Srategic Specialist and Market Liquidity

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Abstract

The empirical literature suggests that the limit order book contains information that might be used by the specialist for his own advantage. We develop a model of insider trading where there is a specialist who has access to the order book and informed traders who receive information about the liquidation value of the asset. The presence of a strategic specialist in the market induces non-monotonicity of market indicators with respect to the variance of liquidation value. Moreover, the existence of private information about supply significantly affects market performance as it induces, among other effects, lower market liquidity. Finally, our model suggests another link between Kyle's (1985, 1989) and Glosten and Milgrom's (1985) models by allowing for strategic behavior of the specialist.

JEL Classification numbers: D82, G12, G14.

Keywords: Insider trading, Imperfect competition, Market liquidity.

*I am very grateful to Jordi Caballé for his valuable comments and kind guidance. I would also like to thank Cecilia Caglio, Amil Dasgupta, Steffen Huck, Carolina Manzano, Giovanna Nicodano, David Pérez-Castrillo, Avi Wohl and seminar participants at Universitat Autònoma de Barcelona, University College London, Universidad de Guanajuato, Universitat Pompeu Fabra, EEA Meeting 2005, FMA Chicago 2005, International Conference in Finance, Copenhagen 2005, FMA Siena 2005, CEPR/Studienzentrum Gerzensee European Summer Symposium in Financial Markets, XII Finance Forum and XXIX Simposio de Análisis Económico for their very helpful comments. Any errors are my own responsability.

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1 Introduction

The creation of new markets over the last years has emphasized the importance of market performance and market design and led to important changes in the regulation and structure of securities markets. One of the most studied problems is the issue of market transparency - the ability of traders to observe information during their trading. An important question is what type of information the traders might observe or have access to different sources of information: information about fundamentals or information about supply. On the one hand, there are agents in the market who acquire information about fundamentals, which are predictors of future prices. On the other, there are agents who, due to their position in the market, might have access to the order book and can therefore gather information about the supply side of the market. These traders are in general intermediaries but their responsibilities might differ depending on the trading system of each exchange. They might be a NYSE specialist, a "Makler" at Frankfurt Stock Exchange or a "Saitori" at Tokyo Stock Exchange. Their main obligations are to maintain a "fair and orderly" market, to increase market performance and attract more investors. However, the most important common feature in all the exchanges is the role played in supplying liquidity. Since these specialists can see the limit order book, they can see the incoming orders before anyone else and therefore they enjoy an informational advantage. It has been shown that in an imperfect competitive setup, traders exploit their informational advantage by taking into account the effect the quantity they choose is expected to have on both the price and the strategy adopted by other traders. The strategic use of this private information is even more important when we consider different types of information. The aim of this paper is to analyze the process through which different types of information are transmitted to prices and the implications of strategic trading on the market performance in this new setup. In order to do so, we develop a model of insider trading in the context of an imperfectly competitive market - similar to Kyle (1989) - where agents have private information either about future prices or about supply. This distinction between value-informed traders and supply-informed traders is designed to capture the different types of information that influence the security prices at any given point in time and the effect of the presence of a strategic specialist on market performance.

In the rational expectations paradigm, traders understand that prices reveal the information they have when they choose the quantities to be traded. The link between information and prices via trades provides an explicit mechanism for information transmission between traders. The existence of private information means that a trader may have incentives to act strategically in order to maximize his profits. Therefore, given his private information, a trader maximizes his conditional expected profits taking into account the effect of his trading on prices and taking as given the strategies other traders use to choose their demand schedules. As in the imperfect competition model of Kyle (1989), we further assume that all traders strategically choose the amounts they trade. Therefore, the specialist also chooses his demand, taking into account the effect of his trading on prices and revealing some information about the shock in supply to other market participants. As a result, in our model both the information about the value of the asset and about supply is revealed through the quantities to be traded.

Kyle (1989), to which our work is closely related, proposes an imperfect competition model in which there are noise traders, price-informed traders and uninformed traders who submit limit orders.[1] He shows that a strategic trader acts as he trades against a residual supply curve. This implies lower quantities by comparison with the competitive rational expectations equilibrium and, consequently, equilibrium prices reveal less information than in the competitive case. As will be emphasized in this paper, the dual role of prices in aggregating information and clearing the market is even more important when we have different types of information.

Our model bears some resemblance with the literature that studies the role of a specialist with market power. Glosten (1989) studies the strategic behavior of the specialist when setting the prices and emphasizes the role played by a monopolistic rather than competitive specialist on social welfare. Hagerty (1991) studies monopolistic competition between specialists when securities are independently distributed. This

[1] Both the theoretical and empirical literature that studies limit orders and their implications on market liquidity are very rich: Rock (1996), Glosten (1994), Chakravarty and Holden (1995), Seppi (1997), Parlour (1998), Parlour and Seppi (2003), Sandas (2001), Foucault et al. (2005), Biais et al (1995), Hollifield et al. (1999), Hasbrouck and Saar (2002).

assumption is relaxed by Gehrih and Jackson, (1998) who isolate the effect of indirect competition between the specialists and the intra-asset competition. Finally, Seppi (1997) studies competition of a specialist in the presence of informative limit orders and public priority rules where the limit order book is common knowledge.[2]

The empirical work of Cao et al. (2003) and Harris and Panchapagesan (2005) provides evidence that the limit order book contains information that can help in predicting future prices and that the specialists use this information to their own advantage when competing with the limit orders for the provision of liquidity. In our model, we emphasize exactly this feature of the specialist: he knows the limit order book, while the other informed traders have only private information about the liquidation value. We stress here the fact that the specialist maintains the limit order book, but also trades for his own account, making a market in that stock. This feature of the specialist together with the mechanism of trading drives the results of our model.

Consequently, our model suggests that allowing the specialist to behave strategically plays an important role in market-making and in information aggregation. Thus the presence of a strategic specialist who has private information about the limit order book worsens off the market performance. Our results capture the intuition of Boehmer et al. (2005) that increased transparency of the limit-order book is beneficial for market performance. Indeed, we find that the strategic specialist decreases market depth and increases the volatility of prices and the amount of information revealed in prices. Moreover, unlike in the perfectly competitive case, this trader also makes positive profits. An important implication of our model is that the presence of different types of information in the market decreases market liquidity.[3] This result is similar to the one in the dual trading literature and this is not at all surprising. Despite initially pos-

[2]The policy changes undertaken recently generated also a large body of literature concerned with the effect of the specialist on market performance. The differences in the performance of specialist firms was studied by Corwin (1999), Cao et al. (1997) and Coughenour and Deli (2002) and they show that the differences in liquidity provision are due to the organizational form, execution costs, the use of trading halts, and market stabilization.

[3]A similar result is obtained by Fishman and Hagerty (1995), who show that contrary to the belief that more information about the informed agents' trades limits their potential profits, mandatory disclosure of trades leads to a less liquid market.

sessing only one type of information, both value-informed and supply-informed traders end up trading on the two types of information, as the brokers-dealers do in the dual trading literature. However, unlike in this literature, in our model we also obtain other important implications with respect to market performance: both the informativeness of prices and volatility of prices being affected.

Another important result of our work is the way the asymmetry of information affects the market performance. On the one hand, we obtain a result consistent with the stylized facts from the empirical literature: the higher the asymmetry of information, the higher is the volume of trading. On the other hand, we find that the strategic specialist induces non-monotonicity of the market depth and other market indicators with respect to the asymmetry of information (variance of liquidation value).

Finally, note that our model is related to Kyle's (1989) but produces results consistent with Glosten and Milgrom's (1985), which shows that more information in the market leads to an increase in the bid-ask spread (i.e. a decrease in the market liquidity). As shown by Krishnan (1992) and Back and Baruch (2004), the two separate strands of literature (to which Kyle (1985) and Glosten and Milgrom (1985) belong, respectively) are in fact intertwined. The suggested link is an equivalence between the extensive forms in Krishnan (1992), and a convergence process of the equilibria in Glosten and Milgrom (1985) to the equilibrium in Kyle (1985) in Back and Baruch (2004). Our work suggests that the compatibility of the results produced by the two families of models may have a dimension other than the ones revealed in Krishnan (1992) and Back and Baruch (2004) by allowing for strategic behavior by informed dealer.

The remainder of this paper is organized as follows. Section 2 presents the model. We establish the information structure and define the imperfect competitive rational equilibrium expectations. We characterize the equilibrium both in the case with a strategic specialist and the benchmark case without a strategic specialist. We find a unique linear imperfect competitive rational expectations price function together with agents' demand functions in equilibrium. Section 3 proceeds with the calculation of some market indicators: volatility of prices, informativeness of prices and expected profits and then Section 4 compares the market indicators in the two cases. Finally, Section 5 summarizes the results. All the proofs appear in the appendix.

2 The Model

We consider a similar framework to the one in Kyle (1989) in which we add the strategic specialist. In order to be able to emphasize the role of the specialist we consider a simpler setup where traders are risk neutral and there are no uninformed traders. As already pointed out by Kyle (1989), the assumption of the existence of uninformed traders does not change the analysis, but their presence leads to an increase in market depth.

The specialist system appeared when traders realized that they could be more successful if they concentrate their trading on a small number of stocks. By understanding the reason of trading, the identity of the traders and the amount they trade the specialist can trade more successfully than other dealers. When a security is listed in NYSE, several specialists are invited to apply, but only one is selected. Since the specialist has this privileged position, the Exchange insures that the one selected satisfies the regulatory guidelines found in Exchange's Allocation Policy and Procedures. In order to emphasize the role played by the specialist we will model the noise by assuming a random supply and that the specialist is the only one who receives information about it.[4] The presence of shocks in supply has a significant price impact. A supply shock leads to a change in prices and this makes investors revise their expectations. However, if the supply shock is observable by some traders, these traders make use of their informational advantage and therefore are willing to adjust their demand. Consequently, we assume that the specialist, by having access to the order book, acts as a supply-informed trader who receives a signal about supply. This approach was used before by Gennotte and Leland (1990) who consider a model where speculators possess private and diverse information.[5] They consider price-taker speculators who gather information

[4] In the Kyle-type models noise is needed in the model to prevent prices from being fully revealing. To overcome this difficulty, several ways of introducing noise were used: adding noise traders, considering uncertainty which has a dimension greater than that of price, or assuming that the aggregate endowment is imperfectly observed.

[5] A similar assumption is that market makers have some information about the uninformed order flow and it can be found in Admati and Pfleiderer (1991) and Madhavan (1992). Palomino (2001) considers also a setup where the informed agents have information both about the liquidation value and the quantity traded by one of the noise traders.

either about prices or about supply and show that these informational differences can cause financial markets to be relatively illiquid. Our model builds on the assumption made by Gennotte and Leland (1990) concerning the existence of a random supply and supply-informed speculators but we consider an imperfect competition setup with both value-informed and supply-informed agents where the agents submit limit orders. The information revelation is increased significantly in our setup since the agents are placing limit orders and therefore, they condition their demands on prices and hence, infer part of the others' information. Moreover, since in general, there is only one specialist trading in the stock, we assume here that there is only one supply-informed trader.

In what follows, we make the following assumptions:

A.1 There is a single security in the market that trades at market clearing price $\widetilde{p}$ and yields an exogenous liquidation value $\widetilde{v}$, which has a normal distribution with mean $\overline{v}$ and variance σ_v^2.

A.2 There are N value-informed traders, indexed $n = 1, ..., N$ and a supply-informed trader - the specialist. The price informed trader n observes a private signal $\widetilde{i}_n = \widetilde{v} + \widetilde{e}_n$. We assume that e_n is distributed $N(0, \sigma_e^2)$ for all $n = 1, ..., N$. We suppose that for any $j \neq n$ $\widetilde{e}_j$ and $\widetilde{e}_n$ are uncorrelated and moreover, they are uncorrelated with all the other random variables in the model. The supply-informed trader observes a private signal $\widetilde{S}$ which is normal distributed with mean 0 and variance $\sigma_S^2 > 0$.

A.3 The net supply $\widetilde{m}$ consists of a fixed amount $\overline{m}$ and a random supply $\widetilde{S}$ distributed $N\left(0, \sigma_S^2\right)$. This liquidity shock $\widetilde{S}$ is observed only by the supply-informed trader.

A.4 Agents are risk neutral and behave strategically taking into account the effect of their trading on prices.

As in Kyle (1989), the n^{th} value-informed trader has a strategy X_n which is a mapping from $\mathbb{R}^2$ (the Cartesian product of the set of asset prices and the set of his signals) to $\mathbb{R}$ (the set of shares he desires to trade), $X_n(\cdot, \cdot) : \mathbb{R}^2 \longrightarrow \mathbb{R}$. After observing his signal i_n, each value-informed trader submits a demand schedule (or generalized limit order) $X_n\left(\cdot, i_n\right)$, which depends upon his signal. Similarly, the supply-informed trader has a strategy Y, which is a mapping from $\mathbb{R}^2$ (the Cartesian product of the set of asset prices and the set of his signals) to $\mathbb{R}$ (the set of shares he wants to trade), $Y(\cdot,\cdot) : \mathbb{R}^2 \longrightarrow \mathbb{R}$. After observing the signal S, the supply-informed trader chooses a demand schedule $Y(\cdot, S)$, which depends upon that signal. Notice that since $\overline{m}$ is known by everyone, this implies that the supply-informed agent actually knows $\widetilde{m}$. Given a market clearing price p, the quantities traded by value-informed traders and supply-informed trader can be written $x_n = X_n(p, i_n)$, $n = 1, ..., N$ and $y = Y(p, S)$. In the above notations, a tilde distinguishes a random variable from its realization. Thus, x_n denotes a particular realization of $\widetilde{x}_n$. The assumption that the value-informed and the supply-informed agents submit limit orders for execution against existing limit orders submitted by the other market participants turns out to be very important. In this context both the value-informed and the supply-informed agents provide liquidity and therefore, play a market-making role.

The price of the asset is set such that the market clears. The traders submit their demand schedules to an auctioneer who aggregates all the schedules submitted, calculates the market clearing price and allocates quantities to satisfy traders' demand. Thus, the market clearing price $\widetilde{p}$ should satisfy with probability one

$$\sum_{n=1}^{N} X_n\left(\widetilde{p}, \widetilde{i}_n\right) + Y\left(\widetilde{p}, \widetilde{S}\right) = \widetilde{m}. \tag{1}$$

To emphasize the dependence of the market-clearing price on the strategies of the traders we write

$$p = p(X, Y),\ x_n = x_n(X, Y),\ y = y(X, Y),$$

where X is the vector of strategies of value-informed traders defined by $X = (X_1, ..., X_N)$ and Y is the strategy of the supply-informed trader.

The traders are risk neutral and maximize expected profits. The profits of the value-informed trader n and supply-informed trader are, respectively, given by

$$\widetilde{\pi}_n^{PI} = (\widetilde{v} - \widetilde{p}(X, Y))\, \widetilde{x}_n(X, Y),\quad \widetilde{\pi}^{SI} = (\widetilde{v} - \widetilde{p}(X, Y))\, \widetilde{y}(X, Y).$$

With these notations, following Kyle (1989) we can proceed to define a rational expectations equilibrium in our setup.

Definition 1 *An imperfectly competitive rational expectations equilibrium is defined as a vector* (X, Y, p)*, where* X *is a vector of strategies of the value-informed agents* $X = (X_1, ..., X_N)$*,* Y *is a strategy of the supply-informed agent and* p *is the equilibrium price such that the following conditions hold:*

1. For all $n = 1, ..., N$ *and for any alternative strategy vector* X' *differing from* X *only in the* n^{th} *component* X_n*, the strategy* X *yields a higher profit than* X'*:*

$$\begin{aligned} &E_n\left[(\widetilde{v} - \widetilde{p}(X, Y))\widetilde{x}_n(X, Y) |\ \widetilde{p}(X, Y) = p,\ \widetilde{i}_n = i\right] \geq \\ &E_n\left[(\widetilde{v} - \widetilde{p}(X', Y))\widetilde{x}_n(X', Y) |\ \widetilde{p}(X', Y) = p,\ \widetilde{i}_n = i\right]. \end{aligned}$$

2. For any alternative strategy Y' *the strategy* Y *yields a higher profit than* Y' *:*

$$\begin{aligned} &E\left[(\widetilde{v} - \widetilde{p}(X, Y))\widetilde{y}(X, Y) |\ \widetilde{p}(X, Y) = p,\ \widetilde{S} = S\right] \geq \\ &E\left[(\widetilde{v} - \widetilde{p}(X', Y))\widetilde{y}(X, Y') |\ \widetilde{p}(X, Y') = p,\ \widetilde{S} = S\right]. \end{aligned}$$

3. The price $p = \widetilde{p}(X, Y)$ *clears the market (with probability one) i.e.*

$$\sum_{n=1}^{N} X_n\left(\widetilde{p}, \widetilde{i}_n\right) + Y\left(\widetilde{p}, \widetilde{S}\right) = \widetilde{m}.$$

This defines a Nash equilibrium in demand functions. Given their private information, traders maximize their conditional expected profits taking into account the effect of their trading on prices and taking as given the strategies other traders use to choose their demand schedules.

We look for a symmetric linear Bayesian Nash Equilibrium as in Kyle (1989), that is, an equilibrium where the strategies X_n and Y are linear functions:

$$\begin{aligned} &X_n\left(\widetilde{p}, \widetilde{i}_n\right) = \alpha^{PI} + \beta^{PI}\widetilde{i}_n - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1, ..., N \text{ and} \\ &Y\left(\widetilde{p}, \widetilde{S}\right) = \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \end{aligned} \tag{2}$$

where $\alpha^{PI}, \beta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI} \in \mathbb{R}$.

With this assumption we can infer from the market clearing condition that the equilibrium price is given by

$$p = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(N\alpha^{PI} + \alpha^{SI} + \beta^{PI}\sum_{n=1}^{N}\widetilde{i}_n + \left(\beta^{SI} - 1\right)\widetilde{S} - \widetilde{m}\right). \tag{3}$$

2.1 Characterization of the Equilibrium

In the following proposition, we describe the equations that characterize the symmetric Bayesian-Nash equilibrium. This equilibrium has linear trading and pricing rules and is shown to be unique among all linear, symmetric Bayesian-Nash equilibria. As in most Kyle-type models, the linearities are not ex-ante imposed in the agents strategy sets: as long as the informed traders use linear trading strategies, the pricing rule will be linear and vice-versa.

Proposition 1 *If* $N(N-2) \geq \frac{\sigma_e^2}{\sigma_v^2}$ *there exists a unique linear symmetric equilibrium where agents' strategies are given by:*

$$\begin{aligned} X_n\left(\widetilde{p}, \widetilde{i}_n\right) &= \alpha^{PI} + \beta^{PI}\widetilde{i}_n - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1, ..., N \text{ and} \\ Y\left(\widetilde{p}, \widetilde{S}\right) &= \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \end{aligned}$$

with $\alpha^{PI}, \beta^{PI}, \gamma^{PI}$, $\alpha^{SI}, \beta^{SI}, \gamma^{SI}$ *given by*

$$\begin{aligned} \alpha^{PI} &= \frac{\sigma_e^2\left(N\left(3N-2\right)\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\overline{v} + \frac{N\left(N-2\right)\sigma_v^2 - \sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} \\ \beta^{PI} &= \frac{\delta^{1/2}}{2N\left(N\sigma_v^2+\sigma_e^2\right)} \\ \gamma^{PI} &= \frac{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)} \\ \alpha^{SI} &= -\frac{\left(N-1\right)\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\sigma_e^2\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{v} + \frac{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m} \\ \beta^{SI} &= \frac{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}{2N\left(N\sigma_v^2+\sigma_e^2\right)} \\ \gamma^{SI} &= -\frac{\left(N-1\right)\sigma_e^2\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)}, \end{aligned} \tag{4}$$

where

$$\delta \equiv \frac{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2}.$$

The condition $N(N-2) \geq \frac{\sigma_e^2}{\sigma_v^2}$ is similar to the usual condition $N > 2$ in all Kyle-type models. It tells us that we need competition in order to alleviate the asymmetric information problem. In our model, the asymmetric information problem is even more important than in Kyle (1989) because we have two different types of information that aggregate in prices. The supply-informed agent acts as an informational monopolist trading on the information about supply and thus, he always extracts some rents. However, since he submits limit orders he also observes the average of the value-informed agents' signals. Since this average is informationally equivalent to observing the whole vector of private signals, he uses this as a private signal about the liquidation value. However, the quality of this signal depends on the homogeneity of the signals received by value-informed traders. The value-informed traders are asymmetrically informed, so increasing their number will make them compete more aggressively against each other and reveal more information. This increased competition will make the average signal more informative and therefore, the supply-informed agent better informed. Consequently, in the case of heterogeneity of the value-informed traders' signals (σ_e^2/σ_v^2 high), we need more competition in order to refine the final information embedded in prices. As we will explain later, we have a bidirectional flow of information between traders (from the supply-informed trader to the value-informed agents and vice-versa). Where the information received by the value-informed traders is very heterogenous (σ_e^2/σ_v^2 high relatively to $N(N-2)$), the signal on liquidation value inferred by the supply-informed agent from prices is poor. On one hand, the supply-informed trader uses this signal when choosing his trading strategy. Since this signal is erroneous, it alters both his strategy and the information revealed by him about supply. On the other hand, the value-informed agents infer from prices information about supply, but they fail in doing that because the information about supply contained in prices is erroneous (it is based on the poor signal). As a result, when there are not enough value-informed agents to increase the quality of the average signal $\left(N(N-2) < \dfrac{\sigma_e^2}{\sigma_v^2}\right)$, the propagation of this poor quality signal might lead to a situation where equilibrium fails to exist.

Once we have determined the equilibrium demand strategies, we can also determine the market clearing price.

Corollary 2 *The equilibrium price is given by*

$$\widetilde{p}=\frac{\sigma_e^2\left(2N-1\right)}{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}\overline{v}+\frac{N\sigma_v^2}{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}\sum_{n=1}^{N}\widetilde{i}_n$$
$$-\frac{N\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\widetilde{S}-\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}. \tag{5}$$

From this corollary we can see that the unconditional expectation of the equilibrium price is

$$E\left(\widetilde{p}\right)=\overline{v}-\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}$$

and it depends on the expected supply $\overline{m}$. If $\overline{m}=0$, the price is an unbiased estimator of $\overline{v}$, but it is biased if $\overline{m}\neq 0$. We also can see that, as expected, the higher the supply (the expected supply $\overline{m}$, or the realization of the liquidity shock $\widetilde{S}$ observed by the supply-informed agent), the lower the price and the greater the signals received by the value-informed agents, the higher the price.

Also note that a change in the different components of the supply has a different impact on price. A change in the known part of supply $\overline{m}$ is absorbed by the agents through the quantity demanded in a proportion of $\frac{N-1}{N}$ (we have seen while calculating the strategies that $\alpha=N\alpha^{PI}+\alpha^{SI}=g(N,\sigma_v^2,\sigma_e^2)+\frac{(N-1)}{N}\overline{m}$, where $g(N,\sigma_v^2,\sigma_e^2)$ is a function which does not depend on $\overline{m}$) and only $\frac{1}{N}$ is reflected in price. Similarly, half of a shock in the component of supply known to supply-informed agent $\widetilde{S}$ is absorbed by this agent through his demand and is partly reflected in price. As explained earlier, the supply-informed trader has a monopolist position and extracts rents that amount to half of the unknown component of supply.

2.2 The Benchmark

To proceed with the analysis it is useful to consider first as a benchmark the imperfect setup without a specialist. Notice that this model is a simplified version of Kyle's (1989) model with the difference that we do not have uninformed agents and we replace the noise agents by a random supply.

Proposition 3 *There is a unique linear symmetric equilibrium defined as:*

$$X_{I,n}\left(\widetilde{p},\widetilde{i}_n\right)=\alpha_I+\beta_I\widetilde{i}_n-\gamma_I\widetilde{p},\ \textit{for any } n=1,...,N$$

where $\alpha_I,\beta_I,\gamma_I$ *are given by*

$$\begin{aligned}
\alpha_I &= \frac{2\sigma_e^2}{N\sigma_v^2}\left(\frac{(N-2)\,\sigma_S^2}{N\,(N-1)\,\sigma_e^2}\right)^{1/2}\overline{v}+\frac{(N-2)}{N\,(N-1)}\overline{m}\\
\beta_I &= \left(\frac{(N-2)\,\sigma_S^2}{N\,(N-1)\,\sigma_e^2}\right)^{1/2}\\
\gamma_I &= \frac{N\sigma_v^2+2\sigma_e^2}{N\sigma_v^2}\left(\frac{(N-2)\,\sigma_S^2}{N\,(N-1)\,\sigma_e^2}\right)^{1/2}.
\end{aligned}$$

The equilibrium price when there is no specialist is

$$\begin{aligned}
\widetilde{p}_I &= \frac{2\sigma_e^2}{N\sigma_v^2+2\sigma_e^2}\overline{v}+\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\sum_{n=1}^{N}\widetilde{i}_n-\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\left(\frac{N\,(N-1)\,\sigma_e^2}{(N-2)\,\sigma_S^2}\right)^{1/2}\widetilde{S}\\
&\quad-\frac{\sigma_v^2}{(N\sigma_v^2+2\sigma_e^2)\,(N-1)}\left(\frac{N\,(N-1)\,\sigma_e^2}{(N-2)\,\sigma_S^2}\right)^{1/2}\overline{m}.
\end{aligned}$$

Notice that the price is an unbiased estimator of $\widetilde{v}$ if and only if $\overline{m}=0$. In order to show the effect the presence of the strategic specialist has on market performance we will compare several market indicators. In what follows, we present the most important ones: price volatility, informativeness of prices, expected volume and expected profit.

Corollary 4 *The market indicators for an economy without a specialist are the following:*

1) The price volatility, measured as the variance of price, is

$$Var\left(\widetilde{p}_I\right)=N\left(\frac{\sigma_v^2}{N\sigma_v^2+2\sigma_e^2}\right)^2\left(N\sigma_v^2+\frac{(2N-3)}{(N-2)}\sigma_e^2\right).$$

2) The information content of prices is

$$Var\left(\widetilde{v}\right)-Var\left(\widetilde{v}\right|\widetilde{p}_I)=N(\sigma_v^2)^2\,(N-2)\left(N\,(N-2)\,\sigma_v^2+(2N-3)\,\sigma_e^2\right)^{-1}.$$

3) The expected volume traded by a value-informed agent is

$$E\left(|x_{I,n}|\right)=\frac{1}{N}\overline{m}+\left(\frac{2}{\pi}\right)^{1/2}\frac{(N-1)}{N^2}\sigma_S^2.$$

4) The expected profit of a value-informed agent is

$$\Pi_{I,n}^{PI} = E\left(\widetilde{\pi}_{I,n}^{PI}\right) = E\left(\left(\widetilde{v} - \widetilde{p}_I\right)\widetilde{x}_n\right) = \frac{\sigma_v^2}{N\left(N\sigma_v^2 + 2\sigma_e^2\right)}\left(\frac{N\left(N-1\right)\sigma_e^2}{\left(N-2\right)\sigma_S^2}\right)^{1/2}\left(\overline{m}^2 + \sigma_S^2\right).$$

3 Market Indicators

We would like to understand the effects of different types of information and the existence of a strategic specialist on market performance. We therefore study the following market indicators in our new setup: market liquidity, informativeness of prices, price volatility, and the ability of informed traders to exploit their private information. We are first interested with market liquidity because it has been recognized as the most important characteristic of well-functioning markets. There are different measures of market liquidity used in the literature: market depth, bid-ask spread and price movement after trade. We will use as a measure of liquidity market depth (as defined by Kyle (1985)), which represents the trading volume needed to move prices one unit. While solving the above system we obtained

$$\gamma = N\gamma^{PI} + \gamma^{SI} = \frac{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}.$$

On the other hand, from the price equation (3) we can see that an increase (decrease) in the known component of supply by γ induces the price to fall (rise) by one dollar. We use the same measure as Kyle and consequently, γ is our measure of market liquidity. As can be seen, market depth γ has two components that have opposite effect. The first component $N\gamma^{PI}$ is attributed to the value-informed agents trading. This is the amount by which they contribute to a change in the price when each of them trades an additional unit. The more value-informed agents are in the market, the higher the liquidity. Similarly, γ^{SI} is the change in price due to an additional unit of trading by the supply-informed agent. The two components have opposite signs and we thus have a trade-off: the value-informed agents increase market liquidity while the supply-informed agent reduces it.

The fact that γ^{SI} is negative is a very important result in our model and it is a consequence of the mechanism of information transmission through prices. In general, with asymmetric information, prices play a dual role of information aggregation and market clearing. The role of information aggregation played by prices is even more important in our economy with asymmetric and different information. We have two important channels through which information flows: through one channel we have a flow of information about the liquidation value from the value-informed traders towards the supply-informed trader and through the other one we have a flow of information about supply from the supply-informed trader towards the value-informed traders. The supply-informed agent puts a positive weight on price ($\gamma^{SI} < 0$) because when he sees an increase in price, he associates it with good news about the liquidation value (he knows the supply, so the price increase cannot be due to a decrease in supply). This mechanism of information transmission actually triggers a decrease in market liquidity. For one additional unit demanded by a value-informed agent, the price goes up. The supply-informed agent associates it with good news about the liquidation value and increases his demand leading to an even higher increase in price. Since the same volume further increases the price, we may conclude that we have a decrease in market liquidity.

Next, let us investigate how the market depth varies with the parameters of the model: the variance of the liquidity shock σ_S^2, the variance of signals σ_e^2, and the variance of the liquidation value σ_v^2.

Corollary 5 *(i) Market depth is increasing in the variance of liquidity shock $\widetilde{S}$, σ_S^2.*

(ii) Market depth is decreasing in the variance of the error of the signal received by value-informed agents σ_e^2.

(iii) Market depth viewed as a function of the variance of liquidation value σ_v^2 has an inverted U-shape.

As we have seen before, the market depth has two components $\gamma = N\gamma^{PI} + \gamma^{SI}$. The first component is the contribution to the market depth of trades executed by value-informed agents while the second one is the contribution to the market depth of trades executed by the supply-informed agent. The two components have opposite effects and thus, the final result on market depth due to the market-making activity of the agents depends on which of the two components dominates. The first result in Corollary 5 is similar to the ones found in the literature (Kyle (1985) and other imperfect competition models) - the greater is the variance of the noise trading, the greater the market depth. Here, the noise trading is modeled as the random supply. As a result, the direct effect is that the higher the variance of the supply, the easier it is for value-informed agents to hide and therefore to make use of their informational advantage (the volume needed to move the price is higher, and this helps them to trade better on their information without revealing too much of it). In addition, in our model the same is true for the supply-informed agents. If the variance of the liquidity shock (or signal of the supply-informed agent) σ_S^2 is high, the supply-informed agent is better camouflaged and can trade more actively on his private information about supply. The second result claims that if the signals of the value-informed agents are very poor, market depth is low. Note that when the difference in the information between the value-informed agents is small, they will compete more strongly against the supply-informed agent and less among themselves. Once their information becomes very different, i.e. σ_e^2 increases, they will also start competing more aggressively against each other (thus reducing their informational advantage).

Finally, we study the behavior of the market depth with respect to the variance of the liquidation value. As can be seen in Figure 1, market depth has an inverted U-shape. This result differs from previous results in the literature and this difference is triggered precisely by the existence of a supply-informed agent. Where there are only value-informed traders, we find that the higher the variance of the liquidation value, the higher their informational advantage and therefore the lower market depth is. The existence of the supply informed-agent affects the informational advantage of the value-informed agents. If the variance σ_v^2 is small, the average signal about the liquidation value inferred by the supply-informed agent is quite good. So the supply-informed agent can infer the private information of the value-informed agents quite well, thus reducing their informational advantage and inducing an increase of market liquidity. However, as the variance of liquidation value σ_v^2 increases, the informational advantage of the value-informed traders increases substantially, offsetting this effect and therefore, reducing market depth.

In what follows, we study the behavior of volatility of prices with respect to the variance of the liquidation value of the asset.

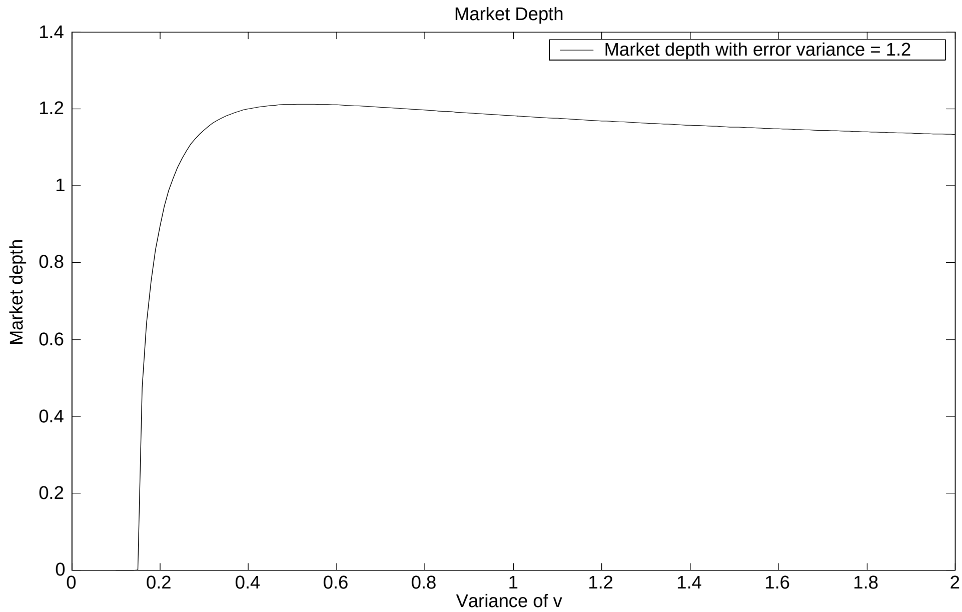

Figure 1: Comparative statics for market depth. Parameters values: $N = 4$, $\sigma_e^2 = 1.2$, $\sigma_S^2 = 2$.

Corollary 6 *The price volatility, measured as the variance of price, is*

$$Var\left(\widetilde{p}\right) = \frac{N^3\left(N-2\right)\left(\sigma_v^2\right)^2 + 2N^2\left(N-2\right)\sigma_v^2\sigma_e^2 - \left(\sigma_e^2\right)^2}{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)}\left(\frac{N\sigma_v^2}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\right)^2.$$

As in the case where there is no supply-informed agent, we find that the volatility of prices does not depend on the noise in supply. If the noise in supply increases all the agents - both the value-informed and the supply-informed - trade more aggressively, making better use of their particular informational advantage. We also find that price volatility has a U shape with respect to the variance of the liquidation value of the asset, σ_v^2. Looking at the way the information is incorporated in prices (see Equation 5) we observe that the weight associated with the information of the value-informed agents increases with σ_v^2, while the weight associated with the information of the supply-informed agent decreases.[6] The reason is the same as in the case of market depth. On the one hand, the higher the variance of the liquidation value of the asset, the higher the

[6]This weight is actually the intensity of trading on information divided by the market depth.

volatility of prices (the traders trade more aggressively and reveal more information in prices). On the other hand, the lower the variance of the liquidation value of the asset, the better the average signal inferred by the supply-informed agent is. As a result, the value-informed traders have to trade more aggressively against the supply informed trader and make him reveal more information about supply.

Next, we would like to find out the amount of private information (both about the liquidation value and supply) that is revealed through prices. We thus define the information content of prices as the difference between the prior variance of the payoff and the variance conditional on prices. Using the normality assumptions, we obtain the expression presented in the following Corollary:

Corollary 7 *The information content of prices is*

$$Var\left(\widetilde{v}\right)-Var\left(\widetilde{v}|\,\widetilde{p}\right)=\frac{N^2(\sigma_v^2)^2\left(N(N-2)\sigma_v^2-\sigma_e^2\right)}{N^3\left(N-2\right)\left(\sigma_v^2\right)^2+2N^2\left(N-2\right)\sigma_v^2\sigma_e^2-\left(\sigma_e^2\right)^2}.$$

As with the previous Corollary, we also find here that price efficiency or the information content of prices does not depend on the variance of supply shock $\widetilde{S}$. Moreover, we obtain that informativeness of prices increases with respect to the variance of the liquidation value σ_v^2 and decreases with respect to the variance σ_e^2. These results tells us that when it is difficult to predict the liquidation value or when the signals of value-informed agents are poor, prices play a very important role in information aggregation. While these results, are qualitatively similar to the case without supply-informed agent, as we will see later, they are quantitatively different.

Let us turn to the expected volume traded by the value-informed agent and supply-informed agent, respectively.

Corollary 8 *The expected volume traded by a value-informed agent is*

$$E\left(|x_n|\right)=\frac{2\left(N-1\right)\sigma_v^2\overline{m}}{N^2\sigma_v^2+\sigma_e^2}+\frac{\left(\frac{2}{\pi}\right)^{1/2}}{4N^2}\left(\frac{\left(N^2\sigma_v^2+\sigma_e^2\right)^2+N\left(N\sigma_v^2+\sigma_e^2\right)^2}{\left(N\sigma_v^2+\sigma_e^2\right)^2\left(N^2\sigma_v^2+\sigma_e^2\right)^2}\left(\sigma_v^2+\sigma_e^2\right)\delta+\sigma_S^2\right).$$

The expected volume traded by the supply-informed trader is

$$E\left(|y|\right)=\frac{2\left(N\sigma_v^2+\sigma_e^2\right)\overline{m}}{\left(N^2\sigma_v^2+\sigma_e^2\right)}+\left(\frac{1}{8\pi}\right)^{1/2}\sigma_S^2\left(1+\frac{\left(N-1\right)\sigma_e^2\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(\sigma_v^2+\sigma_e^2\right)}{N\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)^2}\right).$$

The expected volumes traded by the value-informed agents and the supply-informed agent depend positively on the expected supply $\overline{m}$ and the variance of the supply shock σ_S^2. However, both the effects of an increase in σ_S^2 and in $\overline{m}$ are stronger in the case of a supply-informed trader. This is the role we actually wanted the supply-informed agent to play. Since he has information about supply he captures a big part of the shocks. In the previous literature, where agents only had information about the liquidation value, the trading volume neither depended on the variance of the liquidation value nor on the variance of the errors. In our case, they are dependent and moreover, when the known component in supply $\overline{m}$ is different from 0, the comparative statics with respect to the variance of the liquidation value σ_v^2 and to error σ_e^2 are ambiguous. Where the known component in supply $\overline{m}$ is equal to zero, we find that the expected volume traded by the informed agents increases with respect to the variance of liquidation value σ_v^2 and decreases with the variance of the errors σ_e^2. Unlike in the case without a specialist, this result might explain one of the stylized facts from the empirical literature: the higher the asymmetry of information, the higher the volume of trading. The reasons are the same as before: the higher the variance of liquidation value, the better the informational advantage of the value-informed traders, so the higher the expected volume. Also, the higher the variance of errors, the more heterogeneous are the signals received by the value-informed traders. This implies lower quality of price as a signal about the supply, and therefore lower volume of trading by value-informed traders. On the other hand, from the point of view of the supply-informed agent both high variance of liquidation value σ_v^2 and high variance of the errors σ_e^2 imply high heterogeneity of the signals of value-informed traders and this implies price is a poor signal regarding the liquidation value. However, heterogeneity makes the value-informed traders trade more aggressively against one another. As a result, the expected volume traded by the supply-informed trader is inverted U-shaped, the shape being determined by which of the above mentioned effects dominates.

We next compute the unconditional expected profits for all agents.

Corollary 9 *The unconditional expected profit of the n^{th} value-informed agent is*

$$\Pi_n^{PI} = E\left(\widetilde{\pi}_n^{PI}\right) = \frac{\sigma_v^2\delta^{1/2}(N-1)\sigma_e^2}{2N\left(N^2\sigma_v^2+(2N-1)\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\left(\frac{N\left(N\sigma_v^2+\sigma_e^2\right)}{(N(N-2)\sigma_v^2-\sigma_e^2)}-\frac{(N-1)\sigma_e^2}{(N^2\sigma_v^2+\sigma_e^2)}\right)+\frac{(N-1)}{(N^2\sigma_v^2+\sigma_e^2)}\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+(2N-1)\sigma_e^2\right)\delta^{1/2}}\overline{m}^2.$$

The unconditional profit of the supply-informed agent is

$$\Pi^{SI} = E\left(\widetilde{\pi}^{SI}\right) = \frac{\delta^{1/2}(N-1)\sigma_e^2\sigma_v^2}{2\left(N^2\sigma_v^2+(2N-1)\sigma_e^2\right)}\left(\frac{(N-1)\sigma_e^2}{(N^2\sigma_v^2+\sigma_e^2)(N\sigma_v^2+\sigma_e^2)}+\frac{N}{(N(N-2)\sigma_v^2-\sigma_e^2)}\right)+\frac{2N\sigma_v^2\left(N\sigma_v^2+\sigma_e^2\right)}{\left(N^2\sigma_v^2+(2N-1)\sigma_e^2\right)\delta^{1/2}}\frac{\left(N\sigma_v^2+\sigma_e^2\right)}{(N^2\sigma_v^2+\sigma_e^2)}\overline{m}^2.$$

As we expected, allowing the supply-informed agent to behave strategically allows him to make positive profits (unlike the case of perfect competition where he makes zero profits). As pointed out by Brown and Zhang (1997), despite the fact that dealers may be better informed than other traders, in a competitive market they cannot earn rents from the information on the order flow. This is due to the fact that price informed agents use their informational advantage to make gains at the dealers' expense. However, since the specialist has market power his trade is profitable (see Hasbrouck and Sofianos (1993) for empirical evidence). Note also that since the value-informed traders always absorb $\frac{1}{2N}$ of the shock S, it is actually the different information that they receive that gives them different profits. The non-monotonicity with respect to the variance of liquidation value is also transmitted here, both expected profits having a U shape.

We also want to see what the impact of changes in supply is on the equilibrium price and the quantity demanded by the different agents. Like Gennotte and Leland (1990), we study the two following cases: a supply increase known to all agents $\overline{m}$, and a supply increase known only to supply-informed agent $\widetilde{S}$.

Corollary 10 *A positive shock in supply known to all the agents $\overline{m}$ leads to an increase in the demand of both type of agents, a decrease in the equilibrium price and therefore, to an increase in the expected profits of both types of agents.*

As expected, an increase in the supply known to all agents makes them adjust their demands in accordance with the existing supply, and it also leads to a decrease of the equilibrium price. Here, we find that the value-informed agents always absorb a greater proportion of the shock in supply $\overline{m}$.

Corollary 11 *A positive shock in the component of supply $\widetilde{S}$, known to the supply-informed agent, decreases the equilibrium price and increases the quantities demanded both by the value-informed and supply-informed agents.*

In the case of a positive shock in the supply $\widetilde{S}$, the supply-informed agent increases his demand, making use of the private information he has. This shows the crucial role played by specialist when markets suffer a shock - specialists are obliged to maintain orderly markets when prices are falling by buying shares with their own money. Since the specialist sees the order book, he can manage the supply shock more effectively. Moreover, in our setup, the increase in supply (due to a positive shock in $\widetilde{S}$) absorbed by the supply-informed agent is N times higher than the increase of supply absorbed by any value-informed agents. An interesting result is that the supply-informed agent always absorbs half of the unobservable shock in supply, the other half being absorbed by value-informed agents. This result resembles somewhat the one obtained by Röell (1990), and is explained by the fact that the supply-informed trader acts as a monopolist, extracting half of the rents.[7] Notice that in our model, the supply-informed trader always extracts half of the rents despite the fact that they submit limit orders, while in Röell (1990) this was only possible if either the number of brokers-dealers increased significantly, or the brokers-dealers submitted market orders.

4 Comparison of Market Indicators

We now compare the market indicators in two cases: one in which there is a supply-informed agent, and one where there is none. Let us first study the effect the presence

[7] In a model that examines the effects of dual trading, Röell (1990) considers several broker-dealers who have better information about uninformed traders than the market maker.

of the supply-informed agent has on market depth. We have that

$$\gamma \equiv N\gamma^{PI} + \gamma^{SI} = \frac{\left(N^2\sigma_v^2 + (2N-1)\,\sigma_e^2\right)\sigma_S}{2N^2\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)\sigma_e}\left(\frac{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)\left(N^2\sigma_v^2 + \sigma_e^2\right)}{(N-1)}\right)^{1/2}$$

the market depth where there is a supply-informed agent and

$$\gamma^I \equiv N\gamma_I = \frac{\left(N\sigma_v^2 + 2\sigma_e^2\right)\sigma_S}{\sigma_v^2\sigma_e}\left(\frac{(N-2)}{N\,(N-1)}\right)^{1/2}$$

the market depth where there is none. As we can see in Figure 2, market depth is less where we have a supply-informed agent in the market ($\gamma < \gamma^I$).[8] This result is quite intuitive if one considers that the supply-informed agent plays a dual role in the market. First, he reveals a part of his information in the process of trading. Second, by having the information about supply, he makes the value-informed agents reveal more of their information.[9] Notice that our agents observe only one type of information, but they place limit orders and therefore, through the price, they also trade on the information of the other market participants. This is similar to the literature on dual trading Röell (1990), Fishman and Longstaff (1992), and Sarkar (1995) where dealer-brokers together with information about the liquidation value are able to observe a component of the order flow. However, in our model, with imperfect competition and limit orders, the presence of the supply-informed trader plays a more complex role as we can see by studying the other market indicators.

Finally, the decrease in the market liquidity in the presence of the supply-informed agent captures the intuition of Glosten and Milgrom (1985), that more information in the market decreases market liquidity. In their model, they use the bid-ask spread as a measure of liquidity (low liquidity being equivalent to high bid-ask spread), and an increase in the number of informed agents increases the bid-ask spread.

[8] To understand better the implications of market power on our results, we also consider the case when we have several agents who have information about the supply. As expected, the numerical analysis suggests that the market liquidity will be higher when market power decreases.

[9] Subrahmanyam (1991) also finds that market liquidity decreases when the amount of information in the market increases (when the number of informed traders increases) and the market maker is risk averse. The similitarity of the results is caused by the fact that the supply-informed agent is risk neutral, but he behaves strategically and therefore acts as a risk-averse agent.

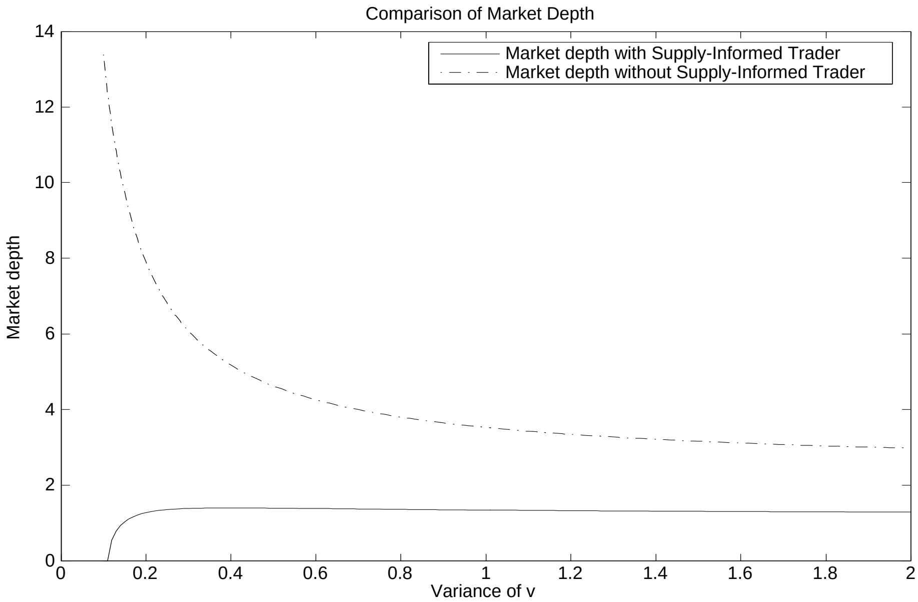

Figure 2: Comparison of market depth with and without supply-informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

We also find that when there is a supply-informed trader in the market, value-informed traders trade more aggressively on their private information $\left(\beta^{PI} > \beta_I\right)$ and they devote less to market-making activity.[10]

The inside information allows value-informed agents to make gains at the expense of market makers. However, when there is a supply-informed agent who has the ability to disentangle the order flow originated by value-informed agents from a shock in supply, the advantage of the value-informed agent diminishes and therefore, so do his market-making gains. A part of the gains that the value-informed agents made from market-making activity is now made by the supply-informed agent. As we have already seen, value-informed agents put a greater weight on market-making activity than the supply-informed agent does. Thus the specialist, even though he may have information about supply, faces strong competition in market-making from the other value-informed traders.

[10] The intensity of trading and the intensity of the market making activity are defined similarly to the literature as the coefficients of the signals (private signal and price) minus the average signals.

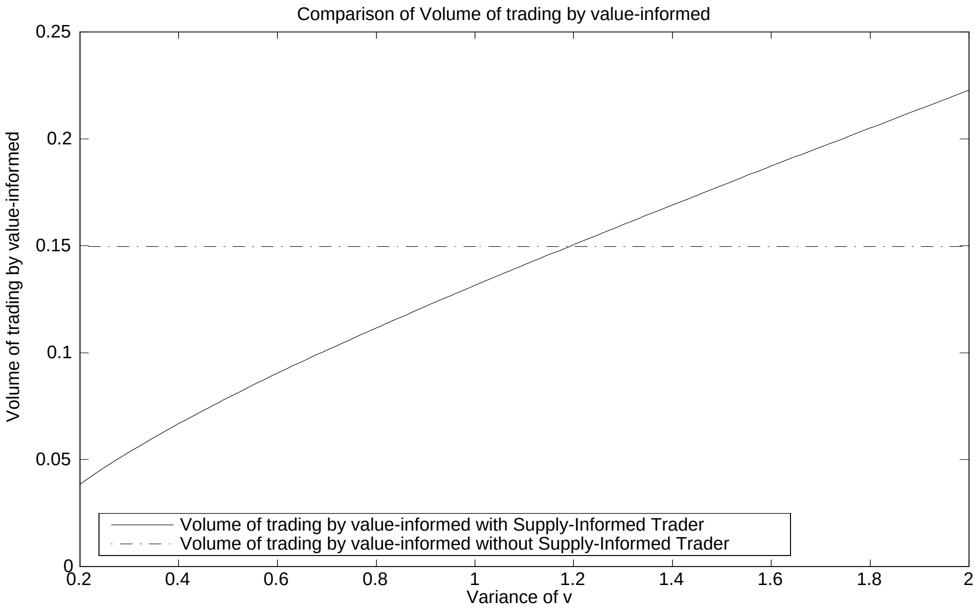

Figure 3: Comparison of volume of trading by the value-informed traders when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 1$, $\sigma_S^2 = 2$.

Another interesting result of the presence of a strategic specialist brings about concerns the volume of trading. We have seen that the volume of trading of value-informed traders where there is no supply-informed agent in the market depends only on the number of informed agents and the variance of the shock in supply. However, where there is a supply-informed agent in the market, the volume of trading depends positively on the variance of the liquidation value. As can be seen in Figure 3, when the variance of liquidation value is small, the volume of trading by value-informed traders is smaller where there is a supply-informed trader. As the variance of the liquidation value increases, the volume of trading by value-informed traders increases when there is a supply-informed trader in the market. So our model explains one of the stylized facts about volume: the higher the asymmetry between's trader information, the greater the volume of trading.

Proposition 12 *The presence of the supply-informed agent in the market leads to higher volatility of prices, lower informativeness of prices and lower expected profits by the value-informed agents (when $\overline{m} = 0$) .*

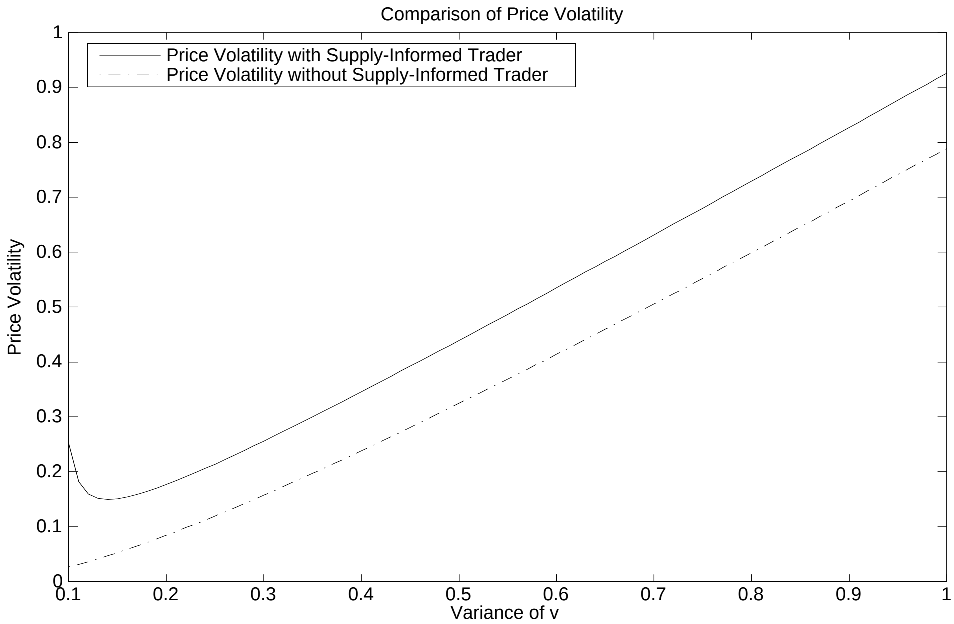

Figure 4: Comparison of price volatility when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

As pointed out above, one should note that the results concerning the trading volume, volatility of prices, informativeness of prices and expected profits by the value-informed agents are very different from the ones in the dual trading literature (Röell (1990) and Sarkar (1995)). As a result of the fact that both types of traders infer the others' information, the reduction in the market depth no longer offsets the impact on order flow. A first implication is that, unlike the other papers, the price informativeness and the volatility of prices are affected by the presence in the market of the supply-informed trader.

The fact that the volatility of prices increases where is a supply-informed agent (see Figure 4), is quite intuitive: the more information is released in prices, the more volatile the prices are. The higher volatility is thus due to two factors. First, the existence in the market of the information about supply forces value-informed agents to reveal more of their information. Second, the supply-informed trader is also revealing information about the supply and the more information is revealed in prices, the more volatile the prices are. Notice that our results are consistent with the empirical evidence found by

Hasbrouck and Saar (2002) that high volatility is associated with low market depth.

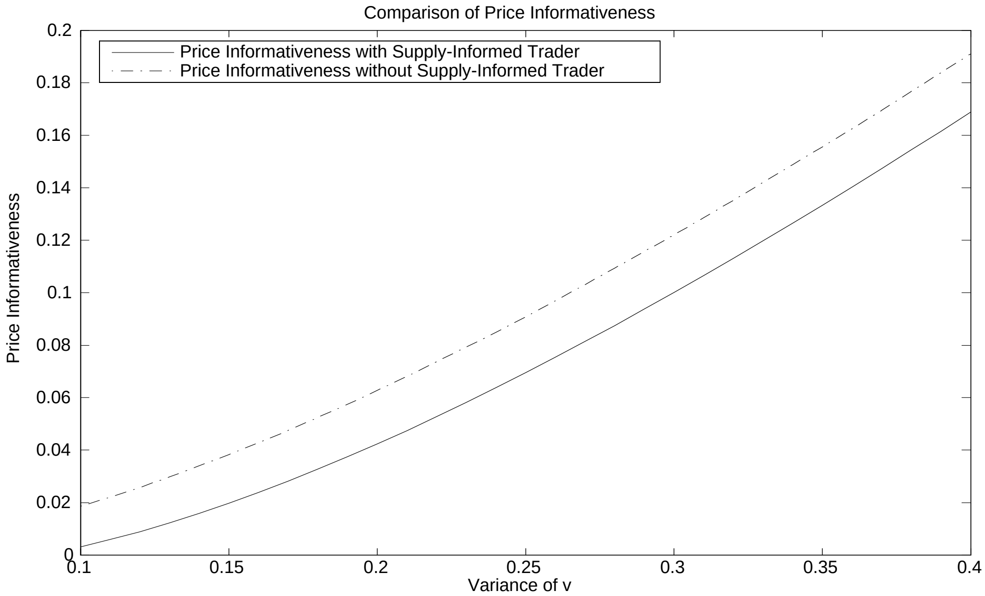

Figure 5: Comparison of price informativeness when there is one or none supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2$.

As it can be seen in Figure 5, the price informativeness decreases when there is a supply-informed agent in the market. Despite more information being aggregated in price, the prices are less informative about the liquidation value of the asset. This is so because the price aggregates two types of information: about the liquidation value of the asset and about supply. Of course, when a trader uses the price as a signal together with another signal (either about liquidation value or about supply) the price reveals more information to him.

Finally, we study the expected profits of the value-informed traders. Where the known component of supply $\overline{m}$ is 0, the expected profits of the value-informed traders decrease when there is a supply-informed trader (Figure 6). Although the total expected profits increase when there is a supply-informed trader, the biggest part of this profits is made now by the supply-informed trader.[11]

[11] We have not modelled the noise traders in the model, but if we had done so this increase in total profits will occur at the expense of the noise traders. Therefore, unlike in Röell (1990) and Sarkar

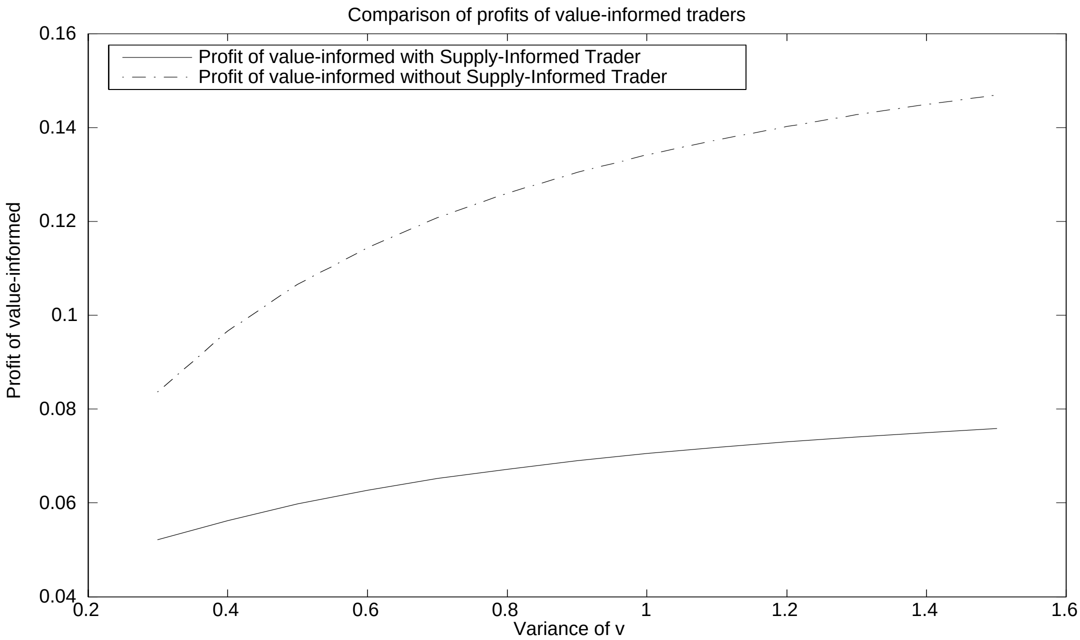

Figure 6: Comparison of expected profits of value-informed when there is one or no supply informed trader. Parameters values: $N = 4$, $\sigma_e^2 = 0.7$, $\sigma_S^2 = 2, \overline{m} = 0$.

As we can see from the previous analysis, the strategic specialist has an important effect on market performance. His access to the information contained in the order book and the strategic use of this information worsens the market (lower market liquidity, higher volatility of prices, lower price informativeness). These results are mainly triggered by the specialist's monopolistic position on the information about the order book and by the dual role of price in market clearing and, aggregating and revealing information. As we have already mentioned, the numerical analysis we performed suggests that we obtain an increase in market liquidity by reducing the market power of the specialist. As a result, the policy implication is that the market power of a specialist who has access to the limit order book should be reduced. One way of doing this is by disclosing the information contained in the limit order book. Our model does not permit us to fully analyze this issue but our results are in line with the empirical study of Boehmer et al. (2005) (who analyzed the impact of the new trend in increasing pre-trade trans-

(1995), the transaction costs of the noise traders would increase when there is a supply-informed trader.

parency, making the order book visible prior to the market opening). Boehmer et al. (2005) study how transparency affects trading by looking at the introduction of the NYSE's OpenBook program.[12] They show that the increase in pre-trade transparency increases the risk of proprietary trading due to the loss of their informational advantage. Also, they obtain a reduction in spreads and a decrease in the participation rate due to the decrease in the profitability of liquidity provision. Although our model is not an open-book model, there are similar implications. In an open book model, the value informed traders receive the information disseminated about the order book. In our model, they do not receive directly the information about the order book. However, since both they and the specialist place limit orders, the value informed trader can infer information about the order book from the price.

5 Conclusions

In this paper we have presented a model of a strategic specialist who has exclusive information about the limit order book. We analyze how the specialist makes use of this private information and how this information is aggregated and partially revealed through the equilibrium price and we conclude that his presence in the market worsens market performance. Allowing the specialist to behave strategically, he makes positive profits (unlike in the case of a dealer in the perfect competitive case) and increases the amount of information revealed in prices. We see that he has a dual role in inducing information transmission in the market: first because he has superior information (which he reveals in the trading process) and second, because he urges value-informed agent to reveal more of their information. However, the most important consequence of his presence in the market is that he decreases market liquidity (this outcome being brought about by the strategic behavior and the mechanism of information transmission through prices). Using a Kyle-type model we find a similar result to Glosten and

[12] The NYSE exchange initiated in 2002 a program called OpenBook through which they started selling information on its limit order book. This system allows the traders to observe the depth in the book for all prices (before only the best bid and ask prices were observed). The specialist still has some private information on the individual orders that make up the book.

Milgrom (1985) (i.e. that more information in the market decreases market liquidity). Modelling the dealer as a specialist (supply-informed trader) who behaves strategically helps us to link the two strands of the literature.

The presence of a strategic specialist who observes the limit order book also induces non-monotonicity of market liquidity and other market indicators with respect to the variance of the liquidation value. As a result, in our case the asymmetry of private information plays a more important role.

Finally, our paper has the following policy implication: decreasing the market power of the specialist increases market performance. This sustains the Security Exchange Commission's recent actions in disseminating the information contained in the limit order book and is in line with the empirical literature concerned with the effect of transparency on market performance.

6 Appendix

Lemma A.1 *In a symmetric linear equilibrium* $N\gamma^{PI} + \gamma^{SI} \neq 0$.

Proof. We look for a symmetric linear equilibrium. Therefore, we use the linear strategies defined in (2) and we can write the market clearing condition (1) as it follows:

$$N\alpha^{PI} + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - N\gamma^{PI}\widetilde{p} + \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p} = \overline{m} + \widetilde{S}. \tag{6}$$

We define $\gamma \equiv N\gamma^{PI} + \gamma^{SI}$ and $\alpha \equiv N\alpha^{PI} + \alpha^{SI}$. Using these definitions, the market clearing condition can be written as

$$\alpha + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - \gamma\widetilde{p} - (1 - \beta^{SI})\widetilde{S} = \overline{m}.$$

We want to prove that $\gamma \neq 0$. Let us suppose that $\gamma = 0$. Then, the above condition becomes

$$\alpha + \beta^{PI} \sum_{n=1}^{N} \widetilde{i}_n - (1 - \beta^{SI})\widetilde{S} = \overline{m}.$$

Since $\widetilde{i}_n$, $n = 1, ..., N$ are independent of $\widetilde{S}$, it results that $\beta^{PI} = 0$. Plugging it in the above equation we obtain that

$$\alpha - (1 - \beta^{SI})\widetilde{S} = \overline{m},$$

which cannot be satisfied because α and $\overline{m}$ are real numbers and $\widetilde{S}$ is a random variable. We obtained therefore, a contradiction. ■

Lemma A.2 *In a symmetric linear equilibrium the optimal demand for the value-informed trader n and for the supply-informed trader are, respectively,*

$$x_n\left(\widetilde{p}, \widetilde{i}_n\right) = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{i}_n\right.\right) - \widetilde{p}\right] \tag{7}$$

$$y(\widetilde{p}, \widetilde{S}) = N\gamma^{PI}\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{S}\right.\right) - \widetilde{p}\right] \tag{8}$$

with $\gamma^{PI} > 0$, and $(N-1)\gamma^{PI} + \gamma^{SI} > 0$.

Proof. Let us first determine the optimal demand for the value-informed traders. The value-informed trader n considers the other players' strategies as given by (2). As a result, he is facing the following residual demand:

$$p = \frac{\alpha - \alpha^{PI} + \beta^{PI}\sum\limits_{j\neq n}\widetilde{i}_j - (1-\beta^{SI})\widetilde{S} - \overline{m}}{(N-1)\gamma^{PI} + \gamma^{SI}} + \frac{x_n}{(N-1)\gamma^{PI} + \gamma^{SI}} \tag{9}$$

and he solves the following maximization problem:

$$\max_{x_n \in \mathbb{R}} E\left((\widetilde{v} - \widetilde{p})\, x_n \left|\widetilde{p}, \widetilde{i}_n\right.\right) \Leftrightarrow$$

$$\max_{x_n \in \mathbb{R}} E\left(\left(\widetilde{v} - \frac{\alpha - \alpha^{PI} + \beta^{PI}\sum\limits_{j\neq n}\widetilde{i}_j - (1-\beta^{SI})\widetilde{S} - \overline{m} - x_n}{(N-1)\gamma^{PI} + \gamma^{SI}}\right) x_n \left| \widetilde{p}, \widetilde{i}_n\right.\right).$$

We write the first order condition for this problem and we find the optimal demand of the value-informed trader n :

$$x_n = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{i}_n\right.\right) - p\right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{(N-1)\gamma^{PI}+\gamma^{SI}}<0 \Leftrightarrow (N-1)\gamma^{PI}+\gamma^{SI}>0.$$

Similarly, the supply-informed trader takes as given the strategies of the value-informed traders and in conformity with (2). The residual demand faced by him is therefore

$$p=\frac{N\alpha^{PI}+N\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{n=1}^{N}\widetilde{e_n}-\overline{m}-\widetilde{S}}{N\gamma^{PI}}+\frac{y}{N\gamma^{PI}}. \tag{10}$$

The supply-informed trader solves the following maximization problem:

$$\max_{x_n\in\mathbb{R}} E\left(\left(\widetilde{v}-\widetilde{p}\right)x_n\left|\widetilde{p},\widetilde{i_n}\right.\right)\Leftrightarrow$$

$$\max_{x_n\in\mathbb{R}} E\left(\left(\widetilde{v}-\frac{\alpha-\alpha^{PI}+\beta^{PI}\sum_{j\neq n}\widetilde{i_j}-(1-\beta^{SI})\widetilde{S}-\overline{m}-x_n}{(N-1)\gamma^{PI}+\gamma^{SI}}\right)x_n\right|\left.\widetilde{p},\widetilde{i_n}\right).$$

and from here we find the optimal demand of supply-informed trader

$$y=N\gamma^{PI}\left(E\left(\widetilde{v}\left|\widetilde{p},\widetilde{S}\right.\right)-p\right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{N\gamma^{PI}}<0\Leftrightarrow N\gamma^{PI}>0.$$

Since $N\geq 1$ it results $\gamma^{PI}>0$. ■

Lemma A.3 *In a symmetric linear equilibrium for any $n=1,...,N$ we have*

$$\begin{aligned} E\left(\widetilde{v}\left|\widetilde{p}=p,\widetilde{i_n}=i_n\right.\right) &= \overline{v}\left(1-A\left(N-1\right)\beta^{PI}-B\right)-A(\alpha-\overline{m}) \\ &\quad +(B-A\beta^{PI})\widetilde{i_n}+A\gamma\widetilde{p}. \end{aligned}$$

Proof. We can rewrite the market clearing condition (6) as

$$\widetilde{p}\gamma-\alpha+\overline{m}-\beta^{PI}\widetilde{i_n}=(N-1)\,\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{j\neq n}\widetilde{e_j}-(1-\beta^{SI})\widetilde{S}. \tag{11}$$

From here it results that $\left(\widetilde{p}, \widetilde{i_n}\right)$ is informationally equivalent to $\left(\widetilde{h_n}, \widetilde{i_n}\right)$ where by definition $\widetilde{h_n} \equiv (N-1)\,\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{j\neq n}\widetilde{e_j} - (1-\beta^{SI})\widetilde{S}$. Consequently, we have $E\left(\widetilde{v}\,\middle|\,\widetilde{p}=p, \widetilde{i_n}=i_n\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{h_n}=h_n, \widetilde{i_n}=i_n\right)$. Applying the projection theorem for normally distributed random variables we obtain that

$$\begin{aligned}
E\left(\widetilde{v}\,\middle|\,\widetilde{h_n}=h_n, \widetilde{i_n}=i_n\right) &= \overline{v} + A\left(\widetilde{h_n} - (N-1)\,\beta^{PI}\overline{v}\right) + B\left(\widetilde{i_n} - \overline{v}\right) \\
&= \overline{v}\left(1 - A(N-1)\,\beta^{PI} - B\right) - A(\alpha - \overline{m}) + (B - A\beta^{PI})\widetilde{i_n} + A\gamma\widetilde{p}, \quad (12)
\end{aligned}$$

where A and B are the solution of the following system of equations:

$$\begin{aligned}
A &= M^{-1}(N-1)\beta^{PI}\sigma_v^2\sigma_e^2 \quad (13)\\
B &= M^{-1}\left[(\beta^{PI})^2(N-1)\,\sigma_v^2\sigma_e^2 + \left(1-\beta^{SI}\right)^2\sigma_S^2\sigma_v^2\right] \\
M &= (\beta^{PI})^2(N-1)\left(N\sigma_v^2 + \sigma_e^2\right)\sigma_e^2 + (1-\beta^{SI})^2\sigma_S^2\left(\sigma_v^2+\sigma_e^2\right).
\end{aligned}$$

■

Lemma A.4 *In a symmetric linear equilibrium we have*

$$E(\widetilde{v}|\widetilde{p}=p, \widetilde{S}=S) = \overline{v}\left(1 - CN\beta^{PI}\right) - C(\alpha-\overline{m}) + (1-\beta^{SI})C\widetilde{S} + C\gamma\widetilde{p}.$$

Proof. We write again the market clearing condition (6) this time in order to find a pair informationally equivalent to $\left(\widetilde{p}, \widetilde{S}\right)$

$$\widetilde{p}\gamma - \alpha + \overline{m} + (1-\beta^{SI})\widetilde{S} = \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n}. \quad (14)$$

We define $\theta \equiv \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n}$. From here it results that $\left(\widetilde{\theta}, \widetilde{S}\right)$ is informationally equivalent to $\left(\widetilde{p}, \widetilde{S}\right)$. Consequently, $E\left(\widetilde{v}\,\middle|\,\widetilde{p}=p, \widetilde{S}=S\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{\theta}=\theta, \widetilde{S}=S\right)$. Applying again the projection theorem for normally distributed random variables we obtain that

$$\begin{aligned}
E\left(\widetilde{v}\,\middle|\,\widetilde{\theta}=\theta, \widetilde{S}=S\right) &= \overline{v} + C\left(\widetilde{\theta} - N\beta^{PI}\overline{v}\right) + D\widetilde{S} \\
&= \overline{v}\left(1 - CN\beta^{PI}\right) - C(\alpha-\overline{m}) + (1-\beta^{SI})C\widetilde{S} + C\gamma\widetilde{p}, \quad (15)
\end{aligned}$$

where

$$C = \sigma_v^2\left(\beta^{PI}\left(N\sigma_v^2+\sigma_e^2\right)\right)^{-1}. \quad (16)$$

$\blacksquare$

Lemma A.5 *The coefficients $\alpha^{PI}, \beta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ are the solution of the following system of equations:*

$$\begin{cases} \alpha^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m})\right) \\ \beta^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(B - A\beta^{PI}\right) \\ \gamma^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(1 - A\gamma\right) \\ \alpha^{SI} = N\gamma^{PI}(\overline{v}\left(1 - CN\beta^{PI}\right) - C(\alpha - \overline{m})) \\ \beta^{SI} = N\gamma^{PI}(\left(1 - \beta^{SI}\right)C \\ \gamma^{SI} = N\gamma^{PI}\left(1 - C\gamma\right) \\ \quad M = \left(\beta^{PI}\right)^2 (N-1)\left(N\sigma_v^2 + \sigma_e^2\right)\sigma_e^2 + (1 - \beta^{SI})^2\sigma_S^2\left(\sigma_v^2 + \sigma_e^2\right) \\ \quad A = M^{-1}(N-1)\beta^{PI}\sigma_v^2\sigma_e^2 \\ \quad B = M^{-1}\left(\left(\beta^{PI}\right)^2 (N-1)\,\sigma_v^2\sigma_e^2 + (1 - \beta^{SI})^2\sigma_S^2\sigma_v^2\right) \\ \quad C = \sigma_v^2\left(\beta^{PI}(N\sigma_v^2 + \sigma_e^2)\right)^{-1}. \end{cases} \tag{17}$$

Proof of Lemma A.5. In Lemma A.3 and Lemma A.4 for we have established the expressions for $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{i_n} = i_n\right)$ and $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{S} = S\right)$. We will use them now to find the expressions for the strategies for the value-informed agents and for the supply-informed agent.

First, since $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{i_n} = i_n\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{h_n} = h_n, \widetilde{i_n} = i_n\right)$ we plug (12) in (7) and we obtain that

$$\begin{aligned} x_n\left(\widetilde{p}, \widetilde{i_n}\right) &= ((N-1)\gamma^{PI} + \gamma^{SI})\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m})\right. \\ &\quad \left. + (B - A\beta^{PI})\widetilde{i_n} + (A\gamma - 1)\widetilde{p}\right). \end{aligned}$$

We identify the coefficients in the definition of the strategy of the value-informed trader n (2) and we get the following equations:

$$\begin{aligned} \alpha^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI})(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A(\alpha - \overline{m})) \\ \beta^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI})(B - A\beta^{PI}) \\ \gamma^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI})(1 - A\gamma), \end{aligned} \tag{18}$$

where A and B are given by (13).

Second, since $E\left(\widetilde{v}\left|\widetilde{p}=p,\widetilde{S}=S\right.\right)=E\left(\widetilde{v}\left|\widetilde{\theta}=\theta,\widetilde{S}=S\right.\right)$ we plug (15) in (8) and we obtain in a similar manner

$$y\left(\widetilde{p},\widetilde{S}\right)=N\gamma^{PI}\left(\overline{v}-C(\alpha-\overline{m})+(1-\beta^{SI})C\widetilde{S}+(C\gamma-1)\widetilde{p}\right).$$

We identify the coefficients in the definition of the strategy of the supply-informed trader (2) and we get the following equations:

$$\begin{aligned}\alpha^{SI}&=N\gamma^{PI}(\overline{v}\left(1-CN\beta^{PI}\right)-C(\alpha-\overline{m}))\\ \beta^{SI}&=N\gamma^{PI}(1-\beta^{SI})C\\ \gamma^{SI}&=N\gamma^{PI}(1-C\gamma),\end{aligned}\tag{19}$$

where C is given by (16).

Putting together the equations (13), (18), (16) and (19) we obtain that α^{PI},β^{PI}, $\gamma^{PI},\alpha^{SI},\beta^{SI},\gamma^{SI}$ are the solution of the above system of equations. ■

Proof of Proposition 1. The equilibrium values of the coefficients α^{PI},β^{PI}, $\gamma^{PI},\alpha^{SI},\beta^{SI},\gamma^{SI}$ are the solution of the system of equations given in the statement of Lemma A.5. After some tedious algebra and defining by

$$\delta\equiv\frac{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)\left(N^2\sigma_v^2+\sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2},$$

we obtain the following coefficients:

$$\begin{aligned}\alpha^{PI}&=\frac{\sigma_e^2\left(N\left(3N-2\right)\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)\left(N\sigma_v^2+\sigma_e^2\right)}\overline{v}+\frac{N\left(N-2\right)\sigma_v^2-\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}\\ \beta^{PI}&=\frac{\delta^{1/2}}{2N\left(N\sigma_v^2+\sigma_e^2\right)}\\ \gamma^{PI}&=\frac{\left(N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}\\ \alpha^{SI}&=\left(-\frac{\left(N-1\right)\sigma_e^2}{\left(N\sigma_v^2+\sigma_e^2\right)}\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}\delta^{1/2}\right)\overline{v}+\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{N\left(N^2\sigma_v^2+\sigma_e^2\right)}\overline{m}\\ \beta^{SI}&=\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N\left(N\sigma_v^2+\sigma_e^2\right)}\\ \gamma^{SI}&=-\frac{\left(N-1\right)\sigma_e^2}{\left(N\sigma_v^2+\sigma_e^2\right)}\frac{N^2\sigma_v^2+\left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N^2\sigma_v^2+\sigma_e^2\right)}\delta^{1/2}.\end{aligned}$$

■

Proof of Corollary 2. From the market clearing condition (6) we obtain that the equilibrium price is

$$\widetilde{p} = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(\alpha + \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n} - (1-\beta^{SI})\widetilde{S} - \overline{m}\right).$$

Using the formulas we have obtained for the equilibrium coefficients we can write that the equilibrium price equals to

$$\begin{aligned}\widetilde{p} = {} & \frac{\sigma_e^2\left(2N-1\right)}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\overline{v} + \frac{N\sigma_v^2}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\sum_{n=1}^{N}\widetilde{i_n} \\ & - \frac{N\sigma_v^2\left(N^2\sigma_v^2 + \sigma_e^2\right)}{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\widetilde{S} - \frac{2N\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}.\end{aligned}$$

Notice that since $\widetilde{i_n} = \widetilde{v} + \widetilde{e_n}$ we can write

$$\begin{aligned}\widetilde{p} = {} & \frac{\sigma_e^2\left(2N-1\right)}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\overline{v} + \frac{N^2\sigma_v^2}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\widetilde{v} + \frac{N\sigma_v^2}{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}\sum_{n=1}^{N}\widetilde{e_n} \\ & - \frac{N\sigma_v^2\left(N^2\sigma_v^2 + \sigma_e^2\right)}{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\widetilde{S} - \frac{2N\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}.\end{aligned}$$

Taking the expectations it results that $E\left(\widetilde{p}\right) = \overline{v} - \dfrac{2N\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}{\left(N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2\right)\delta^{1/2}}\overline{m}$. ■

Proof of Corollary 5. While solving the above system we have obtained that

$$\gamma = N\gamma^{PI} + \gamma^{SI} = \frac{N^2\sigma_v^2 + \left(2N-1\right)\sigma_e^2}{2N^2\sigma_v^2\left(N\sigma_v^2 + \sigma_e^2\right)}\left(\frac{\left(N(N-2)\sigma_v^2 - \sigma_e^2\right)\left(N^2\sigma_v^2 + \sigma_e^2\right)\sigma_S^2}{\left(N-1\right)\sigma_e^2}\right)^{1/2}.$$

We study first how market depth varies when the variance of liquidity shock $\widetilde{S}$ varies. We compute the derivative $\dfrac{\partial\gamma}{\partial\sigma_S^2}$ and we obtain

$$\frac{\partial\gamma}{\partial\sigma_S^2} > 0.$$

Then we calculate $\dfrac{\partial\gamma}{\partial\sigma_e^2}$ and after somehow tedious calculations we obtain that

$$\frac{\partial\gamma}{\partial\sigma_e^2} < 0.$$

Finally, we study how the variance of liquidation value, σ_v^2 affects the market depth. We calculate the derivative $\dfrac{\partial \gamma}{\partial \sigma_v^2}$ and we obtain that this expression has the opposite sign to $f\left(\sigma_v^2\right)$, where

$$\begin{aligned} f\left(\sigma_v^2\right) &= N^4\left(\sigma_v^2\right)^3(N-1)\left(N^2-3N+1\right)-3\sigma_e^2N^2\left(\sigma_v^2\right)^2(2N-1)(N-1) \\ &\quad -3\sigma_v^2\left(\sigma_e^2\right)^2N(2N-1)(N-1)-\left(\sigma_e^2\right)^3(2N-1)(N-1). \end{aligned}$$

We study this function and we obtain that the equation $f'\left(\sigma_v^2\right)=0$,

$$f'\left(\sigma_v^2\right)=3(N-1)N\left[\left(N^3\left(\sigma_v^2\right)^2\left(N^2-3N+1\right)-2\sigma_e^2N(2N-1)\sigma_v^2-\left(\sigma_e^2\right)^2(2N-1)\right)\right],$$

has only one positive solution equal to

$$\sigma_e^2\frac{(2N-1)+(N-1)\left((2N-1)(N-1)\right)^{1/2}}{N^2\left(N^2-3N+1\right)}\equiv k_l(N).$$

We obtain that $k_l(N)>\dfrac{1}{N(N-2)}$. So, it results that the function $f\left(\sigma_v^2\right)$ is decreasing for $\dfrac{\sigma_v^2}{\sigma_e^2}\in\left[\dfrac{1}{N(N-2)},k_l(N)\right]$, and is increasing for $\dfrac{\sigma_v^2}{\sigma_e^2}>k_l(N)$. Since $f(0)=-\left(\sigma_e^2\right)^3(2N-1)(N-1)$, it results that it exists $k^*\left(N,\sigma_e^2\right)$ such that $f\left(k^*\left(N,\sigma_e^2\right)\right)=0$. Therefore, the function $f\left(\sigma_v^2\right)<0$ for any $\sigma_v^2<k^*\left(N,\sigma_e^2\right)$ and is greater than 0 otherwise.

Once we have characterized the behavior of function $f\left(\sigma_v^2\right)$ we can conclude that the market depth is an increasing function of σ_v^2 if $\sigma_v^2<k^*\left(N,\sigma_e^2\right)$ and is decreasing otherwise. ■

Proof of Corollary 6. We have seen that the equilibrium price is given by (5). As a result, we can compute the variance, and after some straightforward algebra we find

$$Var\left(\widetilde{p}\right)=\frac{N^3(N-2)\left(\sigma_v^2\right)^2+2N^2(N-2)\sigma_v^2\sigma_e^2-\left(\sigma_e^2\right)^2}{\left(N(N-2)\sigma_v^2-\sigma_e^2\right)}\left(\frac{N\sigma_v^2}{N^2\sigma_v^2+(2N-1)\sigma_e^2}\right)^2.$$

■

Proof of Corollary 7. We compute now $Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p})$. Due to the normality assumptions we have that

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = (Var(\widetilde{p}))^{-1} (Cov(\widetilde{v},\widetilde{p}))^2 .$$

We calculate the covariance

$$Cov(\widetilde{v},\widetilde{p}) = \frac{(N\sigma_v^2)^2}{N^2\sigma_v^2 + (2N-1)\,\sigma_e^2},$$

and together with the formula for variance $Var(\widetilde{p})$ we obtained before, we plug them above to obtain

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = \frac{N\,(\sigma_v^2)^2\,(N(N-2)\sigma_v^2 - \sigma_e^2)}{N^3\,(N-2)\,(\sigma_v^2)^2 + 2N^2\,(N-2)\,\sigma_v^2\sigma_e^2 - (\sigma_e^2)^2}.$$

■

Proof of Corollary 8. Since the demand of the value-informed agent x_n can be written as the sum of normal variables it results that x_n is also a normal variable. The mean of x_n is $\mu_n = \dfrac{(N-1)\,\sigma_v^2}{(N^2\sigma_v^2 + \sigma_e^2)}\overline{m}$ while the variance $\sigma_{x_n}^2$ is

$$\sigma_{x_n}^2 = Var(x_n) = \frac{(\sigma_v^2+\sigma_e^2)\,\delta}{4N^2}\left(\frac{1}{(N\sigma_v^2+\sigma_e^2)^2} + \frac{N}{(N^2\sigma_v^2+\sigma_e^2)^2}\right) + \frac{\sigma_S^2}{4N^2}.$$

Then, since x_n is $\mathcal{N}(\mu_n, \sigma_{x_n}^2)$ it results that the expected volume of trade by a value-informed trader is

$$E(|x_n|) = \int_{-\infty}^{\infty} |x_n| \frac{1}{\sigma_{x_n}\sqrt{2\pi}} \exp\left(-\frac{(x_n-\mu_n)^2}{2\sigma_{x_n}^2}\right) dx_n = 2\mu_n + \left(\frac{2}{\pi}\right)^{1/2} \sigma_{x_n}^2 =$$

$$\frac{2\,(N-1)\,\sigma_v^2}{(N^2\sigma_v^2+\sigma_e^2)}\overline{m} + \left(\frac{2}{\pi}\right)^{1/2}\left(\frac{(\sigma_v^2+\sigma_e^2)\,\delta}{4N^2}\left(\frac{1}{(N\sigma_v^2+\sigma_e^2)^2} + \frac{N}{(N^2\sigma_v^2+\sigma_e^2)^2}\right) + \frac{1}{4N^2}\sigma_S^2\right).$$

Similarly, the quantity demanded by the supply-informed agent is a normal variable with mean $\mu_y = \dfrac{(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2+\sigma_e^2)}\overline{m}$ and variance

$$\sigma_y^2 = Var(y) = \frac{1}{4}\sigma_S^2\left(1 + \frac{(N-1)\,\sigma_e^2\,(N(N-2)\sigma_v^2 - \sigma_e^2)\,(\sigma_v^2+\sigma_e^2)}{N\,(N^2\sigma_v^2+\sigma_e^2)\,(N\sigma_v^2+\sigma_e^2)^2}\right).$$

Then since y is $\mathcal{N}(\mu_y, \sigma_y^2)$ it results that the expected volume of trade of the supply-informed agent is

$$E(|y|) = \int_{-\infty}^{\infty} |y| \frac{1}{\sigma_y\sqrt{2\pi}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) dy = 2\mu_y + \sqrt{\frac{2}{\pi}}\sigma_y^2$$

$$= 2\frac{(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2+\sigma_e^2)}\overline{m} + \left(\frac{2}{\pi}\right)^{1/2}\frac{1}{4}\sigma_S^2\left(1+\frac{(N-1)\sigma_e^2(N(N-2)\sigma_v^2-\sigma_e^2)(\sigma_v^2+\sigma_e^2)}{N(N^2\sigma_v^2+\sigma_e^2)(N\sigma_v^2+\sigma_e^2)^2}\right).$$

■

Proof of Corollary 9. Let us compute first the unconditional expected profit of the n^{th} value-informed trader.

$$\Pi_n^{PI} = E\left(\widetilde{\pi_n^{PI}}\right) = E\left((\widetilde{v}-\widetilde{p})\,\widetilde{x_n}\right).$$

Using the formulas we have obtained for $\widetilde{p}$ and $\widetilde{x_n}$ we obtain

$$\begin{aligned}\Pi_n^{PI} = {} & \frac{\sigma_v^2\delta^{1/2}(N-1)\sigma_e^2}{2N(N^2\sigma_v^2+(2N-1)\sigma_e^2)(N\sigma_v^2+\sigma_e^2)}\left(\frac{N(N\sigma_v^2+\sigma_e^2)}{(N(N-2)\sigma_v^2-\sigma_e^2)} - \frac{(N-1)\sigma_e^2}{(N^2\sigma_v^2+\sigma_e^2)}\right) \\ & + \frac{(N-1)\sigma_v^2}{(N^2\sigma_v^2+\sigma_e^2)}\frac{2N\sigma_v^2(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2+(2N-1)\sigma_e^2)\delta^{1/2}}\overline{m}^2.\end{aligned}$$

Let us compute now the unconditional expected profit of the supply-informed trader

$$\Pi^{SI} = E\left(\widetilde{\pi^{SI}}\right) = E\left((\widetilde{v}-\widetilde{p})\,\widetilde{y}\right).$$

Similarly, using the formulas we have obtained for $\widetilde{p}$ and $\widetilde{y}$ we can write further

$$\begin{aligned}\Pi^{SI} = {} & \frac{\delta^{1/2}(N-1)\sigma_e^2\sigma_v^2}{2(N^2\sigma_v^2+(2N-1)\sigma_e^2)}\left(\frac{(N-1)\sigma_e^2}{(N^2\sigma_v^2+\sigma_e^2)(N\sigma_v^2+\sigma_e^2)} + \frac{N}{(N(N-2)\sigma_v^2-\sigma_e^2)}\right) \\ & + \frac{2N\sigma_v^2(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2+(2N-1)\sigma_e^2)\delta^{1/2}}\frac{(N\sigma_v^2+\sigma_e^2)}{(N^2\sigma_v^2+\sigma_e^2)}\overline{m}^2.\end{aligned}$$

The total profits in the market are

$$\Pi = N\Pi^{PI} + \Pi^{SI} = E\left((\widetilde{v}-\widetilde{p})\left(\sum_{n=1}^{N}\widetilde{x_n}+\widetilde{y}\right)\right).$$

But from the market clearing condition it results that

$$\begin{aligned}\Pi &= N\Pi^{PI} + \Pi^{SI} = E\left((\widetilde{v}-\widetilde{p})\left(\overline{m}+\widetilde{S}\right)\right) = \\ &= \frac{N\sigma_v^2}{(N^2\sigma_v^2+(2N-1)\sigma_e^2)\delta^{1/2}}\left((N^2\sigma_v^2+\sigma_e^2)\sigma_S^2 + 2(N\sigma_v^2+\sigma_e^2)\overline{m}^2\right).\end{aligned}$$

We can check and see that indeed the profits we have obtained sum up to this amount. ■

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