Evaluating a nonlinear asset pricing model on international data

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Abstract

The paper analyses the ability of a nonlinear asset pricing model suggested by Dittmar (2002) to explain the returns on international value and growth portfolios. For comparison we use some competing pricing models; such as the ICAPM, the exchange rate risk augmented ICAPM and the international two-factor model proposed by Fama and French (1998). All models are evaluated both unconditionally and conditionally. The models are evaluated by applying the Hansen and Jagannathan distance measure. We also employ several alternative measures to ensure a robust comparison of the models. We find support for the model of Dittmar (2002). Evaluated conditionally, this model successfully passes all the different diagnostic tests performed in the analysis.

JEL Classification: G12, G15

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1. Introduction

If international financial markets are integrated, then only global systematic risk variables should affect expected returns. A central, but still unsolved problem is the identification of these global risk variables.

The International Capital Asset Pricing Model, (ICAPM), (see for example Adler and Dumas (1983), Solnik (1983), Karolyi and Stultz (2002)) assumes that the world market portfolio is the only factor that drives the return generating process of all assets, and at the same time restricts the risk-return trade-off to the first two moments of the market portfolio return. The poor empirical performance of the model (see for instance Fama and French (1998)) has cast doubt on the validity of the model. This has usually been interpreted as an evidence for a necessity to include additional factors in model building. Consequently, the original model has been extended to a range of multifactor models to take into consideration other potential common risk factors (see for example Fama and French (1998), Dahlquist and Sällström (2002), Griffin (2002), Cavaglia, Hodrick, Vadim and Zhang (2002) and Zhang (2006)).

Fama and French (1998) perform an unconditional asset pricing test for a large number of countries and suggest a two-factor model, in which the world market portfolio is augmented by a global value portfolio, in order to capture the international growth/value effect. In contrast, Zhang (2006) adopt a conditional approach that relates the time-varying risk premiums to the world business cycle. She finds that the world CAPM augmented with exchange rate risk factors is the best performing model. This result is also supported by Dahlquist and Sällström (2002).

It is however possible that the poor performance of the ICAPM is not due to insufficient information content of the world market portfolio return, but to the restrictive assumptions regarding the relationship between marginal utility and asset returns, i.e. the assumptions of normaly distributed returns and/or a quadratic utility function. Dittmar (2002) introduces a model that takes into account higher co-moments between asset returns and marginal utility growth. He finds that the model can successfully price a set of US industry portfolios. In addition, the paper shows that by including the higher co-moment components in the pricing model, the Fama and French factors add no significant information. It is interesting to see if this model performs well even in an international context. There are two possible benefits if this model is successful in pricing international assets. From a practical point of view, it would be unnecessary to construct additional potential risk factors. This is particularly convenient when the candidate factors are constructed based on firm characteristics. From a theoretical point of view, this model is more consistent with a microeconomic modeling of the risk and return relationship comparing to the anomaly based factor models and offers an economic explanation of the failure of the canonical ICAPM.

The purpose of this paper is to test the pricing ability of the nonlinear model suggested by Dittmar (2002) and to compare the model with some competing asset pricing models; such as the ICAPM, the exchange rate risk augmented ICAPM and the international two-factor model proposed by Fama and French (1998).

To be able to evaluate the pricing ability of different models we need a quantitative measure of the degree of mispricing of each model. Hansen and Jagannathan (1997) propose such a measure, now known as the Hansen and Jagannathan distance, (HJD). They develop a measure for the distance between each suggested asset pricing model and the set of true pricing kernels. Since the parameters of the suggested pricing model can be determined by minimizing this distance, the HJD methodology is suitable both for model estimation and for model evaluation. Therefore, we employ this methodology for comparing the different models. However, to guard us against possible problems with this evaluation method we also use some alternative measures, e.g. the ability of the models to predict the expected returns and if the models are positioned in the admissible region given by the Hansen and Jagannathan bounds (Hansen and Jagannathan (1991)).

The contribution of this paper is two-fold: it is to our knowledge the first study that applies the nonlinear model of Dittmar (2002) on international asset data and it extends the approach of Zhang (2006) to a data set consisting of a wider range of countries.

Our results support the nonlinear model of Dittmar (2002) in the sense that this model successfully passes all the different diagnostic tests performed in the analysis. Although this model is rejected on an unconditional basis, its performance is greatly enhanced once conditional information is incorporated in the estimations. We also show that ICAPM is not sufficient for pricing international assets. Furthermore, our results does not support the exchange rate augmented ICAPM. The success of this model in passing the specification tests may to some extent depend on a large pricing error volatility rather than a good pricing ability. We also find that the Fama and French's two-factor model performs well for portfolios sorted on the Book-to-Market ratio but cannot price the portfolios sorted on the Earnings-to-Price ratio. This is in agreement with the findings of the Fama and French (1998). This result might be due to the fact that one of the factors in the model is also constructed based on the Book-to-Market ratio. Finally, conditioning on the business cycle variable has almost no impact on the Fama and French's two-factor model. This support the conjecture by Zhang (2006) that the Fama and French's value factor also reflects business cycle information.

The outline of the paper is as follows: section 2 describes the tested asset pricing models; section 3 covers the econometric methods and the data used in our analysis; section 4 analyzes the empirical results and there is finally a conclusion in section 5.

2. The asset pricing models and the variables

Asset pricing models are typically specified as linear factor models for expected returns:

$$E_t\left[R_{i,t+1}\right] = \alpha_{i,t} + \lambda'_{f,t}\beta_{i,t}\,, \tag{1}$$

where, $\alpha_{i,t}$ is the intercept term, $\beta_{i,t}$ is a $k\times 1$ vector of sensitivities of the assets to the risk factors and $\lambda_{f,t}$ is a $k\times 1$ vector of factor risk premiums. If prices are arbitrage free, then there exists at least one stochastic discount factor, m_{t+1}, that maps every future and possibly uncertain payoff, x_{t+1}, to its current (time t) price, p_t , according to the formula:

$$p_t = E_t\left(m_{t+1}x_{t+1}\right). \tag{2}$$

One may rewrite equation (1) so that the pricing models are expressed as models for the discount factor:

$$m_t \approx y_t \equiv a_t + b'_t f_t\,, \tag{3}$$

where y_t is the suggested model for the discount factor, a_t is an intercept term, b_t is a $k\times 1$ vector of coefficients which may be interpreted as risk prices and f_t is a $k\times 1$ vector of factor realizations. The factor risk premiums from equation (1) are related to equation (3) via the following expression:

$$\lambda_{\mathrm{t}} = -\frac{1}{E_t\left[y_{t+1}\right]}\operatorname{cov}_t\left(y_{t+1}, y_{t+1}\right), \tag{4}$$

where $1/E_t\,[y_{t+1}]$ is a measure of risk free interest rate.

We investigate four pricing models for international assets. The first three models are adopted from earlier international asset pricing studies, while the fourth model, which is the main focus of this study, is the non-linear model of Dittmar (2002).

Our first model is the International single-beta CAPM as in Grauer, Litzenberger and Stehle (1976). Assuming that the Purchasing Power Parity (PPP) holds, only the exposure to global market risk, represented by innovations to the world market portfolio, is priced. Thus for this model the proxy discount factor is:

$$y_t = a_t + b_t R_{W,t}\,, \tag{5}$$

where $R_{w,t}$ is the excess return on the global market factor.

The second model is the Fama-French (1998) international multifactor model. This model assumes that the Book-to-Market (BM) effect is a common world phenomenon and is driven by global risks other than the global market risk. This model adds the High-Minus-Low (HML) factor to the global market factor (see Fama and French (1998)). For this model the proxy discount factor is:

$$y_t = a_t + b_{1,t} R_{W,t} + b_{2,t} HML_t\,. \tag{6}$$

The third model is the International CAPM with exchange risk. When PPP does not hold, covariances with exchange rates become potential sources of risk as first noted in Adler and Dumas (1983). Our third model adds an exchange risk factor to the market model, as in Dumas and Solnik (1995). For this model the proxy discount factor is:

$$y_t = a_t + b_{1,t} R_{W,t} + b_{2,t} EXC_t\,, \tag{7}$$

where EXC_t is the realization of the exchange risk factor at time t.

The fourth model is due to Dittmar (2002).[1] Since this model is new in international asset pricing tests we will provide a relatively more detailed presentation of the model.

The derivation of the original ICAPM relies on normally distributed returns and/or a quadratic utility function; assumptions that result in "mean-variance" preferences. The failure of the static ICAPM to explain average returns might be due to that it ignores higher co-moments between returns and the growth in investors' marginal utility.

Dittmar (2002) proposes a model that takes into account these higher co-moments. By assuming that consumption equals wealth in each period we can substitute wealth, *W,* for consumption, *C*, in the Marginal Rate of Substitution, (MRS) of a representative consumer:

$$\frac{U'(C_{t+1})}{U'(C_t)} = \frac{U'(W_{t+1})}{U'(W_t)}. \tag{8}$$

By making a Taylor approximation up to the power of *k*, Dittmar (2002) shows that the discount factor can be expressed as:

$$y_t = a_t + \sum_{i=1}^{k} b_{i,t} R_{W,t}^{i}, \text{ for some power } k > 0, \tag{9}$$

where $R_{W,t}^{i}$ is the *i*:th power of the return on the world market portfolio at time *t* and the b_i coefficients are related to the derivatives of the utility function (see Dittmar (2002)). This has two merits. First, we do not need to know the exact functional form of $U(\cdot)$ and second we can relate the signs on the derivatives of $U(\cdot)$ to economic theory.

[1] It should be mentioned that we do not include a measure of human capital in the market portfolio, as in Dittmar (2002). Since data are not available on a monthly basis for the labour income series for many countries, we are not able to construct a good proxy for the returns on human capital.

Dittmar (2002) suggests that imposing restrictions derived from behavioral assumptions of the agent can increase power of the tests. Therefore, the following assumptions are made in the model: the agent is risk averse, his absolute risk aversion is decreasing and his absolute prudence is decreasing.[2] These assumptions in turn lead to the restrictions that $b_{1,t} < 0$, $b_{2,t} > 0$ and $b_{3,t} < 0$. Since the preference theory gives no guidance in determining the sign of additional polynomial terms, Dittmar (2002) argues that the expansion should be truncated at the power of three, assuming that higher order polynomial terms are not important for pricing. We now turn to the econometric methods used in the analysis.

3. Econometric methods

The Hansen and Jagannathan distance

The pricing models we evaluate are suggestions to the functional form of the discount factor m.[3] However, m is not necessarily unique and there may be a set M of m's that satisfies equation (2). Thus, if an asset pricing model is an adequately specified model for one of these m's, it must belong to the set M.

Therefore, to evaluate a candidate pricing model, Hansen and Jagannathan (1997) suggest that one should measure the distance between a suggested discount factor, y, and the nearest valid m in M. In particular, they suggest a second moment distance metric, now well known as the Hansen and Jagannathan distance (HJD):

$$\min \delta \equiv \left\| y - m \right\|. \tag{10}$$

[2] Absolute prudence is defined as $-U'''/U''$ and determines whether an increase in risk to future income increases or decreases current savings.

[3] For simplicity we drop time subscript and present the model for HJD in unconditional terms.

Hansen and Jagannathan (1997) show that this distance is equal to the distance between the result from projecting y on the $N \times 1$ return vector, R, and the unique projection of every m in M onto R, which we denote R^*. R^* can in turn be calculated as the return that prices every return R_j in R by construction:

$$R^* = p' E(RR')^{-1} R \,, \tag{11}$$

where p is the $N \times 1$ price vector. This R^* can be thought of as the portfolio return in R that best mimics the behavior of every m in M. The distance measure then becomes:

$$\delta = \left[E(yR - p)' E(RR')^{-1} E(yR - p) \right]^{1/2} . \tag{12}$$

This distance should be zero for correct models. Since y will typically be a function of parameters, $y(\Theta)$, one needs the estimates of these parameters to calculate the HJD. Hansen and Jagannathan (1997) show that one can estimate the parameters by minimizing the distance with respect to the parameters, that is:

$$\underbrace{\arg\min}_{\Theta} \delta = \left[E(\mathrm{y}(\Theta)R - p)' E(RR')^{-1} E(y(\Theta)R - p) \right]^{1/2} . \tag{13}$$

Thus, this becomes a one step GMM estimation using the inverse of the second moment matrix of the returns as the weighting matrix. This matrix is, in contrast to the optimal weighting matrix suggested by Hansen (1982), invariant across models. Since δ quantifies the degree of mispricing it should also be suitable for model comparisons. As showed by Jagannathan and Wang (1996), the asymptotic sampling distribution of the HJD is:

$$T\delta^2 \xrightarrow{d} \sum_{j=1}^{N-k} w_j v_j \quad \text{as } T \to \infty, \tag{14}$$

where N is the number of assets, k the number of estimated parameters, $v_1,\ldots,v_{N-k}$ are independent $\chi^2(1)$ random variables and w_j is the weight attached to each v_j.[4]

Testing for the relevance of additional factors

To test the relevance of additional factors to the world market portfolio we first conduct a nested test comparing the augmented models with the ICAPM. An additional factor may have a significant contribution in pricing assets even if the factor model is not sufficient to price all the assets. The nested test follows Cochrane (1996) and is computed as:

$$T \times HJD(ICAPM) - T \times HJD(ICAPM + Additional\ factor) \sim \chi^2(1) \tag{15}$$

We next compute the adjustment term, A, introduced by Hansen and Jagannathan (1997), which measures the minimum adjustment needed for a model to become an admissible discount factor:

$$A = \left(R'R\right)^{-1}(yR - p)R\,. \tag{16}$$

The adjustment term should be smaller in magnitude for an augmented ICAPM model comparing to the ICAPM if the additional factor is important for pricing assets. Our metric to quantify this improvement for each model i, is denoted by ΔA_i and is calculated as:

$$\Delta A_i = \sum_{t=1}^{T}\left(\left|A_{ICAPM,t}\right| - \left|A_{i,t}\right|\right), \tag{17}$$

where $A_{ICAPM,t}$ and $A_{i,t}$ are the adjustment terms at time t for the ICAPM and the augmented model i, respectively.

[4] See Hodrick and Zhang (2001) for calculation of the weighting parameter w_j.

Model discrimination test

We also perform a non-nested test suggested by Singleton (1985) to compare models with non-significant HJD. The test discriminates between two competing non-rejected models, where one model is true and the other model is false, but the false model is not rejected due to low power of the specification test (HJD test in our case). Denoting the two competing models as *M*1 and *M*2 respectively, the test procedure consists of constructing a more general model by forming a convex combination of the two models which nests both candidate models as a special case, and then test each separate model against this general model. The test statistic is computed as:

$$NNT(1,2) = T\left(g_1(\theta_1)^{'} A\left(A^{'}\Sigma_1 A\right)^{-1} A^{'} g_1(\theta_1)\right) \tag{18}$$

$$A = S_1^{-1}\left(g_1(\theta_1) - g_2(\theta_2)\right)$$

$$\Sigma_1 = S_1 - G_1\left(G_1^{'} S_1^{-1} G_1\right) G_1^{'},$$

$$G_1 = \partial g_1(\theta_1)/\partial\theta^{'},$$

where $g_i(\theta_i)$, i = 1 and 2, is the vector of average pricing errors for *Mi* based on the estimated parameter vector θ_i and S_1 is a consistent estimator of the variance of $(T^{1/2} g_1(\theta_1))$.

If *M*1 is correct then NN_T (1,2) converges to a $\chi^2(1)$ distribution. The roles of *M*1and *M*2 can be reversed to produces the similar statistic NN_T (2,1). This leaves four possibilities; NN_T (1,2) is insignificant and NN_T (2,1) is significant then *M*1 is selected, NN_T (1,2) is significant and NN_T (2,1) is insignificant then *M*2 is selected, both tests are significant and so no model can be selected and finally both tests are insignificant and hence it is not possible to choose between them on the basis of this test.

HJD diagnostics and alternative evaluation measures

As pointed out earlier, the HJD as an estimation method is a GMM estimator and has, as other GMM based methodologies, some caveats. First, the statistical inference is affected by the sample size. Ahn and Gadarowski (1999) find that in small samples the expected value of the HJD for a correct model can be large instead of zero, and that the HJD test tends to rejects too often. Second, Kan and Zhou (2002) find that the HJD has a tendency to prefer noisy factors and it is not always reliable in telling good models apart from bad ones in finite samples.

As noted by Dittmar (2002), because the distribution of the HJD test statistic is a function of the optimal GMM weighting matrix, a non-significant HJD test (inability to reject that the distance is zero) may be due to highly volatile pricing errors of the model. However, a large pricing error variance should also result in insignificance of the estimated parameters of the model since the distribution of these estimates will be penalized by large pricing errors. To examine this possible problem, we follow Dittmar (2002) and compute Wald statistics for the parameters of the risk price of each factor and each model.

As complements to the HJD we also conduct alternative evaluation measures. Cochrane (1996) plot realized mean returns of the assets against the models predictions of the expected return, calculated as:

$$E\left(R_j\right) \equiv \frac{p_j - \operatorname{cov}\left(\hat{y}, R_j\right)}{E\left(\hat{y}\right)}. \tag{19}$$

According to Cochrane (1996), this may be an important diagnostic as it guards against accepting a poor model with weak pricing ability but large enough standard errors, or rejecting a model due to tiny standard errors that in fact produces fairly small pricing errors.

An additional diagnostic tool is the Hansen and Jagannathan (1991) bound. This bound gives the minimum standard deviation of a stochastic discount factor, (SDF), as a function of its mean and is calculated as:

$$\sigma_m\left(E(m)\right) \geq \left[\mathrm{I} - E(m)E(R)\right]' \Sigma^{-1}\left[\mathrm{I} - E(m)E(R)\right]^{1/2}, \quad (20)$$

where I is a $N \times 1$ vector of ones and Σ is the covariance matrix of the returns. If a candidate SDF, y, is to satisfy the condition $E(y_{t+1}R_{t+1}) = \mathrm{I}$ then its mean and standard deviation must plot above this boundary.

Conditional vs. unconditional estimation

We evaluate the models ability to price assets both unconditionally and conditionally. Campbell (2003) notes that risk aversion, and accordingly expected returns, must vary over time to explain why the variations in excess returns are predictable. In the unconditional model the risk prices are constant:

$$y_t = a + b'f_t. \quad (21)$$

Whereas in the conditional model we let the risk prices vary over time:

$$y_t = a_t + b_t' f_t. \quad (22)$$

We assume that these time variations can be well approximated by letting the parameters be linear functions of a set of lagged instruments:

$$a_t = a_0 + a_1' Z_{t-1} \quad (23)$$

$$b_t = b_0 + b_1' Z_{t-1}. \quad (24)$$

This means that the conditional models to be estimated have the form:

$$y_t = \phi_F' f_t + \phi_z' Z_{t-1} + \phi_{F,Z}' Z_{t-1} \otimes f_t. \quad (25)$$

This method of incorporating conditional information with lagged instruments has been used previously for instance by Ferson and Harvey (1999) and Cochrane (1996). This specification differs somewhat from Zhang (2006) who restricts b_1 to be constant over time.

For the models of Dittmar (2002) we impose the restrictions discussed earlier by using the following form when estimating the unconditional model:

$$y_t = (a_t)^2 - (b_{1,t})^2 R_{w,t} + (b_{2,t})^2 R_{w,t}^2 - (b_{3,t})^2 R_{w,t}^3 . \tag{26}$$

While for the estimation of the conditional model the restricted form of the stochastic discount factor is:

$$y_t = (a' Z_{t-1})^2 - (b_1' Z_{t-1})^2 R_{w,t} + (b_2' Z_{t-1})^2 R_{w,t}^2 - (b_3' Z_{t-1})^2 R_{w,t}^3 . \tag{27}$$

Conditioning variables

The instruments used should in some sense be variables that with some economic intuition capture states of the nature that influence risk aversion and thereby the pricing of risk. The stage of the business cycle is potentially such an instrument, or rather proxy. Therefore we use an estimate of the business cycle as a conditioning instrument. This is estimated by using the Hodrick-Prescott filter (Hodrick and Prescott (1997)) on the US Industrial Production series[5] to retrieve its stochastic trend, and then subtracting this trend from the original series to get the estimate of the cycle.[6] This method follows Hodrick and Zhang (2001) and originates from Daniel and Torous (1995). Daniel and Torous (1995) use a recursive estimation technique to estimate the cycle variable, guaranteeing that it will be in the investors' information

[5] Zhang (2006) uses a global industrial production index constructed by DataStream.

[6] The smoothing parameter is set to be 6400, which is standard when using monthly data.

set. We choose to use the whole sample when estimating the cycle variable. This choice is of course debatable as it might induce look ahead bias. Our standpoint is that the investors can observe the real stage of the business cycle adequately and that we want to find the best possible estimate of the observed cycle. Hopefully, utilizing the complete information of the data increases the possibility of achieving a better estimate.

Data

We consider three sets of test assets for a total of 14 developed countries: national market portfolios and two alternative characteristic sorted portfolios. These characteristic sorted portfolios are formed based on two different fundamental variables: the Book-to-Market ratio (BM) and the Earnings-to-Price ratio (EP). These characteristics are commonly used as value-growth indicators. The choice of these portfolios is motivated by the extensive evidence on the existence of the value premium in international data. The large cross-sectional differences in average returns of these portfolios make them suitable as test assets for model evaluation. Firms with high (low) values of the characteristics are assigned to the high (low) portfolio.[7] All the portfolios are value weighted.

We use monthly USD-based returns. Excess returns are calculated by subtracting the one-month US T-bill rate. All the data are downloaded from Kenneth French's homepage; the sample period is from January 1975 to December 2003 giving a total of 348 monthly observations on each series. We use the Morgan Stanley dollar-based world market index as the proxy for the market portfolio. Data used for estimation of the instruments are from the data base EcoWin.

[7] See Kenneth R. French's homepage for details on the sorting method.

We construct the exchange rate factor by first calculating excess holding period returns for three chosen currencies, Deutsche mark (DM)[8], Japanese Yen (¥) and UK pound (£):

$$r_t^{exc,i} = \frac{S_{i,t-1}}{S_{i,t}}\left(1 + r_{i,t}\right) - \left(1 + r_{usd,t}\right),\ i = \text{DM}, ¥, £\ , \tag{28}$$

and then construct an equally weighted portfolio out of these excess returns:

$$EXC_t = \frac{1}{3}\sum_{\in i} r_t^{exc,i}\ . \tag{29}$$

This differs somewhat from previous studies i.e. Zhang (2006), who includes the excess holding return for each country as a separate risk factor. We choose to construct a portfolio to reduce the number of parameters in the model and make the results easier to interpret, although this might lead to an information loss. We now turn to the empirical analysis.

4. Analysis

The analysis starts by looking at the mean excess returns of the test assets as well as those of the factor portfolios. Then, for each of the three sets of test assets (country indices and two different characteristic-sorted portfolios) we minimize equation (13) to estimate the different discount factors and the related HJD. We first use the HJD and the estimated factor risk prices to evaluate the models. We then perform non-nested test to discriminate between the models. Next we compare the estimated expected returns implied by each model with the sample mean returns. Finally, we analyze the validity of the models with help of the Hansen and Jagannathan bounds approach.

[8] From January 1, 2002 Germany has joined the EMU and the DM has been replaced by the Euro.

Mean excess returns

Table 1 shows that the mean excess returns of the world market portfolio and the global HML portfolio are significantly different from zero but the null hypothesis of zero mean excess return cannot be rejected for the exchange rate factor. All the country indices have significantly positive mean excess returns. Spain has the lowest average return while Hong Kong shows the highest mean excess return among all the indices. The corresponding values for the test assets constructed based on firms' fundamental variables are presented in Table 2. All the means are significant except the means of the low portfolios for Japan. For most of the countries the high portfolios have a larger mean excess returns than the low portfolios. The larger excess returns of the high portfolios relative to the low portfolios might be considered as a relatively higher risk exposure of the firms belonging to the former portfolios. In this case, a proper asset pricing model should be able to explain these risk premiums.

Results of the HJD test

To test the pricing ability of the candidate models, we compute their HJD and the related *p*-values. To obtain the *p*-value, we simulate the test statistic of equation (14) 10000 times.

The results are presented in Table 3. The test results show that we cannot reject any of the models for the estimations based on the country indices. This finding is in accordance with the result of Fama and French (1998).

For the BM portfolios, all the unconditional models are rejected except the model 2 that contains the Fama and French's factor mimicking portfolio, HML. Interestingly, when we turn to the conditional models, no model is rejected. This supports the findings of Zhang (2006) that conditioning on the business cycle variable, the market portfolio is sufficient for pricing the portfolios sorted on the BM characteristics.

For the EP portfolios, all the unconditional models are rejected. However, after conditioning on the business cycle proxy the models 3 and 4 cannot be rejected at the 5% significance level and the model 4 has the smallest distance among all the models.[9] It is worth mentioning that the HJD is approximately the same for model 2 and model 3. Therefore, the difference in their *p*-values might be due to a higher pricing error variance of the model 3.

As we see from the results for both the BM and the EP portfolios in Table 3, conditioning on the business cycle variable has almost no impact on the model 2. This may, to some extent, support the conjecture by Zhang (2006) that the HML factor also reflects business cycle information.

Investigating the impact of the various additional factors

To further examine the relevance of different additional factors to the world market portfolio we conduct a nested test comparing the models 2, 3 and 4 with the ICAPM.[10] The results in Table 4 show that we cannot reject that the additional factors, except for the test based on the country indices, have in general a significant impact on asset pricing. It is worth mentioning that the importance of the nonlinear model (model 4) is only revealed when we apply a conditional setting. This could be due to time variation of the risk price of the additional factor (squared market portfolio) in that model.

[9] The cubic extension of the model does not display any improvement. For the sake of space, the results for this model are not reported but are available upon request.

[10] We use the same restriction on the one factor model (model with only the world market portfolio) when performing the nested test to compare model 4 with the ICAPM. This is to avoid that the changes in the distance would be due to the imposed restriction in the nonlinear model instead of the effect of the additional factor.

Next we compute the improvement in the adjustment term, ΔA_i, (equation (17)). Figure 1.A shows the decrease in the magnitude of the adjustment term when the ICAPM is extended by an additional factor. Comparison of the values of ΔA_i between the three different unconditional models shows that for both the BM and EP portfolios the largest improvement is achieved by model 2. However, it should be noted that all the unconditional models are rejected for the EP portfolios, which means that the decrease in the adjustment term required to make the models feasible is not sufficient. Turning to the conditional estimations the results are different depending on the choice of the test assets. For the BM portfolios, model 2 still gives the largest reduction in the adjustment term, while for the EP portfolios, model 4 decreases the adjustment term more than the other two alternative models.

Figure 1.B illustrates the impact of adding the conditional variable to each model in terms of a reduction in the adjustment term. The result supports the finding from the HJD analysis and shows that conditioning on the business cycle variable has no considerable impact for model 2, while for model 4 it is of crucial importance that the model is estimated conditionally.

Discriminating between the non-rejected models

Next we perform the non-nested test of Singleton (1985) for comparison of the models with non-significant HJD. We do not perform this analysis for the portfolio set based on the country indices, since, as we observed from the results of the HJD tests, this portfolio set is not able to discriminate the pricing ability of the different models. Therefore the non-tested test will only be conducted to compare the conditional models 2, 3 and 4 for the BM portfolios and the conditional model 3 and 4 for the EP portfolios. The results are presented in Table 5. For the BM portfolios the test chooses the model 2 against model 3 and model 4 and chooses model 4 against model 3. For the EP portfolios the test chooses the model 4 against model 3. Note that the model 2 was rejected by the HJD-test for the EP portfolios and it is therefore not compared to the others models for these portfolios.

All in all regarding the results in this section, we can conclude that the standard ICAPM is not sufficient for pricing international assets. Furthermore, the model 3 is not rejected by the HJD but other tests indicate that this result may be due to poor power properties of the test for this model. The model 2 outperforms the other models for BM portfolios but cannot successfully price the EP sorted portfolios. This might to some extent be due to the fact that the BM sorted test assets and one of the candidate factors are both constructed based on the same characteristics (see MacKinlay (1995)). Model 4 shows a strong performance overall; it is not rejected by the HJD on any portfolio set.

Examining the parameter significance

We now look at the significance of the estimated parameters to investigate if the results from the HJD test are possibly driven by large pricing error variance. This analysis should be considered as a complement to the HJD test statistics and not as an independent analysis for comparison of the models.

Table 6, shows the estimated coefficients of the equation (3) for each model and the related *p*-values of the Wald statistics. Since most of the distances for the unconditional models are significant we only focus on the Wald statistics for the conditional models.[11] All the models, except model 4, have only a few significant

[11] In the conditional model, the time varying coefficient of each factor is given by equation (23). The Wald statistics tests whether the parameters of this equation are both zero.

coefficients. Therefore we cannot reject the possibility that insignificant result of the HJD for these models might be a result of large pricing error variance. This motivates applying some additional methods to compare the performance of the different models.

Comparing the model predicted means with the sample means

In this section we compare the estimated mean excess returns based on the different models (see equation (19)) with the realized sample mean excess returns. For each model and each test asset we first examine if there is a significant difference between the average of the estimated values and the average of realized sample mean excess returns. Then we run a regression of the estimated values based on each model on the realized values.

The first panel of the Table 7 shows the estimated *t*-values for the equality of means between the estimated and the realized average returns. For all of the models we have negative *t*-values, but some of these values are not significant. This means that the models generally underestimate the average returns. The conditional model 4 performs best and has no significant *t*-value.

The results of the regression analysis (the second panel of Table 7) are mainly in agreement with the result of the test for equality of means. Most of the models for which the estimated expected returns deviate significantly from the realized mean return have also insignificant slope coefficients. Model 4, in the conditional setting, is the only model that has significant slope coefficient for all the three sets of the test assets. Fama and French's two-factor model (model 2) has a significant slope coefficient for all the assets except the market indices. Summing up the result of the both analyses, the conditional models are in general better than the unconditional models. This supports the results from the HJD of the previous section.

Hansen and Jagannathan bounds diagnostic

Figure 2 shows the plots of the estimated means and standard deviations of the estimated stochastic discount factors for each model and each portfolio set along with the corresponding minimum volatility boundary. As we primarily use this evaluation method as a complementary diagnostic tool we do not perform any formal statistical tests for the significance of the distance between the feasible region and the estimated SDFs.

For the unconditional estimates, all the estimated discount factors lie below the bounds and therefore the time series processes of these estimated models are not volatile enough to be considered as admissible discount factors. For the sake of space we do not show the plots for the unconditional estimations. For the conditional estimates, the models 3 and 4 are inside the feasible region for all the portfolio sets, while model 2 lies outside the bound. It is interesting given the previous results that the nonlinear model passes this diagnostic tool as well.

5. Summary and conclusions

In this paper we evaluate the pricing ability of the nonlinear model suggested by Dittmar (2002) on international equity data and compare this model with several existing international asset pricing models, i.e. the ICAPM, the international two-factor model proposed by Fama and French (1998) and ICAPM augmented by the exchange risk factor. All models are evaluated both unconditionally and conditionally, where the latter allows the risk prices to be time varying. The models are estimated and evaluated by the HJD methodology. However, we also use a number of alternative evaluation methodologies.

Our results support the nonlinear model of Dittmar (2002); allowing for time-varying risk prices, the model successfully passes all the different diagnostic tests performed in the analysis.

We find that ICAPM cannot adequately explain observed average returns of the test assets except for the country indices. The HJD test on the portfolios sorted on the BM characteristics rejects all the unconditional models except the model that contains the Fama and French's factor mimicking portfolio, HML. However, the test cannot reject any conditional factor model, which means that conditioning on the business cycle variable, the market portfolio is sufficient for pricing Book-to-Market sorted portfolios. This is in agreement with finding of Zhang (2006) on a sample of three large countries. For EP sorted portfolios, only the conditional nonlinear model and the conditional ICAPM augmented by the exchange rate factor are not rejected by HJD test.

A non-nested test for comparison between two competing non-rejected models shows that for the BM sorted portfolios the Fama and French two-factor model performs better than the nonlinear model of Dittmar while the latter model outperforms the ICAPM model augmented by the exchange risk factor. For the EP sorted portfolios the nonlinear model is the preferred model.

When comparing, for each model, the estimated expected returns given by the model to the realized mean returns, shows that the conditional nonlinear model gives in general a better prediction of the mean returns than the predictions given by any of the other models.

Our results indicate that the non-rejection of the exchange rate augmented ICAPM depends mostly on the large volatility of pricing error of this model rather than the model's pricing ability. This raises some doubts about the success of this model in previous studies, such as Zhang (2006) and Dahlquist and Sällström (2002). Furthermore, we find that the Fama and French's two-factor model performs better than the other models for BM sorted portfolios but is not able to price the EP sorted portfolios. This might to some extent be due to the fact that the BM sorted test assets and the HML factor are both constructed based on the same characteristics. The only model that performs well in all the conducted tests is the nonlinear model.

It is worth mentioning that the importance of the nonlinear model is only revealed when we apply a conditional setting. On the contrary, conditioning on the business cycle variable has almost no impact on the Fama and French's two-factor model. This might support the conjecture by Zhang (2006) that the HML factor also reflects business cycle information.

We conclude that the nonlinear model is an appropriate international asset pricing model. One important implication is that the world market portfolio return is the only important factor for pricing assets and it is unnecessary to strive for the identification of new risk factors. Instead we need to restate the relationship between the market portfolio and the asset returns in a manner that makes it more consistent with common beliefs about agents' attitude towards risk.

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Table 1. The means and their *t*-values for the factors and country indices

Table shows the mean and the related *t*-values for 14 country indices.

		Means	*t*-values
Factors	Rw	1.1	4.19
	HML	0.5	3.77
	Exchange	0.2	1.41
Country indices	USA	1.19	4.87
	Japan	1.02	2.87
	UK	1.49	4.19
	France	1.30	3.63
	Ger.	1.12	3.46
	Italy	1.09	2.65
	Neth.	1.39	4.87
	Belg.	1.31	4.41
	Switz.	1.20	4.23
	Sweden	1.41	3.84
	Spain	1.01	2.81
	Hongk.	1.80	3.67
	Singa.	1.26	2.92
	Aust.	1.22	3.37

Table 2. The means and their *t*-values for the test assets based on the fundamental variables

Table shows the mean and the related *t*-values for portfolios constructed based on the four fundamental variables, i.e. Book-to-Market ratio, BM, and Earnings-to-Price ratio, EP. The *high* (*low*) portfolios consist of firms with *large* (*small*) values of the variables.

		Means		***t*-values**	
		BM	EP	BM	EP
	USA	1.48	1.57	6.17	6.35
	Japan	1.57	1.36	4.04	3.99
	UK	1.64	1.78	4.23	4.81
	France	1.67	1.61	4.04	3.88
	Ger.	1.58	1.23	4.52	3.61
High	Italy	1.03	1.16	2.23	2.52
	Neth.	1.61	1.58	4.15	4.30
	Belg.	1.63	1.59	4.76	5.08
	Switz.	1.39	1.20	4.02	3.85
	Sweden	1.88	1.87	4.31	4.82
	Spain	1.04	1.50	2.41	3.84
	Hongk.	1.96	2.11	3.27	3.96
	Singa.	1.78	1.58	3.20	3.48
	Aust.	1.70	1.70	4.71	4.91
	USA	1.13	1.09	4.24	4.06
	Japan	0.66	0.63	1.77	1.62
	UK	1.37	1.39	3.75	3.78
	France	1.18	1.10	3.28	2.98
	Ger.	1.09	1.12	3.06	3.14
	Italy	1.15	1.16	2.78	2.72
Low	Neth.	1.28	1.15	4.45	3.71
	Belg.	1.17	1.30	3.79	4.27
	Switz.	1.15	1.14	4.05	3.77
	Sweden	1.29	1.44	3.33	3.63
	Spain	0.85	0.85	2.10	2.16
	Hongk.	1.67	1.66	3.67	3.36
	Singa.	1.12	1.15	2.66	2.42
	Aust.	0.98	0.95	2.45	2.25

Table 3. Evaluation of the models with the Hansen and Jagannathan distance

Table shows the Hansen and Jagannathan distance measure (HJD) and its significance test for portfolios constructed based on the two fundamental variables, i.e. Book-to-Market ratio, BM, and Earnings-to-Price ratio, EP. Values significant at the 5% level are marked in bold. The distance is estimated for the following models:

- Model 1: The model with the excess return on the value-weighted world market index, R_w (ICAPM).
- Model 2: The model with R_w and a portfolio mimicking Book-to-Market factor, HML.
- Model 3: The model with R_w and an exchange rate factor.
- Model 4: The nonlinear model of Dittmar (2002), which includs R_w and the squared R_w.

		Country	indices	BM	portfolios	EP	portfolios
		HJD	*p*-value	HJD	*p*-value	HJD	*p*-value
Uncond.	Model 1	0.16	0.81	0.39	**0.00**	0.41	**0.00**
	Model 2	0.16	0.74	0.32	0.20	0.37	**0.02**
	Model 3	0.15	0.81	0.38	**0.00**	0.39	**0.00**
	Model 4	0.15	0.84	0.39	**0.00**	0.40	**0.00**
Cond.	Model 1	0.14	0.84	0.37	0.11	0.40	**0.00**
	Model 2	0.14	0.79	0.31	0.21	0.37	**0.02**
	Model 3	0.13	0.82	0.36	0.06	0.37	0.05
	Model 4	0.12	0.91	0.34	0.18	0.35	0.16

Table 4. Nested tests for model comparison

The table presents the results of the nested tests if, given the world market portfolio, an additional factor is important for pricing assets. Values significant at the 5% level are marked in bold. See Table 3 for the description of the models.

		Country indices		**BM**		**EP**	
		Uncond	Cond	Uncond	Cond	Uncond	Cond
Model 2/Model 1	Stat.	0.20	1.77	27.07	20.92	14.73	13.40
	p-val	0.65	0.18	**0.00**	**0.00**	**0.00**	**0.00**
Model 3/Model 1	Stat.	2.95	4.09	4.10	4.10	7.24	11.73
	p-val	0.09	**0.04**	**0.04**	**0.04**	**0.01**	**0.00**
Model 4/Model 1	Stat.	3.06	7.77	1.23	14.71	4.13	18.46
	p-val	0.08	**0.01**	0.27	**0.00**	**0.04**	**0.00**

Table 5. Non-nested tests for model comparison

The table presents the results of the non-nested tests of comparison of two models. Values significant at the 5% level are marked in bold. See Table 3 for the description of the models.

	BM portfolios			EP portfolios	
M1/M2	Model 2	Model 3	Model 4	Model 3	Model 4
Model 2		1.72	1.21		
		0.19	0.27		
Model 3	14.62		8.52		10.60
	0.00		**0.00**		**0.00**
Model 4	9.09	0.61		0.30	
	0.00	0.43		0.58	

Table 6. The estimated coefficients given by the conditional models

Table shows the estimated coefficients given by the conditional models and the corresponding *p*-value of the Wald statistics for a joint significance test of all the parameters involved in the related coefficient. The coefficients are computed at the average value of the instruments. Test portfolios are constructed based on the two fundamental variables, i.e. Book-to-Market ratio, BM, and Earnings-to-Price ratio, EP. Values significant at the 5% level are marked in bold. See Table 3 for the description of the models.

		Constant		R_m		HML		Exchange		R^2_m	
		Coefficient	*p*-val.	Coefficient	*p*-val.	Coefficient	*p*-val.	Coefficient	*p*-val.	Coefficient	*p*-val.
Model 1	Indices	0.99	0.00	-2.08	0.15						
	BM	1.07	0.00	-3.05	0.07						
	EP	1.04	0.00	-3.42	**0.03**						
Model 2	Indices	1.01	0.00	-2.36	0.17	3.42	-0.82				
	BM	1.07	0.00	-2.59	0.18	8.80	**-0.01**				
	EP	1.08	0.00	-3.21	0.06	8.95	**-0.03**				
Model 3	Indices	1.02	0.00	-2.95	0.55			2.00	0.66		
	BM	1.08	0.00	-5.51	0.06			7.55	0.41		
	EP	1.04	0.00	-7.71	**0.02**			13.53	0.08		
Model 4	Indices	0.89	0.00	-2.95	**0.02**					42.82	0.09
	BM	0.84	0.00	-3.53	**0.00**					73.88	**0.00**
	EP	0.86	0.00	-3.94	**0.00**					67.93	**0.01**

Table 7. Comparing the estimated expected returns with the sample mean returns

The table compares the estimated mean excess returns based on the different models with the realized sample mean excess returns. The first panel shows the *t*-values of the test for significant difference between the average of the estimated values and the average of realized sample mean excess returns. The second panel illustrates the *t*-values of the slope coefficients from the regression of the estimated values on the realized values. The significant values are marked in bold. For details regarding the models, please refer to Table 3.

***t*-values of equality of two means**

		Country indices	BM portfolios	EP portfolios
Uncond.	Model 1	**-2.87**	**-5.18**	**-3.85**
	Model 2	**-2.67**	-1.95	**-2.18**
	Model 3	**-2.17**	**-3.95**	**-2.07**
	Model 4	-1.89	**-4.26**	**-2.28**
Cond.	Model 1	**-2.41**	**-3.53**	**-3.57**
	Model 2	**-2.10**	-1.32	-1.92
	Model 3	-1.32	**-2.60**	**-2.04**
	Model 4	-0.60	-0.67	-0.09

***t*-values of the slope coefficients**

		Country indices	BM portfolios	EP portfolios
Uncond.	Model 1	0.33	0.10	-0.87
	Model 2	0.69	**4.12**	**2.64**
	Model 3	0.98	1.46	1.10
	Model 4	1.11	0.71	0.70
Cond.	Model 1	1.15	**2.57**	-0.01
	Model 2	1.62	**5.32**	**2.85**
	Model 3	**2.40**	**2.96**	1.35
	Model 4	**4.09**	**3.40**	**2.86**

Figure 1. Comparing the adjustment terms of the different models

Diagram A shows the decrease in the magnitude of the adjustment term of the ICAPM when this model is extended by an additional factor. Diagram B shows the effect of decrease in the magnitude of the adjustment term for each augmented ICAPM model when the models are estimated conditionally. For details regarding the models, please refer to Table 3.

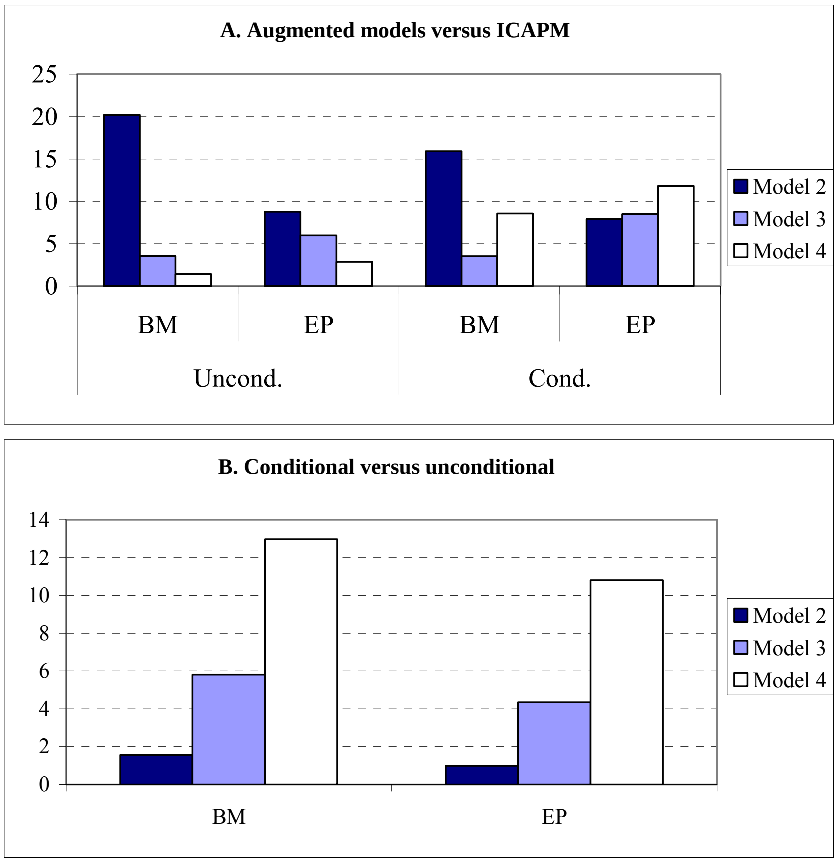

Figure 2. Hansen and Jagannathan bounds for the conditional estimates

The figure plots the means and standard deviations of the estimated stochastic discount factors for each model and each portfolio set together with the corresponding minimum volatility bounds. The bounds are calculated using gross returns of the risky assets. For details regarding the models, please refer to Table 3.