Insider Trading Rules and Price Formation in Securities Markets — An Entropy Analysis of Strategic Trading*

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Abstract

This paper addresses the issue of how insider trading rules affect price formation in securities markets and suggests the application of information theory to market microstructure theory. We analyze a variant of Kyle's (1985) setting by simply introducing a more general criterion for informational efficiency borrowed from information theory — namely maximum information transmission. The analysis shows that both the insider's optimal trading strategy and the market price of the risky security depend on the insider trading restriction. Insider trading restrictions are reported to be detrimental to the liquidity of the securities market. We find that a unique insider trading rule exists which implements semi–strong form informational efficiency of the securities market. Alternative restrictions on insider trading give rise to either underreaction or overreaction in securities prices. Too strict insider trading rules are shown to account for excess volatility in securities prices. Contrary to common notion, the uninformed investors are shown to be hurt by too restrictive insider trading rules. We conclude that loose insider trading rules are preferred by the group of investors as a whole.

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1 Introduction

Regulators of financial markets are concerned about insider trading on the one hand and informational efficiency of the market on the other hand. In order to guarantee a fair market, rules are enacted which shall restrict insider trading activity. The purpose of these restrictive rules on insider trading is to ensure approximately equal opportunities for all market participants.[1] Supervising authorities such as for example the U.S. Securities and Exchange Commission (SEC) permanently monitor the compliance of these rules. The violation of these rules may involve prosecution according to the penal law, ultimately.[2]

This paper adds — on methodologically new grounds — to the large body of literature which is concerned with the issue whether insider trading restrictions are beneficial or detrimental. The prohibition of insider trading is reported to cut both ways by a bulk of theoretical studies. For instance, with respect to the informational efficiency of the securities market Manove (1989), Leland (1992), and Shin (1996) favor the permission of insider trading resulting in more informative securities prices. Thus, these studies identify the dilemma of the regulating bodies which stems from the trade–off between the informational efficiency of the market and the insider trading restriction. The prohibition of trading activity on the basis of information which is not yet reflected in securities prices affects the informational efficiency of securities prices adversely. The opposite observation is due to Fishman and Hagerty's (1992) analysis which documents that allowing for insider trading damages the informational efficiency of securities prices.

The common notion of informational efficiency of financial markets is due to Fama (1970) who coined the differentiation between informational efficiency in the strong form, in the semi–strong form, and in the weak form depending on the level of information — private, public, and historical respectively — which is reflected in securities prices. However, a rich number

[1] According to Treynor and LeBaron (2004) the purpose of insider trading rules is to protect dealers as well as investors. They argue that insider trading rules sustain the dealers in providing liquidity and foster the investors' confidence in the capital market what ultimately generates additional welfare due to higher investment activity.

[2] Cf. Bris (2005, section IV).

of empirical studies collect evidence that even informational efficiency in the weak form often is not present in financial markets. For instance, momentum trading strategies which are based solely on past returns and exploit short–term autocorrelation of returns are documented to be profitable on a risk–adjusted basis that is to produce positive cumulative abnormal returns in the world's leading stock markets.[3] Thus, these studies indicate that the informational efficiency of financial markets seems to be a precious fiction rather than reality.

In the face of the opposing evidence Fama (1998) conjectures the deviation from informational efficiency to vanish once longer time horizons are considered. However, behavioral finance establishes a strand of financial economics which tries to explain the deviation of securities prices from the informationally efficient level. Major recent advances are due to Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1998), and Hong and Stein (1999). These papers provide rationales for underreaction and overreaction in securities prices by relying on an assumption of some non–standard behavior on the part of the economic agents which is at odds with the paradigm of unbounded rationality. More precisely, Daniel, Hirshleifer and Subrahmanyam (1998) model the time–variation of overconfidence due to the investors' self–attribution bias. Barberis, Shleifer and Vishny's (1998) approach relies on the investors' biased inference on grounds of a fictitious dividend model which accounts for the representativeness heuristic and investors' conservatism but does not describe the true random dividend process. Finally, Hong and Stein (1999) explicitly incorporate bounded rationality of two heterogenous investor groups. The group of news–watchers solely acts on the basis of diffusing private information and discards information reflected in securities prices whereas the group of momentum traders solely acts on the basis of historical securities prices.[4]

The present paper — contrary to the behavioral finance models mentioned

[3] Cf. Jegadeesh and Titman (1993) and Rouwenhorst (1998).

[4] Note that each model captures one of the two dimensions of bounded rationality which were identified by Simon (1955) initially. Daniel, Hirshleifer and Subrahmanyam (1998) and Barberis, Shleifer and Vishny (1998) model limited capabilities in information processing whereas Hong and Stein (1999) focus on limited access to information.

above — approaches deviations from informational efficiency as well as underreaction and overreaction in securities markets on grounds of a market imperfection. In essence, the effects of insider trading rules which restrict the trading activity of insiders are scrutinized. Hence, the analysis highlights the trade–off between restrictions on the trading activity of insiders and the informational efficiency of the securities market. We accomplish the analysis in a variant of Kyle's (1985) setting where we replace the market efficiency condition by some broader criterion of informational efficiency. More precisely, instead of imposing explicitly that the price of the risky security becomes semi–strong form informationally efficient we simply claim that the securities markets are regulated such that they allow for maximum information transmission. In turn, the latter means that the price of the risky security shall become maximally informative for the value of the risky security subject to some restriction on the insider's trading activity. Note that meeting this objective is adequate from the regulator's viewpoint.

Methodologically, we approach the regulator's problem of fixing insider trading restrictions from the perspective of information theory. The economic setting of the securities market trading game is similar to that in Kyle's (1985) seminal paper. Thus, this paper establishes a link between market microstructure theory and information theory explicitly. Basically, we exploit Shannon's (1948) pioneering work on communication theory in order to study price formation in securities markets.[5] This approach to price formation seems straightforward given that already Hayek (1945, p. 527) in his pioneering work emphasized the role of the price system as communication device for information. However, some applications of information theory to financial economics already exist. For example Branger (2004), Gulko (1999*a*), and Gulko (1999*b*) represent applications of information theory to selected issues in asset pricing theory.[6]

The major insights delivered by the information theoretic analysis of strategic trading can be summarized briefly. Extending the work of Kyle

[5]Shannon's (1948) work can also be found in Shannon and Weaver (1998).

[6]The reader astonished at the application of information theory to asset pricing is referred to Kullback (1997, p. 4) where already in the first chapter the Radon–Nikodym derivative is introduced which is of paramount importance for the change of measure.

(1985), this paper shows that both the insider's optimal trading strategy and the market price of the risky security explicitly depend on the insider trading restriction enacted by the regulator. Furthermore, it confirms the adverse effects of insider trading restrictions on the liquidity of the securities market as well as the incompatibility of maximum liquidity of the securities market on the one hand and maximum protection of inferiorly informed market participants on the other hand. We report that a unique insider trading rule exists which implements semi–strong form informational efficiency of the securities market.[7] It is shown that alternative bounds on the insider's trading activity account for deviations from semi–strong form informational efficiency and effect either under– or overreaction to the order flow. In the case of too restrictive insider trading rules the securities prices exhibit excess volatility. Instead of benefitting the uninformed investors by cutting back the insider's expected profits too restrictive insider trading rules are shown to make the uninformed investors worse off. Thus, we conclude that the group of investors as a whole naturally prefers loose insider trading rules.

The remainder of the paper proceeds as follows. Section 2 introduces the basic information theoretic concepts of differential entropy and conditional differential entropy which will be employed in later sections. The basic structure of the underlying economy as well as the securities market trading game between the insider and the market makers are described in section 3. Next, in section 4 we provide the solution to the securities market trading game and derive the major results of the paper. Section 5 concludes, formulates policy implications, and outlines further research avenues.

2 Digression on information theory

Since the application of information theory to issues of financial economics is not yet very common this subsection aims at introducing the basic information theoretic concepts of measuring uncertainty and information. Alternatively, these concepts can be found for example in the textbook by Cover

[7] The finding on the semi–strong form informational efficiency demonstrates that Kyle's (1985) approach represents a special case of our analysis. Since we impose a more general condition of informational efficiency our approach encompasses that of Kyle (1985).

and Thomas (1991).

In information theory the uncertainty of a random variable is quantified by a measure which is referred to as the entropy of the random variable. If the random variable is continuous instead of having solely a discrete support the uncertainty of the random variable is measured by differential entropy.[8] However, the information theoretic concept of entropy is not restricted to a single random variable but also applies to a vector of random variables. The differential entropy of a vector of continuous random variables is given in definition 1.

Definition 1 *For $n \in \mathbb{N}$ the differential entropy $h(\boldsymbol{x})$ of a vector of continuous random variables $\boldsymbol{x}^\top = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$ with joint density function $f(x_1, x_2, \ldots, x_n)$ is defined as*

$$h(\boldsymbol{x}) = -\int_{\mathcal{X}} f(x_1, x_2, \ldots, x_n) \cdot \ln f(x_1, x_2, \ldots, x_n)\, dx_1\, dx_2 \ldots dx_n$$

where $\mathcal{X} \subseteq \mathbb{R}^n$ is the support of the random vector $\boldsymbol{x}$.

Note that for $n = 1$ definition 1 also comprises the case of a single continuous random variable. The intuition that the differential entropy truly quantifies the uncertainty of a continuous random variable can hardly be gained from inspection of definition 1. To make the intuition more concrete we specify the differential entropy of a normally distributed random variable in lemma 1.

Lemma 1 *The differential entropy $h(\tilde{x})$ of the normally distributed random variable $\tilde{x} \sim \mathcal{N}(\mu, \sigma^2)$ is*

$$h(\tilde{x}) = \frac{1}{2}\left[\ln\left(2\pi\sigma^2\right) + 1\right].$$

Since the natural logarithm is strictly monotonic the differential entropy of a normally distributed random variable is strictly monotonically increasing

[8] The differential entropy of a continuous random variable results in the limiting case from the entropy of the quantization of that random variable as the discretization becomes infinitely fine. Cf. Cover and Thomas (1991, section 9.3).

in the variance of the random variable. This observation establishes the notion that the differential entropy indeed serves as measure for the uncertainty of a random variable.

A concept closely related to entropy is that of conditional entropy. The conditional entropy measures the uncertainty of a random variable conditionally on another random variable. In the case of continuous random variables the conditional differential entropy is introduced in definition 2.

Definition 2 *Let $\tilde{x}$ and $\tilde{y}$ be two continuous random variables. The differential entropy of $\tilde{x}$ conditionally on $\tilde{y}$ that is the conditional differential entropy $h(\tilde{x}|\tilde{y})$ is defined as*

$$h(\tilde{x}|\tilde{y}) = -\int_{\mathcal{Y}} \int_{\mathcal{X}} f(x, y) \cdot \ln f(x|y) \, dx \, dy$$

where $\mathcal{X} \subseteq \mathbb{R}$ and $\mathcal{Y} \subseteq \mathbb{R}$ are the support of $\tilde{x}$ and $\tilde{y}$ respectively. Furthermore, $f(x, y)$ denotes the joint density function of $\tilde{x}$ and $\tilde{y}$, and $f(x|y)$ represents the density of $\tilde{x}$ conditionally on $\tilde{y}$.

The appearance of the joint density function $f(x, y)$ in definition 2 instead of the conditional density function $f(x|y)$ is motivated by the following remark.

Remark Given a realization y of the random variable $\tilde{y}$ the ex–post remaining uncertainty of the random variable $\tilde{x}$ is captured by the conditional density $f(x|y)$. Thus, the ex–post conditional differential entropy of the random variable $\tilde{x}$ is given by

$$h(\tilde{x}|y) = -\int_{\mathcal{X}} f(x|y) \cdot \ln f(x|y) \, dx. \tag{1}$$

As for each realization of the random variable $\tilde{y}$ a different ex–post conditional differential entropy (1) obtains, the ex–ante conditional differential entropy of the random variable $\tilde{x}$ — that is before a realization of the random variable $\tilde{y}$ is drawn — is given by the average ex–post conditional differential entropy $h(\tilde{x}|y)$. Formally,

$$\int_{\mathcal{Y}} f(y) \cdot h(\tilde{x}|y) \; dy = -\int_{\mathcal{Y}} \int_{\mathcal{X}} f(y) \cdot f(x|y) \cdot \ln f(x|y) \, dx \, dy, \tag{2}$$

where averaging means weighting by the marginal density $f(y)$ of the conditioning random variable $\tilde{y}$. Finally, the application of Bayes' law

$$f(x, y) = f(x|y) \cdot f(y) \tag{3}$$

to the right hand side of (2) yields definition 2. $\square$

Thus, the conditional entropy of a random variable quantifies on an ex–ante basis the uncertainty which is expected to remain by conditioning on some other random variable. That is the conditional entropy accounts ex–ante for all potential realizations of the conditioning random variable.

We close this short digression on information theory by finally providing an intuition which we will exploit in the analysis of the remainder of the paper. We interpret the entropy $h(\tilde{x})$ as a measure of prior uncertainty about the random variable $\tilde{x}$. Consequently, the conditional entropy $h(\tilde{x}|\tilde{y})$ represents a measure of the posterior uncertainty about the random variable $\tilde{x}$ conditionally on the random variable $\tilde{y}$. Then, it is straightforward to take the difference of these two entropies

$$h(\tilde{x}) - h(\tilde{x}|\tilde{y}) \tag{4}$$

as the reduction of the uncertainty about the random variable $\tilde{x}$ from observing the random variable $\tilde{y}$. Alternatively, we interpret the difference (4) as the transmitted information about the random variable $\tilde{x}$ by the observation of the random variable $\tilde{y}$. This intuitive interpretation of that difference is sufficient to follow the upcoming analysis.[9] Furthermore, this interpretation conforms to the common notion that information reduces uncertainty.

3 Setup of the model

In this section we outline the general structure of the economic setting initially. Next, the securities market trading game between the insider and the market makers is described.

[9]The reader familiar with information theory recognizes the difference (4) as the mutual information or the cross entropy of the random variables $\tilde{x}$ and $\tilde{y}$, immediately.

3.1 Economy

The economy under consideration resembles that of the single auction setting in Kyle (1985). A security with uncertain liquidation value is exchanged among three kinds of traders in a single trading round. There is a single risk neutral insider who is perfectly informed about the risky security's liquidation value. Additionally, there are liquidity traders whose demand for the risky security is purely random. Thus, the liquidity traders demand for the risky asset is not related to any kind of information at all. The liquidity traders' demand is presumed to be motivated exogenously. Ultimately, there are competitive risk neutral market makers who set the market price for the risky security. The market makers set the market price such that it is maximally informative for the liquidation value of the risky security subject to some bound on insider trading activity. The market makers are supposed to accept that position in the risky security which offsets the insider's and the liquidity traders' aggregate demand for the risky security at the prevailing market price. Thus, the market makers' price setting behavior is somewhat less restrictive than that presumed by Kyle (1985).[10]

The market trading game spans one period between dates t and $t+1$. At the beginning of the market trading game the liquidation value of the risky security $\tilde{v}$ is drawn from the normal distribution $\mathcal{N}\left(p_t, \sigma_t^2\right)$ where p_t denotes the actual market price of the risky security. Thus, the actual market price of the risky security is presumed to represent the unconditional expectation of the risky security's liquidation value.

The liquidity traders' unobservable random demand for the risky security amounts to $\tilde{u}$ which has law $\mathcal{N}\left(0, \sigma_u^2\right)$ and is uncorrelated to the risky security's liquidation value. That is $\mathrm{E}\left[\tilde{u}\,\tilde{v}\right] = 0$. The insider's demand for the risky security is denoted by $\tilde{x}$ and is assumed to be some linear function

[10] Contrary to Kyle (1985, equilibrium condition (2), p. 1318) we do not impose that the market price is set such that informational efficiency in the semi–strong form is established. Semi–strong form informational efficiency requires that the market price equals the expected liquidation value conditionally on the order flow or the aggregate demand respectively. From the perspective of risk neutral market makers this corresponds to an expected zero profit condition.

of the risky security's liquidation value. That is

$$\tilde{x} = \alpha + \beta \cdot \tilde{v} \tag{5}$$

where α and β are some real numbers to be determined optimally by the insider. Optimality from the insider's perspective means the extraction of a maximum rent on the private information about the risky security's liquidation value. Consequently, the insider's and liquidity traders' aggregate demand for the risky security results as $\tilde{y} = \tilde{x} + \tilde{u}$.

At date $t+1$ the order flow $\tilde{y}$ is cleared by the market makers at the market price $\tilde{p}_{t+1}$. The market price $\tilde{p}_{t+1}$ in turn is assumed to account for the price impact of the order flow. That is

$$\tilde{p}_{t+1} = p_t + \Delta p_t\left(\tilde{y}\right) = p_t + \lambda \cdot \tilde{y} \tag{6}$$

defines the price rule where $\Delta p_t\left(\tilde{y}\right) = \lambda \cdot \tilde{y}$ represents the price impact of the order flow and λ is some positive real number.[11] Similar to Kyle (1985) the price impact is presumed to be proportionate to the order flow. Hence, the parameter λ captures the price impact of the order flow ultimately. Securities markets which are characterized by a small price impact of the order flow are referred to as deep securities markets. Thus, the parameter λ operationalizes the depth of a securities market and its reciprocal serves as measure of liquidity.

The solution of the securities market trading game between the insider and the market makers requires the determination of the insider's optimal demand for the risky security (5) and the price rule (6) which generates a maximally informative market price for the liquidation value of the risky security subject to a restriction on the insider's trading activity. Next we outline the securities market trading game in detail. Its solution is left to section 4.

3.2 Securities market trading game

Drawing on the information theoretic concepts introduced in section 2 we now describe the securities market trading game. As mentioned above the analysis

[11] The restriction on the sign of λ conforms to the common economic intuition that a positive aggregate demand raises the market price and vice versa.

of the securities market trading game aims at the determination of (i.) the insider's optimal demand schedule for the risky security and (ii.) the price rule which enables maximum information transmission for the liquidation value of the risky security subject to a restriction on insider trading. The problem is referred to as securities market trading game since the insider's demand depends on the price rule and vice versa. Thus, the interdependency of the insider's and the market makers' decisions introduces a kind of strategic interaction since both parties pursue individual interests.

Recall, the insider is privately informed about the liquidation value of the risky security and aims at extracting a maximum rent from this informational advantage. Hence, the risk neutral insider solves the optimization problem

$$\max_{\tilde{x}} \quad \mathrm{E}\left[\left(\tilde{v} - \tilde{p}_{t+1}\right) \tilde{x} | \tilde{v}\right] \tag{7}$$

which corresponds to expected profit maximization.

However, the market makers who are concerned about a maximally informative market price subject to some restriction on insider trading solve the optimization problem

$$\max_{\lambda} \quad h\left(\tilde{v}\right) - h\left(\tilde{v}|\tilde{p}_{t+1}\right) \tag{8}$$

$$\text{s.t.} \quad \sigma_x^2 \leq \xi \tag{9}$$

where $\sigma_x^2 \equiv \mathrm{Var}\left[\tilde{x}\right]$, and $\xi > 0$ denotes the upper bound on the insider's trading activity which is set exogenously by the regulator. Note, the notion of σ_x^2 as the insider's trading activity is equivalent to the interpretation of σ_u^2 as the liquidity traders' trading activity. Recalling the intuition of term (4) yields the insight that the objective function (8) strives for maximum information transmission which corresponds to maximizing the reduction of the uncertainty about the liquidation value of the risky security. Put differently, the objective function (8) ensures that the market price becomes maximally informative for the risky security's liquidation value.

Before turning to the solution of the securities market trading game consisting of (7), (8) and (9) some comments on the approach's mechanics are in order. The upper bound ξ on the insider's trading activity adopted by the regulator affects the market price of the risky security which is determined by the market makers. The market price of the risky security in turn affects the insider's demand schedule. To summarize, the restriction on the insider's trading activity affects both the market price and the insider's demand for the risky security. Therefore, the present setting allows to study the impact of insider trading rules on transaction prices and the strategic demand of the insider.

Finally, from the analytical viewpoint ξ truly represents some real number. However, one should not expect a regulator to announce a certain real number as insider trading rule. Rather, one might better think of the upper bound from a conceptual perspective. Then, a low value for ξ is suggestive for very restrictive insider trading rules accompanied by heavy punishment of insider trading according to the penal law and vice versa.

4 Impact of insider trading restrictions

In this section we first provide the solution to the securities market trading game. On grounds of this solution we establish some results concerning the dependency of the insider's demand for the risky security, the insider's expected profits, and the depth of market on the severity of the insider trading rules. Next, we identify the necessary condition which must be met by the regulator in order to guarantee a semi–strong from informationally efficient market price. Thereafter, we derive the results on underreaction and overreaction of securities prices as well as on excess volatility of securities prices triggered by restricting the insider's trading activity. Finally, we analyze how the market makers and the uninformed investors are affected by the severity of the insider trading rules.

The insider's demand schedule and the market makers' price rule which represent the solution to the securities market trading game consisting of (7), (8) and (9) is given in proposition 1.

Proposition 1 *The insider's optimal demand for the risky security is*

$$\tilde{x} \equiv x(\tilde{v};\xi) = \frac{\tilde{v} - p_t}{2\lambda(\xi)}$$

and the market makers' price rule is

$$\tilde{p}_{t+1} \equiv p_{t+1}(\tilde{y};\xi) = p_t + \Delta p_t(\tilde{y};\xi) = p_t + \lambda(\xi)\tilde{y}$$

where

$$\lambda(\xi) = \frac{1}{2}\sqrt{\sigma_t^2 \xi^{-1}}$$

and ξ denotes the upper bound on the insider's trading activity set by the regulator.

The most important insight gained from inspection of proposition 1 is that both the insider's optimal demand for the risky security and the market makers' price rule depend on the bound on the insider's trading activity explicitly. The impact of insider trading rules on the insider's optimal demand for the risky security, on the insider's expected profits, and on the depth of the market is summarized in corollary 1.

Corollary 1 *The looser are the insider trading rules the higher are (i.) the insiders' demand for the risky security, (ii.) the insider's expected profits, and (iii.) the depth of the market and vice versa.*

Corollary 1 simply confirms the common notion of the insider's optimal demand strategy as well as the associated expected profits of the insider. If the insider trading rules are less restrictive the profit maximizing insider's demand for the risky security increases in order to extract a maximum rent on his private information. Thus, the insider is going to exploit his informational advantage inasmuch as the regulator allows. Contrary, the more restrictive are the insider trading rules implemented by the regulator the lower are the insider's expected profits.

For the time being, the finding on the insider's expected profits suggests that the remainder of the market participants — that is the liquidity traders and the market makers — suffer less from insider trading if the restrictions on insider trading are more severe. Consequently, stricter insider trading rules seem to be desirable from the other market participants' perspective. Furthermore, the regulator seems to reach the aim of protecting the other market participants by introducing more severe insider trading rules.[12]

[12] We will revisit these conjectures in the discussion of corollary 5.

Finally, the depth of the market is found to increase if the insider trading rules becomes looser. Thus, the ability of the market to accommodate a large order flow with small price impact increases the less restrictive are the insider trading rules. Hence, regulators who primarily focus on the liquidity of the market are in favor of implementing loose insider trading rules. Note, corollary 1 identifies the regulator's conflict inherent in the choice of insider trading rules. The aims of maximum protection of inferior informed market participants from being picked off by the insider and the provision of maximum liquidity cannot be accomplished simultaneously.

The price of the risky security as given in proposition 1 is determined to be maximally informative for the risky security's liquidation value subject to the restriction on the insider's trading activity. The restriction on the insider's trading activity which establishes a semi–strong form informationally efficient price for the risky security is specified in proposition 2.

Proposition 2 *The market price of the risky security is semi–strong form informationally efficient if and only if the bound on the insider's trading activity equals the liquidity traders' trading activity that is* $\xi^* = \sigma_u^2$.

Semi–strong form informational efficiency means that the market price of the risky security equals the best forecast of the risky security's liquidation value conditionally on the order flow which represents public information. Thorough inspection of proposition 2 yields the insight that the solution to the securities market trading game as given in proposition 1 coincides with Kyle's (1985, Theorem 1) equilibrium if and only if the regulator bounds the insider's trading activity by ξ^*. This insight truly extends Kyle's (1985) analysis. Put differently, the approach of Kyle (1985) simply is a special case of the securities market trading game which we study. This is due to the fact that we employ a more general criterion for informational efficiency borrowed from information theory.[13] However, the semi–strong form informationally

[13]Note, the information theoretic criterion of maximum information transmission solely focuses on the reduction of the variability of the risky security's market price whereas the claim of a semi–strong form informationally efficient price represents a constraint on the mean of the risky security's price. The fact that both approaches coincide for the bound ξ^* yields the insight that the semi–strong form informationally efficient price has minimum

efficient price of the risky security which obtains for the bound ξ^* is useful in serving as benchmark anyway.

Corollary 2 collects the implications for the market price of the risky security if the regulator restricts insider trading activity to some level different from the bound ξ^*.

Corollary 2 *Compared to the semi–strong form informationally efficient price, the market price of the risky security underreacts and overreacts to the order flow for $\xi > \xi^*$ and $\xi < \xi^*$ respectively.*

Note, the notion of underreaction and overreaction of the market price of the risky security as used in corollary 2 corresponds to that which is common in financial economics. According to the efficient market hypothesis the market price of the risky security should be semi–strong form informationally efficient. Hence, underreaction and overreaction of the risky security's market price usually is taken to be any deviation from the semi–strong form informationally efficient price. Thus, the major insight gained from corollary 2 is that if the regulator does not enact the appropriate insider trading rules the market price of the risky security necessarily exhibits underreaction or overreaction to the order flow. Put differently, a bound on the insider's trading activity other than ξ^* produces deviations from the semi–strong form informationally efficient price. Consequently, since a continuum of bounds on the insider's trading activity exists the observation of a semi–strong form informational efficient price — that is the choice of the bound ξ^* on the part of the regulator — is an event of measure zero.

Let us further comment on corollary 2. Strictly speaking, the market price of the risky security solely reacts too strongly or too little to the liquidity traders' demand for the risky security. This can easily be verified from plugging the order flow $\tilde{y} = x\left(\tilde{v}, \xi\right) + \tilde{u}$ into the price impact. According to

variability, too. Thus, the semi–strong form informationally efficient price — which obtains for the bound ξ^* — is the best linear estimator of the risky security's liquidation value.

proposition 1 we obtain

$$\begin{aligned}\Delta p_t(\tilde{y};\xi) &= \lambda(\xi)\left(\frac{\tilde{v}-p_t}{2\lambda(\xi)}+\tilde{u}\right)\\ &= \frac{1}{2}\left(\tilde{v}-p_t\right)+\lambda(\xi)\tilde{u} \end{aligned} \tag{10}$$

for the price impact of the order flow.

Corollary 1 reports that the insider always sizes his demand for the risky security according to the bound on his trading activity. Hence, the insider scales back his order size if the market price of the risky security becomes more sensitive to the order flow that is if the depth of the market is reduced and vice versa. Consequently, the insiders' impact on the risky security's market price reflected in the first term on the right hand side of (10) is independent of the bound ξ and thus constant.

However, this independence does not prevail with respect to the liquidity traders' demand for the risky security which is reflected in the second term on the right hand side of (10). The liquidity traders' price impact truly is a function of the restriction on the insider's trading activity. The dependency of the liquidity traders' price impact on the bound ξ allows to derive corollary 3 immediately.

Corollary 3 *Compared to the volatility of the semi–strong form informationally efficient price, the market price of the risky security exhibits excess volatility if the regulator establishes insider trading rules which are too restrictive that is $\xi < \xi^*$.*

The implications of the corollaries 2 and 3 for the regulators' insider trading policy are straightforward. If the regulator was concerned about semi–strong form informationally efficient prices — instead of solely maximally informative market prices — he should monitor the market for the trading activity of liquidity traders permanently and loosen or tighten the insider trading rules accordingly in order to meet the bound ξ^* instantaneously.

Presumed that the trading activity of the liquidity traders varies throughout different stocks, however the bound ξ on the insider's trading activity set by the regulator is applied to the securities market as a whole, it should be expected that the market prices of some stocks are close to semi–strong form informational efficiency whereas the markets prices of some stocks exhibit either underreaction or overreaction. Hence, this observation yields the insight that one indeed can expect that portfolio–based strategies work which exploit underreaction and overreaction in securities prices if insider trading rules are implemented market wide.

For instance, assume that the regulator has fixed the insider trading rules. Then a shock to the liquidity traders' trading activity might trigger overreaction and excess volatility since ultimately the implemented bound is less than the actual trading activity of the liquidity traders. For example, if the regulator luckily has set the bound ξ^* initially but an unexpected increase of the liquidity traders' trading activity occurs, then the bound fixed initially becomes too strict and both overreaction and excess volatility is caused.

The previous discussion corroborates the view that an unexpected increase of the liquidity traders' trading activity might produce overreaction in a risky security's market price in the presence of restrictions on insider trading. Presumed that an increased trading activity of liquidity traders can be observed in initial public offerings our approach suggests an explanation for overreaction of securities prices in initial public offerings on grounds of a market imperfection.

Finally, both the insider's demand for the risky security and market makers' price rule are — that is the solution to the securities market trading game is — independent of the liquidity traders' trading activity σ_u^2. They solely depend on the bound ξ on the insider's trading activity. Thus, the securities market trading game is invariant to changes in the trading activity of liquidity traders. However, any mismatch of the bound ξ and the liquidity traders' trading activity σ_u^2 produces deviations of the market prices from the semi–strong form informationally efficient level.

We add a final comment on a technical aspect of our approach. Absent any restriction on the insider's trading activity the course of the proof of proposition 1 shows that the risky security's market price becomes maximally informative for the liquidation value if λ converges to zero.[14] In this limiting case the liquidity traders' price impact is washed out as can be verified from inspection of (10). Put differently, since the size of the insider's

[14]Cf. the market makers' problem (41) in the appendix.

demand converges to infinity the relative importance of the liquidity traders demand diminishes. Thus, the market price of the risky security becomes a sufficient statistic for the liquidation value of the risky security or for the insider's private information, respectively. Consequently, observing the market price in this case is then informationally equivalent to knowing the risky security's liquidation value. Hence, the market price of the risky security is fully revealing the insider's private information. Indeed, this limiting case truly provides the maximally informative market price for the liquidation value of the risky security as was claimed initially.

After having explored the impact of the insider trading rules on the insider's expected profits in corollary 1 we now turn to the analysis of the market makers' expected profits which are given in proposition 3.

Proposition 3 *The market makers' unconditional expected profits are*

$$\mathrm{E}\left[\tilde{\pi}(\xi)\right] = \mathrm{E}\left[\tilde{\pi}(\xi^*)\right] + \frac{1}{2}\sqrt{\sigma_t^2 \xi^{-1}} \cdot \left(\sigma_u^2 - \xi\right)$$

where $\mathrm{E}\left[\tilde{\pi}(\xi^*)\right]$ *denotes the market makers' unconditional expected profits in a semi–strong form informationally efficient securities market and* ξ *is the upper bound on the insider's trading activity set by the regulator.*

Inspection of proposition 3 delivers the insight that the market makers' unconditional expected profits may differ from those which obtain in a semi–strong form informationally efficient securities market. Note, if $\xi \lesseqgtr \xi^* = \sigma_u^2$ proposition 3 implies

$$\mathrm{E}\left[\tilde{\pi}(\xi)\right] \gtreqless \mathrm{E}\left[\tilde{\pi}(\xi^*)\right] \tag{11}$$

and equality obtains if and only if $\xi = \xi^* = \sigma_u^2$. Thus, the regulator's choice of the bound ξ on the insider's trading activity affects the profitability of the market making industry. From the market makers' perspective the choice of $\xi < \xi^*$ truly is judged to be favorable since the unconditional expected profits exceed those which are earned in a semi–strong form informationally efficient securities market. Immediately, straightforward calculation yields corollary 4.

Corollary 4 *The stricter are the insider trading rules the higher are the market makers' unconditional expected profits and vice versa.*

Tying together the findings of proposition 3 and corollary 4 we conclude that the market makers prefer tighter bounds $\xi < \xi^*$ on insider trading to looser bounds. Thus, market conditions which foster overreaction and excess volatility of securities prices are preferable from the market makers' perspective since their unconditional expected profits are raised above the level which is realized in a semi–strong form informationally efficient securities market. On grounds of this observation one might doubt if the market making industry benefits the informational efficiency of securities markets at all.

Furthermore, corollary 4 explicitly reveals the conflict between the regulator and the group of the market makers. If the regulator aims at maximizing the informational efficiency of the securities market by implementing looser insider trading rules the profits of the market making industry decrease. Thus, the analysis provides us with the insight that the aims of increasing both the informational efficiency of the securities market and the profitability of the market making industry are incompatible to each other. In contrast, the group of the market makers is hurt by relaxing the insider trading rules in favor of the informational efficiency of the securities market.

From the uninformed investors' perspective the corollaries 1 and 4 immediately show the dual nature of the regulator's decision problem. If the regulator sacrifices the informational efficiency of the capital market in exchange for better protection of the uninformed traders the insider's profits are cut back but the market making industry increases its profits. Whether the regulator by implementing more severe insider trading rules simply shifts profits from the insider to the market making industry is discussed in corollary 5.

Corollary 5 *The stricter are the insider trading rules the higher is the sum of the insider's and the market makers' unconditional expected profits.*

Consequently, corollary 5 reports that the profits are not simply shifted from the insider to the market makers if the bound on the insider's trading activity becomes tighter. Rather, stricter insider trading rules reduce the profits of the insider and increase the profits of the market making industry but the market makers' profits grow at a higher rate compared to the decrease of the insider's profits. Hence, the profits which accrue to the market makers due to stricter insider trading rules exceed the informational rents which are lost by the insider.

Given that the uninformed investors bear the losses which correspond to the insider's and the market makers' profits the uninformed investors' welfare is affected adversely by stricter insider trading rules. Most notably, although the regulator seemingly acts in the uninformed investors' best interest in cutting down the insider's profits by more severe restrictions on insider trading the regulator actually worsens the situation for the uninformed investors. Thus, besides effecting overreaction and excess volatility in the market price of the risky security the regulators' choice of too strict insider trading rules has unfavorable welfare implications from the uninformed investors' perspective. Basically, the uninformed investor's welfare perspective thus reverses the intuition which can be gained from solely studying the insider's profits.[15]

Finally, the discussion of the corollaries 1 and 5 highlights that both the uninformed investors and the insider naturally have a preference for loose insider trading rules and hence an increased informational efficiency of the securities market. However, the increase of both the uninformed investors' welfare and the insider's profits then must be borne by the market making industry whose profits are shrinking the less severe are the insider trading restrictions. Put differently, an increased prosperity of the market making industry from severe restrictions on the insider trading activity comes at the cost of the investors as a whole and vice versa.[16]

5 Conclusion

By borrowing techniques form information theory this paper presented an entropy analysis of strategic trading in securities markets. Thus, it truly extends the body of literature of financial economics on market microstructure.

The analysis was performed in a variant of Kyle's (1985) seminal paper. The claim of semi–strong form informationally efficient securities prices is

[15]Cf. footnote 12.

[16]For instance, Christie and Schultz (1994) and Christie, Harris and Schultz (1994) report that NASDAQ market makers' quoting behavior affected the investors adversely.

replaced by the broader information theoretic criterion of maximally informative securities prices subject to some restriction on insider trading. The restriction on insider trading was captured by an upper bound on the insider's trading activity as quantified by the variance of the insider's order. Basically, the upper bound on the variance of the order size means that the orders on average deviate less form the expected order size. Put differently, extremely large orders are less likely the tighter the bound becomes. This slight variation both confirms and produces additional insights beyond Kyle's (1985) findings.

The analysis reveals that the concepts of semi–strong form informationally efficient securities prices on the one hand and of maximally informative securities prices on the other hand which stem from financial economics and information theory respectively have to be distinguished carefully. That is maximum information transmission does not coincide with semi–strong form informational efficiency generally. More precisely, semi-strong form informationally efficient securities prices are maximally informative but maximally informative securities prices are not necessarily semi–strong from informationally efficient.

Furthermore, if and only if the regulator implements insider trading rules such that the insider's trading activity is bound by the uninformed traders' trading activity the securities prices are semi–strong form informationally efficient. Other bounds on the insider's trading activity produce deviations of the securities prices from the semi–strong form informationally efficient level.

In particular, if the regulator enacts insider trading rules which are too restrictive compared to the uninformed traders' trading activity the depth of the securities market is reduced, and the securities prices overreact and exhibit excess volatility.[17]

Although the regulator truly achieves the reduction of the insider's profits by extremely severe insider trading rules, the uninformed traders then are picked off by the market making industry. In that case the market making

[17]Note that here excess volatility means volatility in excess of that volatility which can be observed in a semi–strong form informationally efficient market. Cf. equation (55). Thus, our notion perfectly conforms to that in Shiller (1981, p. 421).

industry expands its profitability and the uninformed investors are hurt even more severely although the insider's profits are shrunk. Thus, the analysis identifies the negative externality of too restrictive insider trading rules concerning the welfare of the investors as a whole. Consequently, a conflict of interest between the group of investors — both informed and uninformed — and the market makers exists. The former have a natural preference for loose insider trading rules whereas the market makers don't and vice versa. Since it is the regulator who enacts the insider trading rules the open issue remains whether ultimately he acts in the best interest of the market makers or of the investors.

The above results allow to formulate policy implications for regulators. Limiting the insider's trading activity to a level other than the trading activity of the uninformed traders produces produces under– and overreaction compared to the semi–strong form informationally efficient securities prices. Hence, when the trading activity of noise traders varies over time and semi–strong form informational efficiency is desirable the insider trading restriction should be less restrictive in times of broad liquidity trader participation and vice versa. For instance, during initial public offerings when uninformed trading rises the bound on the insider's trading activity should be looser to prevent overreaction.

Rather, regulators should implement different insider trading rules for different segments of the market if the regulators care about the semi–strong form informational efficiency of the securities market. In particular, market segments with a broad participation of uninformed investors should be subject to looser insider trading rules and vice versa. Put differently, a positive role of insider trading exists since it absorbs or neutralizes uninformative trading and thus reduces deviations of the securities prices from the semi–strong form informationally efficient level.

Further research — both theoretical and empirical — is stimulated by the present paper. Theoretically, an information theoretic analysis of a multi–period or continuous–time securities market might produce additional insights concerning the intertemporal impact of insider trading rules on securities markets' characteristics. Additionally, the effects of imperfect competition among informed investors in the spirit of Holden and Subrahmanyam

(1992) could also be analyzed from the perspective of information theory. Empirically, the following hypotheses generated by the present study could be tested. First, the analysis suggests that securities markets with extremely strict insider trading rules are ceteris paribus more prone to overreaction and excess volatility. This hypothesis could be verified by comparison of stock markets with different regulations of insider trading.[18] Finally, the discussion of the results highlights that stocks which face a broad public participation and thus a high trading activity of uninformed investors can be expected to exhibit overreaction and excess volatility. This hypothesis could be tested by measuring the trading activity of both informed and uninformed investors in individual stocks along the lines of Easley, Kiefer, O'Hara and Paperman (1996), first. Then, the hypothesis can be checked from comparison of two groups of stocks which are characterized by high and low trading activity of uninformed investors respectively.

[18] Cf. the design of the Bushman, Piotroski and Smith (2005) cross–country study concerning the effects of insider trading restrictions on analyst coverage. They find that the introduction of insider trading laws increases both the intensity (i.e. the number of analysts per covered firm) and the breadth (i.e. the fraction of covered firms) of coverage.

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A Proofs

A.1 Proof of lemma 1

Proof. The density function $f(x)$ of the normally distributed random variable $\tilde{x}$ with law $\mathcal{N}(\mu, \sigma^2)$ is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{12}$$

Taking the natural logarithm of both sides of (12) yields

$$\ln f(x) = -\ln\left(\sqrt{2\pi}\sigma\right) - \frac{(x-\mu)^2}{2\sigma^2}. \tag{13}$$

Hence, after plugging (13) into definition 1 the differential entropy of the random variable can be calculated as

$$\begin{aligned}
h(\tilde{x}) &= -\int_{-\infty}^{\infty} f(x) \cdot \ln f(x)\, dx \\
&= \int_{-\infty}^{\infty} \left[\ln\left(\sqrt{2\pi}\sigma\right) + \frac{(x-\mu)^2}{2\sigma^2}\right] f(x)\, dx \\
&= \ln\left(\sqrt{2\pi}\sigma\right) \int_{-\infty}^{\infty} f(x)\, dx + \frac{1}{2\sigma^2} \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx \qquad (14) \\
&= \ln\left(\sqrt{2\pi}\sigma\right) + \frac{1}{2\sigma^2}\,\sigma^2 \\
&= \frac{1}{2}\left[2 \cdot \ln\left(\sqrt{2\pi}\sigma\right) + 1\right] \\
&= \frac{1}{2}\left[\ln\left(2\pi\sigma^2\right) + 1\right],
\end{aligned}$$

since the two integrals in (14) equal unity and the variance of the random variable respectively. This completes the proof. □

A.2 Proof of proposition 1

Proof. After plugging (6) into (7) and replacing $\tilde{y} = \tilde{x} + \tilde{u}$ the insider's objective function becomes

$$\begin{aligned}
\mathrm{E}\left[(\tilde{v} - \tilde{p}_{t+1})\,\tilde{x}|\tilde{v}\right] &= \mathrm{E}\left[(\tilde{v} - p_t - \lambda\,(\tilde{x} + \tilde{u}))\,\tilde{x}|\tilde{v}\right] \\
&= (\tilde{v} - p_t - \lambda\,(\tilde{x} + \mathrm{E}\left[\tilde{u}|\tilde{v}\right]))\,\tilde{x} \\
&= (\tilde{v} - p_t - \lambda\,(\tilde{x} + \mathrm{E}\left[\tilde{u}\right]))\,\tilde{x} \qquad (15) \\
&= (\tilde{v} - p_t - \lambda\,\tilde{x})\,\tilde{x}.
\end{aligned}$$

Note, since $\mathrm{E}\left[\tilde{u}\,\tilde{v}\right]=0$ one obtains $\mathrm{E}\left[\tilde{u}|\tilde{v}\right]=\mathrm{E}\left[\tilde{u}\right]$ what is exploited in (15). Then, the first order condition for the insider's optimal demand is

$$\tilde{v}-p_t-2\lambda\,\tilde{x}=0,$$

which is equivalent to

$$\tilde{x}=\frac{1}{2\lambda}\left(\tilde{v}-p_t\right) \tag{16}$$

what yields $\alpha=-p_t\left(2\lambda\right)^{-1}$ and $\beta=\left(2\lambda\right)^{-1}$ by comparison of (5) to (16).

Next, we manipulate the market makers' objective function (8). Let $f\left(v,p_{t+1}\right)$ denote the joint density function of the risky security's liquidation value $\tilde{v}$ and the market price $\tilde{p}_{t+1}$. Furthermore, $f\left(v\right)$ and $f\left(p_{t+1}\right)$ represent the respective marginal densities and $\mathcal{V}$ and $\mathcal{P}$ the respective supports. Hence, Bayes' law

$$f\left(v,p_{t+1}\right)=f\left(v|p_{t+1}\right)\cdot f\left(p_{t+1}\right)=f\left(p_{t+1}|v\right)\cdot f\left(v\right) \tag{17}$$

defines the conditional densities $f\left(v|p_{t+1}\right)$ and $f\left(p_{t+1}|v\right)$.

By definition 2 we have

$$\begin{aligned} h\left(\tilde{v}|\tilde{p}_{t+1}\right) &= -\int_{\mathcal{P}}\int_{\mathcal{V}} f\left(v,p_{t+1}\right)\cdot\ln f\left(v|p_{t+1}\right)dv\,dp_{t+1} \\ &= -\int_{\mathcal{P}}\int_{\mathcal{V}} f\left(v,p_{t+1}\right)\cdot\ln\frac{f\left(v,p_{t+1}\right)}{f\left(p_{t+1}\right)}\,dv\,dp_{t+1}, \end{aligned} \tag{18}$$

where (18) results from the application of Bayes' law (17). Now, since

$$\ln\frac{f\left(v,p_{t+1}\right)}{f\left(p_{t+1}\right)}=\ln f\left(v,p_{t+1}\right)-\ln f\left(p_{t+1}\right)$$

holds (18) becomes

$$\begin{aligned} h\left(\tilde{v}|\tilde{p}_{t+1}\right) = &-\int_{\mathcal{P}}\int_{\mathcal{V}} f\left(v,p_{t+1}\right)\cdot\ln f\left(v,p_{t+1}\right)\;dv\,dp_{t+1} \\ &-\left[-\int_{\mathcal{P}}\int_{\mathcal{V}} f\left(v,p_{t+1}\right)\cdot\ln f\left(p_{t+1}\right)\;dv\,dp_{t+1}\right]. \end{aligned} \tag{19}$$

As the marginal density $f\left(p_{t+1}\right)$ is independent of v the factor $\ln f\left(p_{t+1}\right)$ can be factored out of the $\mathcal{V}$–integral. Hence, the term in brackets in (19) is equivalent to

$$-\int_{\mathcal{P}} \ln f\left(p_{t+1}\right) \int_{\mathcal{V}} f\left(v, p_{t+1}\right)\, dv\, dp_{t+1}$$
$$= -\int_{\mathcal{P}} f\left(p_{t+1}\right) \cdot \ln f\left(p_{t+1}\right)\, dp_{t+1}, \quad (20)$$

where we exploit that the inner integral $\int_{\mathcal{V}} f\left(v, p_{t+1}\right) dv$ defines the marginal density $f\left(p_{t+1}\right)$. Recalling definition 1 yields the insight that the first term in (19) and the right hand side of (20) equal the differential entropy of the random vector $\left(\tilde{v}, \tilde{p}_{t+1}\right)^{\top}$ that is $h\left(\tilde{v}, \tilde{p}_{t+1}\right)$ and the differential entropy of the market price $\tilde{p}_{t+1}$ that is $h\left(\tilde{p}_{t+1}\right)$ respectively. This observation allows to rewrite (18) as

$$h\left(\tilde{v} | \tilde{p}_{t+1}\right) = h\left(\tilde{v}, \tilde{p}_{t+1}\right) - h\left(\tilde{p}_{t+1}\right), \quad (21)$$

finally. Using (21) we obtain

$$h\left(\tilde{v}\right) + h\left(\tilde{p}_{t+1}\right) - h\left(\tilde{v}, \tilde{p}_{t+1}\right) \quad (22)$$

for the market makers' objective function (8), ultimately.

By definition the risky security's liquidation value has law $\tilde{v} \sim \mathcal{N}\left(p_t, \sigma_t^2\right)$. By (16) we obtain

$$\begin{aligned} \tilde{y} &= \tilde{x} + \tilde{u} \\ &= \frac{1}{2\lambda}\left(\tilde{v} - p_t\right) + \tilde{u} \end{aligned} \quad (23)$$

for the order flow. Plugging (23) into (6) yields

$$\tilde{p}_{t+1} = \frac{1}{2}\left(\tilde{v} + p_t\right) + \lambda\, \tilde{u} \quad (24)$$

for the market price of the risky security. Hence, the market price of the risky security has law $\tilde{p}_{t+1} \sim \mathcal{N}\left(p_t, \frac{1}{4}\sigma_t^2 + \lambda^2 \sigma_u^2\right)$. Thus, by lemma 1 we obtain

$$h\left(\tilde{v}\right) = \frac{1}{2}\left[\ln\left(2\pi\sigma_t^2\right) + 1\right] \quad (25)$$

and

$$h\left(\tilde{p}_{t+1}\right) = \frac{1}{2}\left[\ln\left(2\pi\left(\frac{1}{4}\sigma_t^2 + \lambda^2\sigma_u^2\right)\right) + 1\right]. \quad (26)$$

Note,

$$\begin{aligned}\operatorname{Cov}\left[\tilde{v};\tilde{p}_{t+1}\right] &= \operatorname{Cov}\left[\tilde{v};\frac{1}{2}\left(\tilde{v}+p_t\right)+\lambda\,\tilde{u}\right]\\ &= \frac{1}{2}\sigma_t^2 \end{aligned} \tag{27}$$

since $\operatorname{Cov}\left[\tilde{v};\tilde{u}\right] = \operatorname{E}\left[\tilde{u}\,\tilde{v}\right] = 0$. Hence, the random vector $\left(\tilde{v},\tilde{p}_{t+1}\right)^\top$ is bivariate normally distributed having law

$$\begin{pmatrix}\tilde{v}\\ \tilde{p}_{t+1}\end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\mu},\Sigma\right) \tag{28}$$

where

$$\boldsymbol{\mu} = \begin{pmatrix}p_t\\ p_t\end{pmatrix}$$

and

$$\Sigma = \begin{pmatrix}\sigma_t^2 & \frac{1}{2}\sigma_t^2\\ \frac{1}{2}\sigma_t^2 & \frac{1}{4}\sigma_t^2+\lambda^2\sigma_u^2\end{pmatrix}.$$

Calculating the determinant of the variance–covariance matrix Σ yields

$$|\Sigma| = \lambda^2\sigma_t^2\sigma_u^2 \tag{29}$$

and the inverse of the variance–covariance matrix Σ results as

$$\Sigma^{-1} = \begin{pmatrix}\frac{1}{\sigma_t^2}+\frac{1}{4\lambda^2\sigma_u^2} & -\frac{1}{2\lambda^2\sigma_u^2}\\ -\frac{1}{2\lambda^2\sigma_u^2} & \frac{1}{\lambda^2\sigma_u^2}\end{pmatrix}. \tag{30}$$

Note, the density function of the joint normally distributed random vector $\left(\tilde{v},\tilde{p}_{t+1}\right)^\top$ is given by

$$\begin{aligned}&f\left(v,p_{t+1}\right) =\\ &\qquad \frac{1}{2\pi\sqrt{|\Sigma|}}\cdot\exp\left(-\frac{1}{2}\left[\begin{pmatrix}v\\ p_{t+1}\end{pmatrix}-\boldsymbol{\mu}\right]^\top\Sigma^{-1}\left[\begin{pmatrix}v\\ p_{t+1}\end{pmatrix}-\boldsymbol{\mu}\right]\right).\end{aligned} \tag{31}$$

Taking the natural logarithm of both sides of (31) and applying (28), (29) and (30) yields

$$\begin{aligned}\ln f\left(v,p_{t+1}\right) = &-\ln\left(2\pi\lambda\sigma_t\sigma_u\right)\\ &-\frac{1}{2}\begin{pmatrix}v-p_t\\ p_{t+1}-p_t\end{pmatrix}^\top\begin{pmatrix}\frac{1}{\sigma_t^2}+\frac{1}{4\lambda^2\sigma_u^2} & -\frac{1}{2\lambda^2\sigma_u^2}\\ -\frac{1}{2\lambda^2\sigma_u^2} & \frac{1}{\lambda^2\sigma_u^2}\end{pmatrix}\begin{pmatrix}v-p_t\\ p_{t+1}-p_t\end{pmatrix}.\end{aligned} \tag{32}$$

According to definition 1 we calculate the differential entropy of the random vector $(\tilde{v}, \tilde{p}_{t+1})^\top$ as

$$h(\tilde{v}, \tilde{p}_{t+1}) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v, p_{t+1}) \cdot \ln f(v, p_{t+1})\, dv\, dp_{t+1}. \tag{33}$$

By using (32) the right hand side of (33) is equivalent to

$$\begin{aligned} &\ln(2\pi\lambda\sigma_t\sigma_u)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v, p_{t+1})\, dv\, dp_{t+1} \\ &\quad + \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(v, p_{t+1}) \\ &\qquad \cdot \begin{pmatrix} v - p_t \\ p_{t+1} - p_t \end{pmatrix}^\top \begin{pmatrix} \frac{1}{\sigma_t^2} + \frac{1}{4\lambda^2\sigma_u^2} & -\frac{1}{2\lambda^2\sigma_u^2} \\ -\frac{1}{2\lambda^2\sigma_u^2} & \frac{1}{\lambda^2\sigma_u^2} \end{pmatrix} \begin{pmatrix} v - p_t \\ p_{t+1} - p_t \end{pmatrix} dv\, dp_{t+1}. \end{aligned} \tag{34}$$

Since the first double integral in (34) is a full integral of the bivariate normal density $f(v, p_{t+1})$ and hence is equal to unity we obtain

$$\begin{aligned} &\ln(2\pi\lambda\sigma_t\sigma_u) + \frac{1}{2}\left(\frac{1}{\sigma_t^2} + \frac{1}{4\lambda^2\sigma_u^2}\right)\mathrm{E}\left[(\tilde{v} - p_t)^2\right] \\ &\qquad - \frac{1}{2\lambda^2\sigma_u^2}\mathrm{E}\left[(\tilde{v} - p_t)(\tilde{p}_{t+1} - p_t)\right] + \frac{1}{2}\frac{1}{\lambda^2\sigma_u^2}\mathrm{E}\left[(\tilde{p}_{t+1} - p_t)^2\right], \end{aligned} \tag{35}$$

which can be rewritten as

$$\begin{aligned} &\ln(2\pi\lambda\sigma_t\sigma_u) + \frac{1}{2}\left(\frac{1}{\sigma_t^2} + \frac{1}{4\lambda^2\sigma_u^2}\right)\mathrm{Var}[\tilde{v}] \\ &\qquad - \frac{1}{2\lambda^2\sigma_u^2}\mathrm{Cov}[\tilde{v}; \tilde{p}_{t+1}] + \frac{1}{2}\frac{1}{\lambda^2\sigma_u^2}\mathrm{Var}[\tilde{p}_{t+1}], \end{aligned} \tag{36}$$

alternatively. Now, since $\mathrm{Var}[\tilde{v}] = \sigma_t^2$ and $\mathrm{Var}[\tilde{p}_{t+1}] = \frac{1}{2}\sigma_t^2$ as well as by (27) we have

$$\begin{aligned} &\ln(2\pi\lambda\sigma_t\sigma_u) + \frac{1}{2}\left(\frac{1}{\sigma_t^2} + \frac{1}{4\lambda^2\sigma_u^2}\right)\sigma_t^2 \\ &\qquad - \frac{1}{2\lambda^2\sigma_u^2}\frac{1}{2}\sigma_t^2 + \frac{1}{2}\frac{1}{\lambda^2\sigma_u^2}\left(\frac{1}{4}\sigma_t^2 + \lambda^2\sigma_u^2\right). \end{aligned} \tag{37}$$

Simplifying (37) yields

$$h(\tilde{v}, \tilde{p}_{t+1}) = 1 + \ln(2\pi\lambda\sigma_t\sigma_u) \tag{38}$$

for the differential entropy of the random vector $(\tilde{v}, \tilde{p}_{t+1})^\top$. Now, by (25), (26), and (38) the market makers' objective function (22) becomes

$$\begin{aligned} h(\tilde{v}) + h(\tilde{p}_{t+1}) - h(\tilde{v}, \tilde{p}_{t+1}) = \\ \frac{1}{2}\left[\ln\left(2\pi\sigma_t^2\right) + 1\right] + \frac{1}{2}\left[\ln\left(2\pi\left(\frac{1}{4}\sigma_t^2 + \lambda^2\sigma_u^2\right)\right) + 1\right] \\ - \left[1 + \ln\left(2\pi\lambda\sigma_t\sigma_u\right)\right]. \end{aligned} \quad (39)$$

Finally, the right hand side of (39) is equivalent to

$$\frac{1}{2}\ln\left(1 + \frac{\sigma_t^2}{4\lambda^2\sigma_u^2}\right) \quad (40)$$

and the market makers' problem is

$$\max_{\lambda} \quad \frac{1}{2}\ln\left(1 + \frac{\sigma_t^2}{4\lambda^2\sigma_u^2}\right) \quad (41)$$

subject to $\sigma_x^2 \leq \xi$. By inspection of the optimization problem (41) one realizes that the maximum is reached for λ converging to zero since the natural logarithm is strictly monotonically increasing in its argument. However, by exploiting (16) which is known to the market makers, the constraint $\sigma_x^2 \leq \xi$ can be rewritten as

$$\frac{1}{4\lambda^2}\sigma_t^2 \leq \xi \quad (42)$$

which is equivalent to

$$\lambda \geq \frac{1}{2}\sqrt{\sigma_t^2\xi^{-1}}. \quad (43)$$

Thus, inequality (43) establishes a lower bound on λ, and the market maker's optimal decision in the presence of a restriction on the insider' trading activity is

$$\lambda(\xi) = \frac{1}{2}\sqrt{\sigma_t^2\xi^{-1}} \quad (44)$$

which is given in the proposition. This completes the proof. □

A.3 Proof of corollary 1

Proof. Note,

$$\frac{d}{d\xi}\lambda(\xi) = -\frac{1}{4}\xi^{-1}\sqrt{\sigma_t^2\xi^{-1}} < 0 \quad (45)$$

and hence

$$\frac{d}{d\xi}\left|x(\tilde{v};\xi)\right| = -\frac{|\tilde{v}-p_t|}{2\lambda(\xi)^2}\cdot\frac{d}{d\xi}\lambda(\xi) > 0$$

what yields the first part of the corollary. The insider's average expected profits prior to observing a specific realization of the risky security's liquidation value equals

$$\mathrm{E}\big[\mathrm{E}\left[(\tilde{v}-\tilde{p}_{t+1})\,\tilde{x}|\tilde{v}\right]\big] = \mathrm{E}\left[(\tilde{v}-\tilde{p}_{t+1})\,\tilde{x}\right] \tag{46}$$

by the law of iterated expectations. Applying $\tilde{p}_{t+1}$ and $\tilde{x}$ according to proposition 1 as well as $\tilde{y} = \tilde{x} + \tilde{u}$ to the right hand side of (46) leaves us with

$$\mathrm{E}\left[\left(\tilde{v}-p_t-\lambda(\xi)\left(\frac{\tilde{v}-p_t}{2\lambda(\xi)}+\tilde{u}\right)\right)\frac{\tilde{v}-p_t}{2\lambda(\xi)}\right]$$

which in turn is equivalent to

$$\begin{aligned}
\mathrm{E}\left[\left(\frac{\tilde{v}-p_t}{2}-\lambda(\xi)\tilde{u}\right)\frac{\tilde{v}-p_t}{2\lambda(\xi)}\right] &= \mathrm{E}\left[\frac{(\tilde{v}-p_t)^2}{4\lambda(\xi)}-\frac{(\tilde{v}-p_t)\,\tilde{u}}{2}\right] \\
&= \frac{1}{4\lambda(\xi)}\mathrm{E}\left[(\tilde{v}-p_t)^2\right]-\frac{1}{2}\mathrm{E}\left[\tilde{v}\,\tilde{u}\right]+\frac{1}{2}p_t\mathrm{E}\left[\tilde{u}\right] \\
&= \frac{1}{4\lambda(\xi)}\sigma_t^2. \qquad (47)
\end{aligned}$$

Finally, plugging (44) into (47) results in

$$\frac{1}{2}\sqrt{\sigma_t^2\xi}$$

for the insider's a priori expected profits. Hence,

$$\frac{d}{d\xi}\frac{1}{2}\sqrt{\sigma_t^2\xi} = \frac{1}{2}\cdot\frac{1}{2}\left(\sigma_t^2\xi\right)^{-\frac{1}{2}}\cdot\sigma_t^2 > 0$$

what yields the second part of the corollary. Finally, the depth of the market is quantified by the reciprocal of $\lambda(\xi)$. Thus,

$$\frac{d}{d\xi}\frac{1}{\lambda(\xi)} = -\frac{1}{\lambda(\xi)^2}\cdot\frac{d}{d\xi}\lambda(\xi) > 0$$

what gives us the last part of the corollary. This completes the proof. □

A.4 Proof of proposition 2

Proof. Semi–strong form informational efficiency means

$$\tilde{p}_{t+1} = \mathrm{E}\left[\tilde{v}|\tilde{y}\right]. \tag{48}$$

According to (23) we calculate

$$\begin{aligned}\mathrm{E}\left[\tilde{y}\right] &= \mathrm{E}\left[\frac{1}{2\lambda(\xi)}\left(\tilde{v}-p_t\right)+\tilde{u}\right]\\ &= \frac{1}{2\lambda(\xi)}\left(\mathrm{E}\left[\tilde{v}\right]-p_t\right)+\mathrm{E}\left[\tilde{u}\right]\\ &= 0 \end{aligned} \tag{49}$$

and

$$\begin{aligned}\mathrm{Var}\left[\tilde{y}\right] &= \mathrm{Var}\left[\frac{1}{2\lambda(\xi)}\left(\tilde{v}-p_t\right)+\tilde{u}\right]\\ &= \frac{1}{4\lambda(\xi)^2}\sigma_t^2+\sigma_u^2. \end{aligned} \tag{50}$$

Furthermore,

$$\begin{aligned}\mathrm{Cov}\left[\tilde{v};\tilde{y}\right] &= \mathrm{Cov}\left[\tilde{v};\frac{1}{2\lambda(\xi)}\left(\tilde{v}-p_t\right)+\tilde{u}\right]\\ &= \frac{1}{2\lambda(\xi)}\sigma_t^2. \end{aligned} \tag{51}$$

Hence, by (49), (50), and (51) we calculate

$$\begin{aligned}\mathrm{E}\left[\tilde{v}|\tilde{y}\right] &= \mathrm{E}\left[\tilde{v}\right]+\frac{\mathrm{Cov}\left[\tilde{v},\tilde{y}\right]}{\mathrm{Var}\left[\tilde{y}\right]}\left(\tilde{y}-\mathrm{E}\left[\tilde{y}\right]\right)\\ &= p_t+\frac{\frac{1}{2\lambda(\xi)}\sigma_t^2}{\frac{1}{4\lambda(\xi)^2}\sigma_t^2+\sigma_u^2}\tilde{y}. \end{aligned} \tag{52}$$

Plugging both the market makers' price rule from proposition 1 and (52) into (48) yields

$$p_t+\lambda(\xi)\tilde{y} = p_t+\frac{\frac{1}{2\lambda(\xi)}\sigma_t^2}{\frac{1}{4\lambda(\xi)^2}\sigma_t^2+\sigma_u^2}\tilde{y},$$

which is equivalent to

$$\lambda(\xi) = \frac{1}{2}\sqrt{\sigma_t^2\sigma_u^{-2}}. \tag{53}$$

Ultimately, comparison of (53) to (44) yields the proposition. This completes the proof. □

A.5 Proof of corollary 2

Proof. According to (45) we have $\lambda(\xi) < \lambda(\xi^*)$ for $\xi > \xi^*$ and vice versa. This completes the proof. $\square$

A.6 Proof of corollary 3

Proof. Application of (10) to proposition 1 yields

$$\begin{aligned}
\mathrm{Var}\left[p_{t+1}(\tilde{y};\xi)\right] &= \mathrm{Var}\left[p_t + \Delta p_t(\tilde{y};\xi)\right] \\
&= \mathrm{Var}\left[\Delta p_t(\tilde{y};\xi)\right] \\
&= \mathrm{Var}\left[\frac{1}{2}\left(\tilde{v} - p_t\right) + \lambda(\xi)\tilde{u}\right] \\
&= \frac{1}{4}\sigma_t^2 + \lambda(\xi)^2\sigma_u^2 \qquad (54)
\end{aligned}$$

for the variance of the risky security's market price. Hence, for $\xi < \xi^*$ we have $\lambda(\xi) > \lambda(\xi^*)$ according to (45) and thus

$$\mathrm{Var}\left[p_{t+1}(\tilde{y};\xi)\right] > \mathrm{Var}\left[p_{t+1}(\tilde{y};\xi^*)\right] \qquad (55)$$

immediately. This completes the proof. $\square$

A.7 Proof of proposition 3

Proof. The market makers' profits are given by

$$\tilde{\pi}(\xi) \equiv \big(p_{t+1}(\tilde{y};\xi) - \tilde{v}\big) \cdot \tilde{y} \qquad (56)$$

where $\tilde{y} = x(\tilde{v};\xi) + \tilde{u}$ and ξ is the upper bound on the insider's trading activity chosen by the regulator. Define $\Delta p_{t+1}(\xi) \equiv p_{t+1}(\tilde{y};\xi) - p_{t+1}(\tilde{y};\xi^*)$ and $\Delta x(\xi) \equiv x(\tilde{v};\xi) - x(\tilde{v};\xi^*)$. Then the market makers' unconditional expected profits are given by

$$\begin{aligned}
\mathrm{E}\left[\tilde{\pi}(\xi)\right] &= \mathrm{E}\left[\big(p_{t+1}(\tilde{y};\xi^*) - \tilde{v} + \Delta p_{t+1}(\xi)\big) \cdot \big(x(\tilde{v};\xi^*) + \tilde{u} + \Delta x(\xi)\big)\right] \\
&= \mathrm{E}\left[\big(p_{t+1}(\tilde{y};\xi^*) - \tilde{v}\big) \cdot \big(x(\tilde{v};\xi^*) + \tilde{u}\big)\right] \\
&\quad + \mathrm{E}\left[\Delta p_{t+1}(\xi) \cdot \big(x(\tilde{v};\xi^*) + \tilde{u}\big)\right] \\
&\quad + \mathrm{E}\left[\big(p_{t+1}(\tilde{y};\xi^*) - \tilde{v}\big) \cdot \Delta x(\xi)\right] \\
&\quad + \mathrm{E}\left[\Delta p_{t+1}(\xi) \cdot \Delta x(\xi)\right]. \qquad (57)
\end{aligned}$$

Next, the first unconditional expectation on the right hand side of (57) is analyzed in detail. The expectation

$$\mathrm{E}\left[\big(p_{t+1}(\tilde{y};\xi^*) - \tilde{v}\big)\cdot\big(x(\tilde{v};\xi^*) + \tilde{u}\big)\right] = \mathrm{E}\left[\tilde{\pi}(\xi^*)\right] \tag{58}$$

represents the market makers' unconditional expected profits in a semi–strong form informationally efficient market since the market price of the risky security and the insider's demand for the risky security are obtained according to the bound ξ^* on the insider's trading activity which implements semi–strong form informational efficiency.

Note,

$$\begin{aligned}
\Delta p_{t+1}(\xi) &= p_{t+1}(\tilde{y};\xi) - p_{t+1}(\tilde{y};\xi^*) \\
&= p_t + \lambda(\xi)\cdot\big(x(\tilde{v};\xi) + \tilde{u}\big) - p_t - \lambda(\xi^*)\cdot\big(x(\tilde{v};\xi^*) + \tilde{u}\big) \\
&= \lambda(\xi)x(\tilde{v};\xi) + \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\tilde{u} - \lambda(\xi^*)x(\tilde{v};\xi^*) \\
&= \frac{\tilde{v} - p_t}{2} + \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\tilde{u} - \frac{\tilde{v} - p_t}{2} \\
&= \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\tilde{u},
\end{aligned} \tag{59}$$

and

$$\begin{aligned}
\Delta x(\xi) &= x(\tilde{v};\xi) - x(\tilde{v};\xi^*) \\
&= \frac{\tilde{v} - p_t}{2}\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right).
\end{aligned} \tag{60}$$

By using (59) and (60) we calculate the remaining three unconditional expectations on the right hand side of (57). First,

$$\begin{aligned}
&\mathrm{E}\left[\Delta p_{t+1}(\xi)\cdot\big(x(\tilde{v};\xi^*) + \tilde{u}\big)\right] \\
&\quad = \mathrm{E}\left[\big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\tilde{u}\cdot\big(x(\tilde{v};\xi^*) + \tilde{u}\big)\right] \\
&\quad = \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\mathrm{E}\left[\tilde{u}\cdot\left(\frac{\tilde{v} - p_t}{2\lambda(\xi^*)} + \tilde{u}\right)\right] \\
&\quad = \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\left(\frac{1}{2\lambda(\xi^*)}\big(\mathrm{E}\left[\tilde{u}\tilde{v}\right] - p_t\mathrm{E}\left[\tilde{u}\right]\big) + \mathrm{E}\left[\tilde{u}^2\right]\right) \\
&\quad = \big(\lambda(\xi) - \lambda(\xi^*)\big)\cdot\sigma_u^2
\end{aligned} \tag{61}$$

since $\mathrm{E}[\tilde{u}\tilde{v}] = \mathrm{E}[\tilde{u}] = 0$ and $\mathrm{E}[\tilde{u}^2] = \sigma_u^2$ by assumption. Second,

$$
\begin{aligned}
&\mathrm{E}\left[\big(p_{t+1}(\tilde{y};\xi^*) - \tilde{v}\big) \cdot \Delta x(\xi)\right] \\
&\qquad = \mathrm{E}\left[\left(p_t + \lambda(\xi^*)\left(\frac{\tilde{v}-p_t}{2\lambda(\xi^*)} + \tilde{u}\right) - \tilde{v}\right) \cdot \frac{\tilde{v}-p_t}{2}\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right)\right] \\
&\qquad = \left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \mathrm{E}\left[\left(p_t + \frac{\tilde{v}-p_t}{2} + \lambda(\xi^*)\tilde{u} - \tilde{v}\right) \cdot \frac{\tilde{v}-p_t}{2}\right] \\
&\qquad = -\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \mathrm{E}\left[\left(\frac{\tilde{v}-p_t}{2} - \lambda(\xi^*)\tilde{u}\right) \cdot \frac{\tilde{v}-p_t}{2}\right] \\
&\qquad = -\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \mathrm{E}\left[\frac{1}{4}\left(\tilde{v}-p_t\right)^2 - \frac{1}{2}\lambda(\xi^*)\left(\tilde{u}\tilde{v} - p_t\tilde{u}\right)\right] \\
&\qquad = -\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \left(\frac{1}{4}\mathrm{E}\left[(\tilde{v}-p_t)^2\right] - \frac{1}{2}\lambda(\xi^*)\left(\mathrm{E}[\tilde{u}\tilde{v}] - p_t\mathrm{E}[\tilde{u}]\right)\right) \\
&\qquad = -\frac{1}{4}\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \cdot \sigma_t^2
\end{aligned}
\tag{62}
$$

since $\mathrm{E}[\tilde{u}\tilde{v}] = \mathrm{E}[\tilde{u}] = 0$ and $\mathrm{E}\left[(\tilde{v}-p_t)^2\right] = \sigma_t^2$ by assumption. Finally,

$$
\begin{aligned}
\mathrm{E}[\Delta p_{t+1}(\xi) \cdot \Delta x(\xi)] &= \mathrm{E}\left[\big(\lambda(\xi) - \lambda(\xi^*)\big) \cdot \tilde{u} \cdot \frac{\tilde{v}-p_t}{2}\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right)\right] \\
&= \big(\lambda(\xi) - \lambda(\xi^*)\big)\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \cdot \frac{1}{2}\big(\mathrm{E}[\tilde{v}\tilde{u}] - p_t\mathrm{E}[\tilde{u}]\big) \\
&= 0
\end{aligned}
\tag{63}
$$

since $\mathrm{E}[\tilde{u}\tilde{v}] = \mathrm{E}[\tilde{u}] = 0$ by assumption.

Plugging (58), (61), (62), and (63) into (57) yields

$$
\mathrm{E}[\tilde{\pi}(\xi)] = \mathrm{E}[\pi(\xi^*)] + \big(\lambda(\xi) - \lambda(\xi^*)\big) \cdot \sigma_u^2 - \frac{1}{4}\left(\frac{1}{\lambda(\xi)} - \frac{1}{\lambda(\xi^*)}\right) \cdot \sigma_t^2 \tag{64}
$$

for the market makers' unconditional expected profits. Next, applying both (44) and $\xi^* = \sigma_u^2$ to (64) as well as rearranging terms results in the market makers' unconditional profits as given in the proposition. This completes the proof. □

A.8 Proof of corollary 4

Proof. Straightforward calculation yields

$$\begin{aligned}\frac{d}{d\xi}\left(\mathrm{E}\left[\tilde{\pi}(\xi^*)\right]+\frac{1}{2}\sqrt{\sigma_t^2\xi^{-1}}\cdot\left(\sigma_u^2-\xi\right)\right)& \\ =-\frac{1}{4}\xi^{-1}\sqrt{\sigma_t^2\xi^{-1}}\cdot\left(\xi+\sigma_u^2\right)<0.& \qquad (65)\end{aligned}$$

This completes the proof. □

A.9 Proof of corollary 5

Proof. The sum of the insider's and the market makers' unconditional expected profits is

$$\mathrm{E}\left[\tilde{\pi}(\xi^*)\right]+\frac{1}{2}\sqrt{\sigma_t^2\xi^{-1}}\cdot\left(\sigma_u^2-\xi\right)+\frac{1}{2}\sqrt{\sigma_t^2\xi}. \qquad (66)$$

Hence,

$$\begin{aligned}\frac{d}{d\xi}\left(\mathrm{E}\left[\tilde{\pi}(\xi^*)\right]+\frac{1}{2}\sqrt{\sigma_t^2\xi^{-1}}\cdot\left(\sigma_u^2-\xi\right)+\frac{1}{2}\sqrt{\sigma_t^2\xi}\right)& \\ =-\frac{1}{4}\xi^{-1}\sqrt{\sigma_t^2\xi^{-1}}\cdot\sigma_u^2<0& \qquad (67)\end{aligned}$$

can be calculated immediately. This completes the proof. □