# Project Financed Investments, Debt Maturity and Credit Insurance* 

Van Son Lai and Issouf Soumaré ${ }^{\dagger}$

May 1, 2006

[^0]
# Project Financed Investments, Debt Maturity and Credit Insurance 


#### Abstract

This paper studies the impact of credit insurance on both investment and financing decisions of project financed companies. Although, financial guarantees have been portrayed in the extant literature as tools for credit insurance to foster investments, there are other implications for the use of these guarantees, especially for project finance requiring huge amounts of investment. We find that under the value maximizing paradigm, the presence of credit insurance can exacerbate the under-investment problem. We also discuss the effects of guarantee subsidy, agency costs and risk on project investment incentives. Finally, our framework establishes a relationship between the project debt maturity and its investment incentives.


Keywords: Investment incentives, Credit insurance, Debt maturity structure, Project finance. JEL Classification: G11, G14, G31, G38.

## 1 Introduction

Project finance (PF) is an increasingly important method of financing large-scale capitalintensive projects, such as power plants, oil pipelines, ports, tunnels, etc. The demand for financing often exceeds the supply capacity of the project sponsor and of local capital markets (Farrell, 2003). Project finance is an arrangement in which a sponsor creates a new project company through a special purpose vehicle (SPV) and looks to the project future cash flows as the main source of repayment to lenders. Project-financed investments have grown at a compound rate of almost 20 percent over the past 10 years and globally firms financed 234 billion dollar of capital expenditures using project finance in 2004, up from 172 billion dollar in 2003 (Esty, 2004a).

Since project financed investments involve huge amounts of capital and are highly levered, (e.g., Brealey, Cooper and Habib (1996), Esty (2003), Kleimeier and Megginson (2001), Shah and Thakor (1987)), one way for lenders to hedge credit risk is to require credit insurance (or financial guarantee) for the loans they make. A credit insurance is a promise from a third party to make good on payments to the fund provider when the borrower defaults. To have access to funds at lower costs, project companies resort to credit insurance to improve their credit rating and debt capacity. Government agencies and international organizations such as the World Bank Multilateral Investment Guarantee Agency (MIGA) and Export Credit Agencies (For example, US Export-Import, China Export and Credit Insurance Corporation (Sinosure), Export Development Canada (EDC), Export Credits Guarantee Department (ECGD) of the UK, COFACE France) are some of the main providers of credit insurances, especially to back large-scale projects financing (e.g., Dailami and Leipziger (1998), Ehrhardt and Irwin (2004)). Nowadays, the demand for credit insurances is increasingly widespread. More private insurance companies are entering the credit insurance business.

In this paper, we study the effects of credit insurance on project finance investment policies and financing decisions. We analyze the project's investment incentives in presence of not free credit insurance. Although credit insurances have been depicted in the existing literature mostly as tools for credit enhancement and fostering investment, there
are other unexplored implications for their use by firms. For example, Froot, Scharfstein and Stein (1993), Leland (1998), Smith and Stulz (1985), and Smith and Morellec (2005) in the corporate hedging literature, discuss the use of hedging instruments (also known as alternative risk transfer (ART) tools in the insurance literature) by firms as value enhancing tool. ${ }^{1}$ However, Jin and Jorion (2006) found that hedging does not seem to affect market value for the oil and gas industry. We provide the extent of the impact of one of ART instruments, credit insurance (e.g., Banks (2004)), on the capital structure and risk of the project, and study the relationship between the debt maturity and the investment incentives of the project. However, we differ from the above cited papers as they use forward contracts as hedging instruments, thus the firm does not pay explicitly for its hedging, whereas in our framework, the project pays for its insurance contract. Merton and Bodie (1999) outline three mechanism for firms to control their investment policies: diversification, insurance and hedging. Here, we focus on the insurance mechanism.

Several policy implications are raised from our study. They should help project companies in their decision to use credit insurance. Credit insurances allow the project company to have access to more funding at lower costs, which increases the project debt capacity, especially for large investments as it is the case in project financed investments. In other words, as expected, the creditworthiness of the project is enhanced by credit insurance. However, counter to basic intuition, when a larger portion of its debt is guaranteed, a firm by maximizing its shareholders net-wealth can give rise to underinvestment (measured by the investment level vis a vis the investment without credit insurance). Indeed, at the outset, the project pays a fee for credit insurance which reduces its cash flows. Moreover, the possible lowering of the interest rate obtained through credit insurance reduces the relative tax shields even though the project gets tax deductions on its insurance premium expense. At last, the effect of the tax shields reduction outweighs the decrease in the bankruptcy cost. Therefore, credit insurance improves the debt financing terms of the project, but that does not necessarily result into shareholders' wealth increase. Our result contrasts with Mayers and Smith (1987),

[^1]Garven and MacMinn (1993) and Smith and Morellec (2005), who argue that firms may be more likely to hedge to control for their overinvestment incentives. However, in their framework they use a costless hedging instrument.

One may ask why will a project company require credit insurances for its loan if it can result in value destruction. In reality, in most cases for large project financed investments, given the level of risk involved, debtholders will require credit insurances before lending to the project. Moreover, in most cases, the government will intervene (through credit agencies or multilateral guarantors) to get the project a go ahead, otherwise some net present value projects will be abandoned do to the lack of financing support, especially in developing countries. This justifies the use of credit insurances by project companies even if it destroys value, otherwise the project cannot be undertaken especially if a minimum investment is required.

Increasing the risk level of the project can induce less investment and therefore less net-wealth to the project sponsor. The intuition is as follows. When the project risk increases, with perfect information about the project volatility, the marginal borrowing cost increases, therefore it becomes too costly to insure the loan. Based on our numerical experiment with plausible baseline parameters, the relative increase in tax shields following the risk shifting is not enough to compensate for the increase in the costs of eventual default. However, it is important to point out that this specific result is parameter dependent, since the inverse phenomenon can be observed too.

In the case of public guarantee, the insurance subsidy creates more investment incentives. Otherwise, some net positive present value projects could be abandoned resulting in forgone taxes and social benefits for the government. This is in line with Lai and Soumaré (2005), who analyze the investment with government financial guarantees. In addition, by using the degree of overpricing of insurance fee as proxy for the agency costs, we found that more investments are made when the agency costs are lower. Contrarily, the manager will tend to under invest and destroy shareholders value in order to avoid monitoring from debtholders and/or insurance providers. Empirical support for this finding on agency costs can be found in Esty (2003) who argued that "project finance creates value by reducing the agency costs associated with large, transaction-
specific assets, and by reducing the opportunity cost of underinvestment due to leverage and incremental distress costs."

Finally, we study the relationship between the project investments and its debt maturity. Although several theoretical works have investigated the maturity structure of firm debt and its impact on firm capital structure (e.g., Diamond (1991), Flannery (1986), Myers (1977) among others), few have been devoted to the study of the relationship between firm debt maturity and its investment incentives. For example, Zhdanov and Lyandres (2003) study the relationship between firm investment and its debt maturity. However, here we combine credit enhancement and fostering investment in project finance. We find that there is over-investment with low and high maturities and underinvestment with intermediate maturities. Intuitively, for low maturities, the bankruptcy cost is very low almost null, and for high maturities, the project is able to extract more tax benefits through coupon payments which motivates the over-investment and therefore increases the value to shareholders. For lower size investments projets, we observe that the investment level increases for debt maturities over the range $[0,7]$ years and for debt maturities beyond 7 years, the investment level decreases. We observe an inverse trend for higher size investments projects. The empirical support for these findings is the recent work by Aivazian, Ge and Qiu (2005), who test the relationship between debt maturity and firm investment. They find that longer maturity debt is associated with less investment for firms with high growth opportunities. In contrast, debt maturity is not significantly related to investment for firms with low growth opportunities.

Related to our work are previous studies on optimal capital structure and investment flexibility. For example, Parrino, Poteshman and Weisbach (2005) study the investment distortions when risk averse managers decide whether to undertake risky projects. Ju and Ou-Yang (2005) determine jointly the optimal capital structure and debt maturity in a stochastic interest rate environment. Titman and Tsyplakov (2002) propose a model in which the firm can dynamically adjust both its capital structure and its investment choices. Morellec and Smith (2005) analyze the relation between agency conflicts and risk management. However, none of these papers include in their studies alternative risk transfer instruments such as credit insurances commonly used in project financed
investments. We depart from these previous literature, by considering a project finance in which the company can adjust its investment level and has simultaneously access to a not free credit insurance contract when financing the project.

This study also differs from previous studies on credit enhancement (e.g., Chen, Hung and Mazumdar (1994), Chen and Mazumdar (1996), Gendron, Lai and Soumaré (2006), Lai (1992), Merton (1974, 1977), Merton and Bodie (1992)) which analyze the credit enhancement of the project without taking into account its value maximizing objective. In that respect, they focus on the debt capacity of the project by assuming the objective of the project to be its credit enhancement or simply assuming a perfect market.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 presents our numerical results and provides a general discussion of the findings. Section 4 concludes.

## 2 The model

We consider a sponsor undertaking a new project. The project is a stand-alone special purpose vehicle (SPV), meaning that the project is an independent and separate entity. Since the project sponsor has limited wealth, the total investment to undertake the project is done by equity-debt financing mix. The only commitment of the sponsor is its capital contribution. The project cash flows are used to pay its debt. In this financing framework, often referred to as non- or limited recourse financing, lenders depend on the performance of the project itself for repayment rather than the credit of the sponsor.

We assume a simple capital structure for the project, consisting of a single debt and equity contracts. At the outset, the project requires an initial investment $I$ financed partly by the sponsor in the amount $S$ and the rest $I-S$ is financed by debt. In other words, shareholders (proxied by the sponsor) decide to infuse a capital level $S$ and borrow $I-S$ to finance the new project. The equity capital $S$ is entirely financed by the existing shareholders, meaning that no new shares are issued, or simply stated, there are no new shareholders in our model. Thus the initial amount of debt required to start the project is $D=I-S$ and it is financed with a coupon paying debt with coupon rate $c$ and face value $F$. For ease of computation and without loss of generality, we assume the debt to
be issued at par, i.e., the value of the debt $D$ is equal to its face value $F$ (e.g., Leland and Toft (1996)). Later, in the paper, we discuss how the coupon rate $c$ and the face value $F$ of the debt are obtained endogenously from the project's maximization problem and the participation constraint of debtholders.

We also introduce a (private or public) insurer who insures partially or fully the loan payment in case of default by the project, as it is the case in most project financed investments. The project pays for the insurance. This feature of alternative risk transfer through financial innovation in our model departs us from previous works.

The project pays corporate taxes. Hence, with the total investment $I$, the project generates after tax total asset value $V(I)$ characterized by the following stochastic technology

$$
\begin{equation*}
V_{t}(I)=\left(1-\tau_{c}\right) v(I, \theta, t) \tag{1}
\end{equation*}
$$

where $v($.$) is a twice differentiable function with respect to its three arguments and is$ concave with respect to $I$. We denote by $\tau_{c}$ the corporate tax rate. The total value $v$ includes the growth opportunities. The random variable $\theta$ captures the stochastic nature of the price of the assets.

Following Parrino, Poteshman and Weisbach (2005), we denote by $q$ the payout rate by the project in terms of debt repayment and/or dividend payout. It consists of dividend payment plus after tax coupon paid to debtholders by the project and is obtained endogenously from the following equation:

$$
\begin{equation*}
q V(I)=\delta V(I)+\left(1-\tau_{c}\right) c F, \tag{2}
\end{equation*}
$$

where $\delta$ represents the dividend payout rate as percentage of the value of the project. $c F$ is the dollar coupon paid over the time interval $d t$. It is equal to the coupon rate $c$ times the face value $F$ of the debt, both obtained endogenously from the project maximization program.

The dynamic of $\theta$ in equation (1) follows a Ito process and it drives the generating process of the asset value. Therefore, with $Z$ denoting the standard Wiener process in the risk neutral world, the risk-adjusted process for the asset value (net of capital cost)
is assumed to follow a geometric Brownian motion process as follows:

$$
\begin{equation*}
V_{t}(I)=V_{t}(I)\left[(\mu-\lambda \sigma-q) d t+\sigma d Z_{t}\right] \tag{3}
\end{equation*}
$$

where $\mu$ is the instantaneous mean return and $\sigma$ is the instantaneous return volatility which captures the aggregate risk level of the project. We assume that $\sigma$ is chosen or known by the project manager. The parameter $\lambda$ is the market price of risk for the project value (See Hull (2005) and Schwartz and Moon (2000) for the estimation of $\lambda$ and the use of risk neutral valuation respectively, in valuing real options and internet companies). In the case of a traded security, $\mu-\lambda \sigma$ is equal to the risk-free rate $r$. However if the underlying asset is not traded, as may often be the case in capital-budgeting-associated options, its growth rate may actually fall below the equilibrium total expected return required of an equivalent-risk traded financial security, with the difference or "rate of return shortfall" necessitating a dividend-like adjustment in option valuation (see McDonald and Siegel, 1984, 1985).

### 2.1 Debt covenants and the value of the guaranteed debt

For ease of computation and without loss of generality, we assume the debt to be issued at par, i.e., the value of the debt $D$ is equal to its face value $F$. The debt pays instantaneous coupon rate $c$ and matures at time $T$. Thus the yield on the debt is equal to the coupon rate $c$. The value of $c$ and $F$ will be determined endogenously from the value maximization of the project.

The debt has a protective covenant that specifies that if at any time during the life of the debt, $[0, T]$, the project value decreases to a boundary $V_{t}^{-}$, it is forced into bankruptcy by debtholders. At each time $t$, the project defaults in one of the following two situations, either its cash flows are not enough to make the required payment on the debt or its value hits the default boundary. The empirical evidence of barrier provision in debt contracting has been provided by Brockman and Turtle (2003). Thus, similar to Black and Cox (1976) and Ju et al. (2005), we use a bankruptcy triggering boundary with exponential growth as follows

$$
\begin{equation*}
V_{t}^{-}=F e^{g(t-T)} \tag{4}
\end{equation*}
$$

where $g$ is the instantaneous growth rate and is fixed exogenously.
Lets denote by $f(t)$ the probability density function for first hitting the boundary $V_{t}^{-}=F e^{g(t-T)}$. It is the probability density function for the first exit time. For later use, we define $\Phi$ the probability of hitting the boundary over the interval $[0, t]$

$$
\begin{align*}
\Phi(t) & =\int_{0}^{t} f(\tau) d \tau \\
& =N\left(X_{1}(t)\right)+\left(\frac{V(I)}{F e^{-g T}}\right)^{-2 a_{1}} N\left(X_{2}(t)\right), \tag{5}
\end{align*}
$$

and $\Psi$ a variant of this probability with discounting

$$
\begin{align*}
\Psi(t, x) & =\int_{0}^{t} e^{-(r+x) \tau} f(\tau) d \tau \\
& =\left(\frac{V(I)}{F e^{-g T}}\right)^{-a_{1}+a_{2}} N\left(X_{3}(t)\right)+\left(\frac{V(I)}{F e^{-g T}}\right)^{-a_{1}-a_{2}} N\left(X_{4}(t)\right), \tag{6}
\end{align*}
$$

where

$$
\begin{array}{cl}
X_{1}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)-a_{1} \sigma^{2} t}{\sigma \sqrt{t}}, & X_{2}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)+a_{1} \sigma^{2} t}{\sigma \sqrt{t}}, \\
X_{3}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)-a_{2} \sigma^{2} t}{\sigma \sqrt{t}}, & X_{4}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)+a_{2} \sigma^{2} t}{\sigma \sqrt{t}}, \\
a_{1}=\frac{\mu-\lambda \sigma-q-g-\sigma^{2} / 2}{\sigma^{2}}, & a_{2}=\frac{\sqrt{\left(a_{1} \sigma^{2}\right)^{2}+2(r+x) \sigma^{2}}}{\sigma^{2}}
\end{array}
$$

and $N($.$) is the cumulative normal standard distribution function. We refer the interested$ reader to Harrison (1990) and Ju et al. (2005) for the derivations of the closed forms (5) and (6). As we will discuss later, the cumulative distribution functions $\Phi$ and $\Psi$ are affected by the investment and financing policies of the project through the project value, the level of the barrier and the payout rate $q$.

As we have already mentioned, the debt is insured by a third party. We assume that the insurance contract specifies a partial guarantee in the portion $\omega$ of the total debt when the project defaults. The value of $\omega$ is in the interval $[0,1]$, and $\omega=1$ corresponds to the full insurance case. The project pays for the insurance. ${ }^{2}$ Since we have assumed

[^2]the debt to be issued at par, at each instant $t$, the value of the debt is equal to its face value. Thus when the project defaults, the insurer will be asked to pay the remaining amount on the debt. Therefore, the value of the guaranteed debt today is equal to the present value of the expected future payments to be made by the project plus the expected amount to be paid by the insurer in case of default. It is obtained as follows:
\[

$$
\begin{align*}
D= & c F \int_{0}^{T} e^{-r t}(1-\Phi(t)) d t+\int_{0}^{T} e^{-r t}(1-\alpha) F e^{g(t-T)} f(t) d t \\
& +F(1-\Phi(T)) e^{-r T}+\int_{0}^{T} e^{-r t} \min \left(\omega F, F-(1-\alpha) F e^{g(t-T)}\right) f(t) d t \tag{7}
\end{align*}
$$
\]

This expression of the value of the debt has four terms. The first term represents the expected payments of coupons. $1-\Phi(t)$ is the probability of not defaulting over the interval $[0, t]$ since $\Phi(t)$ defined in equation (5) is the probability of hitting the boundary over $[0, t]$. The second term represents the expected salvage value of the project adjusted for the violation of the absolute priority rule when default occurs. Recall, $f(t)$ is the probability density function of first hitting the boundary and $F e^{g(t-T)}$ is that boundary at time $t$. Because $V$ follows a continuous process, when it hits the boundary, its value is equal to $F e^{g(t-T)}$. The coefficient $\alpha$ is the percentage of loss relinquished to debtholders in case of default, and captures the violation of the absolute priority rule. The third term is the expected payment of the debt face value if no default occurs. $1-\Phi(T)$ is the probability of not defaulting during the life of the debt. From equation (5), the function $\Phi$ is a function of the project investment $I$ and the level of its debt $F$. Or these two variables are affected by the insurance contract, therefore, the first three terms of the debt expression are implicitly affected by the guarantee portion $\omega$ of the debt. The fourth term is the value of the guarantee denoted by $G$, and states that, in default state, the payment by the insurer is the minimum between the maximum specified amount in the insurance policy, $\omega F$, and the total default amount on the debt, $F-(1-\alpha) F e^{g(t-T)}$. If we denote by

$$
\begin{equation*}
t_{-}=\min \left(\max \left(\frac{1}{g} \ln \left(\frac{1-\omega}{1-\alpha}\right)+T, 0\right), T\right) \tag{8}
\end{equation*}
$$

the first time the maximum amount, $\omega F$, specified in the guarantee contract becomes superior to the default amount, $F-(1-\alpha) F e^{g(t-T)}$, then the value of the guarantee
becomes ${ }^{3}$

$$
\begin{aligned}
G & =\int_{0}^{t_{-}} e^{-r t} \omega F f(t) d t+\int_{t_{-}}^{T} e^{-r t}\left(F-(1-\alpha) F e^{g(t-T)}\right) f(t) d t \\
& =\omega F \Psi\left(t_{-}, 0\right)+F\left[\Psi(T, 0)-\Psi\left(t_{-}, 0\right)\right]-(1-\alpha) F e^{-g T}\left[\Psi(T,-g)-\Psi\left(t_{-},-g\right)\right] \cdot(9)
\end{aligned}
$$

Using this expression of the guarantee in the debt value expression (7) yields

$$
\begin{align*}
D= & \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right]+F(1-\Phi(T)) e^{-r T} \\
& \omega F \Psi\left(t_{-}, 0\right)+F\left[\Psi(T, 0)-\Psi\left(t_{-}, 0\right)\right]+(1-\alpha) F e^{-g T} \Psi\left(t_{-},-g\right) . \tag{10}
\end{align*}
$$

Note that since the debt is issued at par, $D$ is equal to $F$ and will be determined endogenously from the maximization problem of the project under the participation constraint of debtholders. Because the debt is issued at par, the interest rate on the debt is equal to the coupon rate $c$. Using equation (10), we obtain the value of the credit spread as follows:

$$
\begin{equation*}
c-r=r \frac{(1-\omega) \Psi\left(t_{-}, 0\right)-(1-\alpha) e^{-g T} \Psi\left(t_{-},-g\right)}{1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)} . \tag{11}
\end{equation*}
$$

It can be readily shown that $c-r \geq 0$ by construction of $t_{-}$. This credit spread expression is a semi-closed form solution since to obtained the full closed form solution for $c$ implies solving for a fix point, because the functions $\Phi$ and $\Psi$ contain $q$ the payout rate by the firm, which itself depends on the coupon rate $c$, the face value of the debt $F$ and the level of the investment $I$ obtained from the project maximization. However, all else being equal, the credit spread $c-r$ decreases with the guarantee portion $\omega$. And when $\omega$ reaches 1 (full insurance coverage), the credit spread $c-r$ becomes zero since $t_{-}$becomes zero, which implies $\Psi\left(t_{-}, 0\right)=\Psi\left(t_{-},-g\right)=0$ from equation (6).

Figure 1 plots the debt capacity and the borrowing interest rate when the portion of the guarantee varies. We observe that the debt capacity of the project increases as $\omega$ increases. And for the same level of $\omega$, the debt capacity is even higher with higher borrowing interest rate. However, it is possible for the project to reach the same

[^3]debt capacity by trading off between the borrowing interest rate and the guarantee portion. The borrowing interest rate decreases with the guarantee portion $\omega$. Thus credit insurance allows the project to access to more funding at lower cost.

Next, we describe the project shareholders' net-wealth and the maximization program.

### 2.2 The project shareholders' net-wealth

The objective of the project is to maximize the net-wealth of its shareholders given by the following equation

$$
\begin{align*}
\text { NetWealth }= & V(I)+\text { TaxShields }+ \text { Depreciation } \\
& - \text { BankruptcyCosts }-\left(1-\tau_{c}\right) P-D-S, \tag{12}
\end{align*}
$$

where $D$ is the amount of debt borrowed and $P$ the cost of the guarantee policy paid to the insurer. $P$ is an expense fee for the project, therefore, it benefits from the tax deduction on the amount. The sponsor finances $S$ amount of the total investment and the remaining part $I-S$ constitutes the total value of debt financing, i.e., $D=I-S$. TaxShields are the tax shields obtained from the interest payments on the debt; Depreciation is the capital depreciation tax shield that is included in the value. Tax shields are always assumed to be usable. BankruptcyCosts are the contracting costs of bankruptcy due to credit default. TaxShields is computed as follows

$$
\begin{align*}
\text { TaxShields } & =\int_{0}^{T} e^{-r t} \tau_{c} c F(1-\Phi(t)) d t \\
& =\tau_{c} \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right] \tag{13}
\end{align*}
$$

In this expression, the project obtains tax deductions on the coupon payments on its debt. Depreciation is computed as follows

$$
\begin{align*}
\text { Depreciation } & =\int_{0}^{T} e^{-r t} \tau_{c} e^{-h t} I(1-\Phi(t)) d t+(1-\Phi(T)) \int_{T}^{+\infty} e^{-r t} \tau_{c} e^{-h t} I d t \\
& =\tau_{c} \frac{I}{r+h}(1-\Psi(T, h)), \tag{14}
\end{align*}
$$

where $h$ is a parameter for the tax code depreciation allowance for the capital. Here the project benefits from depreciation on its capital cost allowance. Over the interval $[0, T]$,
the project gets depreciation tax shields if no default occurs, and after $T$, assuming the project survives, the project gets also depreciation tax shields until the investment is depreciated entirely. BankruptcyCosts is obtained as follows

$$
\begin{equation*}
\text { BankruptcyCosts }=\int_{0}^{T} e^{-r t} \alpha F e^{g(t-T)} f(t) d t=\alpha F e^{-g T} \Psi(T,-g) . \tag{15}
\end{equation*}
$$

The bankruptcy cost is the lost of value for the project shareholders when default occurs. It is equal to $\alpha$ times the salvage value of the project where $\alpha$ is the percentage of loss relinquished to debtholders in case of default, and captures the violation of the absolute priority rule.

Putting the pieces together yields

$$
\begin{align*}
\text { NetWealth }= & V(I)-I+\tau_{c} \frac{I}{r+h}(1-\Psi(T, h)) \\
& +\tau_{c} \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right] \\
& -\alpha F e^{-g T} \Psi(T,-g)-\left(1-\tau_{c}\right) P \tag{16}
\end{align*}
$$

The first line of the equation is the net-wealth of an all-equity financed project. The project benefits from the capital depreciation tax shield. The second line is the benefit from tax deduction on interest payments. The last line materializes the losses due to potential contracting costs of bankruptcy and deductible insurance premium payment.

In perfect insurance markets, the guarantee premium should be equal to the present value of the expected guarantee payments, i.e., $P=G$. However, the project will pay less since it can deduct taxes from the premium expenses. Therefore, in absence of other market imperfections, the project would like to insure its debt to hedge against the distribution of default, hence reducing the contracting costs of bankruptcy. Another justification can be the tax benefits.

To capture the case of insurance subsidy, we express the insurance premium as follows: $P=(1-\varepsilon) G$, where $\varepsilon \geq 0$ captures the presence of subsidy. A value of $\varepsilon=1$ materializes a full subsidy, i.e., the project does not pay for its credit insurance premium.

To capture the effect of agency costs, we also use a shortcut through the premium paid for the insurance, by considering a payment schedule of $P=(1+\varepsilon) G$ where $\varepsilon \geq 0$. We interpret $\varepsilon G$ as a proxy for the costs induced by the agency conflicts between the
players. The presence of agency conflicts is costly for the project and is assumed to be proportional to the amount of the value of the guarantee. In that case, the project pays more than the fair price of the guarantee.

### 2.3 The project's optimization problem

As we mentioned above, the project maximizes it shareholders net-wealth given by equation (16). The maximization program is stated as follows:

$$
\begin{aligned}
\max _{I, \omega}[W(I, \omega)= & V(I)-I+\tau_{c} \frac{I}{r+h}(1-\Psi(T, h)) \\
& +\tau_{c} \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right] \\
& \left.-\alpha F e^{-g T} \Psi(T,-g)-\left(1-\tau_{c}\right) P\right]
\end{aligned}
$$

under the financing constraint

$$
F=I-S
$$

The functions $\Phi$ and $\Psi$ are obtained from equations (5) and (6). From the debtholders participation constraint (10), the credit spread is obtained by (11). The insurance premium has the following form

$$
P= \begin{cases}G, & \text { fairly priced } \\ (1-\varepsilon) G, & \text { subsidized insurance } \\ (1+\varepsilon) G, & \text { accounted for agency costs }\end{cases}
$$

with $G$ obtained from equation (9).
Table 1 summarizes the exogenous variables used in the maximization program and the endogenous variables obtained from the optimization.

The full optimization equations are provided in the appendix. The optimization program under constraint is performed numerically using Matlab optimization toolbox. We next discuss the numerical results of our optimization exercise.

## 3 Numerical results and general discussion

### 3.1 Parameters estimation

The parameters values are set based on previous studies such as Ju et al. (2005) and Parrino, Poteshman and Weisbach (2005), and empirical evidence on the characteristics
of project financed investments.
The risk-free interest rate is $r=5 \%$, the growth rate of the barrier $g=1.5 \%$, the dividend payout rate is $\delta=0.5 \%$, the capital depreciation rate parameter $h=2$, the corporate tax rate is $\tau_{c}=35 \%$. The baseline project debt maturity is $T=10$ years, which corresponds to the empirical evidence on project finance average debt maturity between 8 to 12 years. We use $\alpha=20 \%$ as bankruptcy cost coefficient. We assume a decreasing return to scale production technology for the total asset value of the project:

$$
V(I)=\left(1-\tau_{c}\right) \theta I^{\gamma}
$$

where $\gamma=0.80$ and $\theta$ is a random variable capturing the stochastic nature of the assets with initial value $\theta=10$. We assume the dynamics of $\theta$ to follow a geometric Brownian motion with instantaneous mean growth rate $\mu=0.12$ and annualized volatility $\sigma=40 \%$. We set $\lambda=\frac{\mu-r}{\sigma}=0.175$ as if the project's assets were tradable. The baseline parameters values are summarized in Table 2.

Below, we run our numerical simulations with the guarantee portion $\omega$. We also run the same numerical simulations using the fixed amount $H$ of loan to be guaranteed. For example, the insurer can accept to guarantee a fix amount $H$ of the loan instead of a portion $\omega$. The results are qualitatively the same. We report the results for the cases with $\omega$, the results with $H$ can be obtained from the authors upon request.

### 3.2 Financial guarantees as catalyst for credit enhancement

From equation (10), if the total investment level $(I)$ does not change and the sponsor contribution $(S)$ stays the same, then the coupon rate should decrease with guarantee, or equivalently the interest rate charged on the guaranteed loan should be lower compared to that on the non-guaranteed loan.

Figure 2 plots respectively the ratio of the expected guarantee over the total debt $(G / F)$, the borrowing interest rate $(c)$, the debt ratio computed as the ratio of the total debt over the sum of the total debt plus the total equity: $F /(E+F)$, the tax shields amount, the bankruptcy costs, and the net-wealth. Each variable is plotted as function of the percentage investment by the sponsor $(S / I)$ and for three different levels of loan
guarantee portion ( $\omega=0 \%, 20 \%$ and $50 \%$ ). Since the sponsor brings capital $S$, the remaining amount $F=I-S$ is raised in form of debt.

From the graphs of Figure 2, we observe that when the percentage of expected guarantee increases, the borrowing interest rate decreases. Indeed, for interest rates lower than the required compensation, debtholders are less willing to extend more financing to the project unless there is a credit insurance. Therefore, the financial guarantee enhances the creditworthiness of the project by lowering the borrowing interest rate, which allows the project to borrow more, leading to an increase in the project debt ratio. For lower levels of sponsor's contribution $(S / I)$, the sensitivity of the borrowing interest rate to the partial guarantee percentage is very high, and that sensitivity decreases substantially when the sponsor contribution increases. The intuition is that, when the sponsor brings more own capital contribution, the demand for outside financing in forms of debt is lower, therefore less need for credit insurances.

However as we have mentioned in the introduction, using credit insurances affects the net-wealth to project shareholders. The bottom three graphs of Figure 2 show how the sponsor's net-wealth is changing when his capital contribution varies and the guarantee portion changes. First, the project gains from tax deduction on the insurance premium paid. Second, because of the lower interest rate with credit insurance, the project extracts less tax shields and at the same time its bankruptcy cost of default decreases. In the analysis here, the investment level is kept constant. We then observe that, with credit insurance, the project receives less tax shields and at the same time the bankruptcy cost decreases. But the relative change in the tax shields is higher than that of the bankruptcy cost of credit default, which drives the sponsor's net-wealth down. If the investment level does not change, as shown in the graph, the net-wealth of the sponsor decreases when the project insures a larger portion of its debt. The explanation is that, the relative decrease in tax shields obtained with insurance is not compensated by the gain from reduction in the bankruptcy costs.

Recall, in this framework, we have assumed that the project company can always borrow as long as it can pay the interest rate obtained from the participation constraint of the debtholders. In that sense, it is always possible for the project to raise capital
in the debt market. In practice, the project will need to improve its creditworthiness in order to access financing, which justifies the use of credit insurances, especially for large investment projects as it is the case in project financed investments.

One of the constraining assumptions in this section is that the investment level remains the same. Indeed, by changing the insurance contract terms, the project will endogenously adjust its investment, which of course will have an impact on the sponsor's wealth. We discuss the investment incentives of the project in the next section. Our objective is to gauge the severity of under/over-investment which we measure relative to the investment level without credit insurance.

### 3.3 Investment incentives in the presence of credit insurances

Within our proposed framework, the project can react to the presence of loan guarantees in two different ways. Either, it changes the total investment level, or the sponsor changes his capital contribution for given risk posture or investment/financing policy. In both cases, the demand for external financing will be affected and also the net-wealth to the project shareholders will be affected. Since the objective of the project is to maximize its shareholders' net-wealth, studying the investment incentives induced by the presence of credit insurances is of particular interest, especially in project financed investments more likely to use credit insurances.

The project is performing the following maximization:

$$
\max _{I, \omega} W(I, \omega) \quad \text { under the financing constraint } \quad F=I-S \text {. }
$$

The optimal investment level $I^{*}$ and the endogenous borrowing interest rate $c^{*}$ are obtained from this maximization. Below and above the optimal investment $I^{*}$, there will be wealth destruction.

Table 3 presents the values of the optimal policies as a result of the project's maximization for different values of sponsor contributed capital $S$ and guarantee portion $\omega$ of the debt. As depicted in the tables, counter to basic intuition, when the portion of partial guarantee increases, if the project has flexibility over the level of its investment, it tends to invest less in order to maximize its shareholders value. With credit insurances,
undoubtedly the debt capacity of the project is improved but that does not necessary result in an increase in the project shareholders' net-wealth. Indeed, more credit insurances implies less tax shields, all else being equal. Moreover, the project has to pay for the insurance fees which reduces too its value. At the optimum, the presence of credit insurances implies less investment by the project and more credit insurances exacerbate the under-investment problem.

The natural question is why will a project require credit insurances for its loan if it can result in value destruction. In reality, in most cases for large project financed investments, given the level of risk involved, debtholders will require loan guarantees before lending to the project. This constrains the project to seek credit insurance even if it destroys value, otherwise the project cannot be undertaken especially if the project requires a minimum level of investment.

The optimal investment and the net-wealth to project sponsor are increasing functions of the sponsor's contributed capital. When the sponsor brings more contribution to finance the project, the project invests more and also the net-wealth to the project sponsor increases.

In Figure 3, we divide the investment space into two regions. The region of lower investments where the investment levels are below the optimal investment $I^{*}$, and the region of higher investments with investments above the optimal investment $I^{*}$. When the project invests $I<I^{*}$, it can always capture new wealth opportunities by investing up to $I^{*}$. However, if the project invests $I>I^{*}$, there is wealth destruction. In the last case, to increase its shareholders wealth, the project has to decrease the investment level. In the second graph of Figure 3, we observe that the relative increase in the cost of bankruptcy is higher than that of the tax shields for high investments. The wealth loss from bankruptcy is not compensated by the tax shields amount, which results in netwealth destruction. In addition, for very high investments, the total expected tax gain from capital depreciation even decreases because of the increase in the default probability. Indeed, in default states, there is no capital depreciation, therefore an increase in the default probability will have more negative impact on the expected positive gain from capital depreciation. In sum, for high investment levels, the relative increase in the
bankruptcy costs overweights the relative gains from tax shields on coupon payments and capital depreciation.

If the net-wealth of the sponsor is below the maximum net-wealth, there will be two investment solutions: the lower investment and the higher investment. Table 4 gives the investment amounts when the sponsor's net-wealth is fixed exogenously as follows:

$$
\arg _{I}\{W(I)=\bar{W} \quad \text { under the financing constraint } \quad F=I-S\} .
$$

We observe that in the lower investments region, when the guarantee portion increases, the project increases its investment in order to maintain it shareholders net-wealth. Therefore if the project needs to invest more, it has to insures partially its loan in order to preserve its shareholders net-wealth. In the higher investments region, if the project needs to invest more, unless debtholders require loan guarantees, it is beneficial for the project to decrease its credit insurance and pay the high borrowing interest rates. But, unfortunately in practice above a certain risk based interest rate, debtholders will be less willing to extend their funds to the project unless there are credit insurances for the loans. For the project, it will be appropriate to finance the project with credit insurance, otherwise the project will be aborted and shareholders instead of gaining positive non optimal wealth will have nothing.

From this analysis, it is clear that there is a tradeoff between the investment level $I$ and the portion of the loan to be guaranteed $\omega$. Next, we analyze the sensibility of the investment level with respect to the other parameters: the project risk, the guarantee cost subsidy, the agency cost parameter and the maturity of the debt.

### 3.4 Project risk and investment policies

Figure 4 plots the optimal policies as function of the project's volatility. We observe that, at optimum, increasing the risk level of the project implies less investment and therefore less net-wealth to the project sponsor. Indeed, when the project risk increases, in absence of asymmetric information about the project volatility, the financing cost becomes huge and it is too costly for the project to guarantee its debt. Indeed, the relative increase in tax shields following the risk shifting is not enough to compensate for the increase in the
bankruptcy costs of eventual default. Even though the investment level is decreasing, therefore less demand for debt, the borrowing interest rate is increasing. For high risky projects, the marginal tax shield is huge, however, because of the low level of debt, the size of the amount of tax shields is lower also. The project will need huge amounts of credit insurances in order to enhance its credit, which will be too costly.

### 3.5 Subsidy and investment policies

As argued by Esty (2004b) and Chen (2005), many project financed investments involve huge amounts of money and sometimes will require the intervention of the host government in the form of loan subsidies. Some of the main government institutions providing credit insurance are the Export Credit Agencies (ECAs) (Example, U.S. Export-Import Bank, China Export \& Credit Insurance Corporation, Export Credits Guarantee Department (ECGD) of the UK, COFACE France, Export Development Canada (EDC)), the World Bank Multilateral Investment Guarantee Agency (MIGA), among others. The government will intervene in order for the project to go ahead, otherwise some net present value projects will be abandoned do to the lack of financing support, especially in developing countries. The rationale for the government intervention is because of the future tax revenues and social benefits. This is in line with the study by Lai and Soumaré (2005), who analyze project financed investments with government financial guarantees.

We capture the presence of subsidy in the expression of the insurance premium paid by the project as follows: $P=(1-\varepsilon) G$, where $\varepsilon \geq 0$ and $G$ is the expected guarantee amount given by equation (9). In this expression, the parameter $\varepsilon$ captures the presence of subsidy. A value of $\varepsilon=0$ represents a no-subsidy context, and a value of $\varepsilon=1$ materializes a full subsidy, i.e., the project does not pay any insurance premium.

Figure 5 plots the optimal policies as function of the subsidy percentage $\varepsilon$. As depicted in the graphs, the project invests more when it has a subsidy on its guarantee premium and the investment increases with the subsidy percentage, which in turn affects the net-wealth positively. Since the investment increases and the sponsor capital contribution stays at the same level, the demand for external financing increases which increases the borrowing interest rate. The presence of subsidy gives more incentives to
the project to invest, while without subsidy, the project can forgo some net present value projects which will result in forgone taxes and social benefits for the government.

### 3.6 Agency costs and investment policies

Esty (2003) argues that "project finance creates value by reducing the agency costs associated with large, transaction-specific assets, and by reducing the opportunity cost of underinvestment due to leverage and incremental distress costs." To capture the effect of agency costs, we use a shortcut through the premium paid for the guarantee, by considering a payment schedule of $P=(1+\varepsilon) G$ with $\varepsilon \geq 0$ and $G$ the expected value of the guarantee payment by the insurer given by equation (9). We interpret $\varepsilon G$ as a proxy for the costs induced by the agency conflicts between the players. The presence of agency conflicts is costly for the project and is assumed to be proportional to the amount of the value of the guarantee. In that case, the project pays more than the fair price of the guarantee.

Figure 6 plots the optimal policies as function of the agency costs parameter $\varepsilon$. As depicted in the graphs, the project invests more when the agency costs are lower, which in turn affects the shareholders positively. This is consistent with project finance involving huge investment amounts. When the agency costs are high, it is optimal to under invest in order to maximize shareholders net-wealth. Nonetheless, the optimal net-wealth under severe agency conflict is lower than the one with no-agency conflict. Indeed, the manager will tend to under invest and destroy shareholders value in order to avoid monitoring from debtholders or guarantee providers. Since the investment decreases and the sponsor capital contribution stays at the same level, the demand for external financing decreases which decreases the borrowing interest rate.

### 3.7 Debt maturity and investment policies

In this section, we study the relationship between the project investment and the maturity of its debt. Although several theoretical works have investigated the maturity structure of project debt and its impact on project capital structure (e.g., Diamond (1991), Flannery (1986), Myers (1977) among others), few have been devoted to the
study of the relationship between project debt maturity and its investment incentives.
Figure 7 plots the optimal policies as function of the debt maturity. At the optimum, we observe a U shape for the investment, with over-investment for low and high maturities and under-investment for intermediate maturities. The investment decreases with the debt maturity over the maturity range $[0,5]$ years and increases for maturities over 5 years. The expected guarantee amount provided and the borrowing interest rate are increasing with the debt maturity. The net-wealth to project sponsor and the debt ratio have a U shape with high values for low and high maturities and low values for intermediate maturities. Intuitively, for low maturities, the bankruptcy cost is very low almost null and for high maturities, the project is able to extract more tax benefits through coupon payments which motivates the over-investment and therefore increases the value to shareholders.

To gauge for the effects of the investment size and the debt maturity on the project investment policies, we fix the sponsor's net-wealth at a fix amount (for example at 700) and look for the investment amounts needed to maintain this net-wealth level. Since it is not the optimal net-wealth, two investment amounts can be found as shown in the first graph of Figure 3: low and high investment levels. We then divide the investment into two regions: investment levels lower than the optimal investment $I^{*}$ and investment levels higher than the optimal investment $I^{*}$.

Figure 8 plots the investment incentives as function of the debt maturity for the two investment regions. In the first region (low investments region), we observe that the investment level increases for debt maturities over the range $[0,7]$ years and for debt maturities beyond 7 years, the investment level decreases. However, despite the non monotonicity of the investment level, the borrowing interest rate increases as the debt maturity increases. In the second region, instead, the investment level decreases with the debt maturity over the short term, and then increases for longer maturities.

The empirical support for these findings is the recent work by Aivazian, Ge and Qiu (2005) who test the relationship between debt maturity and project investment. They find that longer maturity debt is associated with less investment for projects with high growth opportunities. In contrast, debt maturity is not significantly related to investment
for projects with low growth opportunities. This finding is consistent with our model implications.

## 4 Conclusion

This paper proposes a framework to study the investment incentives of a value maximizing project company in the presence of credit insurance. Our work has several policy implications for structuring project financed investments. Credit insurances allow the project to have access to more funding at lower costs, which increases the project debt capacity, especially for large investment projects as it is the case in project financed investments. However, counter to basic intuition, when the guarantee portion of the total debt increases, the project tends to under invest in order to maximize its shareholders net-wealth. We find that with credit insurances the project can be inclined to invest less and more credit insurances exacerbate the under-investment problem. We also discuss the effect of risk shifting in project finance. Moreover, the presence of subsidy on the insurance premium gives more incentives to the project to invest, otherwise, some net present value projects will be forgone. We also find support for the argument of Esty (2003) that "project finance creates value by reducing the agency costs associated with large, transaction-specific assets, and by reducing the opportunity cost of underinvestment due to leverage and incremental distress costs." Finally, we study the relationship between the project investment incentives and its debt maturity.

## Appendix: Optimization equations

The maximization equations are:

$$
\begin{aligned}
\max _{I, \omega}[W(I, \omega)= & V(I)-I+\tau_{c} \frac{I}{r+h}(1-\Psi(T, h)) \\
& +\tau_{c} \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right] \\
& \left.-\alpha F e^{-g T} \Psi(T,-g)-\left(1-\tau_{c}\right) P\right]
\end{aligned}
$$

under the financing constraint

$$
F=I-S
$$

The other equations are

$$
\begin{gathered}
c=r+r \frac{(1-\omega) \Psi\left(t_{-}, 0\right)-(1-\alpha) e^{-g T} \Psi\left(t_{-},-g\right)}{1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)}, \\
t_{-}=\min \left(\max \left(\frac{1}{g} \ln \left(\frac{1-\omega}{1-\alpha}\right)+T, 0\right), T\right), \\
P= \begin{cases}G, & \text { fairly priced, } \\
(1-\varepsilon) G, & \text { subsidized insurance, } \\
(1+\varepsilon) G, & \text { accounted for agency costs, }\end{cases} \\
G=\omega F \Psi\left(t_{-}, 0\right)+F\left[\Psi(T, 0)-\Psi\left(t_{-}, 0\right)\right] \\
\quad-(1-\alpha) F e^{-g T}\left[\Psi(T,-g)-\Psi\left(t_{-},-g\right)\right] \\
\Phi(t)=N\left(X_{1}(t)\right)+\left(\frac{V(I)}{F e^{-g T}}\right)^{-2 a_{1}} N\left(X_{2}(t)\right), \\
\Psi(t, x)=\left(\frac{V(I)}{F e^{-g T}}\right)^{-a_{1}+a_{2}} N\left(X_{3}(t)\right)+\left(\frac{V(I)}{F e^{-g T}}\right)^{-a_{1}-a_{2}} N\left(X_{4}(t)\right),
\end{gathered}
$$

where

$$
\begin{gathered}
X_{1}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)-a_{1} \sigma^{2} t}{\sigma \sqrt{t}},
\end{gathered} \quad X_{2}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)+a_{1} \sigma^{2} t}{\sigma \sqrt{t}}, ~ \begin{gathered}
X_{3}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)-a_{2} \sigma^{2} t}{\sigma \sqrt{t}}, \\
X_{4}(t)=\frac{-\ln \left(V(I) / F e^{-g T}\right)+a_{2} \sigma^{2} t}{\sigma \sqrt{t}} \\
a_{1}=\frac{\mu-\lambda \sigma-q-g-\sigma^{2} / 2}{\sigma^{2}}, \\
q V(I)=\delta V(I)+\left(1-\tau_{c}\right) c F .
\end{gathered}
$$

## References

[1] Adam, T. R. and C. S. Fernando, 2005, Hedging, Speculation, and Shareholder Value, Forthcoming Journal of Financial Economics.
[2] Aivazian, V. A., Y. Ge, and J. Qiu, 2005, Debt Maturity Structure and Firm Investment, Financial Management, 34, 107-1119.
[3] Banks, E., 2004, Alternative Risk Transfer: Integrated Risk Management Through Insurance, Reinsurance and Capital Markets, Wiley.
[4] Bartram, S. M., G. W. Brown, and F. R. Fehle, 2004, International Evidence on Financial Derivative Usage, SSRN: http://ssrn.com/abstract=471245.
[5] Black, F. and J. C. Cox, 1976, Valuing Corporate Securities: Some Effects of Bond Indenture Provisions, Journal of Finance, 31, 351-367.
[6] Brockman, P. and H. J. Turtle, 2003, A Barrier Option Framework for Corporate Security Valuation, Journal of Financial Economics, 67, 511-529.
[7] Chen, A. H., 2005, It's Time to Correct the Shortfalls of BOT and PPP in Infrastructure Project Finance - and Here is How, Working Paper, Southern Methodist University.
[8] Chen, A. H., Hung, M.-W., and S. C. Mazumdar, 1994, Valuation of Parent Guarantees of Subsidiary Debt: Ownership, Risk and Leverage Implications, PacificBasin Finance Journal, 2, 391-404.
[9] Chen, A. H. and S. C. Mazumdar, 1996, Loan Guarantees and the Optimal Financing and Investment Policies of Multinational Corporations, Research in Finance, 2, 81-104.
[10] Dailami, M. and D. Leipziger, 1998, Infrastructure Project Finance and Capital Flows: A New Perspective, World Development, 26, 1283-1298.
[11] Diamond, D. W., 1991, Debt Maturity Structure and Liquidity Risk, The Quarterly Journal of Economics, 106, 709-737.
[12] Ehrhardt, D. and T. Irwin, 2004, Avoiding Customer and Taxpayer Bailouts in Private Infrastructure Projets: Policy Toward Leverage, Risk Allocation and Bankruptcy, World Bank Policy Research Paper 3274 (April).
[13] Esty, B. C., 2004a, An Overview of Project Finance, SSRN.
[14] Esty, B. C., 2004b, Why Study Large Projects? An Introduction to Research on Project Finance, European Financial Management, 10, 213-224.
[15] Esty, B. C., 2003, The Economic Motivations for Using Project Finance, Working Paper, HBS.
[16] Farrell, L. M., 2003, Principal-Agency Risk in Project Finance, International Journal of Project Management, 21, 547-561.
[17] Flannery, M. J., 1986, Asymmetric Information and Risky Debt Maturity Choice, Journal of Finance, 41, 19-37.
[18] Froot, K. A., D. S. Scharfstein, and J. C. Stein, 1993, Risk Management: Coordinating Corporate Investment and Financing Policies, Journal of Finance, 48, 1629-1658.
[19] GARVEN, J. R. and R. D. MacMinn, 1993, The Underinvestment Problem, Bond Covenants and Insurance, Journal of Risk and Insurance, 60, 635-646.
[20] Gendron, M., V.S. Lai and I. Soumaré, 2006, Effects of Maturity Choices on Loan Guarantee Portfolios. Forthcoming in The Journal of Risk Finance.
[21] Harrison, J. M., 1990, Brownian Motion and Stochastic Flow Systems, Krieger Publishing Company.
[22] Hull, J., 2005, Options, Futures and Other Derivatives (6th edition), Pearson Prentice Hall.
[23] Jin, Y. and P. Jorion, 2006, Firm Value and Hedging: Evidence from US Oil and Gas Producers, Journal of Finance, 61, 893-919.
[24] Ju, N. and H. Ou-Yang, 2005, Capital Structure, Debt Maturity, and Stochastic Interest Rates, Forthcoming Journal of Business.
[25] Ju, N., R. Parrino, A. M. Poteshman, and M. S. Weisbach, 2005, Horses and Rabbits? Trade-Off Theory and Optimal Capital Structure, Journal of Financial And Quantitative Analysis, 40, 259-281.
[26] Kleimeier, S. and W. L. Megginson, 2001, An Empirical Analysis of Limited Recourse Project Finance, Working Paper, The University of Oklahoma.
[27] Lai, V. S., 1992, An Analysis of Private Loan Guarantees, Journal of Financial Services Research, 6, 223-248.
[28] Lai, V. S. and I. Soumaré, 2005, Investment Incentives in Project Finance in the Presence of Partial Loan Guarantees, Research in Finance, 22, 161-186.
[29] Leland, H. E., 1998, Agency Costs, Optimal Risk Management, and Capital Structure, Journal of Finance, 53, 1213-1243.
[30] Leland, H. E. and K. B. Toft, 1996, Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, Journal of Finance, 51, 987-1019.
[31] Mayers, D. and C. Smith, 1987, Corporate Insurance and the Underinvestment Problem, Journal of Risk and Insurance, 54, 45-54.
[32] McDonald, R. and D. Siegel, 1984, Option Pricing When the Underlying Asset Earns a Below-Equilibrium rate of return: A Note, Journal of Finance, 39, 261265.
[33] McDonald, R. and D. Siegel, 1985, Investment and the valuation of Firms When There is an Option to Shut Down, International Economic Review, 26, 331-349.
[34] Merton, R. C, 1974, On The Pricing of Corporate Debt: The Risk Structure of Interest Rates, Journal of Finance, 29, 449-470.
[35] Merton, R. C., 1977, An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory, Journal of Banking and Finance, 1, 3-11.
[36] Merton, R. C. and Z. Bodie, 1992, On the Management of Financial Guarantees, Financial Management, 21 (Winter), 87-109.
[37] Merton, R. C. and Z. Bodie, 1999, Finance, Prentice Hall.
[38] Morellec, E. and C. W., Smith, Jr., 2005, Agency Conflicts and Risk Management, Working Paper, University of Lausanne and University of Rochester.
[39] Myers, S. C., 1977, Determinants of Corporate Borrowing, Journal of Financial Economics, 5, 147-175.
[40] Parrino, R., A. M. Poteshman, and M. S. Weisbach, 2005, Measuring Investment Distorsions when Risk-Averse Managers decide Whether to Undertake Risky Projects, Financial Management (Spring), 21-60.
[41] Schwartz, E. S. and M. Moon, 2000, Rational Pricing of Internet Companies, Financial Analysts Journal (May/June), 62-75.
[42] Shah, S. and A. V. Thakor, 1987, Optimal Capital Structure and Project Financing, Journal of Economic Theory, 42, 209-243.
[43] Smith, C. W. and R. M. Stulz, 1985, The Determinants of Firms' Hedging Policies, Journal of Financial and Quantitative Analysis, 20, 391-405.
[44] Titman, S. and S. Tsyplakov, 2002, A Dynamic Model of Optimal Capital Structure, Working Paper, University of South Carolina.
[45] Zhdanov, A. and E. Lyandres, 2003, Underinvestment or Overinvestment? The Effect of Debt Maturity on Investment, Working Paper, University of Rochester.

## Figure 1: Debt Capacity and Borrowing Interest Rate as Function of $\omega$.

These graphs plot respectively (from left to right) the debt capacity and the borrowing interest rate as function of the portion $\omega$ of the guarantee. They are obtained from the following equation:

$$
\begin{aligned}
F= & \frac{c F}{r}\left[1-(1-\Phi(T)) e^{-r T}-\Psi(T, 0)\right]+F(1-\Phi(T)) e^{-r T} \\
& \omega F \Psi\left(t_{-}, 0\right)+F\left[\Psi(T, 0)-\Psi\left(t_{-}, 0\right)\right]+(1-\alpha) F e^{-g T} \Psi\left(t_{-},-g\right) .
\end{aligned}
$$

For the graph in the left hand side, the debt capacity $(D=F)$ is plotted for three borrowing interest rates. For the right hand side graph, the borrowing interest rate $(c)$ is plotted for three debt levels. The investment level is normalized to $I=100$. The other parameters values are $V(I)=1.5 I, \tau_{c}=0.35, r=5 \%, \alpha=0.30, g=3 \%, \mu=0.12, \sigma=0.40, \lambda=(\mu-r) / \sigma$, $\delta=0.5 \%, T=10$.



## Table 1: Description of the optimization variables.

This table describes the exogenous variables used to perform our optimization exercise and the endogenous variables generated as output.
(a) Exogenous variables

| $r$ | Risk-free interest rate |
| :--- | :--- |
| $\mu$ | Project asset returns growth rate |
| $g$ | Growth rate of the barrier |
| $\delta$ | Dividend payout rate |
| $\tau_{c}$ | Corporate tax rate |
| $h$ | Capital depreciation rate |
| $\alpha$ | Bankruptcy cost coefficient |
| $\varepsilon$ | Subsidy of insurance or agency cost coefficient |
| $\lambda$ | Market price of risk |
| $T$ | Project debt maturity |
| $\sigma$ | Volatility of the project asset returns |

(b) Endogenous variables

| $I$ | Project investment amount |
| :--- | :--- |
| $c$ | Coupon rate/project borrowing interest rate |
| $F$ | Debt value/face value |
| $\omega$ | Guarantee portion of the total debt |

Table 2: Baseline parameters values.
This table summarizes the baseline parameters values. These values are to be used in our optimization program unless otherwise stated.

| $r$ | Risk-free interest rate | 0.05 |
| :--- | :--- | :---: |
| $\mu$ | Project asset returns growth rate | 0.12 |
| $\sigma$ | Volatility of the project asset returns | 0.40 |
| $\lambda$ | Market price of risk | 0.175 |
| $g$ | Growth rate of the barrier | 0.015 |
| $\delta$ | Dividend payout rate | 0.005 |
| $\tau_{c}$ | Corporate tax rate | 0.35 |
| $h$ | Capital depreciation rate | 0.02 |
| $\alpha$ | Bankruptcy cost coefficient | 0.20 |
| $\varepsilon$ | Subsidy of insurance or agency cost coefficient | 0.00 |
| $T$ | Project debt maturity | 10 |
| $\gamma$ | Coefficient of the production technology | 0.80 |
| $\theta$ | Initial output price | 10 |

## Figure 2: Partial Loan Guarantee and Credit Enhancement.

These graphs plot respectively (from left to right, and from top to bottom) the ratio of expected guarantee over the total debt $(G / F)$, the borrowing interest rate $(c)$, the debt ratio (computed as the ratio of the total debt over the sum of the total debt plus the total equity: $F /(E+F)$ ), the tax shields amount, the bankruptcy costs, and the net-wealth to the sponsor as function of the sponsor percentage investment $(S / I)$ for three levels of loan guarantee portion $(\omega=0$, $20 \%, 50 \%$ ). For example, when the sponsor finances $S / I=20 \%$ of the project investment, the remaining $80 \%$ of the amount is financed by debt $F$. The debt is issued at par. The baseline parameters values are $I=5000, \theta=10, \gamma=0.8, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\varepsilon=0, \mu=0.12, \lambda=0.175, \delta=0.5 \%, \sigma=0.40, T=10$.


Table 3: Optimal policies and project maximization.
This table shows the values of the optimal policies for different investment amounts invested by the sponsor. The following optimization is performed: $\max W(I)$ under the financing constraint $F=I-S$ for given levels of $\omega$, which implies $I^{*}$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=0.05, \alpha=0.20, \sigma=0.40$, $g=0.015, \delta=0.005, \varepsilon=0 \%, T=10, \mu=0.12, \lambda=0.175$.
(a) Sponsor investment: $S=100$

| Partial Guarantee Percent $(\omega)$ | 0 | 0.1 | 0.2 | 0.3 |
| :--- | ---: | ---: | ---: | ---: |
| Optimal Investment $\left(I^{*}\right)$ | 4376.2 | 3344.6 | 2585 | 2082.8 |
| Debt Value $(F)$ | 4276.2 | 3244.6 | 2485 | 1982.8 |
| Expected Guarantee $(G)$ | 0 | 263.78 | 376.69 | 402.02 |
| Borrowing Interest Rate $(c)$ | 0.18673 | 0.11119 | 0.071687 | 0.050313 |
| Debt Ratio: $\frac{F}{W+S+F}$ | 0.74046 | 0.72804 | 0.71101 | 0.69115 |
| Sponsor Net-Wealth $(W)$ | 1398.8 | 1112 | 910.01 | 786.03 |
| Tax shields | 528.08 | 320.02 | 190.65 | 120.43 |
| Capital depreciation | 589.3 | 486.44 | 394.25 | 326.81 |
| Bankruptcy costs | 660.9 | 467.71 | 335.49 | 253.6 |

(b) Sponsor investment: $S=500$

| Partial Guarantee Percent $(\omega)$ | 0 | 0.1 | 0.2 | 0.3 |
| :--- | ---: | ---: | ---: | ---: |
| Optimal Investment $\left(I^{*}\right)$ | 4748.7 | 3631.6 | 2839.2 | 2340.4 |
| $\quad$ Debt Value $(F)$ | 4248.7 | 3131.6 | 2339.2 | 1840.4 |
| Expected Guarantee $(G)$ | 0 | 235.39 | 319.33 | 320.53 |
| Borrowing Interest Rate $(c)$ | 0.14912 | 0.094056 | 0.064622 | 0.050095 |
| Debt Ratio: $\frac{F}{W+S+F}$ | 0.67611 | 0.63925 | 0.59568 | 0.55274 |
| Sponsor Net-Wealth $(W)$ | 1535.3 | 1267.3 | 1087.8 | 989.16 |
| Tax shields | 538.84 | 327.07 | 201.3 | 138.14 |
| Capital depreciation | 691.34 | 563.68 | 457.58 | 384.92 |
| $\quad$ Bankruptcy costs | 623.85 | 420.2 | 286.8 | 208.78 |

## Figure 3: Investment regions.

This graph plots the evolution of the sponsor's net wealth for different levels of investment. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\delta=0.5 \%, \varepsilon=0, \sigma=0.40, \omega=0 \%, \mu=0.12, \lambda=0.175$.



Table 4: Investment incentives for given level of sponsor's net wealth.
This table shows the values of the partial guarantee percentage, optimal investment, optimal shareholders' value, borrowing interest rate, the debt value, the total expected guarantee, the sponsor's total investment as portion of the total investment, and the debt ratio for different investment amounts invested by the sponsor. The sponsor's net wealth is fixed at a given level: $W(I, \omega)=\bar{W}=700$ under the financing constraint $F=I-S$, which implies $I$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=0.05$, $\alpha=0.20, \sigma=0.40, g=0.015, \delta=0.005, \varepsilon=0 \%, T=10, \mu=0.12, \lambda=0.175$.
(a) Sponsor investment: $S=100$

|  | Low investment region |  |  |  | High investment region |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\omega$ | 0 | 0.1 | 0.2 | 0.3 | 0 | 0.1 | 0.2 | 0.3 |
| Investment | 763 | 846 | 970 | 1133 | 8660 | 6697 | 4663 | 3221 |
| Debt | 663 | 746 | 870 | 1033 | 8560 | 6597 | 4563 | 3121 |
| Guarantee | 0 | 45 | 107 | 180 | 0 | 601 | 763 | 691 |
| Interest rate | 0.0844 | 0.0729 | 0.0605 | 0.0501 | 0.7947 | 0.1779 | 0.0855 | 0.0506 |
| Debt Ratio | 0.45 | 0.48 | 0.52 | 0.56 | 0.91 | 0.89 | 0.85 | 0.80 |
| Net wealth | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 |
| $\quad$ Tax shields | 92 | 87 | 82 | 77 | 962 | 582 | 302 | 159 |
| Depreciation | 127 | 141 | 161 | 186 | 668 | 783 | 634 | 477 |
| $\quad$ Bankruptcy | 71 | 81 | 97 | 117 | 1453 | 1053 | 673 | 427 |

(b) Sponsor investment: $S=500$

|  | Low investment region |  |  |  | High investment region |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\omega$ | 0 | 0.1 | 0.2 | 0.3 | 0 | 0.1 | 0.2 | 0.3 |
| Investment | 748 | 756 | 766 | 771 | 9895 | 7713 | 5800 | 4574 |
| Debt | 248 | 256 | 266 | 271 | 9395 | 7213 | 5300 | 4074 |
| Guarantee | 0 | 5 | 11 | 14 | 0 | 646 | 871 | 892 |
| Interest rate | 0.05685 | 0.0542 | 0.0514 | 0.0500 | 0.5539 | 0.1603 | 0.0821 | 0.0506 |
| Debt Ratio | 0.17 | 0.18 | 0.18 | 0.18 | 0.89 | 0.86 | 0.82 | 0.77 |
| Net wealth | 700 | 700 | 700 | 700 | 700 | 700 | 700 | 700 |
| Tax shields | 35 | 35 | 34 | 34 | 1071 | 649 | 362 | 213 |
| Depreciation | 128 | 129 | 131 | 132 | 892 | 948 | 811 | 684 |
| $\quad$ Bankruptcy | 9 | 10 | 11 | 11 | 1583 | 1134 | 770 | 553 |

## Figure 4: Optimal Policies as Function of Project Risk.

These graphs plot respectively (from left to right, and from top to bottom) the expected guarantee amount $(G)$, the borrowing interest rate $(c)$, the optimal investment $(I)$, and the net-wealth to the project sponsor $(W)$. The following optimization is performed max $W(I)$ under the financing constraint $F=I-S$ for different values of $\sigma$, which implies $I^{*}$. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\delta=0.5 \%, \varepsilon=0, \omega=20 \%, \mu=0.12, \lambda=0.175$.





Figure 5: Optimal Policies as Function of the Subsidy Percentage.
These graphs plot respectively (from left to right, and from top to bottom) the expected guarantee amount $(G)$, the borrowing interest rate $(c)$, the optimal investment $(I)$, and the net-wealth to the project sponsor $(W)$. The following optimization is performed $\max W(I)$ under the financing constraint $F=I-S$ for different values of $\varepsilon$, which implies $I^{*}$. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\delta=0.5 \%, \sigma=0.40, \omega=20 \%, T=10, \mu=0.12, \lambda=0.175$.


Figure 6: Optimal Policies as Function of the Agency Cost Percentage.
These graphs plot respectively (from left to right, and from top to bottom) the expected guarantee amount $(G)$, the borrowing interest rate $(c)$, the optimal investment $(I)$, and the net-wealth to the project sponsor $(W)$. The following optimization is performed $\max W(I)$ under the financing constraint $F=I-S$ for different values of $\varepsilon$, which implies $I^{*}$. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\delta=0.5 \%, \sigma=0.40, \omega=20 \%, T=10, \mu=0.12, \lambda=0.175$.


## Figure 7: Optimal Policies as Function of Debt Maturity.

These graphs plot respectively (from left to right, and from top to bottom) the expected guarantee amount $(G)$, the borrowing interest rate $(c)$, the optimal investment $(I)$, and the net-wealth to the project sponsor $(W)$. The following optimization is performed $\max W(I)$ under the financing constraint $F=I-S$ for different maturities $T$, which implies $I^{*}$. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%, \alpha=0.20, g=1.5 \%$, $\delta=0.5 \%, \varepsilon=0, \sigma=0.40, \omega=20 \%, \mu=0.12, \lambda=0.175$.


Figure 8: Investment incentives and debt maturity for given level of sponsor net wealth.

These graphs plot respectively (from left to right, and from top to bottom) the investment ( $I$ ), the guarantee $(G)$, the the borrowing interest rate $(c)$ and the debt ratio $(F /(W+S+F))$. The sponsor's net wealth is fixed at a given level: $W(I)=\bar{W}=1000$ under the financing constraint $F=I-S$, which implies $I$. The sponsor's contributed capital is fixed at $S=500$. The debt is issued at par. The baseline parameters values are $\gamma=0.80, \theta=10, h=2, \tau_{c}=0.35, r=5 \%$, $\alpha=0.20, g=1.5 \%, \delta=0.5 \%, \varepsilon=0, \sigma=0.40, \omega=20 \%, \mu=0.12, \lambda=0.175, \mathrm{~T}=10$.

## Lower investments region



## Higher investments region







[^0]:    *We acknowledge the financial support from the Institut de Finance Mathématique of Montreal (IFM2) and the Fonds Québecois de la Recherche sur la Société et la Culture (FQRSC). All errors are the authors' sole responsibility.
    ${ }^{\dagger}$ Laval University, Faculty of Business Administration, Quebec, QC., Canada G1K 7P4; Tel: 1-418-656-2131; Fax: 1-418-656-2624; Email: vanson.lai@fsa.ulaval.ca \& issouf.soumare@fsa.ulaval.ca.

[^1]:    ${ }^{1}$ Adam and Fernando (2005) find empirical support for the value creation with selective hedging. Bartram, Brown and Fehle (2004) provides the international evidence on the use of financial derivatives by firms around the world.

[^2]:    ${ }^{2}$ Note that, $\omega$ is chosen from the viewpoint of the project company, however, the insurer can refuse to insure a certain level of $\omega$, hence a rationing from the insurer. This feature is not modelled explicitly in our model. We assume that, the project can insure $\omega$ portion of its debt as long as it pays for the insurance premium.

[^3]:    ${ }^{3}$ Instead of using the portion $\omega$ guaranteed by the insurer, we could also assume that the insurer guarantees fixed amount $H$, then the expected guarantee amount would be $G=\int_{0}^{T} e^{-r t} \min (H, F-$ $\left.(1-\alpha) F e^{g(t-T)}\right) f(t) d t$, and $t_{-}=\min \left(\max \left(\frac{1}{g} \ln \left(\frac{F-H}{F(1-\alpha)}\right)+T, 0\right), T\right)$. Thus the value of $G$ becomes $G=H \Psi\left(t_{-}, 0\right)+F\left[\Psi(T, 0)-\Psi\left(t_{-}, 0\right)\right]-(1-\alpha) F e^{-g T}\left[\Psi(T,-g)-\Psi\left(t_{-},-g\right)\right]$.

