Time-Varying Risk Exposure of Hedge Funds *

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Abstract

This article aims to investigate risk exposure of hedge funds using switching regime beta models. This approach allows to analyze hedge fund tail event behavior and in particular the changes in hedge fund exposure conditional on different states of various risk factors. We find that in the normal state of the market, the exposure to risk factors could be very low but as soon as the market risk factor captured by S&P500 moves to a crisis state characterized by negative returns and high volatility, the exposure of hedge fund indexes to the S&P500 and other risk factors may change significantly. We further extend the regime switching model to allow for non-linearity in residuals and show that switching regime models are able to capture and forecast the evolution of the idiosyncratic risk factor in terms of changes from a low volatility regime to a distressed state that are not directly related to market risk factors.

Keywords: Hedge Funds; Risk Management; Regime-Switching Models, Liquidity;

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1 Introduction

The last decade has seen an increase in the number of hedge funds and availability of hedge fund data both on individual hedge funds and on hedge fund indexes. Unlike mutual funds, hedge funds engage in dynamic strategies, use leverage, take concentrated bets and have nonlinear payoffs. Colossal losses for hedge funds in Fixed Income strategy in 1998, Long/Short Equity and Global Macro strategies in 2002 and recent losses in Convertible Bond Arbitrage strategy are all attributed to different reasons and risk exposures. Therefore, it is important to understand and model time-varying risk exposures for various strategies and obtain reliable estimates for predicted exposures of hedge fund returns to various market risk factors in different market environments.

Hedge fund strategies greatly differ from each other and have different risk exposures. Fung and Hsieh (2001) analyzed a "trend following" strategy and Mitchell and Pulvino (2001) studied a "risk arbitrage" strategy. Both studies find the risk return characteristics of the hedge fund strategies to be nonlinear and stress the importance of taking into account option-like features while analyzing hedge funds. Moreover, Agarwal and Naik (2004) show that the non-linear option-like payoffs are not restricted just to these two strategies, but are an integral part of payoffs of various hedge fund strategies.

Hedge funds may exhibit non-normal payoffs for various reasons such as their use of options, or option-like dynamic trading strategies or strategies that lose money during market downturns. Unlike most mutual funds (Koski and Pontiff (1999) and Almazan et al. (2001)), hedge funds frequently trade in derivatives. Further, hedge funds are known for their "opportunistic" nature of trading and a significant part of their returns arise from taking state-contingent bets.

Nevertheless, there is still a limited understanding of the real non-linear exposure to risk factors of the different hedge funds strategies. Up to this point, most of the studies on hedge funds performance have been based either on (i) the classical linear factor model applied to mutual funds, (ii) non-parametric models or (iii) linear factor models with option-like factors. We are unaware of any published evaluation of the profitability or risk of hedge funds based on parametric non-linear models. In this paper, in line with the asset pricing perspective proposed by Bekaert and Harvey (1995, 2002), we suggest to analyze the exposure of hedge fund indexes with a factor model based on regime switching, where non-linearity in the exposure is captured by factor loadings that are state dependent. The regime switching approach is able to identify when the market risk factor is characterized by normal conditions, crisis or bubbles and the state dependent factor loading is able to capture the exposure of the

hedge fund to the market risk factor in these different states. The reason of using switching regime models for hedge fund returns is that the link between a hedge fund and market risk factors, due to dynamic strategies employed by hedge funds, is characterized by a phase-locking model¹. For example, we might have that most of the time, market betas are very low, but suddenly market betas become very high. If we use simple linear models with linear factors, then betas will be close to zero, so we are not capturing a small-probability crisis event that would lead to the phase-locking behavior. The regime-switching approach is able to identify when the market risk factor is characterized by normal conditions, crisis or bubbles; and the state dependent factor loading is able to capture the exposure of the hedge fund to the market risk factor in these different states. Simple models with linear factors show hedge fund correlations with the market index or some other risk factors to be close to zero; however, we show the exposure to the market factor goes from negligent to highly positive when the market moves from the normal regime to the crisis regime, as it was observed many times in the last decade.

We further extend the regime switching model to allow for non-linearity in residuals and show that switching regime models are able to capture and forecast the evolution of the idiosyncratic risk factor in terms of changes from a low volatility regime to a distressed state that are not directly related to market risk factors.

The rest of the paper is organized as follows. Related literature review is presented in Section 2. In Section 3 we attempt to define a series of beta switching regime models that could be used to analyze the different hedge fund style indexes. Section 4 describes the data and presents results for the asymmetric beta, beta switching regime models and evolution of the idiosyncratic factor. Section 5 concludes.

2 Literature Review

It is well known that hedge funds implement dynamic strategies. To address this issue, Fung and Hsieh (1997) use factor analysis conditional on market environment to determine the dominant styles in hedge funds. Agarwal and Naik (2004) use a piecewise linear function of the market return to approximate the nonlinear payoffs of different hedge fund strategies.

Similar to Agarwal and Naik (2004), we concentrate on equity-oriented hedge fund strategies. We analyze hedge fund risks conditional on nonlinearity in the factor loading. In par-

¹ "Phase-locking" behavior is borrowed from natural sciences, where otherwise uncorrelated actions suddenly become synchronized. For example, in the summer of 1998 when the default in Russian government debt triggered a global flight to quality that changed hedge fund correlations with market indexes from 0 to 1 overnight.

ticular, we concentrate on dynamic factors that are switching from a regime or state that we call "normal" to two other regimes that could be identified as "crisis" and "bubble" that are usually characterized respectively by (i) largely low returns and high volatility and (ii) high returns. We are not imposing a structure on the risk factor as a call or a put shape like Agarwal and Naik (2004) and Fung and Hsieh (1997) for hedge funds and Glosten and Jagannathan (1994) for mutual funds. Our approach is quite different. We are proposing a factor model that allows for regime switching in expected returns and volatilities. Our model is not strategy-dependent, so the fact that each hedge fund is not implementing the same strategy within the same hedge fund category makes our model more desirable to use. Fung and Hsieh (1997) emphasize limitations of models, including their own, that put structure on the risk factor. However, given that hedge fund data became available only in 1994², Fung and Hsieh (1997) had to resort to building models with a predetermined structure on the risk factor, such as conducting a quintile analysis, due to data limitations. Given current availability of more than 10 years of monthly data, we can conduct a thorough analysis on hedge fund returns that could not be done before due to data limitations. In a completely flexible approach, in the same spirit as Fung and Hsieh (1997), we measure the exposure of the hedge fund indexes to risk factors in different states, i.e. when the market risk factor is facing normal returns and volatility, or when the market is facing low returns and high volatility, or some other states where the states are not exogenously imposed but endogenously determined by the model.

Moreover, another distinctive feature of our research is that nonlinearity is endogenously introduced in our model. We are not imposing specific non-linear structures or option-like approach. Our goal is to analyze hedge fund exposures in different states of the world. Our econometric and methodological contribution is to introduce a model that allows for linear or non-linear factors with non-linear beta. Glosten and Jagannathan (1994) use linear betas with non-linear factors. Assness, Krail and Liew (2001) look at asymmetric betas in hedge fund exposure. Agarwal and Naik (2004) show that a large number of hedge fund indexes show no correlation in up-market conditions, but a positive correlation in downmarket conditions. Fung and Hsieh (1997) define five different states of the market factor, and analyze hedge fund returns for each of the states. This asymmetry of betas or factor loadings in up-market versus down-market conditions confirms the nonlinear nature of hedge fund payoffs. However, this methodology imposes an explicit and exogenous definition of up and down market conditions. In our model, this definition is endogenous; the data allows the model to recognize the state variable that is driving this factor movement: up-

 $^{^{2}}$ CSFB/Tremont database for hedge fund indices starts in January, 1994. Individual hedge funds were available prior to 1994; however, the database is only adjusted for survivorship bias starting January 1994.

market and down-market conditions and the likelihood of transition from one state to the other. Regime-switching models have been used in a number of contexts, ranging from the analysis of the business cycle with Hamilton's (1989) model, Ang and Bekaert's (2002) regime switching asset allocation model and to Billio and Pelizzon's (2005) analysis of contagion among markets. The closest to our paper implementation is by Chan, Getmansky, Haas and Lo (2005) where regime switching models are applied to the CSFB/Tremont hedge fund indexes to obtain a measure of systemic risk, i.e. the possibility to switch from a normal to a distressed regime. Our approach differs from the previous work since we use a switching regime *beta* model to measure the exposure of hedge funds indexes to different regimes that characterize market risk factors. Such exposure cannot be measured with the simple switching regime model used by Chan et al. (2005).

The contribution of our approach is that it sheds a light on an important question in hedge fund literature - understanding risk exposure of hedge funds especially in extreme states of the world. Pierre Saint-Laurent (2005) in a recent article said that it is very important to understand returns and risks in extreme market events. Currently, no models exist in the hedge fund literature that explain a time-varying hedge fund risk exposure without imposing exogenous definition of different states of the world (i.e. up or down markets, declines or rallies). Our contribution is that we are deriving a time-varying hedge fund risk exposure distribution and endogenously measuring time-varying hedge fund risks for different hedge fund strategies.

3 Theoretical Framework

Linear factor models such as the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT) have been the foundation of most of the theoretical and empirical asset pricing literature. Formally, a simple one factor model applied to hedge fund index returns could be represented as:

$$R_t = \alpha + \beta I_t + \omega u_t \tag{1}$$

where R_t is the return of a hedge-fund index in period t, I_t is a factor, for example, S&P500 in period t, and u_t is IID.

In this model, we can identify the exposure of hedge fund returns to a factor I. Unfortunately this theory constrains the relation between risk factors and returns to be linear. Therefore it cannot price securities whose payoffs are nonlinear functions of the risk factors. Researchers have addressed this problem using a nonlinear asset pricing framework (Rubenstein (1973), Kraus and Litzenberger (1976), Dybvig and Ingersoll (1982), Bansal and Viswanathan (1993), Bansal, Hseih, and Viswanathan (1993), and Harvey and Siddique (2000a, 2000b)). Glosten and Jaganathan (1994) show how a value can be assigned to the skill of the manager generating a nonlinear payoff.

As it was mentioned before, the distribution of hedge fund returns is not normal, and is often characterized by negative skewness and fat tails. Therefore, β is different for various regimes and this calls for the introduction of a different model. We can truncate the distribution of R_t , say at the median or zero and look at the difference of betas for "up or down" markets similar to Agarwal and Naik (2004), Mitchell and Pulvino (2001), Assness, Krail and Liew (2004) and Chan, Getmansky, Haas and Lo (2005) who find that the risk arbitrage strategy shows zero correlation with up-market conditions, but a large positive correlation during down-market conditions. Following this approach, we look at asymmetric betas in hedge fund exposure by specifying different beta coefficients for down-markets versus up-markets. Specifically, consider the following regression:

$$R_{it} = \alpha_i + \beta_i^+ I_t^+ + \beta_i^- I_t^- + \epsilon_{it}$$

$$\tag{2}$$

where

$$I_t^+ = \begin{cases} I_t & \text{if } I_t > 0 \\ 0 & \text{otherwise} \end{cases}, \quad I_t^- = \begin{cases} I_t & \text{if } I_t \le 0 \\ 0 & \text{otherwise} \end{cases}$$
(3)

where I_t is the return on the index. Since $I_t = I_t^+ + I_t^-$, the standard linear model in which fund *i*'s market betas are identical in up and down markets is a special case of the more general specification (2), the case where $\beta_i^+ = \beta_i^-$.

The specification (2) essentially tries to capture asymmetries in the index exposure. Given previous results that show that correlations between hedge fund and market returns change conditionally on the state of the market, we propose a more flexible and complete model for capturing this feature: a switching regime model.

A Markov switching-regime model is a model where systematic and un-systematic events may change from the presence of discontinuous shifts in average return and volatility. The change in regime should not be regarded as predictable but as a random event. More formally, the model could be represented as:

$$R_t = \alpha + \beta(S_t)I_t + \omega u_t \tag{4}$$

$$I_t = \mu(S_t) + \sigma(S_t)\epsilon_t \tag{5}$$

where S_t is a Markov chain with n states and transition probability matrix **P**. Each state of the market index I has its own mean and variance and the same applies to the returns of the hedge fund index. More specifically, the hedge fund mean returns and volatility are related to the states of the market index and are given by the parameter α plus a factor loading, β , on the conditional mean of the factor, where β could be different conditional on a state of a factor risk.

For example, if n=2 (state labels are 0 or 1), the model can be represented as follows:

$$R_t = \left\{ \begin{array}{l} \alpha + \beta_0 I_t + \omega u_t \ if \ S_t = 0\\ \alpha + \beta_1 I_t + \omega u_t \ if \ S_t = 1 \end{array} \right\}$$

where the state variable S depends on time t, and β depends on the state variable:

$$\beta(S_t) = \begin{cases} \beta_0 & \text{if } S_t = 0\\ \beta_1 & \text{if } S_t = 1 \end{cases}$$
(6)

and the Markov chain S_t (the regime switching process) is described by the following transition probabilities:

$$\Pr(S_t = 0 | S_{t-1} = 0) = p \quad \Pr(S_t = 1 | S_{t-1} = 0) = 1 - p$$

$$\Pr(S_t = 1 | S_{t-1} = 1) = q \quad \Pr(S_t = 0 | S_{t-1} = 1) = 1 - q$$
(7)

where p and q are elements of the transition probability matrix \mathbf{P} , and \mathbf{P} can be represented as follows:

$$\mathbf{P} = \begin{bmatrix} p & 1-p\\ 1-q & q \end{bmatrix}$$
(8)

The parameters p and q determine the probability to remain in the same regime. This model allows for a change in the variance of returns only in response to occasional, discrete events. Despite the fact that the state S_t is unobservable, it can be estimated statistically (see for example Hamilton (1989,1990)).

Our specification is similar to the well-known "mixture of distributions" model. However, unlike standard mixture models, the regime-switching model is not independently distributed over time unless p and q are equal to 0.5. Indeed, one key aspect of the switching regime model is that if the volatility has been efficiently characterized with different parameters for different periods of data, it will be probable that in the future the same pattern will apply. As the switching regime approach accounts for "short-lived" and "infrequent" events, it provides an accurate representation of the left-hand tail of the return distribution.

The advantage of using a Markov chain as opposed to a "mixture of distributions" is that the former allows for conditional information to be used in the forecasting process. This allows us to: (i) fit and explain the time series, (ii) capture the well known cluster effect, under which high volatility is usually followed by high volatility (in presence of persistent regimes), and (iii) generate better forecasts compared to the mixture of distributions model, since switching regime models generate a time conditional forecast distribution rather than an unconditional forecasted distribution.

The regime-switching approach is also superior to non-parametric approaches. The Markov switching model is more flexible that just using a truncated distribution approach as proposed for hedge funds by Assness, Krail and Liew (2001) as at each time t, we have a mixture of one or more normal distributions, and this mixture changes every time. Using the truncated distribution will lead to non-parametric estimation, which is exogenously imposed, and it is hard to make inferences about beta forecast and conditional expectations. Instead, we use a parametric model to help us to separate the states of the world. We will be able to infer time-varying risk exposures of hedge funds, make forecasts and calculate conditional expectations.

Our approach is in the spirit of Agarwal and Naik (2004) but rather than transforming factors from linear to non-linear, we introduce a mix of linear factors with different betas. In this way we capture, with a formal model, the idea of Fung and Hsieh (1997) to separate factors into different quintiles based on historical performance and try to access the exposure

of hedge fund returns to factors in each of the quintiles. Moreover, the use of quintiles implies the exogenous definition of states. We let the model to determine the states and our formal model allows us to forecast future behaviour.

More specifically, once estimated, forecasts of changes in regime can be readily obtained, as well as forecasts of β_t itself. In particular, because the k-step transition matrix of a Markov chain is simply given by \mathbf{P}^k , the conditional probability of the regime S_{t+k} given date-t data $\mathcal{R}_t \equiv (R_t, R_{t-1}, \ldots, R_1)$ takes on a particularly simple form:

$$\operatorname{Prob}\left(S_{t+k} = 0|\mathcal{R}_t\right) = \pi_1 + (p - (1 - q))^k \left[\operatorname{Prob}\left(S_t = 0|\mathcal{R}_t\right) - \pi_1\right]$$
(9)

$$\pi_1 \equiv \frac{(1-q)}{(2-p-q)} \tag{10}$$

where $\operatorname{Prob}(S_t = 0|\mathcal{R}_t)$ is the probability that the date-*t* regime is 0 given the historical data up to and including date *t* (this is a by-product of the maximum-likelihood estimation procedure). Using similar recursions of the Markov chain, the conditional expectation of β_{t+k} can be readily derived as:

$$E[\beta_{t+k}|\mathcal{R}_t] = \mathbf{a}_t' \mathbf{P}^k \boldsymbol{\beta}$$
(11)

$$\mathbf{a}_{t} = \left[\operatorname{Prob}\left(S_{t} = 0 | \mathcal{R}_{t}\right) \operatorname{Prob}\left(S_{t} = 1 | \mathcal{R}_{t}\right) \right]'$$
(12)

$$\boldsymbol{\beta} \equiv [\beta_0 \ \beta_1]' \tag{13}$$

Time-varying betas can be easily determined using equation 11 by assuming that k=0. This gives us the framework for analyzing time-varying risk exposures for hedge funds for different factors. Moreover, this framework can be used to calculate expected time-varying risk exposures for hedge funds for various factors, by setting k to be more than 0. For example, if k=1, we can calculate the evolution of expected one-month beta exposures to different factors.

The previous model described in equations 4 and 5 could be extended in several ways. For example, we propose a regime switching model with non-linearity in residuals and in the intercept coefficient:

$$R_t = \alpha(Z_t) + \beta(S_t)I_t + \omega(Z_t)u_t \tag{14}$$

$$I_t = \mu(S_t) + \sigma(S_t)\epsilon_t \tag{15}$$

In this model, additional non-linearities are captured by the intercept and the residuals. Z_t proxies for all other non-linearities not captured by non-linearities between hedge fund and the risk factor I.

Usually there are more than one factor that affect hedge fund returns. Our switching regime beta model could be easily extended to a multifactor model.

The first extension is a model in the same spirit as of Agarwal and Naik (2004) with a non-linear exposure to S&P 500 and a linear exposure to other risk factors. More formally:

$$R_t = \alpha(Z_t) + \beta(S_t)I_t + \sum_{k=1}^K \theta_k F_{kt} + \omega(Z_t)u_t$$

$$I_t = \mu(S_t) + \sigma(S_t)\epsilon_t$$
(16)

where θ_k is the linear factor loading of the hedge fund index on the k-th risk factor and F_{kt} is the return on the k-th risk factor at time t.

However, this model does not consider the possibility that the exposure to the other factors could be affected by the regime that characterizes the S&P 500. To capture this feature, we propose a multifactor beta switching model with non-linearity in residuals:

$$R_t = \alpha(Z_t) + \beta(S_t)I_t + \sum_{k=1}^K \theta_k(S_t)F_{kt} + \omega(Z_t)u_t$$

$$I_{1t} = \mu(S_t) + \sigma(S_t)\epsilon_t$$
(17)

This model allows us to detect the exposure of hedge fund indexes to different factors conditional on the state that characterizes the market index factor that in our empirical analysis is represented by the S&P500.

4 Empirical Analysis

4.1 Data

For the empirical analysis in this paper, we use aggregate hedge-fund index returns from CSFB/Tremont database from January 1994 to March 2005. The CSFB/Tremont indexes are asset-weighted indexes of funds with a minimum of \$10 million of assets under management, a minimum one-year track record, and current audited financial statements. An aggregate index is computed from this universe, and 10 sub-indexes based on investment style are also computed using a similar method. Indexes are computed and rebalanced on a monthly frequency and the universe of funds is redefined on a quarterly basis. We use net-of-fee monthly excess return (in excess of LIBOR). This database accounts for survivorship bias in hedge funds (Fung and Hsieh (2000b)). Section A.1 describes each category in detail. Table 1 describes the sample size, S&P 500 β , annualized mean, annualized standard deviation, minimum, median, maximum, skewness and kurtosis for monthly CSFB/Tremont hedge-fund index returns as well as for the S&P 500.

For our empirical analysis, we evaluate the exposure of hedge fund indexes to the market index, S&P 500; therefore, we only concentrate on hedge fund styles that either directly or indirectly have S&P 500 exposure. For example, we concentrate on directional strategies such as Dedicated Shortseller, Long/Short Equity and Emerging Markets as well as nondirectional strategies such as Distressed, Event Driven Multi-Strategy and Risk Arbitrage.

Categories greatly differ. For example, annualized mean for the Dedicated Shortseller category is the lowest: -0.87% and the annualized standard deviation is the highest at the 17.66%. Distressed has the highest mean of 13.00%, but relatively low standard deviation: 6.64%. The lowest annualized standard deviation is reported for the Equity Market Neutral strategy at 2.99% with an annualized mean of 9.80%. Hedge fund strategies also show different third and fourth moments. Specifically, non-directional funds such as Event Driven Multi-Strategy, Risk Arb and Convertible Bond Arb all have negative skewness and high kurtosis. The exception is for the Equity Market Neutral strategy which has a low positive skewness and kurtosis. Directional strategies such as Dedicated Shortseller, Long/Short Equity have positive skewness and small kurtosis. Emerging Markets has a slight negative skewness of -0.62 and a small kurtosis. The market factor is characterized by high annualized return of 11.25% and high standard deviation of 15.14% during our sample period. Moreover, the distribution of the factor is far from normal and is characterized by negative skewness.

Variable	Sample Size	β _{S&P500}	Ann. Mean	Ann. SD	Min	Med	Max	Skew	Kurt
Hedge Funds	135	1.00	10.63	8.07	-7.55	0.80	8.53	0.11	2.12
Convertible Bond Arb	135	0.04	8.94	4.70	-4.68	1.04	3.57	-1.36	3.38
Dedicated Shortseller	135	-0.89	-0.87	17.66	-8.69	-0.41	22.71	0.86	2.03
Emerging Markets	135	0.54	8.83	16.88	-23.03	1.26	16.42	-0.62	4.30
Equity Market Neutral	135	0.07	9.80	2.99	-1.15	0.80	3.26	0.30	0.33
Distressed	135	0.24	13.00	6.64	-12.45	1.21	4.10	-2.85	17.82
Event Driven Multi-	135	0.19	10.31	6.13	-11.52	0.90	4.66	-2.64	17.57
Risk Arb	135	0.12	7.82	4.30	-6.15	0.63	3.81	-1.29	6.41
Long/Short Equity	135	0.41	11.77	10.52	-11.43	0.80	13.01	0.24	3.75
S&P 500	135	1.00	11.25	15.14	-14.46	1.39	9.78	-0.59	0.49

Table 1: Summary statistics for monthly CSFT/T remont hedge-fund index returns from January 1994 to March 2005.

4.2 Asymmetric beta model

As the examples in Section 1 illustrate, hedge-fund returns may exhibit a number of nonlinearities that are not captured by linear methods such as correlation coefficients and linear factor models. We apply specification (2) to essentially try to capture asymmetries in the index exposure.

Using the specification (2), we regress hedge fund returns on the S&P 500 index during up and down conditions. The results are reported in Table 2. Beta asymmetries are quite pronounced especially for Emerging Markets, Distressed, Event Driven Multi-Strategy and Risk Arbitrage. For example, the Distressed index has an up-market beta of 0.05—seemingly market neutral—however, its down-market beta is 0.42! The exposure of the Convertible Bond Arbitrage strategy to the S&P 500 is negligible for both up and down markets; therefore, a more comprehensive model is needed to measure the exposure of this style. These asymmetries are to be expected for certain nonlinear investment strategies (Agarwal and Naik (2004) and Chan, Getmansky, Haas and Lo (2005)).

4.3 Beta switching regime models

4.3.1 One factor model

A shown above, the non-linear relationship between the market risk factor and hedge fund index returns could be captured for some hedge fund strategies using asymmetric beta models. We take this approach further and estimate non-linear exposure of hedge fund returns to different states of the market factor. We present a formal model that characterizes the distribution of the market risk factor, i.e., the S&P 500. Therefore, in this section we propose first the estimation of the switching regime model for the S&P 500 and then analyze the exposure of different hedge fund indexes to the different states of the market. We use the model described in equation 5 and consider three regimes: regime 0 is a bubble regime and has extremely high returns, regime 1 is a normal regime and regime 2 is a crisis or market down-turn regime and is characterized by very low returns and extremely high volatility. The results of the estimation are shown in Table 3.

Table 3 shows that the return pattern of S&P 500 could be easily captured with three regimes, where regime 0 has a mean of 5.79% and a relatively low volatility 1.52%. This regime represents the bubble state and has a very low probability of remaining in the same regime next month: $P_{00}=28\%$. The normal regime is captured by regime 1 with a mean statistically different than zero and equal to 0.85% and a volatility of 2.49%. This is a persistent regime, and the probability of remaining in it is 98%. The last regime captures

Variable	α	β-	β+	β
Convertible Bond Arb	0.29	0.05	0.02	0.04
Dedicated Shortseller	-0.09	-0.99	-0.78	-0.89
Emerging Markets	0.66	0.78	0.29	0.53
Equity Market Neutral	0.15	0.04	0.11	0.07
Long/Short Equity	0.34	0.47	0.35	0.41
Distressed	1.06	0.42	0.05	0.24
Event Driven Multi-Strategy	0.76	0.35	0.04	0.19
Risk Arb	0.32	0.20	0.06	0.12

Table 2: Regressions of monthly CSFB/Tremont hedge-fund index returns on the S&P 500 index return, and on negative and positive S&P 500 index returns, from January 1994 to March 2005. Parameters significantly different from zero at the 10% level are shown in bold type.

Variable	Estimate	t-stat
μο	5.79	15.22
μ_1	0.85	2.53
μ_2	-2.02	-2.25
σ_0	1.52	12.80
σ_1	2.49	25.74
σ_2	4.51	29.46
P ₀₀	0.28	1.93
P ₀₁	0.05	1.17
P ₁₀	0.02	0.94
P ₁₁	0.98	43.45
P ₂₁	0.00	0.00
P ₂₂	0.74	6.54

Table 3: Beta regime switching model for the market risk factor, S&P 500. The following model is estimated: $R_t = \alpha + \beta(S_t)I_t + \omega u_t, I_t = \mu(S_t) + \sigma(S_t)\epsilon_t$. Parameters significantly different from zero at the 10% level are shown in bold type.

market downturns and has a mean of -2.02% and a volatility of 4.51%. The probability of remaining in this regime is 74%.

The model estimation allows us to infer when S&P 500 was in one of the three regimes for each date of the sample using the Hamilton filter and smoothing algorithm (Hamilton, 1994). Figure 1 depicts the resulting series.

From Figure 1 we observe that in the first part of the sample, the S&P 500 returns are frequently characterized by the normal regime 1, in particular from July 1994 to December 1996. The period from 1997 through 2003 is characterized mostly by two other regimes: bubble and crisis. This outcome is generated mainly by high instability of the financial markets starting from the Asian crisis in 1997, well captured by regime 2, the technology and internet boom, well capture by regime 0, the Japanese crisis of March 2001, September 11, 2001 and the market downturn of 2002 and 2003, captured mostly by regime 2. The last part of the sample from 2003 through 2005 is characterized by the normal regime 1.

This analysis of the S&P 500 returns illustrates the ability of regime-switching models to capture changes in return process determined endogenously by the model.

In addition to analyzing the change in the S&P 500 returns and probability of being in a particular regime, we derive both conditional and unconditional distributions for the S&P 500 for all three regimes as well as for the total time series. Figure 2 depicts unconditional distribution of S&P 500 overall, in crisis, normal and bubble regimes. First, it is worthwhile to notice that during the time period analyzed in the paper, market clearly experienced three distinct regimes: bubble, normal and crisis. Moreover, the total distribution is skewed, and distribution of being in a crisis state is characterized by fat tails. Figure 2 also depicts conditional distribution of different regimes, conditioned on starting in regime 2, a crisis regime. The resulting total distribution closely overlaps regime 2 distribution, especially in the left tail. Therefore, once in crisis, the market is more likely to stay in crisis (74%), and both conditional regime 2 and total distribution are fat-tailed.

Figure 3 shows conditional distributions of S&P 500 overall, in crisis, normal and bubble regimes first conditional on a bubble regime and second conditional on a normal regime. Interestingly, that conditioned on being in a bubble, there is a certain probability of staying in a bubble (28%), but there is also a large left-tail probability of moving to a crisis (67%). It looks like the bubble regime is often transitory, followed by a crisis. The probability of going to a normal regime after a bubble is small (5%). Conditional on being in a normal regime, the total distribution is almost identical to the conditional probability of a normal regime. Therefore, if a market is in the normal regime, it is more likely to stay this way (98%). The conditional distributions for all regimes are very close to normal in this case. Nevertheless, there is a small probability of 2% of moving to a bubble regime that is more



Figure 1: Market S&P 500 excess returns and probabilities of being in a particular regime over time. There are 3 states of the market: regime 0 is a bubble regime, regime 1 is a normal regime and regime 2 is a crisis regime.

likely (67%) followed by a crisis.

Overall, the results confirm that during the period of 1994 to March, 2005, S&P 500 was clearly characterized by three separate regimes. In the paper, we are interested in clearly understanding the exposure of each hedge fund strategy to the market in all these regimes. In other words, we are interested in finding the exposure of hedge fund returns to all parts of this distribution. Moreover, most studies analyzed exposure of hedge funds given a normal regime. Using the results in Figure 3, it is clear that this approach underestimates the tail of the distribution and thus biases hedge fund market risk exposure.

After having characterized the process for the S&P 500 we analyze the exposure of different hedge fund strategies to different S&P 500 regimes. The analysis is based on the model presented in equation 14 and results are shown in Table 4.

We find different factor loadings with respect to the S&P 500 regimes for almost all hedge funds indexes. The only exception is the Convertible Bond Arbitrage strategy. More specifically, for Equity Market Neutral we find a positive exposure both in the regime 0 and regime 2, i.e. when the market is rolling up or is facing a down-turn. The exposure is zero in normal times. This result is in line with the fact that the Market Neutral strategy can neutralize the effects of normal movements of the market, but when the market is suddenly moving to another regime facing a phase-locking phenomenon, the exposure becomes positive. This effect is not captured by the asymmetric beta analysis, i.e. the up and down-betas for the Equity Market Neutral strategy in Table 2 are not significant at 10%. Regarding the other more directional strategies (Dedicated Shortseller and Long/Short Equity), we do find significant exposures to the S&P 500 regimes, but again the factor loadings vary a lot for different regimes. In particular, Dedicated Shortseller shows a large negative exposure of -1.13 to S&P 500 in normal times. This relationship is maintained for the crisis period; however, the exposure is reduced in half for the bubble state of the market. Long/Short Equity strategy aims to go both long and short on the market during the normal regime. Our analysis shows that the exposure to the market during the normal regime is three times as high as the exposure during the other two regimes. There is, therefore, an attempt of this strategy to reduce the exposure to the market down-turns, but the exposure remains still positive, as shown in Table 4.

The Emerging Market strategy shows a peculiar positive exposure mostly when the market is characterized by the crisis state and is relatively large in normal time. The result of the large exposure on market down-turns is intuitive since many emerging markets do not allow short-selling. Therefore, the exposure result is similar to writing a put option on the S&P index.

The other three strategies are related to the Event Driven categories. The exposures



Figure 2: First figure describes unconditional distribution of S&P 500 overall, in crisis, bubble and normal regimes. Second figure describes conditional distribution of S&P 500 given a crisis regime, for the overall, crisis, bubble and normal regimes. There are 3 states of the market: regime 0 is a bubble regime, regime 1 is a normal regime and regime 2 is a crisis regime.



Figure 3: First figure describes conditional distribution of S&P 500 given a bubble regime, for the overall, crisis, bubble and normal regimes. Second figure describes conditional distribution of S&P 500 given a normal regime, for the overall, crisis, bubble and normal regimes. There are 3 states of the market: regime 0 is a bubble regime, regime 1 is a normal regime and regime 2 is a crisis regime.

Estimate	Convertible	Dedicated	Emerging	Equity	Long/Short	Distressed	Event	Risk Arb
	Bond Arb	Shortseller	Markets	Market	Equity		Driven	
				Neutral			Multi-	
							Strategy	
α_0	1.27	3.47	1.34	0.26	0.09	1.03	0.91	0.62
α_1	-0.16	-1.11	0.25	1.18	1.47	-3.52	-3.55	0.24
β ₀	0.01	-0.58	0.17	0.12	0.20	0.11	0.15	0.10
β_1	0.05	-1.13	0.38	0.04	0.64	0.36	0.14	0.14
β ₂	-0.01	-0.99	0.51	0.05	0.21	0.12	0.17	0.07
ω ₀	0.53	2.28	1.62	0.60	1.22	1.17	1.13	0.70
ω_1	1.70	2.61	5.26	0.66	3.89	3.75	3.45	1.81
P^{Z}_{00}	0.89	0.69	0.98	0.86	0.99	0.98	0.99	0.89
P [∠] 11	0.85	0.84	1.00	0.83	0.97	0.50	0.76	0.69

Table 4: The exposure of CSFB/Tremont hedge-fund index strategies to different S&P 500 regimes. The following model is estimated: $R_t = \alpha(Z_t) + \beta(S_t)I_t + \omega(Z_t)u_t, I_t = \mu(S_t) + \sigma(S_t)\epsilon_t$. Parameters significantly different from zero at the 10% level are shown in bold type.

to the S&P 500 are positive and quite similar in different states of the market, especially for the Event Driven Multi-Strategy that has a slightly higher exposure during the market down-turn. Distressed security strategy presents a larger exposure in normal times. The Risk Arbitrage Strategy presents a positive exposure in the normal regime and when the market is rolling up and an almost zero exposure in the crisis regime.

Overall, the results are consistent with the asymmetric beta analysis presented above; however, we showed that with the switching regime beta model we are able to capture features of the links with hedge fund returns and S&P 500 that are difficult to capture by simply splitting past returns in up and down market movements. Nevertheless, other risk factors play a role as important as the S&P 500 in characterizing the time-varying hedge fund exposure. This aspect is investigated in the next section with a multifactor model.

Moreover, using regime switching framework allows us to calculate time-varying risk exposure of hedge funds implied by the data, i.e. time-varying betas with respect to various factors including S&P 500 for various hedge fund strategies. So far, this has not been accomplished in previous research due to predominant use of non-parametric models in explaining hedge fund risks. Time-varying betas can be easily determined using equation 11 by assuming that k=0. This gives us the framework for analyzing time-varying risk exposures for hedge funds for different factors. Time-varying market risk betas are depicted for several hedge fund strategies in Figures 4 and 5. First, note that the market exposure changes over time for all strategies, confirming that hedge funds are implementing dynamic strategies. Figure 4 depicts evolution of market betas for Hedge fund index, Long/Short equity and Risk arbitrage strategies. In all cases, starting the middle of 2003, exposure of these strategies to the market dramatically increased. For example, for the Combined Hedge fund index, the forecasted exposure in April, 2004 was 0.07, seemingly market-neutral; however, exposure in March, 2005 increased to 0.37, which is a significant positive market exposure. For the same time period, the exposure of the Long/Short equity increased from 0.20 to 0.64, more than 3-fold. Risk arbitrage strategy exposure more than doubled during this time period to 0.14. It is interesting to note that in all these categories, the market beta is cyclical: it was increasing from 1994 through 1997, then it abruptly dropped and stayed low for 7 years, and started to increase in 2003. Similar behavior is also observed for Convertible bond arbitrage, Distressed, and Dedicated short seller (for this strategy, the exposure is increasing in the negative direction). This cyclical behavior in market beta can be largely attributed to the changes in market regimes. For example, as was found in Figure 1, we observe that in the first part of the sample, the S&P 500 returns are frequently characterized by the normal regime 1, in particular from July 1994 to December 1996. The period from 1997 through 2003 is characterized mostly by two other regimes: bubble and crisis. The last part of the sample from 2003 through 2005 is characterized by the normal regime.

Figure 5 shows a different story compared to results depicted in Figure 4. Here, we show strategies which have a recent decrease either in volatility of the market beta or decrease in the market beta estimate implied from the data. For example, for the Event driven multi-strategy, the exposure decreased from 0.17 in 2003 to 0.14 in March, 2005. For the Emerging markets category, since 1997 to 2003, the exposure fluctuated a lot, from 0.2 to 0.5. However, since the beginning of 2003, up to the end of the data sample, the market exposure equilibrated at 0.41. The same behavior is observed for Equity market neutral strategy, where from 1997 to 2003, the exposure fluctuated from 0.03 to 0.09, equilibrating to 0.07 starting the middle of 2003.

Moreover, this framework can be extended to calculating expected hedge fund exposures to different factors one-month from now, 6-months from now, 1-year from now and so on. Our flexible approach can allow us to calculate expected time-varying betas for t+k periods by using specification 11.

In addition to the derivation of the expected market exposures, the switching regime beta model is able to show the evolution of idiosyncratic risk of hedge funds. In particular our estimation of the Markov chain for the idiosyncratic risk of the hedge funds shows that they are characterized by two different regimes with high and low volatility. The evolution of the probability of being in the high volatility regime for the idiosyncratic risk factor is presented in Figure 6.

Figure 6 plots monthly probabilities from January 1994 to March 2005 of hedge fund indexes facing a high volatility regime for the idiosyncratic factor, i.e. volatility of the hedge fund not related to the volatility of the S&P 500 index. We see that the evolution of the volatility of different strategies is quite different. In particular, we observe that Long/Short Equity and Emerging Market index present a low probability of being in the high volatility regime in the last part of the sample and a high probability in the middle of the sample that corresponds to the series of crises and a bubble from 1997 till 2001 that has amplified the risk faced by S&P 500 already captured by the switching beta. This indicates that not only the link with the S&P 500 is changing, but that the idiosyncratic risk of the hedge fund may switch to the high volatility regime at the same time when the market is characterized by turbulence.

Event Driven Multi-Strategy almost always is characterized by the low volatility regime for its idiosyncratic risk factor; however, the probability of a high volatility regime greatly increases for periods characterized by high illiquidity events and other unexpected shocks not correlated with market returns. For example, in February 1994, the U.S. Federal Reserve started a tightening cycle that caught many hedge funds by surprise, causing significant



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Figure 4: The evolution of market betas for Hedge fund index, Long/Short Equity and Risk Arbitrage strategies from January 1994 to March 2005.



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Figure 5: The evolution of market betas for Emerging Markets, Equity Market Neutral and Event Driven Multi-Strategy strategies from January 1994 to March 2005.



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Figure 6: The evolution of the probability of being in the high volatility regime for the idiosyncratic risk factor from January 1994 to March 2005 for Convertible Bond Arbitrage, Long/Short Equity, Emerging Markets, and Event Driven Multi-Strategy.

dislocation in bond markets worldwide; the end of 1994 witnessed the start of the "Tequila Crisis" in Mexico; in August 1998, Russia defaulted on its government debt leading to a liquidity crunch in worldwide financial markets; the first quarter of 2000 saw a crash of the Internet boom, and in middle of 2002 there was a drying out of merger activities, decrease in defaults and release of news about WorldCom accounting problems. During all of these periods, the probability of a high volatility regime skyrocketed, reaching 1 for the Russian default crisis.

The most interesting indicator is the evolution of being in the high volatility regime by the Convertible Arbitrage index that indicates that it has moved to a large volatility regime from the end of 2003 and is still characterized by this regime at the end of the sample. If we consider jointly the state of the market index (normal time in the last two years) and the state of the idiosyncratic factor for the Convertible Arbitrage index, we see that the switching regime beta model is able to disentangle whether the source of risk is characterized by market conditions or by potential distress in the hedge fund index strategy. Not surprisingly, April of 2005 (not in the sample period) has seen extremely low returns and high liquidations in the Convertible Bond Arbitrage sector. Just tracking market exposure will not lead to this predictive result. We are not plotting the probability for the other hedge fund indexes but the results shows that Risk Arbitrage presents an evolution of probability largely similar to the Event Driven market strategy and Dedicated Shortseller is very similar to Long/Short Equity.

4.3.2 Multifactor model

Multifactor model with non-linear exposure only to S&P 500

As discussed above, other factors are affecting hedge fund index returns and this calls for the use of a multifactor framework. We begin with a comprehensive set of risk factors that will be candidates for each of the risk models, covering stocks, bonds, currencies, commodities and volatility. These factors are presented in Table 5. They are also described by Chan, Getmansky, Haas and Lo (2005). We use step-wise approach to limit the final list of factors for our analysis. Using a combination of statistical methods and empirical judgement, we use these factors to estimate risk models for the 8 hedge fund indexes. In all our analysis, hedge fund returns, S&P 500, USD, Lehman Government Credit and Gold are used in excess of LIBOR returns.

We first consider the model presented in equation (16) and the results for this model are contained in Table 6. Here, we are considering linear factors: Large-Small, Value-Growth,

Variable	Abbreviati	Definition
S&P500	SP	Monthly return of the S&P 500 index including dividends
Large-Small	LS	Monthly return difference between Russell 1000 and Russell 2000 indexes
Value-Growth	VG	Monthly return difference between Russell 1000 Value and Growth indexes
USD	USD	Monthly return on Bank of England Trade Weighted Index
Lehman Government Credit	L.GC	Monthly return of the Lehman U.S. Aggregated Government/Credit index
Term Spread	TS	10-year T Bond minus 6-month LIBOR
VIX	VIX	Implied volatility based on the CBOE's OEX options.
Credit Spread	CS	The difference between BAA and AAA indexes provide by Moody's
Gold	Gold	Monthly return using gold bullion \$/Troy Oz. Price

Table 5: Definitions of aggregate measures of market conditions and risk factors. All variables except VIX are obtained using Datastream. VIX is obtained from the CBOE.

USD, Lehman Government Credit, Term Spread, VIX, Credit Spread and Gold and nonlinear exposure to different states of the S&P 500.

The number of factors F selected for each risk model varies from a minimum of 2 for Risk Arbitrage to a maximum of 6 for the Event Driven Multi-Strategy, not including the S&P 500 index. This pattern is plausible because the Event Driven Multi-Strategy by definition includes a broad set of strategies hence a broad array of risk factors is needed to capture the variation in this category versus other categories.

The statistical significance of the factor loadings on S&P 500 conditional on the different regimes is almost the same as the one obtained in the previous analysis with only the S&P 500 risk factor. The only main difference is the exposure of the Risk Arbitrage strategy in the crisis state to the S&P500 and the exposure of the Emerging Markets strategy in the bubble state to the S&P 500. This indicates that the analysis performed above is robust to the inclusion of other factors that may affect hedge index returns.

Regarding the Fama and French factor: Large Minus Small, we observe that this factor is relevant for almost all the hedge fund index strategies, the only exception is Equity Market Neutral. The exposure to the Large Minus Small factor is negative for almost all the Hedge funds indexes (the only exception is the Dedicated Short Bias) suggesting that returns of these hedge indexes resemble those achieved by going long on small stocks and short on large stocks (as shown already by Agarwal and Naik (2004) and Chan et al (2005)). Another potential explanation is that this factor is capturing liquidity risk as highlighted by Acharya et al (2004).

The hedge fund exposure to another Fama and French factor: Value Minus Growth is positive for Convertible Bond Arbitrage, Dedicated Shortseller and Risk Arbitrage and negative for Long/Short Equity strategy.

A detailed analysis of other factors is presented for each hedge fund strategy. The list of factors and hedge fund exposures to these factors is unique for each hedge fund strategy; therefore, we are analyzing them one by one for each hedge fund strategy.

Convertible Bond Arbitrage

This strategy is characterized by investing in a company bond while shorting the common stock of the same company. Positions are designed to protect the principal from market moves. In all regimes, the convertible bond arbitrage strategy is not correlated with S&P 500 moves. The strategy is doing better when returns on small and value stocks are high. Clearly, because the strategy is designed to profit from upward fixed income moves, the strategy is positively related to the Lehman Brothers Government/Credit bond index returns. The most significant coefficient in the regression is -1.77 effect of Credit Spread. Clearly, when credit spread increases, liquidity decreases and there is a low demand for low-credit

Estimate	Convertible	Dedicated	Emerging	Equity	Long/Short	Distressed	Event	Risk Arb
	Bond Arb	Shortseller	Markets	Market	Equity		Driven	
				Neutral			Multi-	
							Strategy	
α_0	0.79	0.48	1.01	-0.95	0.17	0.66	0.50	0.06
α_1	-0.38	-0.78	0.48	0.37	0.49	-3.71	-4.41	0.48
β_0 (SP)	0.04	-0.87	0.23	0.14	0.23	0.14	0.26	0.09
β_1 (SP)	0.02	-1.13	0.39	0.04	0.48	0.31	0.13	0.15
β_2 (SP)	0.01	-0.79	0.55	0.05	0.27	0.18	0.24	0.17
θ_1 (LS)	-0.05	0.59	-0.23	0.00	-0.34	-0.13	-0.17	-0.14
θ_2 (VG)	0.05	0.20	-0.01	0.00	-0.15	0.03	0.03	0.07
θ_3 (USD)			0.30				0.14	
θ_4 (L.GC)	0.13		0.57		0.31	0.20	0.13	
θ_5 (TS)			0.96	0.16	-0.48			
θ_6 (VIX)						-1.74	0.08	
θ_7 (CS)	-1.77						-2.17	
θ_8 (Gold)			0.12					
ω ₀	0.34	1.51	1.29	0.59	0.92	1.08	0.95	0.74
ω_1	1.65	3.35	4.85	0.71	2.67	3.44	3.73	1.29
p_{00}^{Z}	0.85	0.59	0.98	1.00	0.99	0.98	0.99	0.99
p_{11}^{Z}	0.85	0.42	1.00	0.99	0.97	0.49	0.70	0.96

Table 6: The exposure of CSFB/Tremont hedge-fund index strategies to different S&P 500 regimes. The following model is estimated: $R_t = \alpha(Z_t) + \beta(S_t)I_t + \sum_{k=1}^{K} \theta_k F_{kt} + \omega(Z_t)u_t, I_t = \mu(S_t) + \sigma(S_t)\epsilon_t$. Hedge fund returns, S&P 500, USD, Lehman Government Credit and Gold are used in excess of LIBOR returns. Parameters significantly different from zero at the 10% level are shown in bold type.

Estimate	Convertible	Dedicated	Emerging	Equity	Long/Short	Distressed	Event	Risk Arb
	Bond Arb	Shortseller	Markets	Market	Equity		Driven	
				Neutral			Multi-	
~	0.00	0.71	0.51	0.90	0.01	0.20	Strategy	0.06
α ₀	0.88	-0./1	0.51	0.80	-0.01	0.29	0.54	0.00
α_1	-0.40	-0.01	0.05	-0.05	0.19	0.03	-1.45	0.19
$p_0(SP)$	0.07	-0.//	0.89	0.16	0.63	0.03	0.30	0.14
β_1 (SP)	0.04	-1.04	0.55	0.08	0.62	0.31	0.19	0.17
β_2 (SP)	-0.08	-0.69	0.22	0.03	0.26	0.40	0.19	0.15
θ_{1_0} (LS)	-0.07	0.15	-1.00	-0.16	-0.60	-0.22	-0.31	-0.27
θ_{1_1} (LS)	-0.01	0.82	-0.40	-0.03	-0.40	-0.20	-0.04	-0.10
θ_{1_2} (LS)	-0.12	0.19	-0.23	0.03	-0.35	-0.13	-0.17	-0.18
θ_{2_0} (VG)	0.11	-0.30	0.19	0.16	-0.44	0.08	0.04	0.18
θ_{2_1} (VG)	0.02	0.66	0.59	0.05	0.09	0.24	0.04	-0.15
θ_{2_2} (VG)	0.07	0.29	-0.08	-0.02	-0.13	0.16	0.02	0.11
θ_{3_0} (USD)	0.02	1.67	-0.07		0.41			-0.22
θ_{3_1} (USD)	-0.14	0.32	0.30		0.17			-0.04
θ_{3_2} (USD)	0.13	-0.85	0.35		-0.15			0.09
θ_{4_0} (L.GC)	0.14	1.36	0.28	0.08	-0.05	0.25		-0.23
θ_{4_1} (L.GC)	0.09	0.09	0.61	-0.08	0.05	-0.10		0.07
θ_{4_2} (L.GC)	0.14	-0.93	0.46	0.21	0.64	0.10		0.18
$\theta_{5_0}(TS)$	-0.12	-0.36	1.68	-0.19	-1.31	-0.51		
θ_{5_1} (TS)	0.08	1.20	0.51	-0.38	-0.40	-0.28		
θ_{5_2} (TS)	0.44	-1.60	1.48	0.47	-0.01	0.59		
θ_{6_0} (VIX)	0.06	-0.26	0.66		0.46	-0.09	0.28	0.12
θ_{6_1} (VIX)	-0.06	-0.04	0.48		0.15	0.24	0.01	0.09
θ_{6_2} (VIX)	-0.14	-0.19	-0.30		-0.06	-0.05	-0.02	-0.08
$\theta_{7_0}(CS)$	-0.64	3.07	23.16	0.21	10.77	-6.23	2.87	8.59
θ_{7_1} (CS)	3.02	2.35	-9.74	-0.33	-4.02	-4.14	-5.18	-1.02
θ_{7_2} (CS)	-2.05	-0.47	-0.83	-0.52	-0.32	-3.66	-1.33	0.38
θ_{8_0} (Gold)	-0.05	0.24	-1.02	-0.22		-0.23	-0.33	-0.16
θ_{8_1} (Gold)	0.15	0.19	0.11	0.05		-0.11	-0.01	0.01
θ_{8_2} (Gold)	0.00	-0.48	0.07	-0.04		0.15	0.01	0.05
ω_0	0.24	0.07	0.93	0.38	0.81	0.18	0.82	0.29
ω_1	1.63	3.21	4.61	0.73	2.80	1.54	2.31	0.99
p_{00}^{Z}	0.87	0.53	0.98	0.94	0.98	0.86	0.98	0.97
p ^Z ₁₁	0.89	0.78	1.00	0.98	0.93	0.96	0.84	1.00

Table 7: The exposure of CSFB/Tremont hedge-fund index strategies to different S&P 500 regimes. The following model is estimated: $R_t = \alpha(Z_t) + \beta(S_t)I_t + \sum_{k=1}^{K} \theta_k(S_t)F_{kt} + \omega(Z_t)u_t, I_t = \mu(S_t) + \sigma(S_t)\epsilon_t$. Hedge fund returns, S&P 500, USD, Lehman Government Credit and Gold are used in excess of LIBOR returns. Parameters significantly different from zero at the 10% level are shown in bold type.

securities. Convertible Bond Arbitrage funds mostly hold low-credit securities. At the time of high credit spreads, brokers request a higher hair-cut fee to obtain more leverage. Cost of funding goes up, therefore, the return on the strategy decreases.

Dedicated Shortseller

This strategy is geared to maintain net short position at all times. The highest net negative market exposure is during the normal regime of the market. Dedicated Shortseller strategy is doing well when large and value indexes are performing well.

Emerging Markets

This strategy involves both equity and fixed income investing around the world. The net market exposure is positive in all states of the world, which makes sense because many emerging markets do not allow or have limited short selling and do not offer viable futures or derivative products with which to hedge market exposure. Especially, during crises periods, the exposure is highly positive (0.55) due to inability to hedge this exposure. The strategy is performing well when small stocks are doing better than large stocks, which can be attributed to liquidity premium. The effect of the US Dollar is positive and significant (0.30). Stronger US Dollar increases demand for foreign goods, thus, boosting emerging markets economy. Since many emerging markets funds invest in fixed income, it makes sense that the relationship between the Lehman Brothers Government/Credit index returns and the strategy returns is positive and significant (0.57). Moreover, an increase in the term spread has a positive effect (0.96) on the strategy returns. The strategy is doing well when the yield curve is sloping up.

Equity Market Neutral

The strategy is designed to be market beta neutral, which is confirmed for normal market conditions (0.04 and not significant). However, the strategy is not neutral during market up and down times. Managers are not able to timely put market hedges in place, thus, the strategy is positively exposed to market movements in up (0.14) and down (0.05) conditions. The strategy is also doing well during the periods of the upward sloping yield curve (0.16 effect on the term spread).

Long/Short Equity

The strategy takes both long and short market positions. During the normal times of the market factor, the exposure is 0.48 and it is almost halved in both crisis and bubble periods. The strategy is doing well when small and growth stocks do well. The strategy is doing well during the low interest rate environment (the exposure to the Lehman Brothers Government/Credit index returns is positive = 0.31). Generally, the strategy is doing well when the yield curve is flat. So, if long or short-term rates are changing, then the return of the Long/Short Equity strategy decreases as can be seen by a negative coefficient on the term spread (-0.48).

<u>Distressed</u>

Distressed strategy primarily concentrates on investing in the debt, equity or trade claims of companies in financial distress and generally bankruptcy. There is a modest market exposure during normal times (0.31) and the exposure is halved during crisis and bubble times. The strategy is doing well when small stocks are outperforming their large counterparts. Because the strategy is also investing in fixed income, it is highly positively correlated with the Lehman Brothers Government/Credit index returns (0.20). The strategy is performing well when intra-month volatility of S&P 500 and the implied volatility index are moderate are low and are not increasing, which is confirmed by a negative coefficient (-1.74) on VIX. Generally, a distressed strategy is widely viewed as a short-volatility strategy, which is confirmed by the negative exposure on VIX.

Event Driven Multi-Strategy

This subset refers to hedge funds that draw upon multiple themes, including risk arbitrage, distressed securities, and occasionally others such as investments in micro and small capitalization public companies that are raising money in private capital markets. Fund managers often shift assets between strategies in response to market opportunities. Therefore, the market exposure is positive in all market states. The strategy is doing well when small stocks are outperforming large ones. Event Driven Multi-Strategy managers are opportunistic and therefore when US Dollar is stronger, they have more investing power and can take advantage of more investment opportunities. Therefore, the relationship between the US Dollar and strategy returns is 0.14. Managers in this strategy can also invest in fixed-income assets; therefore, we record a positive exposure (0.13) to the Lehman Brothers Government/Credit index returns. There is a positive, but small exposure to VIX (0.08). The most significant coefficient in the regression is -2.17 effect of Credit Spread. Clearly, when credit spread increases, liquidity decreases and there is a low demand for low-credit securities. Event Driven Multi-Strategy funds mostly hold low-credit securities. At the time of high credit spreads, brokers request a higher hair-cut fee to obtain more leverage. Cost of funding goes up, therefore, the return on the strategy decreases.

Risk Arbitrage

Specialists invest simultaneously in long and short positions in both companies involved in a merger or acquisition. Risk arbitrageurs are typically long the stock of the company being acquired and short the stock of the acquiring company. Market exposure is positive especially in crises periods (0.17), indicating that managers in this strategy take a lot of risk. The strategy is correlated with the performance of small versus large stocks (-0.14). There is a small premium to value stocks (0.07).

Multifactor model with non-linear exposure to all factors

Finally we estimate the multifactor model specified in equation (17) and the results are contained in Table 7. Here, we are considering non-linear exposure to factors: S&P 500, Large-Small, Value-Growth, USD, Lehman Government Credit, Term Spread, VIX, Credit Spread and Gold. Almost all hedge fund indexes have a significant exposure to the Large Minus Small factor in at least two out of three states. All strategies have exposure to the S&P 500 even after accounting for conditional exposure to other risk factors. Generally, we find that the model that accounts for different factors conditional on the state of the market is richer and captures more exposures compared to previous models. Moreover, the model is showing that factor exposure is changing conditional on the state of the market. Below we consider each strategy separately and address time-varying risk exposures for various factors.

Convertible Bond Arbitrage

Compared to previous results, the Convertible Bond Arbitrage strategy has a significant positive exposure to the S&P 500 during bubble times and significant negative exposure to the market during crises times. During normal times, there is no exposure because managers typically perfectly hedge market fluctuations. However, during bubble and crises periods, convertible bond arb. managers cannot perfectly hedge their market exposure, this is especially true for crises times. Therefore, during market up turns, the strategy is positively related to the market and negatively related during market down turns. This result could only be obtained after correctly adjusting for non-linear exposures to all factors. The strategy has a positive exposure to credit spread when market is in the normal times (3.02). The spread reflects investor perception relating to how likely the issuing company will be able to make timely interest payments and pay off the principal at maturity. The larger, or wider, the spread, the more concern investors have regarding the issuing company's ability to make timely interest payments. During normal times of the market, investors are less worried about the increase in credit spread. However, during crises times, this worry is more sound and the strategy is negatively compensated for having a high credit spread (-2.05). During bubble times of the market, investors become worried knowing that convertible funds tend to short stock (-0.64). During normal and crises states of the market, the effect of term spread is positive and significant (0.08 and 0.44 respectively). In a way, the managers implement going long on longer maturity bond and shorting a shorter maturity bond. In crises states of the market, this strategy is very vulnerable to liquidity shocks. If the short-term interest rate is rising, the strategy will lose money in those states. However, in a bubble state of the market, the relationship between the term spread and the strategy is negative. The Convertible bond arbitrage managers tend to go long on short-term bonds and short longerterm bonds. VIX is a variable that needs to be interpreted jointly with different regimes

of the S&P 500 because changes in S&P 500 are captured by varying betas for different regimes of the market. For the Convertible Bond Arbitrage strategy, the effect of VIX is negative in normal and crises markets (-0.06 and -0.14 respectively). The result shows that the strategy is short on volatility after adjusting for S&P 500. During a crisis, managers are short market, so they try to reduce the exposure to the volatility of the S&P 500. This result is also confirmed by Hutchinson and Gallagher (2005) who find negative exposure to VIX in market downturns.

Dedicated Shortseller

This strategy has significant exposure to all factors in different states of the world. Gold is acting as a substitution asset for when the market is in a crisis (-0.48) compared to when the market is in a bubble (0.24). The exposure to credit spread in a crisis period is (-0.47) and positive in both bubble (3.07) and normal (2.35) periods signifying that dedicated shortseller often holds high credit securities that do well in increasing credit spread times.

Emerging Markets

The exposure to credit spread in a crisis period is (-0.83) and in a normal period is (-9.74), whereas, in a bubble period is (23.16). It can be interpreted that generally emerging markets managers tend to buy low liquidity, low credit rated instruments; however, in the up-market, they quickly adjust their exposure to buying high credit securities. The Emerging Markets strategy has a positive exposure to VIX in market up-turns (0.66) and negative exposure to VIX in market down-turns (-0.30). As was found in the previous table, the exposure of the strategy to the US Dollar is 0.30. It is confirmed here for normal market periods. The relationship stays positive (0.35) for market crisis periods; however, the relationship is reversed for market booms. During these times, if US Dollar is highly overvalued, that will lead to a sharp decline in returns of emerging market instruments, thus, leading to the decline in the emerging markets returns (-0.07).

Equity Market Neutral

The Equity Market Neutral strategy is doing well when the short-term rate is at a high level and is not increasing and the long-term rate is declining. Therefore, when term spread is increasing, the strategy is not doing well, which is evidenced by negative exposure to term spread in both bubble (-0.19) and normal market states (-0.38). However, in market crises periods, managers tend to go long on more liquid shorter maturity bonds and shorting longer maturity bonds. Thus, in a crisis state of the market, the relationship between the term spread and the strategy is positive (0.47). During normal and crises states of the world, the increase of credit risk premium signals flight to quality, therefore, there is a negative relationship between credit spread and the hedge fund strategy. However, during upward trends of the market, this relationship is reversed (0.21). Gold has an interesting effect on the strategy. On average, the relationship is insignificant; however, during market bubble or crises states, the relationship is negative and significant.

Long/Short Equity

The US Dollar acts as a substitution asset for this strategy when the market is in a bubble (0.41) compared to when the market is in crisis (-0.15). The Long/Short Equity strategy is doing generally poorly when the long-term yield is increasing. In the crisis state of the market, the effect of the term spread on the Long/Short Equity returns is -1.31. The exposure to credit spread in a crisis period is (-0.32) and in a normal period is (-4.02), whereas, in a bubble period is (10.77). It can be interpreted that generally Long/Short Equity managers tend to buy low liquidity, low credit rated instruments; however, in the up-market, they quickly adjust their exposure to buying high credit securities. This might make sense because managers tend to take concentrated trades in specific sectors or markets.

Distressed

The exposure to credit spread is negative and significant during all regimes of the credit spread, meaning that distressed funds always hold illiquid and low quality securities. This strategy is usually categorized by taking positions in companies that will do better in the future through restructuring and other means. Therefore, an increase in credit spread sends a signal that these companies might have an inability to make timely interest payments. Therefore, the relationship between credit spread and Distressed returns is negative in all states of the market factor. Interestingly, when the market is in a bubble, the exposure of the Distressed category to gold is negative and significant (-0.23). When the market is in a crisis, the exposure of this hedge fund strategy to gold is positive and significant (0.15). Therefore, gold acts as a substitution asset to the market when the market market is in crisis. Distressed funds also hold bonds; therefore, during bubble periods, the exposure to bonds is positive (0.25); however, it is insignificant during crises periods.

Event Driven Multi-Strategy

Similar to previous results, the exposure to VIX is positive and significant, especially during bubble periods (0.28). The strategy has a high positive exposure to credit spread in market bubble (2.87) and negative credit spread exposure during normal (-5.18) and crises time periods (-1.33). Generally, event driven types of strategies do well when credit risk premium is moderate and is declining. However, unlike Distressed strategy managers, managers in this strategy might bet on merger or engage in other strategies during market upturns. Similar to the result for Distressed funds, gold acts as a substitution asset to the market when the market is in crisis. The exposure in crisis is 0.01 and exposure in a bubble is -0.33.

Risk Arbitrage

The exposure to VIX is positive and significant, especially during bubble periods (0.28), but negative during crisis periods (-0.08). Risk Arbitrage strategy is concerned with the success of a merger. Generally, during normal times, the exposure to credit spread is negative (-1.02) as investors are concerned with the ability of a company to make timely interest payments. However, during crises or bubble periods, the strategy is concentrating more on high quality issues and the exposure to credit spread in those states is positive (0.38 and 8.59 respectively). The exposure to gold is positive and significant (0.05) only in the crises state of the world. So, gold acts like a good hedge when the market is in down turn. Also, US Dollar is acting like a hedge when the market is in down turn. During this period, the exposure to US Dollar is 0.09.

Generally, our results show that factor exposures are different for various factors conditional on the state of the market risk factor. A model that accounts for different factors conditional on the state of the market is richer and captures more exposures compared to previous models. Moreover, the model is showing that factor exposure is changing conditional on the state of the market. This confirms our initial hypothesis that the exposures to different risk factors are time-varying and conditional on the state of the market risk factor. Indeed, for many factors we observe that the risk exposure is zero or positive, and suddenly becomes negative or opposite during crises periods. There are two simultaneous relationships that cause this time-varying change in factor exposures. The first link is between the S&P 500 and factors that is changing based on the states of the world of the market factor. The second link is between hedge funds and these factors. The exposure of hedge funds to different factors changes. Our methodology allows us to isolate different factor exposure when the market is moving. We can separate factor exposures during various market conditions (crisis, normal and bubble). In the future work, we can use individual hedge funds to disentangle the two effects that cause time-varying change in factor exposures and conduct an attribution analysis.

5 Conclusion

In this paper we characterized the exposure of hedge fund indexes to risk factors using switching regime beta models. This approach allows to analyze time-varying risk exposure for hedge funds, and in particular, the changes in hedge fund exposure conditional on different states of various risk factors. We have three main results. First, we find that it is important to analyze hedge fund exposure conditional on the different states of the market index because exposures may change when the market index switches from the normal state to the bubble or crisis states. Second, the use of switching beta models that allows for a non-linear relationship among hedge fund returns and risk factors highlights that these hedge funds also exhibit significant non-linear exposure to Fama and French's (1993) size and value factors, bonds, currencies, commodities and volatility. In particular, we show that exposures can be strongly different in the crisis regimes compared to normal times suggesting that risk exposures of hedge funds in the crisis regimes are quite different than those faced during normal regimes. Therefore, investigation of risk exposure of hedge funds requires a deep analysis of the evolution of time-varying risk exposure, and more specifically of hedge fund tail-event behaviour. Third, the extension of the regime switching model to allow for non-linearity in residuals has suggested that switching regime models are able to capture and forecast the evolution of the idiosyncratic risk factor in terms of changes from a low volatility regime to a distressed state not directly related to market risk factors. In particular, our analysis shows that the convertible bond arbitrage distress observed in the recent period is not related to a particular regime of the market index or some other systemic risk factor, but to a switch in the volatility of the idiosyncratic risk factor of such category. Our sample is not including much of the period that has characterized the distress of the Convertible Bond Arbitrage strategy, but our estimation has allowed to forecast this potential evolution of this strategy highlighting already at the beginning of 2004 that this strategy may be entering a challenging period.

A Appendix

This appendix contains the TASS category definitions in Section A.1.

A.1 TASS Category Definitions

The following is a list of category descriptions, taken directly from TASS documentation, that define the criteria used by TASS in assigning funds in their database to one of 11 possible categories:

- **Convertible Arbitrage** This strategy is identified by hedge investing in the convertible securities of a company. A typical investment is to be long the convertible bond and short the common stock of the same company. Positions are designed to generate profits from the fixed income security as well as the short sale of stock, while protecting principal from market moves.
- **Dedicated Shortseller** Dedicated short sellers were once a robust category of hedge funds before the long bull market rendered the strategy difficult to implement. A new category, short biased, has emerged. The strategy is to maintain net short as opposed to pure short exposure. Short biased managers take short positions in mostly equities and derivatives. The short bias of a manager's portfolio must be constantly greater than zero to be classified in this category.
- **Emerging Markets** This strategy involves equity or fixed income investing in emerging markets around the world. Because many emerging markets do not allow short selling, nor offer viable futures or other derivative products with which to hedge, emerging market investing often employs a long-only strategy.
- **Equity Market Neutral** This investment strategy is designed to exploit equity market inefficiencies and usually involves being simultaneously long and short matched equity portfolios of the same size within a country. Market neutral portfolios are designed to be either beta or currency neutral, or both. Well-designed portfolios typically control for industry, sector, market capitalization, and other exposures. Leverage is often applied to enhance returns.
- **Distressed** Fund managers invest in the debt, equity or trade claims of companies in financial distress and generally bankruptcy. The securities of companies in need of legal action or restructuring to revive financial stability typically trade at substantial discounts to par value and thereby attract investments when managers perceive a turn-around will materialize. Managers may also take arbitrage positions within a company's capital structure, typically by purchasing a senior debt tier and short-selling common stock, in the hopes of realizing returns from shifts in the spread between the two tiers.
- **Event Driven Multi-Strategy** This subset refers to hedge funds that draw upon multiple themes, including risk arbitrage, distressed securities, and occasionally others such as investments in micro and small capitalization public companies that are raising money in private capital markets. Fund managers often shift assets between strategies in response to market opportunities.
- **Risk Arbitrage** Specialists invest simultaneously in long and short positions in both companies involved in a merger or acquisition. Risk arbitrageurs are typically long the stock of the company being acquired and short the stock of the acquiring company. The principal risk is deal risk, should the deal fail to close.

Long/Short Equity This directional strategy involves equity-oriented investing on both the long and short sides of the market. The objective is not to be market neutral. Managers have the ability to shift from value to growth, from small to medium to large capitalization stocks, and from a net long position to a net short position. Managers may use futures and options to hedge. The focus may be regional, such as long/short U.S. or European equity, or sector specific, such as long and short technology or healthcare stocks. Long/short equity funds tend to build and hold portfolios that are substantially more concentrated than those of traditional stock funds.

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