaR models, BIS may increase the multiplier further by an increment between 0 and 1. This rule has been criticised by many researchers (Longin, 2000; Danielson et al., 1998) as being too crude giving rise to high MCR values; thus it eliminates any incentives that banks might have to improve their internal models. According to the standard approach the MCR of an equity position must be at least 12% of the position and aims to cover the maximum loss over a period of 10 days[27]. Thus, approximately, dividing by the square root of 10 one could derive the daily capital charge; that is 3.79%. Table 5 contains the daily VaR estimates for the CAC-DS index daily returns on the 19/10/87 which is the day where the minimum daily return of -9.90% occurred. Clearly, the capital charge of 3.79% is much less than the loss of -9.90% implying that the standard approach offers inadequate coverage against extreme events. On the other hand, the use of EVT can provide far better predictions against these rare market movements. For example, the VaR estimates at the 99.90% confidence level provided by the GL-MW-W100 is -10.21% which adequately captures the extreme daily return of -9.90%. The best prediction derived using traditional methods was -5.97% by the HS1000 and HS1500; a far from adequate estimation. However, if a bank were to multiply the VaR estimates derived by EVT by a factor between 3 and 4, the capital charges would be enormous (e.g. between 30% and 40%). Therefore, although EVT can provide accurate tail predictions, the use of the multiplication factor will make these particular MCRs very high, thus deterring banks from considering its use.

INSERT TABLE 5 ABOUT HERE

[26] These estimates should be based on the 99% confidence level, assume a holding horizon of 10 days and use at least one year's data. It is common practise that banks initially calculate daily VaR and scale these estimates by using the rule of the square root of time.

[27] A 4% of capital is charged for the *specific risk* and 8% of capital for the *general market risk*. The BIS defined *specific risk* to be the gross equity position in the market as a whole (the sum of all long and all short equity positions) and *general market risk* to be the net equity position (the difference between the sum of the long and the sum of the short equity positions). The capital charge for specific risk is 8% but if the portfolio is well diversified and liquid reduces to 4%. The index used in this paper can be considered to be a both well diversified and liquid portfolio, therefore a capital charge of 4% for specific risk is assumed.

In addition, Jorion (2002) argued that financial institutions should tend to favour VaR models which generate estimates of low variability because they would not be forced to sell assets or change their trading strategies frequently in order to satisfy regulatory requirements. Tables 6 contains the standard deviation of the VaR estimates across a wide set of confidence levels during the time period 2/1/87 to 31/12/91. The variability of the VaR estimates generated by the EVT methods is, in general, similar to the variability of VaR provided by the other methods. However, at the 99.90% confidence level, the EVT-based VaR estimates are extremely volatile. Taking into account, therefore, the objectives of a financial institution and the volatile and relatively large VaR values that the EVT method provides at the 99.90% confidence level, it could be argued that a financial institution would be reluctant to adopt EVT analysis in VaR modelling unless the multiplication factor was to be reduced or even abolished.

INSERT TABLE 6 ABOUT HERE

8. CONCLUSION

In this paper EVT methods were used to derive VaR estimates related to the lower tail of the daily returns in the French stock market. Regarding the analysis of extremes it was found that the too much celebrated GEV distribution is not the best model for the extreme minima of the daily returns since it was found that a fatter tailed distribution, the GL, offers better descriptions. Considering that current applications of EVT in finance focus only on either the GEV or GP distribution the implication is that the probabilities of the ruinous extreme events maybe underestimated. The results also indicated that the behaviour of extremes is time variant apparently affected by economic and political events in the French stock market.

With respect to VaR, the empirical results indicated that EVT methods can be valuable when the interest is in protecting a portfolio from the really catastrophic events located deep in the lower tail of the returns distribution. At low confidence levels, however, EVT-based VAR did not offer any benefits over less sophisticated methods but this was to be expected since EVT focus on modelling the tails of the returns distribution and not its central part. EVT-based VaR estimates were also found to be larger than those derived by traditional methods, leading to higher MCR. In that respect, one could argue that the BIS multiplication factor is too high thus, discouraging financial institutions from adopting EVT methods when deriving VaR. The results also showed that techniques which capture some of the time variant nature of the extremes distribution have the potential to improve the accuracy of VaR estimates since current market conditions are explicitly taken into account.

APPENDIX

The GEV, GL and GP are three parameter distributions which have the following CDFs, quantile functions and parameter estimates. The parameters k, a and b are called shape, scale and location respectively.

	Generalised Extreme Value (GEV)	Generalised Logistic (GL)	Generalised Pareto (GP)
Cumulative Distribution Function (CDF)			
	$F(x) = e^{-e^{-y}}$	$F(x) = 1/(1+e^{-y})$	$F(x) = 1-e^{-y}$
Quantile function			
	$X(F) = \begin{cases} b + a\{1-(-\log F)^k\}/k, & k \neq 0 \\ b - a\log(-\log F), & k = 0 \end{cases}$	$X(F) = \begin{cases} b + a\left[1-\{(1-F)/F\}^k\right]/k, & k \neq 0 \\ b - a\log\{(1-F)/F\}, & k = 0 \end{cases}$	$X(F) = \begin{cases} b + a\{1-(1-F)^k\}/k, & k \neq 0 \\ b - a\log(1-F), & k = 0 \end{cases}$
Parameter estimates			
	$k = 7.8590c + 2.9554c^2$ where $c = \dfrac{(2b_1 - b_0)}{(3b_2 - b_0)} - \dfrac{\ln 2}{\ln 3}$	$k = -t_3$	$k = \dfrac{l_1}{l_2} - 2$
	$a = \dfrac{l_2 k}{(1-2^{-k})\Gamma(1+k)}$	$a = \dfrac{l_2}{\Gamma(1-k)\Gamma(1+k)}$	$a = (1+k)\,l_1$
	$b = l_1 - \dfrac{a}{k}\{1-\Gamma(1+k)\}$	$b = l_1 - \dfrac{a}{k}\{1-\Gamma(1-k)\Gamma(1+k)\}$	the location parameter b is assumed known (can be estimated as $x_{1:n}$)

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Table 1. Descriptive statistics for the DAX-DS and CAC-DS daily returns

Time period	*n*	Mean (%)	St.Dev (%)	Min (%)	Max (%)	Skewness	Kurtosis	*SW*
1973-2001	7253	0.04	1.13	-9.89	7.97	-0.425*	4.829*	0.000

Note: This table includes descriptive statistics for the CAC-DS index daily returns over the period 1973 to 2001. *n* denotes the number of observations, St.Dev denotes the standard deviation of returns, the minimum and maximum daily returns are indicated as Min and Max and *SW* indicates the Shapiro-Wilk normality test. An * indicates statistical significance at the 5% level.

Table 2. CAC-DS weekly minima GEV and GL parameter estimates and AD *p*-values

		GEV parameter estimates				GL parameter estimates				
Sub-periods (s)	***N***	**ß$_s$**	**a_s**	**?$_s$**	**AD *p*-value**	**ß$_s$**	**a_s**	**?$_s$**	**AD *p*-value**	**Better fit**
***s* = 1**										
1. (02/01/73-28/12/01)	1451	0.007	0.007	-0.094	0.008	-0.009	0.005	0.232	0.043	**GL**
***s* = 10**										
1. (02/01/73-31/12/75)	145	0.009	0.008	0.034	0.323	-0.012	0.005	0.148	0.137	**GEV**
2. (02/01/76-28/11/78)	145	0.007	0.006	-0.022	0.262	-0.010	0.004	0.184	0.065	**GEV**
3. (29/11/78-29/10/81)	145	0.007	0.007	-0.146	0.168	-0.009	0.005	0.267	0.430	**GL**
4. (30/10/81-14/09/84)	145	0.006	0.006	0.123	0.406	-0.008	0.004	0.093	0.321	**GEV**
5. (17/09/84-04/08/87)	145	0.005	0.006	-0.091	0.322	-0.008	0.004	0.230	0.861	**GL**
6. (05/08/87-25/06/90)	145	0.005	0.006	-0.337	0.000	-0.007	0.005	0.406	0.030	**GL**
7. (26/06/90-18/05/93)	145	0.007	0.006	-0.139	0.156	-0.009	0.004	0.263	0.537	**GL**
8. (19/05/93-04/04/96)	145	0.007	0.005	0.118	0.210	-0.009	0.003	0.096	0.078	**GEV**
9. (08/04/96-25/02/99)	145	0.007	0.008	-0.100	0.145	-0.010	0.005	0.236	0.090	**GEV**
10. (26/02/99-28/12/01)	146	0.010	0.008	0.060	0.312	-0.013	0.005	0.132	0.804	**GL**
***s* = 30**										
1. (02/01/73-24/12/73)	48	0.008	0.007	0.010	0.580	-0.010	0.005	0.164	0.568	**GEV**
2. (26/12/73-27/12/74)	48	0.013	0.010	0.220	0.156	-0.017	0.006	0.036	0.233	**GL**
3. (30/12/74-22/12/75)	48	0.007	0.006	0.001	0.448	-0.009	0.004	0.169	0.856	**GL**
4. (23/12/75-13/12/76)	48	0.007	0.005	-0.061	0.120	-0.009	0.003	0.210	0.070	**GEV**
5. (14/12/76-29/11/77)	48	0.008	0.007	-0.075	0.720	-0.011	0.005	0.219	0.590	**GEV**
6. (30/11/77-14/11/78)	48	0.008	0.007	0.134	0.430	-0.010	0.005	0.087	0.341	**GEV**
7. (15/11/78-07/11/79)	48	0.005	0.006	-0.238	0.215	-0.007	0.004	0.332	0.342	**GL**
8. (08/11/79-21/10/80)	48	0.007	0.006	-0.034	0.222	-0.010	0.004	0.192	0.410	**GL**
9. (22/10/80-08/10/81)	48	0.008	0.010	-0.099	0.209	-0.012	0.007	0.235	0.359	**GL**
10. (09/10/81-24/09/82)	48	0.007	0.006	0.134	0.008	-0.009	0.004	0.087	0.188	**GL**
11. (27/09/82-07/09/83)	48	0.006	0.006	0.239	0.650	-0.008	0.003	0.025	0.770	**GL**
12. (08/09/83-17/08/84)	48	0.006	0.006	0.036	0.211	-0.008	0.004	0.147	0.109	**GEV**
13. (20/08/84-02/08/85)	48	0.004	0.004	0.110	0.670	-0.006	0.003	0.101	0.870	**GL**
14. (05/08/85-21/07/86)	48	0.006	0.008	-0.092	0.559	-0.009	0.006	0.230	0.843	**GL**
15. (22/07/86-30/06/87)	48	0.006	0.007	0.007	0.769	-0.009	0.005	0.165	0.901	**GL**
16. (01/07/87-07/06/88)	48	0.007	0.010	-0.335	0.002	-0.011	0.008	0.404	0.039	**GL**
17. (08/06/88-26/05/89)	48	0.005	0.005	0.040	0.632	-0.007	0.003	0.145	0.429	**GEV**
18. (29/05/89-10/05/90)	48	0.004	0.004	-0.306	0.750	-0.006	0.003	0.382	0.886	**GL**
19. (11/05/90-25/04/91)	48	0.008	0.007	0.017	0.167	-0.011	0.005	0.159	0.475	**GL**
20. (26/04/91-09/04/92)	48	0.005	0.004	-0.274	0.049	-0.006	0.003	0.359	0.227	**GL**
21. (10/04/92-26/03/93)	48	0.007	0.005	-0.191	0.298	-0.009	0.004	0.299	0.328	**GL**
22. (29/03/93-10/03/94)	48	0.006	0.005	0.048	0.315	-0.008	0.003	0.140	0.591	**GL**
23. (11/03/94-24/02/95)	48	0.009	0.006	0.343	0.351	-0.011	0.003	-0.032	0.345	**GEV**
24. (27/02/95-08/02/96)	48	0.006	0.005	0.066	0.358	-0.008	0.003	0.128	0.490	**GL**
25. (09/02/96-24/01/97)	48	0.005	0.004	-0.117	0.045	-0.006	0.003	0.247	0.037	**GEV**
26. (27/01/97-12/01/98)	48	0.008	0.008	0.029	0.460	-0.011	0.005	0.151	0.505	**GL**
27. (13/01/98-21/12/98)	48	0.009	0.010	-0.046	0.390	-0.013	0.007	0.200	0.337	**GEV**
28. (22/12/98-01/01/99)	48	0.007	0.007	0.151	0.235	-0.010	0.005	0.077	0.292	**GL**
29. (02/12/99-08/11/00)	48	0.012	0.008	0.054	0.683	-0.015	0.005	0.136	0.409	**GEV**
30. (09/11/00-28/12/01)	59	0.012	0.009	0.048	0.019	-0.015	0.006	0.140	0.361	**GL**

Note: This table includes the PWM parameter estimates and the Anderson-Darling (AD) goodness of fit test *p*-values for the GEV fitted to the reverse weekly minima and for the GL fitted to the weekly minima over the period 1973 to 2001. *N* denotes the number of extreme observations in each sub-period, and ß$_s$, a_s and ?$_s$ denote the location, scale and shape parameters, respectively.

Table 3. CAC-DS daily VaR backtesting for the period 2/1/87 to 31/12/91

Actual daily returns	**1253**					
Confidence level	**90.00%**	**95.00%**	**99.00%**	**99.50%**	**99.75%**	**99.90%**
Expected violations	**125**	**63**	**13**	**6**	**3**	**1**
VC250	83 (0.000)	51 (0.001)	26 (0.000)	22 (0.000)	16 (0.000)	12 (0.000)
VC500	95 (0.001)	53 (0.000)	22 (0.001)	18 (0.000)	15 (0.000)	14 (0.000)
VC1000	76 (0.000)	45 (0.000)	22 (0.001)	16 (0.000)	13 (0.000)	10 (0.000)
VC1500	80 (0.000)	49 (0.000)	26 (0.000)	19 (0.000)	15 (0.000)	12 (0.000)
HS250	118 (0.000)	55 (0.006)	10 (0.698)	3 (0.343)	N/A	N/A
HS500	122 (0.017)	62 (0.001)	15 (0.002)	5 (0.853)	2 (0.787)	N/A
HS1000	131 (0.003)	61 (0.002)	12 (0.880)	8 (0.762)	3 (0.990)	1 (0.972)
HS1500	139 (0.003)	70 (0.005)	16 (0.279)	9 (0.554)	3 (0.990)	1 (0.972)
EWMA250	80 (0.000)	48 (0.000)	20 (0.002)	17 (0.000)	15 (0.000)	11 (0.000)
EWMA500	77 (0.000)	47 (0.000)	20 (0.002)	16 (0.000)	12 (0.000)	10 (0.000)
EWMA1000	79 (0.000)	48 (0.000)	23 (0.001)	17 (0.000)	13 (0.000)	11 (0.000)
EWMA1500	80 (0.000)	49 (0.000)	23 (0.001)	16 (0.000)	13 (0.000)	11 (0.000)
MCS250	87 (0.000)	52 (0.000)	27 (0.000)	23 (0.000)	17 (0.000)	11 (0.000)
MCS500	102 (0.004)	56 (0.000)	22 (0.001)	19 (0.000)	15 (0.000)	14 (0.000)
MCS1000	81 (0.000)	48 (0.000)	23 (0.000)	17 (0.000)	13 (0.000)	12 (0.000)
MCS1500	91 (0.000)	52 (0.000)	26 (0.000)	20 (0.000)	17 (0.000)	12 (0.000)
EVT-Static						
GL-static-W30	158 (0.000)	85 (0.000)	25 (0.000)	13 (0.019)	7 (0.165)	3 (0.415)
GEV-static-W30	158 (0.000)	82 (0.000)	23 (0.001)	12 (0.112)	7 (0.165)	6 (0.009)
EVT-Moving Window						
GL-MW-W50	147 (0.000)	77 (0.001)	20 (0.018)	9 (0.554)	7 (0.165)	2 (0.826)
GL-MW-W100	157 (0.000)	88 (0.000)	17 (0.002)	10 (0.071)	6 (0.346)	0 (0.285)
GL-MW-W200	159 (0.000)	75 (0.005)	14 (0.786)	9 (0.554)	6 (0.346)	1 (0.972)
GL-MW-W300	163 (0.000)	76 (0.002)	17 (0.031)	10 (0.359)	8 (0.068)	1 (0.972)
GEV-MW-W50	148 (0.000)	74 (0.002)	18 (0.028)	9 (0.554)	7 (0.165)	3 (0.415)
GEV-MW-W100	157 (0.000)	84 (0.001)	15 (0.002)	10 (0.071)	6 (0.346)	1 (0.972)
GEV-MW-W200	160 (0.000)	67 (0.004)	13 (0.865)	9 (0.554)	7 (0.165)	1 (0.972)
GEV-MW-W300	164 (0.000)	73 (0.005)	17 (0.031)	10 (0.359)	8 (0.068)	3 (0.415)

Note: This table contains the number of VaR violations by the actual daily returns. For the Variance-Covariance (VC), Historical Simulation (HS), Monte Carlo Simulation (MCS) and the Exponential Weighted Moving Average (EWMA) methods 250, 500, 1000 and 1500 past daily returns were used for calibration. In order for the EWMA method to effectively use 250, 500, 1000 and 1500 past daily returns the parameter ? was set equal to 0.996, 0.998, 0.999 and 0.9993333332, respectively. For the MCS method the normal distribution was assumed and 10000 random scenarios were generated. The heading EVT-Static contains the results derived using parameter estimates for the weekly minima divided into 30 sub-periods. The heading EVT-Moving Window contains the VaR results when a MW of length 50, 100, 200 and 300 weekly minima were used. The numbers in parentheses are the p-values of the Cristoffersen test statistic. The test assesses whether the number of the VaR violations by the actual returns over the corresponding time period is too many or too few and in addition it also assesses whether the violations occurred in clusters.

Table 4. CAC-DS daily VaR backtesting for the period 2/1/97 to 28/12/01

Actual daily returns	**1265**					
Confidence level	**90.00%**	**95.00%**	**99.00%**	**99.50%**	**99.75%**	**99.90%**
Expected violations	**127**	**63**	**13**	**6**	**3**	**1**
VC250	129 (0.008)	80 (0.003)	35 (0.000)	29 (0.000)	17 (0.000)	14 (0.000)
VC500	137 (0.005)	90 (0.000)	36 (0.000)	26 (0.000)	22 (0.000)	16 (0.000)
VC1000	141 (0.001)	94 (0.000)	42 (0.000)	30 (0.000)	24 (0.000)	19 (0.000)
VC1500	160 (0.000)	109 (0.000)	49 (0.000)	36 (0.000)	27 (0.000)	21 (0.000)
HS250	137 (0.010)	80 (0.001)	16 (0.007)	7 (0.929)	N/A	N/A
HS500	151 (0.000)	89 (0.000)	22 (0.039)	12 (0.117)	4 (0.888)	N/A
HS1000	174 (0.000)	100 (0.000)	27 (0.000)	18 (0.002)	10 (0.008)	4 (0.151)
HS1500	195 (0.000)	116 (0.000)	32 (0.000)	16 (0.000)	8 (0.070)	3 (0.420)
EWMA250	136 (0.005)	82 (0.002)	35 (0.000)	26 (0.000)	19 (0.000)	12 (0.000)
EWMA500	142 (0.002)	96 (0.000)	39 (0.000)	29 (0.000)	23 (0.000)	17 (0.000)
EWMA1000	143 (0.002)	101 (0.000)	43 (0.000)	32 (0.000)	25 (0.000)	18 (0.000)
EWMA1500	156 (0.002)	102 (0.000)	40 (0.000)	33 (0.000)	24 (0.000)	19 (0.000)
MCS250	139 (0.004)	82 (0.002)	39 (0.000)	30 (0.000)	20 (0.000)	13 (0.000)
MCS500	144 (0.004)	93 (0.000)	38 (0.000)	29 (0.000)	24 (0.000)	17 (0.000)
MCS1000	157 (0.000)	100 (0.000)	43 (0.000)	34 (0.000)	29 (0.000)	19 (0.000)
MCS1500	168 (0.000)	119 (0.000)	51 (0.000)	39 (0.000)	29 (0.000)	23 (0.000)
EVT-Static						
GL-static-W30	170 (0.000)	106 (0.000)	43 (0.000)	27 (0.000)	14 (0.000)	0 (0.282)
GEV-static-W30	170 (0.000)	100 (0.000)	40 (0.000)	28 (0.000)	17 (0.000)	2 (0.831)
EVT-Moving Window						
GL-MW-W50	156 (0.000)	92 (0.000)	31 (0.000)	17 (0.000)	6 (0.355)	1 (0.970)
GL-MW-W100	169 (0.000)	101 (0.000)	27 (0.000)	16 (0.002)	8 (0.070)	0 (0.282)
GL-MW-W200	188 (0.000)	113 (0.000)	34 (0.000)	18 (0.000)	10 (0.008)	0 (0.282)
GL-MW-W300	210 (0.000)	132 (0.000)	40 (0.000)	20 (0.000)	13 (0.000)	0 (0.282)
GEV-MW-W50	155 (0.000)	87 (0.000)	29 (0.000)	18 (0.000)	10 (0.002)	2 (0.831)
GEV-MW-W100	167 (0.000)	95 (0.000)	26 (0.000)	17 (0.001)	11 (0.002)	1 (0.969)
GEV-MW-W200	186 (0.000)	107 (0.000)	33 (0.000)	21 (0.000)	13 (0.000)	2 (0.830)
GEV-MW-W300	209 (0.000)	125 (0.000)	37 (0.000)	22 (0.000)	13 (0.000)	1 (0.969)

Note: This table contains the number of VaR violations by the actual daily returns. For the Variance-Covariance (VC), Historical Simulation (HS), Monte Carlo Simulation (MCS) and the Exponential Weighted Moving Average (EWMA) methods 250, 500, 1000 and 1500 past daily returns were used for calibration. In order for the EWMA method to effectively use 250, 500, 1000 and 1500 past daily returns the parameter ? was set equal to 0.996, 0.998, 0.999 and 0.9993333332, respectively. For the MCS method the normal distribution was assumed and 10000 random scenarios were generated. The heading EVT-Static contains the results derived using parameter estimates for the weekly minima divided into 30 sub-periods. The heading EVT-Moving Window contains the VaR results when a MW of length 50, 100, 200 and 300 weekly minima were used. The numbers in parentheses are the *p*-values of the Cristoffersen test statistic. The test assesses whether the number of the VaR violations by the actual returns over the corresponding time period is too many or too few and in addition it also assesses whether the violations occurred in clusters.

Table 5. CAC-DS daily VaR (%) on the 19/10/87

Actual return: -9.90%	**Confidence level**					
Method	**90.00%**	**95.00%**	**99.00%**	**99.50%**	**99.75%**	**99.90%**
VC250	-1.18	-1.53	-2.13	-2.36	-2.62	-2.94
VC500	-1.37	-1.80	-2.59	-2.86	-3.09	-3.55
VC1000	-1.18	-1.52	-2.19	-2.54	-2.74	-2.97
VC1500	-1.16	-1.49	-2.17	-2.36	-2.59	-2.92
HS250	-1.08	-1.66	-3.54	-3.59	N/A	N/A
HS500	-1.22	-1.90	-3.54	-3.94	-5.97	N/A
HS1000	-1.06	-1.52	-2.55	-3.54	-3.94	-5.97
HS1500	-1.04	-1.43	-2.29	-3.44	-3.68	-5.97
EWMA250	-1.32	-1.70	-2.40	-2.66	-2.90	-3.19
EWMA500	-1.32	-1.69	-2.40	-2.65	-2.89	-3.18
EWMA1000	-1.35	-1.73	-2.44	-2.70	-2.95	-3.24
EWMA1500	-1.38	-1.77	-2.50	-2.77	-3.02	-3.32
MCS250	-1.18	-1.53	-2.13	-2.36	-2.62	-2.94
MCS500	-1.37	-1.80	-2.59	-2.86	-3.09	-3.55
MCS1000	-1.18	-1.52	-2.19	-2.54	-2.74	-2.97
MCS1500	-1.16	-1.49	-2.17	-2.36	-2.59	-2.92
EVT-Static						
GL-static-W30	-1.08	-1.55	-2.71	-3.29	-3.94	-8.10
GEV-static-W30	-1.08	-1.60	-2.75	-3.24	-3.73	-5.95
EVT-Moving Window						
GL-MW-W50	-1.03	-1.43	-2.39	-2.84	-3.33	-6.29
GL-MW-W100	-1.13	-1.65	-3.03	-3.74	-4.56	-10.21
GL-MW-W200	-0.94	-1.38	-2.54	-3.15	-3.85	-8.76
GL-MW-W300	-0.95	-1.35	-2.36	-2.87	-3.44	-7.17
GEV-MW-W50	-1.04	-1.48	-2.41	-2.78	-3.13	-4.58
GEV-MW-W100	-1.13	-1.70	-3.08	-3.70	-4.35	-7.62
GEV-MW-W200	-0.94	-1.42	-2.59	-3.12	-3.68	-6.57
GEV-MW-W300	-0.95	-1.39	-2.40	-2.83	-3.26	-5.29

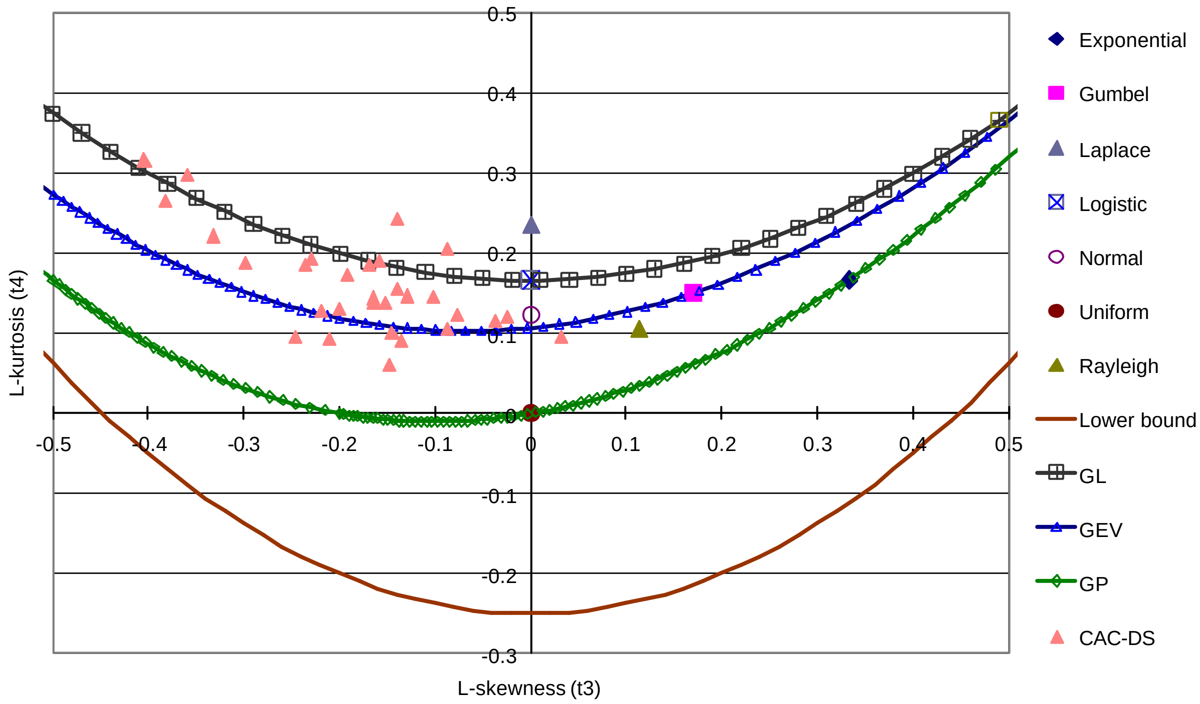
Note: This table contains the daily VaR estimates (%) for the CAC-DS index daily returns on the 19/10/87. The actual return was -9.90% and it can be attributed to the stock markets' collapse.

Table 6. CAC-DS daily VaR standard deviation (%) for the period 2/1/87 to 31/12/91

	Confidence level					
Method	**90.00%**	**95.00%**	**99.00%**	**99.50%**	**99.75%**	**99.90%**
VC250	0.43	0.56	0.79	0.87	0.95	1.05
VC500	0.29	0.37	0.52	0.57	0.63	0.69
VC1000	0.14	0.18	0.25	0.27	0.30	0.33
VC1500	0.08	0.11	0.15	0.17	0.18	0.20
HS250	0.32	0.53	2.23	2.45	N/A	N/A
HS500	0.16	0.26	0.96	2.23	1.80	N/A
HS1000	0.08	0.13	0.42	1.09	1.67	1.50
HS1500	0.06	0.09	0.31	0.71	1.30	1.22
EWMA250	0.26	0.33	0.47	0.52	0.57	0.63
EWMA500	0.15	0.20	0.28	0.31	0.33	0.37
EWMA1000	0.08	0.10	0.15	0.16	0.18	0.20
EWMA1500	0.06	0.07	0.10	0.11	0.12	0.13
MCS250	0.51	0.63	0.86	0.95	1.03	1.13
MCS500	0.32	0.40	0.55	0.61	0.66	0.73
MCS1000	0.16	0.20	0.27	0.30	0.33	0.37
MCS1500	0.09	0.12	0.17	0.18	0.21	0.24
EVT-Static						
GL-static-W30	0.26	0.45	1.22	1.80	2.59	12.33
GEV-static-W30	0.26	0.45	1.27	1.86	2.64	10.94
EVT-Moving Window						
GL-MW-W50	0.29	0.47	1.21	1.75	2.49	11.48
GL-MW-W100	0.17	0.29	0.77	1.12	1.60	7.38
GL-MW-W200	0.05	0.11	0.38	0.58	0.85	4.15
GL-MW-W300	0.03	0.06	0.25	0.39	0.59	2.90
GEV-MW-W50	0.29	0.48	1.26	1.80	2.53	10.15
GEV-MW-W100	0.16	0.29	0.80	1.15	1.63	6.53
GEV-MW-W200	0.05	0.11	0.39	0.60	0.88	3.65
GEV-MW-W300	0.03	0.06	0.26	0.41	0.62	2.50

Note: This table contains the standard deviation (%) of the daily VaR estimates for the CAC-DS index daily returns over the period 2/1/87 to 31/12/91.

Figure 1. L-moments ratios diagram for the CAC-DS weekly minima



Note: This diagram illustrates the L-moments ratios points for the CAC-DS index daily returns weekly minima, divided into 30 sub-periods, over the period 1973 to 2001. The plots of the L-skewness and L-kurtosis are mainly concentrated around the theoretical curves of the GL and the GEV distributions indicating that these two distributions are likely to fit adequately the empirical data.