

Spanning Tests for Replicable Small Cap Indexes as Separate Asset Classes: International Evidence

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ABSTRACT

Traditional and step-down spanning tests are used to assess the behavior of replicable G7 and Asian country small cap indexes as separate asset classes of efficient portfolios for U.S. investors. Empirical tests on different index combinations show that the composition of a benchmark portfolio determines whether or not a small cap index could enlarge the original efficient frontier. The interaction among all assets in a portfolio is the key to the effectiveness and efficiency of a small cap index in efficient portfolios and constraints do not always reduce diversification benefits of the small-cap assets. Most small cap indexes of G 7 countries are separate asset classes to the portfolios consisting of the popular benchmark indexes in G 7 markets in our sample period.

Keywords: portfolio diversification; small cap indexes; spanning tests.

JEL Codes: G32, G34.

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1. Introduction

There is a growing interest amongst academics and practitioners on the performance of small cap indexes and on the benefits of small caps for international diversification. In finance theory, investors with monotonically increasing and strictly concave utility functions prefer higher expected return and lower variance. For arbitrary distributions of asset returns, investors will choose on the basis of mean and variance if the utility functions are quadratic; and for arbitrary preferences, investors will choose on the basis of mean and variance if asset returns are multivariate normally distributed. The restrictions required for mean-variance choice are quite strong. However, since mean-variance analysis can generate empirical predictions that may be testable, mean-variance analysis is widely used in finance. The existence and the magnitude of the benefits from diversifying over small cap stocks has been questioned in recent years. From modern portfolio theory, if small cap returns do not perfectly correlate with returns of other class of assets, investors could gain from size diversification.

Banz (1981) first reported the size anomaly that small cap stocks had substantially higher returns than that of large cap stocks even after controlling for risk over long investment horizons. Banz's paper spawned several studies on the small effect. However, some recent studies show different results on the validity of such an anomaly. For example, Berk (1996) finds that although the returns and firm sizes based on market value have an inverse relationship, there is no significant relationship between average returns and non-market based measures of firm size (book value of assets; book value of property, plant, and equipment; total value of annual sales; total number of employees). Horowitz et al. (2000) find that small cap portfolios underperformed large cap portfolios

for 1980–1996 using data from NYSE, AMEX and NASDAQ.. Dimson and Marsh (1999) find that large cap stocks have higher returns than small cap stocks for the US and UK data also using data for the 1990's. Moreover, Reilly and Wright (2002) also find large cap stocks outperform small cap stocks over the period 1984–2000.

The differential time varying performance of small caps vs. large caps was initially addressed by Brown et al. (1983). In addition, based on US markets data of 1926–1989 and return autocorrelations among different investment horizons, Reinganum (1992) finds that the changes of the size premium for small caps are not random but predictable since the premium reverse over long time horizons.¹

The benefits of international diversification on portfolio management are well documented in the literature and the mean-variance spanning tests have been used to study such benefits. (see e.g. Harvey (1995), Bekaert and Urias (1996), and Errunza et al. (1999), and Driessen et al. (2003)).

¹ A few studies attempt to explore the determinants of the small firm premium. Chan et al. (1985) find that the risk premia in a multi-factor pricing model contributes a large part of the size premium. Since changes in the risk premia are related to business conditions, they argue that the economy expansions and contractions will have big impacts on the returns of small firms. Levis (2002) suggests the cyclical pattern of the size premia may result from changes of economic fundamentals of small cap firms such as the cash flows and discount rates. In addition, Jensen et al. (1997) also find that significant positive premiums of small caps appears when U.S. monetary policy conditions are expansive and insignificant or negative premiums of small caps occurs when the monetary policy is restrictive. Moreover, Stoll and Whaley (1983) find that transaction costs and investors' holding-period are responsible for the higher returns of small caps. Economic factors are commonly believed as the causes for the different return behaviors between small and large caps. Chan and Chen (1991) argue that small cap firms are usually high financially leveraged and face more cash flow problems, or have low production efficiency than big firms do. Small cap firms will react differently from large cap firms to the same macroeconomic impacts. Dimson and Marsh (1999) and Levis (2002) find that small firms rely more on short term financing so that small caps are more influenced by the condition of the credit markets. In addition, since there are more small companies in some industrial sectors than in others, there could have different performance among various industrial sectors. Dimson and Marsh (1999) find that differences of the weights in different industrial sectors between a small cap index and a large cap index are an important reason for the size premium. However, Levis (2002) concludes that although the difference in industrial sectors causes the small cap premium, industrial performance is not a critical factor. Moreover, Dimson and Marsh (1999) and Levis (2002) find that differences in dividend growth between small and large caps and superior earnings growth in some small firms are related to the size premium. More recently, Eun et al. (2004) state that the return behaviors of large cap and small cap stocks are quite different since returns on large-cap stocks are substantially influenced by common global factors while returns on small-cap stocks are primarily driven by local and idiosyncratic factors.

Although the existence of higher returns for small-cap firms is inconclusive over a long horizon, from a strategic asset allocation perspective, small-cap stocks could potentially be an effective vehicle for international diversification. Eun et al. (2004) argue that investors are more prone to investing in well-known, large foreign companies because such companies are highly visible and often multinational. In addition, institutional investors who track national market indices could reinforce this large-cap bias since large-cap stocks dominate market indices. Similarly, in documenting the gains from international diversification, academic studies tend to use large-cap stocks or national stock market indices dominated by the former. However, the fact that large-cap stocks or stock market indices tend to co-move reduces the benefits from international diversification. Therefore, there could be potential benefits of small-cap stocks in international diversification. Eun et al. (2004) compose three value-weighted index funds for ten countries (a large-cap fund is the top 20 percent, a small cap fund is the bottom 20 percent, and a mid-cap fund is from the rest of stocks in each country). and demonstrate that international small-cap funds are not spannable by country stock market indices (except for Hong Kong) for the period 1980-99. Second, the optimal international portfolio tends to comprise the U.S. market index and foreign small-cap funds; neither foreign market indices nor mid-cap funds receive positive weights during the sample period. The extra gains from the augmented diversification with small-cap funds are statistically significant if transaction costs for small-cap funds are not excessive. More recently, Petrella (2005) uses data from December 1998 to December 2002, performs regression-based tests for mean-variance spanning and finds that euro area small and mid

cap stocks that are classified by size quartile and quintile rankings are independent asset classes.

This paper serves to add to the literature on the potential benefits of small-cap stocks as an instrument for portfolio diversification. Previous studies such as Eun et al (2004) and Petrella (2005) base their results on small cap portfolios that are constructed by the authors but may be difficult to mimic by the typical investor, who would have to invest in a large number of stocks, incurring potentially enormous transactions costs, particularly if the portfolios are rebalanced every year. To avoid the problem that different methodologies used to form size-based portfolios could result in different results, we use the indices available in commercial databases that are easily replicable by the investor.

To identify the behavior of different small cap indexes, we include emerging markets into a diversified portfolio because emerging market stocks could also give rise to large diversification opportunities and they make up a disproportionately small share of investors' equity holdings in U.S. investors. Data show that emerging market share in US investors' portfolios is a little higher than 1% of foreign equities (Fernandes 2002). We want to investigate if the small cap indexes could be separate asset classes after the effects of investing in emerging markets and other international assets are considered.

This paper employs the approaches of the traditional spanning test and step-down spanning test on mean-variance frontiers; and the tests results are compared with empirical measures for portfolio optimization. Mean-variance spanning test and step-down test are econometric methods to test the hypothesis that adding new assets (a small cap index in our study) could enlarge the Markowitz efficient frontier of an investment portfolio. The spanning hypothesis is rejected when the efficiency of Markowitz frontier

of a benchmark portfolio is not improved after new assets are included in the benchmark portfolio. Step-down spanning tests provide further statistic measures on the risk reduction of the global minimum variance portfolio on the frontier and the changes in the slope of the capital allocation line when the risk assets are combined with risk free assets.

Test results on indexes from U.S., Japan, emerging markets, Hong Kong, Taiwan, Singapore, Euro-Market and other members of G 7 group show that the compositions of benchmark portfolios, the time of sample data covered, investors' holding period and investment constraints all are factors that determine whether or not a small cap index is a separate asset class and that some small cap indexes are separate asset classes when the benchmark portfolio includes only popular market indexes in the news media for G 7 country markets. Constraints do not necessarily reduce the diversification benefits of a small cap index to a benchmark portfolio. In addition, Pearson correlation and linear relationship among assets in a portfolio cannot be used to explain the results of spanning tests and the correlation, which has been widely used to explore and explain diversification benefits of assets, is ineffective to search and explain the benefits of portfolio diversification. Comparing with traditional spanning test, the step-down spanning test is more powerful and the results of our step-down tests are well consistent with empirical measures on the portfolio efficiency. The step-down spanning test could be used as a standard approach in studying and building Markowitz efficient frontier.

To best of our knowledge, our work is the first attempt that using size-based indexes available in commercial databases to investigate the behavior of small caps in international markets. The rest of the paper is organized as follows. Section 2 describes the data. Section 3 provides a brief review of the Mean-Variance Spanning methodology.

and presents the results on small cap indexes in Asian markets and developed markets and provides evidence on the persistence of small caps as separate classes. Section 4 describes the economic significance of the step-down spanning test and presents results on some small indexes. Further tests that the composition of benchmark assets influences the significance of spanning test by using market indexes of G 7 countries is presented in this section . Section 5 compares the results of traditional spanning test and step-down spanning test with popular empirical measures. The paper concludes in section 6.

2. Data

Financial services in the world create and publish many small-cap stock benchmarks for different countries. Reilly et al. (2002) compare the six small-cap and micro-cap indexes to three large-cap stock indexes (the S&P 500 Index, the Russell 1000 Index, and the Wilshire Large-- Cap 750 Index) and a global stock and bond index from 1984 to 2000. They find that there are strong similarities among the small-cap stock indexes in term of risk and correlation between them or with large cap index. In this paper, the following indexes for U.S. and non- U.S. markets are used in our study.

2.1 Indexes in the U.S. market

Reilly et al. (2002) find that large-cap stock series (S&P 500 Index, the Russell 1000 Index, and the Wilshire Large-Cap 750 Index) behavior similar in risk, return and correlation. In our study, S&P 500 total return index (SP500) is used to represent large-cap stocks in U.S. market and the Russell 2000 Total Return (R2000) of Frank Russell Company Indexes is used as the representative of the small-Cap Index for U.S. market. The Russell 2000 is the smallest 2,000 securities, based upon market cap size, in the

Russell 3000, which is the 3,000 largest (by market cap) U.S.-domiciled stocks from the NYSE, the AMEX, and NASDAQ.

2.2 Indexes in international markets

For European and emerging markets, we collected MSCI Europe Total Return (EUT), MSCI EU Value Total Return (EUV), MSCI Europe Small Cap Total Return (EUS) and S&P/IFCI Emerging Composite Total Return (EMG). For Hong Kong, MSCI Hong Kong Total Return (HKT) and MSCI Hong Kong Small Cap Total Return (HKS) are used. For Japanese market, we collect data of MSCI Japan Total Return (JPT), BARRA/Nikko Large Cap TR Total Return (NIKL), BARRA/Nikko Small Cap Total Return index (NIKS). For Chinese markets, MSCI Zhong Hua Value Total Return (CNV) and MSCI Zhong Hua Growth Total Return (CNG) are used. For Taiwan, MSCI Taiwan Value Total Return (TWV) and MSCI Taiwan Growth Total Return index (TWG) are used. For Singapore, MSCI Singapore Value Total Return (SINV) and MSCI Singapore Small Cap Total Return (SINS) are used. In addition, MSCI value and small cap total return indexes of different G 7 countries are also collected.

The sample periods of most indexes are from January 1999 to December 2004; NIKL, NIKS, EMG and indexes in U.S. market are from January 1989 to December 2004. All the monthly returns of indexes above are from the Ibbotson Database of Ibbotson Associates and converted to U.S. Dollar return. We use monthly data because monthly data are less influenced by bid-ask or thin trading effects (Ferson et al.1993).

Moreover, we also retrieve some popular indexes in the news media for G 7 countries from DataStream for the period of December 1998 to December 2004. Specifically, we get total return indexes in US dollar of S&P TSX (SPTSX) in Canada,

S&P MIB (SPMIB) in Italy, CAC 40 (CAC40) in France, DAX 30 (DAX30) in Germany and FTSE 100 (FTSE100) in U.K. and the index of NIKKIE 300 (NIKI300) in Japan. We get the monthly returns in U.S. dollar for these index series. The characteristics of different indexes in our study are presented in Appendix 1.

3. Regression based tests of spanning

3.1 Mean-variance spanning test

Huberman and Kandel (1987) first introduced the concept of mean-variance spanning. A set of K risky assets spans a larger set of $N + K$ risky assets if the minimum-variance frontier of the K assets is the same as the minimum variance frontier of the $K + N$ assets. In literatures, the first set of K risky assets is often called the benchmark assets and the second set of N risky assets is called the test assets. Following Huberman and Kandel (1987) and Kan and Zhou (2001), we will check if the small-cap index of Hong Kong (HKS), for example, can be spanned by a benchmark portfolio consisted of several benchmark indices: for instance, SP500, R2000, EMG, EUV and EUG. The spanning test involves a procedure that the new assets (each small-cap index to be tested) are regressed on the benchmark assets as follows:

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + \dots + \varepsilon_i \quad (1)$$

where R_i is the return on the small-cap index to be tested from the i -th country, and SP500 (R2000, EMG and EUV etc.) denotes the return on the benchmark index. α_i is the estimated regression intercept for the small-cap index i and β_j ($j = 1, 2, \dots$) is the estimated regression coefficient associated with the benchmark assets j . The null hypothesis of spanning is equivalent to the joint hypothesis that α is equal to zero and the sum of β_j is equal to one:

$$H_0: \alpha_i = 0, \text{ and } \sum \beta_j = 1 \quad (2)$$

Some test statistics have been proposed to test this hypothesis.

3.1.1 Lagrange multiplier (LM) test

For notational brevity, we use the matrix form of model (1) in what follows:

$$R = XB + E, \quad (3)$$

When there is only one new asset, as is the case in our analysis, R is a $T \times 1$ matrix of R_t (T is the length of the time-series); X is a $T \times (K + 1)$ matrix with its typical row as $[1, \text{SP500}, \text{R2000}, \text{EMG}, \text{EUV} \dots]$. $B = [\alpha, \beta]'$, and E is a $T \times 1$ matrix with ε_t as its typical row.

We assume $T \geq K+2$ and $X'X$ is nonsingular. For the purpose of obtaining exact distributions of the test statistics, we also assume that conditional on returns of benchmark assets, the disturbances ε_t are independent and identically distributed as multivariate normal with mean zero and variance Σ .

The unconstrained maximum likelihood estimator of Σ is

$$\Sigma_u = \frac{1}{T} (R - X \hat{B})' (R - X \hat{B}) \quad (4)$$

Let Σ_c be the constrained maximum likelihood estimator of Σ and

$$U = |\Sigma_u| / |\Sigma_c| \quad (5)$$

Huberman and Kandel (1987) proved that the likelihood ratios of unconstrained and constrained estimation of Eq.3 could be used to test the hypothesis of Eq.2. According to Kan and Zhou (2001), when $N=1$, the likelihood ratio test (LR) can then be written as

$$LR = T \ln (1 + \lambda U) \quad (6)$$

and $1/U = (1 + \lambda U) \quad (7)$

Lagrange multiplier test (LM) is given by

$$LM = T\lambda / (1 + \lambda) \sim \chi_2^2 \quad (8)$$

Substituting Eq.7 to Eq.8, we get the Lagrange multiplier

$$LM = T(1-U) \sim \chi_2^2 \quad (9)$$

3.1.2 Small Sample Distribution of Spanning Test

Kan and Zhou (2001) state that asymptotic tests (like Lagrange multiplier) are problematic when applied to finite samples. We will also use the F-test for small samples under the null hypothesis of spanning developed by Kan and Zhou (2001):

$$\left(\frac{1}{U} - 1\right) \left(\frac{T - K - 1}{2}\right) \sim F_{2, T - K - 1} \quad (10)$$

where T is the length of the time-series; U is the same as Eq.5, the ratio of unconstrained and constrained maximum likelihood estimator of variance.

In addition, according to Kan and Zhou (2001), the Lagrange multiplier test are better than the other two (Likelihood ratio LR and Wald test W) in spanning test with small samples. In fact, when N=1, the relationships between any two of the three tests are simple. For example, $LM = W / (1 + W / T)$. We will use Lagrange multiplier in our spanning test.

To detect possible multicollinearity, we will calculate the Variance Inflation Factor (VIF) for all independent variables on OLS estimation of Eq.1 .VIF is a formal method of detecting the presence of multicollinearity and it measures how much the variances of the estimated regression coefficients are inflated as compared to when the predictor variables are not linearly related.

$$(VIF)_i = (1 - R_i^2)^{-1} \quad i = 1, 2, \dots, k \quad (11)$$

where R_i^2 is the coefficient of multiple determination when X_i is regressed on the $K-1$ other X variables in the model. Mean VIF values denoted by $\overline{(VIF)}$ are calculated by:

$$\overline{(VIF)} = \frac{\sum_{i=1}^K (VIF)_i}{K} \quad (12)$$

VIF values of larger than 10 are often used as an indicator of serious multicollinearity problems.

3.2 Mean-variance spanning tests on small cap indexes in Asian markets

We first focus on the small cap indexes in Asian markets. All the indexes selected and the parts of the market they represent are listed in Table 1.

Table 1

Table 2 reports characteristics of monthly returns of different indexes in US and Asian markets. The averages of returns (arithmetic or geometric) have a pattern through our sample period. The average returns of the small-cap index are greater than that of large cap and the indexes with higher return have higher standard deviations in each country or region except for Zhong Hua index, in the latter, the average return of MSCI Zhong Hua Growth TR is less than that of MSCI Zhong Hua Value TR.

Table 2

Table 3 presents the results of correlation structure of the returns of the indexes. First, the correlations between small-size portfolios and large-size portfolios are well less than one. The imperfect correlation between small and large cap returns implies that

small caps could potentially enhance portfolio diversification. Second, the correlation level between US market and European market is greater than the correlation level between the US and Asian markets. For example, the correlation between SP500 and EUV is 0.80 and the correlation between R2000 and EUS is 0.80. However, the correlation between SP500 and NIKL is 0.47 and the correlation between R2000 and NIKS is 0.20. Small cap indexes from Asia might bring great benefits for diversification because their low correlation with U.S. market. Finally, among the Asian markets, the correlation coefficients between indexes show some country specific characteristics. The correlation between HKL and NIKL is 0.46 and the correlation between HKS and NIKS is 0.42; while the correlation between HKL and CNV is 0.91 and the correlation between HKS and CNG is 0.76. Therefore, the imperfect correlation among Asian markets also may provide potentials to portfolio diversification. In addition, although most of the correlations in Table 3 are significant at the level of 5 percent, the correlation between EUT and NIKS (0.1774) and the correlation between TWV and three indexes of Japan market (0.1461 to 0.1636) are not significant at 5 percent level (The critical value at 5 % significant level is 0.1780, see note on Table 3).

Table 3

To provide a formal test of the hypothesis that investing in small caps of Asian markets could enlarge the efficient frontier for U.S. investors, we employ the spanning tests.

Results of LM test statistics and small sample F test statistics on small cap indexes in Asian markets are reported in Table 4. We form several benchmark portfolios in our tests. The small cap indexes from Asia have different spanning test results with these benchmark portfolios.

First, the spanning test results for Japanese small cap index (NIKS) are significant for all our benchmark portfolios and the spanning test results for other small cap indexes of Asian markets are not significant for all our benchmark portfolios. Specifically, we form the basic investment portfolio by four benchmark indexes of SP500, R2000, EMG and EUT or by five benchmark indexes of SP500, R2000, EMG, EUS and EUV. Both Japanese small cap and large cap indexes (NIKL) have significant spanning test. The spanning test for other small indexes from Chinese small cap indexes (HKS, TWG and CNG) and Singapore (SINS) are not significant. In addition, when we include NIKL in the basic portfolio, NIKS still has significant spanning test result although the correlation coefficient between NIKL and NIKS is greater than 0.82;

Moreover, if we include both NIKL and HKS in our basic portfolio, NIKS is still significant. The correlation coefficient between HKS and NIKS is 0.42 and the correlation between HKS and NIKL is 0.54 (Table 3). The situation that an index (NIKS) could be a separate asset class and have significant spanning test even when this index is highly correlated with one of benchmark assets (NIKL) or when this index has a low correlation with one of benchmark assets (HKS) makes pair-wise correlation almost useless in identifying potential independent asset classes.

In fact, correlation is widely used among both academics and professionals to explain diversification benefits and there are a lot of works that try to find factors driving the

correlations. For example, Bruno Gerard et al. (2002) summarize the previous works on international diversification by the correlation method and reach the conclusion that diversifying across countries may yield higher benefits than diversifying across industries because the average correlation between the countries is noticeably lower than the average correlation between the industries. Eun et al. (2004) also use correlation as main tool to explain their finds. However, our results on Asian indexes show that pair-wise correlation is not a good indicator for studying the diversification of a portfolio.

Table 4 and Table 5

The estimated VIF, Mean VIF and R-square of OLS estimation on different index combinations are listed in Table 4 and 5. We find no evidence of serious multicollinearity problem from the results. In addition, the adjusted R-squares of OLS estimation of Japanese small index in different index combinations are from 12% to 69% when the spanning tests of NIKS are significant. However, the adjusted R-squares of other small index combinations are also in the range of 12% to 69%, but their spanning tests are insignificant. This fact means that linear relationship among indexes cannot explain spanning test. The interactions among assets in a portfolio determine if a new asset could be a separate asset.

Our spanning test results of small cap indexes in Asian markets are interesting. Although Japan is a member of the G 7 group, the small cap indexes in Japan still could be a separate asset class for U.S. investors. However, the other small cap indexes in Asian markets are not independent asset classes for diversification when indexes from major markets are available. In addition, Petrella (2005) also reports that small indexes of

all European market combined are independent asset class. To investigate diversification benefits of small cap indexes from different developed countries, we further conduct the spanning test on individual small cap indexes for the markets in G 7 countries.

3.3 Mean-variance spanning tests on small cap indexes in G7 markets

There are numerous indexes for G 7 markets. Considering the consistence of the methods used to build up indexes, we mainly focus on the small cap indexes in different countries from MSCI index family. MSCI value and MSCI small cap total return indexes in different countries are used in our study (Table 6). The spanning test results for these small cap indexes are listed in Table 7. We have the following findings from Table 7.

Table 6 and Table 7

First, MSCI small cap total return indexes for each country are not independent asset class when MSCI value total return indexes (used as proxies for large cap segment of individual markets) of that countries are included in the benchmark portfolio containing SP500, R2000 and EMG indexes. However, if we use NIKL as the proxy for Japanese large cap market, both NIKS and RJpS are significant in the spanning tests with the benchmark portfolio containing SP500, R2000, EMG and NIKL. Even in this situation, MSCI small cap index (JpS) is still not significant in the test.

The insignificance of the tests for MSCI small cap indexes of G7 countries in portfolios containing MSCI value indexes may be due to the compositions of both MSCI value and MSCI small cap indexes. MSCI Small Cap Indexes select the most liquid securities relative to their market capitalization, and targets for index inclusion 40% of the full market capitalization of the eligible small cap universe within each industry

group, within each country. Effective after the close of trading on September 28, 2001, MSCI broadened the eligible companies' full market capitalization range from USD 200 ~ 800 million to USD 200 ~1,500 million. However, the MSCI country value index is formed by dividing constituents of an underlying MSCI Standard Country Index into a value index and a growth index, each targeting 50% of the market capitalization of the underlying country index. Prior to May 30, 2003, the indices used Price/Book Value (P/BV) ratios to divide the standard MSCI country indices into value and growth indices. All securities were classified as either "value" securities (low P/BV securities) or "growth" securities (high P/BV securities), relative to each MSCI country index. Therefore, some companies may be included in both MSCI small cap and MSCI value (or NIKL) indexes at the same time.

Second, U.S. and Japan are the two largest economies in G 7 and the world. When we build the benchmark portfolio by SP500 R2000 EMG NIKL and NIKS, the spanning test are significant for the small cap indexes of Canada and Italy but insignificant for the indexes of Germany, France and UK. These differences in the tests may be due to the characteristics of small companies listed on the stock markets of different countries. For example, there is a lot of nature resource companies listed in the Canadian market.

Moreover, when we compose the benchmark portfolio by nine assets of SP500 R2000 EMG NIKL, NIKS and MSCI small cap indexes of other 4 countries, Canadian small cap and Italian small cap indexes (CaS and ItS) still have significant spanning tests. In addition, when we compose the benchmark portfolio by SP500 R2000 EMG NIKL and other five small indexes of G7 countries, the spanning tests for NIKS and RJP5 are also

significant respectively. These findings indicate that the small cap indexes in Canada, Italy and Japan could be independent asset classes in our sample period.

Most important, similarly to our finding in the tests on indexes from Asian markets, even the small index to be tested has high correlation with some of the benchmark indexes, this small cap index could still have significant spanning test. Table 8 lists the results of correlation calculation. For example, the correlation coefficient between NIKS and NIKL is 0.82, but NIKS still is significant in most of our test. However, the spanning test of NIKS is not significant when JpV is in the benchmark portfolio. The correlation coefficient between NIKS and JpV is 0.85. On the other hand, UKS and GES do not have correlation coefficient of more than 0.80 with any indexes, but the spanning tests on these indexes are insignificant. In addition, the adjusted R-squares of OLS estimations in Table 7 also show no difference between the portfolios with significant spanning test and the combinations with insignificant spanning test. The results of spanning test on small cap indexes from Asian and G 7 countries imply that mean-variance spanning test results depend on the interactions among the assets included in a portfolio. An index to be tested may be a separate asset class under some combination of benchmarks but not be a separate asset class in other benchmark portfolios. Pair-wise correlation is not a good indicator for designing the diversification portfolio and the spanning test could be used as an alternative for portfolio diversification.

Table 8

3.4 Persistence of a small cap index as an asset class

Literatures have well documented that the results of spanning test vary with the change of the time period that the sample data covered. Our previous results show that benchmark portfolios could not span some small cap indexes in the period of January 1999 to December 2004. We are interested in if these small assets could be a separate asset class for another or a long period of time. Unfortunately, most small cap indexes available in databases are from MSCI index family and MSCI small cap indexes have data only back to January 1999. Two non-MSCI Small cap indexes of Japanese market have data back to 1989. Therefore, we use these two small cap indexes, NIKS and RJpS, to conduct our test. The benchmark asset is consisted of SP500, R2000, EMG, EUT and NIKL. The spanning test results are listed in Table 9. We divide the whole sixteen year from January 1989 to December 2004 into three sub-periods. We conduct tests for these sup-periods respectively and the period of the sixteen year as a whole.

Table 9

The spanning tests on Japanese small cap indexes are insignificant for the period of January 1989 to December 1993 and the period of January 1994 to December 1998. However, the spanning test on both small cap indexes are significant at 5 % for the period of January 1999 to December 2004 and the whole period of January 1989 to December 2004. In addition, the mean VIF does not exceed 10, so there is no serious multicollinearity among independent variables in our test. Moreover, there is no difference on adjusted R-squares between portfolios with significant or insignificant spanning test.

Since the spanning tests are significant for the whole sample period of January 1989 to December 2004, and insignificant in the first and second five-year period of January 1989 to December 2004, the results in Table 9 imply that not only the time that the test data covered is important to the test results, but also does the length of the holding period influence the results. A small cap index (asset) cannot be a separate asset class forever.

Moreover, in previous sections we find that the pair-wise correlation is poor indicator to identify an asset class and the spanning test may be used as an alternative for portfolio diversification. However, the results that there is no permanent independent small asset class imply that the spanning test method may be not easy to apply in practice because this method required people to predict the ex ante behavior and interaction of many indexes (small cap indexes and benchmark assets) in a purposed portfolio.

4. Step-down mean-variance spanning test

4.1 Economic significance of the step-down spanning test

Spanning tests also have an economic interpretation. Kan and Zhou (2001) decompose the spanning test in two parts: one is related to the tangency portfolio, and the other to the global minimum variance portfolio on the efficient frontier. To better assess the statistical evidence against the spanning hypothesis, Kan and Zhou (2001) suggest researchers should examine the two components of the spanning hypothesis ($\alpha = 0$ and $\sum \beta_j = 1$) individually, instead of jointly. Following Kan and Zhou (2001), we use the step-down procedure to test the spanning hypothesis.

First, we test $\alpha = 0$.

$$F_1 = (T - k - 1)\left(\frac{1}{U} - 1\right) \quad (13)$$

where U is the ratio of unconstrained estimate of variance and the constrained estimate of variance by imposing only the constraint of $\alpha = 0$ on Eq.5. Under the null hypothesis, F_1 has a central F-distribution with 1 and $T - K - 1$ degrees of freedom.

Second, we test $\Sigma \beta_j = 1$ conditional on $\alpha = 0$.

$$F_2 = (T - K) \left(\frac{1}{U} - 1 \right) \quad (14)$$

where U is the ratio of constrained estimate of variance by imposing only the constraint of $\alpha = 0$ and the constrained estimate of variance by imposing both the constraints of $\alpha = 0$ and $\Sigma \beta_j = 1$ on Eq.5. F_2 has a central F-distribution with 1 and $T - K$ degrees of freedom, and it is independent of F_1 .

Let the level of significance of the first test is α_1 and that of the second test is α_2 . Under the step-down procedure, Kan and Zhou (2001) suggest that we will accept the spanning hypothesis if we accept both tests. The significance level of the step-down test overall is

$$1 - (1 - \alpha_1)(1 - \alpha_2) = \alpha_1 + \alpha_2 - \alpha_1\alpha_2 \quad (15)$$

The step-down test provides us information on what causes the rejection of traditional spanning test. If the rejection is due to the first test (F_1 significant), then the two tangent portfolios on the efficient frontier are statistically different; and if the rejection is due to the second test, the two global minimum-variance portfolios are different statistically.

Table 10

The step-down mean-variance spanning tests on some of our sample indexes are listed on Table 10. The behaviors of these small cap indexes are different. Some of them could improve the tangency portfolio (F1 test is significant), and some of them could improve the globe minimum variance portfolio (F2 test is significant). Moreover, some of the small cap indexes could improve both the tangency portfolio and the global minimum variance portfolio, but others bring no benefits to diversification at all. In addition, the behavior of the small cap indexes also changes with time. NIKS and RJpS, for example, have different results in the period of 1983 to 2004 when they are included in a benchmark portfolio. NIKS and RJpS improve both the tangency portfolio and the global minimum variance portfolio in the period of January 1999 to December 2004; however, NIKS and RJpS could not add benefits to both the tangency portfolio and the global minimum variance portfolio in the period of January 1989 to December 1993.

When we conduct spanning test in the section 3.3, we find that GeS is insignificant and ItS is significant (Table 7). However, the results in Table 10 show that the small cap index from Germany (GeS) has significant F1 test; while the small cap index from Italy (ItS) only has significant F2 test. An asset with significant F2 test reduces risk in the global minimum variance portfolios and this fact does not necessarily mean this asset is economically important; however, the difference in the tangency portfolio can have significant economic meaning (Kan et al. 2001). The significant F1 test is very useful for investors with a low risk aversion because they will benefit from the improvement of the tangency portfolio's characteristics. Since an asset may have insignificant spanning test and at the same time, it may still have significant F1 test (GeS, for example), the step-down test is better than the traditional spanning test. However, even though an index does

have significant F1 or F2 results *ex post*, we still have to find ways to work on indexes *ex ante*.

4.2. Small cap indexes of G 7 countries in popular index portfolios

Our previous results show that the results of spanning test vary with the perspectives of investors. A test portfolio may be an asset class under one combination of benchmarks but may not be an asset class under another combination of benchmark assets. The composition of the benchmark portfolio is an important factor that decides whether a new asset is a separate asset class or not.

To provide more evidences that the significance of spanning test is influenced by the combination of benchmark assets, we use some popular indexes from different markets of G 7 countries in the news media to form our benchmark portfolio. Specifically, we use SPMIB to represent Italian market, SPTSX for Canadian market, FTSE100 for U.K. market, CAC40 for France Market and DAX30 for Germany market. We combine these indexes with SP500, R2000 and EMG to form the benchmark portfolio of nine indexes. The results of spanning test and step-down test on small cap indexes in G 7 countries with this benchmark portfolio are listed on Table 11.

Table 11

First, all small cap indexes, except for UKS, have significant spanning test. Among these indexes with significant spanning test, Cas, FrS and GeS only have significant F2 test and ItS, NIKS and RJpS are significant on both F1 and F2 tests. The Mean VIF shows no serious multicollinearity. Moreover, similar to previous tests, there are no

differences on adjusted R-Squares between the significant and insignificant spanning tests and between step-down tests.

Second, the test results of small cap indexes in Table 11 are different from the results of these small cap indexes in Table 7 and Table 10. For example, FrS has significant LM and F2 test in Table 11, but all these tests are insignificant in Table 7 and Table 10. In addition, GeS has significant spanning test and F2 test when it is tested against this popular indexes portfolio, but it has insignificant spanning test in Table 7 and has significant F1 test and insignificant F2 test in Table 10. These facts show that the combinations of benchmark portfolios do influence the results of the spanning test and the results of the step-down test. Therefore, the combination of benchmark is a very important factor to determine whether or not a small cap asset could bring in diversification benefits and what kind of benefits (F1 or F2 test) it could result in.

Furthermore, because the benchmark portfolio in Table 11 includes indexes that are often reported in the news media to represent the performance of different markets in the world, the significant spanning tests on the small indexes indicate that these popular indexes do not tell all stories about these markets and there is the “large-cap bias” suggested by Eun et al. (2004). Some small cap assets could be separate asset classes if an investment portfolio includes only these popular indexes.

5. Assessment of diversification benefits

5.1 Three measures of portfolio efficiency for diversification

There are many empirical measures that are used by academic and professional works to evaluate effectiveness and efficiency in portfolio diversification or portfolio management. We employ three popular ones to assess the diversification benefits when a

small cap index is added in a benchmark portfolio.

5.1.1 The reduction in the portfolio risk

First, we find the global minimum variance portfolios that are formed by the benchmark assets (GMVB) and benchmark assets plus the testing asset (GMVT). Then the difference of the standard deviations of both global minimum variance portfolios is used as the measure of reduction in the portfolio risk. Let GMV represent the decrease in the standard deviation of the global minimum variance portfolios and ST represent the standard deviation of the global minimum variance portfolios. We have

$$GMV = ST_{GMVB} - ST_{GMVT} \quad (16)$$

where the subscripts represent different portfolios.

5.1.2 Sharpe Ratio

When investors think only about the mean and variance of their portfolios and they can borrow or lend at risk-free rates, the tangency portfolio of risky assets is more important than the global minimum variance in determining the return of their investments. The Sharpe ratio is one of the popular measures for portfolio efficiency. Similarly to the works by Bekaert and Urias, (1996), Eun et al.(2004)and Petrella (2005), we compute the changes in the Sharpe ratio of the tangency portfolio after adding the test asset to the benchmark assets.

Petrella (2005) states that both poor out-of-sample performance of the optimal portfolio and instability of the optimal portfolio's weights are two practical problems in the estimation of risk in mean variance optimization. In addition, positive risk-free rates will result in higher returns per unit of risk for the optimal portfolios than assuming zero of risk free rates and will highlight undesirable characteristics of the tangency portfolio.

Jorion (1985) points out that assuming zero for risk-free rate could reduce bias of the positive risk-free rates. Therefore, we will assume risk-free rate to be zero in our analysis.

Let SP represent the change in the portfolio's Sharpe ratio, R and ST represent the return and standard deviation of the tangency portfolio, respectively.

$$SP = \frac{R_T}{ST_T} - \frac{R_B}{ST_B} \quad (17)$$

where the subscripts of T and B represent test portfolio and benchmark portfolio, respectively.

5.1.3 Expected return change from efficient portfolios

The third measure of the diversification benefit is the gain in expected return of the efficient portfolio by adding new assets into benchmark assets. Since the change of expected return or variance measure of the efficient portfolios will give us the same results to evaluate the diversification benefit, we only present the results on the reduction in the risk of efficient portfolios.

5.2 Investment policy constraints

Investors often face restrictions on their investments. Some restrictions limit the weights (w_i) of the capital that investors could invest in different asset classes. These limitations will have impacts on our analysis. Similarly to the work of Petrella (2005), we use three sets of investment policy constraints in our analysis.

5.2.1 Unconstrained policy

In unconstrained policy, all assets can be long or short up to 100% of total capital and the sum of the weights invested in each asset adds to one.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad -1 \leq w_i \leq 1 \quad (18)$$

where K is the number of benchmark assets and N is the number of test assets. In our study, $N=1$.

5.2.2 No short sale policy

The condition of unconstrained weights in assets could result in large long and short positions in some assets. However, large short positions are difficult to implement in practice, particularly in small and mid cap stocks; and in addition, short sales could be too costly for most of the stocks (Petrella 2005). We use no short sale policy as that portfolio weights are non-negative.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad 0 \leq w_i \leq 1 \quad (19)$$

where K is the number of benchmark assets and N is the number of test assets. In our study, $N=1$.

5.2.3 Upper bound policy

Upper bound policy restricts the investment position in a single asset. In portfolio management, upper and/or lower limits are frequently imposed in solving optimal portfolio problems. We set 0.5 as upper bound in the optimization process.

$$\sum_{i=1}^{K+N} w_i = 1 \quad \text{and} \quad 0 \leq w_i \leq 0.5 \quad (20)$$

where K is the number of benchmark assets and N is the number of test assets. In our study, $N=1$.

5.3 Estimation of Markowitz mean-variance frontier

Markowitz mean-variance frontiers are used to find the global minimum variance portfolios and the tangent portfolios. To compute GMV and SP in equations (16) and (17), we use historical sample estimation of risk and return as inputs to the Optimizer of

Ibbotson to get the efficient frontiers. The results of diversification gains, based on GMV and SP, under constraints of Eq.18, Eq.19 and Eq.20 are listed in Table 12. The empirical measures show the following characteristics.

Table 12

First, most of the small cap indexes with significant spanning tests have bigger diversification gains on both empirical measures of GMV and SP, and some of them have a gain only on one of the two empirical measures. For example, the NIKS has a significant spanning test and SINS is insignificant in our previous spanning tests. In Table 12, under the restriction of unconstrained, NIKS in case 18 has a gain of 0.066 on SP and 0.284 on GMV, and SINS in case 23 has a gain of 0.069 on SP and 0.003 on GMV. We notice that the SP of SINS is slight higher than that of NIKS, but SINS still is insignificant in spanning test because GMV of SINS is very small. GeS is another example. The spanning test of GeS is insignificant in Table 7; however, the SP of GeS in the case 43 is similar to SP of other small cap indexes that are significant in spanning test. In fact, the step-down test (Table 10) shows GeS could bring in diversification benefit in the tangent portfolio. The results clearly show that both the gains on GMV and SP contribute to the significance of spanning tests.

Second, consistent with our previous discussion, the behavior of a small cap index in diversification depends on the composition of the benchmark portfolio and changes with time. NIKS, for example, has GMV of 0.640 and SP of 0.071 in the case 52; and as the composition of benchmark changes, it has gains of 0.071 on GMV and 0.105 on SP in the

case 17. Since the results in Table 12 are consistent with the step- down tests of Table 10, step-down test could provide more information than traditional spanning tests to evaluate the diversification benefits.

Most important, constraints on the investments have impacts on the potential diversification gains that a small cap index could bring in. In most of our cases, constraints reduce the gains of GMV and SP of a small cap index to a benchmark portfolio. However, constraints do not necessarily results in the decrease of GMV and SP simultaneously. For example, NIKS in the case 28, the constraints increase the GMV while decrease the SP at the same time. RJPS in the case 57, the constraints play in a different way; they increase the SP and decrease the GMV. Moreover, with the same constraints, the diversification gains with the same small cap indexes change differently when the benchmark portfolios are different. For instance, NIKS in case 2 and case17 is combined with different benchmark portfolios, no short sale constraint reduce both GMV and SP in the case 2, but no short sale constraint increases GMV in the case 17. Wang (1998) suggests that imposing constraints on portfolio weights and no short selling could reduce the estimation risk and improves the efficiency of optimal portfolios estimated by sample moments. However, the results in Table 12 do not support his argument. The effect of upper bound constraints is mixed. For instance, NIKS in case 2 and 17, the upper bound constraint reduce both GMV and SP in case 2 but increase GMV and reduce SP in case 17. The only difference between case2 and case17 is that the assets included in the benchmark portfolios are different. Therefore, we need to be careful when considering constraints on the diversification benefits of small caps because the effects of

constraints depend on the asset combination of a portfolio and constraints could sometimes even increase the benefit on a specific economic measure.

Fig.1 clearly shows that small cap indexes with significant spanning tests add more benefits to diversification. The points of the cases with significant spanning tests are located on the right upper part in dimensions of GMV and SP in Fig1a to Fig1c. In addition, we can also find that the constraints overall reduce the diversification benefits arising from the small cap indexes (Fig.1d).

Fig. 1

The economic significance of step-down spanning is well related to the empirical measures on portfolio efficiency on the efficient frontier. In Table 10, we find that both F1 test and F2 test are significant for CaS. Fig.2 and results in Table 12 show that adding CaS to the benchmark portfolio does decrease the risk and increase the Sharpe ratio. The efficient frontier is enlarged. Moreover, We find insignificant of both step-down F tests on FrS in Table 10. The results in table 12 and Fig.3 confirm our previous tests. It is clear that FrS does not bring in the diversification benefit. Both the frontiers with/without FrS are identical.

Fig. 2 and Fig. 3

In previous tests, we also find insignificant spanning test for GeS and significant spanning test for ItS in Table 7. The step- down test results of the Table 10 show GeS

could result in some diversification benefits in the tangent portfolio (the F1 test is significant) but cannot reduce the risk of the global minimum variance portfolio on the efficient frontier portfolio. Fig.4 and the results in Table 12 clearly reflect the result of step-down spanning test and empirical measures. In addition, the results in Table 10 and 12 and Fig.5 show that ItS could decrease the risk of the global minimum variance portfolio, but brings in little benefit to the tangent portfolio (F1 is insignificant and F2 is significant in the step-down test). Moreover, Fig. 6 and Table 12 also show that UKS is not a single asset class.

Fig. 4, Fig.5 and Fig. 6

In the step-down mean-variance spanning test on NIKS and RJpS, we find that the diversification benefits resulting from adding the small cap indexes to benchmark portfolios change over time. In the period of 1989 to 1993, NIKS brings in almost no benefit of diversification to the tangent portfolio and little reduction in the risk of global minimum variance portfolio. The p-value of F1 is 0.96 and p-value of F2 test is 0.56 (Table 10). The gain of standard deviation is 0.256 and the Sharpe ratio increases by 0.001 (Table 12). But in 1994-1998, the F2 test of NIKS is significant and GMV is 0.316. Furthermore, in the period of 1999 to 2004, NIKS are significant on both F1 and F2 tests (Table 10), the GMV is 0.640 and Sharpe ratio increases by 0.071. The empirical measure on RJpS in Table 12 are also consistent with the results of the step-down spanning test of Table 10. Fig.7 to Fig.9 illustrates this historical change of diversification benefits that NIKS could bring in. In Fig.7 NIKS has almost no effect on the benchmark

frontier; and Fig.8 shows the effect of NIKS on reducing the risk of the benchmark frontier. The effects of NIKS are the most significant in Fig.9; NIKS enlarges the frontier of the benchmark portfolio on both directions: reducing the standard deviation of the global minimum variance portfolio and increasing the Sharpe ratio of the tangent portfolio on the efficient frontier.

Fig. 7, Fig.8 and Fig. 9

In fact, all our results on empirical measures are consistent with statistical significant analyses in the previous sections. There are clear advantages of the step-down spanning test over traditional spanning tests. By separating the sources of diversification benefits rather than combining the test statistics in traditional spanning test, we could find more potential separate small cap assets that are relevant for the tangency portfolios or global minimum variance portfolios on Markowitz mean-variance frontier.

6. Conclusion

Spanning tests is a provide a useful and intuitive approach for identifying separate asset class for portfolio diversification. The results of spanning tests on small cap indexes in Asian markets and G 7 countries show that the composition of the benchmark portfolio determines whether a small cap index could be a separate asset class or not. In addition, even though a small cap index is an independent asset class in one period, this small cap index may not be a separate asset class in another period with the same

benchmark portfolio. The lengths of holding period and interaction among the assets in a portfolio influence the test results.

Popular indexes in the news media for major stock markets in G 7 countries do not reflect all movements of the markets. Most small cap indexes in G 7 countries could produce diversification benefits to a portfolio that is consisted of these popular indexes.

In general, constraints reduce the diversification benefits of small cap indexes on benchmark portfolios. However, constraints do not always reduce the usefulness of a small cap index; in some cases, constraints could even results in additional benefits. In addition, the effect of constraints is influenced by the composition of assets in a portfolio.

Normally used pair-wise correlation between two assets is a poor indictor to analysis potential diversification benefits of adding a new asset into a portfolio. It is the interaction among all assets in a portfolio that decides whether or not adding a new asset could enlarge the original frontier.

Step-down mean-variance spanning test is a powerful tool that can be used to search a potential separate asset class for a benchmark portfolio and its results are consistent with empirical measures on variance and Shape ratio for portfolio efficiency. Two F tests of step-down mean-variance spanning test provide information on the effect of a new asset on the tangency portfolio and the global minimum variance portfolio on Markowitz mean-variance frontier, respectively. Therefore, based on the needs and characteristics of an investment portfolio, the step-down mean-variance spanning test could be used, as an alternative for correlation analysis, to identify a separate asset class. However, the problem how to predict the behaviors of assets in a portfolio and the interaction among these assets *ex ante* needs further study.

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Table 1
Representative indexes for different markets

Different countries/regions and their corresponding market indexes used in this paper are presented. The market segment in the table is the part of the market the index used in our study to describe that part of the market. The variable names are the names used for abbreviation in our paper. All index data are from Ibbotson Database and in U.S. dollar.

| Country/Region | Index | Market Segment | Variable Name |
|--------------------|---------------------------------|----------------|---------------|
| U.S. | S&P 500 TR | Large Cap | SP500 |
| | Russell 2000 TR | Small Cap | R2000 |
| Emerging countries | S&P/IFCI Emerging Composite TR | Whole market | EMG |
| Europe | MSCI Europe TR | Whole market | EUT |
| | MSCI EU Value TR | Large Cap | EUV |
| | MSCI Europe Small Cap TR | Small Cap | EUS |
| Japan | MSCI Japan TR USD | Whole market | JPT |
| | BARRA/Nikko Large Cap TR USD | Large Cap | NIKL |
| | BARRA/Nikko Small Cap TR USD | Small Cap | NIKS |
| Singapore | MSCI Singapore Value TR USD | Large Cap | SINV |
| | MSCI Singapore Small Cap TR USD | Small Cap | SINS |
| China | MSCI Zhong Hua Value TR | Large Cap | CNV |
| | MSCI Zhong Hua Growth TR | Small Cap | CNG |
| Hong Kong | MSCI Hong Kong TR USD | Whole market | HKT |
| | MSCI Hong Kong Small Cap TR USD | Small Cap | HKS |
| Taiwan | MSCI Taiwan Value TR USD | Large Cap | TWV |
| | MSCI Taiwan Growth TR USD | Small Cap | TWG |

Note: Since there is no small/large cap index available in some markets, we use value indexes to represent large cap segment and growth indexes to represent small cap segment of that markets.

Table 2
Summary statistics for different indexes

This table shows descriptive statistics monthly percentage returns for different indexes in our study. All return data are in U.S. dollar. The holding period used in the calculation is one month. The sample period is January 1999 through December 2004.

| Index | T Periods | Geometric Mean (%) | Arithmetic Mean (%) | Standard Deviation (%) |
|---------------------------------|--------------|-----------------------|------------------------|------------------------------|
| S&P 500 TR | 72 | 0.10 | 0.21 | 4.60 |
| Russell 2000 TR | 72 | 0.71 | 0.90 | 6.15 |
| S&P/IFCI Emerging Composite TR | 72 | 1.13 | 1.33 | 6.42 |
| MSCI Hong Kong TR USD | 72 | 0.63 | 0.86 | 6.94 |
| MSCI Hong Kong Small Cap TR USD | 72 | 0.91 | 1.18 | 7.49 |
| MSCI Japan TR USD | 72 | 0.22 | 0.39 | 5.84 |
| BARRA/Nikko Large Cap TR USD | 72 | 0.29 | 0.48 | 6.21 |
| BARRA/Nikko Small Cap TR USD | 72 | 1.04 | 1.25 | 6.67 |
| MSCI Zhong Hua Value TR | 72 | 0.69 | 0.96 | 7.44 |
| MSCI Zhong Hua Growth TR | 72 | 0.48 | 0.73 | 7.18 |
| MSCI Taiwan Value TR USD | 72 | -0.02 | 0.45 | 9.85 |
| MSCI Taiwan Growth TR USD | 72 | 0.19 | 0.71 | 10.39 |
| MSCI Singapore Value TR USD | 72 | 1.40 | 1.70 | 7.75 |
| MSCI Singapore Small Cap TR USD | 72 | 1.39 | 1.68 | 7.69 |
| MSCI Europe TR | 72 | 0.24 | 0.36 | 4.97 |
| MSCI EU Value TR | 72 | 0.59 | 0.73 | 5.30 |
| MSCI Europe Small Cap TR | 72 | 0.88 | 1.01 | 5.23 |

Table 3
Return correlation matrix of sample indexes

Pearson correlations between the index series are reported. The sample period is January 1999 through December 2004.

| | SP500 | R2000 | EMG | HKT | HKL | HKS | NIKL | NIKS | JPT | CNV | CNG | TWV | TWG |
|-------|-------|-------|------|------|------|------|------|------|------|------|------|------|------|
| SP500 | 1.00 | | | | | | | | | | | | |
| R2000 | 0.69 | 1.00 | | | | | | | | | | | |
| EMG | 0.76 | 0.75 | 1.00 | | | | | | | | | | |
| HKT | 0.59 | 0.64 | 0.77 | 1.00 | | | | | | | | | |
| HKL | 0.66 | 0.67 | 0.79 | 0.96 | 1.00 | | | | | | | | |
| HKS | 0.52 | 0.61 | 0.75 | 0.81 | 0.78 | 1.00 | | | | | | | |
| NIKL | 0.47 | 0.42 | 0.54 | 0.44 | 0.46 | 0.54 | 1.00 | | | | | | |
| NIKS | 0.28 | 0.20 | 0.37 | 0.26 | 0.27 | 0.42 | 0.82 | 1.00 | | | | | |
| JPT | 0.49 | 0.42 | 0.56 | 0.45 | 0.47 | 0.56 | 0.98 | 0.84 | 1.00 | | | | |
| CNV | 0.59 | 0.62 | 0.75 | 0.94 | 0.91 | 0.81 | 0.39 | 0.25 | 0.43 | 1.00 | | | |
| CNG | 0.61 | 0.65 | 0.79 | 0.90 | 0.92 | 0.76 | 0.52 | 0.29 | 0.50 | 0.80 | 1.00 | | |
| TWV | 0.32 | 0.31 | 0.66 | 0.48 | 0.46 | 0.47 | 0.15 | 0.15 | 0.16 | 0.49 | 0.44 | 1.00 | |
| TWG | 0.57 | 0.49 | 0.75 | 0.54 | 0.52 | 0.55 | 0.32 | 0.26 | 0.33 | 0.52 | 0.49 | 0.78 | 1.00 |
| SINV | 0.54 | 0.46 | 0.62 | 0.69 | 0.64 | 0.61 | 0.35 | 0.23 | 0.36 | 0.71 | 0.58 | 0.35 | 0.42 |
| SINS | 0.54 | 0.51 | 0.61 | 0.70 | 0.67 | 0.65 | 0.36 | 0.32 | 0.37 | 0.71 | 0.64 | 0.34 | 0.37 |
| EUV | 0.80 | 0.68 | 0.71 | 0.59 | 0.63 | 0.49 | 0.37 | 0.22 | 0.41 | 0.59 | 0.58 | 0.31 | 0.39 |
| EUS | 0.68 | 0.80 | 0.79 | 0.65 | 0.66 | 0.63 | 0.42 | 0.31 | 0.45 | 0.64 | 0.62 | 0.42 | 0.54 |
| EUT | 0.82 | 0.74 | 0.72 | 0.59 | 0.63 | 0.50 | 0.40 | 0.18 | 0.42 | 0.56 | 0.62 | 0.28 | 0.41 |

Note: The t-stats for correlation $t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2}$. With n=72 and at significant level 5%, the correlation between two indexes is significant when their correlation coefficient is greater than 0.1780.

Table 4

Mean-variance spanning tests for Asian small cap indexes

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to be test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + \dots + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H₀: $\alpha = 0$ and $\sum \beta_j = 1$. The sample period is from January 1999 to December 2004.

| Test asset | Benchmark assets | Alpha | Sum of Beta | Adj-R2 | Mean VIF | LM Test p-value | F Test p-value |
|---------------------------|---------------------------------------|-------|-------------|--------|----------|-----------------------------|----------------------------|
| Panel A: Japan Market | | | | | | | |
| NIKS | SP500 R2000 EMG and EUT | 0.75 | 0.33 | 0.12 | 3.34 | 14.65 0.00 | 8.56 0.00 |
| | SP500 R2000 EMG EUS and EUV | 0.78 | 0.45 | 0.12 | 4.53 | 10.18 0.01 | 5.43 0.01 |
| | SP500 R2000 EMG EUS EUV and NIKL | 0.86 | 0.82 | 0.69 | 4.09 | 6.09 0.05 | 3.01 0.06 |
| | SP500 R2000 EMG EUS EUV NIKL and HKS | 0.86 | 0.83 | 0.69 | 4.07 | 6.07 0.05 | 2.95 0.06 |
| | SP500 R2000 EMG EUS and EUV | -0.09 | 0.60 | 0.26 | 4.53 | 7.11 0.03 | 3.45 0.04 |
| Panel B: Hong Kong Market | | | | | | | |
| HKS | SP500 R2000 EMG and EUT | -0.13 | 0.79 | 0.56 | 3.34 | 2.72 0.26 | 1.32 0.27 |
| | SP500 R2000 EMG EUS and EUV | -0.10 | 0.85 | 0.56 | 4.53 | 1.34 0.51 | 0.63 0.54 |
| | SP500 R2000 EMG EUS EUV and NIKL | -0.08 | 0.94 | 0.58 | 4.09 | 0.22 0.90 | 0.10 0.91 |
| | SP500 R2000 EMG EUS EUV NIKL and NIKS | -0.14 | 0.95 | 0.58 | 4.56 | 0.19 0.91 | 0.09 0.92 |

Table 4 (cont.)
Mean-variance spanning tests for Asian small cap indexes

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUV + \dots + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H₀: $\alpha = 0$ and $\sum \beta_j = 1$. The sample period is from January 1999 to December 2004.

| Test asset | Benchmark assets | Alpha | Sum of Beta | Adj-R ² | Mean VIF | LM Test p-value | F Test p-value |
|---------------------------|---------------------------------------|-------|-------------|--------------------|----------|-----------------|----------------|
| Panel C: Taiwan Market | | | | | | | |
| TWG | SP500 R2000 EMG and EUT | -0.90 | 1.10 | 0.59 | 3.34 | 1.61 0.45 | 0.77 0.47 |
| | SP500 R2000 EMG EUS and EUV | -0.66 | 1.24 | 0.61 | 4.53 | 2.49 0.29 | 1.18 0.31 |
| | SP500 R2000 EMG EUS EUV and NIKL | -0.68 | 1.14 | 0.63 | 4.09 | 1.36 0.51 | 0.62 0.54 |
| | SP500 R2000 EMG EUS EUV NIKL and NIKS | -0.81 | 1.16 | 0.62 | 4.56 | 1.76 0.41 | 0.80 0.45 |
| Panel D: Chinese Market | | | | | | | |
| CNG | SP500 R2000 EMG and EUT | -0.43 | 0.93 | 0.61 | 3.34 | 1.02 0.60 | 0.48 0.62 |
| | SP500 R2000 EMG EUS and EUV | -0.42 | 0.88 | 0.61 | 4.53 | 1.74 0.42 | 0.82 0.45 |
| | SP500 R2000 EMG EUS EUV and NIKL | -0.41 | 0.94 | 0.61 | 4.09 | 0.89 0.64 | 0.41 0.67 |
| | SP500 R2000 EMG EUS EUV NIKL and NIKS | -0.23 | 0.90 | 0.62 | 4.56 | 0.87 0.65 | 0.39 0.68 |
| Panel E: Singapore Market | | | | | | | |
| SINS | SP500 R2000 EMG and EUT | 0.83 | 0.85 | 0.37 | 3.34 | 2.15 0.34 | 1.03 0.36 |
| | SP500 R2000 EMG EUS and EUV | 0.90 | 0.81 | 0.36 | 4.53 | 2.68 0.26 | 1.28 0.29 |
| | SP500 R2000 EMG EUS EUV and NIKL | 0.90 | 0.82 | 0.35 | 4.09 | 2.45 0.29 | 1.15 0.32 |
| | SP500 R2000 EMG EUS EUV NIKL and NIKS | 0.55 | 0.89 | 0.38 | 4.56 | 0.89 0.64 | 0.40 0.67 |

Note: the test values in bold are significant at 10% significant level.

Table 5 VIF Results

VIF results for different combination of test assets are listed in the table. $(VIF)_i = (1 - R_i^2)^{-1}$
 $i = 1, 2, \dots, k$ where R_i^2 is the coefficient of multiple determination when X_i is regressed on the $K - 1$
other X variables in the model. Mean VIF values denoted by $\overline{(VIF)}$ are calculated by:

$$\overline{(VIF)} = \frac{\sum_{i=1}^K (VIF)_i}{K}$$

| Test Asset | Benchmark Assets | | | | | | | | | Mean VIF |
|------------|------------------|-------|------|------|------|------|------|------|------|----------|
| | SP500 | R2000 | EMG | EUT | EUS | EUV | NIKL | NIKS | HKS | |
| NIKS | 3.66 | 2.85 | 3.14 | 3.72 | | | | | | 3.34 |
| | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| | 4.00 | 3.37 | 4.08 | | 6.29 | 5.35 | 1.45 | | | 4.09 |
| | 4.03 | 3.39 | 5.07 | | 6.42 | 5.44 | 1.54 | | 2.61 | 4.07 |
| NIKL | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| HKS | 3.66 | 2.85 | 3.14 | 3.72 | | | | | | 3.34 |
| | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| | 4.00 | 3.37 | 4.08 | | 6.29 | 5.35 | 1.45 | | | 4.09 |
| | 4.00 | 3.72 | 4.09 | | 6.84 | 5.50 | 4.22 | 3.57 | | 4.56 |
| TWG | 3.66 | 2.85 | 3.14 | 3.72 | | | | | | 3.34 |
| | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| | 4.00 | 3.37 | 4.08 | | 6.29 | 5.35 | 1.45 | | | 4.09 |
| | 4.00 | 3.72 | 4.09 | | 6.84 | 5.50 | 4.22 | 3.57 | | 4.56 |
| CNG | 3.66 | 2.85 | 3.14 | 3.72 | | | | | | 3.34 |
| | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| | 4.00 | 3.37 | 4.08 | | 6.29 | 5.35 | 1.45 | | | 4.09 |
| | 4.00 | 3.72 | 4.09 | | 6.84 | 5.50 | 4.22 | 3.57 | | 4.56 |
| SINS | 3.66 | 2.85 | 3.14 | 3.72 | | | | | | 3.34 |
| | 3.93 | 3.37 | 3.77 | | 6.29 | 5.31 | | | | 4.53 |
| | 4.00 | 3.37 | 4.08 | | 6.29 | 5.35 | 1.45 | | | 4.09 |
| | 4.00 | 3.72 | 4.09 | | 6.84 | 5.50 | 4.22 | 3.57 | | 4.56 |

Table 6
Representative indexes for different markets in G7 countries

Different regions and countries in G7 and their corresponding market indexes used in this paper are presented. The market segment in the table is the part of the market the index used in our study to describe that part of the market. The variable names are the names used for abbreviation in our paper. All index data are from Ibbotson Database and in U.S. dollar.

| Country/Region | Index | Market Segment | Variable Name |
|--------------------|-----------------------------------|----------------|---------------|
| U.S. | S&P 500 TR | Large Cap | SP500 |
| | Russell 2000 TR | Small Cap | R2000 |
| Emerging countries | S&P/IFCI Emerging Composite TR | whole market | EMG |
| Canada | MSCI Canada Small Cap TR | Small Cap | CaS |
| | MSCI Canada Value TR | Large Cap | CaV |
| France | MSCI France Small Cap TR | Small Cap | FrS |
| | MSCI France Value TR | Large Cap | FrV |
| Germany | MSCI Germany Small Cap TR | Small Cap | GeS |
| | MSCI Germany Value TR | Large Cap | GeV |
| Italy | MSCI Italy Small Cap TR | Small Cap | ItS |
| | MSCI Italy Value TR | Large Cap | ItV |
| Japan | BARRA/Nikko Large Cap TR USD | Large Cap | NIKL |
| | BARRA/Nikko Small Cap TR USD | Small Cap | NIKS |
| | MSCI Japan Small Cap TR | Small Cap | JpS |
| | MSCI Japan Value TR | Large Cap | JpV |
| U.K. | Russell/NOMURA Japan Small Cap TR | Small Cap | RJpS |
| | MSCI U.K. Small Cap TR | Small Cap | UKS |
| | MSCI U.K. Value TR | Large Cap | UKV |

Table 7

Mean-variance spanning tests for small cap indexes of G7 Countries

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKL + \dots + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis $H_0: \alpha = 0$ and $\sum \beta_j = 1$. The sample period is from January 1999 to December 2004.

| Test asset | Benchmark assets | Alpha | Sum of Beta | Adj-R2 | Mean VIF | LM Test p-value | F Test p-value |
|-------------------------|---|-------|-------------|--------|----------|----------------------------|----------------------------|
| Panel A: Canada Market | | | | | | | |
| CaS | SP500 R2000 EMG and CaV | 0.22 | 0.91 | 0.74 | 2.97 | 1.67 0.43 | 0.80 0.45 |
| | SP500 R2000 EMG NIKL and NIKS | 0.74 | 0.76 | 0.60 | 3.18 | 8.62 0.01 | 4.49 0.01 |
| | SP500 R2000 EMG NIKL NIKS FrS GeS ItS UKS | 0.72 | 0.79 | 0.58 | 3.74 | 6.41 0.04 | 3.03 0.06 |
| Panel B: France Market | | | | | | | |
| FrS | SP500 R2000 EMG and FrV | -0.45 | 0.83 | 0.69 | 2.95 | 3.67 0.16 | 1.80 0.17 |
| | SP500 R2000 EMG NIKL and NIKS | -0.37 | 0.93 | 0.57 | 2.77 | 0.88 0.65 | 0.41 0.67 |
| Panel C: Germany Market | | | | | | | |
| GeS | SP500 R2000 EMG and GeV | -0.93 | 0.88 | 0.56 | 2.86 | 3.54 0.17 | 1.73 0.19 |
| | SP500 R2000 EMG NIKL and NIKS | -1.17 | 0.96 | 0.54 | 3.18 | 3.98 0.14 | 1.93 0.15 |
| Panel D: Italy Market | | | | | | | |
| ItS | SP500 R2000 EMG and ItV | 0.42 | 0.95 | 0.70 | 2.46 | 1.62 0.44 | 0.77 0.47 |
| | SP500 R2000 EMG NIKL and NIKS | 0.57 | 0.68 | 0.37 | 3.18 | 6.52 0.04 | 3.29 0.04 |
| | SP500 R2000 EMG NIKL NIKS FrS GeS CaS UKS | 0.69 | 0.76 | 0.60 | 3.83 | 6.41 0.04 | 3.03 0.06 |

Table 7 (cont.)
Mean-variance spanning tests for small cap indexes of G7 Countries

The results of mean-variance spanning tests on different index portfolios are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKL + \dots + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H₀: $\alpha = 0$ and $\sum \beta_j = 1$. The sample period is from January 1999 to December 2004.

| Test asset | Benchmark assets | Alpha | Sum of Beta | Adj-R2 | Mean VIF | LM Test p-value | F Test p-value |
|-----------------------|--|-------|-------------|--------|----------|----------------------------|----------------------------|
| Panel E: Japan Market | | | | | | | |
| NIKS | SP500 R2000 EMG and JpV | 0.45 | 0.89 | 0.72 | 2.44 | 2.23 0.33 | 1.07 0.35 |
| | SP500 R2000 EMG and NIKL | 0.90 | 0.74 | 0.68 | 2.48 | 9.14 0.01 | 4.87 0.01 |
| | SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS | 0.96 | 0.78 | 0.68 | 3.53 | 7.06 0.03 | 3.37 0.04 |
| JpS | SP500 R2000 EMG and JpV | 0.17 | 1.01 | 0.74 | 2.95 | 0.17 0.92 | 0.08 0.92 |
| | SP500 R2000 EMG and NIKL | 0.66 | 0.85 | 0.68 | 2.48 | 3.74 0.15 | 1.84 0.17 |
| RJpS | SP500 R2000 EMG and JpV | 0.35 | 0.92 | 0.77 | 2.44 | 1.68 0.43 | 0.80 0.45 |
| | SP500 R2000 EMG and NIKL | 0.81 | 0.78 | 0.75 | 2.48 | 9.43 0.01 | 5.05 0.01 |
| | SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS | 0.76 | 0.82 | 0.75 | 3.53 | 6.18 0.05 | 2.91 0.06 |
| Panel F: UK Market | | | | | | | |
| UKS | SP500 R2000 EMG and UKV | 0.16 | 0.95 | 0.76 | 2.90 | 0.68 0.71 | 0.32 0.73 |
| | SP500 R2000 EMG NIKL and NIKS | 0.13 | 0.94 | 0.73 | 3.18 | 0.51 0.78 | 0.23 0.79 |

Note: the test values in bold are significant at 10% significant level.

Table 8
Return correlation matrix of indexes in G7 countries

Pearson correlations between indexes series are reported. The sample period is January 1999 through December 2004.

| | SP500 | R2000 | EMG | CaS | CaV | FrS | FrV | GeS | GeV | ItS | ItV | NIKL | NIKS | JpS | JpV | R |
|-------|-------|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| SP500 | 1.00 | | | | | | | | | | | | | | | |
| R2000 | 0.69 | 1.00 | | | | | | | | | | | | | | |
| EMG | 0.76 | 0.75 | 1.00 | | | | | | | | | | | | | |
| CaS | 0.61 | 0.74 | 0.73 | 1.00 | | | | | | | | | | | | |
| CaV | 0.69 | 0.57 | 0.59 | 0.75 | 1.00 | | | | | | | | | | | |
| FrS | 0.58 | 0.73 | 0.68 | 0.63 | 0.54 | 1.00 | | | | | | | | | | |
| FrV | 0.76 | 0.62 | 0.62 | 0.59 | 0.65 | 0.74 | 1.00 | | | | | | | | | |
| GeS | 0.61 | 0.67 | 0.69 | 0.60 | 0.53 | 0.78 | 0.65 | 1.00 | | | | | | | | |
| GeV | 0.71 | 0.69 | 0.63 | 0.57 | 0.61 | 0.75 | 0.83 | 0.67 | 1.00 | | | | | | | |
| ItS | 0.54 | 0.60 | 0.59 | 0.56 | 0.52 | 0.79 | 0.74 | 0.63 | 0.70 | 1.00 | | | | | | |
| ItV | 0.47 | 0.53 | 0.46 | 0.50 | 0.51 | 0.68 | 0.71 | 0.50 | 0.61 | 0.81 | 1.00 | | | | | |
| NIKL | 0.47 | 0.42 | 0.54 | 0.47 | 0.31 | 0.29 | 0.32 | 0.27 | 0.25 | 0.30 | 0.26 | 1.00 | | | | |
| NIKS | 0.28 | 0.20 | 0.37 | 0.34 | 0.30 | 0.22 | 0.20 | 0.21 | 0.10 | 0.19 | 0.22 | 0.82 | 1.00 | | | |
| JpS | 0.30 | 0.24 | 0.37 | 0.37 | 0.31 | 0.24 | 0.22 | 0.20 | 0.11 | 0.20 | 0.24 | 0.83 | 0.97 | 1.00 | | |
| JpV | 0.41 | 0.27 | 0.43 | 0.42 | 0.44 | 0.21 | 0.32 | 0.21 | 0.20 | 0.26 | 0.28 | 0.89 | 0.85 | 0.86 | 1.00 | |
| RJpS | 0.31 | 0.24 | 0.39 | 0.37 | 0.30 | 0.23 | 0.22 | 0.19 | 0.11 | 0.21 | 0.22 | 0.86 | 0.99 | 0.98 | 0.88 | 1.00 |
| UKS | 0.65 | 0.80 | 0.80 | 0.69 | 0.58 | 0.79 | 0.66 | 0.69 | 0.67 | 0.68 | 0.62 | 0.52 | 0.39 | 0.40 | 0.44 | 0.00 |
| UKV | 0.74 | 0.49 | 0.63 | 0.56 | 0.62 | 0.55 | 0.79 | 0.56 | 0.70 | 0.64 | 0.64 | 0.35 | 0.22 | 0.21 | 0.38 | 0.00 |

Table 9
Mean-variance spanning tests for small cap indexes in different periods

The results of mean-variance spanning tests on the small cap index in different periods are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * EUT + \dots + \varepsilon_i$$

Mean VIF is the average Variance Inflation Factor for all independent variables. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H₀: $\alpha = 0$ and $\sum \beta_j = 1$. The sample period is from January 1989 to December 2004 and monthly total return data in US dollar are used.

| Test asset period | Benchmark assets | Alpha | Sum of Beta | Adj-R2 | Mean VIF | LM Test p-value | F Test p-value |
|---------------------|------------------|-------|-------------|--------|----------|-----------------|----------------|
| Panel A: NIKS | | | | | | | |
| Jan.1989 - Dec.1993 | SP500 R2000 | | | | | 0.38 | 0.17 |
| | EMG NKIL and EUT | -0.03 | 1.09 | 0.80 | 2.50 | 0.83 | 0.84 |
| Jan.1994 - Dec.1998 | SP500 R2000 | | | | | 3.18 | 1.51 |
| | EMG NKIL and EUT | 0.08 | 0.77 | 0.77 | 2.46 | 0.20 | 0.23 |
| Jan.1999 - Dec.2004 | SP500 R2000 | | | | | 10.07 | 5.37 |
| | EMG NKIL and EUT | 0.88 | 0.71 | 0.68 | 3.03 | 0.01 | 0.01 |
| Jan.1989 - Dec.2004 | SP500 R2000 | | | | | 8.81 | 4.47 |
| | EMG NKIL and EUT | 0.48 | 0.80 | 0.75 | 2.23 | 0.01 | 0.01 |
| Panel B: RJPS | | | | | | | |
| Jan.1989 - Dec.1993 | SP500 R2000 | | | | | 0.93 | 0.42 |
| | EMG NKIL and EUT | -0.01 | 1.12 | 0.84 | 2.50 | 0.63 | 0.66 |
| Jan.1994 - Dec.1998 | SP500 R2000 | | | | | 2.09 | 0.97 |
| | EMG NKIL and EUT | -0.03 | 0.84 | 0.79 | 2.46 | 0.35 | 0.38 |
| Jan.1999 - Dec.2004 | SP500 R2000 | | | | | 10.27 | 5.49 |
| | EMG NKIL and EUT | 0.79 | 0.76 | 0.75 | 3.03 | 0.01 | 0.01 |
| Jan.1989 - Dec.2004 | SP500 R2000 | | | | | 7.07 | 3.56 |
| | EMG NKIL and EUT | 0.44 | 0.84 | 0.79 | 2.23 | 0.03 | 0.03 |

Note: the test values in bold are significant at 5 % level

Table 10
Step-down mean variance spanning test on small cap indexes

Step-down spanning tests for small cap indexes are reported. The first step (F1) is an F-test of $H_0: \alpha = 0$, and the second step (F2) is a F-test of $H_0: \sum \beta_j = 1$ conditional on $\alpha = 0$. Significant level for both tests are calculated by $1 - (1 - \alpha_1)(1 - \alpha_2) = \alpha_1 + \alpha_2 - \alpha_1\alpha_2$. The sample periods are the same as Table 4, Table 7 and Table 9.

| Test asset | Benchmark assets | F1-test | | F2-test | | Significant level |
|--|-------------------------------|-------------|-------------|-------------|-------------|-------------------|
| | | Statistic | p-value | Statistic | p-value | |
| Panel A: Asian Market | | | | | | |
| NIKS | SP500 R2000 EMG EUS and EUV | 1.03 | 0.31 | 9.84 | 0.00 | 0.32 |
| NIKL | SP500 R2000 EMG EUS and EUV | 0.02 | 0.89 | 7.33 | 0.01 | 0.90 |
| TWG | SP500 R2000 EMG EUS and EUV | 0.69 | 0.41 | 1.68 | 0.20 | 0.53 |
| HKS | SP500 R2000 EMG EUS and EUV | 0.03 | 0.86 | 1.24 | 0.27 | 0.90 |
| CNG | SP500 R2000 EMG EUS and EUV | 0.59 | 0.44 | 1.05 | 0.31 | 0.62 |
| SINS | SP500 R2000 EMG EUS and EUV | 1.43 | 0.24 | 1.12 | 0.29 | 0.46 |
| Panel B: G7 Market | | | | | | |
| CaS | SP500 R2000 EMG NIKL and NIKS | 3.19 | 0.08 | 5.59 | 0.02 | 0.10 |
| FrS | SP500 R2000 EMG NIKL and NIKS | 0.42 | 0.52 | 0.40 | 0.53 | 0.77 |
| GeS | SP500 R2000 EMG NIKL and NIKS | 3.66 | 0.06 | 0.19 | 0.66 | 0.68 |
| ItS | SP500 R2000 EMG NIKL and NIKS | 1.02 | 0.32 | 5.55 | 0.02 | 0.33 |
| UKS | SP500 R2000 EMG NIKL and NIKS | 0.11 | 0.74 | 0.36 | 0.55 | 0.88 |
| NIKS | SP500 R2000 EMG and NIKL | 3.75 | 0.06 | 5.76 | 0.02 | 0.08 |
| RJpS | SP500 R2000 EMG and NIKL | 4.13 | 0.05 | 5.71 | 0.02 | 0.06 |
| Panel D: Index history | | | | | | |
| A: NIKS vs. SP500 R2000 EMG NKIL and EUT | | | | | | |
| Jan.1989 - Dec.1993 | | 0.00 | 0.96 | 0.35 | 0.56 | 0.98 |
| Jan.1994 - Dec.1998 | | 0.02 | 0.88 | 3.06 | 0.09 | 0.89 |
| Jan.1999 - Dec.2004 | | 3.66 | 0.06 | 6.81 | 0.01 | 0.07 |
| Jan.1989 - Dec.2004 | | 2.64 | 0.11 | 6.24 | 0.01 | 0.12 |
| B: RJPS vs. SP500 R2000 EMG NKIL and EUT | | | | | | |
| Jan.1989 - Dec.1993 | | 0.00 | 0.99 | 0.86 | 0.36 | 0.99 |
| Jan.1994 - Dec.1998 | | 0.00 | 0.95 | 1.98 | 0.17 | 0.96 |
| Jan.1999 - Dec.2004 | | 4.03 | 0.05 | 6.65 | 0.01 | 0.06 |
| Jan.1989 - Dec.2004 | | 2.74 | 0.10 | 4.33 | 0.04 | 0.13 |

Note: the test values in bold are significant at 10 % level

Table 11

Mean-variance spanning tests of small cap indexes of G7 in popular benchmarks

The results of mean-variance spanning tests and step-down spanning tests for small cap indexes on different small cap indexes in G7 countries are reported. The test methodologies are described in the text of the paper. *Test asset* is the asset to test if it could enlarge the efficient frontier formed by popular *Benchmark assets*. Alpha, Sum of Beta and Adj-R² are from the estimation of Eq.1.

$$R_i = \alpha_i + \beta_1 * SP500 + \beta_2 * R2000 + \beta_3 * EMG + \beta_4 * NIKI300 + \dots + \varepsilon_i$$

The benchmark assets are combination of popular indexes of SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30. Lagrange multiplier (LM) test and small sample F test are used to test the hypothesis H₀: $\alpha = 0$ and $\sum \beta_j = 1$. The first step (F1) is an F-test of H₀: $\alpha = 0$, and the second step (F2) is a F-test of H₀: $\sum \beta_j = 1$ conditional on $\alpha = 0$. The sample period is from January 1999 to December 2004.

| Test asset | Alpha | Sum of Beta | Adj-R2 | LM Test p-value | F Test p-value | F1 test p-value | F2 test p-value |
|------------|-------|-------------|--------|-----------------------------|----------------------------|----------------------------|----------------------------|
| CaS | 0.53 | 0.77 | 0.66 | 6.71 0.03 | 3.19 0.05 | 1.88 0.18 | 4.43 0.04 |
| FrS | 0.28 | 0.73 | 0.71 | 4.91 0.09 | 2.27 0.11 | 0.35 0.55 | 4.23 0.04 |
| GeS | -0.50 | 0.66 | 0.64 | 6.59 0.04 | 3.12 0.05 | 0.84 0.36 | 5.41 0.02 |
| ItS | 0.89 | 0.76 | 0.66 | 8.33 0.02 | 4.05 0.02 | 4.43 0.04 | 3.49 0.07 |
| NIKS | 1.06 | 0.69 | 0.72 | 11.38 0.00 | 5.82 0.00 | 5.68 0.02 | 5.55 0.02 |
| JpS | 0.73 | 0.80 | 0.75 | 5.24 0.07 | 2.43 0.10 | 2.58 0.11 | 2.23 0.14 |
| RJpS | 0.89 | 0.74 | 0.79 | 10.82 0.00 | 5.49 0.01 | 5.49 0.02 | 5.12 0.03 |
| UKS | 0.37 | 1.07 | 0.79 | 1.96 0.38 | 0.87 0.42 | 1.09 0.30 | 0.65 0.42 |

Note: 1. Mean VIF (the average Variance Inflation Factor for all independent variables) is 5.62.

2. The test values in bold are significant at 10 % level

Table 12
Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17. The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

| Case | Test asset | Benchmark assets | Span test | Unconstrained | | No short | | Upper bound | |
|---|------------|---------------------------------------|-----------|---------------|-------|----------|-------|-------------|-------|
| | | | | GMV | SP | GMV | SP | GMV | SP |
| <i>Panel A: Small cap indexes in Asia</i> | | | | | | | | | |
| 3 | HKS | SP500 R2000 EMG and EUT | 0 | 0.083 | 0.006 | 0.015 | 0.000 | 0.027 | 0.008 |
| 4 | TWG | SP500 R2000 EMG and EUT | 0 | 0.011 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 |
| 5 | CNG | SP500 R2000 EMG and EUT | 0 | 0.011 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| 6 | SINS | SP500 R2000 EMG and EUT | 0 | 0.029 | 0.053 | 0.010 | 0.030 | 0.028 | 0.047 |
| 2 | NIKS | SP500 R2000 EMG and EUT | 1 | 0.463 | 0.056 | 0.380 | 0.032 | 0.418 | 0.050 |
| 17 | NIKS | SP500 R2000 EMG EUS EUV and NIKL | 1 | 0.071 | 0.105 | 0.135 | 0.031 | 0.134 | 0.031 |
| 20 | HKS | SP500 R2000 EMG EUS and EUV | 0 | 0.015 | 0.034 | 0.000 | 0.000 | 0.002 | 0.000 |
| 21 | TWG | SP500 R2000 EMG EUS and EUV | 0 | 0.006 | 0.048 | 0.000 | 0.000 | 0.000 | 0.000 |
| 22 | CNG | SP500 R2000 EMG EUS and EUV | 0 | 0.003 | 0.039 | 0.000 | 0.000 | 0.000 | 0.000 |
| 23 | SINS | SP500 R2000 EMG EUS and EUV | 0 | 0.003 | 0.069 | 0.002 | 0.031 | 0.019 | 0.031 |
| 18 | NIKS | SP500 R2000 EMG EUS and EUV | 1 | 0.284 | 0.066 | 0.281 | 0.031 | 0.339 | 0.031 |
| 19 | NIKL | SP500 R2000 EMG EUS and EUV | 1 | 0.199 | 0.034 | 0.146 | 0.000 | 0.205 | 0.000 |
| 24 | HKS | SP500 R2000 EMG EUS EUV and NIKL | 0 | - 0.008 | 0.035 | 0.000 | 0.000 | 0.000 | 0.000 |
| 25 | TWG | SP500 R2000 EMG EUS EUV and NIKL | 0 | - 0.006 | 0.049 | 0.000 | 0.000 | 0.000 | 0.000 |
| 26 | CNG | SP500 R2000 EMG EUS EUV and NIKL | 0 | - 0.008 | 0.042 | 0.000 | 0.000 | 0.000 | 0.000 |
| 27 | SINS | SP500 R2000 EMG EUS EUV and NIKL | 0 | 0.011 | 0.069 | 0.000 | 0.031 | 0.000 | 0.031 |
| 28 | NIKS | SP500 R2000 EMG EUS EUV NIKL and HKS | 1 | 0.070 | 0.106 | 0.135 | 0.031 | 0.134 | 0.031 |
| 29 | HKS | SP500 R2000 EMG EUS EUV NIKL and NIKS | 0 | - 0.004 | 0.020 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 12 (Cont.)
Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17. The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

| Case | Test asset | Benchmark assets | Span test | Unconstrained | | No short | | Upper bound | |
|---|------------|--|-----------|---------------|-------|----------|-------|-------------|-------|
| | | | | GMV | SP | GMV | SP | GMV | SP |
| <i>Panel A: Small cap indexes in Asia</i> | | | | | | | | | |
| 31 | CNG | SP500 R2000 EMG EUS EUV NIKL and NIKS | 0 | 0.007 | 0.022 | 0.000 | 0.000 | 0.000 | 0.000 |
| 32 | SINS | SP500 R2000 EMG EUS EUV NIKL and NIKS | 0 | 0.000 | 0.037 | 0.000 | 0.021 | 0.000 | 0.021 |
| <i>Panel B: Small cap indexes in G 7</i> | | | | | | | | | |
| 1 | CaS | SP500 R2000 EMG and CaV | 0 | 0.041 | 0.042 | 0.009 | 0.027 | 0.018 | 0.047 |
| 7 | FrS | SP500 R2000 EMG and FrV | 0 | 0.065 | 0.007 | 0.010 | 0.000 | 0.013 | 0.000 |
| 8 | GeS | SP500 R2000 EMG and GeV | 0 | 0.032 | 0.045 | -0.001 | 0.060 | 0.025 | 0.000 |
| 9 | ItS | SP500 R2000 EMG and ItV | 0 | 0.012 | 0.041 | 0.003 | 0.027 | 0.017 | 0.034 |
| 10 | NIKS | SP500 R2000 EMG and JpV | 0 | 0.036 | 0.038 | 0.024 | 0.030 | 0.019 | 0.035 |
| 11 | JpS | SP500 R2000 EMG and JpV | 0 | 0.000 | 0.005 | 0.000 | 0.009 | 0.000 | 0.011 |
| 12 | RJpS | SP500 R2000 EMG and JpV | 0 | 0.025 | 0.031 | 0.016 | 0.024 | 0.011 | 0.029 |
| 14 | JpS | SP500 R2000 EMG and NIKL | 0 | 0.061 | 0.057 | 0.069 | 0.012 | 0.074 | 0.025 |
| 13 | NIKS | SP500 R2000 EMG and NIKL | 1 | 0.186 | 0.101 | 0.170 | 0.032 | 0.204 | 0.050 |
| 15 | RJpS | SP500 R2000 EMG and NIKL | 1 | 0.185 | 0.097 | 0.162 | 0.027 | 0.196 | 0.044 |
| 16 | UKS | SP500 R2000 EMG and UKV | 0 | 0.015 | 0.016 | 0.000 | 0.011 | 0.000 | 0.027 |
| 42 | FrS | SP500 R2000 EMG NIKL and NIKS | 0 | 0.010 | 0.003 | 0.002 | 0.000 | 0.007 | 0.000 |
| 43 | GeS | SP500 R2000 EMG NIKL and NIKS | 0 | 0.002 | 0.056 | 0.000 | 0.000 | 0.002 | 0.000 |
| 45 | UKS | SP500 R2000 EMG NIKL and NIKS | 0 | 0.012 | 0.009 | 0.000 | 0.005 | 0.000 | 0.005 |
| 41 | CaS | SP500 R2000 EMG NIKL and NIKS | 1 | 0.188 | 0.085 | 0.109 | 0.077 | 0.158 | 0.061 |
| 44 | ItS | SP500 R2000 EMG NIKL and NIKS | 1 | 0.173 | 0.040 | 0.140 | 0.029 | 0.189 | 0.029 |

Table 12 (Cont.)

Diversification benefits with small indexes

Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17. The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

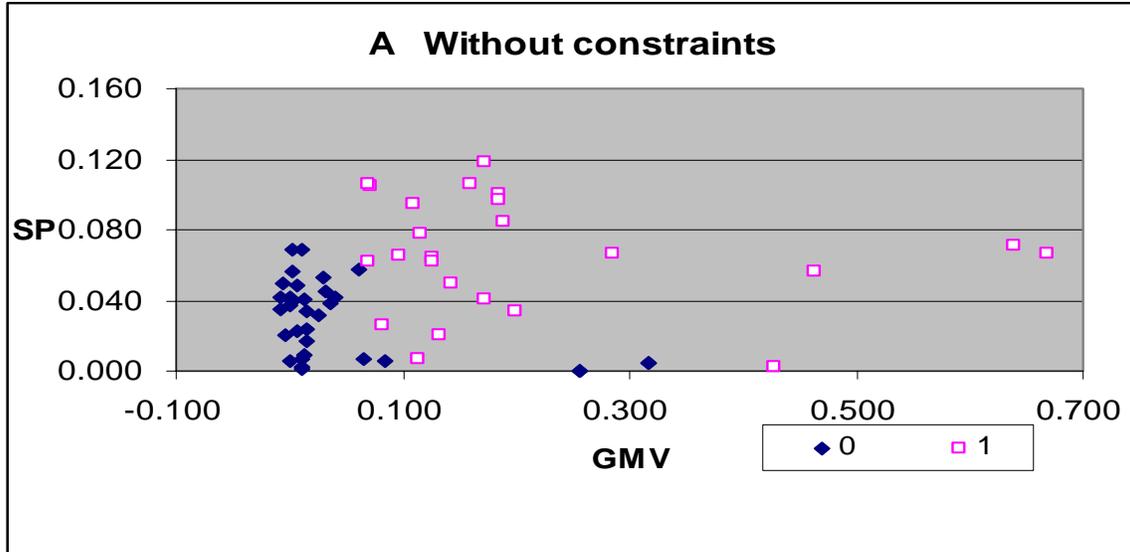
| Case | Test asset | Benchmark assets | Span test | Unconstrained | | No short | | Upper bound | |
|--|------------|---|-----------|---------------|-------|----------|-------|-------------|-------|
| | | | | GMV | SP | GMV | SP | GMV | SP |
| <i>Panel B: Small cap indexes in G 7</i> | | | | | | | | | |
| 46 | NIKS | SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS | 1 | 0.115 | 0.078 | 0.128 | 0.013 | 0.128 | 0.025 |
| 47 | RJpS | SP500 R2000 EMG NIKL FrS GeS ItS CaS UKS | 1 | 0.097 | 0.065 | 0.118 | 0.009 | 0.118 | 0.020 |
| 48 | ItS | SP500 R2000 EMG NIKL NIKS FrS GeS CaS UKS | 1 | 0.142 | 0.050 | 0.077 | 0.005 | 0.082 | 0.016 |
| 49 | CaS | SP500 R2000 EMG NIKL NIKS FrS GeS ItS UKS | 1 | 0.127 | 0.062 | 0.047 | 0.053 | 0.050 | 0.050 |
| <i>Panel C: Small cap indexes in G 7 popular benchmark</i> | | | | | | | | | |
| 40 | UKS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 0 | 0.015 | 0.023 | 0.000 | 0.008 | 0.000 | 0.008 |
| 33 | CaS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.127 | 0.064 | 0.064 | 0.085 | 0.068 | 0.055 |
| 34 | FrS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.114 | 0.007 | 0.000 | 0.000 | 0.000 | 0.000 |
| 35 | GeS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.133 | 0.021 | 0.000 | 0.000 | 0.000 | 0.000 |
| 36 | ItS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.110 | 0.095 | 0.034 | 0.024 | 0.038 | 0.024 |
| 37 | NIKS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.171 | 0.118 | 0.168 | 0.030 | 0.168 | 0.030 |
| 38 | JpS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.070 | 0.062 | 0.104 | 0.009 | 0.097 | 0.009 |

Table 12 (Cont.)
Diversification benefits with small indexes

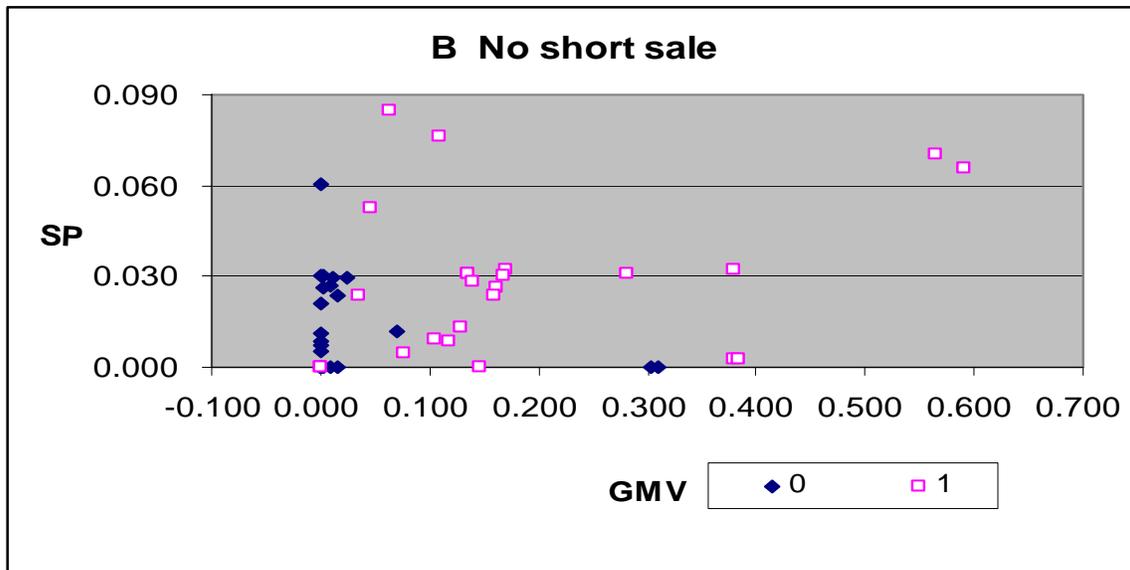
Two popular empirical measures for diversification benefits on different cases are reported. GMV is the decrease in the standard deviation on the globe minimum variance portfolio after a small cap index is included in the benchmark portfolio, and SP is the increase in the Sharpe ratio on the tangent portfolio when the small cap index is included. The formula used to calculate GMV and SP are Eq.16 and Eq.17. The policy constraints are defined in Eq.18 to Eq.20. Span test indicates the results of spanning test, 1 significant and 0 insignificant. For panel A to panel C, the sample period is from January 1999 to December 2004. The sample period of panel D is from January 1989 to December 2004.

| Case | Test asset | Benchmark assets | Span test | Unconstrained | | No short | | Upper bound | |
|---|------------|--|-----------|---------------|-------|----------|-------|-------------|-------|
| | | | | GMV | SP | GMV | SP | GMV | SP |
| <i>Panel C: Small cap indexes in G 7 popular benchmark</i> | | | | | | | | | |
| 39 | RJPS | SP500 R2000 EMG NIKI300 SPTSX SPMIB CAC40 FTSE100 DAX30 | 1 | 0.159 | 0.106 | 0.159 | 0.024 | 0.158 | 0.024 |
| <i>Panel D: History performance of Japanese small cap indexes</i> | | | | | | | | | |
| 50 | NIKS 89-93 | SP500 R2000 EMG NKIL and EUT | 0 | 0.256 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 |
| 51 | NIKS 94-98 | SP500 R2000 EMG NKIL and EUT | 0 | 0.316 | 0.005 | 0.304 | 0.000 | 0.304 | 0.000 |
| 54 | RJPS 89-93 | SP500 R2000 EMG NKIL and EUT | 0 | 0.255 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 55 | RJPS 94-98 | SP500 R2000 EMG NKIL and EUT | 0 | 0.317 | 0.004 | 0.309 | 0.000 | 0.000 | 0.000 |
| 52 | NIKS 99-04 | SP500 R2000 EMG NKIL and EUT | 1 | 0.640 | 0.071 | 0.565 | 0.071 | 0.567 | 0.074 |
| 53 | NIKS 89-04 | SP500 R2000 EMG NKIL and EUT | 1 | 0.082 | 0.026 | 0.380 | 0.003 | 0.389 | 0.003 |
| 56 | RJPS 99-04 | SP500 R2000 EMG NKIL and EUT | 1 | 0.668 | 0.067 | 0.590 | 0.066 | 0.592 | 0.069 |
| 57 | RJPS 89-04 | SP500 R2000 EMG NKIL and EUT | 1 | 0.428 | 0.002 | 0.385 | 0.002 | 0.394 | 0.003 |

Fig.1 Gains on reduce in the risk of the global minimum variance portfolio and increase in the Sharpe ratio on the tangent portfolio. This figure provides graphical evidence of the increase of GMV and SP when spanning test is significant. Fig. 1a refers to the unconstrained optimization process, Fig. 1b to the no short sale constrained optimization process, Fig. 1c to the upper bound constrained optimization process and Fig. 1d to the unconstrained and two constrained optimization processes on the cases with significant spanning test. All the data are from Table 12. The SP and GMV are estimated by Optimizer of Ibbotson Association.

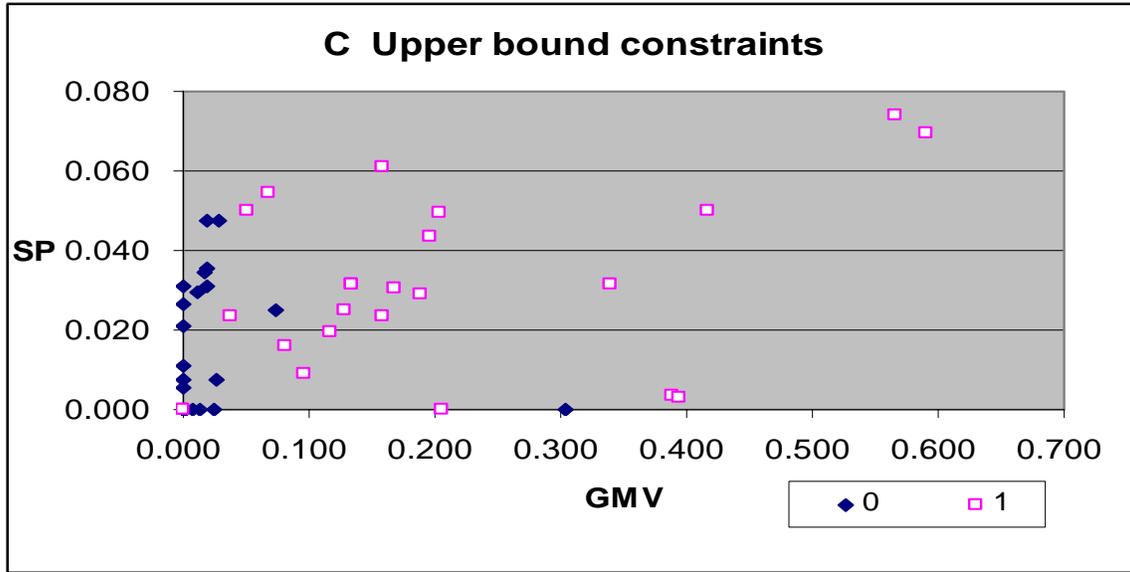


Note: Series 1 has a significant spanning test and series 0 has an insignificant spanning test.

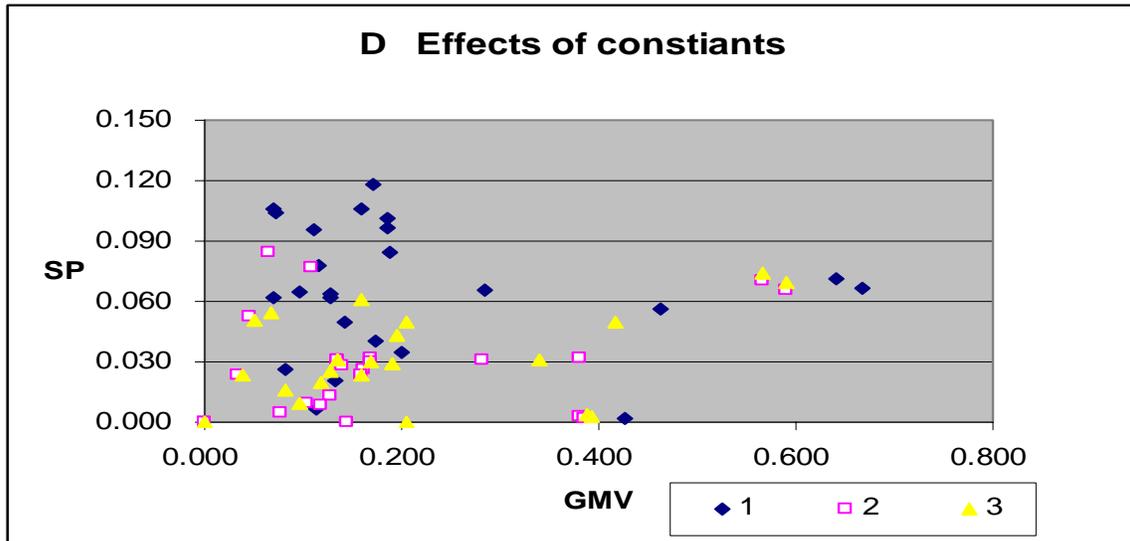


Note: Series 1 has significant spanning test and series 0 has an insignificant spanning test.

Fig.1(cont.) Gains on reduce in the risk of the global minimum variance portfolio and increase in the Sharpe ratio on the tangent portfolio. This figure provides graphical evidence of the increase of GMV and SP when spanning test is significant. Fig.1a refers to the unconstrained optimization process, Fig.1b to the no short sale constrained optimization process, Fig.1c to the upper bound constrained optimization process and Fig.1d to the unconstrained and two constrained optimization processes on the cases with significant spanning test. All the data are from Table 12.



Note: Series 1 has a significant spanning test and series 0 has an insignificant spanning test.



Note: Series 1 is under no constraints, Series 2 is under the condition of no short sale and series 3 is under upper bound condition.

Fig.2 Efficient Frontier – case 41 CaS. This figure provides graphical evidence of the shift of the frontier when CaS is included in a benchmark portfolio. Case 41 refers to the case No.41 in Table 12.No constraints are applied to estimate the frontier..

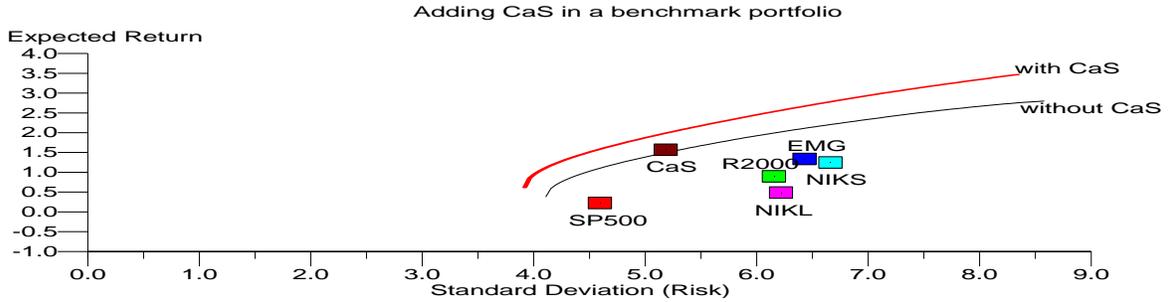


Fig.3 Efficient Frontier – case 42 FrS. This figure provides graphical evidence of the shift of the frontier when FrS is included in a benchmark portfolio. Case 42 refers to the case No.42 in Table 12.No constraints are applied to estimate the frontier.

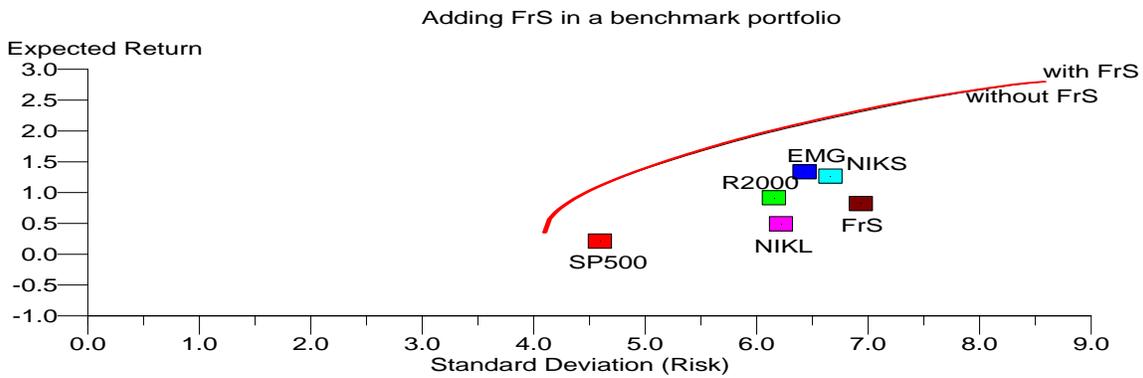


Fig.4 Efficient Frontier – case 43 GeS. This figure provides graphical evidence of the shift of the frontier when GeS is included in a benchmark portfolio. Case 43 refers to the case No.43 in Table 12.No constraints are applied to estimate the frontier.

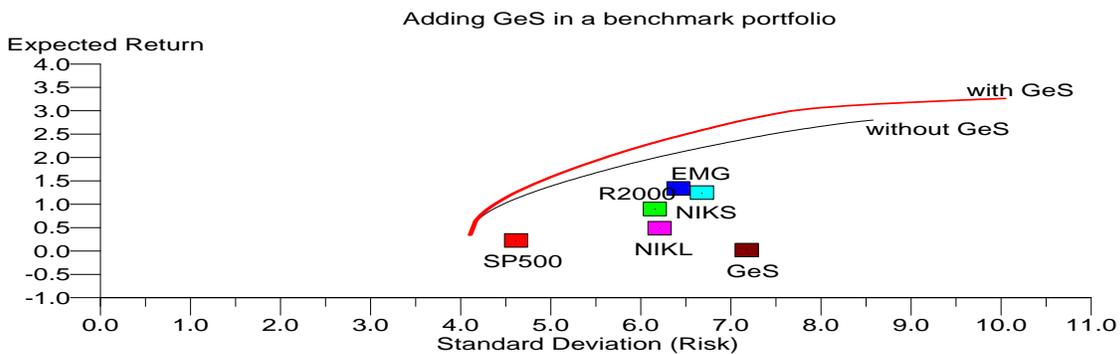


Fig.5 Efficient Frontier – case44 ItS. This figure provides graphical evidence of the shift of the frontier when ItS is included in a benchmark portfolio. Case 44 refers to the case No.44 in Table 12.No constraints are applied to estimate the frontier.

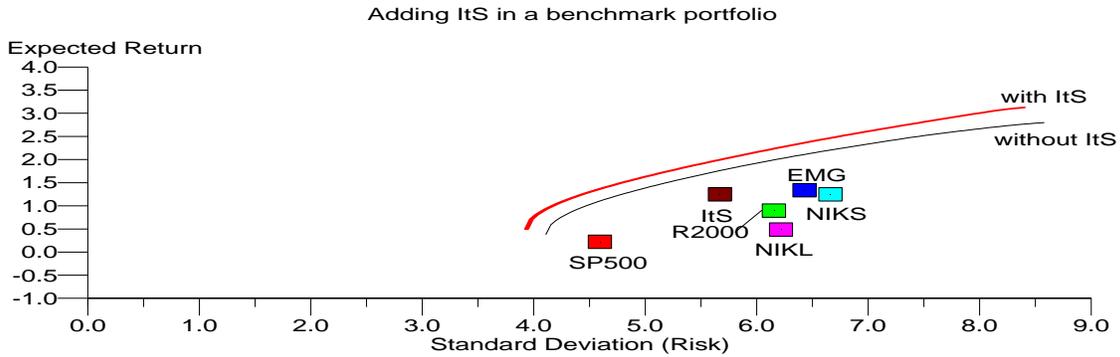


Fig.6 Efficient Frontier – case45 UKS. This figure provides graphical evidence of the shift of the frontier when UKS is included in a benchmark portfolio. Case 45 refers to the case No.45 in Table 12.No constraints are applied to estimate the frontier.

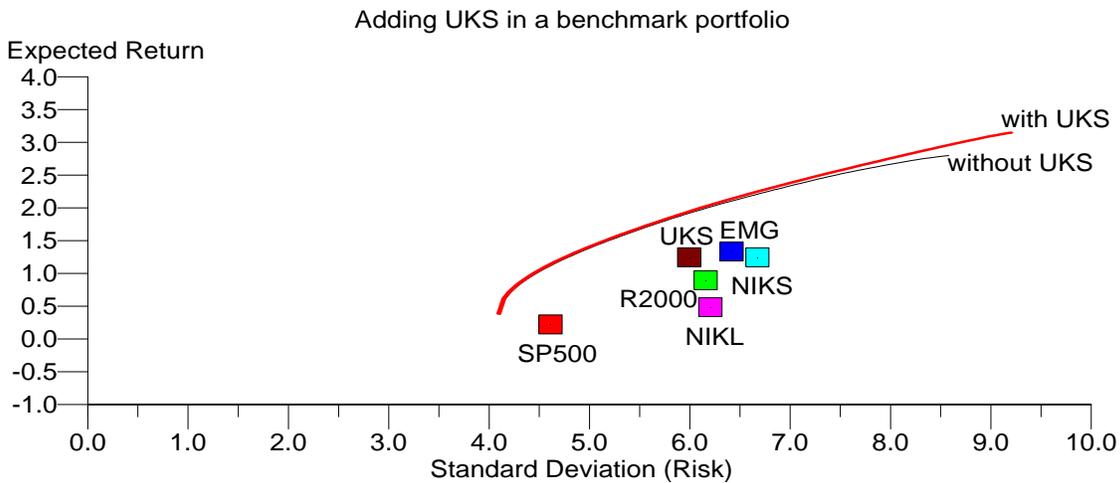


Fig.7 Efficient Frontier - case 50 NIKS 1989-1993. This figure provides graphical evidence of the shift of the frontier when NIKS of 1989-1993 is included in a benchmark portfolio. Case 50 refers to the case No.50 in Table 12.No constraints are applied to estimate the frontier.

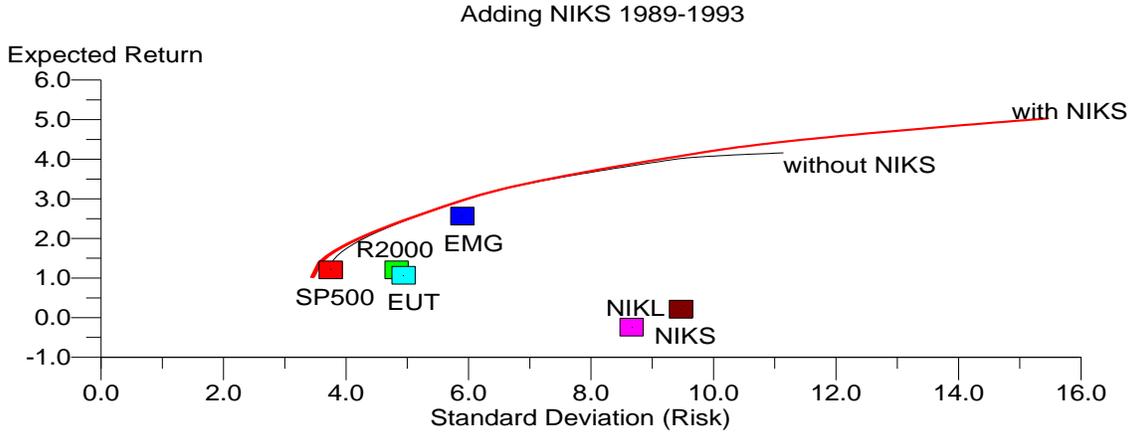


Fig.8 Efficient Frontier - case 51 NIKS 1994-1998. This figure provides graphical evidence of the shift of the frontier when NIKS of 1994-1998 is included in a benchmark portfolio. Case51 refers to the case No51 in Table 12.No constraints are applied to estimate the frontier.

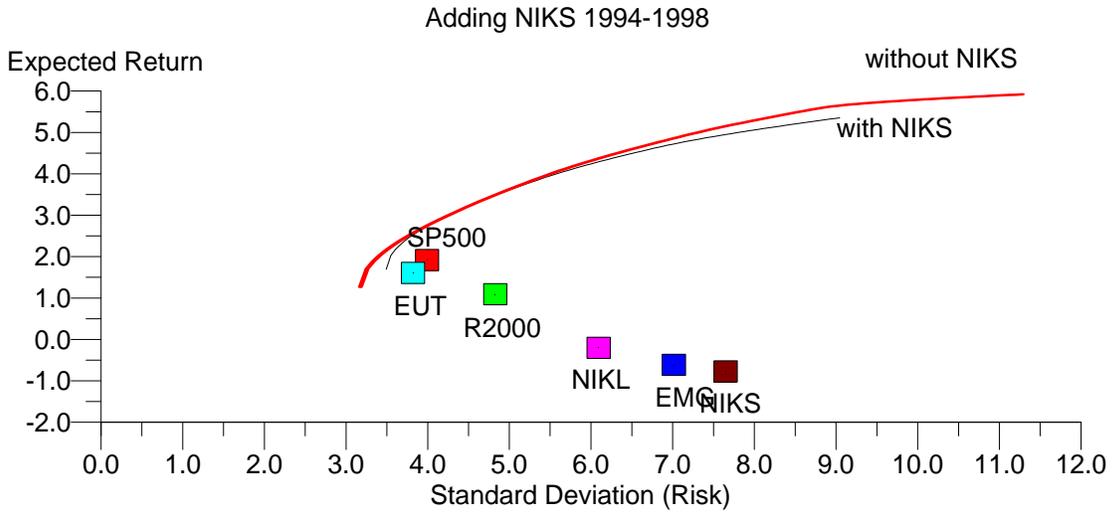
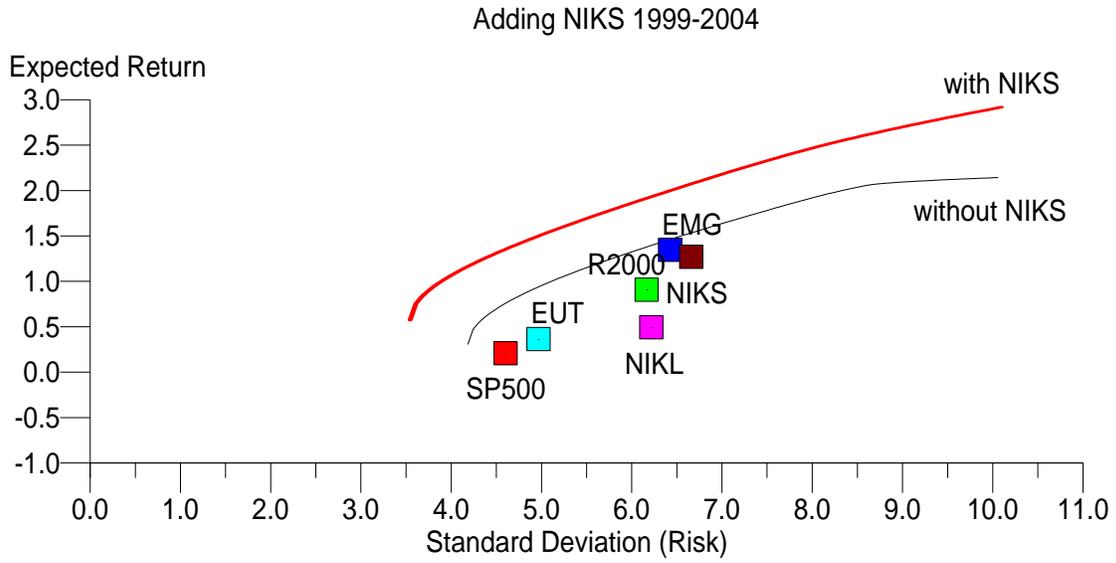


Fig.9 Efficient Frontier - case 52 NIKS 1999-2004. This figure provides graphical evidence of the shift of the frontier when NIKS of 1999-2004 is included in a benchmark portfolio. Case 52 refers to the case No.52 in Table 12.No constraints are applied to estimate the frontier.



Appendix 1 Description of sample indexes

1. BARRA/Nikko Style Index

Development: BARRA, INC. and the Nikko Securities Co., LTD.

Universe: All Japanese stocks listed on all the Stock Exchanges and traded on the OTC in Japan.

Base Date: The end of 1979 (=100)

Rebalance: Twice a year. End of June (The constituents of styles have been decided on the end of May) End of December (The constituents of styles have been decided on the end of November)

Dividends: Individual issue rates of return are adjusted for dividends and rights. It is assumed that dividend payments are reinvested.

Style: Large: Upper 85% of the market capitalization.
Small: Lower 15% of the market capitalization.

Source: <http://www.nikko.jp/NRC/Index/style/manual.html>

2. MSCI Index

Development: Morgan Stanley Capital International Inc.

Source: <http://www.msci.com/equity/index2.html>

2.1 MSCI Small Cap Indices

Universe: 40% of the full market capitalization of the eligible small cap universe within each industry group, within each country. All listed equity securities of companies that have a company full market capitalization in the range of USD 200 – 1,500 million and a minimum free float-adjusted security market capitalization of USD 100 million comprise the small cap equity universe in each country.

Date: Most began January 1999 in our sample.

Maintenance: Semi-annual index reviews, intended to reconstitute the Small Cap Index Series on the basis of a new eligible small cap.

Quarterly index reviews, aimed at promptly reflecting significant market. Ongoing event-related changes are generally implemented in the indices as the events occur.

Dividends: The Monthly Total Return methodologies continue to form the official index series until December 29, 2000. The Daily Total Return is re-based to the Monthly Total Return index levels of December 29, 2000 (the last trading day of 2000). MSCI's Daily Total Return methodology reinvests dividends in indices the day the security is quoted ex-dividend (ex-date).

2.2 MSCI Standard Index Series

- Universe: MSCI Standard Index Series adjusts the market capitalization of index constituents for free float and targets for index inclusion 85% of free float-adjusted market capitalization in each industry group, in each country.
- Maintenance: Annual full country index reviews that systematically re-assess the various dimensions of the equity universe for all countries and are conducted on a fixed annual timetable.
Quarterly index reviews, aimed at promptly reflecting other significant market events.
Ongoing event-related changes, such as mergers and acquisitions, are generally implemented in the indices promptly as they occur.
- Dividends: The Monthly Total Return methodologies continue to form the official index series until December 29, 2000. The Daily Total Return is re-based to the Monthly Total Return index levels of December 29, 2000 (the last trading day of 2000). MSCI's Daily Total Return methodology reinvests dividends in indices the day the security is quoted ex-dividend (ex-date).

2.3 MSCI Growth and Value Indices

- Universe: MSCI Value and Growth Index Series design is to divide constituents of an underlying MSCI Standard Country Index, into a value index and a growth index, each targeting 50% of the free float-adjusted market capitalization of the underlying index. The market capitalization of each constituent should be fully represented in the combination of the value index and the growth index, and, at the same time, should not be double-counted. A security may, however, be represented in both the value index and the growth index at a partial weight. From 1997 to May 2003, the value and growth indices have been constructed based on a single-dimensional framework that allocates securities in a MSCI Standard Country Index into either value or growth based on their Price to Book Value ratios (P/BV). Effective as of the close of May 30, 2003, MSCI applies a two-dimensional framework for style segmentation in which value and growth securities are categorized using different attributes. In addition, multiple factors are used to identify value and growth characteristics.
- Maintenance: Semi-annual style index reviews and style review outside of the semi-annual style index reviews.

2.4 MSCI Zhong Hua

- Universe: Aggregate of the MSCI Hong Kong Index and the MSCI China Free Index. The MSCI China Free Index represents the universe of opportunities for investment in the China equities market available to non-domestic investors. The index contains 31 stocks and has a market capitalization of USD 76.7 billion, as of November 23, 2000. MSCI has made changes to the MSCI China Free Index with effect from 1 June 2000. The new index is expanded to include Hong Kong listed companies

owned by the People's Republic of China (PRC) or by companies incorporated in the PRC.

2.5 MSCI Europe Index.

Universe: As of May 2005, the MSCI Europe Index consisted of the following 16 developed market country indices: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the United Kingdom. MSCI captures approximately 85% of the market cap of each country.

2.6 MSCI Euro

Universe: The construction of the MSCI Euro indices begins with the securities included in the MSCI Europe Index. The less liquid securities in this broad benchmark are screened-out based on the following liquidity screen: for each security within a country, daily traded-value statistics for the latest quarter are calculated and the securities are ranked in order of decreasing liquidity. The bottom 5% of the least liquid securities, in terms of total market capitalization of the country in the MSCI Europe Index, is screened-out. This screening is performed on a country-by-country basis, since liquidity figures are not directly comparable across markets. From the list of securities that have passed the liquidity screen the largest securities in each country are selected until approximately 90% of the total market capitalization of the country in the MSCI Europe Index is captured. Starting January 1, 1999, ten are currently eligible for the MSCI Euro Index: Austria France Netherlands Belgium Germany Portugal Finland Ireland Spain and Italy.

Maintenance: The MSCI Euro indices are reviewed annually in November to coincide with the November quarterly structural changes of the broader MSCI Europe Index. In addition, the MSCI Euro Indices are monitored continually and may be reviewed outside of the annual review in response to market-driven changes such as new issues, mergers, acquisitions, bankruptcies, and other similar corporate events.

3. Russell Indexes

Development: Frank Russell Company.

Source: <http://www.russell.com/US/default.asp>

3.1 Russell 3000 Index

Universe: The 3,000 largest U.S. companies based on total market capitalization, these companies represent approximately 98% of the invest-able U.S. equity market. As of the latest reconstitution, the average market capitalization was approximately \$4.8 billion; the median market

capitalization was approximately \$944.7 million. The index had a total market capitalization range of approximately \$386.9 billion to \$182.6 million.

3.2 Russell 2000 Index

Universe: The 2,000 smallest companies in the Russell 3000 Index, which represents approximately 8% of the total market capitalization of the Russell 3000 Index. As of the latest reconstitution, the average market capitalization was approximately \$664.9 million; the median market capitalization was approximately \$539.5 million. The largest company in the index had an approximate market capitalization of \$1.8 billion.

3.3 Russell/Nomura Indexes

Russell Investment Group and Nomura Securities Co., Ltd. produce Russell/Nomura Japan Equity Indexes that serve to measure performance based on various investment policies. The indexes are value weighted and include only common stocks domiciled in Japan.

3.3.1 Russell/Nomura Total Market Index

Universe: Represents approximately 98% of the invest-able Japan equity market. As of December 2004, this index consisted of approximately 1,700 of the largest securities in Japan based on available (float adjusted) market capitalization. Over historical periods, the number of stocks included in the Total Market Index has varied from approximately 1,100 stocks in 1979, to 1,854 stocks in 2000. As of December 2004, the adjusted market capitalization range of companies in this index ranged from approximately 8.95 trillion yen to 6.2 billion yen. All other Russell/Nomura indexes are subsets of this universe of stocks.

3.3.2 Russell/Nomura Small Cap Index

Universe: Represents approximately the smallest 15% of companies ranked market value of the firms in the Russell/Nomura Total Market Index. Currently 1300 securities make up this index but this number will vary from year to year. The largest company in the index had an approximate adjusted market capitalization of 79.4 billion yen and the smallest company 6.2 billion yen as of December 2004.

4. S&P Indexes

Development: Standard & Poor's, a division of The McGraw-Hill Companies, Inc.

Source:

<http://www2.standardandpoors.com/servlet/Satellite?pagename=sp/Page/HomePg&r=1&l=EN&b=10>

4.1 S&P 500

Universe: 500 leading companies in leading industries of the U.S. economy. Although the S&P 500 focuses on the large-cap segment of the market, with over 80% coverage of U.S. equities it is also an ideal proxy for the total market. The index includes U.S. companies with market cap in excess of \$4 billion. This market cap minimum is reviewed from time to time to ensure consistency with market conditions. The Index Committee strives to maintain a balance for the S&P 500 in line with the sector balance of the universe of eligible companies greater than \$4 billion. The company in the index must be an operating company. Closed-end funds, holding companies, partnerships, investment vehicles and royalty trusts are not eligible. Real Estate Investment Trusts (REITs) are eligible for inclusion.

Maintenance: The S&P Index Committee on a regular basis, following a set of published guidelines for maintaining the index, maintains the S&P 500.

4.2 S&P/IFCI

Universe: Subsets of S&P/IFCG indices, measure the returns of stocks that are legally and practically available to foreign investors. Indices in S&P/IFCG target an aggregate market capitalization of 70-80% of the total capitalization of all exchange-listed shares. S&P/IFCI indices typically cover a high percentage of the stocks in the S&P/IFCG indices. To qualify for S&P/IFCG, a company typically must be domiciled in an emerging market and among the most actively traded securities in that market. To qualify for inclusion in S&P/IFCI, a company must have a minimum average invest-able market capitalization of US \$125 million and trade at least US \$50 million in the 12 months prior to addition. All stocks in the S&P Emerging Markets are mapped according to the Global Industry Classification System (GICS®), which was implemented in 1999. S&P/IFCI Composite now includes: Czech Republic Egypt Hungary China Poland Taiwan Russia India Turkey Indonesia Argentina Korea Brazil Malaysia Chile Philippines Mexico Thailand Peru Morocco Israel and South Africa.

Reconstitution: Once each year on November 1. Index constituents of S&P/IFCG and S&P/IFCI are reconstituted based on the index inclusion criteria. Share changes greater than 10% of a company's market capitalization or changes impacting a constituent's weight in the index by more than 20 basis points are made with two weeks notice. Changes that do not meet these thresholds are made on a daily basis.

4.3 S&P/TSX Composite Index

Universe: Issuers of Index Securities must be incorporated under Canadian federal, provincial, or territorial jurisdictions and listed on TSX. Securities issued by Limited Partnerships, Royalty Trusts, Real Estate Investment Trusts, and Mutual Fund Corporations, and preferred shares, exchangeable shares, warrants, installment receipts and other securities deemed inappropriate by the Committee from time to time are not eligible for inclusion in the Index. The security must represent a minimum weight of 0.025% of the Index, after including the QMV for that security in the total Float capitalization for the Index. In the event that any Index Security has a weight of more than 10% at any month-end, the minimum weights for the purpose of inclusion will be based on the S&P/TSX Capped Composite Index. Moreover, the index comprises approximately 71% of market capitalization for Canadian-based, Toronto Stock Exchange listed companies. The size of the S&P/TSX Composite (C\$913.3 Billion in float market capitalization as of October, 2000) and its broad economic sector coverage has made the S&P/TSX Composite the premier indicator of market activity for Canadian equity markets since its launch on January 1, 1977.

Maintenance: The S&P/TSX Canadian Index Policy Committee maintains the Index. Meetings are held on a monthly basis and from time to time, as needed.

4.4 S&P /MIB

Development: Standard & Poor's and Borsa Italiana

Universe: The S&P/MIB currently measures the performance of 40 equities in Italy and seeks to replicate the broad sector weights of the Italian stock market. The index is derived from the universe of stocks trading on Borsa Italiana exchanges. The S&P/MIB is market cap-weighted after adjusting constituents for free float, capturing approximately 80% of the domestic market capitalization. All stocks traded on Borsa Italiana exchanges are eligible for inclusion except savings shares (azioni di risparmio) and preferred shares. Stocks from Nuovo Mercato and foreign listed stocks are also eligible. The Index Committee strives to include the most liquid and sector-representative stocks in the Italian market. As a result, it is possible that not all 10 GICS® sectors will always be represented. GICS® methodology classifies a company according to its primary line of business as measured by revenues, earnings and/or the market perception of the stock. The S&P/MIB index is based on free float market capitalization, and other factors including liquidity. The Mib30 is based on total market capitalization, as well as liquidity considerations.

Maintenance: The Index Committee maintains the S&P/MIB. The Committee meets quarterly and on an as-needed basis.

5. Nikkei 300

Development: Nihon Keizai Shimbun, Inc.

Source: http://www.nni.nikkei.co.jp/FR/SERV/nikkei_indexes/nifaq300.html

Universe: Market value-weighted index of the 300 major issues selected to represent listed stocks on the first section of the Tokyo Stock Exchange. The index is comprised of stocks with the largest market value in 36 industrial sectors. The 36 industrial sectors are chosen by Nihon Keizai Shimbun, Inc..

Maintenance: The index is reviewed every September. Changes, if any, become effective from early October. A review does not necessarily result in changes.

6. CAC 40

Source: http://www.euronext.com/editorial/wide/0,5371,1732_1203647,00.html

Universe: This index is made up of 40 shares, selected from the one hundred biggest companies listed on Euronext Paris, measured in terms of market capitalization. As the CAC40 is the benchmark for Euronext Paris, changes in the index are closely correlated to changes in the market as a whole. The index is widely used by portfolio managers to measure performance.

7. FTSE 100

Development: FTSE Group.

Universe: 100 most highly capitalized blue chip companies by full market value, representing approximately 80% of the UK market. Used extensively as a basis for investment products, such as derivatives and exchange-traded funds. This index is recognized as the measure of the UK financial markets.

Rebalance: A security will be inserted at the periodic review if it rises to 90th or above the position ranked by market value. A security will be deleted at the periodic review if it falls to 111th or below the position ranked by market value. The indices are reviewed using data from the close of the index calculation on the Tuesday after the first Friday of December for those indices reviewed annually; and the Tuesday after the first Friday of March, June, September and December for those reviewed quarterly.

Dividends: The Total Returns Indices are calculated daily.

Source: <http://www.ftse.com/index.jsp>

8. DAX 30

Development: Deutsche Boerse Group

Universe: A total return index of 30 selected German Prime Standard's 30 largest German companies, in terms of order book volume and market capitalization, traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation. As of June 18, 1999 only XETRA equity prices are used to calculate all DAX indices.

Maintenance: The index is based on prices generated in the electronic trading system Xetra®. Its calculation starts at 9.00 a.m. and ends with the prices from the Xetra closing auction at 5.30 p.m. The percentages of the individual shares in the index, together with the computation factors for the current and the next trading day, are published every evening for the next trading day.

Sources: http://deutsche-boerse.com/dbag/dispatch/en/kir/gdb_navigation/home

http://www.bloomberg.com/markets/stocks/movers_index_dax.html