Return Uncertainty and the Appearance of Biases in Expected Returns*

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Abstract

We study the relationship between return uncertainty and behavioral finance by combining multiple return forecasts for a single asset into an estimate of its unknown expected return. The uncertainty surrounding this expected return estimate is minimized by an optimal *information portfolio* which aggregates the return forecasts. The expected return implied by this minimization exhibits the appearance of overconfidence, biased self-attribution, representativeness, conservatism and limited attention. However, these characteristics as well as return predictability result from expected return uncertainty, and are induced by the information portfolio weights assigned to an individual asset's return forecasts rather than behavioral biases. Moreover, our optimal information portfolio yields testable implications distinct from psychological theories which we verify empirically using analyst earnings forecasts and revisions.

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1 Introduction

When testing market efficiency using historical data, the empirical asset pricing literature usually assumes an asset's expected return was agreed upon by all market participants. For example, the Fama-French (1993) three factor model computes market, SMB and HML sensitivities from ex-post returns to generate a *single* expected return estimate. However, a consensus regarding the correct formulation of expected returns remains elusive since the number of required factors and their composition is controversial. In addition, uncertainty regarding future factor returns and their beta coefficients for individual assets imply expected returns are unknown ex-ante. Price targets as well as intrinsic value measures, such as the residual income valuation model in Lee, Myers and Swaminathan (1999), also provide expected return forecasts which are not necessarily in agreement with those from multifactor asset pricing models.

Motivated by this uncertainty, we examine a combination of *multiple* return forecasts to estimate an *individual* asset's unknown expected return. Return forecasts are issued by information sources who interpret state variables such as the firm's projected earnings or industry conditions. Public information sources include analysts and the firm itself, while the investor generates private return forecasts. The historical accuracy of an information source is measured according to its time series of prior forecast errors, which are defined as the difference between realized and forecasted returns. Historical covariances between the forecast errors of different information sources are also analyzed.

The information portfolio combines the return forecasts for an individual asset into an estimate of its expected return. This is accomplished by assigning each information source a portfolio weight. In contrast to existing portfolio theory for multiple assets with known expected returns, our information portfolio applies to multiple return forecasts for a single asset whose expected return is unknown. Specifically, our optimal information portfolio minimizes the aggregate uncertainty of an asset's estimated expected return by assigning higher portfolio weights to information sources with greater historical accuracy.[1] The expected return estimate implied by the optimal information portfolio is labeled the investor's *perceived* return.

[1]After imposing a common distributional assumption on the set of return forecasts, this minimization is equivalent to solving for the *best linear unbiased estimate* (BLUE) of an asset's true expected return. However, we refrain from referring to the information portfolio weights as linear regression coefficients since their optimality is independent of any distributional assumptions and does not require the return forecasts to be unbiased.

With regards to behavioral finance, the perceived return exhibits the appearance of overconfidence and biased self-attribution as well as representativeness and conservatism. These two pairs of psychological biases have previously been incorporated into the behavioral finance literature by Daniel, Hirshleifer and Subrahmanyam (1998) and Barberis, Shleifer and Vishny (1998) respectively. For example, the optimal information portfolio emphasizes private sources of information which have been historically accurate, while downplaying the investor's less accurate private information sources. Furthermore, even in the absence of any theoretical justification, state variables with trends in their dynamics receive larger information portfolio weights. A property which mimics limited attention is also instilled into the perceived return since return forecasts which are positively correlated with those from more accurate information sources are underweighted by the investor's information portfolio. All of these perceived characteristics are induced by our optimal information portfolio weights rather than psychology.

Unlike Bayesian models in behavioral finance which incorporate psychological biases by imposing assumptions on the investor's prior distribution, we examine the optimal combination of return forecasts when computing their expected return. Therefore, attributes of the investor's perceived return which mimic psychological biases are outputs from information portfolio theory rather than inputs. This important distinction yields several testable implications unique to information portfolio theory. In contrast, Brav and Heaton (2002) demonstrate the difficulty of distinguishing between behavioral and rational explanations for return anomalies using Bayesian techniques.[2]

Periods of high uncertainty surrounding an asset's expected return imply more disparate return forecasts. Return predictability and return characteristics that mimic psychological biases both become more pronounced when expected return uncertainty is high. To illustrate our notion of expected return uncertainty, a BusinessWeek survey conducted at the end of 2005 reported year-end 2006 return forecasts for the S&P 500 ranging between -29.5% and 31.0% with a standard deviation of 7.61%. Provided the 76 forecasters included in the survey have unequal historical accuracies, the average forecast of 7.87% is not the optimal expected return for the S&P 500. When forecasting returns for individual stocks, idiosyncratic changes in a firm's capital structure or investment strategy could increase expected return uncertainty. However, our framework does not assume the return implications of such events are immediately understood and agreed upon by all information sources. Indeed, every

[2]Section 4 contains further details on the distinction between information portfolio theory and the Bayesian approach.

return forecast would be identical and without error under this simplifying assumption. Instead, dispersion between the return forecasts results from parameter uncertainty, previously examined in Lewellen and Shanken (2002), as well as disagreement surrounding the interpretation of available information.

An information source's historical accuracy is determined by the dynamics of an underlying state variable along with its return implications. Predictability in either of these two components improves an information source's historical accuracy. Consequently, after controlling for state variable uncertainty, information portfolio theory asserts that investors focus their attention on state variables which have experienced the highest correlation with realized returns. Indeed, a state variable with perfectly predictable dynamics and a deterministic relationship with the asset's true expected return creates a very accurate information source.

Empirically, Jackson and Johnson (2006) document that momentum and post-earnings announcement drift both coincide with firm-specific events that alter a firm's earnings, while the composite share issuance variable of Daniel and Titman (2005) also indicates return predictability. In addition, Kumar (2005) and Zhang (2005) report that behavioral biases appear stronger during periods of high uncertainty. Besides event and time dependence, Baker and Wurgler (2005) report that cross-sectional firm characteristics such as size and age explain a firm's sensitivity to investor sentiment. These empirical regularities are consistent with information portfolio theory as well as psychological biases. However, information portfolio theory posits that even when an asset's expected return is very uncertain, a relative ranking of the information sources by their historical accuracies is equivalent to the existence of an information portfolio. Thus, for a given level of uncertainty, the investor's perceived return focuses on the most accurate sources of information available. In contrast, this optimal weighting is not predicted by psychological theories.

Empirically, we verify the main testable implications of information portfolio theory by studying earnings momentum. For a given level of uncertainty, behavioral theory predicts stronger earnings momentum when earnings are less informative regarding future returns, while information portfolio predicts the opposite. The first aspect of our empirical study pertains to the return implications of earnings. Specifically, we measure the *sensitivity* of returns to earnings revisions by computing firm-specific correlations between these variables. We find earnings momentum profits increase monotonically from low to high sensitivity stocks by 50%. This evidence is consistent with investors focusing on earnings when this state variable has been informative. The second aspect of our empirical study considers the role of earnings *uncertainty*. As documented in Zhang (2005), momentum profits are larger for stocks with higher earnings dispersion. Most importantly, portfolios derived from double sorts on the sensitivity and uncertainty measures continue to display both relationships with earnings momentum. Consequently, after controlling for earnings uncertainty, firms whose returns are more sensitive to earnings revisions experience greater earnings momentum. This finding is consistent with accurate information sources having greater influence over the perceived return, which is the central prediction of information portfolio theory. Several robustness checks verify that our sensitivity and uncertainty measures are not driven by factors such as book-to-market, size and analyst coverage.

Nonetheless, if knowledge of investor psychology increases the accuracy of return forecasts, then information portfolio theory and psychology are compatible. Indeed, the exact decomposition of the perceived return into the effects of psychology versus information portfolio theory is ultimately an empirical question. Our empirical implementation demonstrates that the contribution of information portfolio theory to the formation of expected returns cannot be ignored. Information portfolio theory also enhances applications of utility maximization. For example, we prove an investor with exponential utility reduces their exposure to the risky asset when uncertainty regarding its expected return is high.

The remainder of this paper begins with the introduction of the optimal information portfolio in Section 2. Section 3 illustrates the impact of state variable predictability on the historical accuracy of an information source and examines its ability to induce return predictability. Section 4 links the optimal information portfolio with expected return characteristics that have previously been attributed to psychology. Testable implications of information portfolio theory are provided in Section 5 along with an empirical implementation. Our conclusions and suggestions for further research are contained in Section 6.

2 Information Portfolio Theory

As in Daniel, Hirshleifer and Subrahmanyam (1998) as well as Barberis, Shleifer and Vishny (1998), we consider a single-investor, single-asset model. Thus, we restrict our attention to an investor functioning as a price-setter who does not "free-ride" on market prices.

Underlying our framework are state variables, examples of which include forecasts for the earnings or sales of an individual firm as well as macroeconomic and industry conditions among many other possibilities. Each state variable forecast is interpreted by an information source who expresses its estimated return implications for a particular asset.[3] In practice, an individual analyst can issue earnings forecasts and long term growth rate projections along with price targets and buy versus sell recommendations, while firms often disclose their earnings and sales figures in conjunction with "guidance" for these state variables. Therefore, multiple information sources can originate from an individual analyst or the firm depending on the amount of information they release.

To simplify the exposition of our framework but without loss of generality, each return forecast is generated by a single state variable.[4] From an academic perspective, this structure enables our framework to address issues related to which sources of information influence expected returns. For example, Brav and Lehavy (2003) examine the marginal importance of analyst price targets to the price formation process in the presence of earnings forecast revisions and stock recommendations. Furthermore, this structure allows the information portfolio to aggregate over the *widest* possible array of information sources. Specifically, the information portfolio aggregates across the return implications of every state variable forecast. Although the economic intuition is identical if information sources interpret multiple state variables before issuing return forecasts, this modification reduces the amount of aggregation performed by the information portfolio.

Certain return forecasts possess private as well as public characteristics. For example, state variables such as earnings forecasts, while public when issued by sell-side analysts, require additional interpretation by the investor to become return forecasts. Therefore, these return forecasts may be considered private along with those originating from buy-side analysts. Conversely, the conversion of analyst price targets into return forecasts is immediate, implying these sources of information are public. Prior returns for the asset also constitute a source of public information. For emphasis, information sources are only assumed to issue return forecasts. The mechanism for computing their historical accuracy is addressed in the next subsection.

In summary, we consider $J > 1$ return forecasts for a single asset originating from J unique

[3]Although sales are usually reported in millions of dollars and earnings stated on a per share basis, information portfolio theory abstracts from these scale complications by aggregating across their return implications.

[4]The next section demonstrates that this structure is able to replicate the return forecasts from multifactor asset pricing models such as Fama-French (1993).

information sources who evaluate the return implications of $K \geq 1$ unique state variable forecasts.[5] The inequality $J \geq K$ enables information sources to disagree on the return implications of a state variable forecast.

2.1 Historical Forecast Accuracy

The historical accuracy of each return forecast is critical to the information portfolio's solution and is computed from the previous forecast errors of an information source. Specifically, at time $t-1$, the time series of forecast errors for the j^{th} information source consists of the following vector

$$\begin{bmatrix} \epsilon_{t-1}^{j} \\ \vdots \\ \epsilon_{t-n}^{j} \end{bmatrix} = \begin{bmatrix} y_{t-1} \\ \vdots \\ y_{t-n} \end{bmatrix} - \begin{bmatrix} \mu_{j,t-1} \\ \vdots \\ \mu_{j,t-n} \end{bmatrix} \quad \text{for } j = 1, 2, \ldots, J \tag{1}$$

over the previous n periods. At time $t-1$, the j^{th} information source issues the return forecast $\mu_{j,t}$ for the $(t-1, t]$ horizon, while y_t denotes the asset's realized return at time t. The calendar time corresponding to the $(t-1, t]$ interval is arbitrary.

At $t-1$, the historical accuracy of the j^{th} information source equals

$$\sigma_{j,t}^{2} = \frac{1}{n} \sum_{i=1}^{n} \left(\epsilon_{t-i}^{j} \right)^{2} , \tag{2}$$

according to their previous forecast errors ϵ_{t-i}^{j} over the last n periods. Let $\sigma_{j,*}^{2}$ denote the unknown true variance associated with the return forecasts of the j^{th} information source, which proxies for their *skill* at forecasting an asset's expected return. The historical accuracy in equation (2) is the investor's estimate of $\sigma_{j,*}^{2}$ based on the information source's prior n forecast errors in equation (1). From the investor's perspective, this estimate represents the uncertainty of the $\mu_{j,t}$ return forecast issued at $t-1$. Statistically, equation (2) calculates the mean-squared error (MSE) of the previous n forecast errors for the j^{th} information source.[6]

[5] When state variable dynamics are random, K represents the number of state variable forecasts rather than the number of actual state variables with each of the former able to yield a distinct return forecast.

[6] This property follows from $E\left[\epsilon^2\right] = Var\left[\epsilon\right] + \left(E\left[\epsilon\right]\right)^2$ with the bias in a forecast equaling $E[\epsilon]$. Information sources may employ Bayesian methods when generating their return forecasts with the usual tradeoff between variance and bias arising from an informative prior. By computing the mean-squared error of prior forecast errors, the potential for optimism to bias analyst earnings forecasts, price targets, and stock recommendations is addressed.

Similarly, the covariance between the time series of forecast errors for the j^{th} and k^{th} information source is estimated as

$$\sigma_{j,k,t} \;=\; \frac{1}{n} \sum_{i=1}^{n} \epsilon^{j}_{t-i}\, \epsilon^{k}_{t-i} \,, \tag{3}$$

for $j \neq k$. Equation (3) represents the investor's estimate of the true but unknown covariance $\sigma_{j,k,*}$ between the return forecasts of two information sources at $t-1$. For emphasis, since the estimates in equations (2) and (3) are calculated using realized forecast errors over the last n periods, they should be denoted as $\hat{\sigma}^2_{j,t}$ and $\hat{\sigma}_{j,k,t}$ respectively but the hats are omitted for notational simplicity. Moreover, the historical accuracy of an information source pertains entirely to its time series of prior return forecasts, with state variable forecasts serving an intermediate role.

The value of n in equations (2) and (3) is specific to an individual asset.[7] Intuitively, established firms in stable industries have a large n. Conversely, initial public offerings, firms undergoing a significant corporate restructuring or undertaking a large investment and those operating in industries that experience major technological innovations have a small n.

Overall, the μ_t vector of return forecasts at time $t-1$ equals

$$\mu_t \;=\; \begin{bmatrix} \mu_{1,t} \\ \mu_{2,t} \\ \vdots \\ \mu_{J,t} \end{bmatrix} . \tag{4}$$

A time series of these vectors $\mu_{t-1}, \ldots, \mu_{t-n}$ over the last n periods yields a Θ_t matrix summarizing the historical accuracies of the J information sources as well as their historical covariances, described by equations (2) and (3) respectively. The Θ_t matrix is a historical estimate of the true but unknown variance-covariance matrix for the J return forecasts in equation (4).

The cross-sectional dispersion across the J forecasts of the μ_t vector at $t-1$

$$\sigma^2_{\mu,t} \;=\; \frac{1}{J-1} \sum_{j=1}^{J} \left(\mu_{j,t} - \bar{\mu}_t\right)^2 \;, \tag{5}$$

[7] When n is information source dependent, a j subscript would be added to form n_j. For example, n could proxy for the experience of an information source. Chen, Liu and Qian (2005) document the importance of experience to the credibility of buy-side analyst forecasts, while Nicolosi, Peng and Zhu (2004) report that experienced individual investors earn higher returns. However, for ease of exposition, all J information sources are evaluated using n previous forecast errors since our initial focus is on a firm-specific information environment.

where $\bar{\mu}_t$ is defined as the average return forecast

$$\bar{\mu}_t \;=\; \frac{1}{J}\sum_{j=1}^{J}\mu_{j,t}\,, \tag{6}$$

does not have an explicit role in our solution for the information portfolio. Nonetheless, the dispersion in equation (5) has an important economic interpretation by offering a concise definition for expected return uncertainty that is economically intuitive. Note that $\bar{\mu}_t$ in equation (6) is not the optimal estimate for an asset's expected return unless every information source has an identical true but unknown forecast accuracy, implying $\sigma^2_{j,*}$ equals a common σ^2_* value. To simplify our notation, we suppress the t subscripts on μ and Θ for the remainder of this paper.

2.2 Optimal Information Portfolio

The investor minimizes the aggregate mean-squared error of the asset's expected return when combining the J return forecasts.[8] Therefore, the optimization problem which solves for the optimal information portfolio W is

$$\begin{aligned} &\min_{W} \quad \frac{1}{2}W^T\Theta W \\ &\text{subject to:} \quad W^T\mathbf{1} = 1\,, \end{aligned} \tag{7}$$

where $\mathbf{1}$ denotes a J-dimensional vector of ones. As proven later in this section, after imposing a common distributional assumption on every return forecast, the objective function in equation (7) is equivalent to finding the best linear unbiased estimator (BLUE) of the asset's expected return given available forecasts. Therefore, equation (7) is consistent with linear regression models used throughout the empirical finance literature. The optimal information portfolio is solved in the following proposition whose proof is contained in Appendix A.

Proposition 1. *The solution for the optimal information portfolio W in equation (7) equals*

$$W \;=\; \frac{\Theta^{-1}\mathbf{1}}{\mathbf{1}^T\Theta^{-1}\mathbf{1}}\,. \tag{8}$$

[8]This approach is related to Peng and Xiong (2004)'s minimization for the variance of beliefs regarding subsequent dividends, while Hong, Scheinkman and Xiong (2005) invoke mean-variance preferences when analyzing different information sources.

When private information sources are evaluated by the investor, the information portfolio is investor-specific in addition to being firm-specific.

2.3 Regression Interpretation of Optimal Information Portfolio

Denote the asset's true return distribution as $\mathcal{N}(\eta, \nu)$ with η being its unknown expected return. Corporate or macroeconomic events that generate expected return uncertainty may also cause η to vary over time but this parameter is written as a constant for notational simplicity.

The main result of this subsection is that after imposing a common distributional assumption on every return forecast

$$\mu \;\stackrel{d}{\sim}\; \mathcal{N}\left(\eta\mathbf{1}, \Theta\right) , \tag{9}$$

the objective function in equation (7) is equivalent to finding the best linear unbiased estimate of η. Specifically, from a linear regression perspective, the true model for the asset's return is described by

$$y \;=\; \eta + e\,, \tag{10}$$

where the error terms e are i.i.d. random variables from a $\mathcal{N}(0, \nu)$ distribution. Therefore, the asset's realized return y is emitted by the true $\mathcal{N}(\eta, \nu)$ distribution. Appendix B considers a special case of equation (10) which has η generated by a N-factor model

$$y \;=\; \left[\beta_0 + \sum_{j=1}^{N} \beta_j\, f_j\right] + e\,. \tag{11}$$

However, regardless of η's specification, its corresponding linear estimator $\hat{y}$ equals

$$\hat{y} \;=\; W^T \mu\,. \tag{12}$$

A linear regression procedure minimizes the mean-squared error of the $y - \hat{y}$ deviations

$$y - \hat{y} \;=\; \eta - W^T \mu + e\,, \tag{13}$$

by choosing the optimal *coefficients* W given a set of *independent variables* which are the return forecasts μ in our framework. The coefficients are required to produce an unbiased estimator which implies

$$
\begin{aligned}
0 &= E\left[y-\hat{y}\right] \\
&= \eta - E\left[W^T\mathcal{N}\left(\eta\mathbf{1},\Theta\right)\right] \\
&= \eta - \eta W^T\mathbf{1}\,.
\end{aligned}
\tag{14}
$$

The $W^T\mathbf{1}=1$ constraint is an immediate consequence of equation (14) which follows from the distributional assumption in equation (9).[9] With $W^T\mu$ being an unbiased estimate of η, minimizing the mean-squared error in equation (13) is equivalent to minimizing

$$
\begin{aligned}
Var\left[y-\hat{y}\right] &= Var\left[\eta - W^T\mu + e\right] \\
&= Var\left[W^T\mathcal{N}\left(\eta\mathbf{1},\Theta\right)+e\right] \\
&= W^T\Theta W + \nu\,,
\end{aligned}
\tag{15}
$$

since the $\mathcal{N}(0,\nu)$ distribution for the error terms is independent of the normal distribution in equation (9) while η is not random. Equation (15) implies the investor minimizes $W^T\Theta W$ since ν is not a function of W. In summary, the best linear unbiased estimate of the asset's expected return minimizes $W^T\Theta W$ subject to the $W^T\mathbf{1}=1$ constraint.[10] Consequently, statistical justification underlying linear regression models also applies to our objective function in equation (7). To clarify, $W^T\Theta W$ is not an estimate of ν. Indeed, even if $W^T\Theta W$ equals zero or η is known (as in classical portfolio theory), the asset is not riskless provided ν is non-zero.

To determine the asset's ex-ante return distribution next period, consider the *prediction* interval for

$$
\tilde{y}_p = W^T\mu + e\,,
\tag{16}
$$

which is conditioned on a set of estimated W coefficients and return forecasts. Equation (16) is not intended to calibrate the W coefficients since $\tilde{y}_p$ is the asset's unobserved (random) return next period.

[9] Equation (9) implies $W^T\mu$ is an unbiased estimator of η although the converse does not hold. The distributional assumption in equation (9) ensures *all* of the return forecasts are not above or below η but does not imply that $W^T\mu$ equals the asset's true expected return. Indeed, confidence intervals and hypothesis tests are required to evaluate point estimates from linear regression models.

[10] Minimizing $W^T\Theta W$ in equation (15) is equivalent to minimizing $\frac{1}{2}W^T\Theta W$ in equation (7).

Instead, conditional on estimated W coefficients, the asset's ex-ante return distribution equals

$$\tilde{y}_p \overset{d}{\sim} \mathcal{N}\left(W^T\mu, \nu + W^T\Theta W\right) . \tag{17}$$

Thus, ex-ante return uncertainty reflects the asset's true variability denoted ν as well as the aggregate uncertainty $W^T\Theta W$ of the J forecasts. Consequently, equation (17) provides further justification for the minimization of $W^T\Theta W$ in equation (7).

For emphasis, the objective function in equation (7) which defines the optimal information portfolio is independent of the distributional assumption in equation (9) and does not require the forecasts to be unbiased. Specifically, equations (2) and (3) evaluate the mean-squared error of an information source. In addition, the time-varying information portfolio weights in Proposition 1 are crucial to our interpretation of the investor's perceived return in Section 4. Therefore, we refrain from referring to our information portfolio weights as linear regression coefficients.

2.4 Perceived Return and Aggregate Return Uncertainty

By aggregating across the return forecasts, the optimal information portfolio immediately generates an estimate for the asset's expected return. This estimate is referred to as the investor's perceived return, and summarizes the information provided by the J return forecasts. Proposition 2 below computes the perceived return and its aggregate uncertainty using the optimal information portfolio.

Proposition 2. *The perceived return for an asset implied by the investor's optimal information portfolio weights in Proposition 1 equals*

$$W^T\mu = \frac{\mathbf{1}^T\Theta^{-1}\mu}{\mathbf{1}^T\Theta^{-1}\mathbf{1}} , \tag{18}$$

while

$$W^T\Theta W = \frac{1}{\mathbf{1}^T\Theta^{-1}\mathbf{1}} , \tag{19}$$

is the aggregate uncertainty of the expected return estimate in equation (18).

Proof: The perceived return follows immediately from equation (8) while the aggregate forecast error is computed as[11]

$$\begin{aligned} W^T \Theta W &= \frac{1}{\mathbf{1}^T \Theta^{-1} \mathbf{1}} \mathbf{1}^T \Theta^{-1} \, \Theta \, \Theta^{-1} \mathbf{1} \frac{1}{\mathbf{1}^T \Theta^{-1} \mathbf{1}} \\ &= \frac{1}{\mathbf{1}^T \Theta^{-1} \mathbf{1}} \, . \end{aligned} \tag{20}$$

Ex-ante, the investor is unaware of the asset's true expected return denoted η. As a consequence, the investor is compelled to aggregate the J return forecasts and rely on the perceived return in equation (18) which has the lowest mean-squared error amongst all other alternative estimates.

2.5 Important Information Portfolio Properties

We begin with the following corollary of Proposition 2 which offers an explicit expression for the information portfolio between two independent information sources.

Corollary 1. *For $J = 2$ and Θ being the diagonal matrix*

$$\begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} ,$$

the information portfolio W equals

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \frac{1}{\sigma_2^2 + \sigma_1^2} \begin{bmatrix} \sigma_2^2 \\ \sigma_1^2 \end{bmatrix} . \tag{21}$$

Therefore, the investor's perceived return equals

$$\frac{\sigma_2^2 \, \mu_1 + \sigma_1^2 \, \mu_2}{\sigma_1^2 + \sigma_2^2} , \tag{22}$$

while

$$\frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} , \tag{23}$$

is the aggregate uncertainty of the asset's estimated expected return in equation (22).

[11] A negative portfolio weight implies the investor reverses the sign of this information source's return forecast when the perceived return is computed.

According to equation (22), the return forecast issued by a more accurate information source has a larger portfolio weight and greater influence on the investor's perceived return. Intuitively, historical accuracy enhances the credibility of an information source. The utility maximization approach in Cheng, Liu and Qian (2005) produces a pair of weights similar to equation (21) for signals issued by sell-side versus buy-side analysts.

The next corollary of Proposition 2 extends Corollary 1 by examining correlated return forecasts.

Corollary 2. *For $J = 2$, let Θ equal*

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} .$$

Under this structure, the portfolio weights are

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \frac{1}{\sigma_2^2 + \sigma_1^2 - 2\sigma_{12}} \begin{bmatrix} \sigma_2^2 - \sigma_{12} \\ \sigma_1^2 - \sigma_{12} \end{bmatrix} . \tag{24}$$

The perceived return for the asset equals

$$\frac{\sigma_2^2\, \mu_1 + \sigma_1^2\, \mu_2 - \sigma_{12}\, (\mu_1 + \mu_2)}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} , \tag{25}$$

while

$$\frac{\sigma_1^2 \sigma_2^2 - (\sigma_{12})^2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}} , \tag{26}$$

is the aggregate uncertainty of the asset's estimated return in equation (25).

A negative covariance, $\sigma_{12} < 0$, between two information sources represents "offsetting" forecast errors. Appendix C proves that a negative covariance reduces the aggregate uncertainty in equation (26). We utilize this property in Section 4 to demonstrate that our optimal information portfolio weights generate perceived returns which exhibit the appearance of several behavioral biases.

3 Historical Accuracy and Return Predictability

Recall that an information source's historical accuracy encompasses forecast variability associated with a state variable as well as uncertainty regarding its return implications. Thus, limited time series data corresponding to a small n would exacerbate disagreements between information sources regarding the correct techniques for estimating a state variable's dynamics as well as its relationship with a firm's expected return. These disagreements would also be compounded by parameter uncertainty in the forecast procedures which is captured by the *estimation errors* in our analysis below.[12]

Expected return uncertainty is also higher when information sources interpret distinct state variables, have unique forecasting techniques for their state variable, and interpret its return implications differently. Furthermore, an information source does not necessarily disclose these components of their return forecast. Instead, at each point in time, the investor observes a collection of return forecasts. Forecast errors over the last n periods in equation (1) for each information source define the Θ matrix underlying our optimal information portfolio.

For simplicity, we begin by examining one information source to investigate the impact of n on its accuracy. We then study a two-information source environment to examine return predictability.

3.1 Uncertainty in the Return Implications of a State Variable

Assume the j^{th} information source utilizes a linear model for converting a known state variable V_t into its associated return forecast

$$\mu_{j,t} \;=\; \hat{\alpha} + \hat{\beta} V_t \,. \tag{27}$$

The hats signify the unknown coefficients of the transformation, while the state variable V_t in equation (27) is not random. As mentioned above, other information sources may employ a transformation different than equation (27) or interpret a different state variable when issuing their return forecast.

According to equation (28) below, the j^{th} information source calibrates the α and β coefficients in equation (27) using realized returns and state variables

$$y_{t-i} \;=\; \alpha + \beta V_{t-i} + \xi_{t-i} \,, \tag{28}$$

over the previous $i = 1, \ldots, n$ periods where ξ_{t-i} is an i.i.d. error term distributed $\mathcal{N}\left(0, \sigma_{\xi}^{2}\right)$. After obtaining the estimates $\hat{\alpha}$ and $\hat{\beta}$ from equation (28), the information source invokes equation (27) to convert V_t into $\mu_{j,t}$ at $t-1$ which simply equals the predicted value $\hat{y}_t$ from this linear regression.

[12] *Herding* by information sources induces positive correlation between a subset of return forecasts, and further reduces the optimal portfolio weights assigned to less accurate information sources according to Subsection 4.3. This property mitigates the impact of herding on the investor's perceived return.

To illustrate the importance of n, the forecast error ϵ_t^j below contributes another observation to the time series of historical forecast errors in equation (1) at time t. In particular, the realization of $\left(\epsilon_t^j\right)^2$ at time t augments equation (2) when computing the j^{th} information source's historical accuracy. In equation (29) below, the expectation of this squared forecast error at $t-1$ is evaluated[13]

$$\begin{aligned}
E\left[\epsilon_t^j\right]^2 &= Var\left[y_t - \mu_{j,t}\right] \\
&= Var\left[\xi_t\right] + \left\{Var\left[\hat{\alpha}\right] + (V_t)^2\, Var\left[\hat{\beta}\right] + 2\, V_t\, Cov\left[\alpha - \hat{\alpha}, \beta - \hat{\beta}\right]\right\} \qquad (29) \\
&= \text{Transformation Uncertainty} + \text{Estimation Error in Transformation}\,.
\end{aligned}$$

For large n, the $\hat{\alpha}$ and $\hat{\beta}$ estimates converge to α and β respectively, implying equation (29) reduces to $Var\left[\xi_t\right]$. Therefore, equation (29) converges to σ_ξ^2 which equals the unknown variance $\sigma_{j,*}^2$ of the j^{th} information source estimated by equation (2). Recall that the investor cannot compute equation (29) at time $t-1$ since the transformation in equation (27) is not disclosed by the information source. Instead, they rely on the j^{th} information sources's historical accuracy computed by equation (2).

However, when n is small, estimation error in $\hat{\alpha}$ and $\hat{\beta}$ is severe. Lewellen and Shanken (2002) examine the asset pricing implications of parameter uncertainty and demonstrate that return predictability cannot necessarily be exploited by investors. In our framework, a small n may undermine the credibility of a knowledgeable information source or a truly relevant state variable. For example, Jagannathan and Wang (2005) find that consumption explains the role of the SMB and HML factors in cross-sectional returns. However, SMB and HML dominate consumption in empirical applications due to the limitations of consumption data.

Equation (29) also illustrates the importance of predictability in the return implications of a state variable. If the conversion of V_t into $\mu_{j,t}$ is perfectly predictable, implying the ξ_{t-i} error terms in equation (28) are identically zero, then the coefficients in equation (27) are known.[14] Conversely,

[13] The $\mu_{j,t}$ return forecast is unbiased since $E\left[y_t - \mu_{j,t}\right] = E\left[\alpha - \hat{\alpha} + V_t\left[\beta - \hat{\beta}\right] + \xi_t\right]$ is zero provided $E\left[\hat{\alpha}\right]$ and $E\left[\hat{\beta}\right]$ equal α and β respectively. These equalities follow from the linear regression in equation (28) providing unbiased coefficient estimates. Thus, equations (27) and (28) imply the j^{th} information source issues unbiased return forecasts. However, this simplification is not a requirement of information portfolio theory since equation (2) evaluates the mean-squared error of return forecasts.

[14] Transforming an analyst's price target into a return forecast involves a perfectly predictable function, although not

when the relationship between a stock's expected return and a state variable is unpredictable, the information source's historical accuracy in equation (2) is likely to be poor.

Finally, even the idealized environment in equation (29) has two important complications. First, the α and β parameters may be time-varying, which would complicate their estimation even for large n values. Second, as discussed in the next subsection, V_t could be a forecast for the state variable. For example, employing the Fama-French (1993) model to generate an asset's expected return requires forecasts for the SMB and HML returns as well as the market.

3.2 Uncertainty in State Variable Dynamics

Jackson and Johnson (2006) document a *post-event drift* in analyst forecasts following seasoned equity offerings, stock re-purchases, equity-financed mergers and dividend initiations as well as omissions. This persistence in analyst forecasts suggests the impact of such events on a firm's earnings dynamics are not immediately understood.

Suppose the j^{th} return forecast is derived from an information source's forecast for a state variable, denoted $\tilde{V}_t$, which is a linear function of its previous realization

$$\tilde{V}_t = \hat{a} + \hat{b} V_{t-1} . \tag{30}$$

The dynamics of V_t are estimated by the information source at $t-1$ as

$$V_{t-i} = a + b V_{t-i-1} + \zeta_{t-i} , \tag{31}$$

using data over the previous $i = 1, \ldots, n$ periods where ζ_{t-i} is another i.i.d. error term whose distribution is $\mathcal{N}\left(0, \sigma^2_\zeta\right)$. Equation (31) is utilized to estimate the a and b coefficients, while the ζ_{t-i} error terms signify the random evolution of the state variable. The $\tilde{V}_t$ notation contains a tilde to emphasize that the information source is forecasting this state variable, in contrast to equation (27) where V_t is known.

When equation (27) with known α and β parameters is combined with equation (30), the following return forecast is generated by the j^{th} information source

$$\begin{aligned} \mu_{j,t} &= \alpha + \beta \tilde{V}_t \\ &= \alpha + \beta \left[\hat{a} + \hat{b} V_{t-1} \right] . \end{aligned} \tag{32}$$

the linear relationship in equation (27).

For clarification, the conversion of the state variable into its return forecast continues to be specified by equation (27). However, for simplicity, the α and β coefficients are assumed to be known since our attention is currently focused on the contribution of state variable uncertainty to the j^{th} information source's historical accuracy. In addition, the j superscript applies to the α, β, a and b coefficients as well as the ξ and ζ error terms but is omitted for notational simplicity.

Inserting the true dynamics of the state variable in equation (31) into equation (28) implies that returns evolve as

$$y_t = \alpha + \beta \left[a + bV_{t-1} + \zeta_t \right] + \xi_t \, . \tag{33}$$

When combined, equations (32) and (33) at time $t-1$ imply the following expectation[15]

$$\begin{aligned} E\left[\epsilon_t^j\right]^2 &= Var\left[y_t - \mu_{j,t}\right] \\ &= Var\left[\xi_t\right] + \beta^2 Var\left[\zeta_t\right] \\ &+ \left\{ \beta^2 Var\left[\hat{a}\right] + \beta^2 \left(V_{t-1}\right)^2 Var\left[\hat{b}\right] + 2\beta^2 V_{t-1} Cov\left[a - \hat{a}, b - \hat{b}\right] \right\} \\ &= \text{Transformation Uncertainty} + \text{State Variable Uncertainty} \\ &+ \text{Estimation Error in State Variable Dynamics} \, . \end{aligned} \tag{34}$$

To clarify, $Var\left[\zeta_t\right]$ corresponds to state variable uncertainty, while $Var\left[\xi_t\right]$ represents randomness in the return implications of the state variable. The estimation error in the second line of equation (34) tends toward zero as $n \to \infty$, implying the variance of next period's forecast error converges to $\sigma_\xi^2 + \beta^2 \sigma_\zeta^2$. This limit equals the j^{th} information source's true accuracy $\sigma_{j,*}^2$ which is estimated by the investor using equation (2).

An important property of equation (34) invoked in the next subsection is that after controlling for $Var\left[\xi_t\right]$, information sources which condition their return forecasts on predictable state variable are

[15] The linearity of equations (30), (32) and (33) imply $\mu_{j,t}$ is an unbiased return forecast. Specifically, $E\left[\epsilon_t^j\right]$ equals zero since the linear regression in equation (31) ensures $E\left[\hat{a}\right] = a$ and $E\left[\hat{b}\right] = b$ under the assumptions imposed on ζ_{t-i} and ξ_{t-i}. However, this property is not a requirement of information portfolio theory since equation (2) minimizes mean-squared error. Note that an economy in which the α, β, a and b coefficients all require calibration produces a complicated estimation error in equation (34) involving cross-products.

likely to have historical accuracies which are superior to information sources who condition on unpredictable state variables. For example, consider two state variables with the first being unpredictable and the second highly predictable. Given identical $Var[\xi_t]$ terms, return forecasts conditioned on highly predictable state variable are likely to be more accurate. More formally, suppose V_t is perfectly predictable and follows a known deterministic process. For this special case, $Var[\zeta_t]$ equals zero while the a and b coefficients are known, implying equation (34) reduces to $Var[\xi_t]$.

In general, state variables and their return implications are not required to arise from linear relationships with mean zero i.i.d. error terms as in our previous illustrations. Indeed, no assumptions are imposed on the conversion of state variables into return forecasts when solving for the information portfolio.

3.3 Return Predictability

To examine return predictability, we consider a two-period economy with an optimistic and pessimistic return forecast for a firm which has recently initiated a large investment. The profitability (earnings / cashflow) of this investment represents the relevant state variable that causes the investor to appear as if they extrapolate from past returns.

At the initial timepoint t_1, the high return forecast is denoted $\mu_{H,1}$ while its low return counterpart is denoted $\mu_{L,1}$. For simplicity but without loss of generality, assume these return forecasts are independent and issued by information sources with equal historical accuracies, implying $\sigma^2_{H,1}$ equals $\sigma^2_{L,1}$. According to equation (22) in Corollary 1, the investor's perceived return over the $(t_1, t_2]$ horizon is the average of the two forecasts, $\mu_1 = \frac{1}{2}[\mu_{H,1} + \mu_{L,1}]$.

During the $(t_1, t_2]$ interval, information regarding the success of the investment is revealed, with the firm's realized return at t_2 equaling $r_{1,2}$. In particular, there are two scenarios, the first indicating success and the second failure. The ex-ante probability attached to these scenarios is irrelevant when the ex-post return sequence is studied at t_3. Furthermore, over the $(t_2, t_3]$ interval, assume $\mu_{H,2}$ continues to exceed $\mu_{L,2}$ with the disparity between these forecasts depending on the uncertainty prevailing at time t_2 surrounding the firm's expected return.

To illustrate return extrapolation in our framework, consider the following two scenarios defined by $r_{1,2}$.

Investment appears to be successful over $(t_1, t_2]$ interval:

$$\begin{cases} r_{1,2} \text{ is high} \\ \sigma^2_{H,2} < \sigma^2_{L,2}, \text{ high return forecast is more accurate since } r_{1,2} \text{ is high} \\ \underline{\text{Perceived return } \mu_2 \text{ over } (t_2, t_3] \text{ horizon is closer to } \mu_{H,2}} \\ \mu_2 \text{ reflects extrapolation from high } r_{1,2} \end{cases}$$

Investment appears to be a failure over $(t_1, t_2]$ interval:

$$\begin{cases} r_{1,2} \text{ is low} \\ \sigma^2_{L,2} < \sigma^2_{H,2}, \text{ low return forecast is more accurate since } r_{1,2} \text{ is low} \\ \underline{\text{Perceived return } \mu_2 \text{ over } (t_2, t_3] \text{ horizon is closer to } \mu_{L,2}} \\ \mu_2 \text{ reflects extrapolation from low } r_{1,2} \end{cases}$$

A small n would increase the disparity between $\sigma^2_{H,2}$ and $\sigma^2_{L,2}$ at t_2 since there are few forecast errors available to assess the skill of the two information sources. Consequently, a small n causes either $\mu_{H,2}$ or $\mu_{L,2}$ to exert more influence on μ_2 for a given realized return $r_{1,2}$. Thus, μ_2 is more extreme when n is small.

One may argue that the empirical evidence concerning long term reversals motivates a mean-reverting prior distribution when issuing return forecasts. However, if the optimistic (pessimistic) information source at t_1 decreases (increases) its return forecast at t_2, then expected return uncertainty is reduced. Indeed, if $\mu_{H,1}$ and $\mu_{L,1}$ converge to a common return forecast μ_* at t_2, then the uncertainty created by the investment is resolved during the $(t_1, t_2]$ horizon and equation (5) is zero. Hence, return extrapolation *attributable* to information portfolio theory continues as long as there is uncertainty regarding the firm's expected return. As a consequence, return predictability can be induced by expected return uncertainty since the information portfolio weights assigned to return forecasts are time-varying. In particular, the information portfolio is updated each period to reflect changes in the historical accuracy of each information source which are larger when n is small.

Intuitively, an sequence of returns may exhibit predictability due to events whose return implications are not immediately understood and agreed upon by all information sources. This finding does not prevent the return forecasts from being updated by the information sources at intermediate timepoints. Instead, only the continuation of forecast dispersion beyond one period is required.[16] Furthermore, greater predictability in the profitability of the investment implies the disparity between the return forecasts would narrow more rapidly. In particular, our discussion in the previous subsection implies predictability would reduce the firm's expected return uncertainty. Nonetheless, provided the investment's profitability is not perfectly predictable, expected return uncertainty persists beyond the initial $(t_1, t_2]$ period.

In summary, high (low) return forecasts have greater historical accuracy following high (low) realized returns. Therefore, according to information portfolio theory, return forecasts are assigned larger information portfolio weights when they are similar to previous return realizations. Consequently, the perceived return appears to be *extrapolated* from past returns. Intuitively, many return sequences exist ex-ante, with the investment's success determining a realized return sequence. Similarly, Bondarenko and Bossaerts (2000) provide an excellent description of the return bias induced by conditioning on an option's eventual in-the-money or out-of-the-money status.

When conducting tests of market efficiency using historical return data, one must be careful that evidence of return predictability accounts for the range of return forecasts underlying an asset's estimated expected return. High or low realized returns result from large state variable fluctuations or high return sensitivities to state variable movements. Either of these effects can increase expected return uncertainty if they undermine the ability of information sources to forecast state variables or their return implications. In addition, when the return forecasts are unbiased, equation (17) implies that assets with higher expected return uncertainty are riskier from the investor's perspective.[17]

Finally, return reversals may coincide with lower variability in state variables forecasts and their transformation into return forecasts. For example, prior returns can constitute the lowest variance source of information when state variables such as earnings are difficult to forecast. However, as n increases, the influence of prior returns would diminish if alternative information sources begin to offer more accurate return forecasts.[18]

[16] The horizon between the issuance of forecasts is important. Longer intervals allow more uncertainty to be resolved before the investor's perceived return is adjusted.

[17] This property holds even when η is constant and is therefore uncorrelated with any risk factor. For emphasis, the asset's true expected return may be time-varying but is written as a constant for notational simplicity.

[18] A practical implication of this property is that conflicts of interest which compromise the accuracy of affiliated analyst forecasts have a greater impact on an IPO's return before non-affiliated analysts establish their credibility.

3.4 Conditional Expectations and Forecast Heterogeneity

The law of iterated expectations is usually invoked to conclude that the "error" separating an expected conditional return and its realization has zero mean. However, information portfolio theory allows different information sources to utilize distinct statistical methodologies when forecasting state variables or ascertaining their return implications. Disparate return forecasts also originate from information sources analyzing different state variables. Consequently, the law of iterated expectations does not ensure homogenous return forecasts across the J information sources. The potential for disagreement regarding future state variables and their impact on the asset's return justifies the existence of multiple information sources in our framework. From a practical perspective, we assume the asset's expected return is sufficiently complex to prevent information sources from obtaining identical estimates for η.

Our framework's structure allows an asset's return forecasts to be determined by multifactor asset pricing models when factors such as the market return are interpreted as state variables. An individual asset's true expected return is unknown for several reasons in these formulations; randomness in the dynamics of the factors, estimation error in the factor loadings for individual assets, and uncertainty regarding the number of required factors as well as their composition.[19] Alternative return forecasts could be generated by price targets and intrinsic value measures which are studied in Brav and Lehavy (2003) and Lee, Myers and Swaminathan (1999) respectively.

However, the standard econometric approach when testing market efficiency restricts itself to a single expected return estimate, which fails to account for uncertainty surrounding the interpretation of available information. Thus, the standard methodology ignores disagreement regarding forecasts of the factor returns. In particular, the market's expected return is assumed to be known ex-ante, in contrast to the BusinessWeek survey mentioned in the introduction which has year-end return forecasts for the S&P 500 ranging from -29.5% to 30.0%. Furthermore, the beta coefficient for each factor is assumed to be known and agreed upon by all information sources, while each information source is further assumed to employ the same multifactor model. Statistically, the beta coefficients in multifactor models are estimated using time series data and fixed for a given horizon to produce a single estimate for an asset's expected return. In contrast, our information portfolio weights are time-varying and consider a cross-section of return forecasts. Indeed, the optimal information portfolio is updated

[19] Alternatively, an information source could utilize a state variable to predict the market's return next period.

each period to reflect changes in the historical accuracy of each information source. Fluctuations in the information portfolio weights are more pronounced when n in equation (1) is small. Appendix B discusses these issues in more detail.

One concern regarding information portfolio theory may be the appearance of *systematic* expected return biases that appear to indicate the presence of psychological biases. In the next section, we demonstrate that our optimal information portfolio weights induce return characteristics that mimic biases utilized in the behavioral finance literature.

4 Properties of the Perceived Return

This section connects our optimal information portfolio with several characteristics of the perceived return previously attributed to investor psychology. In particular, we demonstrate that the *appearance* of overconfidence, biased self-attribution, representativeness, conservatism and limited attention are induced by the optimal information portfolio. However, none of the information sources nor the investor are assumed to be influenced by psychological biases.

Several empirical studies link firm characteristics and periods of uncertainty with behavioral biases originating from the psychology literature.[20] In the context of information portfolio theory, return characteristics induced by the optimal information portfolio which mimic behavioral biases are strongest when expected return uncertainty is high. At the opposite end of the uncertainty spectrum, if all information sources issue identical return forecasts for an asset, then the information portfolio is irrelevant since any combination of these forecasts yields the same perceived return.

4.1 Appearance of Overconfidence and Biased Self-Attribution

To analyze the appearance of overconfidence in the perceived return, we examine two information sources. This first information source is private and the second public, with their return forecasts

[20] These studies include Zhang (2005) and Kumar (2005). Baker and Wurgler (2005) report that young, small firms are more sensitive to investor sentiment, while Jackson and Johnson (2006) find that momentum and post-earnings announcement drift both result from events that significantly alter a stock's earnings. The composite share issuance variable of Daniel and Titman (2005) also indicates return predictability, while Vassalou and Apedjinou (2004) report that momentum strategies are most profitable for firms with high levels of corporate innovation.

and historical accuracies denoted by pr and pb subscripts respectively. The investor's perceived return appears to exhibit overconfidence *whenever* the information portfolio weight w_{pr} for a private information source exceeds the information portfolio weight w_{pb} of a public information source. Later in this subsection, we demonstrate that the appearance of overconfidence can occur even if the private and public information sources have identical theoretical accuracies, especially when n is small.

Interpretation 1. *Appearance of Overconfidence*

Corollary 1 implies the following information portfolio weights for private and public information

$$\begin{bmatrix} w_{pb} \\ w_{pr} \end{bmatrix} = \frac{1}{\sigma^2_{pr} + \sigma^2_{pb}} \begin{bmatrix} \sigma^2_{pr} \\ \sigma^2_{pb} \end{bmatrix} . \tag{35}$$

Consequently, private information is overweighted with w_{pr} exceeding w_{pb} whenever $\sigma^2_{pr} < \sigma^2_{pb}$. Furthermore, the perceived return equals

$$\frac{1}{\sigma^2_{pr} + \sigma^2_{pb}} \left[\sigma^2_{pr}\, \mu_{pb} + \sigma^2_{pb}\, \mu_{pr} \right] , \tag{36}$$

which emphasizes μ_{pr} more than μ_{pb}.

According to equation (36), whenever a private information source is more accurate than its public counterpart, the investor's perceived return mimics overconfidence. Recall that several private information sources can originate from the investor since forecasts for state variables such as earnings require further interpretation by the investor to become return forecasts. In contrast, price targets yield explicit return forecasts which constitute public information sources. Overall, let the return forecast μ_{pr} in equation (36) be associated with the investor's most accurate source of private information. Provided this private source is more accurate than the public information source over the last n periods, the investor appears to exhibit overconfidence. We formalize this property after introducing a characteristic of the perceived return which mimics biased self-attribution.

In the context of information portfolio theory, the investor exhibits the appearance of biased self-attribution when one of their private information sources is more accurate than a public information source according to equation (2), while another private information source is less accurate.[21] Consider

[21] Intuitively, to connect historical accuracy with terminology in the psychology literature, *confirming* private information sources are more accurate than a public information source according to equation (2), while *disconfirming* private information sources have been less accurate than all public information sources over the last n periods.

two private information sources along with the original public information source. The private information source which is more accurate than the public information source is denoted by a c subscript, while the less accurate private information source has a d subscript.

Interpretation 2. *Appearance of Overconfidence with Biased Self-Attribution*
Consider the variance-covariance matrix

$$\begin{bmatrix} \sigma_c^2 & 0 & 0 \\ 0 & \sigma_d^2 & 0 \\ 0 & 0 & \sigma_{pb}^2 \end{bmatrix},$$

with the property that $\sigma_d^2 > \sigma_{pb}^2 > \sigma_c^2$*. The corresponding information portfolio equals*

$$[w_c,\, w_d,\, w_{pb}] \;=\; \frac{1}{D}\left[\sigma_d^2\,\sigma_{pb}^2,\, \sigma_c^2\,\sigma_{pb}^2,\, \sigma_c^2\,\sigma_d^2\right],$$

where D *is defined as* $D = \sigma_d^2\,\sigma_c^2 + \sigma_d^2\,\sigma_{pb}^2 + \sigma_c^2\,\sigma_{pb}^2$*. Therefore, the perceived return* $W^T\mu$ *equals*

$$\frac{[\sigma_c^2\,\mu_d + \sigma_d^2\,\mu_c]\,\sigma_{pb}^2 + \sigma_c^2\,\sigma_d^2\,\mu_{pb}}{D}, \tag{37}$$

which is influenced more by μ_c *than* μ_d*.*

The $\sigma_d^2 > \sigma_c^2$ property ensures the information portfolio weight for μ_c exceeds the information portfolio weight of μ_d. Thus, historically accurate private information sources have more influence over the investor's perceived return. Interestingly, the investor's perceived return may exhibit the appearance of overconfidence even when their private information sources are inaccurate on average since their less accurate private information sources receive smaller information portfolio weights. For example, if the investor successfully predicts the return implications of industry characteristics, but cannot reliably interpret a firm's earnings, then the importance of industry data is accentuated by the information portfolio at the expense of earnings. By implication, the investor pursues trading strategies derived from private information sources which have provided them with *individual* success, regardless of the technique's generality.

Overall, the investor's perceived return gravitates towards their most accurate private information sources and away from those which are less accurate. This tendency causes the investor's perceived return to exhibit the appearance of overconfidence and biased self-attribution as a result of the optimal information portfolio rather than psychology. These return characteristics are formalized below with two private information sources whose historical accuracies are denoted $\sigma^2_{pr,1}$ and $\sigma^2_{pr,2}$ respectively. Let p_j represent the probability that the j^{th} private information source is less accurate than its public counterpart, $\sigma^2_{pr,j} > \sigma^2_{pb}$, after n periods according to equation (2) which captures the *relative* skill of the information sources from the investor's perspective. Recall from the previous section that assessing an information source's skill is more difficult when n is small, implying $p_j \approx \frac{1}{2}$. The p_j probabilities are identically one-half when the return forecasts originate from a common distribution, with the information sources having identical theoretical accuracies. The following four scenarios summarize the comparative historical accuracies of the three information sources after n periods.

Scenario	Historical Accuracies					Probability	Investor Appears to Exhibit
A	$\sigma^2_{pr,1}\,,\,\sigma^2_{pr,2}$	$<$	σ^2_{pb}	$<$	Neither	$(1-p_1)\,(1-p_2)$	Overconfidence from both private sources
B	$\sigma^2_{pr,1}$	$<$	σ^2_{pb}	$<$	$\sigma^2_{pr,2}$	$(1-p_1)\,p_2$	Overconfidence from 1^{st} private source and biased self-attribution
C	$\sigma^2_{pr,2}$	$<$	σ^2_{pb}	$<$	$\sigma^2_{pr,1}$	$p_1\,(1-p_2)$	Overconfidence from 2^{nd} private source and biased self-attribution
D	Neither	$<$	σ^2_{pb}	$<$	$\sigma^2_{pr,1}\,,\,\sigma^2_{pr,2}$	$p_1\,p_2$	No Overconfidence

Observe that the perceived return exhibits the appearance of overconfidence in scenarios A, B and C, with a cumulative probability of $1 - p_1\,p_2$. Therefore, when private and public information sources are equally accurate, with $p_1 = p_2 = \frac{1}{2}$, the probability that the investor's perceived return appears to exhibit overconfidence is 75%. The investor appears to exhibit the greatest amount of overconfidence in scenario A where both private information sources are more historically accurate than the public information source. Furthermore, in scenarios B and C, a historically accurate (inaccurate) private information source is assigned a larger (smaller) portfolio weight than the public information source. Therefore, the probability that biased self-attribution appears to influence the investor's perceived return equals 50%. Nonetheless, the appearance of overconfidence and biased self-attribution occurs despite the two private and public information sources possessing identical levels of skill.

Finally, our results continue to apply when there are more public than private information sources.[22] When two public information sources and one private information source are available, the following four scenarios are relevant after n periods.

Scenario	Historical Accuracies					Probability	Investor Appears to Exhibit
A	$\sigma^2_{pb,1}$, $\sigma^2_{pb,2}$	$<$	σ^2_{pr}	$<$	Neither	$p_1\, p_2$	No overconfidence
B	$\sigma^2_{pb,1}$	$<$	σ^2_{pr}	$<$	$\sigma^2_{pb,2}$	$p_1\,(1-p_2)$	Limited overconfidence from 2^{nd} public source and biased self-attribution
C	$\sigma^2_{pb,2}$	$<$	σ^2_{pr}	$<$	$\sigma^2_{pb,1}$	$p_2\,(1-p_1)$	Limited overconfidence from 1^{st} public source and biased self-attribution
D	Neither	$<$	σ^2_{pr}	$<$	$\sigma^2_{pb,1}$, $\sigma^2_{pb,2}$	$(1-p_1)\,(1-p_2)$	Overconfidence from both public sources

The concept of limited overconfidence in scenarios B and C reflects the private information source's larger portfolio weight relative to *one* of the two public information sources. Indeed, the investor's private information sources may be inaccurate on average. Only in scenario A when the private information source is less accurate than both public information sources is there no evidence of overconfidence.

[22]The relationship between the number of *private* information sources and their accuracy is ambiguous. More private information sources could increase the likelihood of at least one private information source being more accurate than the public information source. Conversely, additional private information sources may diminish the resources allocated to generating each return forecast and thereby decrease their accuracy.

4.2 Appearance of Representativeness and Conservatism

Recall from the previous section that the variance of an information source's forecast error decreases as a result of predictability in state variable dynamics as well as predictability in their return implications. These properties imply that *trends* are capable of increasing an information source's portfolio weight since trends imply predictability. For example, a strong trend in a binomial sequence consists predominately of either up or down movements, implying the estimated binomial probability over the last n observations would be near 0 or 1.

Consider two information sources, labeled consistent and inconsistent, with the former arising from predictability in the dynamics of a state variable or its return implications. The return forecasts as well as historical accuracies associated with consistent and inconsistent information sources are denoted by c and d subscripts respectively, with the property $\sigma_I^2 > \sigma_C^2$ induced by predictability.

Interpretation 3. *Appearance of Representativeness*
Let $\mu = \begin{bmatrix} \mu_C \\ \mu_I \end{bmatrix}$ *and* $\Theta = \begin{bmatrix} \sigma_C^2 & 0 \\ 0 & \sigma_I^2 \end{bmatrix}$. *From Corollary 1, the information portfolio equals*

$$\begin{bmatrix} w_C \\ w_I \end{bmatrix} = \frac{1}{\sigma_I^2 + \sigma_C^2} \begin{bmatrix} \sigma_I^2 \\ \sigma_C^2 \end{bmatrix} , \tag{38}$$

which implies the perceived return

$$\frac{1}{\sigma_C^2 + \sigma_I^2} \left[\sigma_C^2 \, \mu_I + \sigma_I^2 \, \mu_C \right] , \tag{39}$$

is influenced more by μ_C *than* μ_I*.*

Hence, consistent information sources have more influence on the investor's perceived return than their inconsistent counterparts. However, trends can produce consistency without an information source possessing any superior knowledge regarding the true dynamics of a state variable or its relationship with future returns. Indeed, when n is small, the consistency of an information source could be short-lived. For example, prior returns or industry characteristics may generate consistent sources of information for an IPO until its earnings dynamics and return implications can be reliably estimated.

Assume the return forecasts from two information sources both emanate from the true return distribution, which is further assumed to be stationary. These two information sources have identical levels of theoretical accuracy. Therefore, any trend that causes one of the two information sources to be more historically accurate after n periods is statistically insignificant. Nonetheless, for a finite n, equation (2) implies one of the information sources is more accurate as in the following two scenarios.[23]

Scenario	Consistent		Inconsistent	Probability	Investor Appears to Exhibit
A	σ_1^2	$<$	σ_2^2	$\frac{1}{2}$	Representativeness; 1^{st} source consistent, 2^{nd} inconsistent
B	σ_2^2	$<$	σ_1^2	$\frac{1}{2}$	Representativeness; 2^{nd} source consistent, 1^{st} inconsistent

Observe that the appearance representativeness occurs in both scenarios, while its apparent magnitude is proportional to the disparity $|\sigma_1^2 - \sigma_2^2|$. With both historical accuracies computed according to equation (2), this distance decreases as n increases since the return forecasts from both information sources arise from the same distribution.

Furthermore, the investor's perceived return may appear insensitive to the release of new information. As illustrated below, even for the simplest case where two information sources are available, the perceived return has four degrees of freedom.

Interpretation 4. *Appearance of Conservatism*
According to Corollary 1, an infinite number of μ_C, μ_I, σ_C^2 and σ_I^2 combinations result in the same perceived return,

$$\frac{\sigma_C^2\,\mu_I + \sigma_I^2\,\mu_C}{\sigma_C^2 + \sigma_I^2}\,.$$

Therefore, conservatism cannot be established without evaluating multiple sources of information since the investor's perceived return is an aggregate quantity.

As an example, suppose the return implications of earnings and sales are negatively correlated after a large investment or period of price discounting. As demonstrated in the next subsection, negatively

[23] The return forecasts originate from a (normal) continuous distribution. Consequently, the probability that σ_1^2 equals σ_2^2 after n periods is zero.

correlated return forecasts receive larger information portfolio weights and have more influence on the investor's perceived return. Consequently, examining earnings or sales information in isolation creates the impression that the investor's perceived return exhibits conservatism.

4.3 Appearance of Limited Attention

The $\sigma_{1,2}$ covariance term in Corollary 2 incorporates the appearance of limited attention into the perceived return. Barber, Odean and Zhu (2003) present empirical evidence of this bias for individual investors.

Figure 1 illustrates the response of the perceived return and its uncertainty in equations (25) and (26) respectively as a function of the correlation between two forecasts. Observe that forecast correlation has a dramatic impact on the investor's aggregate uncertainty but less influence on their perceived return. Appendix C formalizes this assertion by computing the partial derivatives of the perceived return and its aggregate uncertainty in Corollary 2 with respect to σ_{12}. Intuitively, the investor ignores an information source whose return forecasts are positively correlated with more accurate information sources. This behavior parallels the removal of independent variables in linear regression models due to multicollinearity.

For example, if two analysts are simultaneously optimistic or pessimistic, then the investor may limit their attention to a single *representative* information source, where optimism (pessimism) is associated with positive (negative) forecast errors in equation (1). In contrast, if their return forecasts offer alternative perspectives on the asset's expected return, then the investor benefits from analyzing both information sources. More formally, consider the portfolio weights in Corollary 2

$$\begin{aligned} w_1 &= \frac{\sigma_2^2 - \sigma_{12}}{\sigma_2^2 + \sigma_1^2 - 2\sigma_{12}} \\ w_2 &= \frac{\sigma_1^2 - \sigma_{12}}{\sigma_2^2 + \sigma_1^2 - 2\sigma_{12}} \, . \end{aligned} \tag{40}$$

If the two forecasts are independent, then σ_{12} equals zero and both portfolio weights are positive. However, when σ_{12} equals σ_1^2, the Cauchy-Schwartz inequality implies $\sigma_1^2 \leq \sigma_2^2$ with the first information source being more accurate than the second.[24] From an economic perspective, when $\sigma_{12} = \sigma_1^2$, the

[24]The Cauchy-Schwartz inequality provides an upper bound on the covariance, $(\sigma_{12})^2 \leq \sigma_1^2 \sigma_2^2$. Therefore, when $\sigma_{12} = \sigma_1^2$, this inequality implies $\sigma_1^2 \leq \sigma_2^2$.

portfolio weights in equation (40) become

$$w_1 = \frac{\sigma_2^2 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2} = 1 \tag{41}$$

$$w_2 = \frac{\sigma_1^2 - \sigma_1^2}{\sigma_2^2 - \sigma_1^2} = 0\,. \tag{42}$$

Consequently, a large positive covariance between the two information sources eliminates the second return forecast from the perceived return, with a visual illustration in Figure 2. Thus, investors tend to ignore return forecasts which are positively correlated with more accurate information sources, while negatively correlated forecasts have the greatest influence over the investor's perceived return. Therefore, when attempting to detect conservatism, it is essential to evaluate the aggregate impact of contradictory information.[25]

In summary, the number of return forecasts the investor processes depends on their correlation structure. Thus, the investor may rely on broadly defined sector information rather than firm-specific characteristics if the latter are positively correlated within an industry. For example, during the Internet bubble, the returns of dot-com firms appear to have been driven by industry characteristics. In addition, earnings forecasts issued by analysts who herd are less likely to influence an asset's perceived return.

4.4 Rational versus Behavioral Interpretations

Although the perceived return is derived from the optimal information portfolio, this estimate of the asset's expected return is not referred to as being *rational* since the return forecasts may incorporate investor psychology. In particular, the most accurate information sources could be those which incorporate investor psychology into their return forecasts. As a consequence, information portfolio theory does not preclude behavioral biases from influencing the perceived return.

For example, suppose all J return forecasts are identical and equal to μ_*, with this common expectation further *assumed* to be the result of at least one psychological bias. The investor's perceived

[25] Moreover, limited attention can magnify the appearance of overconfidence since the investor focuses on a subset of public signals which are negatively correlated. As a consequence, when n is small, if one of the public information sources has been historically accurate, another public source of information is likely to have been inaccurate. At a minimum, a private information source is assigned a larger portfolio weight than the inaccurate source of public information.

return equals μ_* regardless of the information portfolio and reflects investor psychology. Conversely, the use of psychology could increase forecast dispersion due to disagreements over the exact nature of the biases committed by investors. As a result, the relevance of information portfolio theory is enhanced by differences of opinion regarding investor psychology. Overall, decomposing the perceived return into the effects of psychology versus the optimal information portfolio is ultimately an empirical question. Our objective in this paper is to demonstrate that expected return uncertainty instills the appearance of psychological biases into an investor's perceived return.

By estimating an investor's expected return, information portfolio theory enhances rather than contradicts utility maximization. Indeed, the investor's perceived return and its aggregate uncertainty are critical inputs in further asset pricing applications. Proposition 3 below, whose proof is in Appendix D, provides a utility maximizing application of information portfolio theory.

Proposition 3. *Assume the investor has a negative exponential utility function,* $U(M) = 1 - e^{-\gamma M}$*, with initial wealth* M*. Under the return distribution in equation (17), the optimal fraction of wealth* f *invested in the risky asset equals*

$$f \;=\; \frac{\mathbf{1}^T \Theta^{-1} \left(\mu - r_f \mathbf{1}\right)}{\gamma\, M \,\left[1 + \nu \mathbf{1}^T \Theta^{-1} \mathbf{1}\right]}\,, \tag{43}$$

where r_f *represents the riskfree interest rate.*

As an explicit illustration, consider two correlated identical return forecasts issued by information sources with identical historical accuracies ($\mu_1 = \mu_2 = \mu_*$, $\sigma_1^2 = \sigma_2^2 = \sigma_*^2$) with σ_{12} describing the off-diagonal sample covariance element as in Corollary 2. Under these specifications, the solution for f in equation (43) reduces to

$$f \;=\; \frac{2\,\left(\mu_* - r_f\right)}{\gamma\, M} \left(\frac{1}{2\nu + \sigma_*^2 + \sigma_{12}}\right). \tag{44}$$

Observe that when information sources forecast higher returns or are more accurate historically, the investor increases their exposure to the risky asset. When the return forecasts are negatively correlated, the investor also purchases more of the risky asset. According to equation (44), accurate return forecasts offset the investor's risk aversion. Therefore, it is difficult to distinguish between the influence of time-varying risk aversion from the asset's expected return uncertainty.

As a special case of equation (44), the fraction of wealth allocated to the risky asset equals $\frac{\eta - r_f}{\gamma\, M\, \nu}$ when there is no uncertainty regarding the asset's expected return since σ_*^2 and σ_{12} are zero while $\mu_* = \eta$.

4.5 Contrast with Bayesian Methods

When an asset's expected return is *unstable*, Brav and Heaton (2002) demonstrate that representativeness and conservatism result from Bayesian priors which underweight past or recent observations respectively. In their model, these biases arise from uncertainty regarding a random *change point* which initiates a different economic regime. Therefore, they highlight the difficulty posed by different possible priors when attempting to disentangle rational from behavioral explanations of return patterns. Furthermore, overconfidence may be inserted directly into the prior distribution of a private return forecast by assuming the investor underestimates its variability. Alternatively, the attribution bias in Gervais and Odean (2001) utilizes improper Bayesian updating to create overconfidence.

In contrast, perceived return characteristics induced by our optimal information portfolio arise from aggregating across multiple return forecasts. As outputs of information portfolio theory, the enable us to provide testable implications independent of any prior distribution. Although Bayesian updating is applicable to multiple forecasts, a prior distribution(s) remains an integral part of the posterior and therefore the investor's expected return. In contrast, the weights assigned to information sources in our framework are derived entirely from return forecasts and realizations.

5 Empirical Implementation

Information portfolio theory does not assume that investor beliefs are influenced by psychological biases. Instead, uncertainty surrounding an asset's expected return causes the optimal information portfolio to induce these return characteristics. In this section, testable implications of information portfolio theory distinct from psychological theories are discussed and verified empirically.

5.1 Testable Implications

There are several testable implications of information portfolio theory, including hypotheses that enable us to distinguish our framework from psychology.

First, the return characteristics induced by information portfolio theory are more pronounced when an asset's expected return is uncertain. Therefore, in the aftermath of events which undermine the relevance of previous return forecasts, the appearance of return characteristics that mimic overconfidence, biased self-attribution, representativeness, conservatism and limited attention in the perceived return is more prevalent, as is return predictability. Corporate restructurings, significant investments as well as technological innovations all reduce the relevance of previous observations and increase the uncertainty surrounding a firm's expected return. Second, predictable state variables which are correlated with returns, regardless of their theoretical justification, influence the investor's perceived return. This tendency is also aggravated when expected return uncertainty is high. Third, negatively correlated return forecasts reduce the investor's aggregate forecast error. Consequently, contradictory sources of information have greater influence over the investor's perceived return.

Two testable hypotheses involving private return forecasts are also available. First, investors overweight their accurate private information sources (successes) at the expense of their less accurate private information sources (failures). Consequently, the trading strategies implemented by an investor are determined by the success of their private return forecasts. Second, less experienced investors have a greater propensity to exhibit overconfidence and biased self-attribution since they have produced fewer forecast errors to assess their ability. This implication arises from n being specific to an information source rather than being common to all information sources.

Overall, to distinguish between the implications of psychological theories versus information portfolio theory, the accuracy associated with each information source is crucial. Evaluating information sources by their historical accuracy facilitates empirical tests of information portfolio theory. Specifically, even in poor information environments, information portfolio theory posits that investors focus on historically accurate sources of information.

However, psychological biases and information portfolio theory are not necessarily incompatible. The extent to which they both influence the perceived return is ultimately an empirical question. For example, if the most accurate information sources incorporate investor psychology into their return forecasts, then psychology undeniably impacts the investor's perceived return. In this economy, uncertainty surrounding the asset's expected return causes the optimal information portfolio to augment, rather than completely explain, characteristics of the investor's perceived return which mimic behavioral biases.

5.2 Hypotheses and Data

In our empirical implementation, there are implicitly two information sources; earnings and everything unexplained by earnings. Testing information portfolio theory requires us to examine the transformation of earnings into returns and earnings variability. We examine the profitability of earnings momentum strategies based on analyst forecast revisions to test both aspects of information portfolio theory.

The relationship between returns and earnings forecasts generates our first hypothesis. Intuitively, $Var[\xi_t]$ in equation (34) is being referenced. The first hypothesis is derived from the information portfolio weights assigned to return forecasts that arise from earnings. Information portfolio theory predicts that investors focus their attention on a firm's earnings when this state variable has experienced a stronger relationship with the stock's realized returns.

Hypothesis 1. *Earnings momentum is stronger for stocks when the return implications of their earnings are more certain.*

Our second hypothesis concerns earnings uncertainty and refers intuitively to $Var[\zeta_t]$ in equation (34). Higher earnings uncertainty translates into greater expected return uncertainty which causes return predictability.

Hypothesis 2. *Earnings momentum is stronger for stocks with higher earnings uncertainty.*

The first hypothesis is critical to verifying information portfolio theory, while the second hypothesis also has a behavioral interpretation (Zhang (2005)). Behavioral theory (e.g. Hirshleifer (2001)) posits that psychological biases are strongest in environments with high uncertainty as well as poor information. Consequently, behavioral theory and our framework are both consistent with the second hypothesis. However, for a given level of uncertainty, behavioral theory predicts stronger earnings momentum when earnings are less informative (low sensitivity of returns to earnings), while information portfolio predicts the opposite. Therefore, in contrast to the optimal information portfolio which predicts investors attempt to find the "best" available sources of information, even during periods of high expected return uncertainty, psychology does not predict that investors shun uninformative state variables. Hence, the first hypothesis is crucial to distinguishing between our framework and psychological explanations for return predictability.

Our empirical tests consider all domestic primary stocks listed on the NYSE, AMEX and NASDAQ with analyst coverage. The monthly stock return and market capitalization data are obtained from CRSP while analyst forecasts are from the I/B/E/S Summary History dataset. The intersection of the CRSP and I/B/E/S datasets over the January, 1976 to December, 2004 sample period is utilized. The start date is determined by the beginning of the I/B/E/S Summary History dataset. Forecast revisions are scaled by stock prices retrieved from I/B/E/S to account for adjustments such as stock dividends and stock splits. Finally, we obtain book-to-market ratios (B/M) from Compustat.

We construct an uncertainty measure to proxy for the return dispersion in equation (5) as well as a sensitivity measure to gauge the relative informativeness of earnings versus everything else when forecasting returns.

5.3 Sensitivity of Returns to Forecast Revisions

Each month, we estimate stock price sensitivities to earnings information by computing the correlation coefficient between stock returns and forecast revisions over the previous twelve months.[26] These correlations proxy for the return implications of analyst forecasts. In particular, stocks with higher correlations are more influenced by earnings since our sensitivity measure parallels the transformation from earnings state variables into return forecasts.

I/B/E/S contains summary statistics on analyst forecasts for the third Thursday of each month (referred to as the I/B/E/S compilation date hereafter). We define the forecast revision for firm i in month t as

$$rev_{i,t} = \frac{FY1_{i,t} - FY1_{i,t-1}}{P_{i,t}}, \tag{45}$$

where $FY1_{i,t}$ and $FY1_{i,t-1}$ are the mean analyst forecast for fiscal year 1 in month t and $t-1$ respectively, while $P_{i,t}$ is the stock price provided by I/B/E/S on the compilation date in month t.[27]

[26] We also estimate the correlation coefficient using observations from the previous 6 and 24 months. Our results are robust to these alternative estimates of the correlation coefficient.

[27] Additional adjustments on $rev_{i,t}$ are performed in the month when a firm announces its fiscal year earnings since analyst forecasts switch to the subsequent fiscal year after the announcement. Thus, the $FY1$ estimates in two consecutive months could be forecasts for two different fiscal years. For example, suppose a firm announces its fiscal year earnings in month t. If the announcement date is before the I/B/E/S compilation date in that month, $rev_{i,t}$ is defined as its mean $FY1$ estimate in month t minus its mean $FY2$ estimate in month $t-1$. Conversely, if the announcement

For each $rev_{i,t}$, we compute the contemporaneous stock return $ret_{i,t}$ defined as the return of stock i between two I/B/E/S compilation dates in month $t-1$ and month t. Once again, the stock prices on the I/B/E/S compilation dates are extracted from I/B/E/S.

Using the monthly forecast revisions and stock returns, we then find the return-forecast sensitivity of stock i in month t by computing the correlation coefficient between rev_i and ret_i over the past 12 months. Based on this sensitivity measure, the stocks are sorted into three groups every month consisting of the bottom 30%, middle 40% and top 30% respectively. For ease of illustration, these three groups are labeled low sensitivity (S1), medium sensitivity (S2) and high sensitivity (S3) stocks.

5.4 Earnings Uncertainty

Our theory also asserts that return predictability is more pronounced when state variables are more uncertain. The uncertainty of earnings information is measured using the standard deviation of analyst forecasts scaled by stock price[28]

$$stdev_{i,t} = \frac{\sigma_{i,t}}{P_{i,t}}. \tag{46}$$

Along with the sensitivity classifications, we divide the stocks into three uncertainly groups each month according to equation (46) which are comprised of the bottom 30%, middle 40% and top 30%. These three groups are referred to as low uncertainty (U1), medium uncertainty (U2) and high uncertainty (U3) stocks.

Table 1 provides an overview of the sensitivity and uncertainty portfolios. Furthermore, we investigate whether there are significant differences among the portfolios in terms of value/growth and large/small characteristics as well as analyst coverage. The Spearman rank correlation coefficients among the sensitivity measure, the uncertainty measure, B/M, size and the number of analysts are computed each month, with their time series average reported in Panel A. Each month we also compute the average rankings of B/M, size and number of analysts for the stocks in the sensitivity and

occurs after the I/B/E/S compilation date in that month, then $rev_{i,t}$ remains defined as the difference in the mean $FY1$ estimates between month t and $t-1$. However, $rev_{i,t+1}$ is defined as the mean $FY1$ estimate in month $t+1$ minus the mean $FY2$ estimate in month t.

[28]As a robustness test, the mean analyst forecast is also used to normalize $\sigma_{i,t}$ instead of the stock price. The results under this alternative normalization are nearly identical to those using equation (46). Consequently, for brevity, they are unreported but available upon request.

uncertainty portfolios. The ranking is normalized to [0, 1]. Thus, a ranking of 0.5 is the median and mean observation. Their time series averages are recorded in Panel B.

The statistics indicate low correlation between the sensitivity measure and the uncertainty measure (0.062), B/M ratio (0.013) and size (0.016). The uncertainty measures correlation with size is also very low (-0.018). On the other hand, the uncertainty measure has a positive correlation with B/M (0.265). In other words, higher dispersion stocks tend to be high B/M or value stocks which is consistent with the findings in Doukas, Kim and Pantzalis (2004). The correlation between the uncertainty measure and B/M is confirmed in Panel B as the average ranking of B/M for the stocks in the low uncertainty portfolio (U1) is 0.40, while the average ranking for the medium (U2) and high (U3) uncertainty portfolios are 0.51 and 0.59 respectively. The pattern is also consistent in the double-sorted portfolios (e.g. S1U1 is the portfolio of the stocks belonging to both S1 and U1). Besides this relationship, the sensitivity and uncertainty portfolios are unrelated to B/M, size and analyst coverage factors. The average rankings of the three variables (B/M, size and number of analysts) for the stocks in each sensitivity and uncertainty portfolio are all close to 0.5 (with the exception of B/M and the uncertainty portfolios). Therefore, the portfolios have similar B/M, size and analyst coverage characteristics, and are well represented by an average stock.

5.5 Earnings Momentum Strategies

When the first two hypotheses are combined, the result is the following prediction for the profitability of earnings momentum strategies. This third hypothesis states that these cross-sectional returns are largest when earnings are informative during periods of high expected return uncertainty.

Hypothesis 3. *Earnings momentum is strongest for stocks with high (previous) uncertainty and sensitivity measures.*

Earnings momentum is implemented as in Jegadeesh and Titman (1993), but with forecast revisions over the past 6 months instead of stock returns. The forecast revision for firm i in month t is defined as

$$REV6_{i,t} = \sum_{j=0}^{5} rev_{i,t-j} \, , \tag{47}$$

where $rev_{i,t}$ is defined in equation (45). We rank the stocks according to equation (47) and assign them to one of five quintile portfolios each month. The bottom quintile portfolio contains stocks with the most unfavorable earnings forecast revision, while the top quintile contains those with the most favorable revision. Overlapping portfolios are then constructed to compute equally-weighted returns each month. For instance, the portfolio having the most favorable revision (E5) consists of six overlapping portfolios from the previous six ranking months. The return for this portfolio is the simple average return of the six portfolios formed over the past six months. If a stock's return is missing during the holding period, it is replaced with the corresponding value-weighted market return. The earnings momentum portfolio is the zero-investment portfolio that buys the most favorable revision portfolio and sells the least favorable revision portfolio, E5-E1, each month.

Our earnings momentum strategy differs slightly from the standard price momentum strategy in another respect. After ranking stocks according to their past returns, Jegadeesh and Titman (1993) skip one month before buying stocks to avoid bid-ask spread and short-term stock price reversal. This one month gap is not inserted into our strategies for two reasons. First, we rank stocks based on their earnings which, unlike past returns, is not subject to the bid-ask spread problem. Second, almost all earnings consensus estimates are available between the 10^{th} and the 20^{th} day of the month. Consequently, about half a month has already been omitted before we start holding positions at the beginning of next month.

5.6 Earnings Momentum Conditioned on Sensitivity and Uncertainty

Chan, Jegadeesh and Lakonishok (1996) document strong earnings momentum profits and suggest that earnings momentum is caused by the slow response of market participants to earnings information. If earnings momentum is caused by market under-reaction to earnings information, our theory would predict that earnings momentum is stronger for stocks whose earnings information is more credible, and those with more uncertain earnings. Thus, we hypothesize that earnings momentum strategies are more profitable for stocks in the high sensitivity and high uncertainty portfolios.

Table 2 reports earnings momentum profits and illustrates the importance of return sensitivity to earnings and earnings uncertainty. When the earnings momentum strategy is implemented using the full sample, the strategy generates an average return of 0.69% per month with a t-statistic of 4.38.

Next, we implement the strategy separately for the three sensitivity groups (S1, S2 and S3). The momentum profit remains significant in each of the three groups. More interestingly, the profit increases monotonically from the low sensitivity group (S1) to the high sensitivity group (S3), with the profit of the latter being about 50% higher than the former (0.79% vs. 0.52%). To clarify, the grouping of S1, S2 and S3 is determined before the stocks are assigned to the earnings momentum portfolios (E1 to E5), and thus before the buying or selling of stocks.

The momentum profit pattern is identical in the three uncertainty groups, increasing monotonically from U1 to U3, the profit of U3 being approximately 70% higher than U1 (0.74% vs. 0.44%). When the earnings momentum strategy is applied to double-sorted portfolios on sensitivity and uncertainty, the monotonic increasing pattern of the momentum profits continues. Within each sensitivity group, the profit increases monotonically from U1 to U3 (e.g. within the medium sensitivity group, the profit is 0.49%, 0.64% and 0.79% for S2U1, S2U2 and S2U3 respectively). In addition, within each uncertainty group, the profit increases monotonically from S1 to S3 (e.g. within the medium uncertainty group, the profit is 0.45%, 0.64% and 0.73% for S1U2, S2U2 and S3U2 respectively).

There is existing evidence that momentum profits are affected by factors such as the B/M ratio, documented in Daniel and Titman (1999), along with size and analyst coverage, as reported in Hong, Lim and Stein (2000). Our descriptive statistics in Table 1 indicate that our sensitivity and uncertainty results are not manifestations of these factors.

In particular, our uncertainty measure is positively correlated with B/M, implying low uncertainty stocks tend to be growth stocks. Daniel and Titman (1999) find stronger momentum among growth stocks, and attribute this finding to investor overconfidence. If uncertainty is irrelevant, the positive correlation between uncertainty and B/M would indicate higher momentum profit amongst low rather than high uncertainty stocks. Therefore, our ability to find increasing momentum profits from U1 to U3 attests to the importance of conditioning on uncertainty.

The sensitivity and uncertainty measures are also weakly positively correlated with analyst coverage, although this feature is not found in Panel B of Table 1. Hong, Lim and Stein (2000) report higher momentum profits for stocks with less analyst coverage, consistent with the slow diffusion of information. Their findings also predict less momentum profits for the high sensitivity and high uncertainty stocks, while we find increasing momentum profits from S1 to S3 and U1 to U3. Consequently, the sensitivity and uncertainty measures both contain important conditional information that is not captured by the existing literature.

Overall, we can reasonably conclude that our earnings momentum results, which are derived from sensitivity and uncertainty measures for the return implications of earnings and variability in earnings respectively, are not driven by book-to-market, size and analyst coverage effects documented in the existing literature.

6 Conclusions

To estimate an individual asset's unknown true expected return, we introduce an optimal information portfolio which minimizes the aggregate uncertainty of multiple return forecasts. Each return forecast is issued by an information source after interpreting a relevant state variable forecast. The information portfolio weight assigned to a return forecast depends on the historical accuracy of its information source. The estimated expected return arising from our optimal information portfolio exhibits the appearance of overconfidence, biased self-attribution, representativeness and conservatism as well as limited attention. Therefore, the investor's expected return displays characteristics that have previously been attributed to investor psychology.

The return characteristics induced by our optimal information portfolio as well as return predictability are strongest when the uncertainty surrounding an asset's expected return is high. However, testable implications of information portfolio theory distinct from psychology are available. In contrast to Bayesian frameworks, these implications are independent of any assumed prior distribution. Specifically, even in poor information environments, investors focus on the most informative state variables.

By examining the profits of earnings momentum strategies, we document the importance of return sensitivity to earnings as well as earnings uncertainty. The two pillars of information theory are verified since momentum profits increase monotonically from low to high sensitivity stocks, and from low to high uncertainty stocks. More importantly, the sensitivity results continue after controlling for the effects of information uncertainty. Thus, investors condition their beliefs in accordance with information portfolio theory with historically accurate information sources having more influence on expected returns. The significance of our sensitivity and uncertainty measures is not driven by factors such as book-to-market, size and analyst coverage.

We also demonstrate that a utility maximizing investor reduces their exposure to the risky asset when uncertainty regarding its expected return increases, which complicates the distinction between risk aversion and return uncertainty. Future applications of information portfolio theory could study return volatility and trade volume arising from fluctuations in the information portfolio weights. Extending our framework to incorporate multiple assets would also enable its cross-sectional return implications to be explored.

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Appendices

A Proof of Proposition 1

Denote the Lagrangian of equation (7) as

$$L(W, \lambda) = \frac{1}{2} W^T \Theta W + \lambda (W^T \mathbf{1} - 1)\,, \tag{48}$$

which generates two equations

$$\frac{\partial L(W, \lambda)}{\partial W} = \Theta W + \lambda \mathbf{1} = 0 \tag{49}$$

$$\frac{\partial L(W, \lambda)}{\partial \lambda} = W^T \mathbf{1} - 1 = 0 \tag{50}$$

involving two unknowns; W and the Lagrangian multiplier λ. Equation (49) is equivalent to

$$W = -\lambda \Theta^{-1} \mathbf{1}\,. \tag{51}$$

Multiplying the transpose of equation (51) by the $\mathbf{1}$ vector yields

$$W^T\mathbf{1} \;=\; -\lambda\mathbf{1}^T\Theta^{-1}\mathbf{1} \tag{52}$$

which implies

$$1 \;=\; -\lambda\mathbf{1}^T\Theta^{-1}\mathbf{1}\,, \tag{53}$$

due to the $W^T\mathbf{1} = 1$ constraint. Therefore, the λ parameter is solved as

$$-\lambda \;=\; \frac{1}{\mathbf{1}^T\Theta^{-1}\mathbf{1}}\,. \tag{54}$$

Substituting equation (54) into equation (51) produces the final result

$$W \;=\; \left(\frac{1}{\mathbf{1}^T\Theta^{-1}\mathbf{1}}\right)\Theta^{-1}\mathbf{1}\,, \tag{55}$$

which satisfies the constraint

$$W^T\mathbf{1} \;=\; \left(\frac{1}{\mathbf{1}^T\Theta^{-1}\mathbf{1}}\right)\mathbf{1}^T\Theta^{-1}\mathbf{1} = 1\,. \tag{56}$$

B Multifactor Models and the Information Portfolio

A three factor version of equation (11) describes the asset's true expected return as

$$\eta \;=\; \beta_0 + \beta_1\,f_1 + \beta_2\,f_2 + \beta_3\,f_3\,, \tag{57}$$

from which a vector μ of three return forecasts may be formed to represent the return implications of each individual factor

$$\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} \;=\; \begin{bmatrix} \alpha_{0,1} + \alpha_{1,1}\,f_1 \\ \alpha_{0,2} + \alpha_{1,2}\,f_2 \\ \alpha_{0,3} + \alpha_{1,3}\,f_3 \end{bmatrix}. \tag{58}$$

The elements of μ in equation (58) arise from single factor versions of equation (11)

$$\begin{aligned} y \;&=\; [\alpha_{0,j} + \alpha_{1,j}\,f_j] + \epsilon^j \quad \text{for } j\text{=1,2,3} && (59)\\ &=\; \mu_j + \epsilon^j\,, && (60) \end{aligned}$$

where ϵ^j are mean zero error terms whose variances and covariances define the Θ matrix. The $\alpha_{0,j}$ intercepts ensure equation (59) provides three unbiased return estimates for η which conform to the distributional assumption in equation (9). The three intercept terms are[29]

$$\alpha_{0,j} \;=\; \beta_0 + \sum_{k=1}^{3} \beta_k\, f_k + [\beta_j - \alpha_{1,j}]\; f_j \quad \text{for } k \neq j\,. \tag{61}$$

With the return forecasts in equation (59) conforming to equation (9), equation (14) implies $W^T\mu$ offers an *unbiased* estimate of the asset's true expected return η in equation (57). Therefore, regardless of N, multifactor models for an asset's expected return are incorporated into information portfolio theory when individual factor returns represent distinct state variables. The return forecasts μ_j associated with each factor in equation (60) replace the single expected return arising from the multifactor model in equation (11). For emphasis, the $\alpha_{0,j}$ and $\alpha_{1,j}$ coefficients are estimated using a time series regression, along with the β_0 and β_j coefficients for $j = 1, \ldots, N$. However, the information portfolio weights W differ from the α coefficients in equation (59) as well as the β coefficients in equation (57). In particular, the portfolio weights sum to one and are derived from the sample variances and covariances, computed according to equations (2) and (3) respectively over the last n periods, for the ϵ^j errors in equation (60) which define the Θ matrix.

In practice, the number of return forecasts J would exceed the number of factors N since their returns are random and the asset's factor loadings are estimated quantities. These sources of uncertainty illustrate the generality of information portfolio theory which is not restricted to a single return forecast for each asset. For example, if the Fama-French (1993) model is utilized to generate expected returns, then information sources can disagree on the return prospects for small-cap and growth stocks as well as the overall market. Thus, the cross-sectional dispersion in equation (5) is larger than the dispersion across the three elements of equation (58) which assumes the random factor returns next period and the asset's factor loadings are known. This property applies to any specification for η and allows for state variables that are not factor returns, including industry and macroeconomic trends, firm-specific earnings forecasts and the projected profitability of their investments, among other possibilities.

[29] When the factors are orthogonal, estimates for the β_j coefficients in equation (11) equal the $\alpha_{1,j}$ regression estimates in equation (59), which reduces equation (61) to $\alpha_{0,j} = \beta_0 + \sum_{k=1}^{3} \beta_k\, f_k$ for $k \neq j$ by eliminating the $[\beta_j - \alpha_{1,j}]\; f_j$ term.

C Covariances and the Perceived Return

The partial derivative of the investor's perceived return in equation (25) with respect to σ_{12} equals

$$\begin{aligned}\frac{\partial \text{Perceived Return}}{\partial \sigma_{12}} &= \frac{-(\mu_1+\mu_2)\left[\sigma_1^2+\sigma_2^2-2\sigma_{12}\right]+2\left[\sigma_2^2\,\mu_1+\sigma_1^2\,\mu_2-\sigma_{12}\,(\mu_1+\mu_2)\right]}{\left(\sigma_1^2+\sigma_2^2-2\sigma_{12}\right)^2} \\ &= \frac{(\mu_2-\mu_1)\,(\sigma_1^2-\sigma_2^2)}{\left(\sigma_1^2+\sigma_2^2-2\sigma_{12}\right)^2}\,. \end{aligned} \tag{62}$$

The sign of this derivative may be either positive or negative. According to the numerator of equation (62), when either the return forecasts or their historical accuracies are identical, the investor's perceived return is invariant to σ_{12}. Figure 1 illustrates the insensitivity of the perceived return to σ_{12} over a range of values.

The partial derivative of the perceived return's aggregate uncertainty in equation (26) with respect to σ_{12} equals

$$\begin{aligned}\frac{\partial \text{Aggregate Uncertainty of Perceived Return}}{\partial \sigma_{12}} &= \frac{-2\sigma_{12}\left[\sigma_1^2+\sigma_2^2-2\sigma_{12}\right]+2\left[\sigma_1^2\sigma_2^2-(\sigma_{12})^2\right]}{\left(\sigma_1^2+\sigma_2^2-2\sigma_{12}\right)^2} \\ &= \frac{2\sigma_{12}\left[\sigma_{12}-(\sigma_1^2+\sigma_2^2)\right]+2\sigma_1^2\sigma_2^2}{\left(\sigma_1^2+\sigma_2^2-2\sigma_{12}\right)^2}\,. \end{aligned} \tag{63}$$

As illustrated in Figure 1, aggregate uncertainty is sensitive to the sign of the sample covariance. In particular, when σ_{12} is negative, equation (63) is large and positive. Thus, uncertainty regarding an asset's expected return decreases as σ_{12} becomes more negative.

D Proof of Proposition 3

Recall from equation (17) that the asset's ex-ante return is distributed $\mathcal{N}\left(W^T\mu, \nu + W^T\Theta W\right)$ under the distributional assumption in equation (9). To prove Proposition 3, the following utility maximization problem is solved

$$\begin{aligned} & \max_f E\left\{U\left[M\,\left((1-f)\,(1+r_f)+f\,\left(1+W^T\mu\right)\right)\right]\right\} \\ =\; & \max_f -E\left[\exp\left\{-\gamma\,M\left(1+r_f+f\,\left(W^T\mu-r_f\right)\right)\right\}\right] \\ =\; & \max_f -\exp\left\{-\gamma\,M\,f\,W^T\mu+\gamma\,M\,f\,r_f+\frac{\gamma^2\,M^2\,f^2}{2}\,\left[\nu+W^T\Theta W\right]\right\}\,, \end{aligned} \tag{64}$$

where the last equality results from the moment generating function of a normal distribution. This maximization involves setting the partial derivative of equation (64) with respect to f

$$\left(-\gamma\, M\, W^T\mu + \gamma\, M\, r_f + \gamma^2\, M^2\, f\, \left[\nu + W^T\Theta W\right]\right)\left(-e^{\left\{-\gamma\, M\, f\, W^T\mu + \gamma\, M\, f\, r_f + \frac{\gamma^2\, M^2\, f^2}{2}\left[\nu + W^T\Theta W\right]\right\}}\right) \tag{65}$$

to zero. This requires the first term in the above product to be zero

$$-\gamma\, M\, W^T\mu + \gamma\, M\, r_f + \gamma^2\, M^2\, f\, \left[\nu + W^T\Theta W\right] \;=\; 0\,. \tag{66}$$

Therefore, the optimal investment in the risky asset equals

$$f \;=\; \frac{W^T\mu - r_f}{\gamma\, M\, \left[\nu + W^T\Theta W\right]}\,, \tag{67}$$

which becomes

$$\begin{aligned} f \;&=\; \frac{1}{\gamma\, M}\frac{\left(\frac{\mathbf{1}^T\Theta^{-1}\mu}{\mathbf{1}^T\Theta^{-1}\mathbf{1}} - r_f\right)}{\nu + \frac{1}{\mathbf{1}^T\Theta^{-1}\mathbf{1}}} \\ &=\; \frac{\mathbf{1}^T\Theta^{-1}\left(\mu - r_f\mathbf{1}\right)}{\gamma\, M\, \left[1 + \nu\mathbf{1}^T\Theta^{-1}\mathbf{1}\right]}\,, \end{aligned} \tag{68}$$

after substituting in the results of Proposition 2.

Observe that the optimal portfolio weights from Proposition 1 transform equation (67) into equation (68). When there is no uncertainty regarding the asset's true expected return, equation (67) implies $f = \frac{\eta - r_f}{\gamma\, M\, \nu}$ since $W^T\mu = \eta$ and $W^T\Theta W = 0$.

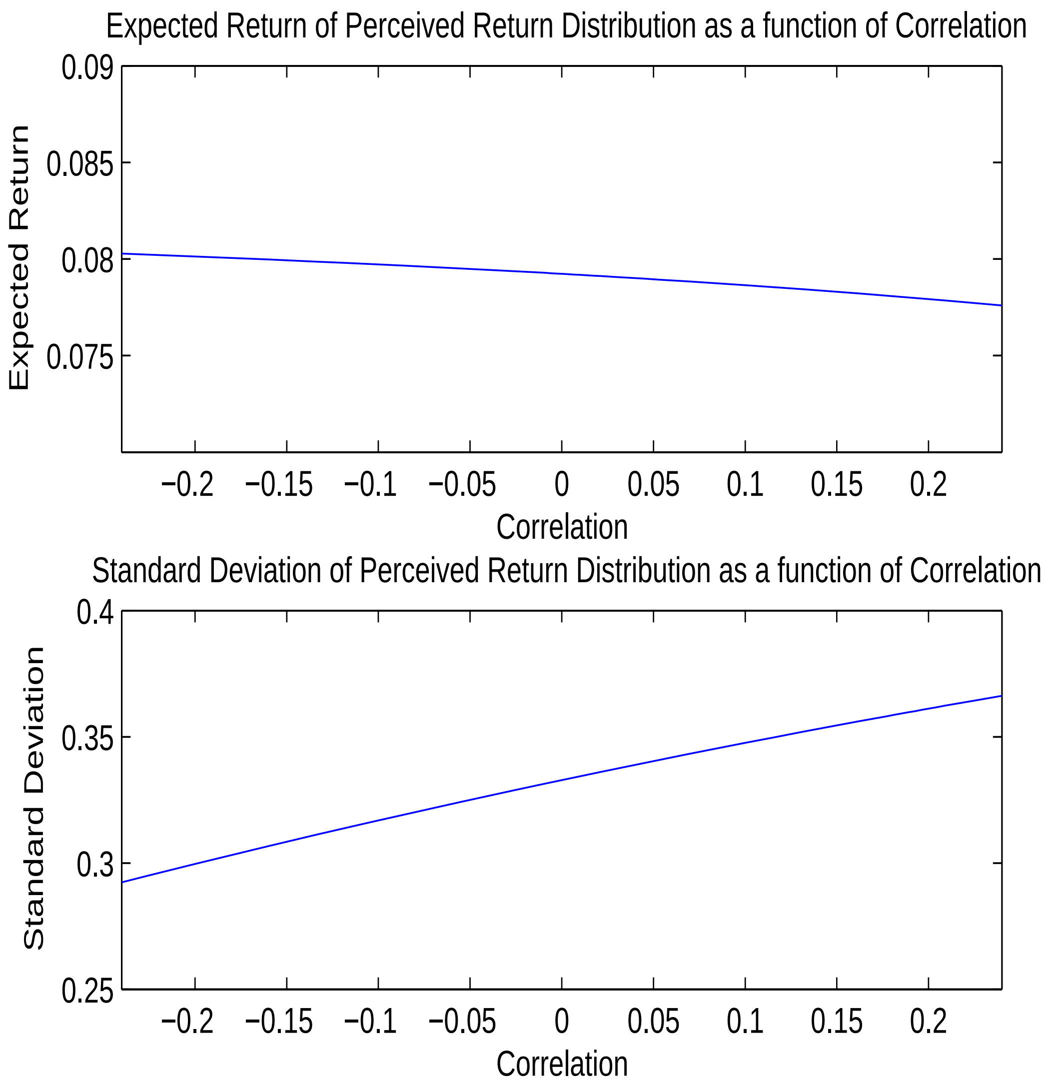

Figure 1: Impact of correlation between two information sources on the perceived return and its aggregate uncertainty according to equations (25) and (26) respectively. These plots are derived from the following parameter values; $\mu_1 = 0.07$, $\mu_2 = 0.10$, $\sigma_1 = 0.40$ and $\sigma_2 = 0.60$. The σ_j parameter denotes the square root of the j^{th} information source's historical accuracy computed in equation (2) for $j = 1, 2$. These root mean-squared error (RMSE) estimates represent the uncertainty of the corresponding return forecasts.

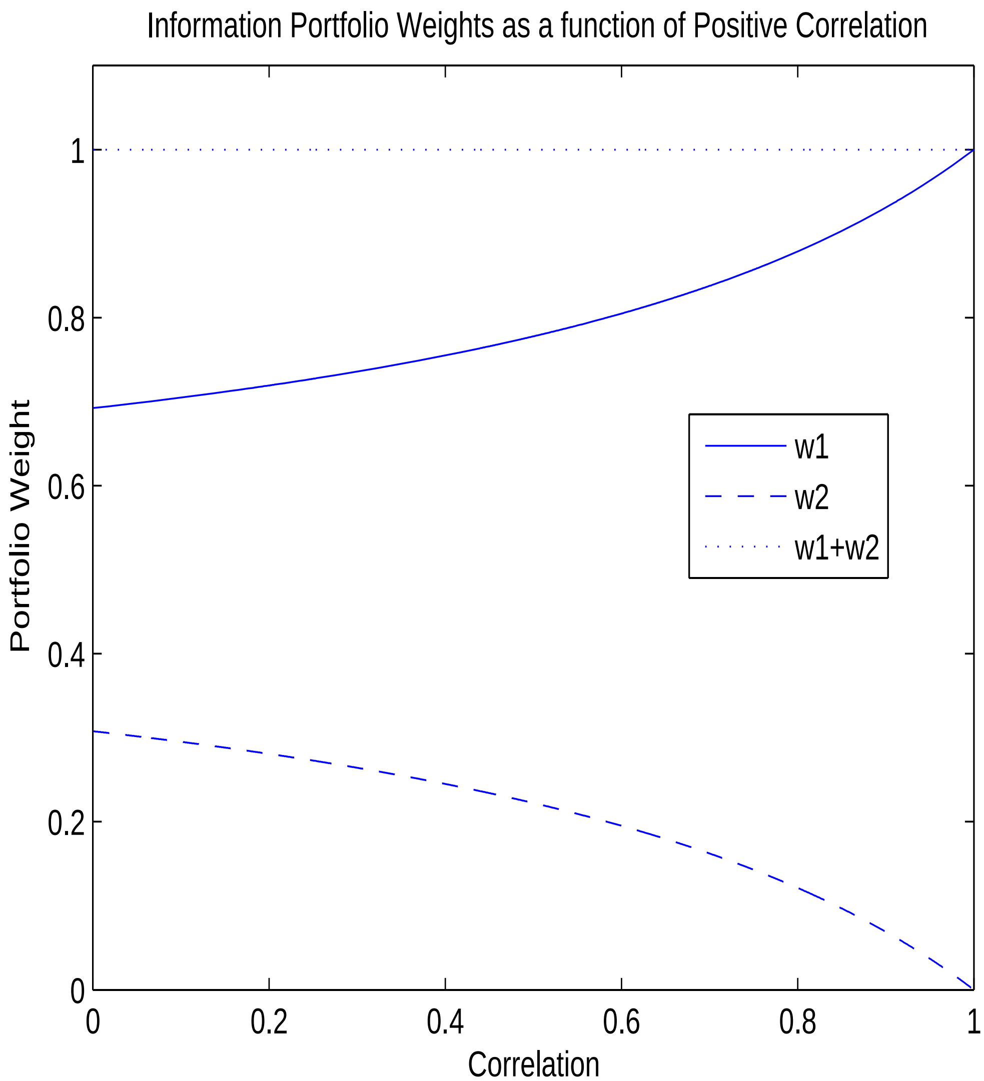

Figure 2: Impact of correlation between two information sources on their corresponding information portfolio weights. The plot above is derived from the following parameter values; $\mu_1 = 0.07$, $\mu_2 = 0.10$, $\sigma_1 = 0.40$ and $\sigma_2 = 0.60$ (as in Figure 1). Observe that higher positive correlation reduces the portfolio weight of the second information source which is less accurate than the first.

Table 1: Descriptive Statistics

This table describes our sensitivity and uncertainty measures as well as the characteristics of our dataset pertaining to B/M, size and number of analysts. The sensitivity measure is estimated monthly for each stock by computing the correlation coefficient between returns and price-scaled analyst forecast revisions over the previous 12 months. The uncertainty measure represents the price-scaled standard deviation of analyst forecasts for every stock each month. The sensitivity measure, uncertainty measure and number of analysts are derived from the I/B/E/S Summary History dataset, while B/M is the book-to-market ratio using the most recent quarterly data from Compustat. Size denotes the stock's market capitalization as reported in CRSP. The Spearman rank correlation coefficients among the five variables are computed each month from January 1976 to December 2004. Panel A reports the time series average of the Spearman correlation coefficients. Panel B reports growth/value, big/small and analyst coverage characteristics for the sensitivity and uncertainty portfolios. The sensitivity (uncertainty) portfolios denoted S1, S2 and S3 (U1, U2 and U3) represent the bottom 30%, middle 40% and top 30% of stocks ranked according to their sensitivity (uncertainty) measures. Double-sorted portfolios are also formed (e.g., S1U1 consists of stocks that belong to both S1 and U1). Each month, stocks are also ranked by B/M, size and number of analysts. This ranking is then normalized to the [0,1] interval. The average ranking for B/M, size and number of analysts in each sensitivity and uncertainty portfolio is computed monthly. The numbers in Panel B are the time series average for these monthly rankings in each sensitivity and uncertainty portfolio.

Panel A: Spearman Rank Correlation Coefficients

	Sensitivity	Uncertainty	B/M	Size	# of Analysts
Sensitivity	1	0.062	0.013	0.016	0.111
Uncertainty		1	**0.265**	-0.018	0.129
B/M			1	-0.275	-0.091
Size				1	0.833
# of Analyst					1

Panel B: Characteristics of Sensitivity and Uncertainty Portfolios

	B/M	Size	# of Analysts
S1	0.49	0.51	0.50
S2	0.50	0.50	0.50
S3	0.51	0.50	0.51
U1	**0.40**	0.50	0.48
U2	**0.51**	0.52	0.52
U3	**0.59**	0.48	0.50
S1U1	**0.39**	0.51	0.49
S1U2	**0.51**	0.52	0.51
S1U3	**0.58**	0.49	0.50
S2U1	**0.40**	0.50	0.48
S2U2	**0.51**	0.51	0.51
S2U3	**0.59**	0.48	0.49
S3U1	**0.40**	0.49	0.47
S3U2	**0.50**	0.51	0.52
S3U3	**0.60**	0.48	0.51

Table 2: Earnings Momentum Strategies

This table describes the profitability of earnings momentum strategies applied to stocks with varying levels of earnings uncertainty and return sensitivity to earnings. At the end of each month from July 1977 to December 2004, stocks from the intersection of the CRSP and I/B/E/S datasets are ranked on the basis of changes in consensus analyst earnings forecasts, measured by cumulative price-deflated revisions in the past six months. Stocks are assigned to five quintile portfolios, and equally weighted returns are computed for each portfolio. The bottom 20% is assigned to the E1 portfolio and the top 20% denotes the E5 portfolio. The trading strategy 6-0-6 in Jegadeesh and Titman (1993) is then implemented. Each month, the portfolio containing the most favorable (unfavorable) past revisions is an overlapping portfolio consisting of the E5 (E1) portfolios during the previous six months. Returns for the favorable (unfavorable) overlapping portfolios are the average returns over the six E5 (E1) portfolios. If a stock's return is missing during the holding period, it is replaced with the corresponding value-weighted market return. The earnings momentum portfolio (E5-E1) is the zero-cost portfolio that buys the most favorable revision portfolio and sells the least favorable revision portfolio (E5-E1) every month. Panel A reports the results for the strategy using the full sample. Panel B reports the results for stocks sorted on their sensitivity to analyst forecast revisions (S1, S2 and S3). Stocks are assigned to these groups before the earnings momentum portfolios are formed. Panel C reports the results when stocks are grouped according to their price-scaled standard deviation of analyst forecasts (U1, U2 and U3). These uncertainty groups are also constructed prior to the formation of the earnings momentum portfolios. Panel D reports our results after double-sorting by the sensitivity and uncertainty measures (e.g. S1U1 represents the group of stocks belonging to S1 and U1).

Panel A: Strategy using full sample

	E1	E2	E3	E4	E5	**E5-E1**	***t-stat***
All	1.09	1.22	1.25	1.48	1.79	**0.69**	***4.38***

Panel B: Strategy conditional on sensitivity of stock price to earnings information

	E1	E2	E3	E4	E5	**E5-E1**	***t-stat***
S1	1.24	1.28	1.23	1.38	1.76	**0.52**	***3.36***
S2	1.12	1.24	1.25	1.46	1.81	**0.69**	***3.76***
S3	1.11	1.26	1.34	1.54	1.90	**0.79**	***4.20***

Panel C: Strategy conditional on uncertainty of earnings information

	E1	E2	E3	E4	E5	**E5-E1**	***t-stat***
U1	1.21	1.14	1.17	1.40	1.65	**0.44**	***2.33***
U2	0.96	1.10	1.16	1.32	1.59	**0.64**	***4.34***
U3	0.81	1.05	1.25	1.29	1.55	**0.74**	***5.04***

Panel D: Strategy conditional on both sensitivity and uncertainty

	E1	E2	E3	E4	E5	**E5-E1**	***t-stat***
S1U1	1.61	1.38	1.25	1.37	1.72	**0.11**	***0.40***
S1U2	1.18	1.22	1.23	1.27	1.62	**0.45**	***2.50***
S1U3	0.95	1.30	1.28	1.46	1.66	**0.71**	***4.42***
S2U1	1.43	1.31	1.25	1.42	1.93	**0.49**	***2.24***
S2U2	1.08	1.25	1.28	1.45	1.72	**0.64**	***3.66***
S2U3	0.87	1.20	1.35	1.46	1.66	**0.79**	***4.53***
S3U1	1.41	1.35	1.27	1.62	1.92	**0.52**	***2.28***
S3U2	1.15	1.25	1.44	1.50	1.88	**0.73**	***4.53***
S3U3	0.82	1.11	1.40	1.44	1.68	**0.86**	***3.94***