# Trading the Forward Bias: Are there Limits to Speculation? \*

Markus Hochradl<sup>†</sup> Christian Wagner<sup>‡</sup>

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#### Abstract

In this paper we offer new approaches to investigate whether deviations from uncovered interest rate parity allow for economically significant excess returns. We use downside risk (and hence capital charge) constrained deposit portfolios as well as strategies with currency option combinations to exploit the forward bias. The results indicate that the puzzle does not only exist statistically but that betting against uncovered interest rate parity is profitable even after adjusting for transaction costs. Overall, our empirical findings lead us to the conclusion that the limits to speculation hypothesis, despite its intuitive appeal, should be handled with care.

**Keywords:** forward bias, trading strategies, limits to speculation, currency options, heuristic optimization

JEL classification: F31, G11

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<sup>&</sup>lt;sup>†</sup>Visiting scholar at the Department of Finance at the NYU Leonard N. Stern School of Business. E-mail: markus.hochradl@stern.nyu.edu

<sup>&</sup>lt;sup>‡</sup>Vienna Graduate School of Finance (VGSF). E-mail: christian.wagner@vgsf.ac.at.

## I Introduction

Following the influential work by Bilson (1981) and Fama (1984), an enormous amount of literature on testing whether the forward rate is an unbiased predictor of the future spot rate accumulated over the last years. Excellent surveys are Hodrick (1987), Engel (1996), and Sarno and Taylor (2002). Although results vary depending on how exchange rates are modeled, the common finding in the overwhelming majority of past research is that the forward rate is not an unbiased predictor of the future spot rate. This forward bias implies the apparent predictability of excess returns over uncovered interest rate parity (UIP).

Tests of the UIP are frequently based on the 'Fama regression' which relates the change in the spot rate to a constant and the lagged forward premium. The null hypothesis is that the constant should equal zero and the slope coefficient unity, i.e. that the movement in the exchange rate compensates for the (interest rate) differential. However, the common finding is that the slope coefficient is less than unity and often negative, indicating not only that UIP does not hold but also that the higher interest rate currency tends to appreciate rather than depreciate.

Motivated by the limited success of statistical or economical explanations, Lyons (2001) proposes an approach based on limited trader participation. The limits to speculation hypothesis (LSH) postulates that deviations from UIP (might) occur and persist because nobody is willing to trade on these deviations since other investment opportunities yield higher Sharpe ratios. Recent papers provide evidence consistent with the LSH. Villanueva (2005) argues that exchange rate undershooting is an empirical implication of the LSH. Inspired by the LSH, Sarno, Valente, and Leon (2005) and Baillie and Kiliç (2005) model exchange rates in a smooth transition regression (STR) framework and both report strong evidence for such non-linearity in the relationship between spot and forward rates. Moreover, the results of Sarno, Valente, and Leon (2005) indicate that a simple single currency strategy would not be of interest for traders. In the present paper we investigate whether sophisticated trading strategies have the potential to attract speculative capital.

While most papers investigating the forward bias are more concerned with modelling rather than how deviations from UIP can be exploited, papers dealing with trading strategies in the foreign exchange market (not directly related to UIP) mainly focus on neural network, genetic programming, or simple technical trading rule approaches. See e.g. Neely, Weller, and Dittmar (1997), Okunev and White (2003), and Olson (2004). We consider trading strategies explicitly aimed at exploiting deviations from UIP, therefore the differential serves as a natural starting point. In particular, we develop two classes of strategies.

For the first we take the perspective of a levered proprietary trader who can choose to allocate funds

to a bias exploiting strategy or some other investment opportunity. Recall, that the LSH postulates that speculative capital can only be attracted by strategies with Sharpe ratios higher than a certain threshold. However, we stress that for the comparison of investment opportunities downside risk should be taken into account as well, especially, in the light of the capital requirements financial institutions have to fulfill for market risks of positions in their trading books. Thus, we analyze downside risk (and hence capital charge) constrained strategies based on taking positions in deposits in different currencies. We apply a heuristic portfolio optimization algorithm in which the trader chooses a certain level of volatility and determines expected returns either through the differentials or on the basis of forecasts for the future spot rate generated from a vector error correction model (VECM).

Furthermore, since currency options are priced through a replication argument including the (biased) forward rate, we also analyze strategies using combinations of plain vanilla and digital options. Since pure options strategies result in extreme payoffs we take the perspective of active portfolio managers seeking to outperform their benchmarks by enhancing their investments with the optionsbased strategy.

The results of our trading strategies indicate that the puzzle does not only exist statistically but that betting against uncovered interest rate parity yields economically significant excess returns even after adjusting for transaction costs. Overall, our empirical findings lead us to the conclusion that the limits to speculation hypothesis, despite its intuitive appeal, should be handled with care. The remainder of the paper is organized as follows. Section II reviews the relevant literature on the forward puzzle. Section III describes the development of two approaches aimed at exploiting the forward bias. Section IV contains the empirical results and section V concludes. Appendices A and B provide details on the VECM specification and the heuristic optimization algorithm applied respectively.

## II Uncovered Interest Rate Parity, Nonlinear Deviations, and Limits to Speculation

Uncovered interest rate parity (UIP) postulates that the (expected) change in the exchange rate just offsets the differential in interest rates, i.e. the (expected) change in the exchange rate equals the forward premium. The UIP condition is commonly tested (based on assuming risk-neutrality and rational expectations) using

$$\Delta s_{t+1} = \alpha + \beta (i_{t,t+1} - i_{t,t+1}^*) + \varepsilon_{t+1}$$
 or equivalently (1)

 $\Delta s_{t+1} = \alpha + \beta (f_{t,t+1} - s_t) + \varepsilon_{t+1}$ 

where  $s_t$  denotes the natural logarithm of the spot exchange rate,  $f_{t,t+1}$  the log forward exchange rate from t to t+1, and  $i_{t,t+1}$  and  $i_{t,t+1}^*$  denote the domestic respectively foreign interest rate. The null hypothesis that UIP holds is given by  $\alpha = 0$  and  $\beta = 1$ . However, the common finding is that the slope coefficient is less than unity, indicating that the higher interest rate currency tends to not depreciate as much as predicted by UIP, and often negative, indicating not only that UIP does not hold but also that the higher interest rate currency tends to appreciate rather than depreciate; excellent surveys of the literature are Hodrick (1987), Froot and Thaler (1990), Engel (1996), and Sarno and Taylor (2002). This finding implies the apparent predictability of excess returns over UIP. To investigate this predictability it is convenient to reparameterize the regression given in equation (1) in terms of excess returns,  $ER_{t+1}$ ,

$$ER_{t+1} = \alpha + (\beta - 1)(f_{t,t+1} - s_t) + \varepsilon_{t+1}$$
(2)

where  $ER_{t+1} \equiv \Delta s_{t+1} - (f_{t,t+1} - s_t) \equiv s_{t+1} - f_{t,t+1}$ . If UIP holds, the excess return is zero. However, the finding that  $\beta$  is usually below unity suggests to take a long deposit in the currency with the higher interest rate financed by a short deposit in the lower interest rate currency. Equivalently, on could sell forward the foreign currency if the forward premium is positive and vice versa.

A substantial amount of research has been devoted to investigating the forward bias in the last two decades aiming at shedding some light on the puzzle. From an economic perspective the statistical rejection of the forward premium being an unbiased predictor of the future change in the spot rate might point to market inefficiency or a risk premium. So far, however, research based on (among others) learning and peso problems, see e.g. Lewis (1995), consumption-based asset pricing, see e.g. Backus, Gregory, and Telmer (1993), and term-structure models, see e.g. Backus, Foresi, and Telmer (2001), has not been able to convincingly solve the puzzle. For further references, we refer to the above cited surveys again.

The limited success of these attempts to find an explanation for the forward bias might be the consequence of research generally relying on linear frameworks despite evidence indicating the presence of nonlinearities in the relationship between the spot and forward exchange rate. Reasons for such nonlinearities include transaction costs, see e.g. Baldwin (1990), Dumas (1992), Hollifield and Uppal (1997), Sercu and Wu (2000), central bank intervention see e.g. Mark and Moh (2002), Moh (2002), and limits to speculation (see below).

In the present paper we focus on the limits to speculation hypothesis (LSH) as put forward by Lyons (2001). The basic idea is that deviations from UIP (might) persist because nobody is willing to trade on these deviations since other investment opportunities yield higher Sharpe ratios. The rationale behind the LSH relies on a central link between price adjustment and order flow and on institutional realities assumed irrelevant within more traditional approaches. Unless both of the following conditions are true, order flow will play a role in price adjustment. First, all information relevant for exchange rates is publicly known, and, second, the mapping of this information into prices is publicly known as well. The second condition is patently violated, but even the first is unlikely to hold, see e.g. Evans and Lyons (2002).

Lyons (2001) reports that financial institutions usually allocate speculative capital based on Sharpe ratios and he stresses that this empirical fact is essential to the LSH rather than a theoretical rationale for why this behavior arises. He claims that no capital would be allocated to strategies with Sharpe ratios below 0.4, the average realized Sharpe ratio for a US buy-and-hold equity investment over the last 50 years<sup>1</sup>. Referring to interviews with practitioners, Lyons (2001) reports that traders' interest would be limited for strategies yielding Sharpe ratios lower than 0.5.

Recent papers find evidence consistent with the LSH. Villanueva (2005) employs a vector error correction model (VECM) to derive the implications of over- and undershooting for the joint spot and forward rate dynamics and uses generalized impulse response analysis to test them. He stresses that undershooting is an empirical implication of the LSH. Sarno, Valente, and Leon (2005) and Baillie and Kiliç (2005) investigate some of the general predictions of the LSH in a smooth transition regression (STR) framework. Both use the Fama regression as starting-point for their analysis. While the former use the expected deviation from UIP as transition variable and an exponential transition function (ESTR) the latter use the lagged forward premium and a logistic function (LSTR). Both report strong evidence for such nonlinearity in the relationship between spot and forward rates. Moreover, the results of Sarno, Valente, and Leon (2005) indicate that a Sharpe ratio of 0.4 is a useful threshold level.

For the purpose of our paper we take a closer look at the work of Sarno, Valente, and Leon (2005). The authors provide empirical evidence that bilateral exchange rate changes are linked nonlinearly to forward premia. The model allows for a time-varying forward bias with nonlinearly mean reverting deviations from UIP. In the limits, the model has two extreme regimes: one with persistent but small deviations from UIP and one where UIP holds. Hence, UIP might not hold most of the time

<sup>&</sup>lt;sup>1</sup>This figure is also reported by Sharpe (1994).

but deviations are too small to attract speculative capital. Therefore, the authors conclude that deviations from UIP do not necessarily indicate market inefficiency but rather that the assumption of linearity used in standard literature might help to understand the puzzle. While the authors motivated their specific nonlinear model by the limits to speculation hypothesis they do not claim to provide a direct test of this specific hypothesis, but rather a test of its general predictions for the relationship between spot and forward exchange rates and can not preclude that the actual source of the nonlinearity might be different. Furthermore, they show that if the true data generating process of UIP deviations were of the nonlinear form they consider this would lead to the bias documented in the literature.

The nonlinear model considered by Sarno, Valente, and Leon (2005) is given by the smooth transition regression (STR) model in equation (3).

$$\Delta s_{t+1} = \left[\alpha_1 + \beta_1 \left(f_t^1 - s_t\right)\right] + \left[\alpha_2 + \beta_2 \left(f_t^1 - s_t\right)\right] \Phi \left[ER_{t+1}^e, \gamma\right] + \varepsilon_{t+1} \tag{3}$$

with  $\Phi$  denoting the exponential transition function given in (4) with the expected excess return,  $ER_{t+1}^e$ , being the transition variable being, and  $\gamma$  being the reversion speed.

$$\Phi\left[ER^{e}_{t+1},\gamma\right] = \left\{1 - \exp\left[-\gamma\left(ER^{e}_{t+1}\right)^{2}\right]\right\}.$$
(4)

The exponential transition function  $\Phi[\cdot]$  is symmetrically inverse-bell shaped around zero with  $\Phi[0] = 0$  and  $\lim_{x\to\pm\infty} \Phi[x] = 1$ . Hence, the model allows for a smooth transition between the upper regime ( $\Phi[\cdot] = 1$ ) and the lower regime ( $\Phi[\cdot] = 0$ ). Motivated by the LSH, the authors introduce the restrictions  $\alpha_2 = -\alpha_1$  and  $\beta_2 = 1 - \beta_1$  which imply that in the upper regime of the model in (3) UIP exactly holds while it becomes a standard linear 'Fama-regression' for the lower regime.

In the context of our paper we are interest in the implications that finding nonlinearity of this type would have for trading strategies aimed at exploiting the forward bias. As outlined above, empirical evidence on the 'Fama-regression', equations (1) and (2), suggests to invest in deposits in currencies with high interest rates by financing in low interest rate currencies; equivalently positions in the forward market could be entered. Analogously to equation (2), the nonlinear model in (3) can be rewritten in terms of excess returns

$$ER_{t+1} = \left[\alpha_1 + \left(\beta_1 - 1\right)\left(f_t^1 - s_t\right)\right] + \left[\alpha_2 + \beta_2\left(f_t^1 - s_t\right)\right]\Phi\left[ER_{t+1}^e, \gamma\right] + \varepsilon_{t+1}$$
(5)

Rearranging and combining equation (5) with the restrictions on  $\alpha_2$  and  $\beta_2$  yields

$$ER_{t+1} = \alpha_1 \left( 1 - \Phi \left[ ER_{t+1}^e, \gamma \right] \right) + \underbrace{(\beta_1 - 1)}_{\beta^*} \left( f_t^1 - s_t \right) + \underbrace{(1 - \beta_1) \Phi \left[ ER_{t+1}^e, \gamma \right]}_{\beta^{**}} \left( f_t^1 - s_t \right) + \varepsilon_{t+1}.$$
(6)

To derive trading decisions corresponding to those based on the linear model given in (2) one has to consider the forward premium,  $\beta^*$ , and  $\beta^{**}$ . Recalling that  $\Phi[\cdot]$  is bounded by zero and unity, the following is true:

- 1.  $\beta^*$  is only equal to  $\beta^{**}$  if UIP holds, in which case  $\beta^* = \beta^{**} = 0$ .
- 2. If  $\beta_1 \neq 1$ ,  $\beta^{**}$  always has the opposite sign of  $\beta^*$ , with  $\beta^* < 0$  for  $\beta_1 < 1$ .
- 3.  $|\beta^*| \ge |\beta^{**}|$ , in particular  $\beta^* = -\beta^{**}$  if  $\Phi[\cdot] = 1$  and  $\beta^* > -\beta^{**}$  if  $\Phi[\cdot] < 1$

As a consequence the overall sign of  $ER_{t+1}$  is determined by the sign of the forward premium and the sign of  $\beta^*$ . In their empirical results, Sarno, Valente, and Leon (2005) find that, in line with their priors based on the LSH,  $\beta_1$  is typically negative or at least statistically significantly less than unity. As a consequence, if trading decisions are based on the excess returns derived from the nonlinear model, one would short the foreign currency if the forward premium is positive and take a long position if the premium is negative. Hence, the implications for traders speculating for the forward bias are independent of whether they presume that exchange rate movements are governed by the linear model in (1) or the nonlinear STR-model in (3).

## **III** Trading Strategies

In the following we describe two approaches for trading rules aimed at generating excess returns from UIP deviations. As is outlined in the description of the data set in section IV, the whole empirical analysis is carried out under consideration of transaction costs through bid-ask spreads. However, for the ease of notation, we omit this detail in the equations below.

For the first approach we take the perspective of a levered proprietary trader who can choose to allocate funds to a bias exploiting strategy or some other investment. The LSH stipulates that capital can only be attracted by trading rules with Sharpe ratios higher than a certain threshold, which Lyons (2001) and Sarno, Valente, and Leon (2005) quantify with 0.4. However, in addition to the risk-adjusted performance, downside risk plays an important role from a bank's perspective (as well). Financial institutions face capital requirements for positions in their trading book which are usually determined through Value-at-Risk (VaR) calculations. Hence, we stress that when choosing how to allocate funds, not only Sharpe ratios but also downside risk should be compared in order to account for differences in the costs arising from capital requirements. We therefore consider an approach to exploit the bias through a heuristically optimized portfolio of deposits in multiple foreign currencies with the portfolio being constrained to have less or equal downside risk than the investment opportunity it is compared to. In this optimization procedure, we assume that the trader chooses a certain level of volatility and determines expected returns either through the interest rate differentials or on the basis of VECM forecasts for the future spot rate.

The second trading rule approach is based on the use of currency options, motivated by the fact that currency options are priced through a replication argument including the (biased) forward rate. Since pure options trading strategies result in extreme payoffs we take the perspective of active money managers seeking to outperform their benchmarks by enhancing their investments with the options-based strategy.

Although the optimized deposit approach is described from the perspective of a (levered) proprietary trader and the options approach from the perspective of a fund manager, both approaches could be applied (with slight modifications) by other market participants as well.

#### A Optimized Portfolios of Zero-Investment Deposit Strategies

Based on the findings of previous research, documenting the existence of the forward bias and thus indicating that an investment in the currency with a higher interest rate yields a higher total return, we use the differential to create a simple deposit strategy betting against UIP.

To start with, consider a setting with two currencies, where one currency is the home and the other is a foreign currency, denoted with j. A simple way trying to exploit the forward bias is to go long a deposit in the currency with the higher interest rate and to go short a deposit in the other currency. The return of such a zero-investment strategy is given by,

$$R_{t,T}^{j} = \mathbb{I}_{\{i_{t,T}^{j} > i_{t,T}\}} (r_{t,T}^{j} - r_{t,T}) + (1 - \mathbb{I}_{\{i_{t,T}^{j} > i_{t,T}\}}) (r_{t,T} - r_{t,T}^{j})$$
with
$$r_{t,T}^{j} = (1 + i_{t,T}^{j}) \frac{S_{T}^{j}}{S_{t}^{j}} \quad \text{and}$$

$$r_{t,T} = (1 + i_{t,T}),$$
(7)

where  $S_t$  denotes the spot exchange rate as number of home currency units per unit of foreign currency at time t,  $i_{t,T}$  and  $i_{t,T}^j$  are the domestic respectively foreign interest rate for deposits from time t to T, and  $\mathbb{I}_{\{i_{t,T}^j > i_{t,T}\}}$  denotes an indicator function that takes the value one if  $i_{t,T}^j$  is greater than  $i_{t,T}$  and is zero otherwise.

A natural thought would be that diversification across multiple currencies could improve the performance compared to limiting trading activity to a single foreign currency strategy. Although already Bilson (1981) considered a mean-variance optimization in his work on the forward bias, surprisingly little effort has been devoted to this issue in recent papers. We apply a heuristic optimization algorithm called "Threshold Accepting" to a constrained portfolio choice problem; this is an innovative approach in research related to the forward bias.

Since it is an empirical fact that financial institutions commonly use Sharpe ratios to choose and to measure the performance of their trading strategies, our starting point is the conventional meanvariance approach. However, we stress that, when comparing different investment opportunities, downside risk should be taken into account as well in order to get a fuller picture of the properties of the investment opportunities under consideration. This is especially relevant in the context of capital charges financial institutions face for market risks for items in their trading books. As described in the Amendment to the Capital Accord to incorporate market risks released by the Basel Committee on Banking Supervision (1996), banks fulfilling certain criteria are allowed to use internal models to measure their market risks. The particular type of VaR-model is not prescribed. We use the historical simulation method which samples from historical data to calculate the VaR; see e.g. Jorion (2001). Consider a setting with  $j = 1, \dots J$  foreign currencies. The time period from t-Z to t spans the window for which historical data is considered.  $\xi_t^z$  is a  $J \times 1$  vector of historical returns at observation  $z, \forall z = 1, ..., Z$ , summarized in  $\xi_t$ , which is thus a  $J \times Z$  matrix. The distribution of portfolio returns is given by  $\omega'_t \xi^z_t$ , each with probability  $\frac{1}{Z}$ , where  $\omega'_t$  is a  $1 \times J$  vector of portfolio weights at time t. The function  $Q_{[q:m]}(u^1, \ldots u^m)$  denotes the q-th largest element among  $(u^1, \ldots u^m)$ , i.e.  $Q_{[1:m]}$  denotes the minimum and  $Q_{[m:m]}$  denotes the maximum. Thus, the empirical  $\alpha$ -quantile of the portfolio return distribution and hence the VaR is given by  $VaR^{\xi_t}(\alpha) = Q_{[\alpha m:m]}(\omega'_t \xi_t).$ 

For the optimization we assume that the trader has a certain level of risk in terms of annualized volatility,  $\sigma^{trader}$ , that he wishes or is allowed to take. With respect to downside risk we assume that the trader does not want to take a downside risk higher than the VaR of some alternative investment opportunity,  $VaR_t^{benchmark}(\alpha)$ , implying not only downside risk itself but also the capital charge being constrained to that of the benchmark.

Furthermore, traders will typically allocate their funds in even proportions. Therefore, we add a restriction that makes sure that portfolio weights are allocated in 5 percent units.

Thus, the optimization problem is to find portfolio weights  $^{2}$  that maximize the expected return

 $<sup>^{2}</sup>$ Recall that the strategy described is a zero-investment strategy. The weights indicate the relative proportion of notional values allocated to the individual foreign currency strategies.

subject to the above constraints,

$$\max_{\omega_t'} E_t[\omega_t' R_{t,T}]$$

subject to

$$\omega_t' \Sigma_t \omega_t = \sigma^{trader},$$

$$VaR_t^{\xi_t}(\alpha) \le VaR_t^{benchmark}(\alpha),$$

$$\omega_t^j \in \{0, 0.05, 0.1, \dots 0.95, 1\}, \forall j = 1, \dots J,$$

$$\omega_t' \mathbf{1} = 1.$$
(8)

The nonlinear VaR-constraint poses a serious problem for conventional optimization approaches. Heuristic optimization algorithms which do not compute exact optima, but find solutions sufficiently close to the global optimum and which provably converge to the global optimum are an easy implementable way out. The heuristic optimization algorithm "Threshold Accepting" (TA) enables solving portfolio optimization problems subject to nearly arbitrary constraints and almost every utility function. It is a refined local search algorithm which is able to escape local minima by accepting solutions which are not worse than the current solution by more than a given threshold. During the course of the algorithm this threshold is successively reduced, eventually reaching the value of zero. For a comprehensive introduction to the concept of TA see Winker (2001), the application to portfolio optimization has been introduced by Dueck and Winker (1992) and has recently been applied by Gilli and Kellezi (2001). We describe the TA procedure in some detail in appendix B, however, since heuristic optimization is not the main focus of this paper we refer to the aforementioned papers for an in-depth treatment.

The optimization problem outlined above requires the specification of expected returns. The payments related to the deposits are fixed in the respective currencies, however, point estimates of the future spot rate will rarely be exact. We therefore generate two sets of simple proximate expected strategy returns. The first set of expected returns consists of the differentials for the relevant period,  $|(i_{t,T}^j - i_{t,T})|$ , i.e. we expect to earn the differential while not specifying an expectation with respect to the exchange rate. The second set of expected returns we generate builds on spot rate forecasts generated from VEC-models; since this type of model has been extensively used in past research related to the UIP, see e.g. Brenner and Kroner (1995) and Zivot (2000), we leave the details of the VECM for appendix A. Basically, we plug the VECM forecast of the future spot rate into equation (7) to calculate expected returns. However, before doing so, we assess the predictive power of VECM forecast despot rate changes by regressing the actual spot rate change on the VECM forecast over rolling five years windows. While the sizes of the resulting

coefficients vary considerably, the signs of the coefficients are fairly persistent. In order to capture this additional information we accordingly sign the VECM forecasted spot rate change in equation (7).

### **B** Options Strategies

The application of the Black and Scholes (1973) framework to exchange rates by Garman and Kohlhagen (1983) yields the following well-known pricing equation for an European call option,

$$C_{t} = F_{t,T} \exp\left[-i_{t,T}(T-t)\right] \Phi(d_{1}) - X \exp\left[-i_{t,T}(T-t)\right] \Phi(d_{2})$$
with
$$d_{1} = \frac{\ln\left(F_{t,T}/X\right) + \frac{1}{2}\sigma^{2}(T-t)}{\sigma\sqrt{(T-t)}},$$

$$d_{2} = d_{1} - \sigma\sqrt{(T-t)},$$
(9)

where  $F_{t,T}$  denotes the forward rate at time t maturing T and  $\Phi$  denotes the cumulative standard normal distribution function. From put-call-parity one gets the pricing equation for the European put option,

$$P_t = X \exp\left[-i(T-t)\right] \Phi(-d_2) - F_{t,T} \exp\left[-i(T-t)\right] \Phi(-d_1).$$
(10)

Given the pricing formulas for the European call and put, one sees that  $\frac{\partial C_t}{\partial F_{t,T}} \ge 0$  and  $\frac{\partial P_t}{\partial F_{t,T}} \le 0$ . Thus, the empirically observed deviations from UIP would result in the call being underpriced and the put being overpriced if  $i_{t,T}^j > i_{t,T}$ . The reverse is true if  $i_{t,T}^j < i_{t,T}$ . Therefore, a simple trading rule could be to hold a call when the foreign interest rate is above the domestic interest rate and hold a put if the domestic interest rate is the higher one. However, we do not consider this basic strategy as will be outlined below.

As a starting point for our options strategies, consider a European call and a European put on the same currency pair with the same maturity and set the strike prices for both equal to the forward rate. From equations (9) and (10) we see that the expected payoffs and hence prices are the same,

$$C_{t} = P_{t} = F_{t,T} \exp\left[-i_{t,T}(T-t)\right] \left[\Phi(d_{1}) - \Phi(d_{2})\right]$$
with
$$d_{1} = \frac{1}{2}\sigma\sqrt{(T-t)},$$

$$d_{2} = -\frac{1}{2}\sigma\sqrt{(T-t)}.$$
(11)

Our initial approach requires an explicit estimate for the next period bias,  $\widehat{bias_{t,T}}$ , with the bias

being defined as  $bias_{t,T} = S_T - F_{t,T}$ . If the domestic interest rate is below the foreign interest rate, evidence on the forward bias would indicate that the spot rate will not drop as much as predicted by the forward rate, i.e. one expects  $bias_{t,T} > 0$ . Having an estimate for the bias, we would expect the future spot rate only to drop until  $F_{t,T} + bias_{t,T}$ . Consider now the call and put with same properties and strikes being equal to the forward rate. From the above argumentation we would presume that the probability for the call to be in the money at maturity is higher than for the put. In fact, we expect that only a put with a strike greater than  $F_{t,T} + bias_{t,T}$  will be in the money. Hence, a trader could go long a presumingly cheap call with strike being equal to the forward rate and at the same time sell a presumingly overvalued put with a strike of  $F_{t,T} + bias_{t,T}$  at a higher price, since (depending on the quality of the bias estimate) he considers the probability of the put ending up in the money to be low. To limit the exposure one can add a (partly) offsetting short position in a forward contract. Thus, at initiation, one would collect the difference between the option premia, at maturity the outflow will range from the difference in the strike prices in the worst case up to 0 in the best case.

If the domestic interest rate is higher than the foreign interest rate, i.e. one expects  $bias_{t,T} < 0$ , one would analogously buy a put with  $X = F_{t,T}$ , sell a call with  $X = F_{t,T} + \widehat{bias_{t,T}}$ , and enter a forward purchase. Again the difference between the option premia is obtained at initiation and the analogue outflows occur at maturity.

The resulting combinations of instruments resemble the well known bull spread formation for  $i_{t,T}^j > i_{t,T}$  respectively the bear spread for  $i_{t,T}^j < i_{t,T}$ . If one constructs the bull and bear spread as motivated above, there is an initial inflow and an outflow at maturity. However, both, the bull and bear spread, can also be constructed such that there is an initial payment and thus an inflow at maturity. Comparing such a bull spread to a standard call with  $X = F_{t,T}$ , the payoff at maturity is the same for both if  $S_T \leq F_{t,T} + \widehat{bias_{t,T}}$ . For  $S_T > F_{t,T} + \widehat{bias_{t,T}}$ , the payoff from the spread is capped by the difference of the strike prices while it is unlimited for the call. However, this upside potential of the call comes at the price of a much higher option premium compared to that of the combination. If the bias estimate is adequate, it is unlikely that the additional initial cost will be compensated by higher future payoffs. Analogue arguments apply for the bear spread.

However, predicting the bias so accurately as to determine the strike prices for the strategy outlined before is very difficult. Inspired by the payoff scheme of the combinations described above, we consider an approach where exact specification of a bias estimate is not necessary. In particular we use digital calls  $(DC_{t,T})$  and digital puts  $(DP_{t,T})$  with payoffs being

$$DC_{t,T} = \exp\left[-i_{t,T}(T-t)\right]\Phi(d_2) \qquad DC_T = \begin{cases} 1 & \text{if } S_T > X = F_{t,T} \\ 0 & \text{otherwise.} \end{cases}$$
(12)

$$DP_{t,T} = \exp\left[-i_{t,T}(T-t)\right]\Phi(-d_2) \qquad DP_T = \begin{cases} 1 & \text{if } S_T < X = F_{t,T} \\ 0 & \text{otherwise.} \end{cases}$$
(13)

Following the argumentation from above, we investigate the proceeds of a strategy that finances a long position in a digital call if  $i_{t,T}^j > i_{t,T}$  while an investment in the digital put would occur if  $i_{t,T}^j < i_{t,T}$ . The return calculations based on the option premium being the notional value yield

$$R_{t,T}^{j} = \mathbb{I}_{\{i_{t,T}^{j} > i_{t,T}\}} \frac{DC_{T} - (1 + i_{t,T})DC_{t}}{DC_{t}} + (1 - \mathbb{I}_{\{i_{t,T}^{j} > i_{t,T}\}}) \frac{DP_{T} - (1 + i_{t,T})DP_{t}}{DP_{t}}.$$
 (14)

Trading rules based purely on the use of options may result in extreme returns, i.e. the potential of high returns is accompanied by great risk. Although the reward for taking this risk might be adequate, traders might not consider such a strategy as an appealing alternative investment opportunity due to the level of risk itself. Nevertheless, such options strategies could be an attractive approach for active portfolio managers seeking to beat their benchmark. We consider real-money managers investing some fraction of their funds,  $\lambda^{trader}$ , in the digital options strategy and the remainder,  $1 - \lambda^{trader}$ , in a tracking portfolio. The distribution of funds depends on the traders' tolerance of risk and confidence in the options strategy.

## **IV** Empirical Results

Our data set comprises monthly observations of spot rates, 1-month forward rates, 1-month interest rates, and 1-month implied volatilities for the following currencies: AUD, CAD, CHF, DKK, GBP, JPY, EUR, and USD. As the home currency we use the USD. Data are close bid-ask prices and end of month data, taken from *Datastream*, *Global Financial Data* and *Reuters*. As benchmarks we use a JP Morgan US Government Bond Index with maturities 7 to 10 years (denoted JPMGB), the S&P 500 Index (denoted SP500), and the MSCI World Total Return Index (denoted MSCI), all denominated in USD. Due to data availability constraints the sample period spans from April 1985 to May 2005.

Since we are evaluating how certain trading strategies would have worked in the past, we have to comment on the issue of data snooping. We do not believe that this plays a major role in our analysis. We are not fitting any model to our sample, all results presented are out-of-sample in the sense that only information that was available at initiation of the strategy is used. Furthermore, at the beginning of our evaluation period in 1995, more than a decade had already been spent on researching the forward bias. Hence, traders have back then already been well aware of the signals that we use for our strategies. In order to reduce the problem of finding spurious relationships specific to single periods, we do not only present results for the full evaluation period but also for three subsamples. The full evaluation period spans from 05/1995 to 12/1998, subsample 2 from 01/1999 to 12/2001, and subsample 3 from 01/2002 to 05/2005.

#### A Optimized Portfolios of Zero-Investment Deposit Strategies

To gain some initial insight we start with the results from the underlying single foreign currency strategy, equation (7), and the naive portfolio in Table 1.

#### [Insert Table 1 about here.]

While the full sample p.a. Sharpe ratios after transaction costs are around or above the threshold of 0.4 for all currencies except the CHF, this varies widely when considering the subsamples. Note, however, that the Sharpe ratios of the naive portfolio significantly exceed the threshold level even in the subsamples. Furthermore, the naive portfolio has the highest Sharpe ratio in the full sample and all subsamples, indicating the benefits of cross-currency diversification.

The results from the optimized-deposit portfolios for all three benchmarks, a historical 95%-VaR (based on data from the previous 10 years), both sets of expected returns (the differential and the VECM-based expected returns), and different levels of  $\sigma^{trader}$  are reported in Table 2.

#### [Insert Table 2 about here.]

The full sample p.a. Sharpe ratios after transaction costs of the portfolios are significantly above those of the benchmarks and the threshold of 0.4. The Sharpe ratios of the portfolios based on the differential expected returns vary between 0.91 and 1.31 and those on the VECM-based expected returns between 1.10 and 1.40, with one outlier equal to 0.63. A comparison of the Sharpe ratios of the portfolios based on the differential and VECM-based expected returns shows that the latter performs better for  $\sigma^{trader}$  being 3% and 5% while for the highest risk level,  $\sigma^{trader}$  being 7%, the situation is vice versa. The pattern in the subsamples is very similar, i.e. the majority of the Sharpe ratios of the portfolios are significantly above those of the benchmarks and the threshold level. Only in 4 out of 54 cases the Sharpe ratio of the portfolio is lower than that of the benchmark but still above the threshold level and only in 2 out of 54 cases we have a Sharpe ratio below the threshold level but still above the benchmark. The use of VECM-based expected returns turns out to be beneficial only in subsample 1 and 2. However, the VECM-based expected returns appear to produce less volatile Sharpe ratios across subsamples. Furthermore, while the portfolios are constrained to have a VaR less than or equal to that of the benchmark, the benchmark was outperformed by the portfolios in almost all cases.<sup>3</sup>

Since the profitability of our approach is conditional on deviations from UIP, our findings seem to suggest that the threshold of 0.4, i.e. the value of the Sharpe ratio causing reversion to UIP, needs to be increased in order to confirm the LSH.

#### **B** Currency Options Trading Strategies

Due to the nature of digital options, the returns of the single foreign currency options strategy are not normal distributed and in general not even symmetrically.<sup>4</sup> Therefore, standard deviations (and hence Sharpe ratios) would not be meaningful. Thus, we prefer to report other figures illustrating the performance of the strategy and especially the distribution of returns. The monthly results are reported Table 3.

#### [Insert Table 3 about here.]

The full sample p.m. average returns after transaction costs vary between 9 and 36 percent. The reason for these amazing result is the fact that positive returns are obtained in 55 to 68 percent of the cases for the single foreign currency approach and in almost 70 percent of the cases for the naive portfolio. In the subsamples we have only 2 out of 20 cases for the single foreign currency approach where the number of positive returns is smaller than 50 percent and consequently the resulting returns are negative. The other results are significantly positive but quite volatile across currencies. Note, however, that the results of the naive portfolio are highly persistent with positive returns in 64 to 71 percent of the cases and returns between 17 and 22 percent. Furthermore, in the case of the naive portfolio it is especially striking that average positive returns are substantially higher than the absolute average negative returns, with the number of positive returns being between 68 and 71 percent.

For the implementation of the options strategy as enhancement to another investment, we assume the following. Suppose a US fund manager wants to beat a benchmark, which he can perfectly track. Conditional on his risk aversion he chooses to invest a fraction  $\lambda^{trader}$  of the funds in the options

<sup>&</sup>lt;sup>3</sup>VaR figures are not reported in order to not overload Table 2, but are available from the authors upon request. <sup>4</sup>The downside is limited by the loss of the option premiums plus the interest that has to be paid for borrowing money to enter long positions. Of course, the upside potential is substantially higher.

strategy and the remainder,  $1 - \lambda^{trader}$ , in the tracking portfolio. We assume that  $\lambda^{trader}$  remains constant over time, i.e. we assume constant relative risk aversion. The results for  $\lambda^{trader} = 0.01$ and all three benchmarks are reported in Table 4. Note that since the fraction of funds invested in the options strategy is only one percent, we do not reject normal distributed strategy returns and thus Sharpe ratios can be reported.

#### [Insert Table 4 about here.]

The full sample p.a. Sharpe ratios after transaction costs show that the single foreign currency and naive portfolio approaches have the potential to significantly beat the benchmarks. This proves to be persistent through the subsamples in the majority of the cases with exceptions being the DKK (and the CHF) in subsample 1, the GBP in subsample 2, and the CHF and the JPY in subsample 3. Note, however, that the Sharpe ratios of the naive portfolio significantly exceed those of the benchmarks in all subsamples. The Sharpe ratio improvement is around 0.3 to 0.4 for the bond index and 0.13 to 0.18 for the stock indices without relevant changes in the volatility.

Thus, our results provide evidence that allocating a relatively small fraction of funds to the options strategy allows for substantial performance improvement. Hence, we would expect that traders show some interest in investing a fraction of their funds (depending on their risk tolerance and their confidence in the strategy) in the bias-exploiting trading strategy.

## V Conclusion

The present paper offers innovative approaches for trading the forward bias documented in past research. The results of our trading strategies indicate that the puzzle does not only exist statistically but that betting against uncovered interest rate parity yields economically significant excess returns even after adjusting for transaction costs. In particular, we find that downside risk (and hence capital charge) constrained strategies based on taking positions in deposits in multiple currencies produce Sharpe ratios which have the potential to attract speculative capital when compared to other investment opportunities. Furthermore, we provide evidence that active portfolio managers could substantially outperform their benchmarks by enhancing their investments with an currency options based approach. Overall, the empirical results lead us to the conclusion that the limits to speculation hypothesis, despite its intuitive appeal, should be handled with care.

## A Appendix: Vector Error Correction Model

Forward contracts are usually priced using covered interest rate parity (CIP), which, as every linear no-arbitrage pricing formula, duplicates one asset with a combination of other assets. So if the original asset has a stochastic trend, then the duplicated asset should have the same stochastic trend.<sup>5</sup> As shown in Engle and Granger (1987) the presence of common stochastic trends, i.e. cointegration, requires the employment of VECMs, which can be interpreted as models in which this period's price change depends on how far the system was out of long-run equilibrium last period.

Suppose that the spot rate has a stochastic trend (in particular, suppose that the spot rate follows a n-factor geometric Brownian motion) and that CIP holds. Then, as shown in Brenner and Kroner (1995), if the natural log of the differential does not have a stochastic trend, the natural logs of the spot and forward rates at any lead or lag must be cointegrated with cointegrating vector (1,-1). However, if the natural log of the differential has a stochastic trend, but is not cointegrated with the natural log of the spot rate with cointegrating vector (1,-1), then any leads or lags of the natural logs of the spot rate, the forward rate, and the differential will form a trivariate cointegrated system with cointegrating vector (1,-1,1).<sup>6</sup> Thus, if the differential has a stochastic trend, then spot and forward rates will not be cointegrated by themselves; the differential must be included in the system to find cointegration.<sup>7</sup>

Although the above argumentation implies cointegration at any lead or lag of the spot and forward rates, Zivot (2000) argues that simple models of cointegration between  $S_t$  and  $F_{t,T}$  more easily capture the important stylized facts of typical monthly exchange rate data than simple models of cointegration between  $S_T$  and  $F_{t,T}$ .

Following the argumentation so far, we utilize 5-year rolling models of cointegration between  $S_t$ and  $F_{t,T}$  and, given that there does not exist cointegration between  $S_t$  and  $F_{t,T}$ , we include the differential to forecast the exchange rate behavior. In particular, we apply the conventional Johansen procedure, Johansen (1995), to estimate the following multivariate VECM specification for each 5year window,

$$\Delta y_{t} = \alpha(\beta y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_{i} \Delta y_{t-i} + \gamma + \tau t + e_{t},$$
(15)

 $<sup>{}^{5}\</sup>mathrm{A}$  variable has a stochastic trend if its first difference has a stationary invertible ARMA representation plus a deterministic component.

<sup>&</sup>lt;sup>6</sup>Please note this argumentation assumes that the time to expiration of the forward contract, T - t, is fixed.

<sup>&</sup>lt;sup>7</sup>The differential will likely not have a stochastic trend, since most probably the same sets of underlying economic forces drive interest rates in both countries, i.e. the interest rates are likely to be cointegrated, and hence the common stochastic trend is eliminated in the differential. Alternatively, the differential will also not have a stochastic trend if both interest rates have no stochastic trend (whether or not they share underlying economic forces).

where  $\Delta y_t$  is a  $K \times 1$  vector of potentially cointegrated variables,  $\alpha$  and  $\beta$  are  $K \times r$  matrices of rank r, with r being the number of linearly independent cointegrating vectors,  $\mu$  and  $\rho$  are  $r \times 1$ vectors of parameters, p is the lag order of the underlying vector autoregressive model (VARM),  $\Gamma_i = -\sum_{j=i+1}^p A_j$ , with  $A_j$  being  $K \times K$  matrices of parameters of the underlying VARM,  $\gamma$  and  $\tau$  are  $K \times 1$  vectors of parameters, and  $e_t$  is a  $K \times 1$  vector of disturbances which has mean zero, covariance matrix  $\Sigma$ , and is i.i.d. normal over time.

We start with  $\Delta y_t = (\Delta ln S_t, \Delta ln F_{t,T})'$ . To select p we use Lütkepohl (1993) versions of Schwartz's Bayesian information criterion (SBIC). Given p, we use Johansen's trace statistic method to determine r. Engle and Granger (1987) show that if the variables  $y_t$  cointegrate we have 0 < r < K. Thus, if r = 1 we estimate the above model otherwise we include the differential and again select p, determine r, and estimate the above model if r = 1 or r = 2.8

## **B** Appendix: Threshold Accepting

Dueck and Scheuer (1990) introduced threshold accepting (TA) as a deterministic analog to simulated annealing. TA is a refined local search algorithm which can be easily implemented and adapted to a variety of problems. After choosing a random starting point, the neighborhood of this point is searched. The new solution is accepted if it is not worse by more than a given threshold,  $T_i \in T$ . The procedure is repeated for a fixed number of steps, *nrsteps*, during a fixed number of iterations, *nriter*. For subsequent iterations, the threshold is successively decreased, reaching the value of zero in the last iteration. By accepting solutions which are not worse by more than a given threshold, the algorithm is able to escape local minima. Even though the algorithm does not necessarily stop with a proved global optimum, convergence of TA is assured; see Althöfer and Koschnick (1991). A pseudo-code for the TA algorithm is given below.

The first paper to apply TA to a portfolio optimization was Dueck and Winker (1992). Recent work includes e.g. Gilli and Kellezi (2001). Given our optimization problem in (8), the starting point  $x_0$ would be given by a randomly chosen portfolio. Within each step a random change in the portfolio composition is performed. For doing so the neighborhood is defined that, if feasible, the weight of one currency will be reduced by five percent and the weight of another currency will be increased by five percent. The new portfolio will be accepted if the evaluation of the new composition is

<sup>&</sup>lt;sup>8</sup>Please note that placing restrictions on the trend terms in the above model yields the following five cases: i) unrestricted trend, ii) restricted trend,  $\tau = 0$ , iii) unrestricted constant,  $\tau = 0$  and  $\rho = 0$ , iv) restricted constant  $\tau = 0$ ,  $\rho = 0$ , and  $\gamma = 0$ , and v) no trend  $\tau = 0$ ,  $\rho = 0$ ,  $\gamma = 0$ , and  $\mu = 0$ . Since a quadratic trend in the levels seems to be economically unreasonable we a priori exclude the unrestricted trend case. Thus, we determine r for each of the remaining 4 cases. Given that the variables  $y_t$  cointegrate, i.e. 0 < r < K, we use SBIC to determine which trend assumption is most plausible.

not worse than that of the old by more than  $T_i$ . Since we start from the mean-variance approach, evaluation is based on quadratic utility and a penalty function for violation of the VaR constraint.

Pseudo-code for the TA algorithm
. Determine <i>nriter</i> and <i>nrsteps</i>
. Determine the thresholds, $T = \{T_1,, T_{nriter}\}$
. Choose a random starting point $x_0$
. For $i = 1$ to <i>nriter</i>
. For $s = 1$ to $nrsteps$
. Choose randomly $x_s$ from the neighborhood of $x_0$
. If $f(x_s) < f(x_0) + T_i$ Then
$f(x_0) = (x_s)$
. End If
. End For
. End For

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Table 1: Deposit Strategy: Single Foreign Currency and Naive Portfolio

		1 un	or 11. 1 un ,	Sample (00	71000 00	/ 2000)		
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive
R	7.42%	3.84%	2.12%	8.09%	3.28%	4.83%	8.32%	5.42%
$\sigma$	10.18%	6.11%	10.25%	9.31%	7.52%	12.28%	9.03%	4.62%
SR	0.73	0.63	0.21	0.87	0.44	0.39	0.92	1.17
		Pan	el B: Subsa	mple 1 (05)	5/1995 - 1	2/1998)		
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive
R	6.86%	4.38%	7.33%	2.37%	1.28%	11.10%	0.90%	4.89%
$\sigma$	8.90%	4.28%	10.05%	8.70%	7.13%	15.46%	7.87%	4.81%
SR	0.77	1.02	0.73	0.27	0.18	0.72	0.11	1.02
		Pan	el C: Subsa	mple 2 (01)	1/1999 - 1	2/2001)		
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive
R	0.28%	-1.54%	7.95%	7.33%	-0.63%	8.47%	10.08%	4.56%
$\sigma$	12.34%	5.42%	9.92%	10.08%	6.85%	11.48%	9.88%	5.55%
SR	0.02	-0.28	0.80	0.73	-0.09	0.74	1.02	0.82
		Pane	el D: Subsa	ample 3 $(01)$	1/2002 - 0	5/2005)		
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive
$\overline{R}$	14.28%	7.99%	-8.58%	14.90%	8.86%	-5.08%	14.74%	6.73%
$\sigma$	9.18%	7.94%	10.29%	9.10%	8.35%	8.17%	9.16%	3.45%
SR	1.56	1.01	-0.83	1.64	1.06	-0.62	1.61	1.95

Panel A: Full Sample (05/1995 - 05/2005)

Notes: R denotes the average p.a. return,  $\sigma$  the p.a. volatility, and SR the annualized Sharpe ratio over the period indicated in the Panel headers; all figures are net of transaction costs.

	Panel A: Full Sample (05/1995 - 05/2005)										
		Benchmark		Differential VECM							
$\sigma^{trader}$		Investment	3%	5%	7%	3%	5%	7%			
JPMGB	$R/\sigma$	7.68%/6.37%	3.78%/3.95%	5.83%/5.49%	6.51%/7.16%	5.31%/4.27%	5.67%/5.14%	4.44%/7.01%			
	$\mathbf{SR}$	0.55	0.96	1.06	0.91	1.24	1.10	0.63			
SP500	$R/\sigma$	10.07%/15.87%	3.90%/3.93%	6.17%/5.65%	9.44%/7.19%	6.47%/4.63%	6.83%/5.05%	8.34%/7.03%			
	$\mathbf{SR}$	0.37	0.99	1.09	1.31	1.40	1.35	1.19			
MSCI	$R/\sigma$	6.39%/14.40%	3.90%/3.93%	6.23%/5.66%	9.41%/7.17%	6.07%/4.61%	7.12%/5.11%	8.85%/7.06%			

1.31

1.32

1.39

1.25

1.10

ŚR

0.15

0.99

Table 2: Optimized Deposit Portfolios vs. Benchmark Investment Opportunities

	Panel B: Subsample 1 (05/1995 - 12/1998)											
		Benchmark		Differential		VECM						
$\sigma^{trader}$		Investment	3%	5%	7%	3%	5%	7%				
JPMGB	$R/\sigma$	10.17%/5.84%	4.47%/3.88%	5.48%/6.40%	4.28%/8.66%	6.20%/4.56%	5.33%/6.02%	4.92%/8.52%				
	$\mathbf{SR}$	0.78	1.15	0.86	0.49	1.36	0.89	0.58				
SP500	$R/\sigma$	21.95%/15.62%	3.88%/3.54%	5.58%/6.52%	8.66%/8.66%	6.68%/5.00%	7.64%/5.45%	10.16%/6.95%				
	$\mathbf{SR}$	1.04	1.10	0.86	1.00	1.34	1.40	1.46				
MSCI	$R/\sigma$	15.90%/13.50%	3.88%/3.54%	5.66%/6.49%	8.66%/8.71%	5.96%/5.03%	8.00%/5.45%	10.30%/6.94%				
	$\mathbf{SR}$	0.76	1.10	0.87	0.99	1.18	1.47	1.48				

	Panel C: Subsample 2 (01/1999 - 12/2001)										
		Benchmark		Differential			VECM				
$\sigma^{trader}$		Investment	3%	5%	7%	3%	5%	7%			
JPMGB	$R/\sigma$	5.09%/5.49%	1.22%/4.35%	2.83%/4.50%	3.90%/5.64%	4.83%/3.88%	6.68%/4.47%	1.62%/5.96%			
	$\mathbf{SR}$	-0.03	0.28	0.63	0.69	1.24	1.50	0.27			
SP500	$R/\sigma$	4.37%/17.28%	2.28%/4.74%	4.21%/5.27%	6.92%/5.82%	7.51%/4.40%	7.51%/4.85%	7.99%/6.97%			
	$\mathbf{SR}$	-0.05	0.48	0.80	1.19	1.71	1.55	1.15			
MSCI	$R/\sigma$	-3.36%/15.50%	2.28%/4.74%	4.48%/5.32%	6.31%/5.78%	7.05%/4.29%	8.04%/5.04%	9.14%/7.01%			
	SR	-0.55	0.48	0.84	1.09	1.64	1.60	1.30			

	Panel D: Subsample 3 (01/2002 - 05/2005)										
		Benchmark		Differential		VECM					
$\sigma^{trader}$		Investment	3%	5%	7%	3%	5%	7%			
JPMGB	$R/\sigma$	7.29%/7.60%	5.27%/3.65%	8.83%/5.21%	11.20%/6.52%	4.77%/4.35%	5.13%/4.78%	6.41%/6.12%			
	$\mathbf{SR}$	0.75	1.44	1.69	1.72	1.10	1.07	1.05			
SP500	$R/\sigma$	2.32%/14.54%	5.34%/3.59%	8.52%/5.01%	12.49%/6.60%	5.31%/4.52%	5.35%/4.87%	6.71%/7.30%			
	$\mathbf{SR}$	0.05	1.49	1.70	1.89	1.17	1.10	0.92			
MSCI	$R/\sigma$	4.75%/14.17%	5.34%/3.59%	8.38%/5.04%	12.94%/6.46%	5.35%/4.51%	5.35%/4.87%	7.03%/7.37%			
	$\mathbf{SR}$	0.22	1.49	1.66	2.00	1.19	1.10	0.95			

Notes: R denotes the average p.a. return,  $\sigma$  the p.a. volatility, and SR the annualized Sharpe ratio over the period indicated in the Panel headers; all figures are net of transaction costs. The column Benchmark Investment gives the results for the investment opportunities the bias-trading strategy is compared to: JP Morgan US Government Bond Index with maturities 7 to 10 years (JPMGB), S&P 500 Index (SP500), MSCI World Total Return Index (MSCI). The results in the columns headed Differential and VECM are for the optimized portfolios using interest differential respectively VECM-based expected returns for different  $\sigma^{trader}$ .

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 Table 3: Options Strategy: Single Foreign Currency and Naive Portfolio

	Panel A: Full Sample (05/1995 - 05/2005)										
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive			
$\overline{R}$	30.39%	15.23%	9.03%	23.51%	14.55%	19.01%	36.60%	20.14%			
$R_{min}$	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%			
$R_{max}$	104.22%	102.44%	99.29%	106.57%	104.64%	141.48%	102.67%	103.26%			
# of $R > 0$ [%]	65.29%	57.85%	55.37%	61.98%	57.02%	55.37%	68.42%	69.42%			
# of $R<0~[%]$	34.71%	42.15%	44.63%	38.02%	42.98%	44.63%	31.58%	30.58%			
$\overline{R}$ if $R > 0$	99.89%	99.44%	97.17%	99.48%	101.16%	115.18%	99.77%	43.32%			
$\overline{R}$ if $R < 0$	-100.35%	-100.35%	-100.33%	-100.36%	-100.36%	-100.32%	-100.26%	-32.49%			

Panel B: Subsample 1 (05/1995 - 12/1998)									
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive	
$\overline{R}$	35.40%	12.44%	7.42%	-5.53%	19.17%	34.59%		17.25%	
$R_{min}$	-100.51%	-100.51%	-100.51%	-100.51%	-100.51%	-100.48%		-100.45%	
$R_{max}$	102.06%	102.44%	99.29%	106.57%	104.64%	141.48%		103.26%	
# of $R > 0$ [%]	68.18%	56.82%	54.55%	47.73%	59.09%	61.36%		68.18%	
# of $R < 0$ [%]	31.82%	43.18%	45.45%	52.27%	40.91%	38.64%		31.82%	
$\overline{R}$ if $R > 0$	98.80%	98.25%	97.33%	98.46%	102.00%	119.62%		40.97%	
$\overline{R}$ if $R < 0$	-100.47%	-100.47%	-100.47%	-100.47%	-100.47%	-100.46%		-33.58%	

	Panel C: Subsample 2 (01/1999 - 12/2001)										
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive			
$\overline{R}$	15.86%	4.69%	31.21%	37.44%	-6.17%	32.85%	29.87%	20.70%			
$R_{min}$	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%			
$R_{max}$	104.22%	101.67%	98.91%	103.17%	102.61%	135.09%	102.45%	102.48%			
# of $R > 0$ [%]	58.33%	52.78%	66.67%	69.44%	47.22%	61.11%	65.71%	63.89%			
# of $R < 0$ [%]	41.67%	47.22%	33.33%	30.56%	52.78%	38.89%	34.29%	36.11%			
$\overline{R}$ if $R > 0$	98.91%	98.73%	97.04%	98.10%	99.19%	117.68%	97.83%	53.19%			
$\overline{R}$ if $R < 0$	-100.43%	-100.40%	-100.45%	-100.41%	-100.43%	-100.45%	-100.40%	-36.78%			

	Panel D: Subsample 3 (01/2002 - 05/2005)										
	AUD	CAD	CHF	DKK	GBP	JPY	EUR	Naive			
$\overline{R}$	29.21%	18.36%	8.81%	39.35%	13.40%	8.84%	36.60%	22.08%			
$R_{min}$	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%	-100.56%			
R <sub>max</sub>	104.22%	101.84%	98.91%	103.17%	102.98%	135.09%	102.67%	102.48%			
# of $R > 0$ [%]	64.47%	59.21%	55.26%	69.74%	56.58%	51.32%	68.42%	71.05%			
# of $R < 0$ [%]	35.53%	40.79%	44.74%	30.26%	43.42%	48.68%	31.58%	28.95%			
$\overline{R}$ if $R > 0$	100.56%	100.10%	97.09%	99.93%	100.65%	112.35%	99.77%	44.63%			
$\overline{R}$ if $R < 0$	-100.29%	-100.28%	-100.24%	-100.26%	-100.30%	-100.25%	-100.26%	-33.26%			

Notes:  $\overline{R}$  denotes the average monthly return,  $R_{min}$  the minimum monthly return,  $R_{max}$  the maximum monthly return over the period indicated in the Panel headers. # of R > 0 [%] denotes the percentage of returns being positive, # of R > 0 [%] for returns being negative.  $\overline{R}$  if R > 0 is the average of positive returns,  $\overline{R}$  if R < 0 the average of negative returns. All figures are net of transaction costs.

	Naive	10.02%/6.49% 0.90	$\frac{12.38\%/15.84\%}{0.52}$	8.74%/14.33% 0.32		Naive	$\frac{12.14\%/5.89\%}{1.11}$	$\frac{23.80\%/15.55\%}{1.17}$	$\frac{17.81\%/13.38\%}{0.91}$		Naive	7.52%/5.46%	0.43	6.81%/17.22% 0.09	-0.84%/15.16% -0.40
	EUR*	10.61%/7.84% 0.93	7.38%/16.03% 0.25	$5.03\%/15.02\%\ 0.11$		EUR					EUR	8.64%/6.18%	00.0	7.38%/17.76% 0.12	-0.54%/15.63% -0.37
	JPY	9.89%/7.07% 0.82	$\frac{12.25\%/16.02\%}{0.51}$	$8.61\%/14.10\%\ 0.32$		ЪЧ	14.22%/6.97% 1.24	$\frac{25.88\%/16.10\%}{1.26}$	$\frac{19.89\%/13.52\%}{1.05}$		JPY	8.98%/6.59%	0.57	8.27%/16.59% $0.18$	0.62%/14.26%- $0.32$
()	GBP	9.35%/7.53% 0.69	$\frac{11.71\%/15.84\%}{0.48}$	8.07%/14.53% 0.27	8	GBP	12.37%/6.52% 1.03	24.03%/15.44% 1.19	$\frac{18.04\%/13.59\%}{0.91}$	1)	GBP	4.30%/6.98%	-0.14	3.59%/17.52%-0.09	-4.06%/15.40% -0.60
(05/1995 - 05/2005	DKK	10.43%/7.27% 0.86	$\frac{12.79\%/16.40\%}{0.53}$	9.15%/14.96% 0.33	(05/1995 - 12/1998	DKK	9.40%/6.41% 0.59	$rac{21.07\%/17.01\%}{0.91}$	$\frac{15.08\%/14.81\%}{0.64}$	(01/1999 - 12/200)	DKK	9.53%/6.02%	0.72	8.82%/17.65% 0.20	1.17%/15.72%- $0.26$
nel A: Full Sample	CHF	8.69%/6.68% 0.68	$\frac{11.05\%/16.98\%}{0.41}$	$\frac{7.41\%/15.18\%}{0.21}$	tel B: Subsample 1	CHF	10.96%/7.05% 0.76	$\frac{22.62\%/16.87\%}{1.01}$	$\frac{16.63\%/14.57\%}{0.75}$	el C: Subsample 2	CHF	8.78%/5.25%	0.08	8.07%/18.61% 0.15	0.42%/16.42% -0.29
Pai	CAD	9.43%/7.31% 0.72	11.79%/15.47% $0.49$	$8.15\%/14.18\%\ 0.28$	Par	CAD	11.56%/6.77% 0.88	$\frac{23.23\%/14.46\%}{1.22}$	$\frac{17.23\%/12.48\%}{0.93}$	Par	CAD	5.60%/6.34%	0.00	4.89%/16.67% -0.02	-2.76%/14.81% -0.54
	AUD	11.25%/6.97% 1.02	$\frac{13.61\%/16.20\%}{0.58}$	9.97%/15.02% 0.39		AUD	14.31%/6.12% 1.42	$\frac{25.98\%/15.07\%}{1.35}$	$\frac{19.99\%/13.26\%}{1.08}$		AUD	6.94%/5.67%	0.30	6.23%/17.54% $0.06$	-1.42%/15.79% -0.42
	Benchmark	7.68%/6.37% 0.55	$\frac{10.07\%/15.87\%}{0.37}$	6.39%/14.40% 0.15		Benchmark	$\frac{10.17\%/5.84\%}{0.78}$	$\frac{21.95\%/15.62\%}{1.04}$	$\frac{15.90\%/13.50\%}{0.76}$		Benchmark	5.09%/5.49%	-0.03	4.37%/17.28% - $0.05$	-3.36%/15.50% -0.55
		$_{ m SR}^{ m R/\sigma}$	$ m _{SR}^{R/\sigma}$	$ m R/\sigma SR$			$ m ^{R/\sigma}_{SR}$	$ m _{SR}^{R/\sigma}$	$_{ m SR}^{ m R/\sigma}$			$\mathrm{R}/\sigma$	ЯХ	$ m _{SR}^{R/\sigma}$	$_{ m SR}^{ m R/\sigma}$
		JPMGB	SP500	MSCI			JPMGB	SP500	MSCI			JPMGB		SP500	MSCI

Table 4: Options as Extension to Other Investment:  $\lambda^{trader} = 0.01$ 

 SR
 -0.55

 Table continued on next page

	Naive	9.95%/7.89% 1.06	5.02%/14.65% 0.23	7.43%/14.42% $0.40$
	EUR	12.30%/9.07% 1.18	7.37%/14.62% $0.39$	$9.78\%/14.54\%\ 0.56$
	λdſ	6.04%/7.52% $0.59$	1.11%/14.90% - $0.04$	$3.51\%/14.27\%\ 0.13$
15)	GBP	10.56%/8.89% 1.00	5.63%/14.33% $0.28$	$8.03\%/14.38\% \\ 0.44$
3(01/2002 - 05/200	DKK	12.31%/9.07% 1.18	7.38%/14.62% 0.39	9.79%/14.54% $0.56$
nel D: Subsample 5	CHF	6.17%/7.42% 0.61	1.25%/15.31% - $0.03$	$3.65\%/14.64\%\ 0.14$
Paı	CAD	10.52%/8.62% 1.03	5.59%/15.19% $0.26$	$8.00\%/15.07\%\ 0.42$
	AUD	11.75%/8.66% 1.17	6.82%/15.86% 0.33	$9.23\%/15.85\% \\ 0.48$
	Benchmark	7.29%/7.60% 0.75	2.32%/14.54% 0.05	$rac{4.75\%}{14.17\%}$
		$_{ m SR}^{ m R/\sigma}$	$^{ m R/\sigma}_{ m SR}$	$_{ m SR}^{ m R/\sigma}$
		JPMGB	SP500	MSCI

Table 4: continued: Options as Extension to Other Investment:  $\lambda^{trader} = 0.01$ 

Notes: R denotes the average p.a. return,  $\sigma$  the p.a. volatility, and SR the annualized Sharpe ratio over the period indicated in the Panel headers; all figures are net of transaction costs. The column Benchmark gives the results for the portfolio manager's benchmark: JP Morgan US Government Bond Index with maturities 7 to 10 years (JPMGB), S&P 500 Index (SP500), MSCI World Total Return Index (MSCI). The remaining columns give the results for the approach of investing a fraction  $\lambda^{trader} = 0.01$  in the digital options strategy and the remainder in the tracking portfolio. \* Note that the option strategy for EUR is only evaluated for the period from 01/1999 to 05/2005 due to data constraints.