Implications of Optimal Investment Policies for Hybrid Pension Plans: Sponsor and Member Perspectives

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Abstract

This paper analyzes pension plan costs and investment strategies in the context of alternative hybrid pension plans which are optimal either from the perspective of the plan sponsor or the beneficiaries. The focus is in particular on how the introduction of minimum and maximum limits for pension benefits as well as minimum guarantees and caps on the return of the members' individual investment accounts affect investment decisions and plan costs. Within a comparative static analysis framework, it is shown that for low to medium risk portfolios, minimum benefit guarantees tend to be more expensive than minimum return guarantees while for the latter costs increase exponentially with investment risk. The study also finds that portfolio choices of the sponsor and the beneficiaries show substantial differences depending on the exact plan design and the beneficiaries' risk aversion. Combining minimum return guarantees and caps on investment returns emerged as a possible means to reduce such differences, to share investment risks and returns more equally between sponsor and beneficiaries, and to keep pension plan costs under control.

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Introduction

First introduced in the United States by the Bank of America in 1985, hybrid types of pension plans altered the traditional form of pension plan design.[1] The term "hybrid" pension plan subsumes plans with elements of both defined contribution (DC) and defined benefit (DB) plans. The motivation for hybrid plans is to combine the best characteristics of DB and DC plans while circumventing their major disadvantages. Most hybrid plans include a DC-type individual account, but also provide minimum and/or maximum annuity benefits at retirement using a DB-type formula. Additionally, investment returns credited to the individual accounts may be subject to return guarantees and/or return caps.[2]

When setting up and running a pension plan, the costs implied by the specific plan design, as well as the asset allocation decision for the accumulated funds, are of major importance. In a pure DC plan, plan members have extensive control over their accounts' investment strategy (usually subject to the investment menu they are offered). This enables participants to shape their portfolio's risk/return profile to their individual risk preferences. The sponsor only promises to make a certain contribution to the account, so the investment risk is therefore completely borne by the members; consequently, the plan sponsor tends to be rather indifferent toward the individual's investment policy, as it poses no cost implications. By contrast, in a pure DB plan, the sponsor is obliged to provide adequate funds to cover the plan liabilities, so he is fully exposed to capital market risk. The asset allocation decision has direct cost implications for his funding situation. In a hybrid pension plan, both parties have an interest in influencing the plan's investment policy. This can result in a conflict of interest, which is the object of investigation of this chapter. At the same time, we scrutinize the costs inherent in different DB-type elements.

To do so, we construct several hypothetical hybrid pension plans, make assumptions about key parameters, and derive optimal investment strategies for particular objective functions. Objective functions analyzed include cost minimization from the perspective of plan sponsor versus maximizing risk adjusted pension benefits from the perspective of plan members. Although the design of the plan and the assumed parameters do not exactly match a particular pension plan, the models draw on real-world elements. In particular, the ones presented in this chapter draw on prior analysis of the European Central Bank's retirement plan,[3] though the model plan developed here has different characteristics and is in particular less complex.[4]

This paper is organized as follows. First we discuss the main elements of the hybrid plans evaluated including a minimum pension guarantee, maximum pension limits, a minimum return guarantee, and a maximum return cap. Next, we focus on technical aspects of the model and the decision-making process assumed. Finally we analyze the optimal investment strategy, both from the perspective of the plan sponsor as well as plan members.

Designing a Hybrid Pension Plan

The pension arrangement analyzed in this study is taken to be a mandatory plan whose members do not contribute to any other public or private pension scheme. It is a non-contributory funded pension plan, consisting of two types of accounts. First, every plan member owns an individual account endowed by the plan sponsor with an assumed payment of 17 percent of the members' annual salary,[5] representing the employer's regular plan contributions. In addition to this, the plan sponsor owns a separate account, called the *contingency reserve*, which plays the role of a settlement account for transfers to or from the individual accounts. The funds in both the individual accounts as well as the contingency reserve represent total plan assets. All plan funds must be invested in the same asset allocation, and the return on this portfolio is credited *pro rata* to the individual accounts and the contingency reserve.

Complementary to the plan sponsor's pledge to finance the individual accounts with regular contributions, the plan design includes a combination of additional guarantees and/or limits. These are related to the level of benefits at retirement and/or to the asset return credited to the individual accounts. Incorporating these elements affects plan liabilities, and may require additional payments from the sponsor (in addition to regular contributions). These supplementary contributions may be triggered in two cases. First, when guaranteeing a minimum return on the plan assets, the plan sponsor must cover shortfalls below the target return by replenishing the individual accounts through supplementary contributions. Second, supplementary contributions may be needed if there are guaranteed minimum pension benefits. Specifically, we posit that if the market value of the total plan assets falls below 90 percent of the actuarial present value of the plan liabilities (i.e. the solvency ratio falls below 0.9), the plan sponsor must immediately endow the contingency reserve with enough funds to re-establish a solvency ratio of one.

In this plan, participants cannot withdraw funds during the accumulation phase. Members leaving the plan before retirement (e.g. due to workforce turnover) may either leave their funds in the plan, or receive the balance of their individual account as a lump sum payment (limited to the actuarial value of the maximum pension where applicable). At the retirement age of 65, the available funds are converted into a life annuity. This conversion may be subject to the guaranteed minimum pension benefits and to maximum pension limits, depending on the exact design of the benefit structure, to be discussed subsequently.

Benefit Structure. In what follows, six distinct hybrid pension plan benefit designs are scrutinized. Every plan is characterized by a unique combination of the elements mentioned earlier.

The reason to compare these designs is to investigate their differential effects on plan costs and pension benefit levels, and implications for optimal plan asset allocation.

In Case I, which we term the "benchmark" design, the pension plan consists of an individual account for every plan member endowed by the plan sponsor with regular contributions of 17 percent of the current salary. These funds are invested in the capital markets. Beneficiaries are protected from return shortfalls by an annual capital guarantee, i.e. a guaranteed yearly minimum return of 0 percent.[6] In case the funds earn less than 0 percent in any given year, the sponsor must make additional contributions. If the funds accumulated over a plan member's career are insufficient to pay for an adequate pension, this plan also guarantees a minimum level of pension benefits, corresponding to two percent of the career-average salary per year of service. Furthermore, this plan limits the maximum level of benefits to two percent of the beneficiaries' final salary per year of service.[7] In the event of a member either leaving the plan or retiring, any funds in the individual account that exceed the actuarial value of the maximum benefits are transferred to the contingency reserve.

The subsequent Cases II to V are constructed by eliminating certain plan elements, compared to the benchmark case. Case II excludes the capital guarantee, and in Case III the maximum benefits are also removed. Case IV eliminates the minimum benefit from the benchmark case, while Case V only includes the annual capital guarantee. Case VI includes the annual capital guarantee alongside a return cap of 10 percent per year, but provides no further benefit elements relating to salary and years of service. If the asset return on the funds in the individual accounts in any year exceeds the 10 percent level, the excess return will be credited to the contingency reserve. Case VII does not include any guarantees or caps and, therefore, can be interpreted as a pure defined contribution plan. Table 1 summarizes the various plan designs.

Table 1 here

The minimum rate of return guarantee increases the complexity of the pension plan substantially. More specifically, the minimum rate of return guarantee *may* introduce an asymmetric dependency between assets and liabilities. Suppose in Case I, the value of a given investment account corresponds to a pension payment in-between the minimum and the maximum pension limit. In this situation, a high asset return in any given year permanently increases the sponsors' liabilities for the current and future years. Negative returns in subsequent years do not decrease the liability as the minimum rate of return guarantee requires the sponsor to replenish the investment account. Thus the high asset return in the first year had a permanent effect on the liabilities.[8] However in a situation where the investment account corresponds to a pension payment either below the minimum pension guarantee or above the maximum pension limit, asset returns do not have an immediate effect on the sponsor's liabilities.

Asset Liability Modeling and the Pension Decision-Making Process. Next we evaluate the asset liability model and decision making process needed to determine the fund's investment strategy. For that purpose, we describe the key assumptions on how assets and liabilities are projected forward, and subsequently specify decision rules used either by the plan sponsor or by the beneficiaries to identify the optimal asset allocation. Regardless of whether the asset allocation decision is made by the sponsor or by the beneficiaries, a two-step heuristic method is applied, which is often found in practical decision-making formats. In the first step, the set of mean-variance efficient asset allocations is determined using a standard Markowitz-type portfolio optimization. In the second step, all portfolios from the efficient frontier are assessed against a projection of asset and liabilities over a horizon of 30 years.

To project the return and risk effects of a certain asset allocation over time, it is necessary to specify the stochastic processes governing asset class returns, the long-term interest rate (10 years), as well as the inflation rate. The difference between the nominal 10 year interest rate and the inflation rate (i.e. the real 10-year interest rate) is used to discount future pension liabilities. The stochastic dynamics of the (uncertain) market values of the assets are modeled as geometric Brownian motion, which implies that the log return of every asset is independent and identically normally distributed. The long-term interest rate as well as the inflation rate are modeled using the multi-dimensional Ornstein/Uhlenbeck process, to cover the empirically observable mean reversion characteristics in these time series.[9]

The investment universe comprises the broad asset classes, long and short-term euro area bonds, world-wide diversified equities, and emerging market equities. A regime-switching model is used to derive expected returns for the fixed income asset classes. This technique allows consistent generation of yield curve projections contingent on expectations about economic activity (Bernadell et al., 2005). In the long-term projection of the macro economic environment, we rely on the Economist Intelligence Unit (EIU) as an external provider of forecasts for the Euro area, the US and Japan.[10] Expected returns on equity investments are approximated by add-ons to the long-term yields on government bonds. In the analysis the equity risk premium is fixed at 2.5 percent annually for world-wide diversified equity. Reflecting higher risk of emerging market investments we assume an equity risk premium of 4 percent for this asset class. All asset classes are subject to short selling constraints and, in addition, the investment in emerging market equity is restricted to a maximum of 5 percent of overall investments.

Table 2 here

The projection of liabilities is based on a discontinuance valuation method usually applied by plan actuaries; this relies on the assumption that service of each participant ceases on the respective valuation date. It assumes that at a given valuation date the individual investment accounts are translated into a (usually deferred) life annuity with inflation-adjusted payments, whereby the minimum and maximum pension limits laid out earlier are applied. The real discount rates used for this exercise are the real 10-year interest rates determined by the asset model. Discontinuance valuation is performed for each year over the 30-year analysis horizon (Bacinello, 2000). The valuation of liabilities requires projecting population dynamics comprising the evolution of the number and composition of staff, salaries, number of retirees and dependents. For this purpose a hypothetical population comprising initially of 1000 staff members is constructed. The population is evolved forward on the basis of an inhomogeneous, discrete-time Markov chain. Transition probabilities and parameters are derived using assumptions for the company's recruitment, promotion and turnover patterns, evolution of salaries as a function of consumer price inflation, and mortality rates.

Comparing the value of liabilities with the projected value of assets at the respective valuation date allows for the evolution of the plan's solvency ratio to be determined and supplementary contributions to be made by the sponsor and average benefits received by the beneficiaries. Given the complexity of the plan design, solutions are determined using Monte Carlo simulation over 1000 simulation runs. In the process, we make a number of specific assumptions about selection criteria used to determine the plan's optimal asset allocation. To this end, two different regimes are introduced. Under the first regime, arguably the standard for hybrid pension plans, the plan sponsor is solely responsible for the investment strategy. Correspondingly, the second regime assumes that decisions are made by the beneficiaries. In both cases, investment decisions apply simultaneously to all individual investment accounts and the contingency reserve.

For the sponsor, we assume that the objective is to minimize the costs of running the plan. More specifically the sponsor's behavior is modeled as minimizing the worst-case value of discounted supplementary contributions, where the worst-case value is defined as the five percent quantile of the distribution of the sum of discounted supplementary contributions over the 30-year investment horizon. Thus, decision criteria other than costs (such as plan solvency) are not considered explicitly. Plan funding is accounted for by the solvency rule, according to which the funding ratio cannot fall short of 90 percent in any single year. More formally, let SC_t be the total amount of supplementary contributions to be made by the plan sponsor in period t and r the appropriate discount rate, then the objective function is given by:

$$\min\ VaR_{5\%}\left[\sum_{t=1}^{30}\frac{SC_t}{(1+r)^t}\right] \tag{1}$$

For the beneficiaries, investment decisions for the plan are made collaboratively for all investment accounts. These decisions may be made in the context of an investment committee composed of staff representatives. Such a body is assumed to maximize the expected value of the constant-relative-risk-aversion (CRRA) utility function $u(PBF)$ with risk aversion parameter $g > 0$, which is assumed to be equal for all beneficiaries.

$$\max\ E[u(PBF)] = \max E\left[\left(\frac{PBF^{1-g}}{1-g}\right)\right] \tag{2}$$

Utility is defined over the pension benefit factor PBF which refers to pension payments per year service expressed as the percentage of final salary at time of retirement. Factor PBF comprises all simulation runs and all plan members retiring over the 30-year investment horizon.

The Plan Sponsor's Investment Decision

We next take the perspective of the plan sponsor, to evaluate the interrelation between asset allocation in the individual pension accounts and the resulting plan costs measured in terms of supplementary contributions by the plan sponsor. Figure 1 depicts the worst-case supplementary contributions for Cases I to IV for different portfolio allocations, and Figure 2 for Cases V and VI. Worst-case costs are measured as the five percent value at risk of the supplementary contributions, i.e. the present value of contributions by the plan sponsor exceeding the regular payments of 17 percent of the salaries. Portfolio allocations are represented by the mean-variance efficient portfolio returns. Details of the cost-optimal asset allocations, including the asset weights for short- and long-term euro area bonds, global and emerging market equities appear in Panel 1 of Table 3. Panel 2 contains the distributional characteristics of the discounted supplementary contributions for the cost-optimal asset allocations. Finally, Panel 3 reports the pension benefits for these allocations in terms of certainty equivalents.[11] These certainty equivalents are calculated according to the utility function stated above and for four different parameters of relative risk aversion, gamma=0 (i.e. mean pension benefits), gamma=1, 5, and 10.

Table 3 here

Focusing first on the benchmark, Figure 1 shows that the worst-case plan costs for Case I follow a U-shaped curve. With increasing expected portfolio returns, the costs first decrease and then rise, resulting in minimum supplementary contributions for an asset allocation with an expected return of 5.57 percent. This portfolio consists of about 84 percent long-term bonds and 16 percent equities. The minimum supplementary contributions amount to 43 percent of the expected regular contributions. Hence, for every discounted euro the sponsor regularly paid into the plan, additional payments of 43 discounted cents are required to cover the costs of the plan. The U-shape pattern of the costs can be directly related to the guarantees included in Case I. Investing in portfolios mainly consisting of short- or long-term bonds will result in assets not being able to generate enough return to cover the costs of the guaranteed minimum benefits. These costs have to be borne by the plan sponsor. As the expected return on the portfolio increases, it becomes increasingly likely that the funds will suffice to at least pay the minimum pension without further contributions by the plan sponsor. The rise in expected portfolio return is in turn accompanied by an increase in return volatility, which induces costs resulting from falling short of the guaranteed minimum annual asset return of zero percent. From a certain level of volatility onwards, these newly induced costs overcompensate the cost savings related to the minimum pension benefits and the overall costs increase again.

Figure 1 here

Changing the design of the plan has some interesting effects. Eliminating the annual return guarantee for the individual accounts in Case II cuts the amount of supplementary contributions, especially in the case of a more risky asset allocation. However the asset allocation which minimizes costs is only slightly different compared to Case I (i.e. about five percent less bonds and more equities). The minimum worst-case costs decrease marginally from 43 to 42 percent of regular contributions. This can be attributed to the fact that for a low risk asset allocation the costs from the annual return guarantee are relatively low.[12]

In Case III, not only the capital guarantee but also the maximum pension regulation is eliminated; now the plan is basically DC but the beneficiaries are protected by a DB minimum pension benefit in the event of adverse capital market developments. Therefore the amount of supplementary contributions increases compared to Case II, as there are no longer funds in excess of those needed to provide maximum pension benefits which could be credited to the plan sponsor. Looking at the cost minimizing asset allocation, the equity exposure is slightly further increased to about 23 percent with overall worst case supplementary contributions of 44 percent.

Case IV shows a quite different cost function. The previously observed U-shape form is maintained showing a minimum of supplementary contributions at a portfolio return of 5.6 percent, which corresponds to an asset allocation of 82 percent bonds and 18 percent equities. While the asset allocation is comparable to Cases I to III, the level of supplementary contributions with 27 percent of regular contributions is substantially lower than in the previous cases. Additionally, the left branch of the cost function (low risk portfolios) is nearly flat while the right branch (more risky portfolios) shows a strong increase. Economically, this can be explained as follows.

As discussed for Case I, the predominant source of costs, especially when investing in low risk allocations, is the minimum benefit guarantee. This guarantee is not included in Case IV, leading to substantially lower costs compared to Case I. Increasing the expected return and the volatility of the portfolio now has two opposing effects. The higher the (expected) portfolio return, the more often the plan sponsor will profit from cashing in funds from the individual retirement accounts that exceed the amount necessary to cover the maximum pension benefits. Contrarily, the higher the return volatility, the more often supplementary contributions will be triggered due to the annual capital guarantee. Since the former effect dominates the latter for less risky portfolios, the overall costs first decrease with increasing expected portfolio return. For more risky allocations the latter effect dominates the former, which leads to rapidly growing contributions. As the cost impact of the minimum benefit guarantee is diminishing for increasing portfolio returns, Cases I and IV hardly differ for highly risky portfolios.

We now turn to the cases with no explicit defined minimum or maximum benefit, depicted in Figure 2. Case V shows a plan with unlimited upside potential but with a shortfall protection resulting from the annual return guarantee. It is obvious that this plan design implies increasing supplementary contributions, the higher the equity exposure. Hence, minimizing the costs in terms of supplementary contributions leads to the minimum volatility portfolio, consisting of 82 percent short-term and 17 percent long-term bonds, and only 1 percent equities. The resulting costs amount to 27 percent of regular contributions.

Figure 2 here

Similar to Case V, Case VI offers an annual capital guarantee and therefore protection against return shortfalls. However, the upside potential is limited due to the 10 percent return cap. This return cap has a significant impact on the shape of the cost curve. While the amount of supplementary contributions in Cases V and VI is approximately equal for low risk asset allocations, the costs in Case VI initially decrease for increasing portfolio return and volatility. For these allocations the increasing costs resulting from the capital guarantee are overcompensated by the profits the plan sponsor can generate through cashing in any returns that exceed the 10 percent cap. From a certain point onwards, however, the costs of the capital guarantee rise disproportionately with further increasing volatility, leading to overall growing supplementary contributions. At minimum worst case costs of 26 percent, resulting from investing in about 69 percent bonds and 31 percent equities, this case proves to be the cheapest of all hybrid plan designs discussed. This holds for the minimum cost asset allocation and especially for all portfolios with high expected returns and volatilities.

Implications for Plan Beneficiaries. The implications of investing in the cost-minimizing portfolios for expected pension benefits can now be derived (see Figure 3). Panel 3 of Table 3 summarizes expected pension benefit factors and their standard deviation, expressing the pension benefits as a percentage of final salary per year of service. In order to relate the probability distribution of the pension benefit factors to the risk aversion of a representative beneficiary, the pension factor certainty equivalents are calculated for a range of risk aversion parameters using the standard CRRA utility function. This allows a direct evaluation of the cost-optimal asset allocation for the various plan designs, Case I to VI, from the perspective of plan members with different levels of risk aversion. Figure 3 depicts the certainty equivalents for all parameters of risk aversion from one to 10 in half steps. Additionally, numerical results for selected levels of risk aversion (gamma = 1, 5, and 10) are presented in Panel 3 of Table 3.

Figure 3 here

The Figure shows that Case III results in the highest pension benefit factors for all levels of risk aversion under scrutiny, with the mean benefit factor being 2.26 percent (see also Table 3, Panel 3). At the same time, Case V always produces the lowest factors, on average 1.52 percent.

This is an interesting result, as both cases show structural similarities. Cases III and V both offer downside protection to the beneficiaries, Case III by means of guaranteed minimum pension benefits, Case V with the annual capital guarantee for the individual accounts. Neither case limits the upside potential.

An explanation for this can be found when looking at the different cost-minimal asset allocations. Optimizing the amount of supplementary contributions in Case V, the plan sponsor will only invest in short- and long-term bonds, resulting in the lowest risk exposure with respect to the capital guarantee. With highly conservative risk and return profile of the assets simultaneously low pension benefits are expected. Such an asset allocation, however, is not appropriate in Case III, since its return expectations are insufficient to cover the costs resulting from the guaranteed minimum benefits, i.e. 2 percent of the career-average salary per year of service. Rather, it is necessary to implement a portfolio strategy that offers higher mean returns, coming at the cost of higher volatility. This, in turn, leads to substantially higher supplementary contributions, since the plan sponsor fully bears the downside volatility while only the beneficiaries profit from the upside volatility.

Implementing a maximum benefit cap (2 percent of final salary per year of service) results in considerably reduced volatilities of the pension benefit factors, i.e. 0.05 percent for Cases I and II, and 0.10 percent for Case IV (see Table 3, panel 3). Consequently, the certainty equivalents of the pension benefit factors are nearly constant for the various risk aversion coefficients reported in Figure 3. Among these cases, Case I offers the highest pension benefits but is also the most costly design. Case II only offers slightly lower benefits combined with slightly lower costs.

In general, it can be concluded that hybrid plans that offer the highest expected pension benefits tend to cause the highest amount of supplementary contributions. Yet there are two exceptions: Case V offers by far the lowest pension benefits, but even given optimal asset allocation patterns, additional costs are not small. By contrast, Case VI has the lowest supplementary contribution, and will lead to expected pensions benefits that exceed all but one other case. The rather high volatility of the pension benefit factor, however, causes the certainty equivalents to drop below those of most other cases for higher levels of risk aversion.

Beneficiaries' Investment Decisions

In this section we assume that the asset allocation decisions are made by the plan participants, rather than the plan sponsor; here the plan members' objective function is to maximize the expected utility of pension benefits by choosing an appropriate asset allocation. This analysis is undertaken for Cases I-VI and also for Case VII, a pure defined contribution plan. Our interest here is to look at the resulting pension benefits for plan members with different levels of risk aversion, as well as the composition of the optimal asset allocation. For simplicity, we assume that the asset allocation decision made by the beneficiaries and their cost impact have no repercussive effects on plan member salaries; neither will rising supplementary contributions lead to lower salaries/salary increases nor will reductions of plan costs be passed on to the workers.

As above, we derive the benefit-optimal portfolio allocations from the set of mean-variance efficient portfolio returns. Details of the optimal investment weights (i.e. the mean and volatility of asset returns, mean and certainty equivalents of pension benefit factors for plan members as well the resulting costs in terms of supplementary contributions for the plan sponsor) are shown in Table 4. The first Panel contains the results for a representative plan member with a low coefficient of risk aversion (gamma = 1), while the other panels show findings for a medium (gamma = 5) and a high (gamma = 10) coefficient. Table 5 provides details regarding the investment weights.

Table 4 and 5 here

The results show that, independent of risk aversion, plan beneficiaries would opt for the asset allocation that offers the highest or almost the highest expected return and the highest or almost the highest volatility in Cases I-V. Table 5 indicates that the optimal asset allocation consists of 100 percent stocks. This is because beneficiaries are protected against downside volatility of the equity and bond markets by guaranteed minimum pension benefits and the annual capital guarantee. The value of this downside protection *ceteris paribus* increases with return volatility. By analogy to option pricing theory, the minimum pension benefit and the capital guarantee can be interpreted as a put option, for which the value is also positively related to the volatility of the underlying.

Analyzing the level of supplementary contributions associated with these asset allocations, it appears that costs for the plan sponsor would be prohibitively high. This is particularly true for Case V, in which the members' individual accounts are protected against negative fluctuations in the capital markets while at the same time offering full participation in positive returns. Here, the certainty equivalents of the pension benefit factors vary between 5.99 percent for a low risk aversion (gamma = 1) and 2.94 percent for a higher risk aversion (gamma = 10). These high pension benefits are associated with worst-case (mean) supplementary contributions of 253 percent (127 percent).

Cases I, II, and IV limit the upside potential available to the beneficiaries by incorporating the maximum pension benefit restriction. This results in lower benefits and lower costs compared to Case V, yet plan members still have the incentive to choose portfolios with very high volatility. Even though the costs are substantially reduced, they are still intolerably high. For example in Case IV, i.e. Case V with incorporated maximum benefit limit, the worst case (expected) supplementary contributions amount to 85 percent (27 percent), being about three times as high as in case the plan sponsor chooses the asset allocation.

Case VI produces a different picture: now, the annual capital guarantee and a 10 percent cap on the maximum annual asset return limit the credits to the beneficiaries' individual accounts. Beneficiaries with a low level of risk aversion (gamma = 1) will still invest in the maximum expected return/maximum volatility portfolio, but more risk averse plan members will choose an asset allocation with substantially reduced exposure to capital market risk. For a medium (high) level of risk aversion (gamma = 5 vs. 10), the allocation to bonds will increase from none (at gamma = 1), to 72 (77) percent (see Table 5). We compare this optimal asset allocation from the members' perspective (with moderate to high risk aversion) to the cost optimal asset allocation from the sponsor's perspective in Panel 1 of Table 3. It is noteworthy that from both perspectives, the optimal investment strategy is nearly identical – a high exposure to bonds and low exposure to equities. This results in quite similar cost implication in terms of supplementary contributions. If the plan sponsor were to set the asset allocation, the five percent Value-at-Risk of supplementary contributions would be 26 percent (panel 2 of Table 3); if the representative member with a medium or high risk aversion selected the optimal asset allocation, this would equally result in supplementary contributions for the plan sponsor of 26 percent. Hence, if the benefit structure of the pension plan is designed according to Case VI, members' and sponsor's interests are consistent, at least with respect to the asset allocation decision for a given plan design.

Case VI might seem to be most suitable for a hybrid pension plan, as it combines acceptable cost consequences for the sponsor and attractive pension benefit factors for the plan members. Nevertheless a superior plan design exists. Case VII is a pure defined contribution plan with no capital guarantee and no return cap. By construction, this plan causes no additional costs in term of supplementary contributions for the sponsor. Additionally, as shown in Table 4, a pure DC plan provides higher mean pension benefits as well as certainty equivalents for all levels of risk aversion. This result is due to the specification of the floor/cap structure, i.e. a minimum rate of return of 0 percent per year and a return maximum of 10 percent per year. Alternatively, setting the cap to an annual return of 12.5 percent and leaving the floor constant at 0 percent implies the following results: For members with a low level of risk aversion (gamma = 1), the certainty equivalent for the benefit factor of the hybrid plan is still lower than in the case of a pure DC plan (2 percent compared to 3 percent). For members with medium to high levels of risk aversion (gamma = 5 or 10), the hybrid plan is more attractive than the pure defined contribution plan.[13] Yet when increasing the cap to 12.5 percent, the plan members will again choose the maximum volatility portfolio (i.e.100 percent equities), independent of their level of risk aversion. Unfortunately this will cause unacceptable costs in terms of supplementary contributions for the plan sponsor.

Conclusions

This chapter evaluates key properties of hypothetical hybrid pension plans in terms of their implications for plan costs and pension benefits, by adapting a pure defined contribution scheme to include minimum and maximum limits for pension benefits, as well as minimum guarantees and return caps on individual investment accounts. In particular we explore optimal investment strategies from the perspectives of both plan sponsor and beneficiaries. We find that introducing DB elements substantially increases the overall costs of running the pension plan and has a major impact on the resulting optimal portfolios.

The investment strategy chosen and the additional plan costs show strong interrelationship. If only minimum rate of return guarantees are included in the plan design, additional costs increase exponentially as a function of higher expected asset return and volatility. Consequently, plan sponsors choose the minimum risk portfolio consisting of around 82 percent short-term bonds, around 17 percent long-term bonds, and only 1 percent equities. Enhancing this plan with a cap on returns credited to the individual accounts leads to a U-shaped cost curve for a broad range of possible asset allocations. Here, the optimal portfolio consists of about 69 percent bonds and 31 percent equities, i.e. about 50 percent more equities than for any other plan design optimized from the sponsor's perspective. At the same time, with this design the additional costs are reduced to 26 percent from 27 percent in the case without the return cap.

For plan designs that guarantee minimum pension benefits, the implied additional costs (expected and worst case values) are also U-shaped as a function of expected investment returns. Therefore, assuming the objective to minimize the worst case value of additional costs, the sponsor will opt for asset allocations which deviate from the minimum risk allocation as well. These portfolios comprise between 77 and 84 percent bonds and between 16 and 23 percent equities. The additional costs for these plans lie in the range of 42 and 44 percent of regular contributions, i.e. about 50 percent above the costs of a plan only guaranteeing a minimum rate of return.

Taking the beneficiaries' perspective, we first evaluate the utility implications of alternative defined benefit elements based on the sponsor's optimal asset allocation decisions. To this end, certainty equivalents of pension benefits are calculated for a range of risk aversion parameters. Generally higher additional costs imply higher expected pension benefits, but the introduction of caps on credited asset returns allows cost reductions with only slightly lower certainty equivalents of random pension benefits. We also evaluate optimal investment choices from the beneficiaries' perspective, and we find that almost independent of risk aversion, plan members tend to select maximum return, maximum risk asset allocations where the plan either guarantees minimum pension benefits or minimum return guarantees. However, if the minimum rate of return guarantees are combined with a cap on the maximum return credited to individual accounts, risk averse members opt for less risky asset allocations. In this case, the optimal asset allocation includes about 72 percent in bonds and 28 percent in equities.

Our results are directly relevant to the moral hazard problem faced by agencies offering insurance against pension plan defaults, including the Pension Benefit Guaranty Corporation (PBGC) in the United States, and the newly established Pension Protection Fund in the United Kingdom (see, e.g., Warshawsky et al. 2005; Coronado and Liang 2005; McCarthy and Neuberger 2005). Like the plan sponsor in this paper, those organizations issue a put option on the value of the assets invested in the insured pension plans. Therefore, they should be interested in rather conservative pension fund asset allocations mainly concentrated in bonds. If, as for the beneficiaries in this chapter, the price of such an option (i.e. the insurance premium) is set independently of its value, and if the insured party can influence the value, there is a chance that the insured party will seek to boost the probability of exercising the option – by investing in high-risk assets or by underfunding the pension plan. A possible solution to this moral hazard problem is to implement funding requirements that take into account both current level of funding as well as investment risk, as for example is done for the German Individual Investment Accounts (“Riester” accounts; c.f. Maurer and Schlag 2004).

The analysis presented in this study can be useful when discussing possible designs of hybrid pension plans. Some plan designs appear to be Pareto-inefficient (e.g. minimum and maximum pension benefits in combination with minimum rate of return guarantee) as they are dominated by others which imply lower additional costs and higher expected utility for plan members. Furthermore, if plan sponsors and beneficiaries are jointly responsible for investment decision, caps on investment returns may reduce conflicts of interest as asset allocations will diverge less between the parties.

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Table 1: Summary of Hybrid Pension Plan Designs

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Individual Account	√	√	√	√	√	√	√
Minimum Benefits	√	√	√				
Maximum Benefits	√	√		√			
Capital Guarantee	√			√	√	√	
Return Cap						√	

Source: Authors' compilations.
Notes: Minimum benefits are defined as 2 percent of career-average salary per year of service; maximum benefits are defined as 2 percent of final salary per year of service; capital guarantee refers to a guaranteed minimal return of 0 percent per year for the individual accounts; the return cap limits the annual return credited to the individual accounts to 10 percent.

Table 2: Parameter Assumptions for Asset Returns, Interest Rates, and Inflation Dynamics

				Correlations			
	Mean (%)	Volatility (%)		Short-term Euro area bonds	Long-term Euro area bonds	Global Equities	EM Equities
Short-term Bonds	4.3	2.5		1			
Long-term Bonds	5.1	3.7		0.43	1		
Global Equities	7.6	17.9		-0.01	0.21	1	
EM Equities	9.1	27.5		0.08	0.1	0.73	1
				Correlations of Innovations			
	θ	κ	σ	10y Interest Rate		Inflation Rate	
10y Interest Rate	0.05	0.08	0.01	1			
Inflation Rate	0.02	0.29	0.01	0.81		1	
Short-term Bonds	-	-	-	-0.07		-0.05	
Long-term Bonds	-	-	-	0.14		0.07	
Global Equities	-	-	-	0.00		0.01	
EM Equities	-	-	-	0.00		0.01	

Source: Authors' calculations.
Notes: Return expectations are derived using yield curve projections as laid down in Bernadell et al. (2005) as well as the assumption of equity risk premia of 2.5% and 4% for Global Equities and Emerging Markets (EM) Equities respectively. Furthermore, the return dynamics are assumed to follow a geometric Brownian motion. The ten-year interest rate as well as the inflation rate are modeled using the process specified by $dX_t = k(q - X_t)dt + s\ dW_t$, where X_t is the value of the Ornstein/Uhlenbeck process in *t*, kappa (κ) is the speed of mean-reversion, theta (θ) is the long-run mean, and sigma (σ) is the volatility of changes of the process. dW_t is the increment of a standard Wiener process. For the above estimates we rely on data provided by JP Morgan European Bond indices (Euro area bonds), MSCI World ex EMU index (Global Equities) and MSCI Emerging Markets index (EM Equities), as well as German inflation rates, and German long-term interest rates.

Table 3: Optimal Investment Decisions: The Plan Sponsor's Perspective

	Case I	Case II	Case III	Case IV	Case V	Case VI
Panel 1: Cost-Optimal Asset Allocation (%)						
Mean Return	5.57	5.70	5.76	5.63	4.48	5.95
Volatility	4.68	5.23	5.53	4.94	2.40	6.53
Short-term Euro area bonds	0.00	0.00	0.00	0.00	81.51	0.00
Long-term Euro area bonds	84.27	79.16	76.61	81.72	17.19	68.95
Global Equities	10.73	15.84	18.39	13.28	1.30	26.05
Emerging Markets Equities	5.00	5.00	5.00	5.00	0.00	5.00
Panel 2: Distributional Characteristics of DSC with Optimal Asset Allocation (%)						
Mean DSC	19.64	18.44	22.14	9.24	12.76	7.00
Std. DSC	13.03	12.89	12.60	9.62	7.76	9.17
5%-VaR DSC	42.49	41.50	43.95	27.01	27.04	25.87
25%-Q DSC	10.27	9.04	12.71	0.00	7.23	0.00
50%-Q DSC	17.56	16.12	19.87	6.70	11.47	2.77
75%-Q DSC	26.92	25.65	29.41	15.17	17.00	11.39
Panel 3: Distributional Characteristics of PB with Optimal Asset Allocation (%)						
Mean PB	1.86	1.87	2.26	1.79	1.52	1.94
Std. PB	0.05	0.05	0.42	0.10	0.35	0.31
Certainty Equivalent ($\gamma = 1$)	1.87	1.87	2.22	1.79	1.49	1.92
Certainty Equivalent ($\gamma = 5$)	1.87	1.87	2.12	1.78	1.37	1.83
Certainty Equivalent ($\gamma = 10$)	1.87	1.87	2.04	1.75	1.27	1.73

Source: Authors' calculations.
Notes: Asset weights in percent; DSC (i.e. Discounted Supplementary Contributions): contributions required on top of fixed regular contributions to fully fund the pension plan (in percent of expected discounted regular contributions); PB (i. e. Pension Benefits): attainable income replacement factor (in percent of final salary per year of service); Gamma (γ): parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type: $u(W) = W^{1-g} / (1-g)$; Objective function: Minimize the 5 percent VaR of DSC; Q: quantile; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Table 4: Optimal Investment Decision: The Plan Participants' Perspective

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Panel 1: Low Level of Risk Aversion (Gamma = 1) (%)							
Mean Return	7.68	7.68	7.68	7.68	7.68	7.68	7.68
Volatility	17.27	17.27	17.27	17.27	17.27	17.27	17.27
Mean DSC	30.91	24.60	29.80	26.58	126.73	14.29	0.00
5%-VaR DSC	87.93	75.71	79.28	84.52	253.47	51.33	0.00
Mean PB	1.97	1.92	3.94	1.95	6.86	1.95	3.79
Certainty Equivalent	1.97	1.91	3.36	1.95	5.99	1.92	3.09
Panel 2: Medium Level of Risk Aversion (Gamma = 5)(%)							
Mean Return	7.68	7.61	7.68	7.68	7.68	5.89	6.33
Volatility	17.27	16.83	17.27	17.27	17.27	6.18	8.74
Mean DSC	30.91	24.14	29.80	26.58	126.73	7.35	0.00
5%-VaR DSC	87.93	74.14	79.28	84.52	253.47	26.06	0.00
Mean PB	1.97	1.92	3.94	1.95	6.86	1.94	2.54
Certainty Equivalent	1.97	1.91	2.45	1.95	4.00	1.83	2.01
Panel 3: High Level of Risk Aversion (Gamma = 10) (%)							
Mean Return	7.68	7.61	7.61	7.68	7.68	5.76	5.76
Volatility	17.27	16.83	16.83	17.27	17.27	5.53	5.53
Mean DSC	30.91	24.14	29.22	26.58	126.73	7.98	0.00
5%-VaR DSC	87.93	74.14	77.68	84.52	253.47	26.31	0.00
Mean PB	1.97	1.92	3.85	1.95	6.86	1.93	2.16
Certainty Equivalent	1.97	1.90	2.16	1.94	2.94	1.73	1.76

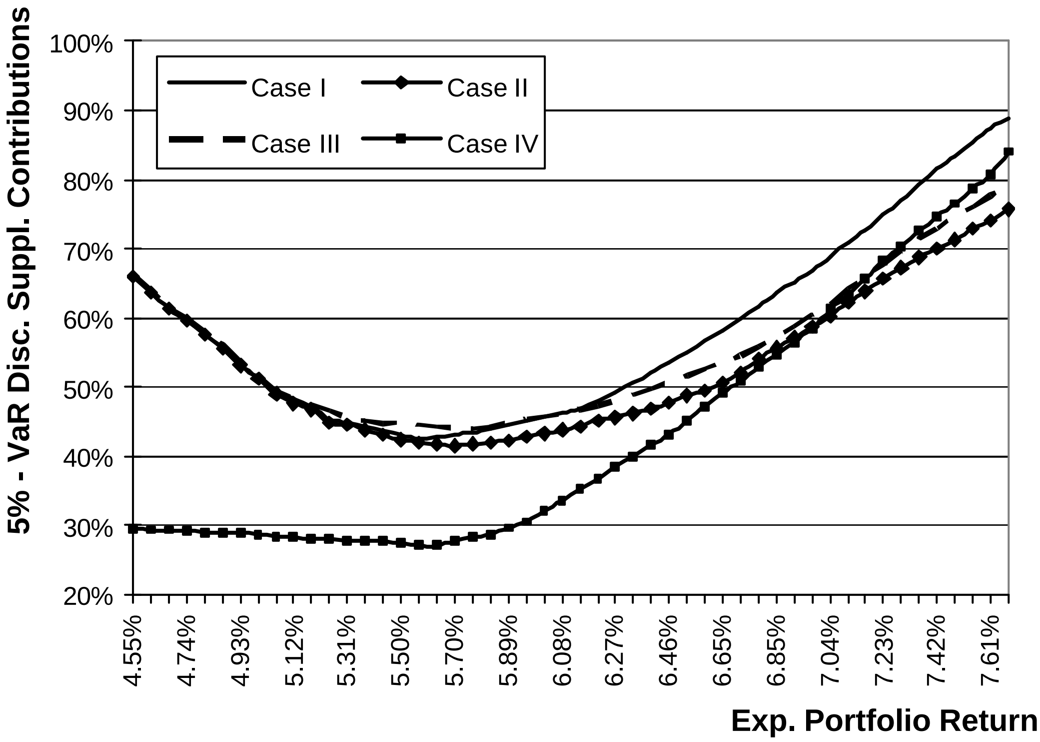
Source: Authors' calculations.
Notes: DSC (i.e. Discounted Supplementary Contributions): contributions required on top of fixed regular contributions to fully fund the pension plan (in percent of expected discounted regular contributions); PB (i.e. Pension Benefits): attainable income replacement factor (in percent of final salary per year of service); Gamma (?): parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type: $u(W) = W^{1-g}/(1-g)$; Objective function: Maximize the expected utility of pension benefits using a CRRA utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

Table 5: Optimal Asset Allocations: Participant Perspectives

	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Panel 1: Low Level of Risk Aversion (Gamma = 1) (%)							
Short-term Euro area bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Long-term Euro area bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Global Equities	95.00	95.00	95.00	95.00	95.00	95.00	95.00
EM Equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00
Panel 2: Medium Level of Risk Aversion (Gamma = 5) (%)							
Short-term Euro area bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Long-term Euro area bonds	0.00	2.55	0.00	0.00	0.00	71.50	53.63
Global Equities	95.00	92.45	95.00	95.00	95.00	23.50	41.37
EM Equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00
Panel 3: High Level of Risk Aversion (Gamma = 10) (%)							
Short-term Euro area bonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Long-term Euro area bonds	0.00	2.55	2.55	0.00	0.00	76.61	76.61
Global Equities	95.00	92.45	92.45	95.00	95.00	18.39	18.39
EM Equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00

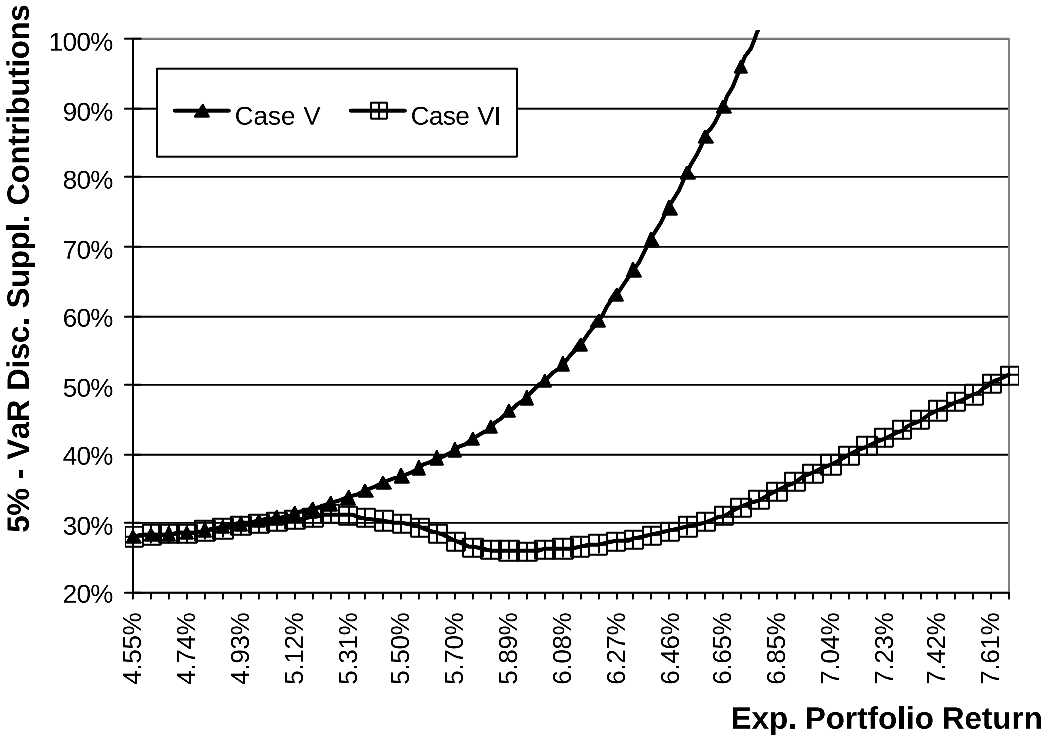
Source: Authors' calculations.
Notes: Objective function: Maximize the expected utility of pension benefits using a CRRA utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

Figure 1: Worst-Case Plan Costs vs. Asset Allocation



Source: Authors' calculations.
Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee

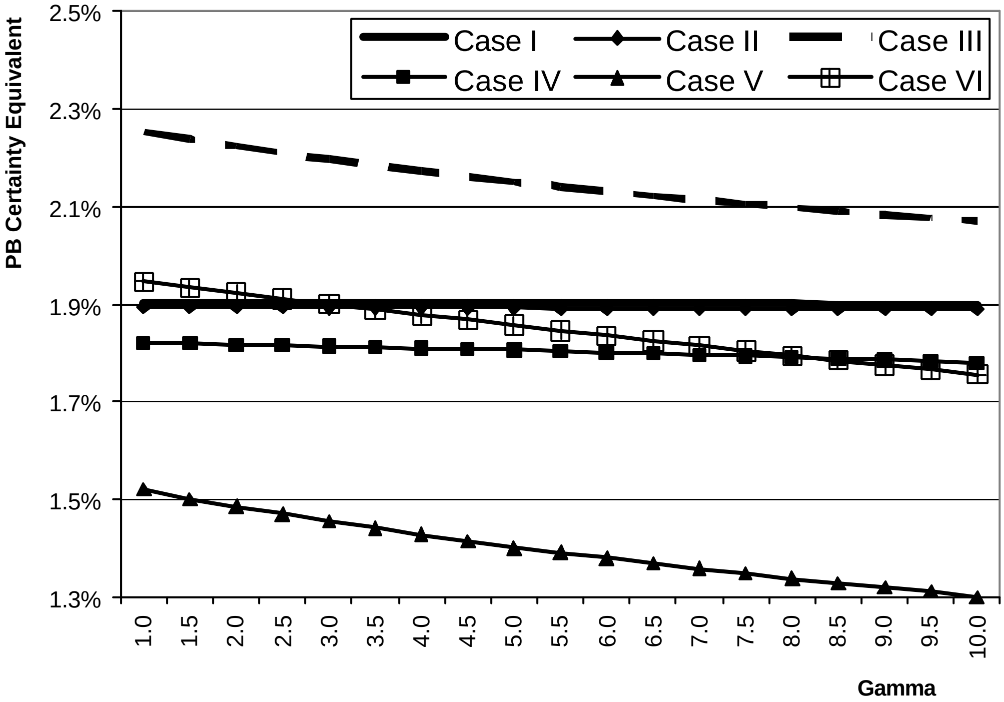
Figure 2: Worst-Case Plan Costs vs. Asset Allocation



Source: Authors' calculations.

Notes: Discounted Supplementary Contributions (DSC) in percent of expected discounted regular contributions; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Figure 3: Certainty Equivalents of Pension Benefit Factors for different Plan Designs with Sponsor's Optimal Asset Allocation



Source: Authors' calculations.

Notes: PB Certainty Equivalent: Certain Pension Benefit (PB) Factor, i.e. retirement income in percent of final salary per year of service, that has the same utility to the beneficiary as the random Pension Benefit Factor provided by the pension plan. Gamma: parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type: $u(W) = W^{1-g} /(1-g)$; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Endnotes

[1] Pension promises in the US have traditionally been either of the pure defined benefit (DB) or pure defined contribution (DC) type (Schieber, 2003). In a DB scheme, the plan sponsor promises to the plan beneficiaries a final level of pension benefits. This level is usually defined according to a benefit formula, as a function of salary trajectory and years of service. Benefits are usually paid as a life annuity rather than as a lump sum. As Bodie et al. (1988) note, the foremost advantage of a DB plan is that it offers stable income replacement rates to retired beneficiaries. The major drawbacks of DB schemes include the lack of benefit portability when leaving the company and the complex valuation of plan liabilities. Moreover, the plan sponsor is exposed to substantial investment and longevity risk, which could result in significant contribution expenses. In a DC scheme, by contrast, the plan sponsor commits to paying funds into the beneficiaries' individual accounts according to a specified formula, e.g. a fixed percentage of annual salary. The most prominent feature of a DC scheme is its inherent flexibility: by construction, it is fully funded in individual accounts. The value of the pension benefits is simply determined as the market value of the backing assets. Therefore, the pension benefits are easily portable in case of job change. Additionally, the beneficiaries have control over their funds' investment strategy and at retirement can usually take the money as a life annuity, a phased withdrawal plan, a lump sum payment, or some combination of these. While the employer is only obliged to make regular contributions, the employee bears the risk of uncertain replacement rates, especially caused by fluctuations in the capital markets (Bodie and Merton, 1992).

[2] An in-depth discussion of the implications of introducing hybrid pension plans in the United States can – among others – be found in Clark and Schieber (2004), Coronado and Copeland (2004), Johnson and Steuerle (2004), and Mitchell and Mulvey (2004).

[3] The European Central Bank (ECB) operates a hybrid pension scheme; plan assets, which exist solely for the purpose of providing benefits for members of the plan and their dependents, are included in the other assets of the ECB. Benefits payable, resulting from the ECBs contributions, have minimum guarantees underpinning the defined contribution benefits.

[4] For example, we do not handle dependent benefits and we assume a simplified population model.

[5] A contribution rate of 17 percent can be considered as reasonable assumption given the typical structure of European pension plans. For example, in Germany, contributions to the state-run pay-as-you-go pension system currently amount to 19.5 percent of salaries. As provisions for dependents' pensions are neglected in this study, reducing the contribution rate by 2.5 percent compared to the German state pension system seems a reasonable assumption.

[6] Alternatively to a focus on absolute return, a minimum fixed rate of return guarantee could be applied to a relative rate of return. For example, Chile's private pension funds were long required to earn an annual real rate of return that depended on the average annual real rate of return earned by all of Chile's private pension funds (Pennacchi 1999). Or the guarantee may be applied to the account balance at the time of retirement, instead of the assumed annual basis.

[7] The maximum pension benefits generally exceed the guaranteed minimum pensions for two reasons. Typically the wage path until retirement is non-decreasing and thus the career-average salary will usually be lower than the final salary. Moreover we impose a limit on the number of years in service that may be considered in the calculation of the minimum and maximum pension benefits. When calculating the maximum pension benefits, we allow for five more years in service to be included than in the calculation of the minimum pension benefits.

[8] This link between assets and liabilities is in contrast to the analogy developed by Bodie and Davis (2000) who compare a pension plan to an equipment trust such as those set up by an air-

line to finance the purchase of airplanes. Here the equipment serves as specific collateral for the associated debt obligation. The borrowing firm's liability is not affected by the value of the collateral. So, for instance, if the market value of the equipment were to double, this would greatly increase the security of the promised payments, but it would not increase their size. As opposed to this scenario, in the scheme developed in this paper, the value of the assets may well affect the liabilities as a high return in a given year may increase the value of the liabilities as outlined above.

[9] A drawback of the Ornstein/Uhlenbeck process is the theoretically positive probability of negative nominal interest rates, but this is eliminated in the simulation procedure by cutting off the negative nominal interest rates.

[10] The EIU forecasts are constructed with the aid of an econometric world model, maintained by the UK based Oxford Economic Forecasting.

[11] The certainty equivalent of a lottery is defined as the fixed payment that provides the same utility as the random lottery.

[12] Analyzing the costs of Individual Account guarantees, Lachance and Mitchell (2004) argue that guarantee costs tend to be insensitive to the asset allocation in cases where the exercise of the guarantees is either extremely likely or extremely unlikely.

[13] The certainty equivalents are 2.10 percent compared to 2.01 percent for gamma = 5, and 1.91 percent compared to 1.76 percent for gamma = 10.